# Bundling and Joint Marketing by Rival Firms* 

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#### Abstract

We study joint marketing arrangements by competing firms who engage in price discrimination between consumers who patronize only one firm (single purchasing) and those who purchase from both competitors (bundle purchasers). Two types of joint marketing are considered. Firms either commit to a component-price that applies to bundle-purchasers and then firms set stand-alone prices for single purchasers; or firms commit to a rebate off their stand alone price that will be applied to bundle-purchasers, and then firms set their stand alone prices. Both methods allow firms to raise prices and earn higher profits. However, the effect of price discrimination on social welfare depends on how prices are chosen. The rebate joint marketing scheme increases joint purchasing, whereas bundle pricing diminishes bundle purchases. If the marginal social value of a bundle over a single purchase is large, the former increases total welfare. However, welfare can also increase with bundle pricing compared to non-discriminatory pricing.


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## 1 Introduction

Joint marketing arrangements involving separate firms in which customers are charged differentially when they patronize multiple firms are not new. Thus, the unilateral cancellation of a joint marketing agreement that involved discounted pricing to skiers who bought passes to ski resorts run by separate operators gave rise to claims of illegal refusal to deal in the famed Aspen Skiing antitrust suit. ${ }^{1}$ In recent years similar joint marketing arrangements involving separate firms have been on the rise. CityPASS, for example, is a package that bundles multiple tourist attractions in nine popular destinations in North America. The package for Chicago allows admission to five attractions: the Shedd Aquarium, the Field Museum, Skydeck Chicago, either of the Adler Planetarium or the Art Institute of Chicago, and either of the John Hancock Observatory or the Museum of Science and Industry. The attractions compete against each other for time-constrained travelers who cannot visit more than a few places. At the same time, the participating venues offer discounted pricing by selling the CityPASS. ${ }^{2,3}$

In this paper we investigate the incentives and welfare implications of instances in which companies choose pricing strategies that target consumers who make joint purchases across firms. The firms offer horizontally differentiated products and consumers view the firms' goods as imperfect substitutes. However, each firm's product has unique features and attributes that give a consumer who has already purchased a unit an added utility from buying the competing product as well, and so consumers are endogenously divided into two groups. While some consumers purchase a single product from either firm, others purchase both products.

We show that firms are able to leverage their joint marketing schemes into higher prices and higher profits compared to both uniform pricing and independent price discrimination. However, the mechanism through which prices and profits are raised depend on the nature of the joint marketing scheme used. When firms market to joint purchasers by each setting a price for their contribution to the bundle (bundle pricing), firms commit to a high bundle price, which drives consumers into single-purchasing. This enables the firms to capture more surplus from singlepurchasing customers. In contrast, if joint marketing takes the form of each firm setting a rebate

[^1]offer that applies to the the stand-alone price when a consumer makes a joint purchase, a generous rebate is offered. This draws consumers into joint purchasing. The increased demand for the bundle is reinforced by charging high stand alone prices, which yields higher profits because the fixed rebate then applies to a high price.

Welfare is affected differently in the two cases. If incremental values associated with purchasing a second unit are high, then the rebate scheme which induces more joint purchasing increases welfare; whereas bundle pricing, which reduces joint purchasing, tends to decrease welfare. However, there are also more subtle welfare implications of joint marketing. When a firm lowers its price to bundle-purchasers then this has both a demand-inducing effect in that some new customers are attracted to the firm; as well as a demand-shifting effect, as existing customers of the firm obtain the second good in order to secure an effective price reduction on the first good. An implication of the latter effect is that consumption decisions are induced that do not lead to increases in welfare, so there are situations in which joint marketing reduces total welfare regardless of which scheme is used.

There is an extensive literature on bundling for the purpose of price discrimination. ${ }^{4}$ Early papers in this literature, however, restrict attention to the case where bundle discounts are offered by a multi-product monopolist (e.g., Adams and Yellen, 1976; McAfee et al., 1989; Armstrong, 1996; Rochet and Choné, 1998), rather than independent firms. Gans and King (2006) are the first to study the situation where bundle discounts are offered by different firms through joint marketing. In contrast to our setting, the two products sold are independent, yet all consumers must purchase both goods so that there are no single purchasers. They show that the unilateral bundling by the pair of firms against other firms is profitable, whereas bundle rebates by both pairs of firms do not increase their profits. At the same time, mutual joint marketing diminishes social welfare substantially. In our setting, with firms offering partial substitutes, price discrimination through joint marketing is always profitable and its impact on social welfare depends on the mechanics of the joint marketing. ${ }^{5}$

The most closely related paper to ours is Armstrong (2013), who studies incentives to offer bundled discounts by separate sellers in a very general setup. He shows that when product valuations are negatively correlated and/or sub-additive (so that products are partial substitutes), firms benefit by offering independent discounts to consumers. He extends the analysis to show that co-

[^2]ordinated discounts implemented by firms reduce competition by mitigating the substitutability of products and reduce total welfare. We also focus on sub-additive and negatively correlated values; but we consider asymmetric firms and contrast discounts with a policy of firms committing to prices for their contribution to the bundle. Compared to setting discounts, firms committing to bundle prices before setting stand-alone prices actually increases profits. The discount policy induces more purchases, however, and total welfare is tied to total consumption whenever goods are not too closely substitutable so that welfare can increase due to joint marketing.

There is also a nascent literature on the impact of joint purchases to which our paper contributes. Gabszewicz and Wauthy (2003) analyze how joint purchases affect price competition and show that various types of equilibria arise depending on the value of incremental utilities from joint purchasing. Kim and Serfes (2006) and Anderson et al. (2012) extend Gabszewicz and Wauthy (2003) by investigating joint purchasing in a horizontally differentiated market. Kim and Serfes (2006) ask under what conditions the "Principle of Minimum Differentiation" is restored when firms choose their location on the Hotelling line; and Anderson et al. (2012) find a nonmonotonic relationship between equilibrium prices and qualities under joint purchasing. There, the additional gain by joint purchasing is valued more by closer consumers so firms have an incentive to sacrifice some sales and set high prices to prevent joint purchases. In contrast to our work, these papers abstract from price discrimination as a motivation for joint marketing. ${ }^{6}$

Somewhat related to joint marketing arrangements are several other recent papers on joint pricing in the context of patents and patent pooling (e.g., Lerner and Tirole, 2004; Cheng and Nahm, 2007; Choi, 2010; Rey and Tirole, 2013; Jeitschko and Zhang, 2014). While this work generally does not consider price discrimination, Lerner and Tirole (2004) and Rey and Tirole (2013) examine how individually set royalty rates interact with pricing in patent pools. Our findings shed further light on this issue and provide additional insights as in our setting we allow for the firms' contributions to the bundle to asymmetric and we consider alternative joint marketing structures.

The paper is organized as follows. In Section 2 we introduce the base model and solve for equilibrium uniform prices for both single and joint purchasing regimes and also study independent price discrimination. Section 3 contains the two forms of joint marketing: bundle pricing and bundle rebates; and draws some initial comparisons to uniform pricing and joint price discrimination in terms of prices and consumption patterns. Profits, consumer surplus, and total welfare are

[^3]presented in Section 4, which then also establishes the main findings between the different welfare and consumption implications of the pricing schemes. The final section concludes. Detailed derivations and proofs are relegated to the Appendix .

## 2 The Model and the Benchmarks

We consider two firms that offer partially substitutable products. Regardless of which firm's product is purchased, when consumers make a purchase, the base utility from consuming the product is given by $V$. However, firms are asymmetric in that each firm's product has idiosyncratic features that are only obtained when purchasing that particular product. We let $v_{i}$ denote the consumer's gross added value from using product $i$ 's unique features. That is, a consumer who purchases product $i$ obtains the base value of having purchased an item $(V)$ and then the added marginal value of product $i$ 's features $\left(v_{i}\right)$ for a total (gross) utility of $V+v_{i}$. In contrast, a consumer who purchases both products receives the base utility and then each product's additional marginal value for a total (gross) utility of $V+v_{i}+v_{j}$. For example, PlayStation offers free online games, whereas Wii is more family friendly and has more games that are suitable for childrenfamilies who care about both of these attributes may purchase both systems, whereas others may be satisfied with simply having a high-end gaming system.

We further assume that the two products are horizontally differentiated, e.g., some have an intrinsic preference for one product over the other (prefer the Wii over PlayStation or vice versa), which we capture through an extension of the Hotelling (1929) model. Thus, when a consumer located at $x \in[0,1]$ purchases product 1 only, her payoff is given by $U_{1}\left(x ; p_{1}\right) \equiv V+v_{1}-t x-p_{1}$, where $t$ denotes the (linear) transportation costs. When she purchases product 2 only, her payoff is given by $U_{2}\left(x ; p_{2}\right) \equiv V+v_{2}-t(1-x)-p_{2}$. And because the common attributes of the product is captured by $V$, the payoff from a joint purchase is given by $U_{12}\left(p_{12}\right) \equiv V+v_{1}+v_{2}-t-p_{12}$, where $p_{12}$ is the price paid by a consumer who purchases products 1 and 2 together. ${ }^{7}$

Note that as the idiosyncratic value from either of the two products increases, more consumers undertake a joint purchase-all else equal. This can lead to a corner solution, in which all consumers purchase both products. Also, if the incremental values of the idiosyncratic characteristics are too small, then an equilibrium in which some consumers purchase more than one unit my fail to exist. To rule out these trivial cases we make the following assumption that holds throughout

[^4]the paper.
Assumption 1. $v_{i} \in\left(t-v_{j}, 2 t\right), i, j=1,2$ and $i \neq j$.

### 2.1 Uniform Pricing and Joint Purchasing

We begin by assuming uniform pricing, so consumers who purchase from both firms pay a price of $p_{12}=p_{1}+p_{2}$.

Letting $\widehat{x_{i}}$ denote a consumer who is indifferent between buying from firm $i$ only and buying from both firms the indifference condition $U_{i}\left(\widehat{x_{i}} ; p_{i}\right)=U_{12}\left(p_{12}\right)$ provides $\widehat{x_{1}}=1-\left(v_{2}-p_{2}\right) / t$ and $\widehat{x_{2}}=\left(v_{1}-p_{1}\right) / t$. This implies that the mass of consumers who buy good $i$ only is

$$
n_{i}=1-\frac{v_{j}-p_{j}}{t}, \quad i=1,2
$$

and, assuming a positive measure of joint purchasers, the mass of consumers who buy both goods is given by

$$
n_{12}=\widehat{x_{2}}-\widehat{x_{1}}=\frac{v_{1}+v_{2}-p_{1}-p_{2}}{t}-1 .
$$

This gives each firm's demand function as

$$
q_{i}\left(p_{i}, p_{j}\right)= \begin{cases}\frac{1}{2}+\frac{1}{2 t}\left(v_{i}-v_{j}+p_{j}-p_{i}\right) & \text { if } p_{i} \geq v_{i}+v_{j}-t-p_{j}  \tag{1}\\ \frac{1}{t}\left(v_{i}-p_{i}\right) & \text { if } p_{i} \leq v_{i}+v_{j}-t-p_{j}\end{cases}
$$



Figure 1: Firm $i$ 's inverse demand function

Note that demand has an inward kink at $v_{i}+v_{j}-t-p_{j}$ (see Figure 1). Above the kink prices are so high that there are no joint purchasers, yielding the standard Hotelling model. Below the kink prices are low enough to have joint purchasers. Here consumers are more price sensitive (demand is flatter), because the value-added of a second unit lies below the value of purchasing the first unit. Note also that the other firm's price is irrelevant on this segment and each firm
behaves as a monopolist with regard to providing their idiosyncratic value $v_{i}$-this is because when pricing to the joint-purchasers on the margin, there is no business stealing from the rival.

Given demand, firm $i$ 's profit function is $\pi_{i}\left(p_{i}, p_{j}\right)=p_{i} q_{i}\left(p_{i}, p_{j}\right)$; the Appendix shows that this yields best responses of

$$
\phi_{i}\left(p_{j}\right)= \begin{cases}\frac{1}{2}\left(t+v_{i}-v_{j}+p_{j}\right) & \text { if } p_{i} \geq \hat{p}_{j}=(\sqrt{2}-1) v_{i}+v_{j}-t \\ \frac{1}{2} v_{i} & \text { if } p_{i} \leq \hat{p}_{j}=(\sqrt{2}-1) v_{i}+v_{j}-t\end{cases}
$$

The key implication of the kink in demand is that each firm's marginal revenue is non-monotonic, and therefore the first-order condition may be satisfied twice-possibly permitting two different pricing strategies: pricing high to compete for single purchasers with less elastic demand; or pricing low to attract joint purchasers-whose demand is more elastic. As a result, firms' best response correspondences are not continuous. Figure 2 depicts the three possible configurations.


Figure 2: Best response correspondences (here: $v_{1}>v_{2}$ )

The following proposition gives the equilibrium configurations, the proof is in the Appendix.
Proposition 1. Let $\Phi^{S}:=\left\{\left(v_{1}, v_{2}\right) \mid(3-\sqrt{2}) v_{1}+\sqrt{2} v_{2} \leq 3 \sqrt{2} t\right.$ and $\left.(3-\sqrt{2}) v_{2}+\sqrt{2} v_{1} \leq 3 \sqrt{2} t\right\}$, and $\Phi^{I}:=\left\{\left(v_{1}, v_{2}\right) \left\lvert\,(\sqrt{2}+1)\left(t-\frac{1}{2} v_{2}\right) \leq v_{1} \leq 2 t\right.\right.$ and $\left.(\sqrt{2}+1)\left(t-\frac{1}{2} v_{1}\right) \leq v_{2} \leq 2 t\right\}$; then if
(i) $\left(v_{1}, v_{2}\right) \in \Phi^{S}$, there is a single-purchasing regime in which

$$
\begin{gathered}
p_{i}^{U_{S}}=\frac{1}{3}\left(3 t+v_{i}-v_{j}\right) \\
n_{i}^{U_{S}}=\frac{1}{6 t}\left(3 t+v_{i}-v_{j}\right) \quad \text { and } \quad n_{12}^{U_{S}}=0, \\
\pi_{i}^{U_{S}}=\frac{1}{18 t}\left(3 t+v_{i}-v_{j}\right)^{2} ;
\end{gathered}
$$

(ii) $\left(v_{1}, v_{2}\right) \in \Phi^{I}$, there is a regime with joint purchases in which

$$
\begin{gathered}
p_{i}^{U_{J}}=\frac{1}{2} v_{i} \quad \text { and } \quad p_{12}^{U_{J}}=\frac{1}{2}\left(v_{i}+v_{j}\right) \\
n_{i}^{U_{J}}=\frac{1}{2 t}\left(2 t-v_{j}\right) \quad \text { and } \quad n_{12}^{U_{J}}=\frac{1}{2 t}\left(v_{1}+v_{2}-2 t\right), \\
\pi_{i}^{U_{J}}=\frac{v_{i}^{2}}{4 t} .
\end{gathered}
$$

As shown in Figure 3, when the additional gain from a second purchase is small, i.e., $\left(v_{1}, v_{2}\right) \in$ $\Phi^{S} \backslash \Phi^{J}$, there is a unique equilibrium in which firms charge high prices to single purchasers. As idiosyncratic values increase, $\left(v_{i}, v_{j}\right) \in \Phi^{S} \cap \Phi^{I}$, firms may price low to attract joint-purchasers. However, the reduction in profit from single purchasers due to the lower prices is greater than the gain from additional sales so, assuming coordination on the preferred equilibrium, firms keep prices high and only target single-purchasers. Lastly, when a joint purchase adds a large additional gain, $\left(v_{1}, v_{2}\right) \in \Phi^{J} \backslash \Phi^{S}$, there is again a unique equilibrium as secondary customers are an attractive prospect and firms lower their prices to capture these consumers.


Figure 3: Single- and Joint-Purchasing Configurations under Uniform Pricing

### 2.2 Joint Purchasing and Independent Price Discrimination

Customers who are located close to a firm purchase from that firm; and those far removed from this firm will not purchase from the firm. Thus, the purchase decision reveals something about the consumer's location on the line. Firms can use this information to segment the market and price discriminate even when they do not engage in joint marketing efforts. This scenario yields
the second benchmark.
Specifically, suppose that firms provide a rebate $\rho_{i}$ off their price to consumers who buy the rival's good-i.e., who purchase both goods. The joint purchaser's price is $p_{12}=\left(p_{1}-\rho_{1}\right)+\left(p_{2}-\right.$ $\left.\rho_{2}\right)$, yielding a net payoff of $U_{12}\left(p_{12}\right)=V+v_{1}+v_{2}-t-\left(p_{1}-\rho_{1}\right)-\left(p_{2}-\rho_{2}\right)$.

The locations of the marginal consumers follow from $U_{i}\left(x_{i} ; p_{i}\right)=U_{12}\left(p_{12}\right)$, implying that $n_{i}=1-\left(v_{j}-p_{j}+\rho_{i}+\rho_{j}\right) / t$ and $n_{12}=\left(v_{1}+v_{2}-p_{1}-p_{2}+2 \rho_{1}+2 \rho_{2}\right) / t-1$.

Firm $i$ 's profit maximization problem is given by

$$
\begin{equation*}
\max _{p_{i}, \rho_{i}} \pi_{i}=p_{i} n_{i}+\left(p_{i}-\rho_{i}\right) n_{12} . \tag{2}
\end{equation*}
$$

While the problem entails a more complex pricing strategy than uniform pricing, the solution is in some sense easier in that the first order conditions are necessary and sufficient and there is a unique equilibrium.

Proposition 2. Under independent price discrimination,

$$
\begin{gathered}
p_{i}^{I P D}=\frac{1}{3}\left(3 t+v_{i}-v_{j}\right), \quad \rho_{i}^{I P D}=\frac{1}{3}\left(2 t-v_{j}\right) \quad \text { and } \quad \rho_{12}^{I P D}=\frac{1}{3}\left(2 t+v_{1}+v_{2}\right) \\
n_{i}^{I P D}=\frac{1}{3 t}\left(2 t-v_{j}\right) \quad \text { and } \quad n_{12}^{I P D}=\frac{1}{3 t}\left(v_{1}+v_{2}-t\right), \\
\pi_{i}^{I P D}=\frac{1}{9 t}\left(v_{i}^{2}+v_{j}^{2}+2 t v_{i}-4 t v_{j}+5 t^{2}\right) .
\end{gathered}
$$

Note that firm $i$ 's choice of the bundle rebate is independent of $v_{i}$ and is decreasing in $v_{j}$, implying that as firm $j$ 's unique features are more valuable, firm $i$ can reduce the bundle rebate, as the rival product's attractiveness serves to increase the demand.

Compared to uniform pricing, the stand alone price under independent price discrimination is identical to the equilibrium price in the single purchasing regime. That is, the price charged to the relatively inelastic (i.e., the 'captured') consumers is the same as when only these are targeted. This follows readily, since the margin on which this price operates is the same across the two cases.

On the other hand, the price paid by joint-purchasers under uniform pricing is lower than the price paid by joint-purchasers under independent price discrimination, i.e., $p_{i}^{U_{J}}=v_{i} / 2<$ $\left(2 t+v_{1}+v_{2}\right) / 3=p_{12}^{I P D}$. This is because when the firm price discriminates and lowers its price to joint purchasers, it cannibalizes its high-profit sales to inelastic consumers who otherwise single-purchase-this limits the amount the firm is willing to lower the price to joint purchasers compared to the case of uniformly low prices (that is, $n_{i}$ is decreasing in the rebate $\rho_{i}$ ).

An implication of consumers being able to obtain rebates upon purchasing the second product is that despite the bundle price being higher than the uniform price under joint purchasing, the
mass of consumers who joint-purchase increases when the firms price discriminate. This can be seen by comparing $n_{12}^{I P D}=\left(v_{1}+v_{2}-t\right) / 3 t$ to $n_{12}^{U_{J}}=\left(v_{1}+v_{2}-2 t\right) / 2 t$ and $n_{12}^{U_{S}}=0$. That is, the high stand alone price pushes people into making a joint purchase in order to obtain the price break.

Thus, independent price discrimination leads to (some) joint purchasing and increased profits when otherwise there would only be single purchasing under uniform pricing. Moreover, independent price discrimination also raises profits when under uniform pricing there are some joint purchasers, because both single purchasers and joint-purchasers now face higher prices, and there are more purchases in total as some consumers shift from single to joint purchasing to avoid the higher non-discounted price that single purchasers face.

This result is similar to one in Armstrong (2013, Proposition 3). He shows that if there are separate sellers, then each seller has an unilateral incentive to offer bundle discounts for consumers who buy goods from both sellers whenever the demand for the bundle product is more elastic than total demand for one firm's product.

## 3 Bundling and Joint Marketing

Consider now joint marketing. Firms act non-cooperatively in their pricing decisions so joint marketing does not take the form of price collusion. Joint marketing is done in two stages, with the firms first committing to their pricing strategies vis-á-vis joint purchasers, and then setting their stand alone prices in light of the offer that is made to joint purchasers.

We consider two types of joint marketing schemes. First we suppose that firms non-cooperatively set the price for their product in the bundle, $\widetilde{p}_{i}$, and then choose stand-alone prices $p_{i}$ in light of the bundle price of $p_{12}^{P} \equiv \widetilde{p}_{1}+\widetilde{p}_{2}$. The second scheme we consider is a rebate scheme in which firms first announce a rebate offer $\rho_{i}$, and then choose the stand alone price in light of the bundle rebate of $\rho:=\rho_{i}+\rho_{j} .{ }^{8}$

In principle either scheme might be implemented by the firms, possibly through a third party facilitator, or, for instance, through marketing on the internet. In practice, however, it could be that for logistical reasons either one or the other scheme is more readily instituted.

[^5]
### 3.1 Bundle Price Marketing

With bundle pricing, each firm first chooses the price $\widetilde{p}_{i}$ for its contribution to the bundle. This yields the bundle price $p_{12}^{P} \equiv \widetilde{p}_{1}+\widetilde{p}_{2}$ that will be marketed to consumers. The firms' stand-alone prices $p_{i}$ are then set in light of this bundle price. Given the choice of prices, consumers make their purchasing decisions, with a firm receiving $\widetilde{p}_{i}$ for each bundle that is sold, and obtaining $p_{i}$ from consumers that purchase only firm $i$ 's product.

Facing a bundle price of $p_{12}^{P}=\widetilde{p}_{1}+\widetilde{p}_{2}$, a joint purchaser's net payoff is $U_{12}\left(p_{12}^{P}\right) \equiv V+v_{1}+$ $v_{2}-t-p_{12}^{P}$, resulting in $n_{i}^{P}=1-\left(v_{j}-p_{12}^{P}+p_{i}\right) / t$ and $n_{12}^{P}=\left(v_{1}+v_{2}+p_{1}+p_{2}-2 p_{12}^{P}\right) / t-1$. We assume that $n_{12}^{P} \geq 0$, which yields the candidate pricing structure. We then confirm the conditions on the candidate pricing structure that entails a positive measure of joint purchasers.

Given the bundle price $p_{12}^{P}$, each firm chooses $p_{i}$ to maximize

$$
\max _{p_{i}} \pi_{i}^{P}=p_{i} n_{i}^{P}+\widetilde{p}_{i} n_{12}^{P} .
$$

From the first-order conditions, each firm's stand-alone price is:

$$
p_{i}\left(\widetilde{p}_{i}, \widetilde{p}_{j}\right)=\frac{\left(t-v_{j}\right)+2 \widetilde{p}_{i}+\widetilde{p}_{j}}{2} .
$$

Note that $p_{i}$ is increasing in $\widetilde{p}_{i}$ and $\widetilde{p}_{j}$. Thus, commitment to the component price leads to raising the stand-alone price. Moreover, $p_{i}$ rises with $t$ and falls with $v_{j}$, because an increase in $t$ and a decrease in $v_{j}$ make demand for firm $i$ 's product less elastic.

In anticipation of the stand alone prices as a function of the bundle price, the firms price their individual contribution to the bundle:

$$
\max _{\widetilde{p}_{i}} \pi_{i}^{P}=\frac{t-v_{j}+2 \widetilde{p}_{i}+\widetilde{p}_{j}}{2} \cdot \frac{t-v_{j}+\widetilde{p}_{j}}{2 t}+\widetilde{p}_{i} \cdot \frac{v_{i}+v_{j}-\widetilde{p}_{i}-\widetilde{p}_{j}}{2 t} .
$$

Joint purchases take place provided that the idiosyncratic values of the two firms are sufficiently high, yielding the following equilibrium.

Proposition 3. When $v_{i}+v_{j} \geq 2 t$, there exits a joint marketing equilibrium in which firms set bundle
component prices with

$$
\begin{gathered}
p_{i}^{P}=\frac{1}{4}\left(5 t+2 v_{i}-v_{j}\right), \quad \widetilde{p}_{i}=\frac{1}{2}\left(t+v_{i}\right) \quad \text { and } \quad p_{12}^{P}=\frac{1}{2}\left(2 t+v_{1}+v_{2}\right) \\
n_{i}^{P}=\frac{1}{4 t}\left(3 t-v_{j}\right) \quad \text { and } \quad n_{12}^{P}=\frac{1}{4 t}\left(v_{1}+v_{2}-2 t\right) \\
\pi_{i}^{P}=\frac{1}{16 t}\left(11 t^{2}+2\left(2 v_{i}-3 v_{j}\right) t+\left(2 v_{i}^{2}+v_{j}^{2}\right)\right)
\end{gathered}
$$

Compared to the case of uniform pricing with joint purchasers the price paid by joint purchasers is now higher, i.e., $p_{12}^{P}>p_{12}^{U_{J}}$; and so is the price paid by single purchasers when compared to the case of uniform pricing with only single purchasing, $p_{i}^{P}>p_{i}^{U_{s}}$, (which, of course, is higher than the uniform joint purchasing price, i.e., $p_{i}^{U_{S}}>p_{i}^{U_{J}}$ ). That is, when jointly marketing the bundle price, both prices (those applied to single purchasers and those applied to joint purchasers) are above the the uniform prices-despite all prices being set non-cooperatively.

The fact that the bundle price is higher under joint marketing suggests that fewer consumers purchase the bundle. However, the fact that the stand alone price is also high, makes the bundle attractive in that it leads to a price discount. The former effect dominates the latter so that fewer joint purchases take place when compared to the case of uniform pricing with joint purchases, i.e., $n_{12}^{P}=\left(v_{1}+v_{2}-2 t\right) / 4 t<n_{12}^{U_{J}}=\left(v_{1}+v_{2}-2 t\right) / 2 t$, given $v_{1}+v_{2} \geq 2 t$; (naturally, more joint purchases take place when compared to uniform pricing with no joint purchases).

Intuitively, by committing to high bundle component prices the firms push demand towards single purchasing and use this increased demand to charge higher stand alone prices. Overall then, profits are greater compared to either of the uniform pricing cases, i.e., $\pi_{i}^{P}>\pi_{i}^{U_{S}}, \pi_{i}^{U_{J}}$.

Consider now the comparison to independent price discrimination. Writing $\widetilde{p}_{i}=p_{i}-\rho_{i}$, the independent price discrimination optimization problem for firm $i$, Equation (2), is:

$$
\max _{\widetilde{p}_{i}, p_{i}} \pi_{i}^{I P D}\left(p_{i}, p_{j}, \widetilde{p}_{i}, \widetilde{p}_{j}\right)
$$

The first-order condition with respect to the bundle component price is

$$
\begin{equation*}
\frac{\partial \pi_{i}^{I P D}}{\partial \widetilde{p}_{i}}=p_{i} \frac{\partial n_{i}}{\partial \widetilde{p}_{i}}+n_{12}+\widetilde{p}_{i}^{I P D} \frac{\partial n_{12}}{\partial \widetilde{p}_{i}}=0 \tag{3}
\end{equation*}
$$

where $\widetilde{p}_{i}^{I P D}=p_{i}^{I P D}-\rho_{i}^{I P D}$.
In contrast, with the joint marketing scheme, firm $i^{\prime}$ s choice of its bundle component price is implied by

$$
\max _{\widetilde{p}_{i}} \pi_{i}^{P}\left(p_{i}^{P}\left(\widetilde{p}_{i}, \widetilde{p}_{j}\right), p_{j}^{P}\left(\widetilde{p}_{i}, \widetilde{p}_{j}\right), \widetilde{p}_{i}, \widetilde{p}_{j}\right)
$$

From the envelope theorem, the effect of changes in the stand alone price $p_{i}$ on $\pi_{i}^{P}$ is of second order $\left(\partial \pi_{i} / \partial p_{i}=0\right)$, so the optimal bundle price component of firm $i, \widetilde{p}_{i}$, satisfies the following first-order condition:

$$
\begin{align*}
\frac{d \pi_{i}}{d \widetilde{p}_{i}} & =\frac{\partial \pi_{i}}{\partial \widetilde{p}_{i}}+\frac{\partial \pi_{i}}{\partial p_{i}} \frac{\partial p_{i}^{P^{\prime \prime}}}{\partial \widetilde{p}_{i}}+\frac{\partial \pi_{i}}{\partial p_{j}} \frac{\partial p_{j}^{P}}{\partial \widetilde{p}_{i}} \\
& =\underbrace{\left[p_{i} \frac{\partial n_{i}}{\partial \widetilde{p}_{i}}+n_{12}+\widetilde{p}_{i} \frac{\partial n_{12}}{\partial \widetilde{p}_{i}}\right]}_{\text {Direct Effect }}+\underbrace{\left[p_{i} \frac{\partial n_{i}^{i}}{\partial p_{j}}+\widetilde{p_{i}} \frac{\partial n_{12}}{\partial p_{j}}\right] \frac{\partial p_{j}^{P}}{\partial \widetilde{p}_{i}}}_{\text {=Indirect Effect }}=0 . \tag{4}
\end{align*}
$$

The first term captures the direct effect that firm $i$ 's component price has on profits. It mirrors the first order condition that the firm has in independent price discrimination, namely Equation (3). The second term is the indirect effect that firm $i$ 's component price has on profits by affecting firm $j$ 's stand alone price that is set in light of the bundle prices. The indirect effect is positive, because committing to a bundle component price leads to the rival raising its stand-alone price. Evaluating the first-order condition (4) at $\widetilde{p}_{i}=\widetilde{p}_{i}^{I P D}$ yields

$$
\left.\frac{d \pi_{i}}{d \widetilde{p}_{i}}\right|_{\tilde{p}_{i}=\tilde{p}_{i}^{I P D}}=\left.\widetilde{p}_{i} \frac{\partial n_{12}}{\partial p_{j}} \frac{\partial p_{j}^{P}}{\partial \widetilde{p}_{i}}\right|_{\tilde{p}_{i}=\tilde{p}_{i}^{I P D}}>0
$$

which implies that $\widetilde{p}_{i}^{P}>\widetilde{p}_{i}^{I P D}$.
Thus, the firms charge a higher bundle price when they commit to the bundle component prices prior to the stand-alone prices. As a consequence, the mass of joint purchaser is smaller in the bundle price commitment case than in the case of independent price discrimination, i.e., $n_{12}^{P}<$ $n_{12}^{I P D}$. This makes single-purchasing relatively more attractive and allows the firms to raise the stand alone price as well, $p_{i}^{P}>p_{i}^{I P D}$, and so profits on each unit sold exceed those of independent price discrimination, leading to an overall increase in profit, $\pi_{i}^{P}>\pi_{i}^{I P D}$.

### 3.2 Rebate Marketing

In the case of non-cooperative joint rebate marketing, each firm determines a discount or rebate of $\rho_{i}$ that will be applied to their stand alone price to any consumer who purchases both goods. Together the discounts constitute a rebate offer of $\rho:=\rho_{i}+\rho_{j}$ that will be jointly marketed to consumers. The firms' stand alone prices are then set in light of the rebate offer. Joint purchasers pay a price of $p_{12}^{R} \equiv p_{i}+p_{j}-\rho$ so that firm $i$ receives a net price of $p_{i}-\rho_{i}$ for its own product in the bundle, and obtains $p_{i}$ from single purchasers.

Given the joint marketing rebate of $\rho$, a joint purchaser's net payoff is $U_{12}\left(p_{12}^{R}\right) \equiv V+v_{1}+v_{2}-$ $t-\left(p_{1}+p_{2}-\rho\right)$ implying $n_{i}^{R}=1-\left(v_{j}-p_{j}+\rho\right) / t$ and $n_{12}^{R}=\left(v_{1}+v_{2}-p_{1}-p_{2}+2 \rho\right) / t-1$, with joint purchasing increasing as the rebate offer $\rho$ increases.

In light of the joint marketing rebate $\rho$, firm $i$ solves

$$
\max _{p_{i}} \pi_{i}^{R}=p_{i} n_{i}^{R}+\left(p_{i}-\rho_{i}\right) n_{12}^{R}
$$

where $\rho_{i}$ can be thought of as the cost of having a product in the bundle. The first-order condition yields the individual firm's stand-alone price

$$
p_{i}\left(\rho_{i}, \rho_{j}\right)=\frac{v_{i}+2 \rho_{i}+\rho_{j}}{2}
$$

To compare with the benchmark case of uniform pricing with joint purchasing, the price can be written as $p_{i}\left(\rho_{i}, \rho_{j}\right)=p_{i}^{U_{J}}+\rho_{i}+\rho_{j} / 2$. This shows the advantage of the joint marketing rebate scheme: each firm raises the stand-alone price to consumers by its rebate and by half the rival's rebate as well. That is, since the firms choose the rebate before choosing their stand-alone prices, their choice of rebate serves as a commitment to raise the stand-alone prices.

Given the optimal stand-alone prices as a function of the joint rebate, each firm selects their partial rebate non-cooperatively:

$$
\max _{\rho_{i}} \pi_{i}^{R}=\frac{v_{i}+2 \rho_{i}+\rho_{j}}{2} \cdot \frac{2 t-v_{j}-\rho_{i}}{2 t}+\frac{v_{i}+\rho_{j}}{2} \cdot \frac{v_{i}+v_{j}+\rho_{i}+\rho_{j}-2 t}{2 t}
$$

The second-order conditions are satisfied, and the problem has the unique solution.
Proposition 4. When firms undertake a joint marketing arrangement in which a rebate is given to bundle purchasers,

$$
\begin{gathered}
p_{i}^{R}=\frac{1}{4}\left(6 t+v_{i}-2 v_{j}\right), \quad \rho_{i}^{R}=\frac{1}{2}\left(2 t-v_{j}\right) \quad \text { and } \quad p_{12}^{R}=\frac{1}{4}\left(4 t+v_{1}+v_{2}\right) \\
n_{i}^{R}=\frac{1}{4 t}\left(2 t-v_{j}\right) \quad \text { and } \quad n_{12}^{R}=\frac{1}{4 t}\left(v_{1}+v_{2}\right) \\
\pi_{i}^{R}=\frac{1}{16 t}\left(12 t^{2}+4\left(v_{i}-2 v_{j}\right) t+\left(v_{i}^{2}+2 v_{j}^{2}\right)\right)
\end{gathered}
$$

Compared to the uniform pricing regime with joint purchasing, joint marketing again leads to an increase in the price for joint purchasers. That is, joint purchasers pay $p_{12}^{R}=\left(4 t+v_{1}+\right.$ $\left.v_{2}\right) / 4>\left(v_{1}+v_{2}\right) / 2=p_{12}^{U_{J}}$. And, as was also the case for joint marketing with bundle component pricing, with a rebate being jointly marketed, the stand alone prices are higher than they are when a uniform price is charged to single purchasers, $p_{i}^{R}>p_{i}^{U_{S}}$. In contrast to the case of joint marketing
through bundle pricing, in this case the higher stand alone price drives purchasers to buy the bundle in order to obtain the rebate-despite the effective bundle price being higher than before, so $n_{12}^{R}>n_{12}^{U_{J}}$. Once again, joint marketing leads to increased profits compared to uniform pricing, that is $\pi_{i}^{R}>\pi_{i}^{U_{s}}, \pi_{i}^{U_{J}}$.

Turning to the comparison with independent price discrimination, without joint marketing of the rebate, firm $i$ solves

$$
\max _{\rho_{i}, p_{i}} \pi_{i}^{I P D}\left(p_{i}, p_{j}, \rho_{i}, \rho_{j}\right) .
$$

The first-order condition with respect to the individual rebate in this case is

$$
\begin{equation*}
\frac{\partial \pi_{i}^{I P D}}{\partial \rho_{i}}=p_{i} \frac{\partial n_{i}}{\partial \rho_{i}}+\left(p_{i}-\rho_{i}\right) \frac{\partial n_{12}}{\partial \rho_{i}}-n_{12}=0 . \tag{5}
\end{equation*}
$$

In contrast, when jointly marketing the bundle rebate, the optimal individual rebate is derived by solving

$$
\max _{\rho_{i}} \pi_{i}^{R}\left(p_{i}^{R}\left(\rho_{i}, \rho_{j}\right), p_{j}^{R}\left(\rho_{i}, \rho_{j}\right), \rho_{i}, \rho_{j}\right),
$$

with the following first-order-condition

$$
\begin{align*}
\frac{d \pi_{i}}{d \rho_{i}} & =\frac{\partial \pi_{i}}{\partial \rho_{i}}+\frac{\partial \pi_{i}}{\partial p_{i}} \frac{\partial p_{i}^{R}}{\partial \rho_{i}}+\frac{\partial \pi_{i}}{\partial p_{j}} \frac{\partial p_{j}^{R}}{\partial \rho_{i}} \\
& =\underbrace{\left[p_{i} \frac{\partial n_{i}}{\partial \rho_{i}}-n_{12}+\left(p_{i}-\rho_{i}\right) \frac{\partial n_{12}}{\partial \rho_{i}}\right]}_{=\text {Direct Effect }}+\underbrace{\left[p_{i} \frac{\partial n_{i}}{\partial p_{j}}+\left(p_{i}-\rho_{i}\right) \frac{\partial n_{12}}{\partial p_{j}}\right] \frac{\partial p_{j}^{R}}{\partial \rho_{i}}}_{=\text {Indirect Effect }}=0 . \tag{6}
\end{align*}
$$

Again, the direct effect reflects the condition obtained in independent price discrimination, Equation (5); and the indirect effect captures the effect that one's own rebate has on the stand-alone pricing decision of the rival. Evaluating the first order condition (5) at $\rho_{i}=\rho_{i}^{I P D}$ yields

$$
\left.\frac{d \pi_{i}}{d \rho_{i}}\right|_{\rho_{i}=\rho_{i}^{I P D}}=\left.\left[p_{i} \frac{\partial n_{i}}{\partial p_{j}}+\left(p_{i}-\rho_{i}\right) \frac{\partial n_{12}}{\partial p_{j}}\right] \frac{\partial p_{j}^{R}}{\partial \rho_{i}}\right|_{\rho_{i}=\rho_{i}^{I P D}}>0,
$$

implying that $\rho_{i}^{R}>\rho_{i}^{I P D}$.
That is, firm $i$ increases its rebate when it commits to jointly marketing the bundle rebate prior to setting the stand-alone price. The effect of this is to increase the attractiveness of the bundle relative to the stand alone price. For this reason, there are more joint purchasers when rebates are jointly marketed compared to the case of independent price discrimination, i.e., $\eta_{12}^{R}=$ $\left(v_{1}+v_{2}\right) / 4 t>\left(v_{1}+v_{2}-t\right) / 3 t=n_{12}^{I P D}$.

However, because firms set stand alone prices only after the rebates have been fixed, this commitment leads to the ability to increase the stand alone prices, $d p_{i}^{R} / d \rho_{j}>0$. This commitment is sufficiently strong so that joint purchasers end up paying more with joint marketing then with independent price discrimination, despite the former getting higher rebates, i.e., $p_{12}^{R}=\left(4 t+v_{1}+v_{2}\right) / 4>\left(2 t+v_{1}+v_{2}\right) / 3=p_{12}^{I P D}$.

In sum, when the firms jointly market the bundle rebate they give generous rebates, as this allows very high stand alone prices that drive consumers to joint purchasing, who despite large rebates are very profitable, because the rebate is taken off very high initial (i.e., stand-alone) prices.

## 4 Welfare Analysis

We now consider the welfare effects of the various pricing schemes derived. We begin with profits, then move to consumer surplus, and finally compare total welfare for the cases analyzed.

### 4.1 Profit Comparisons

Profits are derived in the previous section and are given in the propositions covering the equilibrium configurations. Here, we discuss their relation across the different pricing schemes. Firms' profits are increasing in their ability to engage in and coordinate price discrimination. ${ }^{9}$ Thus, profits are lowest under uniform pricing and firms do better when they independently price discriminate. And regardless of whether the firms choose a bundle price or a rebate scheme when they launch a joint marketing strategy, their profits increase over and above what independent price discrimination achieves. This occurs despite the non-cooperative nature of the agreement, because joint marketing commits the firms to the bundle strategy-a commitment that is leveraged into higher prices and profits.

Given that profit comparisons are quite intuitive when it comes to the degree of commitment and coordination, the intriguing question is whether joint marketing through bundle pricing or through rebates is preferred. While it may be the case that for logistical reasons one method or the other may not be practical or available, whenever firms can engage in either scheme it is worth knowing which yields the greater profit.

Comparing Proposition 3 and Proposition 4 shows that the answer to which scheme dominates

[^6]is not unambiguously clear. In particular, when firms use a joint marketing scheme in which the bundle is priced, then the effective price for both the bundle and the stand alone products are higher when compared to the case of a rebate scheme, that is, $p_{12}^{P}=\widetilde{p}_{1}^{P}+\widetilde{p}_{2}^{P}=\left(v_{1}+v_{2}+2 t\right) / 2>$ $\left(v_{1}+v_{2}+4 t\right) / 4=p_{1}^{R}+p_{2}^{R}-\rho=p_{12}^{R}$, and $p_{i}^{P}=\left(5 t+2 v_{i}-v_{j}\right) / 4>\left(6 t+v_{i}-2 v_{j}\right) / 4=p_{i}^{R}$.

On the flip side, in the case of rebates the mass of joint purchasers is larger when compared to the bundle price marketing arrangement, $n_{12}^{R}=\left(v_{1}+v_{2}\right) / 4 t>\left(v_{1}+v_{2}-2 t\right) / 4 t=n_{12}^{P}$, so the rebate scheme leads to a greater sales volume when compared to the bundle pricing regime.

Which of the two arrangements is more profitable depends on the size of the idiosyncratic values of the two products. Specifically, note from Proposition 3 and Proposition 4 that $\pi_{i}^{R}-$ $\pi_{i}^{P}=\left(\left(v_{j}-t\right)^{2}-v_{i}^{2}\right) / 16 t$, from which it follows that if the idiosyncratic values are similar to oneanother (close in size), then the bundle pricing is preferred whenever the idiosyncratic values are sufficiently large.

When the idiosyncratic values differ (are asymmetric), constellations can arise in which the firm with the smaller $v$ would prefer the rebating scheme whereas the other firm would prefer to institute a bundle price. This suggests that firms that are very different may find it difficult to decide on a joint marketing scheme. However, even in these cases it turns out that the overall profit ranking is unambiguous in favor of bundle pricing. Thus, when idiosyncratic values are large, joint profits are higher under bundle pricing, even if firms' individual idiosyncratic values differ substantially. As a consequence, if side payments are permissible, then even under noncooperative joint marketing, firms can agree on which scheme to employ.

In sum, letting $\Pi$ denote industry profits, we have the following theorem.
Theorem 1 (Profits under Price Discrimination and Joint Marketing). Firms' profits are increasing in the extent of price discrimination in that independent price discrimination increases profits above uniform pricing, and price discrimination through joint marketing raises profits even more:

$$
\Pi^{U_{s}}, \Pi^{U_{J}} \leq \Pi^{I P D}<\Pi^{R}, \Pi^{P}
$$

Moreover, the bundle pricing scheme is preferred to the rebate scheme whenever the idiosyncratic values are large enough to implement the bundle price; otherwise the rebate scheme is chosen:

$$
\Pi^{P}>\Pi^{R} \Longleftrightarrow v_{1}+v_{2} \geq 2 t
$$

### 4.2 Consumer Surplus

Let $C S^{X}$ be the level of consumer surplus for $X \in\left\{U_{S}, U_{J}, I P D, P, R\right\}$, defined by

$$
\begin{aligned}
C^{X}= & \int_{0}^{\widehat{x}_{1}^{X}} U_{1}\left(x ; p_{1}^{X}\right) d x+\int_{\widehat{x}_{1}^{X}}^{\widehat{x}_{2}^{X}} U_{12}\left(x ; p_{12}^{X}\right) d x+\int_{\widehat{x}^{X}}^{1} U_{2}\left(x ; p_{2}^{X}\right) d x \\
= & {\left[V+v_{1} n_{1}^{X}+\left(v_{1}+v_{2}\right) n_{12}^{X}+v_{2} n_{2}^{X}\right]-t\left[\int_{0}^{n_{1}^{X}} x d x+n_{12}^{X}+\int_{0}^{n_{2}^{X}} x d x\right] } \\
& -\left[p_{1}^{X} n_{1}^{X}+p_{12}^{X} n_{12}^{X}+p_{2}^{X} n_{2}^{X}\right] .
\end{aligned}
$$

The first term is the gross utility from consumption, the second term is the transportation costs, and the final term is the total expenditures. ${ }^{10}$

It is natural to conjecture that consumer surplus is directly related to total consumption, because consumers only purchase the second product when the added utility exceeds their transportation costs. In that case consumer surplus would be directly proportional to $n_{12}^{X}$. Indeed, this logic applies when comparing uniform pricing with single purchasing to independent price discrimination. The stand alone price is the same for both cases, but under independent price discrimination some obtain the lower price and more consumers are drawn into purchasing each firm's product. Thus, $n_{12}^{I P D}>n_{12}^{U_{S}}$ and $C S^{I P D}>C S^{U_{S}}$.

However, comparing uniform pricing with joint purchasing to independent price discrimination reveals that even though there are more purchases under price discrimination, $n_{12}^{I P D}>n_{12}^{U_{J}}$, consumer surplus is actually lower, $C S^{I P D}<C S^{U_{J}}$. In this case some consumers make the second purchase even if the additional marginal value from consuming the second unit is smaller than the added transportation costs, simply because the second purchase allows them to take advantage of the lower prices for both the first and the second purchase. And because the stand alone price is higher with price discrimination, consumer surplus is lower, despite more total purchases.

This dynamic, that consumers base their decision on purchasing a second unit in part on how this determines the effective price paid on the first unit, is important in understanding the impact of joint marketing on consumer surplus. In particular, as was shown in Section 3, firms use their commitment to the joint marketing scheme to increase overall prices. An unambiguous result of this is that consumer surplus is always lower under a joint marketing scheme compared to independent pricing schemes with or without price discrimination. That is, CS $^{U_{s}}, C S^{U_{J}}, C S^{I P D}>$ $C S^{P}, C S^{R}$. And yet, because a joint marketing rebate scheme increases joint purchases, total con-

[^7]sumption is larger under this scheme compared to any other: $n_{12}^{R}=\max _{X} n_{12}^{X}$.
Finally, the positive relationship between consumption volume and consumer surplus holds again when comparing the two joint marketing schemes. In particular, consumer surplus is always larger under a rebate scheme compared to the bundle pricing arrangement, $C S^{R}>C S^{P}$, and so is total consumption: $n_{12}^{R}>n_{12}^{P}$.

The main findings concerning consumer surplus are recapped in the following theorem:
Theorem 2 (Consumer Surplus under Price Discrimination and Joint Marketing). All forms of price discrimination harm consumers, unless firms engage in independent price discrimination and thereby induce joint purchases, when otherwise consumers only purchase a single good given uniform pricing, $C S^{I P D}>\mathrm{CS}^{U_{S}}$.

Joint marketing raises prices compared to any of the other settings and as a result leads to lower con-


Finally, consumers are better off under a rebate joint marketing scheme compared to the bundle pricing scheme, CS $^{R}>$ CS $^{P}$.

### 4.3 Total Welfare

We take the sum of consumer surplus and industry profits as a measure of social welfare as usual. Because firms' costs are normalized to zero, consumer expenditures are equal to industry profits, so it is immediate that social welfare is the difference between gross utility and transportation costs:

$$
\begin{equation*}
T W^{X}=\left[V+v_{1} n_{1}^{X}+\left(v_{1}+v_{2}\right) n_{12}^{X}+v_{2} n_{2}^{X}\right]-t\left[\int_{0}^{n_{1}^{X}} x d x+n_{12}^{X}+\int_{0}^{n_{2}^{X}} x d x\right] \tag{7}
\end{equation*}
$$

again with $X \in\left\{U_{s}, U_{J}, I P D, P, R\right\}$.
There are two opposing effects of joint purchases on social welfare. First, more joint purchases increases social welfare because extra surplus is realized from the consumption of additional features. In the case of the standard Hotelling model with inelastic consumer demand, there is no demand creation effect with pricing, as long as the market is covered. Here, however, the demand for a firm's product increase when it charges a relatively lower price for joint purchasers. The first term in (7) captures the base surplus associated with consuming either good, as well as the added gain associated with the idiosyncratic features of the two goods.

The second effect of joint purchasing is that it increases total transportation costs, which is captured in the second term in (7). Recall from the discussion on consumer surplus that added surplus through additional consumption need not be enough to offset the added transportation costs, because the consumption decision is based on the price difference between joint and single
purchasing due to price discrimination. As a result, the net effect on social welfare depends on the relative size of the surplus creation and transportation cost effects.

While the interests of firms are often directly opposed to those of consumers, assessing overall welfare requires closer scrutiny in all but one case: compared to uniform pricing in which consumers only purchase one good, independent price discrimination increases consumer surplus while also raising firm profits. So total welfare is clearly greater under independent price discrimination compared to uniform pricing when consumers do not engage in joint purchasing, $T W^{I P D}>T W^{U_{S}}$.

In contrast, independent price discrimination lowers consumer surplus when compared to uniform pricing with joint purchases. The reason for this is that the increased surplus from added consumption is more than offset by the higher prices paid to firms by those who only purchased a single good. This latter effect, however, is a welfare-neutral price transfer from consumers to firms. Therefore total welfare is also greater under independent price discrimination compared to uniform pricing with joint purchases, $T W^{I P D}>T W^{U_{J}}$.

This result is somewhat reminiscent of welfare effects under third degree price discrimination, in which welfare increases when consumption (output) increases (see Varian, 1985), which is the case here: $n_{12}^{I P D}>n_{12}^{U_{I}}>n_{12}^{U_{S}}=0$. However, note that there is a critical difference in our analysis compared to third degree price discrimination: in our model it is the prices that segment the markets into single and joint purchasers, and consumers self-select into whether they are single or bundle purchasers. Hence the comparison to pricing across distinct markets without an arbitrage possibility is not apt in our setting.

Nevertheless, the association between greater consumption and higher surplus also holds across joint marketing schemes. Thus, total consumption under a rebate scheme is greater than total consumption under bundle pricing, $n_{12}^{R}>n_{12}^{P}$, and total welfare is also greater in the former case, $T W^{R}>T W^{P}$.

A positive association between consumption and total welfare holds more generally, provided that the products' idiosyncratic values are sufficiently large and not too asymmetric. Thus,

Theorem 3 (Total Consumption and Total Welfare). Whenever the idiosyncratic values are not too small $\left(23 v_{1}^{2}+23 v_{2}^{2}-90 t v_{1}-90 t v_{2}+54 t^{2}+80 v_{1} v_{2}>0\right)$ and not too asymmetric, $\left(\left(v_{1}-22 t / 17\right)^{2}+\right.$ $\left.\left(v_{2}-22 t / 17\right)^{2} \leq 288 / 289\right)$, then increases in total equilibrium consumption imply increases in total welfare:

$$
\begin{gathered}
n_{12}^{X}>n_{12}^{Y} \Rightarrow T W^{X}>T W^{Y}, \text { with } \\
T W^{R}>T W^{I P D}>T W^{U_{J}}>T W^{P}>T W^{U_{S}} .
\end{gathered}
$$

The area identified in the Theorem 3 is given in Figure 4.


Figure 4: Social Welfare

The reason for the existence of a threshold that values must exceed is intuitive. Firms are balancing their pricing decisions across two margins, namely the margin at which they capture new customers who otherwise only purchase from the rival, and the margin at which their own customers decide to become joint purchasers or remain single purchasers. As a result, the firms' pricing decisions induce transportation costs that need not be offset by marginal increases in consumer surplus tied to consumption of the second product. However, greater consumption leads to greater surplus provided that the value from added consumption is sufficiently high to offset added transportation costs that are incurred by consumers whose second purchase is partly driven by the desire to lower their expenditure on their first purchase.

Two aspects of the theorem are particularly noteworthy. First, Theorem 3 demonstrates that total welfare is greatest when firms engage in joint marketing that sets a rebate for bundle purchasers compared to any of the other schemes, so joint marketing can increase welfare above what firms can do independently. While consumer surplus is lower under this scheme than when compared to independent price discrimination, the increase in profits more than offsets the reduction in consumer surplus. However, there is an important caveat to note here. From the second part of Theorem 1 we know that when idiosyncratic values are high, firms actually prefer the bundle pricing scheme over the rebate scheme.

Nonetheless, the finding points in the direction of which type of joint marketing arrangements should be viewed as beneficial-especially as the case of bundle pricing leads to the lowest mass of joint purchasers, and the lowest welfare of any equilibrium configuration in which there is joint
purchasing. Because of this, the comparison between surplus through added consumption and losses through additional transportation is relatively easy for the case of joint marketing: if a joint marketing agreement leads to greater consumption then it likely also increases total welfare.

The second noteworthy finding in Theorem 3 is that compared to uniform pricing with (only) single purchasing, (even) joint marketing through bundle pricing can raise total welfare. And, thus, the welfare implications of joint marketing are not clear ex ante, but whenever idiosyncratic values are high and not too asymmetric, there's a chance that either type of joint marketing may increase total welfare.

There is, of course, an immediate important corollary to Theorem 3. If the idiosyncratic values of the products are not that large, then equilibrium consumption need not be positively correlated with equilibrium welfare. The intuition for this is straightforward: if the added value from consuming a second good is not that large, then pricing schemes that induce the added purchase may generate less additional surplus than the added transportation that the second purchase entails, thus reducing total welfare. In fact, when the idiosyncratic values are very small, then no pricing scheme generates more total welfare than simple uniform pricing with single purchasing.

Related to this is the case where idiosyncratic values are large yet very asymmetric. Adverse welfare effects occur on the margin between joint and single purchasing when it comes to the relatively lower-valued good when switching from independent price discrimination to bundle joint marketing. That is, bundle joint marketing will induce additional purchases of the relatively lower valued good for which the added transportation costs are not offset by the added consumption value.

## 5 Conclusion

We have studied the effect of price discrimination through joint marketing by firms producing imperfectly substitutable goods. The main finding is that the impact on overall purchasing and welfare depend on whether firms offer a bundle price or offer rebates off individual prices to consumers who buy from both firms. Firms may want to increase joint purchasing by setting bundle rebates before determining stand-alone prices. Alternatively, they may want to mitigate price competition by pricing the bundle components before setting the individual prices.

Loosely speaking, when a second purchase adds little in terms of incremental utility, firms prefer to offer discounts off their prices. This has the effect of increasing the mass of customers to both firms, which is beneficial because rent-extraction is limited by the added value of the second unit. In contrast, if consumers place a high value on each of the goods and therefore the second
unit is relatively valuable, firms prefer to directly price and market the bundle, as this allows them to increase their stand-alone prices and extract more rents from their captured customers. However, we show that the firms interests need not be aligned in this respect if their incremental values to the consumers vary greatly.

With respect to welfare we find that if second purchases add a lot of additional utility, then price discrimination that induces added purchases raises total welfare. However, as firms find bundle pricing more profitable compared to the rebate scheme when incremental values are high, total purchasing can actually decrease with high additional values when both joint marketing options are available to the firms. Nevertheless, there are parameter values for which total welfare increases due to joint marketing compared to uniform pricing regardless of which joint marketing scheme is used. Indeed, whenever both idiosyncratic values are sufficiently high joint marketing raises total welfare; otherwise independent price discrimination is better from a total welfare perspective.

## Appendix A Derivation of best response correspondences

We will derive firm 1's best response correspondence first. Firm 2's problem will be solved in a similar manner. Given the demand function (1), the profit function of firm 1 is given by,

$$
\pi_{1}= \begin{cases}\left(\frac{1}{2}+\frac{v_{1}-v_{2}+p_{2}-p_{1}}{2 t}\right) p_{1} & \text { if } p_{1} \geq v_{1}+v_{2}-t-p_{2} \\ \frac{1}{t}\left(v_{1}-p_{1}\right) p_{1} & \text { if } p_{1} \leq v_{1}+v_{2}-t-p_{2}\end{cases}
$$

Consider first the case in which $p_{1} \geq v_{1}+v_{2}-t-p_{2}$. Using the first order necessary condition, we identify the candidate candidate best reply $b_{1}\left(p_{2}\right)=\frac{1}{2}\left(t+v_{1}-v_{2}+p_{2}\right)$. It yields a payoff $\pi_{1}^{S}\left(p_{2}\right)=\left(t+v_{1}-v_{2}+p_{2}\right)^{2} / 8 t$. Notice that this case is valid only when $p_{1} \geq v_{1}+v_{2}-t-p_{2}$, so we need to have the condition:

$$
p_{1} \geq v_{1}+v_{2}-t-p_{2} \Longleftrightarrow p_{2} \geq \frac{1}{3}\left(v_{1}+3 v_{2}-3 t\right) .
$$

For the case in which $p_{1} \leq v_{1}+v_{2}-t-p_{2}$, firm 1 sets the price at $p_{1}^{J}=v_{1} / 2$, yielding a payoff $\pi_{1}^{J}=v_{1}^{2} / 4 t$. This payoff is valid only when $p_{1} \leq v_{1}+v_{2}-t-p_{2}$, requiring the following condition:

$$
p_{1} \leq v_{1}+v_{2}-t-p_{2} \Longleftrightarrow p_{2} \leq \frac{v_{1}}{2}+v_{2}-t .
$$

Thus, for $p_{2} \in\left[\frac{1}{3}\left(v_{1}+3 v_{2}-3 t\right), \frac{1}{2} v_{1}+v_{2}-t\right]$ both choices satisfy the requirements. In order
to identify 'true' best response of firm 1, it remains to compare the payoffs in these two cases. From solving $\pi_{1}^{S}\left(p_{2}\right) \geq \pi_{1}^{J}$ we have

$$
\frac{\left(t+v_{1}-v_{2}+p_{2}\right)^{2}}{8 t} \geq \frac{v_{1}^{2}}{4 t} \Longleftrightarrow p_{2} \geq \hat{p}_{2}=(\sqrt{2}-1) v_{1}+v_{2}-t .
$$

Thus, firm 1 switches from the joint-purchasing regime to the single-purchasing regime at $\hat{p}_{2}$. A similar argument applies for firm 2. We therefore summarize firm $i^{\prime}$ 's best response correspondences $\phi_{i}$ as follows

$$
\phi_{i}\left(p_{i}, p_{j}\right)= \begin{cases}b_{i}\left(p_{j}\right)=\frac{1}{2}\left(t+v_{i}-v_{j}+p_{j}\right) & \text { if } p_{j} \geq \widehat{p}_{j}=(\sqrt{2}-1) v_{i}+v_{j}-t \\ \frac{1}{2} v_{i} & \text { if } p_{j} \leq \widehat{p}_{j}=(\sqrt{2}-1) v_{i}+v_{j}-t\end{cases}
$$

Figure 5 illustrates best response correspondence for firm $i$.


Figure 5: Firm $i$ 's best response correspondence

## Appendix B The Omitted Proofs

Proof of Proposition 1 We first identify two critical regions in the space prices according to their resulting demands and denote each set as $\mathcal{J}$ and $\mathcal{S}$, respectively, i.e.,

$$
\begin{aligned}
\mathcal{J} & =\left\{\left(p_{1}, p_{2}\right) \in \mathbb{R}_{+}^{2} \mid p_{1}+p_{2} \leq v_{1}+v_{2}-t\right\}, \\
\mathcal{S} & =\left\{\left(p_{1}, p_{2}\right) \in \mathbb{R}_{+}^{2} \mid p_{1}+p_{2} \geq v_{1}+v_{2}-t\right\} .
\end{aligned}
$$

There are two candidate Nash equilibria according to the region.

$$
\begin{array}{ll}
\left(p_{1}^{U_{J}}, p_{2}^{U_{J}}\right)=\left(\frac{v_{1}}{2}, \frac{v_{2}}{2}\right) & \text { if }\left(p_{1}, p_{2}\right) \in \mathcal{J} \\
\left(p_{1}^{U_{S}}, p_{2}^{U_{S}}\right)=\left(t+\frac{v_{1}-v_{2}}{3}, t+\frac{v_{2}-v_{1}}{3}\right) & \text { if }\left(p_{1}, p_{2}\right) \in \mathcal{S}
\end{array}
$$

First, consider $\left(p_{1}^{U_{J}}, p_{2}^{U_{J}}\right)$. Suppose one firm, say firm 1, deviates to the region $\mathcal{S}$ then its profits becomes

$$
\pi_{1}^{S}\left(p_{1}, p_{2}^{U_{J}}\right)=p_{1}\left(\frac{1}{2}+\frac{v_{1}-v_{2}+p_{2}^{U_{I}}-p_{1}}{2 t}\right)
$$

given $p_{1} \geq v_{1}+v_{2}-t-p_{2}^{U_{J}}=v_{1}+\frac{1}{2} v_{2}-t$. Then the optimal deviating price for firm 1 is given by

$$
p_{1}^{d e v}= \begin{cases}\frac{1}{2} v_{1}-\frac{1}{4} v_{2}+\frac{1}{2} t & \text { if } 2 v_{1}+3 v_{2} \leq 6 t \text { and } v_{1}+v_{2} \geq 2 t \\ v_{1}+\frac{1}{2} v_{2}-t & \text { if } 2 v_{1}+3 v_{2} \geq 6 t\end{cases}
$$

This implies the optimal deviation payoff for firm 1 as follows:

$$
\begin{aligned}
\pi_{1}^{d e v} & =p_{1}^{d e v}\left(\frac{1}{2}+\frac{v_{1}-v_{2}+p_{2}^{U_{I}}-p_{1}^{d e v}}{2 t}\right) \\
& = \begin{cases}\frac{1}{32 t}\left(2 t+2 v_{1}-v_{2}\right)^{2} & \text { if } 2 v_{1}+3 v_{2} \leq 6 t \text { and } v_{1}+v_{2} \geq 2 t, \\
\frac{1}{4 t}\left(2 t-v_{2}\right)\left(2 v_{1}+v_{2}-2 t\right) & \text { if } 2 v_{1}+3 v_{2} \geq 6 t\end{cases}
\end{aligned}
$$

In contrast, the profit in the proposed equilibrium in region $\mathcal{J}$ is $\pi_{1}^{U_{J}}=v_{1}^{2} / 4 t$. Thus, the condition for first candidate to be a Nash equilibrium is that $\pi_{1}^{U_{J}} \geq \pi_{1}^{d e v}$, and computations show that this is equivalent to $\left(v_{1}, v_{2}\right) \in \Phi^{J}$.

We can apply same logic for candidate $\left(p_{1}^{U_{s}}, p_{2}^{U_{s}}\right)$. Suppose firm 1 deviates to the region $\mathcal{J}$. Then, its profit becomes

$$
\pi_{1}^{U_{J}}\left(p_{1}\right)=p_{1}\left(\frac{v_{1}-p_{1}}{t}\right),
$$

given $p_{1} \leq v_{1}+v_{2}-t-p_{2}^{U_{S}}=\frac{4}{3} v_{1}+\frac{2}{3} v_{2}-2 t$. Then, the optimal deviating price for firm 1 is given by

$$
p_{1}^{d e v}= \begin{cases}\frac{4}{3} v_{1}+\frac{2}{3} v_{2}-2 t & \text { if } 5 v_{1}+4 v_{2} \leq 12 t \\ \frac{1}{2} v_{1} & \text { if } 5 v_{1}+4 v_{2} \geq 12 t \text { and } v_{1}+v_{2} \leq 3 t\end{cases}
$$

Thus, the optimal deviation payoff of firm 1 is

$$
\begin{aligned}
\pi_{1}^{d e v} & =p_{1}^{d e v}\left(\frac{v_{1}-p_{1}^{d e v}}{t}\right) \\
& = \begin{cases}\frac{2}{t}\left(\frac{2}{3} v_{1}+\frac{1}{3} v_{2}-t\right)\left(2 t-\frac{1}{3} v_{1}-\frac{2}{3} v_{2}\right) & \text { if } 5 v_{1}+4 v_{2} \leq 12 t \\
\frac{1}{4 t} v_{1}^{2} & \text { if } 5 v_{1}+4 v_{2} \geq 12 t \text { and } v_{1}+v_{2} \leq 3 t .\end{cases}
\end{aligned}
$$

In contrast, the profit in the candidate equilibrium in region $\mathcal{S}$ is given by $\pi_{1}^{U_{S}}=\left(3 t+v_{1}-\right.$ $\left.v_{2}\right)^{2} / 18 t$, so the condition for $\left(p_{1}^{U_{S}}, p_{2}^{U_{S}}\right)$ is that $\pi_{1}^{U_{S}} \geq \pi_{1}^{d e v}$. Computations show that this is satisfied whenever $\left(v_{1}, v_{2}\right) \in \Phi^{S}$.

Proof of Theorem 1 Recall that we have derived the firm's profits under various scenarios (see Proposition 1 - Proposition 4). From the information in each propositions, we have following industry profits: $\Pi^{U_{S}}=\frac{1}{9 t}\left(v_{1}^{2}+v_{2}^{2}-2 v_{1} v_{2}+9 t^{2}\right), \Pi^{U_{J}}=\frac{1}{4 t}\left(v_{1}^{2}+v_{2}^{2}\right), \Pi^{I P D}=\frac{1}{9 t}\left(2 v_{1}^{2}+2 v_{2}^{2}-\right.$ $\left.2 t v_{1}-2 t v_{2}+10 t^{2}\right), \Pi^{P}=\frac{1}{16 t}\left(3 v_{1}^{2}+3 v_{2}^{2}-2 t v_{1}-2 t v_{2}+22 t^{2}\right)$ and $\Pi^{R}=\frac{1}{16 t}\left(3 v_{1}^{2}+3 v_{2}^{2}-4 t v_{1}-\right.$ $\left.4 t v_{2}+24 t^{2}\right)$.

Independent price discrimination increases profits above uniform pricing, which is simply shown as

$$
\Pi^{I P D}-\Pi^{U_{s}}=\frac{1}{9 t}\left(v_{1}^{2}+v_{2}^{2}+2 v_{1} v_{2}-2 t v_{1}-2 t v_{2}+t^{2}\right)=\frac{1}{9 t}\left(v_{1}+v_{2}-t\right)^{2}>0
$$

and

$$
\Pi^{I P D}-\Pi^{U_{J}}=-\frac{1}{36 t}\left(v_{1}^{2}+v_{2}^{2}+8 t v_{1}+8 t v_{2}-40 t^{2}\right) \geq 0
$$

The last inequality above comes from the fact that $\Pi^{I P D}-\Pi^{U_{I}}$ is decreasing in $v_{i}$ and it takes the value of 0 when $v_{1}=v_{2}=2 t$.

The comparison between joint marketing regime and independent price discrimination shows

$$
\begin{aligned}
& \Pi^{I P D}-\Pi^{R}=\frac{1}{144 t}\left(5 v_{1}^{2}+5 v_{2}^{2}+4 t v_{1}+4 t v_{2}-56 t^{2}\right)<0 \\
& \Pi^{I P D}-\Pi^{P}=\frac{1}{144 t}\left(5 v_{1}^{2}+5 v_{2}^{2}-14 t v_{1}-14 t v_{2}-38 t^{2}\right)<0 .
\end{aligned}
$$

Finally, we check whether the bundle pricing scheme is better for the firms when $v_{1}+v_{2} \geq 2 t$.

The difference between the bundle pricing scheme and the rebate scheme is given by

$$
\Pi^{P}-\Pi^{R}=\frac{1}{8}\left(v_{1}+v_{2}-t\right)>0
$$

which holds provided that $v_{1}+v_{2} \geq 2 t$.

Proof of Theorem 2 Computation shows that

$$
\begin{aligned}
C S^{I P D}-C S^{U_{S}} & =\frac{\left(v_{1}+v_{2}-t\right)^{2}}{36 t}>0 \\
C S^{R}-C S^{U_{S}} & =\frac{1}{288 t}\left(\left(v_{1}+v_{2}\right)^{2}+36\left(v_{1}+v_{2}\right) t+14 v_{1} v_{2}-144 t^{2}\right)<0, \\
C S^{R}-C S^{U_{J}} & =-\frac{1}{32 t}\left(3 v_{1}^{2}+3 v_{2}^{2}-20 t v_{1}-20 t v_{2}+56 t^{2}\right)<0, \\
C S^{R}-C S^{P} & =\frac{5}{16}\left(v_{1}+v_{2}-t\right)>0 .
\end{aligned}
$$

Proof of Theorem 3 From the results in Proposition 1 - Proposition 4, we can obtain the measure of joint purchasing consumers in each case:

$$
\begin{array}{lll}
n_{12}^{U_{S}}=0 & n_{12}^{U_{J}}=\frac{1}{2 t}\left(v_{1}+v_{2}-2 t\right) & n_{12}^{I P D}=\frac{1}{3 t}\left(v_{1}+v_{2}-t\right) \\
n_{12}^{R}=\frac{1}{4 t}\left(v_{1}+v_{2}\right) & n_{12}^{P}=\frac{1}{4 t}\left(v_{1}+v_{2}-2 t\right) &
\end{array}
$$

Provided that $v_{1}+v_{2}>2 t$, the rank of measure is

$$
n_{12}^{R}>n_{12}^{I P D}>n_{12}^{U_{J}}>n_{12}^{P}>n_{12}^{U_{S}}=0 .
$$

First, consider the case in which $\left(v_{1}, v_{2}\right) \in \Phi^{J}$. Computation shows the following relationship of total welfare in each regime:

$$
\begin{aligned}
T W^{R}-T W^{I P D} & =-\frac{1}{288 t}\left(17 v_{1}^{2}+17 v_{2}^{2}-44 t v_{1}-44 t v_{2}+40 t^{2}\right)>0, \\
T W^{I P D}-T W^{U_{J}} & =-\frac{1}{72 t}\left(7 v_{1}^{2}+7 v_{2}^{2}-16 t v_{1}-16 t v_{2}+8 t^{2}\right)>0, \\
T W^{U_{J}}-T W^{P} & =\frac{1}{32 t}\left(5 v_{1}^{2}+5 v_{2}^{2}-6 t v_{1}-6 t v_{2}+2 t^{2}\right)>0 .
\end{aligned}
$$

Thus, we have

$$
T W^{R}>T W^{I P D}>T W^{U_{J}}>T W^{P}
$$

Next, consider the case in which $\left(v_{1}, v_{2}\right) \in \Phi^{S}$. Define the subsets

$$
\begin{aligned}
& \Phi_{1}^{S}:=\left\{\left(v_{1}, v_{2}\right) \mid T W^{P}-T W^{U_{S}} \geq 0\right\} \\
& \Phi_{2}^{S}:=\left\{\left(v_{1}, v_{2}\right) \mid T W^{R}-T W^{I P D} \geq 0\right\} .
\end{aligned}
$$

After some algebra one can see that $T W^{I P D}-T W^{P}>0$ for any $\left(v_{1}, v_{2}\right)$. So if $\left(v_{1}, v_{2}\right) \in \Phi_{1}^{s} \cap \Phi_{2}^{S}$, we obtain the following relationship:

$$
T W^{R}>T W^{I P D}>T W^{P}>T W^{U_{S}}
$$

## References

Adams, W. J., and J. L. Yellen. 1976. "Commodity Bundling and the Burden of Monopoly." Quarterly Journal of Economics, 90(3): 475-498.

Anderson, S. P., O. Foros, and H. J. Kind. 2012. "Product Quality, Competition, and MultiPurchasing." Center for Economics Policy Research Discussion Paper 8923.

Armstrong, M. 1996. "Multiproduct Nonlinear Pricing." Econometrica, 64(1): 51-75.
——. 2006. "Recent Developments in the Economics of Price Discrimination." In Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress. eds. by R. Blundell, W. Newey, and T. Persson, Cambridge: Cambridge University Press, 97-141.
__ . 2013. "A More General Theory of Commodity Bundling." Journal of Economic Theory, 148(2): 448-472.

Brito, D., and H. Vasconcelos. Forthcoming. "Inter-Firm Bundling and Vertical Prodcut Differentiation." Scandinavian Journal of Economics.

Caminal, R., and C. Matutes. 1990. "Endogenous Switching Costs in a Duopoly Model." International Journal of Industrial Organization, 8(3): 353-373.

Cheng, L. K., and J. Nahm. 2007. "Product Boundary, Vertical Competition, and the Double MarkUp Problem." RAND Journal of Economics, 38(2): 447-466.

Choi, J. P. 2010. "Patent Pools and Cross-Licensing in the Shadow of Patent Litigation." International Economic Review, 51(2): 441-460.

Gabszewicz, J. J., N. Sonnac, and X. Y. Wauthy. 2001. "On Price Competition with Complementary Goods." Economics Letters, 70(3): 431-437.
__ , and X. Y. Wauthy. 2003. "The Option of Joint Purchase in Vertically Differentiated Markets." Economic Theory, 22(4): 817-829.

Gans, J. S., and S. P. King. 2006. "Paying for Loyality: Product Bundling in Oligopoly." Journal of Industrial Economics, 54(1): 43-62.

Hotelling, H. 1929. "Stability in Competition." Economic Journal, 39(153): 41-57.
Jeitschko, T. D., and N. Zhang. 2014. "Adverse Effects of Patent Pooling on Product Development and Commercialization." B.E. Journal of Theoretical Economics, 14(1): 1-31.

Kim, H., and K. Serfes. 2006. "A Location Model with Preference for Variety." Journal of Industrial Economics, 54(4): 569-595.

Lerner, J., and J. Tirole. 2004. "Efficient Patent Pools." American Economic Review, 94(3): 691-711.
McAfee, R. P., J. McMillan, and M. D. Whinston. 1989. "Multiproduct Monopoly , Commodity Bundling , and Correlation of Values." Quarterly Journal of Economics, 104(2): 371-383.

Rey, P., and J. Tirole. 2013. "Cooperation vs. Collusion: How Essentiality Shapes Co-opetition." IDEI Working Paper 801.

Rochet, J.-C., and P. Choné. 1998. "Ironing, Sweeping, and Multidimensional Screening." Econometrica, 66(4): 783-826.

Stole, L. A. 2007. "Price Discrimination and Competition." In Handbook of Industrial Organization, Vol 3. eds. by M. Armstrong, and R. H. Porter, Amsterdam: North-Holland, 2221-2299.

Thisse, J.-F., and X. Vives. 1988. "On The Strategic Choice of Spatial Price Policy." American Economic Review, 78(1): 122-137.

Varian, H. R. 1985. "Price Discrimination and Social Welfare." American Economic Review, 75(4): 870-875.

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Haucap, Justus and Klein, Gordon J., How Regulation Affects Network and Service Quality in Related Markets, May 2012.
Published in: Economics Letters, 117 (2012), pp. 521-524.
Dewenter, Ralf and Heimeshoff, Ulrich, Less Pain at the Pump? The Effects of Regulatory Interventions in Retail Gasoline Markets, May 2012.

Böckers, Veit and Heimeshoff, Ulrich, The Extent of European Power Markets, April 2012.

Barth, Anne-Kathrin and Heimeshoff, Ulrich, How Large is the Magnitude of FixedMobile Call Substitution? - Empirical Evidence from 16 European Countries, April 2012.
Forthcoming in: Telecommunications Policy.
Herr, Annika and Suppliet, Moritz, Pharmaceutical Prices under Regulation: Tiered Co-payments and Reference Pricing in Germany, April 2012.

Haucap, Justus and Müller, Hans Christian, The Effects of Gasoline Price Regulations: Experimental Evidence, April 2012.

Stühmeier, Torben, Roaming and Investments in the Mobile Internet Market, March 2012.
Published in: Telecommunications Policy, 36 (2012), pp. 595-607.
Graf, Julia, The Effects of Rebate Contracts on the Health Care System, March 2012, Forthcoming in: The European Journal of Health Economics.

44 Pagel, Beatrice and Wey, Christian, Unionization Structures in International Oligopoly, February 2012.
Published in: Labour: Review of Labour Economics and Industrial Relations, 27 (2013), pp. 1-17.

43 Gu, Yiquan and Wenzel, Tobias, Price-Dependent Demand in Spatial Models, January 2012.
Published in: B. E. Journal of Economic Analysis \& Policy, 12 (2012), Article 6.
42 Barth, Anne-Kathrin and Heimeshoff, Ulrich, Does the Growth of Mobile Markets Cause the Demise of Fixed Networks? - Evidence from the European Union, January 2012.
Forthcoming in: Telecommunications Policy.
41 Stühmeier, Torben and Wenzel, Tobias, Regulating Advertising in the Presence of Public Service Broadcasting, January 2012.
Published in: Review of Network Economics, 11/2 (2012), Article 1.
Older discussion papers can be found online at:
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[^1]:    ${ }^{1}$ Aspen Skiing Co. v. Aspen Highlands Skiing Corp., 472 U.S. 585, (1985).
    ${ }^{2}$ As of April 2014, the total price for visiting five attractions is $\$ 187.95$, while the discounted CityPASS price is \$93.95. See http://www.citypass.com/chicago for more information.
    ${ }^{3}$ Other examples cover tied promotions and other joint marketing efforts, including entertainment venues, such as movie theaters, restaurants, or museums, and tour-operators giving reciprocal discounts involving potential competitors; or newspapers jointly marketed under the Newspaper Preservation Act of 1970. Also, a very similar arrangement to the one in the Aspen case is currently in place for multiple ski resorts across four countries that are tied together through joint marketing of the epic-pass.

[^2]:    ${ }^{4}$ Armstrong (2006) and Stole (2007) provide excellent surveys.
    ${ }^{5}$ Brito and Vasconcelos (Forthcoming) build on the investigation of Gans and King (2006) by considering firms that produce vertically differentiated goods. Their main insight is that bundled discounts may induce a decrease in consumer surplus and always induce a reduction in total welfare.

[^3]:    ${ }^{6}$ Gabszewicz et al. (2001) also consider competition between firms that produce complementary goods. There, too, multiple equilibria emerge only for intermediate degrees of complementarity.

[^4]:    ${ }^{7}$ We assume that $V$ is sufficiently large so that all consumers buy at least one product. This is not a particularly restrictive assumption; indeed if the added values that each firm provides are sufficiently high then even with $V=0$ the market is covered.

[^5]:    ${ }^{8}$ Caminal and Matutes (1990) make a similar distinction between prices and rebates. In their model, however, customers are distinguished by purchasing across periods.

[^6]:    ${ }^{9}$ This is in contrast to Thisse and Vives (1988) who find that price discrimination in the single-purchasing Hotelling model decreases profits. The reason for the difference is that price discrimination without joint purchasing leads to fiercer Bertrand competition, whereas with the possibility of joint purchasing there isn't head-to-head competition and business stealing.

[^7]:    ${ }^{10}$ Note that ${\widehat{x_{1}}}^{U_{S}}={\widehat{x_{2}}}^{U_{S}}$ so $n_{12}^{U_{S}}=0$.

