Energy Efficiency Optimization in MIMO Interference Channels: A Successive Pseudoconvex Approximation Approach

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Abstract—In this paper, we consider the (global and sum) energy efficiency optimization problem in downlink multi-input multi-output multi-cell systems, where all users suffer from multi-user interference. This is a challenging problem due to several reasons: 1) it is a nonconvex fractional programming problem; 2) the transmission rate functions are characterized by (complex-valued) transmit covariance matrices; and 3) the processing-related power consumption may depend on the transmission rate. We tackle this problem by the successive pseudoconvex approximation approach, and we argue that pseudoconvex optimization plays a fundamental role in designing novel iterative algorithms, not only because every locally optimal point of a pseudoconvex optimization problem is also globally optimal, but also because a descent direction is easily obtained from every optimal point of a pseudoconvex optimization problem. The proposed algorithms have the following advantages: 1) fast convergence as the structure of the original optimization problem is preserved as much as possible in the approximate problem solved in each iteration, 2) easy implementation as each approximate problem is suitable for parallel computation and its solution has a closed-form expression, and 3) guaranteed convergence to a stationary point or a Karush-Kuhn-Tucker point. The advantages of the proposed algorithm are also illustrated numerically.

Index Terms—Energy Efficiency, Interference Channel, MIMO, Nonconvex Optimization, NOMA, Pseudoconvex Optimization, Successive Convex Approximation, Successive Pseudoconvex Approximation

I. INTRODUCTION

In the era of 5G and Internet of Things by 2020, the number of connected devices is predicted to reach 50 billions [1]. On one hand, as compared to current systems, the data rate should be 1000x higher to serve these devices simultaneously. On the other hand, the significant increase in the data rate is expected to be achieved at the same or even a lower level of energy consumption. Therefore the so-called energy efficiency (EE) is a key performance indicator that should be considered in the design of transmission schemes.

In this paper, we adopt the notion of EE as the ratio between the transmission rate and the consumed energy, which has a unit of bits/Joule, and we study the EE maximization problem

in a downlink multi-input multi-output (MIMO) multi-cell system, where the base stations (BSs) are transmitting in the same frequency band to allow full frequency reuse and the users suffer from multi-user interference. This problem is challenging due to several practical difficulties:

(D1) The transmission rate in the interference channel is a nonconcave function of the transmit covariance matrices.

(D2) In MIMO systems, the transmission rate functions are characterized by (complex-valued) transmit covariance matrices.

(D3) The energy consumption depends not only on the transmission power but also on the processing power that increases with the transmission rate.

For the sake of an intuitive understanding of the challenging nature, consider the sum rate maximization problem, which is a special case of the EE optimization problem if the power consumption is a constant: it has been proved in [2] that the sum rate maximization problem in interference channels is nonconvex and finding its globally optimal point is NP-hard. Due to the high complexity of global optimization, we are mainly interested in iterative algorithms with parallel implementations that can efficiently find stationary points.

In a multi-cell network, multiple transmission links coexist that negatively influence each other through the multi-user interference. The conflicting interests of different links make the EE maximization problem a multi-objective optimization problem and there are several commonly adopted design metrics with different rationale. For example, the global energy efficiency (GEE), which is defined as the ratio between the sum transmission rate and the total power consumption, is a meaningful measure for the EE of the whole network. Nevertheless, it may not be relevant in a heterogeneous network, where different transmission links may have different priorities. The EE of this network is better captured by the (weighted) sum energy efficiency (SEE), defined as the sum of all individual EE.

Related work. The EE optimization problem has received considerable attention in recent years and it has been studied from different perspectives. For example, to address (D1), orthogonal transmission schemes based on user selection or interference cancellation, are adopted in some of the early works [3, 4, 5, 6] so that the transmission rate functions are concave in the transmit covariance matrices. However, this scheme is not optimal due to the inefficient reuse of spectrum, especially considering the large number of devices in future

networks and the existing frequency bandwidth limitations.

Along the direction of nonorthogonal multiple access, the GEE maximization in MISO systems has been studied in [7], where the authors considered additional Quality-of-Service (QoS) constraints, in terms of each link's guaranteed minimum transmission rate. The SEE optimization problem with QoS constraints is studied in [8]. Compared with the GEE function, the SEE function is more difficult to optimize because it is the sum of multiple fractional functions, while each individual fractional function is the ratio of a nonconcave function and a nonconvex function. In MISO systems, the transmission rate is a function of the SINR which is a scalar quantity and the algorithms proposed in [7, 8, 9] are built upon this property. These algorithms can be generalized to MIMO systems by treating the multiple data streams intended for the same user independently (i.e., no joint transmit precoding). Although this scheme has been empirically observed to achieve good performance, it is in principle a suboptimal approach to maximize the EE in MIMO systems where the transmission rate is a function of the (complex-valued) transmit covariance matrices, cf. (D2).

The sequential pricing algorithm for SEE maximization in MISO systems proposed in [10, Alg. 1] is a variant of the block coordinate descent (BCD) algorithm. Although this approach extends to MIMO systems, the approximate problems solved in each iteration do not exhibit any convexity and are thus not easy to solve, making the iterative algorithm not suitable for practical implementation.

A successive lower bound maximization (SLBM) algorithm is proposed in [11] to find a KKT point of a large class of EE optimization problems. The central idea therein is to maximize in each iteration an approximate function that is a global lower bound of the original GEE function. On the one hand, the SLBM algorithm converges even when the objective function is nondifferentiable. On the other hand, the requirement that the approximate function must be a global lower bound of the original objective function may bring limitations. Firstly, the solution of the approximate function may not have a closed-form expression and it can only be found iteratively by a general purpose convex optimization solver. Secondly, such an approximate function may not even exist for some applications, for example, the SEE maximization problem. Note that a global optimization technique is also proposed in [11] for the GEE and SEE maximization problems, which may serve as a benchmark in small problem instances only due to the exponential complexity.

An iterative algorithm is proposed in [12] to maximize the SEE in MIMO systems (without QoS constraints). However, it has two limitations. Firstly, it is a two layer algorithm for which the inner layer consists of a BCD type algorithm which suffers from a high complexity and a slow convergence rate. Secondly, only convergence in function value is established and the convergence to a stationary point is still left open. Besides, the algorithm is not applicable when the rate-dependent processing power consumption (due to, e.g., coding and decoding, cf. [8, 13, 14]) is considered.

The popular technique in literature to address (D3) is to majorize the rate-dependent signal-related power consumption

paper	D1	D2	D3	GEE	SEE	QoS
Tervo et al. [7]	✓			✓		✓
Tervo et al. [8]	✓		✓	✓	✓	✓
Pan et al. [10]	✓	✓			✓	✓
Zappone et al. [11]	✓	✓		✓		✓
He et al. [12]	✓	✓			✓	
This paper	✓	✓	✓	✓	✓	✓

Table I
SUMMARY OF RELATED WORK: ADDRESSED DIFFICULTIES

by a convex function, because it is possible to integrate such a function into existing algorithms such as the SLBM algorithm [11]. This approach is explained in detail in [8] where the signal-related power consumption is assumed to be a convex and monotonic function with respect to the transmission rate. Nevertheless, in MIMO interference channels, it is nontrivial to find such a convex upper bound, if possible at all.

Contributions. In this paper, we study the GEE and SEE optimization problems in multi-cell MIMO interference channels and propose novel iterative algorithms that address the practical difficulties (D1)-(D3) (see Table I), first without and then with per-link QoS constraints; see Table I for a comparison with some of the related works discussed above. The proposed algorithms have the following attractive features:

- fast convergence as the structure of the original optimization problem is preserved as much as possible in the approximate problem solved in each iteration;
- low complexity as each approximate problem is suitable for parallel computation and its solution has a closed-form expression;
- guaranteed convergence to a stationary point or a Karush-Kuhn-Tucker (KKT) point.

The proposed algorithms are based on the successive pseudoconvex approximation (SPCA) framework [15]. In each iteration, an approximate problem is solved, and the approximate problem only needs to exhibit a weak form of convexity, namely, pseudoconvexity. Among others, pseudoconvex optimization problems have two notable properties: firstly, some special cases of pseudoconvex objective functions (e.g., the ratio of positive convex and concave functions) can be easily optimized and every stationary point is globally optimal, and secondly, any direction pointing to an optimal point of a pseudoconvex optimization problem is a descent direction of the objective function. While the first property has been recognized and exploited under the framework of fractional programming (see [3, 4, 11, 16, 17, 18] and the references therein), the second property has largely been overlooked. In this paper, we argue that it plays a fundamental role in designing novel iterative algorithms with provable convergence to maximize differentiable EE functions, by showing repeatedly that it paves the way to define an approximate problem that preserves as much structure available in the original EE function as possible, e.g., the partial concavity (convexity) in the numerator (denominator) function and the division operator. Therefore, the proposed algorithms present a

fast convergence behavior and enjoy an easy implementation.[1]

The SPCA framework was firstly proposed in [15] to design efficient iterative algorithms to minimize a nonconvex objective function over a convex constraint set. On the one hand, the algorithms for GEE and SEE maximization without QoS constraints in this paper are obtained by exploiting the unique problem structures in GEE and SEE maximization and tailoring the theoretical SPCA framework accordingly. On the other hand, the proposed algorithms for GEE and SEE maximization with QoS constraints are a nontrivial extension of the SPCA framework as the nonconcave QoS functions make the constraint set nonconvex. We overcome this difficulty by employing the so-called inner approximation, where the nonconvex constraint set is approximated by a convex subset.

We mention for the completeness of this paper that another popular design metric is to maximize the minimum EE among all links. This problem has been studied in [11, 21], under the framework of SLBM, where no rate-dependent processing power consumption is considered. Our method proposed in this paper cannot be applied to maximize the minimum EE, because the minimum EE is a nondifferentiable function. To our best knowledge, the minimum EE maximization problem with rate-dependent processing power consumption is still an open problem.

Paper structure. The rest of the paper is organized as follows. In Sec. II we introduce the system model and problem formulation. The novel iterative algorithms are proposed in Sections III-VI for the following four problems: GEE maximization without QoS constraints, SEE maximization without QoS constraints, GEE maximization with QoS constraints, and SEE maximization with QoS constraints. Numerical results are reported in Section VII and the paper is concluded in Sec. VIII.

Notation: We use x, $\mathbf{x}$ and $\mathbf{X}$ to denote a scalar, vector and matrix, respectively. We use $\mathbf{X}^H$ and $\mathbf{X}^*$ to denote the Hermitian of $\mathbf{X}$ and the complex conjugate of $\mathbf{X}$, respectively. The inner product of two matrices $\mathbf{X}$ and $\mathbf{Y}$ is defined as $\mathbf{X} \bullet \mathbf{Y} \triangleq \Re(\mathrm{tr}(\mathbf{X}^H \mathbf{Y}))$. The operator $[\mathbf{X}]^+$ returns the projection of $\mathbf{X}$ onto the cone of positive semidefinite matrices. The gradient of $f(\mathbf{X})$ with respect to $\mathbf{X}^*$ and $\mathbf{X}_k^*$ is denoted as $\nabla_{\mathbf{Q}^*} f(\mathbf{X})$ and $\nabla_{\mathbf{Q}_k^*} f(\mathbf{X})$, respectively. $\nabla_{\mathbf{Q}^*} f(\mathbf{X})$ and $\nabla f(\mathbf{X})$ ($\nabla_{\mathbf{Q}_k^*} f(\mathbf{X})$ and $\nabla_k f(\mathbf{X})$) are used interchangeably when there is no ambiguity. When there are multiple matrix variables $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_K$, we use $\mathbf{X}$ as a compact notation to denote all of them: $\mathbf{X} \triangleq (\mathbf{X}_k)_{k=1}^K$. We also use $\mathbf{X}_{-k}$ to denote all matrix variables except $\mathbf{X}_k$: $\mathbf{X}_{-k} \triangleq (\mathbf{X}_j)_{j=1, j \neq k}^K$. The notation $\mathbf{0} \preceq \mathbf{X} \perp \mathbf{Y} \succeq \mathbf{0}$ denotes that $\mathbf{X} \succeq \mathbf{0}$, $\mathbf{Y} \succeq \mathbf{0}$ and $\Re(\mathrm{tr}(\mathbf{X}^H \mathbf{Y})) = 0$. Similarly $0 \leq \mathbf{x} \perp \mathbf{y} \geq \mathbf{0}$ denotes that $\mathbf{x} \geq \mathbf{0}$, $\mathbf{y} \geq \mathbf{0}$ and $\mathbf{x}^H \mathbf{y} = 0$.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a downlink MIMO multi-cell system as depicted in Figure 1, where the number of cells is K. We assume
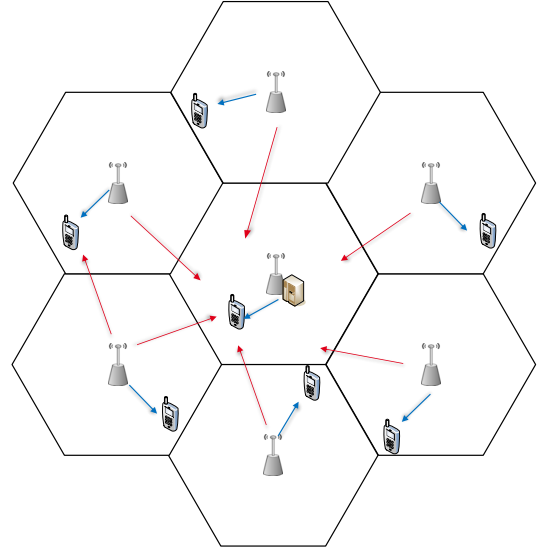

Figure 1. System topology of 1-tiered small cell interferers (with a central processing unit in CRAN). The BSs and users may be equipped with multiple antennas. (This figure is reproduced from [19] ⓒ2017 IEEE).

for simplicity that each cell is serving one user[2]. The number of transmit antennas at the BS of cell k is M_k, and the number of receive antennas of user k served by cell k is N_k. We denote $\mathbf{H}_{kk}$ as the channel matrix from BS k to user k, and $\mathbf{H}_{kj}$ as the downlink channel matrix from BS j to user k. We assume that all K users are active and the multi-user interference is treated as noise, so the downlink transmission rate of the k-th user is:

$$r_k(\mathbf{Q}_k, \mathbf{Q}_{-k}) \triangleq \log \det \left(\mathbf{I} + \mathbf{R}_k(\mathbf{Q}_{-k})^{-1} \mathbf{H}_{kk} \mathbf{Q}_k \mathbf{H}_{kk}^H \right), \quad (1)$$

where $\mathbf{Q}_k \triangleq \mathbb{E} \left[\mathbf{x}_k \mathbf{x}_k^H \right]$ is BS k's transmit covariance matrix (based on the circularly symmetric Gaussian signaling[3]), $\mathbf{Q}_{-k}$ is a compact notation denoting all transmit covariance matrices except $\mathbf{Q}_k$: $\mathbf{Q}_{-k} = (\mathbf{Q}_j)_{j \neq k}$, and $\mathbf{R}_k(\mathbf{Q}_{-k}) \triangleq \sigma_k^2 \mathbf{I} + \sum_{j \neq k} \mathbf{H}_{kj} \mathbf{Q}_j \mathbf{H}_{kj}^H$ is the noise plus interference covariance matrix experienced by user k.

The power consumption at BS k can be approximated by the following equation:

$$p_k(\mathbf{Q}) \triangleq P_{0,k} + \rho_k \mathrm{tr}(\mathbf{Q}_k) + g_k(r_k(\mathbf{Q})),$$

where $\mathbf{Q} \triangleq (\mathbf{Q}_k)_{k=1}^K$, $P_{0,k}$ is the circuit power consumption and $\rho_k \geq 1$ is the inverse of the power amplifier efficiency at the transmitter, while $g_k(x)$ is a nonnegative differentiable function of x with $g_k(0) = 0$, which reflects the rate-dependent processing power consumption, e.g., required for coding and decoding [8, 13, 14]. The typical values of $P_{0,k}$ and ρ_k depend on the types of the cell, e.g., macro cell, remote radio head, and micro cell. Interested readers are referred to [23, Table 8] for its typical values.

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[1]Some preliminary results of this journal paper are reported in our previous conference papers (without proof or with shortened proof only): the GEE maximization problem without QoS constraints and with QoS constraints is studied in [19] and [20], respectively, while the SEE maximization problem without QoS constraints is studied in [21]. Besides this, the rate-dependent processing-related power consumption is not considered in [19, 20, 21].

[2]All results can be generalized to the MIMO interfering broadcast channel where each cell is serving multiple users that generate intra-cell interfere to each other. A concise description for the GEE and SEE maximization problem without QoS constraints is contained in this paper's supplementary materials.

[3]The use of the more general circularly asymmetric Gaussian signaling may bring additional benefits as shown in [22], but it is outside this paper's scope and will be left for future work.

Optimizing the EE of multiple links in the network simultaneously is a typical multi-objective optimization problem, which could be modeled in several different ways. For example, the GEE, which is defined as the ratio between the sum transmission rate and the total consumed power, models the EE of the whole network:

$$\underset{\mathbf{Q}}{\text{maximize}} \quad f_G(\mathbf{Q}) \triangleq \frac{\sum_{k=1}^{K} r_k(\mathbf{Q})}{\sum_{k=1}^{K} p_k(\mathbf{Q})}$$
$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \le P_k, \forall k, \qquad (2)$$

where P_k is BS k's (predefined) sum transmission power budget and the optimization variable is the (complex-valued) transmit covariance matrices $\mathbf{Q} = (\mathbf{Q}_k)_{k=1}^{K}$.

To maximize the GEE, users suffering from bad channel conditions may not be able to transmit, because increasing the transmit power in the denominator may not lead to a notable increase in the transmission rate in the numerator. Another popular design approach is the so-called socially optimal approach, which aims at finding the optimal point that maximizes the sum EE (SEE) over all users:

$$\underset{\mathbf{Q}}{\text{maximize}} \quad f_S(\mathbf{Q}) \triangleq \sum_{k=1}^{K} \frac{r_k(\mathbf{Q})}{p_k(\mathbf{Q})}$$
$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \le P_k, \forall k. \qquad (3)$$

Note that the objective function $f_S(\mathbf{Q})$ in (3) is a sum of multiple fractional functions, each is the ratio of the nonconcave function $r_k(\mathbf{Q})$ and the nonconvex function $p_k(\mathbf{Q})$.

In the previous formulations (2) and (3), there are no QoS constraints specifying each link's minimum guaranteed transmission rate. To incorporate the QoS constraints into the EE optimization problems, we modify the GEE optimization problem (2) as follows:

$$\underset{\mathbf{Q}}{\text{maximize}} \quad f_G(\mathbf{Q}) = \frac{\sum_{k=1}^{K} r_k(\mathbf{Q})}{\sum_{k=1}^{K} p_k(\mathbf{Q})}$$
$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \le P_k, r_k(\mathbf{Q}) \ge R_k, \forall k, \quad (4)$$

and we assume that the solution set of (4) is nonempty. In contrast to problem (2), problem (4) has a nonconvex constraint set due to the nonconvex QoS constraints and is thus more challenging. Therefore we study (2) and (4) separately.

Similarly, the SEE optimization problem subject to QoS constraints is modeled as follows:

$$\underset{\mathbf{Q}}{\text{maximize}} \quad f_S(\mathbf{Q}) = \sum_{k=1}^{K} \frac{r_k(\mathbf{Q})}{p_k(\mathbf{Q})}$$
$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \le P_k, r_k(\mathbf{Q}) \ge R_k, \forall k. \quad (5)$$

In Sections III-VI, we propose novel iterative algorithms that can efficiently find a stationary/KKT point of problems (2)-(5).

III. THE PROPOSED ALGORITHM FOR GLOBAL ENERGY EFFICIENCY MAXIMIZATION

To design an iterative algorithm for problem (2) that enjoys a low complexity but at the same time a fast convergence behavior, we need on the one hand to address the issue of the nonconvexity in the objective function, and, on the other

hand, to preserve the original problem's structure as much as possible. Towards this end, we propose an iterative algorithm based on the successive pseudoconvex approximation framework developed in [15].

To start with, we introduce the definition of pseudoconvex functions: a function $f(\mathbf{x})$ is said to be *pseudoconvex* if [24]

$$f(\mathbf{y}) < f(\mathbf{x}) \implies (\mathbf{y} - \mathbf{x})^T \nabla f(\mathbf{x}) < 0. \qquad (6)$$

In other words, $f(\mathbf{y}) < f(\mathbf{x})$ implies $\mathbf{y} - \mathbf{x}$ is a *descent direction* of $f(\mathbf{x})$ [25]. A function $f(\mathbf{x})$ is pseudoconcave if $-f(\mathbf{x})$ is pseudoconvex. Some properties of pseudoconvex functions that are important to the subsequent development are briefly summarized below.

- A pseudoconvex optimization problem consists of minimizing a pseudoconvex objective function (or maximizing a pseudoconcave objective function) over a convex constraint set.
- Every stationary point of a pseudoconvex optimization problem is also globally optimal.
- The (strong) convexity of a function implies that the function is pseudoconvex, which in turn implies that the function is quasiconvex, but the reverse is generally not true; see [15, Figure 1].

We refer the interested reader to [24, Ch. 9] and [18, Ch. 2] for a comprehensive treatment of pseudoconvex functions and pseudoconvex optimization.

The proposed iterative algorithm for problem (2) consists of solving a sequence of successively refined approximate problems. In iteration t, the approximate problem defined around a given point $\mathbf{Q}^t$ consists of maximizing an approximate function, denoted as $\tilde{f}(\mathbf{Q}; \mathbf{Q}^t)$, under the same constraints as (2). The lack of concavity in the objective function should be properly compensated so that the approximate problems are much easier to solve than the original problem (2).

The numerator functions $(r_k(\mathbf{Q}))_{k=1}^{K}$ are not concave and the denominator functions $(p_k(\mathbf{Q}))_{k=1}^{K}$ are not convex in $\mathbf{Q}$. Meanwhile, the function $r_k(\mathbf{Q})$ is concave in component $\mathbf{Q}_k$, and the function $P_{0,k} + \rho_k \text{tr}(\mathbf{Q}_k)$ in $p_k(\mathbf{Q})$ is convex in component $\mathbf{Q}_k$. Exploiting this partial concavity may notably accelerate the convergence, as shown in [26] and other works. Therefore, we approximate the numerator function $\sum_{j=1}^{K} r_j(\mathbf{Q})$ with respect to (w.r.t.) $\mathbf{Q}_k$ at the point $\mathbf{Q}^t$ by a function denoted as $\tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)$, which is obtained by fixing the other variables $\mathbf{Q}_{-k}$ in $r_k(\mathbf{Q}_k, \mathbf{Q}_{-k})$ and linearizing only the functions $\{r_j(\mathbf{Q})\}_{j \neq k}$ that are not concave in $\mathbf{Q}_k$:

$$\tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) \triangleq r_k(\mathbf{Q}_k, \mathbf{Q}_{-k}^t) + \sum_{j \neq k}(\mathbf{Q}_k - \mathbf{Q}_k^t) \bullet \nabla_k r_j(\mathbf{Q}^t), \qquad (7)$$

where $\mathbf{X} \bullet \mathbf{Y} \triangleq \Re(\text{tr}(\mathbf{X}^H \mathbf{Y}))$ and $\nabla_k r_j(\mathbf{Q})$ is the Jacobian matrix of $r_j(\mathbf{Q})$ with respect to $\mathbf{Q}_k^*$ (the complex conjugate of $\mathbf{Q}_k$). Since $\tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)$ is concave in $\mathbf{Q}_k$, $\sum_{k=1}^{K} \tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)$ is concave in $\mathbf{Q}$. Similarly, we approximate the denominator function $p_k(\mathbf{Q})$ by a convex function $\tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)$ which is obtained by keeping $P_{0,k} + \rho_k \text{tr}(\mathbf{Q}_k)$ and linearizing the nonconvex part $p_k(\mathbf{Q})$ w.r.t. $\mathbf{Q}_k$ at the point

$\mathbf{Q} = \mathbf{Q}^t$:

$$\tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) \triangleq P_{0,k} + \rho_k \text{tr}(\mathbf{Q}_k) + g_k(r_k(\mathbf{Q}^t))$$
$$+ \sum_{j=1}^{K}(\mathbf{Q}_k - \mathbf{Q}_k^t) \bullet \nabla_k g_j(r_j(\mathbf{Q}^t)), \quad (8)$$

and $\tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)$ is positive and convex. This paves the way to define the following approximate function of the original objective function $f(\mathbf{Q})$ at point $\mathbf{Q}^t$, denoted as $\tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$:

$$\tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t) \triangleq \frac{\sum_{k=1}^{K} \tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)}{\sum_{k=1}^{K} \tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)}, \quad (9)$$

The approximate function $\tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$ has some important properties as we outline.

Firstly, the approximate function $\tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$ is still nonconcave, but it is a fractional function of a nonnegative concave function $\sum_{k=1}^{K} \tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)$ and a positive linear function $\sum_{k=1}^{K} \tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)$, which is thus pseudoconcave [15].

Secondly, the approximate function $\tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$ is differentiable and its gradient is the same as that of the original function $f_G(\mathbf{Q})$ at the point $\mathbf{Q}^t$ where the approximate function $\tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$ is defined. To see this, we remark that $\nabla_{\mathbf{Q}_k^*} \tilde{r}_j(\mathbf{Q}_j; \mathbf{Q}^t)\big|_{\mathbf{Q}=\mathbf{Q}^t} = 0$ if $j \neq k$, and

$$\nabla_k \tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)\big|_{\mathbf{Q}=\mathbf{Q}^t} = \nabla_k \left(\sum_{j=1}^{K} r_j(\mathbf{Q}) \right)\Big|_{\mathbf{Q}=\mathbf{Q}^t}, \quad (10a)$$

$$\tilde{r}_{G,k}(\mathbf{Q}_k^t; \mathbf{Q}^t) = r_k(\mathbf{Q}^t). \quad (10b)$$

Similarly, $\nabla_k \tilde{g}_{G,j}(\mathbf{Q}_j; \mathbf{Q}^t) = 0$ if $j \neq k$ and

$$\nabla_k \tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)\big|_{\mathbf{Q}=\mathbf{Q}^t} = \nabla_k \left(\sum_{j=1}^{K} p_j(\mathbf{Q}) \right)\Big|_{\mathbf{Q}=\mathbf{Q}^t}, \quad (11a)$$

$$\tilde{p}_{G,k}(\mathbf{Q}_k^t; \mathbf{Q}^t) = p_k(\mathbf{Q}^t). \quad (11b)$$

Based on the observations in (10)-(11), it can be verified that the gradient of the approximate function $\tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$ is the same as that of the original function $f_G(\mathbf{Q})$ at the point $\mathbf{Q}^t$:

$$\nabla_k \tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)\Big|_{\mathbf{Q}=\mathbf{Q}^t}$$
$$= \frac{\nabla_k \tilde{r}_k(\mathbf{Q}_k^t; \mathbf{Q}^t)}{\sum_{j=1}^{K} \tilde{p}_j(\mathbf{Q}_j^t; \mathbf{Q}^t)} - \frac{(\sum_{j=1}^{K} \tilde{r}_j(\mathbf{Q}_j^t; \mathbf{Q}^t)) \nabla_k \tilde{p}_k(\mathbf{Q}_k^t; \mathbf{Q}^t)}{(\sum_{j=1}^{K} \tilde{p}_j(\mathbf{Q}_j^t; \mathbf{Q}^t))^2}$$
$$= \frac{\nabla_k(\sum_{j=1}^{K} r_j(\mathbf{Q}^t))}{\sum_{j=1}^{K} p_j(\mathbf{Q}^t)} - \frac{(\sum_{j=1}^{K} r_j(\mathbf{Q}^t)) \nabla_k(\sum_{j=1}^{K} p_j(\mathbf{Q}^t))}{(\sum_{j=1}^{K} p_k(\mathbf{Q}^t))^2}$$
$$= \nabla_k f_G(\mathbf{Q})|_{\mathbf{Q}=\mathbf{Q}^t}, \quad (12)$$

where the first and third equality is the expression of $\nabla_k \tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$ and $\nabla_k f_G(\mathbf{Q})$, respectively, and the second equality follows from (10)-(11).

At iteration t of the proposed algorithm, the approximate problem defined at the point $\mathbf{Q}^t$ is to maximize the approximate function $\tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$ defined in (9) subject to the same constraints as in the original problem (2):

$$\underset{\mathbf{Q}}{\text{maximize}} \quad \tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$$
$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k, \ k = 1, \dots, K. \quad (13a)$$

and its (globally) optimal point is denoted as $\mathbb{B}\mathbf{Q}^t$:

$$\mathbb{B}\mathbf{Q}^t \triangleq \underset{(\mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k)_{k=1}^{K}}{\arg\max} \tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t). \quad (13b)$$

Since problem (13a) is pseudoconvex, all of its stationary points are globally optimal [24, Th. 9.3.3]. As we will show shortly, $\mathbb{B}\mathbf{Q}^t$ is unique.

Due to the above mentioned pseudoconcavity, differentiability and equal gradient condition (12) at $\mathbf{Q}^t$ of the approximate function $\tilde{f}(\mathbf{Q}; \mathbf{Q}^t)$ defined in (9), it follows from [15, Prop. 1] that solving the approximate problem (13) yields an ascent direction of the original objective function $f_G(\mathbf{Q})$ at $\mathbf{Q}^t$, unless $\mathbf{Q}^t$ is already a stationary point of problem (2), as stated in the following proposition.

Proposition 1 (Stationary point and ascent direction). *A point $\mathbf{Q}^t$ is a stationary point of (2) if and only if $\mathbf{Q}^t = \mathbb{B}\mathbf{Q}^t$. If $\mathbf{Q}^t$ is not a stationary point of (2), then $\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t$ is an ascent direction of $f_G(\mathbf{Q})$ in the sense that*

$$(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t) \bullet \nabla f_G(\mathbf{Q}^t) > 0.$$

Proof: The proof follows the same line of analysis of [15, Prop. 1] and is thus not duplicated here. ∎

Since $\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t$ is an ascent direction of $f_G(\mathbf{Q})$ at $\mathbf{Q} = \mathbf{Q}^t$ according to Proposition 1, there exists a scalar $\gamma^t \in (0, 1]$ such that $f_G(\mathbf{Q}^t + \gamma^t(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t)) > f_G(\mathbf{Q}^t)$ [27, 8.2.1]. This motivates us to update the variable $\mathbf{Q}$ as

$$\mathbf{Q}^{t+1} = \mathbf{Q}^t + \gamma^t(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t). \quad (14)$$

This also implies that a constant unit stepsize (i.e., $\gamma^t = 1$) does not necessarily lead to an increase in the objective function value, unless $\tilde{f}_G(\mathbf{Q}; \mathbf{Q}^t)$ defined in (9) satisfies an additional assumption that it is a global lower bound of $f_G(\mathbf{Q})$ [15, Assumption (A6)], which is however not the case. In practice, the stepsize γ^t is usually obtained by either the exact line search or the successive line search. Performing the exact line search consists of solving an optimization problem

$$\max_{0 \leq \gamma \leq 1} f(\mathbf{Q}^t + \gamma(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t)).$$

Since the objective function $f(\mathbf{Q})$ is nonconcave, the above optimization problem is nonconvex and not trivial to solve. Therefore, we adopt the successive line search to calculate the stepsize γ^t. That is, given two scalars $0 < \alpha < 1$ and $0 < \beta < 1$, γ^t is set to be $\gamma^t = \beta^{m_t}$, where m_t is the smallest nonnegative integer m satisfying the following inequality:

$$f_G(\mathbf{Q}^t + \beta^m(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t)) \geq$$
$$f_G(\mathbf{Q}^t) + \alpha\beta^m \nabla f_G(\mathbf{Q}^t) \bullet (\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t). \quad (15)$$

Note that the successive line search is carried out over the original objective function $f(\mathbf{Q})$ defined in (2).

The resulting sequence $\{f_G(\mathbf{Q}^t)\}_t$ is increasing:

$$f_G(\mathbf{Q}^{t+1}) = f_G(\mathbf{Q}^t + \beta^{m_t}(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t))$$
$$\geq f_G(\mathbf{Q}^t) + \alpha\beta^{m_t} \nabla f_G(\mathbf{Q}^t) \bullet (\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t)$$
$$\geq f_G(\mathbf{Q}^t), \ \forall t,$$

where the first and second inequality comes from the definition of the successive line search (15) and Proposition 1, respectively.

Algorithm 1 The successive pseudoconvex approximation method for GEE maximization (2)

S0: $\mathbf{Q}^0 = \mathbf{0}$, $t = 0$, and a stopping criterion ε.

S1: Solve problem (13) to compute $\mathbb{B}\mathbf{Q}^t$ by the following steps:

> **S1.0:** $s^{t,0} = \sum_{k=1}^{K} \tilde{r}_{G,k}(\mathbf{Q}_k^t; \mathbf{Q}^t) / \sum_{k=1}^{K} \tilde{p}_{G,k}(\mathbf{Q}_k^t; \mathbf{Q}^t)$, $\tau = 0$, and a stopping criterion ϵ.
> **S1.1:** Compute $\mathbf{Q}_k^{\star}(s^{t,\tau})$ by (17) for all $k = 1, \dots, K$.
> **S1.2:** Compute $s^{t,\tau+1}$ by (18).
> **S1.3:** If $|s^{t,\tau+1} - s^{t,\tau}| < \epsilon$, then $\mathbb{B}\mathbf{Q}^t = \mathbf{Q}^{\star}(s^{t,\tau})$ and go to **S2**. Otherwise $\tau \leftarrow \tau + 1$ and go to **S1.1**.

S2: Compute γ^t by the successive line search (15).

S3: Update $\mathbf{Q}^{t+1}$ according to (14).

S4: If $\|\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t\| \leq \varepsilon$, then STOP; otherwise $t \leftarrow t + 1$ and go to **S1**.

The proposed algorithm is formally summarized in Algorithm 1 and its convergence properties are given in the following theorem.

Theorem 2 (Convergence to a stationary point). *The sequence $\{\mathbf{Q}^t\}$ generated by Algorithm 1 has a limit point, and every limit point is a stationary point of problem (2). Furthermore, the sequence $\{f_G(\mathbf{Q}^t)\}$ is monotonically increasing.*

Proof: The constraint set of problem (2), namely, $\{(\mathbf{Q}_k)_{k=1}^K : \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k\}$, is nonempty and bounded. The sequence $\{\mathbf{Q}^t\}_t$ is thus bounded and has a limit point. Then the latter statement can be proved following the same line of analysis as [15, Theorem 1] and is thus not duplicated here. ∎

In Step 1 of Algorithm 1, a constrained pseudoconvex optimization problem, namely, problem (13) must be solved. Since the optimal point $\mathbb{B}\mathbf{Q}^t$ does not have a closed-form expression, we apply the Dinkelbach's algorithm [17] to solve problem (13) iteratively: at iteration τ of Dinkelbach's algorithm, the following problem is solved for a given and fixed $s^{t,\tau}$ ($s^{t,0}$ can be set to 0):

$$\underset{\mathbf{Q}}{\text{maximize}} \quad \sum_{k=1}^{K} \tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) - s^{t,\tau} \sum_{k=1}^{K} \tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)$$

$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k, \forall k. \tag{16}$$

Since problem (16) is well decoupled across different variables, it can be decomposed component-wise into many smaller optimization problems that can be solved in parallel: for all $k = 1, \dots, K$,

$$\underset{\mathbf{Q}}{\text{maximize}} \quad \tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) - s^{t,\tau} \tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)$$

$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k. \tag{17a}$$

This problem is convex and its (unique) optimal point has a closed-form expression based on the generalized waterfilling solution [28, Lemma 2]:

$$\mathbf{Q}_k^{\star}(s^{t,\tau}) \triangleq \underset{\mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k}{\arg\max} \left\{ \begin{array}{c} \tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) \\ -s^{t,\tau} \tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) \end{array} \right\}$$

$$= \mathbf{V}[\mathbf{I} - \boldsymbol{\Sigma}^{-1}]^+ \mathbf{V}^H, \tag{17b}$$

where $[\mathbf{X}]^+$ denotes the projection of $\mathbf{X}$ onto the cone of positive semidefinite matrices, $(\mathbf{V}, \boldsymbol{\Sigma})$ is the generalized eigenvalue decomposition of $(\mathbf{H}_{kk}^H \mathbf{R}_k (\mathbf{Q}_{-k}^t)^{-1} \mathbf{H}_{kk}, (s^{t,\tau} \rho_k + \mu^{\star})\mathbf{I} + s^{t,\tau} \nabla_{\mathbf{Q}_k^{\star}}(\sum_{j=1}^K g_j(r_j(\mathbf{Q}^t))) - \sum_{j \neq k} \nabla_k r_j(\mathbf{Q}^t))$, and $\mu^{\star}$ is the Lagrange multiplier such that $0 \leq \mu^{\star} \perp \text{tr}(\mathbf{Q}_k^{\star}(s^{t,\tau})) - P_k \leq 0$, which can easily be found by bisection.

After $(\mathbf{Q}_k^{\star}(s^{t,\tau}))_{k=1}^K$ is obtained, $s^{t,\tau}$ is updated as follows:

$$s^{t,\tau+1} = \frac{\sum_{k=1}^{K} \tilde{r}_{G,k}(\mathbf{Q}_k^{\star}(s^{t,\tau}); \mathbf{Q}^t)}{\sum_{k=1}^{K} \tilde{p}_{G,k}(\mathbf{Q}_k^{\star}(s^{t,\tau}); \mathbf{Q}^t)}. \tag{18}$$

It follows from the convergence properties of the Dinkelbach's algorithm (cf. [17]) that

$$\lim_{\tau \to \infty} \mathbf{Q}^{\star}(s^{t,\tau}) = \mathbb{B}\mathbf{Q}^t$$

at a superlinear convergence rate. Note that $\mathbb{B}\mathbf{Q}^t$ is unique, because both $\lim_{\tau \to \infty} s^{t,\tau}$ and $\mathbf{Q}^{\star}(s^{t,\tau})$ are unique. This iterative procedure (17)-(18) is nested under Step 1 of Algorithm 1 as Steps 1.0-1.3.

In the following, we discuss some properties and implementation aspects of the proposed Algorithm 1.

The proposed algorithm presents a fast convergence behavior. The approximate function in (9) is constructed in the same spirit as [15, 26] by keeping as much concavity as possible, namely, $r_k(\mathbf{Q}_k, \mathbf{Q}_{-k})$ in $\mathbf{Q}_k$ and $\sum_{j=1}^{K}(P_{0,k} + \rho_k \text{tr}(\mathbf{Q}_k))$ in $\mathbf{Q}$, and linearizing only the nonconcave functions in the numerator and the nonconvex functions in the denominator, namely, $\sum_{j \neq k} r_j(\mathbf{Q})$ and $\sum_{j=1}^{K} g_j(r_j(\mathbf{Q}))$. Besides this, the division operator is also kept. Therefore, the problem structure is preserved to a large extent and the proposed algorithm is expected to exhibit a fast convergence behavior, as we shall later illustrate numerically.

The proposed algorithm enjoys a low complexity and an easy implementation. In iterative algorithms, the major computational complexity lies in solving the approximate problem in each iteration. In the proposed algorithm, the approximate problem can be decomposed into multiple independent subproblems and is thus suitable for parallel computation. The optimal point of each subproblem has a closed-form expression; by contrast, a generic convex optimization problem must be solved in each iteration in [8, 11].

The proposed algorithm presents a broad applicability. Firstly, it does not require the approximate function to be a global lower bound of the original function, see, e.g., the sequential programming framework proposed in [11]. Such an approximate function may not even exist for some choices of the power consumption models. Secondly, the proposed algorithm is applicable for MIMO systems, where the design variables are complex-valued matrices, and the rate-dependent processing power consumption function $g_k(\cdot)$ does not have to be convex, as assumed in [8].

The proposed algorithm can, e.g., be implemented on a central processing unit which has the channel state information of all direct-link and cross-link channels, namely, $(\mathbf{H}_{kj})_{j,k}$. In practical systems, this central unit could be embedded in the centralized radio access network (CRAN), cf. Figure 1: each BS k sends the direct-link channel $\mathbf{H}_{kk}$ and cross-link channels $(\mathbf{H}_{kj})_{j \neq k}$ to the central unit in the CRAN. Then

the central unit invokes Algorithm 1 and informs each BS k about the optimal transmit covariance matrix $\mathbf{Q}_k$. The incurred latency is mainly due to the signaling exchange between the central unit and the BSs, and the execution of the variable updates. Due to the algorithm's low complexity, the central unit is not required to have a strong computational capability.

IV. THE PROPOSED ALGORITHM FOR SUM ENERGY EFFICIENCY MAXIMIZATION

In this section, we propose an iterative algorithm for problem (3), which consists in solving a sequence of successively refined approximate problems. In iteration t, we approximate the nonconcave function $f_S(\mathbf{Q})$ with respect to $\mathbf{Q}_k$ at the point $\mathbf{Q}^t$ by a function denoted as $\tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)$:

$$\tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t) \triangleq \frac{\tilde{r}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)}{\tilde{p}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)}, \tag{19}$$

where

$$\tilde{r}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t) \triangleq r_k(\mathbf{Q}_k, \mathbf{Q}_{-k}^t) + (\mathbf{Q}_k - \mathbf{Q}_k^t) \bullet \mathbf{\Pi}_k(\mathbf{Q}^t), \tag{20a}$$

$$\mathbf{\Pi}_k(\mathbf{Q}^t) \triangleq p_k(\mathbf{Q}^t) \cdot \nabla_k \left(\sum_{j \neq k} \frac{r_j(\mathbf{Q})}{p_j(\mathbf{Q})} \right) \Bigg|_{\mathbf{Q} = \mathbf{Q}^t}, \tag{20b}$$

and

$$\tilde{p}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t) \triangleq P_{0,k} + \rho_k \text{tr}(\mathbf{Q}_k) + g_k(r_k(\mathbf{Q}^t)) + (\mathbf{Q} - \mathbf{Q}_k^t) \bullet \nabla_k g_k(r_k(\mathbf{Q}^t)). \tag{20c}$$

In (20a)-(20b), we fix $\mathbf{Q}_{-k}$ in $r_k(\mathbf{Q}_k, \mathbf{Q}_{-k})$ and linearize the nonconcave function $r_j(\mathbf{Q})/p_j(\mathbf{Q})$ with respect to $\mathbf{Q}_k$. In (20c), the nonconvex function $g_k(r_k(\mathbf{Q}))$ is linearized. As a result, the numerator and denominator function of $\tilde{f}_k(\mathbf{Q}_k; \mathbf{Q}^t)$ is concave and convex in $\mathbf{Q}_k$, respectively, and $\tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)$ is thus pseudoconcave in $\mathbf{Q}_k$. Besides, it is not difficult to verify that

$$\tilde{r}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t) = r_k(\mathbf{Q}^t), \tag{21a}$$

$$\nabla_k \tilde{r}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)\big|_{\mathbf{Q}=\mathbf{Q}^t} = \nabla_k r_k(\mathbf{Q})|_{\mathbf{Q}=\mathbf{Q}^t} + \mathbf{\Pi}_k(\mathbf{Q}^t), \tag{21b}$$

and

$$\tilde{p}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t) = p_k(\mathbf{Q}^t), \tag{22a}$$

$$\nabla_k \tilde{p}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)\big|_{\mathbf{Q}=\mathbf{Q}^t} = \nabla_k p_k(\mathbf{Q})|_{\mathbf{Q}=\mathbf{Q}^t}. \tag{22b}$$

Then we can show that $\tilde{f}_k(\mathbf{Q}_k; \mathbf{Q}^t)$ and $f(\mathbf{Q})$ have the same gradient w.r.t. $\mathbf{Q}_k$ at the point $\mathbf{Q} = \mathbf{Q}^t$:

$$\nabla_k \tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)\Big|_{\mathbf{Q}=\mathbf{Q}^t}$$
$$= \frac{\nabla_k \tilde{r}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t)}{\tilde{p}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t)} - \frac{\tilde{r}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t) \nabla_k \tilde{p}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t)}{\tilde{p}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t)^2}$$
$$= \frac{\nabla_k r_k(\mathbf{Q}^t) + \mathbf{\Pi}_k(\mathbf{Q}^t)}{p_k(\mathbf{Q}^t)} - \frac{r_k(\mathbf{Q}^t) \nabla_k p_k(\mathbf{Q}^t)}{p_k(\mathbf{Q}^t)^2}$$
$$= \frac{\nabla_k r_k(\mathbf{Q}^t)}{p_k(\mathbf{Q}^t)} - \frac{r_k(\mathbf{Q}^t) \nabla_k p_k(\mathbf{Q}^t)}{p_k(\mathbf{Q}^t)^2}$$
$$+ \sum_{j \neq k} \nabla_k \left(\frac{r_j(\mathbf{Q}^t)}{p_j(\mathbf{Q}^t)} \right) = \nabla_k f_S(\mathbf{Q})|_{\mathbf{Q}=\mathbf{Q}^t}, \tag{23}$$

where the first and third equality is the expression of $\nabla_k \tilde{f}_S(\mathbf{Q}; \mathbf{Q}^t)$ and $\nabla_k f_S(\mathbf{Q})$, respectively, and the second equality follows from (21)-(22).

Given point $\mathbf{Q}^t$ in iteration t, we define an approximate problem of the following form:

$$\begin{aligned} \text{maximize}_{\mathbf{Q}} \quad & \sum_{k=1}^K \tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t) \\ \text{subject to} \quad & \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k, \ k = 1, \ldots, K, \end{aligned} \tag{24}$$

and we denote as $\mathbb{B}\mathbf{Q}^t = (\mathbb{B}_k \mathbf{Q}^t)_{k=1}^K$ the optimal point. Since problem (24) is well decoupled across different variables, it can be decomposed into many smaller optimization problems that can be solved in parallel:

$$\mathbb{B}\mathbf{Q}^t = \underset{(\mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k)_{k=1}^K}{\arg\max} \sum_{k=1}^K \tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t) \tag{25}$$
$$\Updownarrow$$
$$\mathbb{B}_k \mathbf{Q}^t = \underset{\mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k}{\arg\max} \tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t), \ k = 1, \ldots, K. \tag{26}$$

Note that $\mathbb{B}_k \mathbf{Q}^t$ is unique, which can be shown by the same line of argument used in the previous section.

We remark that although $\tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)$ is pseudoconcave in $\mathbf{Q}_k$, the approximate function $\sum_{k=1}^K \tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)$ in (24) is not necessarily pseudoconcave in $\mathbf{Q}$, because, unlike concave functions, the sum of pseudoconcave functions is not always pseudoconcave. Despite the lack of pseudoconcavity in the approximate function $\sum_{k=1}^K \tilde{f}_k(\mathbf{Q}_k; \mathbf{Q}^t)$ in (24), $\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t$ is still an ascent direction of the original objective function $f(\mathbf{Q})$ at $\mathbf{Q} = \mathbf{Q}^t$. To see this, we note that

$$(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t) \bullet \nabla f_S(\mathbf{Q}^t) = \sum_{k=1}^K (\mathbb{B}_k \mathbf{Q}^t - \mathbf{Q}_k^t) \bullet \nabla_k f_S(\mathbf{Q}^t)$$
$$= \sum_{k=1}^K (\mathbb{B}_k \mathbf{Q}^t - \mathbf{Q}_k^t) \bullet \nabla_k \tilde{f}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t).$$

As both the objective function and the constraint set of the approximate problem (24) is well decoupled among the different block variables and each subproblem (26) is pseudoconvex, we have $(\mathbb{B}_k \mathbf{Q}^t - \mathbf{Q}_k^t) \bullet \nabla_k \tilde{f}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t) > 0$ if $\mathbb{B}_k \mathbf{Q}^t \neq \mathbf{Q}_k$ and $(\mathbb{B}_k \mathbf{Q}^t - \mathbf{Q}_k^t) \bullet \nabla_k \tilde{f}_{S,k}(\mathbf{Q}_k^t; \mathbf{Q}^t) = 0$ otherwise. This is formally stated in the following proposition.

Proposition 3 (Stationary point and ascent direction). *A point $\mathbf{Q}^t$ is a stationary point of (3) if and only if $\mathbb{B}\mathbf{Q}^t = \mathbf{Q}^t$. If $\mathbf{Q}^t$ is not a stationary point of (3), then $\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t$ is an ascent direction of $f_S(\mathbf{Q})$ in the sense that*

$$(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t) \bullet \nabla f_S(\mathbf{Q}^t) > 0.$$

Proof: The proof follows the same line of analysis of [15, Th. 3] and is thus not duplicated here. ∎

According to Proposition 3, $\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t$ is an ascent direction of $f_S(\mathbf{Q})$ at $\mathbf{Q} = \mathbf{Q}^t$, and we calculate the stepsize by the successive line search: given two scalars $0 < \alpha < 1$ and $0 < \beta < 1$, γ^t is set to be $\gamma^t = \beta^{m_t}$, where m_t is the smallest nonnegative integer m satisfying the following inequality:

$$f_S(\mathbf{Q}^t + \beta^m(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t)) \geq$$
$$f_S(\mathbf{Q}^t) + \alpha\beta^m \nabla f_S(\mathbf{Q}^t) \bullet (\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t). \tag{27}$$

Note that the successive line search is carried out over the original objective function $f(\mathbf{Q})$ defined in (3). After the

Algorithm 2 The successive pseudoconvex approximation method for SEE maximization (3)

S0: $\mathbf{Q}^0 = \mathbf{0}$, $t = 0$, and a stopping criterion ε.

S1: Solve problem (26) to compute $\mathbb{B}\mathbf{Q}^t$ by the following steps:

> **S1.0:** $s_k^{t,0} = \tilde{r}_{S,k}(\mathbf{Q}^t;\mathbf{Q}^t)/\tilde{p}_{S,k}(\mathbf{Q}^t;\mathbf{Q}^t)$ for all $k = 1,\ldots,K$, $\tau = 0$, and a stopping criterion ϵ.
> **S1.1:** Compute $\mathbf{Q}_k^\star(s^{t,\tau})$ by (29) for all $k = 1,\ldots,K$.
> **S1.2:** Compute $s_k^{t,\tau+1}$ by (30) for all $k = 1,\ldots,K$.
> **S1.3:** If $\left\| \mathbf{s}^{t,\tau+1} - \mathbf{s}^{t,\tau} \right\| < \epsilon$, then $\mathbb{B}\mathbf{Q}^t = \mathbf{Q}^\star(\mathbf{s}^{t,\tau})$ and go to **S2**. Otherwise $\tau \leftarrow \tau + 1$ and go to **S1.1**.

S2: Compute γ^t by the successive line search (27).

S3: Update $\mathbf{Q}^{t+1}$ according to (28).

S4: If $\|\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t\| \leq \varepsilon$, then STOP; otherwise $t \leftarrow t+1$ and go to **S1**.

---

stepsize γ^t is found, the variable $\mathbf{Q}$ is updated as

$$\mathbf{Q}^{t+1} = \mathbf{Q}^t + \gamma^t(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t). \tag{28}$$

The above steps are formally summarized in Algorithm 2. From Proposition 3 and (27)-(28) it can be verified that the sequence $\{f_S(\mathbf{Q}^t)\}_t$ is monotonically increasing. Moreover, the sequence $\{\mathbf{Q}^t\}$ has a limit point and every limit point is a stationary point of (3), whose proof follows the same line of analysis as [15, Th. 3] and thus not duplicated here.

In Step 1 of Algorithm 2, a constrained pseudoconvex optimization problem, namely, problem (26), must be solved, and we apply the Dinkelbach's algorithm to find $\mathbb{B}_{S,k}\mathbf{Q}^t$ iteratively. At iteration τ of Dinkelbach's algorithm, the following problem is solved for a given $s_k^{t,\tau}$ ($s_k^{t,0}$ can be set to 0):

$$\underset{\mathbf{Q}_k}{\text{maximize}} \quad \tilde{r}_{S,k}(\mathbf{Q}_k;\mathbf{Q}^t) - s_k^{t,\tau}\tilde{p}_{S,k}(\mathbf{Q}_k;\mathbf{Q}^t)$$

$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k, \ \forall k. \tag{29}$$

Similar to problem (17), the optimal point of problem (29), denoted as $\mathbf{Q}_k^\star(s_k^{t,\tau})$, has a closed-form expression based on the generalized waterfilling solution (cf. (17) in Section III). After $(\mathbf{Q}_k^\star(s_k^{t,\tau}))_{k=1}^K$ is obtained, $s_k^{t,\tau}$ is updated as follows:

$$s_k^{t,\tau+1} = \frac{\tilde{r}_{S,k}(\mathbf{Q}_k^\star(s_k^{t,\tau});\mathbf{Q}^t)}{\tilde{p}_{S,k}(\mathbf{Q}^\star(s_k^{t,\tau});\mathbf{Q}^t)}. \tag{30}$$

It follows from the convergence properties of the Dinkelbach's algorithm that $\lim_{\tau\to\infty}\mathbf{Q}_k^\star(s_k^{t,\tau}) = \mathbb{B}_k\mathbf{Q}^t$ for all k. This iterative procedure (29)-(30) is nested under Step 1 of Algorithm 2 as Steps 1.0-1.3.

The proposed Algorithm 2 for the SEE maximization problem (3) has the same attractive features as those of Algorithm 1 for the global EE maximization problem (2), namely, the fast convergence, the broad applicability and the low complexity; see the discussion at the end of Sec. III. We complement the discussion by emphasizing that Algorithm 2 is the first parallel Jacobi-type algorithm designed for the maximization of the sum EE function, and pseudoconvexity plays a fundamental role that has not been fully recognized nor exploited by existing techniques. This also marks a notable relaxation in state-of-the-art convergence conditions for Jacobi algorithms.

V. THE PROPOSED ALGORITHM FOR GLOBAL ENERGY EFFICIENCY MAXIMIZATION WITH QoS CONSTRAINTS

In this section, we propose an iterative algorithm to maximize the GEE subject to the QoS constraints defined in (4).

The nonconcave QoS constraints in (4) make the constraint set nonconvex and Algorithm 1 proposed in Sec. III for problem (2) is no longer applicable, because 1) the approximate problem is difficult to solve, and 2) the new point updated according to (14) is not necessarily feasible. To design an iterative algorithm for problem (4) that enjoys a low complexity but at the same time a fast convergence behavior, we need on the one hand to overcome the nonconcavity/nonconvexity in the objective function/the constraint set, and, on the other hand, to preserve the original problem's structure as much as possible. Towards this end, we extend the successive pseudoconvex approximation framework developed in [15] for minimizing a nonconvex function over a convex constraint set to solve problem (4) where the objective function/the constraint set is nonconcave/nonconvex.

In iteration t, the approximate problem defined around the point $\mathbf{Q}^t$ consists of maximizing an approximate function, denoted as $\tilde{f}(\mathbf{Q};\mathbf{Q}^t)$, over an approximate set, denoted as $\tilde{\mathcal{Q}}(\mathbf{Q}^t)$. We first note that the nonconcave function $r_k(\mathbf{Q})$ in (4) can be rewritten as the difference of two concave functions:

$$\begin{aligned} r_k(\mathbf{Q}) &= \log\det\left(\mathbf{I} + \mathbf{R}_k(\mathbf{Q}_{-k})^{-1}\mathbf{H}_{kk}\mathbf{Q}_k\mathbf{H}_{kk}^H\right) \\ &= \log\det\left(\sigma_k^2\mathbf{I} + \sum_{j=1}^K \mathbf{H}_{kj}\mathbf{Q}_j\mathbf{H}_{kj}^H\right) \\ &\quad - \log\det\left(\sigma_k^2\mathbf{I} + \sum_{j\neq k}\mathbf{H}_{kj}\mathbf{Q}_j\mathbf{H}_{kj}^H\right). \end{aligned}$$

Introducing auxiliary variables $\mathbf{Y}_k$ such that $\mathbf{Y}_k = \sum_{j=1}^K\mathbf{H}_{kj}\mathbf{Q}_j\mathbf{H}_{kj}^H$, we reformulate problem (4) as follows:

$$\underset{\mathbf{Q},\mathbf{Y}}{\text{maximize}} \quad f_G(\mathbf{Q}) \tag{31a}$$

$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k, \tag{31b}$$

$$r_k^+(\mathbf{Y}_k) - r_k^-(\mathbf{Q}_{-k}) \geq R_k, \tag{31c}$$

$$\mathbf{Y}_k = \sum_{j=1}^K\mathbf{H}_{kj}\mathbf{Q}_j\mathbf{H}_{kj}^H, \ \forall k, \tag{31d}$$

where $r_k^+(\mathbf{Y}_k) \triangleq \log\det(\sigma_k^2\mathbf{I} + \mathbf{Y}_k)$ and $r_k^-(\mathbf{Q}) \triangleq \log\det(\sigma_k^2\mathbf{I}+\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{Q}_j\mathbf{H}_{kj}^H)$. As we will see later, such a reformulation is beneficial because the resulting approximate problem can be efficiently solved by parallel algorithms.

Approximate function. The nonconcave numerator function $\sum_{k=1}^K r_j(\mathbf{Q})$ is approximated in the same way as in (7). We also approximate the nonconvex denominator function $\sum_{j=1}^K p_j(\mathbf{Q})$ w.r.t. $\mathbf{Q}_k$ by $\tilde{p}_{G,k}(\mathbf{Q}_k;\mathbf{Q}^t)$ defined in (8). The approximate function $\tilde{f}(\mathbf{Q};\mathbf{Q}^t)$ is of the following form:

$$\tilde{f}_G(\mathbf{Q},\mathbf{Y};\mathbf{Q}^t,\mathbf{Y}^t) \triangleq \frac{\sum_{k=1}^K\left(\tilde{r}_{G,k}(\mathbf{Q}_k;\mathbf{Q}^t) - c\left\|\mathbf{Y}_k - \mathbf{Y}_k^t\right\|_F^2\right)}{\sum_{k=1}^K \tilde{p}_{G,k}(\mathbf{Q}_k;\mathbf{Q}^t)}, \tag{32}$$

with $\mathbf{Y}_k^t = \sum_{j=1}^K\mathbf{H}_{kj}\mathbf{Q}_j^t\mathbf{H}_{kj}^H$, while $c \geq 0$ is a given constant. When $c = 0$, the approximate function (32) is the same as (9). However, when $c > 0$, the quadratic regularization term makes the numerator function strongly concave in $\mathbf{Y}$ and the benefit will become clear later. The approximate function $\tilde{f}_G(\mathbf{Q},\mathbf{Y};\mathbf{Q}^t,\mathbf{Y}^t)$ is pseudoconcave in $(\mathbf{Q},\mathbf{Y})$ for any given

and fixed $(\mathbf{Q}^t, \mathbf{Y}^t)$, and the gradient of $\tilde{f}_G(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t, \mathbf{Y}^t)$ and that of $f_G(\mathbf{Q})$ are identical at the point $(\mathbf{Q}^t, \mathbf{Y}^t)$:

$$\nabla_{\mathbf{Q}^\star} \tilde{f}_G(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t, \mathbf{Y}^t)\big|_{\mathbf{Q}=\mathbf{Q}^t, \mathbf{Y}=\mathbf{Y}^t} = \nabla_{\mathbf{Q}^\star} f_G(\mathbf{Q}^t),$$
$$\nabla_{\mathbf{Y}^\star} \tilde{f}_G(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t, \mathbf{Y}^t)\big|_{\mathbf{Q}=\mathbf{Q}^t, \mathbf{Y}=\mathbf{Y}^t} = \mathbf{0} = \nabla_{\mathbf{Y}^\star} f_G(\mathbf{Q}^t). \tag{33}$$

As we have seen repeatedly, these properties are essential in establishing the convergence of the proposed algorithm.

Approximate set. It follows from the definition of concave functions that $r_k^-(\mathbf{Q})$ is upper bounded by its first order approximation at the point $\mathbf{Q}^t$:

$$r_k^-(\mathbf{Q}) \leq r_k^-(\mathbf{Q}^t) + \sum_{j \neq k}(\mathbf{Q}_j - \mathbf{Q}_j^t) \bullet \nabla_{\mathbf{Q}_j^\star} r_k^-(\mathbf{Q}^t)$$
$$\triangleq \bar{r}_k^-(\mathbf{Q}; \mathbf{Q}^t), \tag{34}$$

where

$$r_k^-(\mathbf{Q}^t) = \bar{r}_k^-(\mathbf{Q}^t; \mathbf{Q}^t) \text{ and } \nabla_{\mathbf{Q}^\star} r_k^-(\mathbf{Q}^t) = \nabla_{\mathbf{Q}^\star} \bar{r}_k^-(\mathbf{Q}^t; \mathbf{Q}^t). \tag{35}$$

Thus $r_k^+(\mathbf{Y}_k) - \bar{r}_k^-(\mathbf{Q}; \mathbf{Q}^t)$ is a global lower bound of $r_k(\mathbf{Q})$:

$$r_k(\mathbf{Q}) = r_k^+(\mathbf{Y}_k) - r_k^-(\mathbf{Q}) \geq r_k^+(\mathbf{Y}_k) - \bar{r}_k^-(\mathbf{Q}; \mathbf{Q}^t), \tag{36}$$

where equality holds at $\mathbf{Q} = \mathbf{Q}^t$.

We then define the (inner) approximate constraint set $\tilde{\mathcal{Q}}(\mathbf{Q}^t)$ by replacing the nonconcave functions $r_k(\mathbf{Q})$ with its lower bound $\underline{r}_k(\mathbf{Q}; \mathbf{Q}^t)$:

$$\tilde{\mathcal{Q}}(\mathbf{Q}^t) \triangleq \left\{ (\mathbf{Q}, \mathbf{Y}) : \begin{array}{c} \mathbf{Q}_k \succeq \mathbf{0}, \mathrm{tr}(\mathbf{Q}_k) \leq P_k, \\ r_k^+(\mathbf{Y}_k) - \bar{r}_k^-(\mathbf{Q}_{-k}; \mathbf{Q}^t) \geq R_k, \\ \mathbf{Y}_k = \sum_{j=1}^K \mathbf{H}_{kj} \mathbf{Q}_j \mathbf{H}_{kj}^H, \forall k \end{array} \right\}. \tag{37}$$

The set $\tilde{\mathcal{Q}}(\mathbf{Q}^t)$ is convex as $r_k^+(\mathbf{Y}_k) - \bar{r}_k(\mathbf{Q}; \mathbf{Q}^t)$ is concave.

Approximate problem. In iteration t, the approximate problem defined at the point $\mathbf{Q}^t$ is to maximize the approximate function $\tilde{f}(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t, \mathbf{Y}^t)$ defined in (32) over the approximate set $\tilde{\mathcal{Q}}(\mathbf{Q}^t)$ defined in and (37):

$$\underset{(\mathbf{Q}, \mathbf{Y}) \in \tilde{\mathcal{Q}}(\mathbf{Q}^t)}{\text{maximize}} \quad \tilde{f}_G(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t, \mathbf{Y}^t), \tag{38}$$

and its optimal point is denoted as $(\mathbb{B}_Q \mathbf{Q}^t, \mathbb{B}_Y \mathbf{Q}^t))$. Note that its dependence on $\mathbf{Y}^t$ is suppressed for notation simplicity.

It turns out that $\mathbb{B}_Q \mathbf{Q}^t - \mathbf{Q}^t$ is an ascent direction of the original objective function $f(\mathbf{Q})$ at $\mathbf{Q} = \mathbf{Q}^t$, unless $\mathbf{Q}^t$ is already a KKT point[4] of problem (4), as stated in the following proposition.

Proposition 4 (KKT point and ascent direction). *A point $\mathbf{Q}^t$ is a KKT point of (4) if and only if $\mathbf{Q}^t = \mathbb{B}_Q \mathbf{Q}^t)$. If $\mathbf{Q}^t$ is not a stationary point of (4), then $\mathbb{B}_Q(\mathbf{Q}^t, \mathbf{Y}^t) - \mathbf{Q}^t$ is an ascent direction of $f_G(\mathbf{Q})$ in the sense that*

$$(\mathbb{B}_Q \mathbf{Q}^t - \mathbf{Q}^t) \bullet \nabla_{\mathbf{Q}^\star} f_G(\mathbf{Q}^t) > 0.$$

Proof: See Appendix. ∎

Given the ascent direction $\mathbb{B}_Q \mathbf{Q}^t - \mathbf{Q}^t$, we calculate the stepsize γ^t by the successive line search as explained in (15) and update the variable $\mathbf{Q}$ accordingly. The proposed

[4]For an optimization problem with a nonconvex constraint set, a stationary point is defined as a KKT point, see [29, Definition 2].

algorithm is summarized in Algorithm 3 and its convergence properties are given in the following theorem.

Theorem 5 (Convergence to a KKT point). *Given a feasible initial point $\mathbf{Q}^0 \in \mathcal{Q}$, the sequence $\{\mathbf{Q}^t\}$ generated by Algorithm 3 has a limit point, and every limit point is a KKT point of problem (4).*

Proof: Although the constraint set $\mathcal{Q}$ of problem (4) is nonconvex, the sequence $\{\mathbf{Q}^t\}$ generated by Algorithm 3 is always feasible. To see this, we check if $\mathbf{Q}^{t+1}$ satisfies the QoS constraint $r_k(\mathbf{Q}^{t+1}) \geq R_k$:

$$r_k(\mathbf{Q}^{t+1}) = r_k(\mathbf{Q}^t + \gamma(\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t))$$
$$\geq r_k^+(\mathbf{Y}^t + \gamma(\mathbb{B}_Y \mathbf{Q}^t - \mathbf{Y}^t))$$
$$- \bar{r}_k^-(\mathbf{Q}^t + \gamma(\mathbb{B}_Q \mathbf{Q}^t - \mathbf{Q}^t); \mathbf{Q}^t)$$
$$\geq (1 - \gamma)(r_k^+(\mathbf{Y}^t) - \bar{r}_k^-(\mathbf{Q}^t; \mathbf{Q}^t))$$
$$+ \gamma(r_k^+(\mathbb{B}_Y \mathbf{Q}^t) - \bar{r}_k^-(\mathbb{B}_Q \mathbf{Q}^t; \mathbf{Q}^t))$$
$$\geq (1 - \gamma)r_k(\mathbf{Q}^t) + \gamma R_k,$$

where the first inequality follows from the fact that $r_k^+(\mathbf{Y}) - \bar{r}_k^-(\mathbf{Q}; \mathbf{Q}^t)$ is a global lower bound of $r_k(\mathbf{Q})$, cf. (36), the second inequality from the concavity of $r_k^+(\mathbf{Y}) - \bar{r}_k^-(\mathbf{Q}; \mathbf{Q}^t)$, and the third inequality from the feasibility of $(\mathbb{B}_Q \mathbf{Q}^t, \mathbb{B}_Y \mathbf{Q}^t)$, i.e., $(\mathbb{B}_Q \mathbf{Q}^t, \mathbb{B}_Y \mathbf{Q}^t) \in \tilde{\mathcal{Q}}(\mathbf{Q}^t)$. Therefore $r_k(\mathbf{Q}^{t+1}) \geq R_k$ if $r_k(\mathbf{Q}^t) \geq R_k$. Since $\mathbf{Q}^0$ is feasible, $\mathbf{Q}^{t+1}$ is feasible by induction.

Since the constraint set $\mathcal{Q}$ is closed and bounded, the sequence $\{\mathbf{Q}^t\}_t$ is bounded and thus has a limit point. The proof for the latter argument follows the same line of analysis as [15, Theorem 1]. ∎

On solving the approximate problem (38). Proposition 4 and Theorem 5 hold for any choice of nonnegative c, even when $c = 0$. Since problem (38) is pseudoconcave, its globally optimal point can be found either by standard gradient-based methods or by the interior-point method proposed in [30].

From now on, we assume $c > 0$. As we will show shortly, the choice of a positive c brings numerical benefits when we apply the Dinkelbach's algorithm to solve problem (38) iteratively. At iteration τ of Dinkelbach's algorithm, the following problem is solved for a given and fixed $s^{t,\tau}$:

$$\underset{\mathbf{Q}, \mathbf{Y}}{\text{maximize}} \quad \sum_{k=1}^K \left(\tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) - c \left\| \mathbf{Y}_k - \mathbf{Y}_k^t \right\|_F^2 \right)$$
$$- s^{t,\tau} \sum_{k=1}^K \tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) \tag{39a}$$

$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \mathrm{tr}(\mathbf{Q}_k) \leq P_k, \tag{39b}$$
$$r_k^+(\mathbf{Y}_k) - \bar{r}_k^-(\mathbf{Q}; \mathbf{Q}^t) \geq R_k, \tag{39c}$$
$$\mathbf{Y}_k = \sum_{j=1}^K \mathbf{H}_{kj} \mathbf{Q}_j \mathbf{H}_{kj}^H, \ k = 1, \ldots, K. \tag{39d}$$

We denote the solution of problem (39) as $(\mathbf{Q}^\star(s^{t,\tau}), \mathbf{Y}^\star(s^{t,\tau}))$. Then $s^{t,\tau}$ is updated as follows:

$$s^{t,\tau+1} = \frac{\sum_{k=1}^K \left(\tilde{r}_{G,k}(\mathbf{Q}_k^\star(s^{t,\tau}); \mathbf{Q}^t) - c \left\| \mathbf{Y}_k^\star(s^{t,\tau}) - \mathbf{Y}_k^t \right\|_F^2 \right)}{\sum_{k=1}^K \tilde{p}_{G,k}(\mathbf{Q}_k^\star(s^{t,\tau}); \mathbf{Q}^t)}. \tag{40}$$

It follows from the convergence properties of the Dinkelbach's algorithm that $\lim_{\tau \to \infty} \mathbf{Q}^\star(s^{t,\tau}) = \mathbb{B}_Q \mathbf{Q}^t$ and

Algorithm 3 The successive pseudoconvex approximation method for GEE maximization with QoS constraints (4)

S0: $\mathbf{Q}^0 \in \mathcal{Q}$, $t = 0$, and a stopping criterion ε.
S1: Compute $\mathbb{B}_Q\mathbf{Q}^t$ by solving problem (38):

> **S1.0:** $s^{t,0} = 0$, $\tau = 0$, and a stopping criterion ϵ.
> **S1.1:** Solve problem (39) to compute $\mathbf{Q}^\star(s^{t,\tau})$ by the following steps:
>
>> **S1.1.0:** $\upsilon = 0$, $\boldsymbol{\lambda} = \mathbf{0}$, $\boldsymbol{\Sigma} = \mathbf{0}$, and a stopping criterion σ.
>> **S1.1.1:** Compute $\mathbf{Q}_k^L(\lambda_k^\upsilon)$ and $\mathbf{Y}_k^L(\boldsymbol{\Sigma}_k^\upsilon)$ by (44) and (45), respectively, for all $k = 1, \ldots, K$.
>> **S1.1.2:** Update λ_k and $\boldsymbol{\Sigma}_k$ by (46) for all k.
>> **S1.1.3:** If $\left\|(\boldsymbol{\lambda}^{\upsilon+1}, \boldsymbol{\Sigma}^{\upsilon+1}) - (\boldsymbol{\lambda}^\upsilon, \boldsymbol{\Sigma}^\upsilon)\right\| \leq \sigma$, then $\mathbf{Q}_k^\star(s^{t,\tau}) = \mathbf{Q}_k^L(\lambda_k^\upsilon)$. Otherwise $\upsilon \leftarrow \upsilon + 1$ and go to **S1.1.1**.
>
> **S1.2:** Compute $s^{t,\tau+1}$ by (40).
> **S1.3:** If $\left\|(\boldsymbol{\lambda}^{\upsilon+1}, \boldsymbol{\Sigma}^{\upsilon+1}) - (\boldsymbol{\lambda}^\upsilon, \boldsymbol{\Sigma}^\upsilon)\right\| \leq \sigma$, then $\mathbf{Q}_k^\star(s^{t,\tau}) = \mathbf{Q}_k^L(\lambda_k^\upsilon)$. Otherwise $\upsilon \leftarrow \upsilon + 1$ and go to **S1.1.1**.

S2: Compute γ^t by the successive line search (15).
S3: Update $\mathbf{Q}$ and $\mathbf{Y}$ by $\mathbf{Q}^{t+1} = \mathbf{Q}^t + \gamma^t(\mathbb{B}_Q\mathbf{Q}^t - \mathbf{Q}^t)$ and $\mathbf{Y}^{t+1} = \mathbf{Y}^t + \gamma^t(\mathbb{B}_Y\mathbf{Q}^t - \mathbf{Y}^t)$, respectively.
S3: If $\|\mathbb{B}\mathbf{Q}^t - \mathbf{Q}^t\| \leq \varepsilon$, then STOP; otherwise $t \leftarrow t + 1$ and go to **S1**.

$\lim_{\tau \to \infty} \mathbf{Y}^\star(s^{t,\tau}) = \mathbb{B}_Y\mathbf{Q}^t$. This iterative procedure (39)-(40) is nested under Step 1 of Algorithm 3.

On solving problem (39). Problem (39) is convex and the coupling constraints have a separable structure, which can readily be exploited in the standard dual decomposition method. To see this, the Lagrangian of (39) is:

$$
\begin{aligned}
L(\mathbf{Q}, \mathbf{Y}, \boldsymbol{\lambda}, \boldsymbol{\Sigma}; \mathbf{Q}^t, \mathbf{Y}^t, s^{t,\tau}) &= \sum_{k=1}^K \tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) \\
&\quad - \sum_{k=1}^K \left(c\left\|\mathbf{Y}_k - \mathbf{Y}_k^t\right\|_F^2 + s^{t,\tau}\tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t)\right) \\
&\quad - \sum_{k=1}^K \boldsymbol{\Sigma}_k \bullet \left(\mathbf{Y}_k - \sum_{j=1}^K \mathbf{H}_{kj}\mathbf{Q}_j\mathbf{H}_{kj}^H\right) \\
&\quad + \sum_{k=1}^K \lambda_k(r_k^+(\mathbf{Y}_k) - \bar{r}_k^-(\mathbf{Q}; \mathbf{Q}^t) - R_k),
\end{aligned}
\tag{41}
$$

where λ_k and $\boldsymbol{\Sigma}_k$ are the Lagrange multipliers associated with the constraints (39c)-(39d). The dual function $d(\boldsymbol{\lambda}, \boldsymbol{\Sigma})$ is

$$
d(\boldsymbol{\lambda}, \boldsymbol{\Sigma}) = \max_{(\mathbf{Q}_k \succeq \mathbf{0}, \mathrm{tr}(\mathbf{Q}_k) \leq P_k, \mathbf{Y}_k \succeq \mathbf{0})_{k=1}^K} L(\mathbf{Q}, \mathbf{Y}, \boldsymbol{\lambda}, \boldsymbol{\Sigma}),
\tag{42}
$$

where the dependence of $L(\mathbf{Q}, \mathbf{Y}, \boldsymbol{\lambda}, \boldsymbol{\Sigma})$ on $(\mathbf{Q}^t, \mathbf{Y}^t, s^{t,\tau})$ is dropped in (42) for notation simplicity. The dual problem of (39) is

$$
\underset{\boldsymbol{\lambda} \geq \mathbf{0}, \boldsymbol{\Sigma}}{\text{minimize}} \quad d(\boldsymbol{\lambda}, \boldsymbol{\Sigma}).
\tag{43}
$$

Since the Lagrangian $L(\mathbf{Q}, \mathbf{Y}, \boldsymbol{\lambda}, \boldsymbol{\Sigma})$ is well decoupled across different variables for fixed dual variable $(\boldsymbol{\lambda}, \boldsymbol{\Sigma})$, the maximization problem in (42) can be decomposed into many smaller optimization problems that can be solved in parallel:

for all $k = 1, \ldots, K$,

$$
\mathbf{Q}_k^L(\boldsymbol{\Sigma}_k, \lambda_k) \triangleq
$$
$$
\underset{\mathbf{Q}_k \succeq \mathbf{0}, \mathrm{tr}(\mathbf{Q}_k) \leq P_k}{\arg\max} \left\{
\begin{array}{l}
\tilde{r}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) - s^{t,\tau}\tilde{p}_{G,k}(\mathbf{Q}_k; \mathbf{Q}^t) \\
-\mathbf{Q}_k \bullet \left(\sum_{j=1}^K \mathbf{H}_{jk}^H \boldsymbol{\Sigma}_k \mathbf{H}_{jk}\right) \\
-\mathbf{Q}_k \bullet \left(\sum_{j \neq k} \lambda_j \nabla_{\mathbf{Q}_k^*} r_j^-(\mathbf{Q}^t)\right)
\end{array}
\right\}
\tag{44}
$$

and

$$
\mathbf{Y}_k^L(\boldsymbol{\Sigma}_k) \triangleq \underset{\mathbf{Y}_k \succeq \mathbf{0}}{\arg\max} \left\{
\begin{array}{l}
\lambda_k \log\det(\sigma_k^2\mathbf{I} + \mathbf{Y}_k) - \boldsymbol{\Sigma}_k \bullet \mathbf{Y}_k \\
-c\left\|\mathbf{Y}_k - \mathbf{Y}_k^t\right\|_F^2
\end{array}
\right\},
\tag{45}
$$

where "L" in the superscript stands for "Lagrangian". Since $c > 0$, $\mathbf{Q}_k^L(\lambda_k)$ in (44) and $\mathbf{Y}_k^L(\boldsymbol{\Sigma}_k)$ in (45) exist and are unique, and they have a closed-form expression, cf. [28, Lem. 2] and [31, Lem. 7].

Remark 6. Since $c > 0$, the optimization problem in (45) is strongly concave, which implies that a solution always exists and it is unique [32]. This is however not necessarily the case when $c = 0$ because the constraint set in (45) is unbounded, and this justifies the quadratic regularization in (32).

The dual problem (43) can be solved by the gradient projection algorithm and its gradient of $d(\boldsymbol{\lambda}, \boldsymbol{\Sigma})$ is

$$
\nabla_{\lambda_k} d(\boldsymbol{\lambda}, \boldsymbol{\Sigma}) = \log\det(\sigma_k^2\mathbf{I} + \mathbf{Y}_k^L(\boldsymbol{\Sigma}_k)) - \bar{r}_k^-(\mathbf{Q}_{-k}^L(\lambda_k)) - R_k,
$$
$$
\nabla_{\boldsymbol{\Sigma}^*} d(\boldsymbol{\lambda}, \boldsymbol{\Sigma}) = \sum_{j=1}^K \mathbf{H}_{kj}\mathbf{Q}_j^L(\lambda_k)\mathbf{H}_{kj}^H - \mathbf{Y}_k^L(\boldsymbol{\Sigma}_k).
$$

In iteration υ to solve problem (43), the dual variable is updated as follows:

$$
\lambda_k^{t,\tau,\upsilon+1} = \left[\lambda_k^{t,\tau,\upsilon} + \zeta^{t,\tau,\upsilon}\nabla_{\lambda_k} d(\boldsymbol{\lambda}^{t,\tau,\upsilon}, \boldsymbol{\Sigma}^{t,\tau,\upsilon})\right]^+, \tag{46a}
$$
$$
\boldsymbol{\Sigma}_k^{t,\tau,\upsilon+1} = \boldsymbol{\Sigma}_k^{t,\tau,\upsilon} + \zeta^{t,\tau,\upsilon}\nabla_{\boldsymbol{\Sigma}^*} d(\boldsymbol{\lambda}^{t,\tau,\upsilon}, \boldsymbol{\Sigma}^{t,\tau,\upsilon}), \tag{46b}
$$

where $\boldsymbol{\lambda}^{t,\tau,0}$ and $\boldsymbol{\Sigma}^{t,\tau,0}$ can be set to $\mathbf{0}$. When $c > 0$ and the stepsizes $\{\zeta^{t,\tau,\upsilon}\}_\upsilon$ are properly selected, e.g., $\sum_\upsilon \zeta^{t,\tau,\upsilon} = \infty$ and $\sum_\upsilon (\zeta^{t,\tau,\upsilon})^2 < \infty$, then $\lim_{\upsilon \to \infty} \mathbf{Q}^l(\lambda_k^{t,\tau,\upsilon}) = \mathbf{Q}^\star(s^{t,\tau})$ and $\lim_{\upsilon \to \infty} \mathbf{Y}^l(\lambda_k^{t,\tau,\upsilon}) = \mathbf{Y}^\star(s^{t,\tau})$ [33, Cor. 28.1.1]. This iterative procedure (44)-(46) is nested under Step 1.1 of Algorithm 3.

The Algorithm 3 consists of three layers: the outer layer with index t, middle layer with index τ, and inner layer with index υ, and $\mathbf{Q}^\star = \lim_{t \to \infty} \lim_{\tau \to \infty} \lim_{\upsilon \to \infty} \mathbf{Q}^l(\lambda_k^{t,\tau,\upsilon})$, where $\mathbf{Q}^\star$ is a KKT point of (4) and the limit with respect to t is in the sense of subsequence convergence specified by Theorem 5. Note that both the middle and inner layers converge very fast (typically convergence is observed after a few iterations), and the proposed algorithm exhibits a faster convergence than state-of-the-art algorithm [11], because the proposed approximate problem can be solved by parallel algorithms based on the dual decomposition and all updates have closed-form expressions, while the approximate problem proposed in [11] must be solved by a generic convex optimization solver. Furthermore, we remark that the state-of-the-art algorithm [11] is not applicable when there is rate-dependent processing-related power consumption, cf. Table I.

VI. THE PROPOSED ALGORITHM FOR SUM ENERGY EFFICIENCY MAXIMIZATION WITH QoS CONSTRAINTS

In this section, we propose an iterative algorithm to maximize the SEE subject to the QoS constraints defined in (5).

Given $\mathbf{Q}^t$ at iteration t, it is tempting to define the approximate function as (24), which is proposed for problem (3), where the approximate function is the sum of multiple component functions $(\tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t))_{k=1}^K$, while each component function $\tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)$ is pseudoconcave in $\mathbf{Q}_k$. However, such an approximate function is not necessarily pseudoconcave, and it cannot be decomposed into multiple independent pseudoconcave optimization problems either, cf. (25)-(26), as the QoS constraints introduce coupling among different optimization variables $(\mathbf{Q}_k)_{k=1}^K$ in the constraint set in problem (5). To overcome this difficulty, we define an approximate function that is concave as concavity is preserved under addition and a concave function is also pseudoconcave.

Firstly, we reformulate problem (5) as follows:

$$\underset{\mathbf{Q},\mathbf{Y}}{\text{maximize}} \quad f_S(\mathbf{Q}) \tag{47a}$$

$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \, \text{tr}(\mathbf{Q}_k) \leq P_k, \tag{47b}$$

$$r_k^+(\mathbf{Y}_k) - r_k^-(\mathbf{Q}_{-k}) \geq R_k, \tag{47c}$$

$$\mathbf{Y}_k = \sum_{j=1}^K \mathbf{H}_{kj}\mathbf{Q}_j\mathbf{H}_{kj}^H, \, \forall k, \tag{47d}$$

On the one hand, we approximate the original objective function $f_S(\mathbf{Q})$ by an approximate function $\tilde{f}_S(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t)$:

$$\tilde{f}_S(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t) = \sum_{k=1}^K \tilde{f}_{S,k}(\mathbf{Q}_k, \mathbf{Y}_k; \mathbf{Q}^t), \tag{48a}$$

$$\tilde{f}_{S,k}(\mathbf{Q}_k, \mathbf{Y}_k; \mathbf{Q}^t) \triangleq \frac{r_k(\mathbf{Q}_k, \mathbf{Q}_{-k}^t)}{p_k(\mathbf{Q}^t)} + (\mathbf{Q}_k - \mathbf{Q}_k^t) \bullet \mathbf{\Pi}_k(\mathbf{Q}^t)$$
$$- c \left\| \mathbf{Y}_k - \mathbf{Y}_k^t \right\|_F^2, \tag{48b}$$

where

$$\mathbf{\Pi}_k(\mathbf{Q}) = -\frac{r_k(\mathbf{Q}^t)}{p_k(\mathbf{Q}^t)^2}\nabla_{\mathbf{Q}_k^*}p_k(\mathbf{Q}^t) + \sum_{j \neq k}\nabla_{\mathbf{Q}_k^*}\left(\frac{r_j(\mathbf{Q}^t)}{p_j(\mathbf{Q}^t)}\right).$$

In contrast to (19), $\tilde{f}_{S,k}(\mathbf{Q}_k, \mathbf{Y}_k; \mathbf{Q}^t)$ defined in (48b) is no longer a fractional function, and it is concave in $(\mathbf{Q}_k, \mathbf{Y}_k)$. Therefore, $\tilde{f}_S(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t)$ is concave in $(\mathbf{Q}, \mathbf{Y})$. Furthermore, its gradient at the point $(\mathbf{Q}, \mathbf{Y}) = (\mathbf{Q}^t, \mathbf{Y}^t)$ is the same as that of the original function $f_S(\mathbf{Q})$:

$$\nabla_{\mathbf{Q}_k^*}\tilde{f}_S(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t)\Big|_{\mathbf{Q}=\mathbf{Q}^t} = \nabla_{\mathbf{Q}_k^*}\tilde{f}_{S,k}(\mathbf{Q}_k, \mathbf{Y}_k; \mathbf{Q}^t)\Big|_{\mathbf{Q}=\mathbf{Q}^t}$$
$$= \frac{\nabla_{\mathbf{Q}_k^*}r_k(\mathbf{Q}^t)}{p_k(\mathbf{Q}^t)} - \frac{r_k(\mathbf{Q}^t)}{p_k(\mathbf{Q}^t)^2}\nabla_{\mathbf{Q}_k^*}p_k(\mathbf{Q}^t) + \sum_{j \neq k}\nabla_{\mathbf{Q}_k^*}\frac{r_j(\mathbf{Q}^t)}{p_j(\mathbf{Q}^t)}$$
$$= \nabla_{\mathbf{Q}_k^*}\tilde{f}_S(\mathbf{Q})\Big|_{\mathbf{Q}=\mathbf{Q}^t},$$

and $\nabla_{\mathbf{Y}_k^*}\tilde{f}_S(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t)\Big|_{\mathbf{Y}=\mathbf{Y}^t} = \mathbf{0} = \nabla_{\mathbf{Y}_k^*}\tilde{f}_S(\mathbf{Q})$.

On the other hand, the nonconvex constraint set in (47) is approximated by its inner approximation $\tilde{\mathcal{Q}}(\mathbf{Q}^t)$ defined in (37). Then in iteration t, the approximate problem consists of maximizing the approximate function $\tilde{f}_S(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t)$ over the

---

Algorithm 4 The successive pseudoconvex approximation method for SEE maximization with QoS constraints (5)

S0: $\mathbf{Q}^0 = \mathbf{0}$, $t = 0$, and a stopping criterion ε.
S1: Compute $(\mathbb{B}_Q\mathbf{Q}^t, \mathbb{B}_Y\mathbf{Q}^t)$ by problem (49):
S2: Compute γ^t by the successive line search (27).
S3: Update $\mathbf{Q}$ and $\mathbf{Y}$ by $\mathbf{Q}^{t+1} = \mathbf{Q}^t + \gamma^t(\mathbb{B}_Q\mathbf{Q}^t - \mathbf{Q}^t)$ and $\mathbf{Y}^{t+1} = \mathbf{Y}^t + \gamma^t(\mathbb{B}_Y\mathbf{Q}^t - \mathbf{Y}^t)$, respectively.
S4: If $\|\mathbb{B}_Q\mathbf{Q}^t - \mathbf{Q}^t\| \leq \varepsilon$, then STOP; otherwise $t \leftarrow t + 1$ and go to **S1**.

---

approximate set $\tilde{\mathcal{Q}}(\mathbf{Q}^t)$:

$$\underset{\mathbf{Q},\mathbf{Y}}{\text{maximize}} \quad \sum_{k=1}^K \tilde{f}_{S,k}(\mathbf{Q}_k; \mathbf{Q}^t)$$

$$\text{subject to} \quad \mathbf{Q}_k \succeq \mathbf{0}, \text{tr}(\mathbf{Q}_k) \leq P_k, r_k^+(\mathbf{Y}_k) - \overline{r}_k^-(\mathbf{Q}_k; \mathbf{Q}^t) \geq R_k,$$
$$\mathbf{Y}_k = \sum_{j=1}^K \mathbf{H}_{kj}\mathbf{Q}_j\mathbf{H}_{kj}^H, \, k = 1, \ldots, K, \tag{49}$$

and let $(\mathbb{B}_Q\mathbf{Q}^t, \mathbb{B}_Y\mathbf{Q}^t)$ denote the optimal point. By following the same line of analysis in Proposition 4, we can show that $\mathbb{B}_Q\mathbf{Q}^t - \mathbf{Q}^t$ is an ascent direction of $f_S(\mathbf{Q})$ at $\mathbf{Q} = \mathbf{Q}^t$, unless $\mathbf{Q}^t$ is already a KKT point of (47). To update the variable, the stepsize could be calculated by the successive line search as explained in (27). Following the same line of analysis in Theorem 5, we could claim that the sequence $\{\mathbf{Q}^t\}$ has a limit point and any limit point is a KKT point of (47).

The above iterative procedure is summarized in Algorithm 4. In Step 1, the convex problem (49) can be solved by parallel algorithms based on the dual decomposition, as its objective function and constraint set have a separable structure (the discussion is similar to that of problem (39) and thus omitted here). Therefore, the complexity of the proposed Algorithm 4 is much lower than [10, Alg. 1], where the nonconvex approximate problem is difficult to solve in each iteration.

VII. SIMULATIONS

In this section, we compare numerically the proposed algorithms with state-of-the-art algorithms. In particular, we consider a 7-user MIMO IC, where the number of transmit antennas is $M_T = 8$ and the number of receive antennas is $M_R = 4$. The power dissipated in hardware is $P_{0,k} = 16$W, and the power budget normalized by the number of transmit antennas and noise covariance is 10dB, i.e., $10\log_{10}(P_k/M_T/\sigma^2) = 10$dB. The inverse of the power amplifier efficiency is $\rho = 2.6$, the noise covariance is $\sigma^2 = 1$, the antenna gain is 16dB, and the path loss exponent is 2. The results are averaged over 20 i.i.d. random channel realizations. All algorithms are tested under identical conditions under Matlab R2017a on a PC equipped with an operating system of Windows 10 64-bit, an Intel i7-7600U 2.80GHz CPU, and a 16GB RAM. All of the Matlab codes are available online at http://orbilu.uni.lu/handle/10993/34787.

A. GEE Maximization

We compare the proposed Algorithm 1 based on the successive pseudoconvex approximation for problem (2) with the successive lower bound minimization (SLBM) algorithm
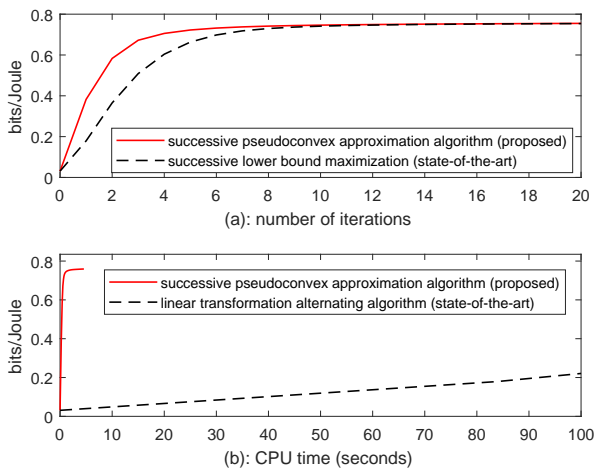
Figure 2. GEE maximization: achieved GEE vs. the number of iterations and CPU time (seconds)

Figure 3. SEE maximization: achieved SEE vs. the number of iterations and CPU time (seconds)

proposed in [11, Prop. 6], which we briefly describe here. At iteration t, $\mathbf{Q}^{t+1}$ is obtained by solving the following problem:

$$\max_{\mathbf{Q} \succeq \mathbf{0}} \left\{ \frac{\sum_{k=1}^{K} (r_k^+(\mathbf{Q}) - r_k^-(\mathbf{Q}^t) - (\mathbf{Q} - \mathbf{Q}^t) \bullet \nabla r_k^-(\mathbf{Q}^t))}{\sum_{k=1}^{K} P_{0,k} + \rho_k \mathrm{tr}(\mathbf{Q}_k)} \right\}, \tag{50}$$

subject to the power constraints ($\mathrm{tr}(\mathbf{Q}_k) \leq P_k$ for all k), where $r_k^+(\mathbf{Q}) \triangleq \log \det(\sigma_k^2 \mathbf{I} + \sum_{j=1}^{K} \mathbf{H}_{kj} \mathbf{Q}_j \mathbf{H}_{kj}^H)$, and this optimization problem is solved iteratively by the Dinkelbach's algorithm. The SLBM algorithm bears its name from the fact that the objective function in (50) is a global lower bound of the original objective function $f_G(\mathbf{Q})$ defined in (2). We do not consider the rate-dependent processing power consumption here because the SLBM algorithm is not applicable otherwise.

As we see from Figure 2 (a), given the same initial point ($\mathbf{Q}^0 = P_T / M_T \mathbf{I}$), both algorithms achieve the same GEE, and the proposed algorithm converges in fewer number of iterations than the SLBM algorithm. In Figure 2 (b), the total CPU time is plotted. Note that all operations are counted, and they consists of solving the approximate problem (13), performing the line search (15), and updating the variable (14), which corresponds to Step **S1**, **S2**, and **S3** of Algorithm 1, respectively. We see from Figure 2 (b), the proposed algorithm needs much less time to converge to a stationary point than that the SLBM algorithm needs. This is because the variable update at each iteration of the proposed algorithm can be implemented in closed-form expressions, while a generic convex optimization problem in the form of (50) must be solved (by CVX [34] in our simulations) for the SLBM algorithm. Finally we remark that the SLBM algorithm cannot handle rate-dependent processing power consumption.

B. SEE Maximization

We compare the proposed Algorithm 2 based on the successive pseudoconvex approximation for problem (3) with the linear transformation alternating (LTA) algorithm proposed in [12, Alg. 1]. Note that we do not consider the rate-
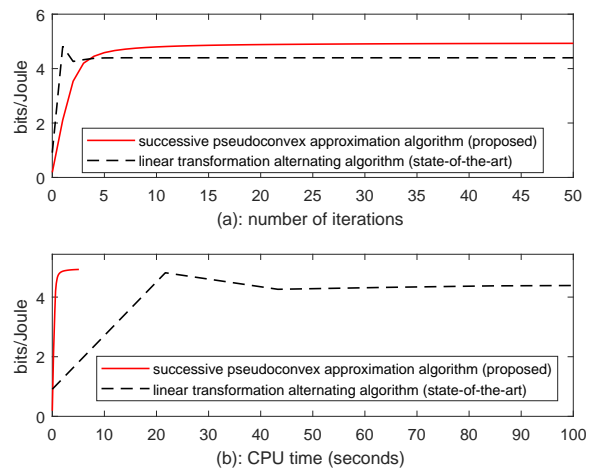
dependent processing power consumption here because the LTA algorithm is not applicable otherwise.

We can draw several observations from Figure 3, where the achieved SEE versus the number of iterations and the total CPU time is plotted, respectively. Firstly, Figure 3 (a) shows that the proposed algorithm achieves a better SEE than the LTA algorithm. Secondly, as we can see from Figure 3 (b), the proposed algorithm converges to the stationary point in less than 1 second and is thus suitable for real time applications. This is because the variable update at each iteration of the proposed algorithm can be implemented in closed-form expressions and the Dinkelbach's algorithm in the inner converges superlinearly. Although the variable updates of the LTA algorithm are also based on closed-form expressions, the inner layer is a BCD type algorithm which suffers from slow asymptotic convergence and typically needs many iterations before convergence. Finally we remark that the LTA algorithm cannot handle rate-dependent processing power consumption.

In Figure 4 we consider the rate-dependent processing power consumption and compare the proposed algorithm with the SCA algorithm in [8, Algorithm 3], where $g_k(x) = 0.1x^2$. Note that the SCA algorithm was originally designed for MISO systems and thus for a single data stream, but it could be extended to the MIMO system by treating the multiple data streams ($\min(M_k, N_k) = 4$ in this example) intended for the same users as interfering streams, which is essentially a MISO IBC. We see from Figure 4 that both the proposed algorithm and the SCA algorithm asymptotically converge to the same SEE, but the proposed algorithm converges in much fewer number of iterations. We also remark that the proposed algorithm has a guaranteed convergence to a stationary point while the SCA algorithm may not necessarily converge. Nevertheless, the SCA algorithm can be implemented in a distributed fashion and is thus more suitable when there is no centralized controller.

C. GEE and SEE maximization with QoS constraints

In this subsection, we test Algorithm 3-4 for problems (4) and (5). In particular, the achieved GEE by Algorithm 3 and
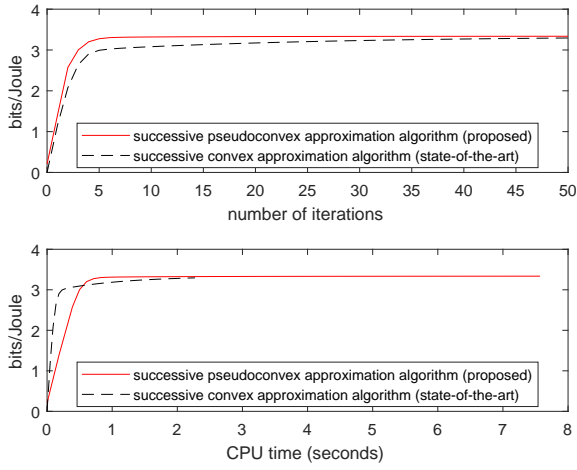
Figure 4. SEE maximization with rate-dependent processing power consumption: achieved SEE vs. the number of iterations and CPU time (seconds)
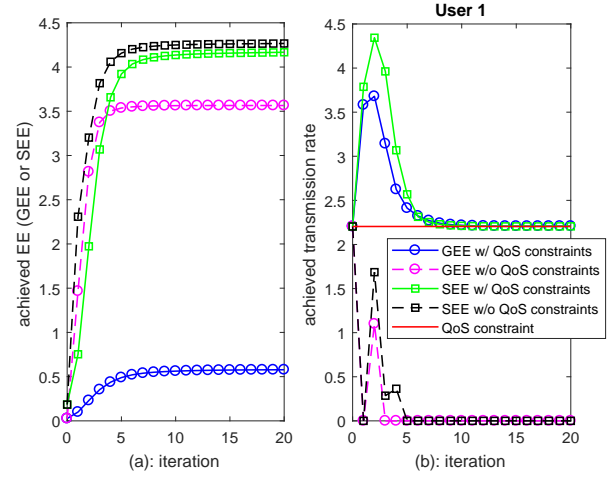

Figure 5. GEE and SEE maximization with QoS constraints: achieved EE and achieved transmission rate of a particular user vs. the number of iterations

the achieved SEE by Algorithm 4 is plotted in Figure 5 (a). As a benchmark, we also plot the achieved GEE by Algorithm 1 and the achieved SEE by Algorithm 2, which are designed for the GEE and SEE maximization problems without QoS constraints, namely, (2)-(3). On the one hand, we can see from Figure 5 (a) that the achieved EE by Algorithms 3-4 is, as expected, monotonically increasing w.r.t. the number of iterations. The achieved GEE/SEE is smaller than that achieved by Algorithm 1/2, because the feasible set of problem (4)/(5) is only a subset of the feasible set of problem (2)/(3). On the other hand, the transmission rate of a particular user is plotted in Figure 5 (b) and we can see that this particular user is guaranteed a minimum transmission rate by Algorithms 3-4, while such a guarantee is not provided by Algorithms 1-2. This is because as long as the QoS constraints are not enforced, the users with bad channel conditions may not be able to transmit in order to maximize the GEE/SEE.

VIII. Concluding Remarks

In this paper, we have proposed novel iterative algorithms based on the successive pseudoconvex approximation framework for the GEE and SEE maximization problem, possibly with nonconcave QoS constraint functions. As we have shown, pseudoconvexity plays a fundamental role, because it enables us to design an approximate function that is not necessarily a global lower bound of the original function. This makes it possible to design new approximate functions that have more flexibility (e.g., rate-dependent processing power consumption) and that can be efficiently optimized. In particular, the proposed algorithms have the following attractive features: 1) fast convergence as the structure of the original optimization problem is preserved as much as possible in the approximate problem solved in each iteration, 2) easy implementation as each approximate problem is suitable for parallel computation and its solution has a closed-form expression, and 3) guaranteed convergence to a stationary point or a KKT point. These advantages of the proposed algorithms are also numerically illustrated.

Appendix

Proof of Proposition 4: Suppose $\mathbf{Q}^t = \mathbb{B}_Q \mathbf{Q}^t$. The Lagrangian of (38) is

$$\tilde{L}(\mathbf{Q}, \mathbf{Y}, \mathbf{\Pi}, \mathbf{\Sigma}, \boldsymbol{\lambda}, \boldsymbol{\mu}; \mathbf{Q}^t, \mathbf{Y}^t)$$
$$= \tilde{f}_G(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t) + \sum_{k=1}^K \mathbf{\Pi}_k \bullet \mathbf{Q}_k^t - \sum_{k=1}^K \lambda_k(\mathrm{tr}(\mathbf{Q}_k) - P_k)$$
$$+ \sum_{k=1}^K \mu_k(r_k^+(\mathbf{Y}_k) - \overline{r}_k^-(\mathbf{Q}_{-k}; \mathbf{Q}^t))$$
$$+ \sum_{k=1}^K \mathbf{\Sigma}_k \bullet (\mathbf{Y}_k - \sum_{j=1}^K \mathbf{H}_{kj} \mathbf{Q}_j \mathbf{H}_{kj}^H),$$

where $(\mathbf{\Pi}, \mathbf{\Sigma}, \boldsymbol{\mu}, \boldsymbol{\lambda})$ are the dual variables. By definition $(\mathbf{Q}^t, \mathbf{Y}^t)$ solves the optimization problem (38). Since $(\mathbf{Q}, \mathbf{Y})$ is a regular point [31], there exists $(\mathbf{\Pi}^t, \mathbf{\Sigma}^t, \boldsymbol{\mu}^t, \boldsymbol{\lambda}^t)$ such that $(\mathbf{Q}^t, \mathbf{Y}^t)$ and $(\mathbf{\Pi}^t, \mathbf{\Sigma}^t, \boldsymbol{\mu}^t, \boldsymbol{\lambda}^t)$ together satisfy the KKT conditions [25, Prop. 4.3.1]:

$$\nabla_{\mathbf{Q}^*} \tilde{L}(\mathbf{Q}^t, \mathbf{Y}^t, \mathbf{\Pi}^t, \mathbf{\Sigma}^t, \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t; \mathbf{Q}^t, \mathbf{Y}^t) = \mathbf{0}, \quad (51)$$

$$\mathbf{0} \preceq \mathbf{\Pi}_k^t \perp \mathbf{Q}_k^t \succeq \mathbf{0}, 0 \leq \mu_k^t \perp \mathrm{tr}(\mathbf{Q}_k^t) - P_k \leq 0, \quad (52)$$

$$\mathbf{Y}_k^t = \sum_{j=1}^K \mathbf{H}_{kj} \mathbf{Q}_j^t \mathbf{H}_{kj}^H, \mathbf{\Sigma}_k^t \bullet (\mathbf{Y}_k^t - \sum_{j=1}^K \mathbf{H}_{kj} \mathbf{Q}_j^t \mathbf{H}_{kj}^H), \quad (53)$$

$$0 \leq \lambda_k^t \perp r_k^+(\mathbf{Y}_k^t) - \overline{r}_k^-(\mathbf{Q}_{-k}^t; \mathbf{Q}^t) - R_k \geq 0, \forall k. \quad (54)$$

Substituting (33) and (35) into (51) yields

$$\nabla_{\mathbf{Q}^*} L(\mathbf{Q}^t, \mathbf{Y}^t, \mathbf{\Pi}^t, \mathbf{\Sigma}^t, \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t) = \mathbf{0}, \quad (55)$$

where $L(\mathbf{Q}, \mathbf{Y}, \mathbf{\Pi}, \mathbf{\Sigma}, \boldsymbol{\lambda}, \boldsymbol{\mu})$ is the Lagrangian of (31):

$$L(\mathbf{Q}, \mathbf{Y}, \mathbf{\Pi}, \mathbf{\Sigma}, \boldsymbol{\lambda}, \boldsymbol{\mu})$$
$$= f_G(\mathbf{Q}) + \sum_{k=1}^K \mathbf{\Pi}_k \bullet \mathbf{Q}_k^t - \sum_{k=1}^K \lambda_k(\mathrm{tr}(\mathbf{Q}_k) - P_k)$$
$$+ \sum_{k=1}^K \mu_k(r_k^+(\mathbf{Y}_k) - r_k^-(\mathbf{Q}))$$
$$+ \sum_{k=1}^K \mathbf{\Sigma}_k \bullet (\mathbf{Y}_k - \sum_{j=1}^K \mathbf{H}_{kj} \mathbf{Q}_j \mathbf{H}_{kj}^H)$$

Similarly, substituting (35) into (54) yields

$$0 \leq \lambda_k \perp r_k^+(\mathbf{Y}_k^t) - r_k^-(\mathbf{Q}^t) - R_k \geq 0, \forall k. \quad (56)$$

Therefore $(\mathbf{Q}^t, \mathbf{Y}^t, \mathbf{\Pi}^t, \mathbf{\Sigma}^t, \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t)$ satisfies the KKT conditions of problem (31), namely, (52), (53), (55) and (56).

If, reversely, there exist $(\mathbf{\Pi}^t, \boldsymbol{\mu}^t, \boldsymbol{\lambda}^t)$ and $(\mathbf{Q}^t, \mathbf{Y}^t)$ satisfying the KKT conditions of problem (31), namely, (52), (53), (55) and (56), we can see that $(\mathbf{\Pi}^t, \boldsymbol{\mu}^t, \boldsymbol{\lambda}^t)$ and $(\mathbf{Q}^t, \mathbf{Y}^t)$ also satisfies the KKT conditions of (38), namely, (51)-(54). Since the objective function in (38) is pseudoconcave and the constraint set $\bar{\mathcal{Q}}(\mathbf{Q}^t)$ is convex, it follows from [24, Th. 10.1.1] that $(\mathbf{Q}^t, \mathbf{Y}^t)$ is an optimal point of (38), i.e., $\mathbb{B}_Q \mathbf{Q}^t = \mathbf{Q}^t$.

If $\mathbb{B}_Q \mathbf{Q}^t \neq \mathbf{Q}^t$, then

$$\tilde{f}_G(\mathbb{B}_Q \mathbf{Q}^t, \mathbb{B}_Y \mathbf{Q}^t; \mathbf{Q}^t) < \tilde{f}_G(\mathbf{Q}^t, \mathbf{Y}^t; \mathbf{Q}^t).$$

Since $\tilde{f}_G(\mathbf{Q}, \mathbf{Y}; \mathbf{Q}^t)$ is pseudoconcave,

$$
\begin{aligned}
0 &< (\mathbb{B}_Q \mathbf{Q}^t - \mathbf{Q}^t) \bullet \nabla_{\mathbf{Q}^*} \tilde{f}_G(\mathbf{Q}^t, \mathbf{Y}^t; \mathbf{Q}^t) \\
&\quad + (\mathbb{B}_Y \mathbf{Q}^t - \mathbf{Y}^t) \bullet \nabla_{\mathbf{Y}^*} \tilde{f}_G(\mathbf{Q}^t, \mathbf{Y}^t; \mathbf{Q}^t) \\
&= (\mathbb{B}_Q \mathbf{Q}^t - \mathbf{Q}^t) \bullet \nabla_{\mathbf{Q}^*} f_G(\mathbf{Q}^t),
\end{aligned}
$$

where the equality follows from (33). Thus $\mathbb{B}_Q \mathbf{Q}^t - \mathbf{Q}^t$ is an ascent direction of $f_G(\mathbf{Q})$ at $\mathbf{Q} = \mathbf{Q}^t$. ∎

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