Operation stability analysis of district heating substation from the control perspective

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Highlights

- ✓ Oscillatory of flow rate observed in district heating substation
- ✓ Mathematical model describing thermal dynamics of heating substation
- ✓ Control theory based criterion for operation stability of heating substation
- ✓ Conditions that leads to operation instability of district heating substation
- ✓ Controller tuning of the plate heat exchanger to ensure robust stability

Keywords

District heating substation; Mathematical model; Operation instability; Stability analysis; Feedback control; Plate heat exchanger

Abstract

Since the heating substation plays a key role in transferring the thermal energy from the primary network to the secondary network and controlling the heat output of district heating system to meet the thermal load, high operation performance of heating substation is essential for energy conservation, cost saving and emission reduction. The dynamic operation stability of heating substation is a very important dynamic characteristic of heating substation and largely affects the operation efficiency of district heating system. The operation instability of heating substation mainly manifest as flow rate and pressure oscillations, which will deteriorate the network hydraulic condition, break the network thermal balance, reduce the consumer comfort and increase the energy cost of the pumping system. Since heating substations will easily operate unstably under some conditions, this paper presents a theoretical method to analyze the stability and retune the feedback controller for operation stability of heating substation. Mathematical model of the plate heat exchanger was established and the feedback control theory was adopted to study the operation stability of heating substation. Based on the mathematical model and feedback control theory, a stability criterion was proposed for analyzing the operation stability of district heating substation effectively. The dynamic model of plate heat exchanger was validated with measured data. Simulation results show that controller tuned at certain operating condition can’t ensure operation stability of heating substation, when operating condition varies in large range. The stability analysis method proposed in this paper can be applied to analyzing the operation stability and tuning the controller of heating substation to enhance the operation stability.

Nomenclature	
A	matrix of state space model
b	width of the plate heat exchanger flow channel (m)
$B_1,\ B_2,\ B_3,\ B_4$	matrices of linearized state space model
C	matrix of linearized state space model
c_p	specific heat capacity of water (W/kg · K)
C_v	flow capacity of control valve
C_{Nu}	the empirical parameter
d	distance between neighboring plates (m)
D	hydraulic diameter (m)
$G(s),\ G_{d,1}(s),\ G_{d,2}(s),$ $G_{d,3}(s),\ G_{d,4}(s),$	transfer function of linearized plate heat exchanger model
i	$\sqrt{-1}$
k	overall heat transfer coefficient (W/m^2 · K)
K	transfer function of controller
k_c	controller gain
k_v	valve gain
l	length of flow channel
$L(\mathrm{i}\omega)$	loop transfer function
M	number of flow channels in each side
N	number of the control volumes in a flow channel
$n_1,\ n_2$	the empirical parameter
Nu	the Nusselt number
Pr	the Prantl number
q	volume flow rate (m^3/s)
R	the rangeability of the valve
Re	the Reynolds number
s	the Laplace variable
t	time (s)
T	temperature (^{o}C)
x	coordinate along the flow channel (m)
z	controller zero
λ	heat conductivity coefficient (W/m^2 · K)
μ	the dynamic viscosity
ρ	density of the water (kg/m^3)
τ	time delay of temperature sensor (s)
δ	small deviation & increment symbol of variable
ω	frequency (rad/s)

Nomenclature	
Δx	length of a control volume (m)
Subscripts	
h	high temperature side
in	inlet
l	low temperature side
out	outlet

1. Introduction

In China, The total energy consumption of district heating systems in northern areas covers 24% of the total energy cost of building energy systems [1]. Therefore, improving the operation efficiency of district heating system is important to reducing energy cost and enhancing room comfort. In large scale district heating systems, the heating substations are the terminals, which control the heat outputs to the secondary networks. Efficient regulation of the district heating network relies on effective operation and control of the heating substation.

There have been numerous researches on improvement of heating substation efficiency by applying new control strategy. Gustafsson et al. [2] developed a new control approach for indirectly connected district heating substations based on a physical model, which maximizes the ΔT of the district heating network. They also verified the control method experimentally through implementation of the control method in a real district heating substation; the results confirms that it is possible to control the radiator system based on the primary supply temperature while maintaining comfort; however, conclusions regarding improvements in ΔT were hard to distinguish [3]. Since high return temperature will lead to large amount of overall distribution energy cost, the temperature difference faults can be detected and eliminated by using fault detection approaches. Gadd and Werner [4, 5] presented a fault detection based method to achieve low return temperatures in district heating substations.

Modeling the heating substation is important to analyzing and evaluating the operation performance of heating substation. Brand et al. [6] developed a numerical model for heating substation, which takes into consideration the effect of service pipes. With this model, they studied the effects of service pipe on waiting time for DHW, heat loss, and overall cost. Brand et al. [7] also used the commercial software IDA-ICE and Termis to model and analyze various solutions for controlling the redirected bypass flow and evaluated their performance and the effect on the DH network in heating substation. Kuosa [8] developed a numerical model for a district heating system with ring network, with which the variations of flow rates, pressure losses and overall heat transfer coefficients of plate heat exchanger in heating substation were simulated and analyzed on selected days. Dobos and Abonyi [9] developed the nonlinear dynamic model of the district heating network including the heat exchanger in heating substations, heat production units and pipelines to study the nonlinear model predictive control of district heating network.

Since plate heat exchanger is the core component of heating substation, mathematical modeling and control performance analysis of plate heat exchanger is important to improve the operation efficiency of district heating substation. Feedback control analysis and design of plate heat exchangers have been paid attentions.

Al-Dawery [10] established a first order model with time delay to suggest the transient responses of a plate heat exchanger, and a fuzzy logic controller of the plate heat exchanger was designed to achieve less settling time and oscillatory behavior. Michel and Kugi [11] developed a control strategy without knowledge of the heat transfer of plate heat exchanger based on controlling the total thermal energy stored in the heat exchanger and a Kalman Filter to estimate the states.

However there were few studies concerning the dynamic operation stability of heating substation, which is a very important dynamic characteristic for heating substation and largely affects the operation efficiency of district heating system. The operation instability of heating substation mainly manifest as flow rate and pressure oscillations, which will deteriorate the network hydraulic condition, break the network thermal balance, reduce the consumer comfort and increase the energy cost of the pumping system. Since heating substations will easily operate unstably under some conditions, this paper presents a theoretical method to analyze the stability and retune the feedback controller for operation stability of heating substation. The theoretical method presented in this paper mainly utilized the techniques developed in feedback control theory [12]. The feedback control theory has been effectively applied to analyze the operation stability and elimination of oscillations in a central heating system using pump control [13]. Tahersima et al. utilized the feedback control theory to study the stability performance and developed a gain scheduling controller of radiator heating system in a room [14]. In our previous work, the control oriented approaches were adopted to establish an accurate low order model of room heating system and propose a two-degrees-of-freedom H_∞ loop-shaping controller [15]. Research on the operation stability of district heating system focuses on dynamic variation and fluctuation of flow rates, pressures and temperatures in the district heating network, which is very important and applicable in improving the operating efficiency of the district heating network. In this paper, the operation stability of district heating substation was studied from the control perspective. An analytical tool was developed to analyzing the operation stability of district heating substation.

2. Control levels of district heating system

Fig. 1 shows the schematic of district heating system. The hot water is generated from the heat source and delivered to the heating substations by the primary circulation pump along the primary pipelines. The heating substations are usually located near the center of load regions. The main components of a heating substation are the plate heat exchanger, primary control valve, secondary circulation pump and the control system.

In order to provide sufficient thermal energy effectively, the district heating system is usually regulated under three control levels. The first level is named the centralized control; this level functions at the heat sources, which controls the primary supply temperature and the pump speed to meet the total heating load variations of the network. The second level is called the local control; this level functions at each heating substation, which controls the secondary supply temperature and secondary pump speed to satisfy the variable heating load of the heat consumers. The secondary supply temperature is controlled by adjusting the control valve of the plate heat exchanger at the primary side. The third level is personal control; this level functions at each radiator, which controls the flow rate of radiator according to the room temperature difference between the desired and value to maintain the room air temperature around the desired value. In district heating system operation, the three control levels work simultaneously to allocate heat to each consumer. Fig. 1 shows the control structure of district heating network. As is shown, local control plays a key role in controlling the heat output of the primary network.

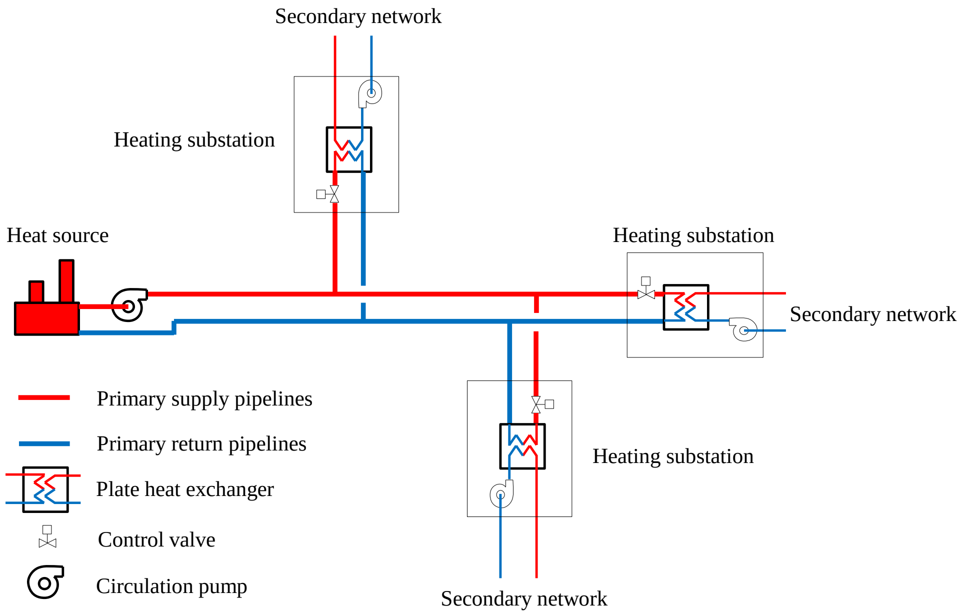


Fig. 1. Schematic of district heating system and heating substation

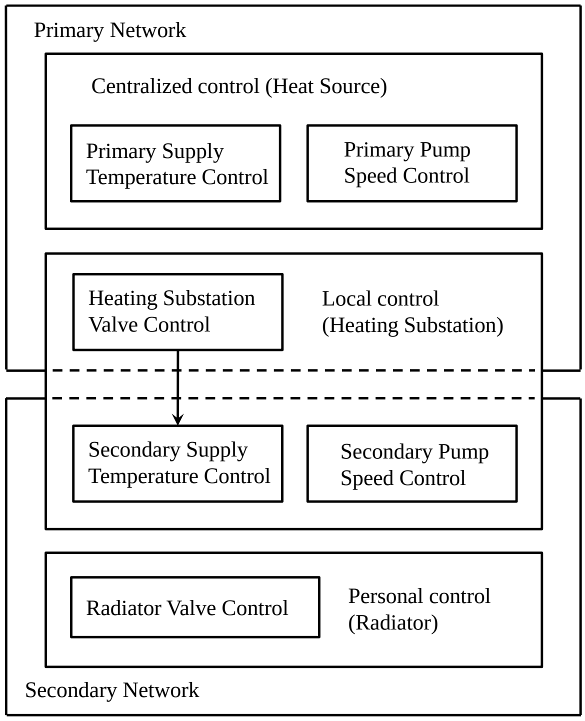


Fig. 2. Control levels of the district heating network.

Fig.3 shows the schematic of the feedback control structure in heating substation, which composes the main part of the local control. Efficient operation of the heating substation requires the feedback control loop to be stable. However, numerous heating substations are working in unstable conditions, and oscillations of flow rate and pressure always occur. The instability is resulted from the nonlinearities of valve and plate heat exchanger, sensor delay and improperly-tuned controller.

Fig.4 shows the measured primary side flow rate, supply temperature and outdoor temperature of a district heating substation in Tianjin, China. The secondary side of the heating substation is a commercial building with 22 floors. These measured data are to illustrate the operation instability of a district heating substation. As is shown, oscillation of primary side flow rate occurs when supply temperature is high. Such oscillation may deteriorate the hydraulic condition of the district heating network, increase the energy cost of pumping system and reduce the lifetime of control valve. In order to investigate the instability analytically and stabilize the heating substation with properly-tuned controller, mathematical models of the heat exchanger and the feedback controller are established.

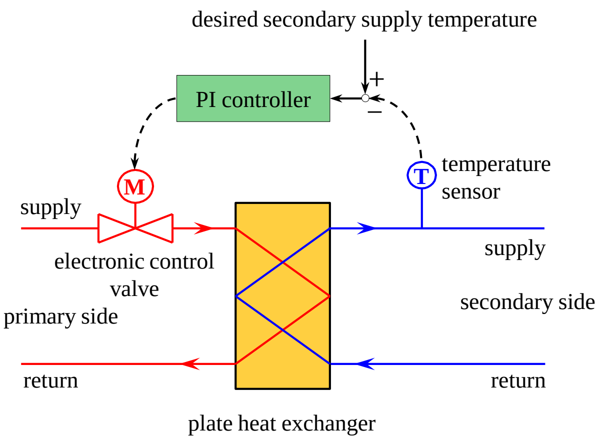
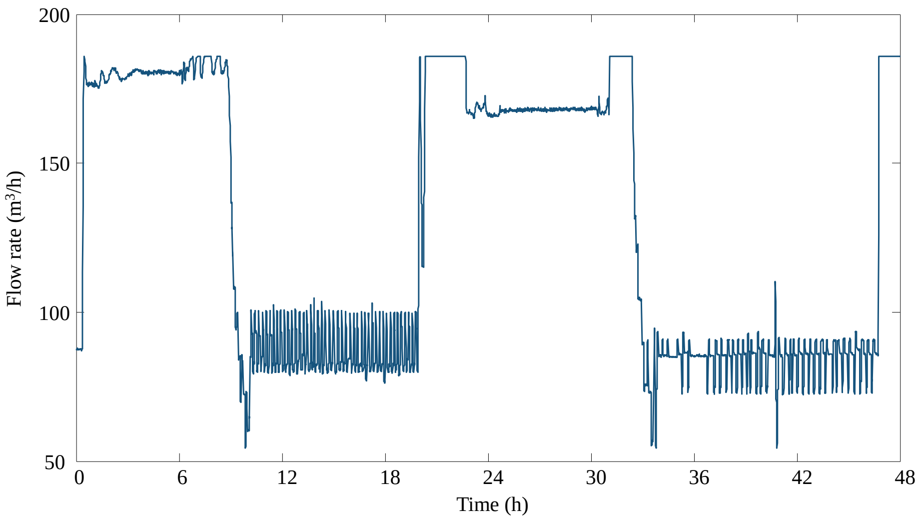
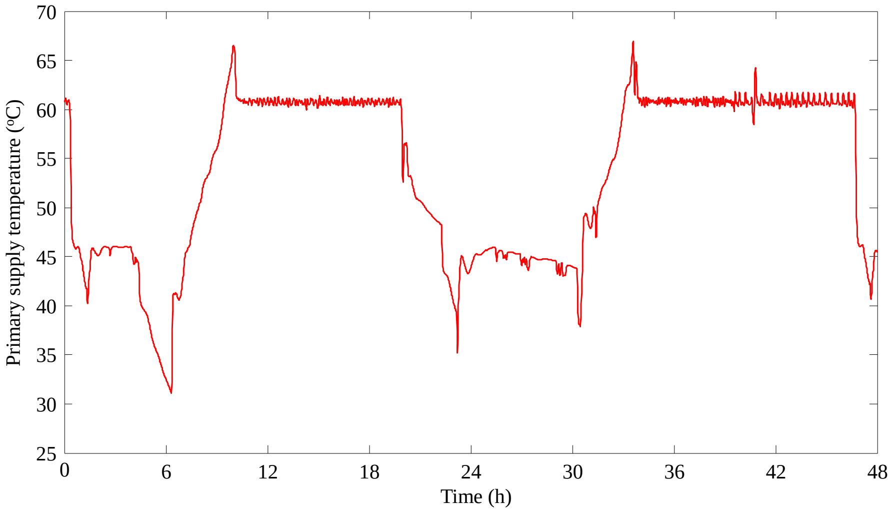


Fig. 3. Schematic of heating substation control system



(a)



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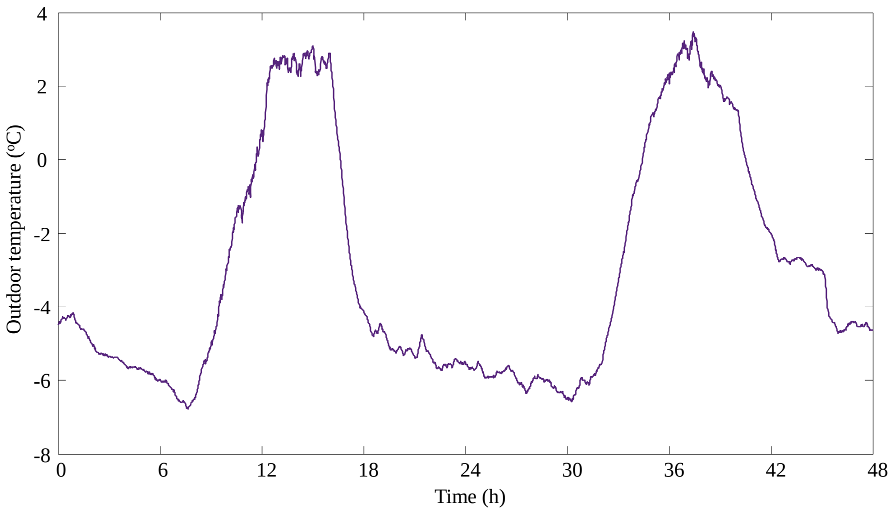


(c)

Fig.4. Measured primary side data of a heating substation in Tianjin, China. (a) Measured primary side flow rate. (b) Measured primary supply temperature. (c) Measured outdoor temperature.

3. Modeling the plate heat exchanger

The thermal dynamics of plate heat exchanger can be described by a pair of partial differential equations (PDEs) [11]:

$$\frac{\partial T_h}{\partial t} = \frac{q_h}{Mbd}\frac{\partial T_h}{\partial x} + \frac{k}{\rho c_p d}(T_l - T_h) \tag{3}$$

$$\frac{\partial T_l}{\partial t} = -\frac{q_l}{Mbd}\frac{\partial T_l}{\partial x} + \frac{k}{\rho c_p d}(T_h - T_l) \tag{4}$$

where T_h and T_l are the temperature distribution of the high temperature side and the low temperature side, respectively. M is the number of flow channels in each side. b is the width of each flow channel. d is the distance between neighboring plates. q_h and q_l are the flow rates of the high temperature side and low temperature side, respectively. ρ is the water density. c_p is the specific thermal capacity. k is the overall heat transfer coefficient of the plate heat exchanger, which is the function of flow rates of the two sides. Calculation of k is summarized in Appendix A.

The PDE model can be reduced to an ordinary differential equation (ODE) model with the finite volume/difference method. Fig. 4 illustrates the finite volume division of the plate heat exchanger. The ODE model of plate heat exchanger can be derived as:

$$\frac{dT_{h,j}}{\partial t} = \frac{q_h}{Mbd\Delta x}\left(T_{h,j+1} - T_{h,j}\right) + \frac{k}{\rho c_p d}\left(T_{l,j} - T_{h,j}\right) \tag{5}$$

$$\frac{\partial T_{l,j}}{dt} = -\frac{q_l}{Mbd\Delta x}\left(T_{l,j} - T_{l,j-1}\right) + \frac{k}{\rho c_p d}\left(T_{h,j} - T_{l,j}\right) \tag{6}$$

where $\Delta x = l/N$; $j = 1, \dots, N$; l is the channel length; N is the number of volumes divided in each channel; $T_{h,N+1} = T_{h,in}$ and $T_{l,0} = T_{l,in}$ are the inlet temperatures of the high temperature and low temperature sides, respectively; $T_{h,1} = T_{h,out}$ and $T_{l,N} = T_{l,out}$ are the outlet temperatures of the high temperature and low temperature sides, respectively.

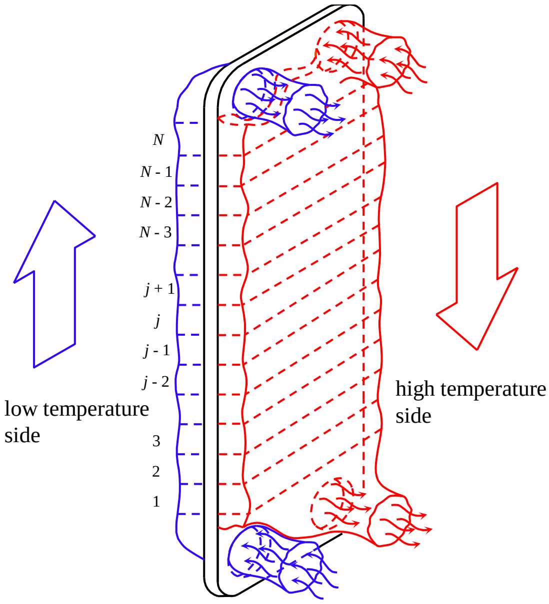


Fig. 4. Finite volume division of the plate heat exchanger flow channel.

The feedback control theory based method adopted in this paper generally includes three steps. The first step is to develop the dynamic model of plate heat exchanger, control valve and controller. The dynamic model of plate heat exchanger is a pair of nonlinear ordinary differential equations describing the thermal dynamics of the heat transfer process between the high temperature and low temperature sides. Since the models of plate heat exchanger and control valve are nonlinear, they should be linearized at an equilibrium point for stability analysis of the heating substation control loop with the feedback control theory. The second step is to do Laplace transform for the linearized models and derive the loop transfer function of the whole system $L(s)$ (transfer function from primary flow rate to secondary supply temperature) [12]. The third step is to draw the curve of $L(s)$ on the complex plane for s varying along the imaginary axis from 0 to $i\infty$. This curve is called the

Nyquist curve [12]. The operation stability of heating substation can be judged with the relation between the Nyquist curve and the point $(0, -1)$ on the complex plane. The Nyquist curve method for analyzing stability of dynamic systems named the Nyquist criterion was developed in 1930s, which has become a core concept and technique of classical control theory [12]. The Nyquist criterion is very applicable to practical problems and has been extended to more modern control technique [17].

The rest of this paper is organized as follows. The next section illustrates the control levels of the whole district heating system. The primary flow rate oscillation was observed from the measured data of a heating substation in Tianjin, China. Then the nonlinear ordinary differential equation model of plate heat exchanger was derived. The stability analysis method for heating substation was developed with the linearized model of heating substation and the Nyquist criterion. The nonlinear plate heat exchanger model was validated with measured data from the literature. The dynamic responses and operation stability of a heating substation were studied for application of the proposed stability analysis method.

4. Operation stability analysis method

4.1. Model linearization

Dynamic model of plate heat exchanger Eq. (5) and (6) is nonlinear. In order to analyze the operation stability with the frequency domain method, model linearization is required [17]. The nonlinear model described by Eq. (5) and (6) can be linearized into the following linear state space form:

$$\frac{dT}{dt} = AT + B_1 q_h + B_2 q_l + B_3 T_{h,in} + B_4 T_{l,in} \quad (7)$$

$$T_{l,out} = CT \quad (8)$$

where $T = [\,T_{h,1} \quad T_{h,2} \quad \cdots \quad T_{h,N} \quad T_{l,1} \quad T_{l,2} \quad \cdots \quad T_{l,N}\,]^{\mathrm{T}}$. A, B_1, B_2, B_3, B_4 and C are constant matrices (details for derivation of the matrices are listed in **Appendix A**). Doing Laplace transform to Eq. (7) and (8), the input-output model of plate heat exchanger can be written in the following form:

$$T_{l,out} = G(s)q_h + G_{d,1}(s)q_l + G_{d,2}(s)T_{h,in} + G_{d,3}(s)T_{l,in} \quad (9)$$

where $G(s)$, $G_{d,1}(s)$, $G_{d,2}(s)$ and $G_{d,3}(s)$ are transfer functions, that can be calculated with the matrices A, B_1, B_2, B_3, B_4 and C (see **Appendix A**). When linearizing a nonlinear system like Eq. (5) and (6), an equilibrium point should be specified. The equilibrium point of Eq. (5) and (6) is the solution of the steady state Eq. (5) and (6) (of which the time derivatives are made zero), with specified steady state inputs: q_h, q_l, $T_{h,in}$ and $T_{l,in}$. Therefore, with different steady state inputs, different equilibrium points can be derived. Since the matrices: A, B_1, B_2, B_3 and B_4 of the linearized model Eq. (7) and (8) are dependent on the equilibrium point. Different selection of equilibrium points will lead to different linearized models. However, all of the possible equilibrium points of the nonlinear system (Eq. (5) and (6)) lead to a set of linearized models, with which it is sufficient to study the robust stability of the nonlinear system [17].

The input-output structure of plate heat exchanger is illustrated in Fig. 5. The inputs can be divided into two categories: manipulated inputs and disturbance inputs. In district heating substation, the secondary supply temperature $T_{l,o}$ is controlled by adjusting the primary flow rate q_h. Therefore q_h is the manipulated input, and the primary supply temperature $T_{h,in}$, secondary flow rate q_l and secondary return temperature $T_{l,in}$ are disturbance inputs .

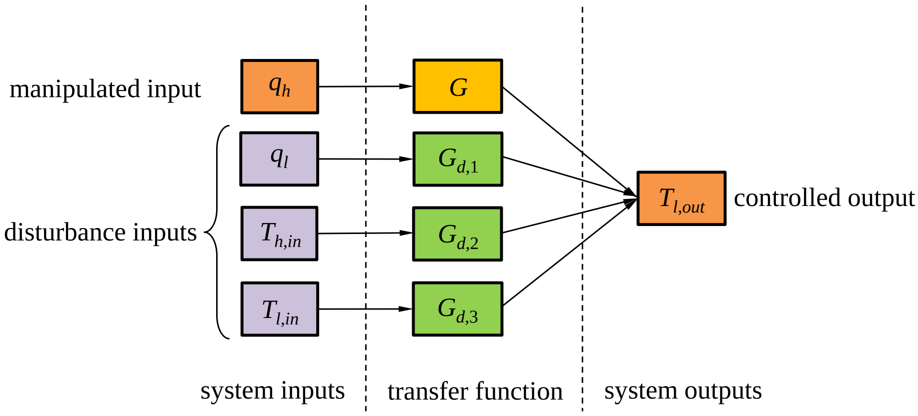


Fig. 5. Input-output structure of the plate heat exchanger.

4.2. Models of the controller and valve

The PI control law is usually adopted as feedback controller K in district heating substation, and K can be represented as the Laplace transform form:

$$K = k_c \frac{s+z}{s} \tag{10}$$

where $k_c > 0$ is the controller gain and $z > 0$ is controller zero.

The equal percentage valve is usually used in heating substation. The characteristic for equal percentage valve is nonlinear. The relation between flow rate and valve opening of equal percentage valve can be characterized by the following formula [13, 18]:

$$q = C_{\mathrm{v}} \sqrt{\Delta p} R^{x-1} \tag{11}$$

where C_{v} is the flow capacity of the valve; R is the rangeability of the valve; Δp is the pressure difference of the valve. Fig. 6 shows the characteristic of an equal percentage valve with $C_\mathrm{v} = 450$, $\Delta p = 0.23\mathrm{atm}$ and $R = 50$. Fig. 6 also shows the tangent line of valve characteristic line. The slope of the tangent line can be derived by:

$$k_v = \left(\frac{\partial q}{\partial x}\right)_{x=x_0} \tag{12}$$

x_0 is the operating point.

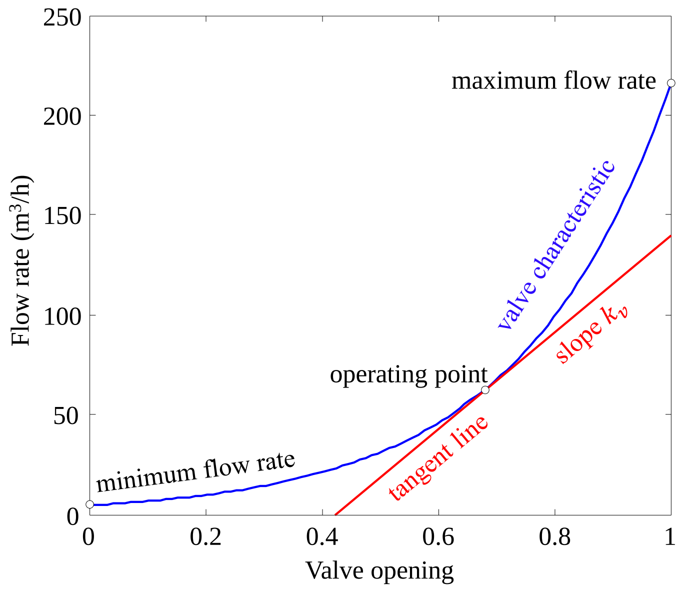


Fig. 6. Characteristic of an equal percentage valve.

The block diagram of heating substation feedback control system can be illustrated as Fig. 7. Since the valve characteristic is nonlinear, the valve gain k_v is varying with the operating point x_0 changing. The varying range of valve gain $k_{v,min} \le k_v \le k_{v,max}$ can be derived from the valve characteristic Eq. (11) and (12) as follows.

$$k_{v,min} = \left(\frac{\partial q}{\partial x}\right)_{x=0} = C_\mathrm{v}\sqrt{\Delta p}R^{-1}\ln R$$

$$k_{v,max} = \left(\frac{\partial q}{\partial x}\right)_{x=1} = C_{\mathrm{v}}\sqrt{\Delta p}\ln R$$

Therefore, the varying valve gain satisfies:

$$C_{\mathrm{v}}\sqrt{\Delta p}R^{-1}\ln R < k_v \leq C_{\mathrm{v}}\sqrt{\Delta p}\ln R \tag{13}$$

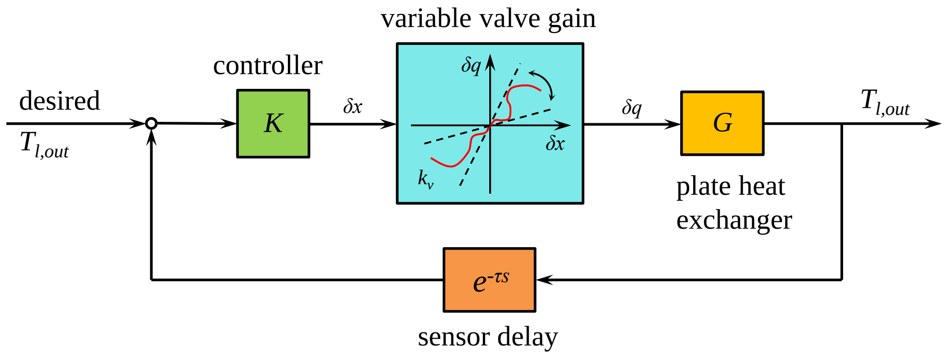


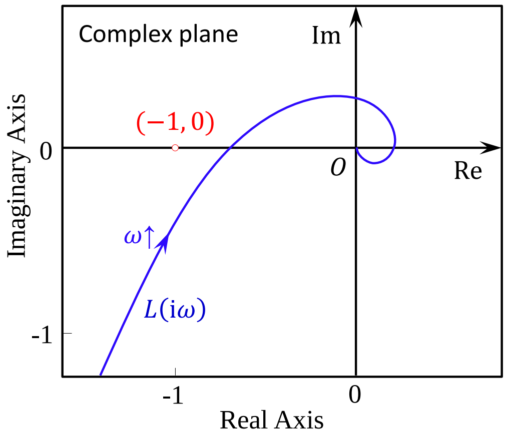
Fig. 7. Block diagram of heating substation control system.

4.3. Stability criterion of heating substation

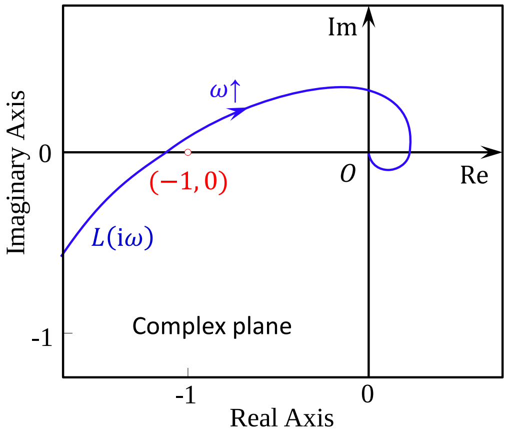
According to the feedback control theory [17], the operation stability of feedback control system can be judged by the Nyquist stability criterion. If introducing this criterion to heating substation system, the operation stability of district heating substation can be judged by the following criterion:

If the curve of $L(s) = k_v K(s) G(s) e^{-\tau s}$ (s is varying along the imaginary axis from 0 to i∞) encircles or crosses the point (-1, 0) on the complex plane, the heating substation control system will be unstable.

Here the complex variable s can be replaced by the pure imaginary variable $\mathrm{i}\omega$ with $0 \leq \omega < +\infty$, where $\mathrm{i} = \sqrt{-1}$. Fig. 8 shows three cases of the relation between the curve of $L(\mathrm{i}\omega)$ and the point (-1, 0). For cases (b) and (c), the heating substation will be unstable.



(a)



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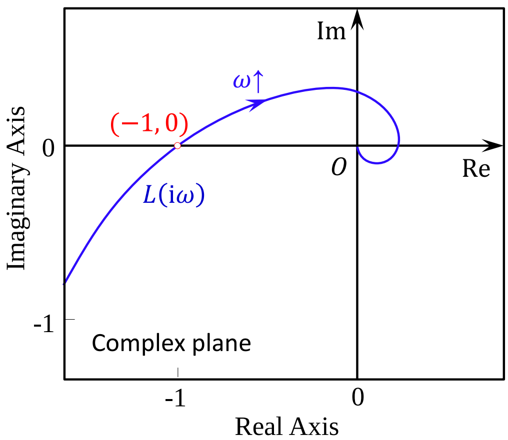


(c)

Fig. 8. Nyquist plots for operation stability analysis of heating substation. (a) The curve of $L(\mathrm{i}\omega)$ doesn't encircle point (-1, 0). (b) The curve of $L(\mathrm{i}\omega)$ encircles point (-1, 0). (c) The curve of $L(\mathrm{i}\omega)$ crosses the point (-1, 0).

Since the dynamics of plate heat exchanger is nonlinear, the linearized model G will be perturbed to G' ($G' \neq G$), if the operating condition (equilibrium point) is changed. And the perturbation may lead to instability of the heating substation control system. According to the proceeding criterion and Fig.8, operation stability of heating substation will be damaged, if the perturbed system G' causes the curve of $L'(\mathrm{i}\omega) = k_v{}'K(\mathrm{i}\omega)G'(\mathrm{i}\omega)e^{-\mathrm{i}\omega\tau}$ to encircle or cross the point (-1, 0). The conversion from stability to instability may happen, when operating condition of plate heat exchanger in heating substation changes largely. As is observed in Fig. 3, with the primary supply temperature increasing to a high level, the operation stability of heating substation will be damaged, and the oscillatory will occur.

Therefore, case (a) for a certain operating condition doesn't mean that the heating substation will be stable for all operating conditions. To ensure robust stability of heating substation at all conditions, the case (a) should be held for all perturbed models G' at any operating conditions. This criterion also indicates that robust stability for all operating conditions can be ensured with a small loop gain: $|L(\mathrm{i}\omega)|$, which means that if the absolute value of $L(\mathrm{i}\omega)$ is small enough, operation stability can always be satisfied. This can be intuitively observed from Fig. (8). Fig. 9 shows the Nyquist plots of all possible operating conditions (equilibrium points). As is shown if all of these curves do not encircle or cross point (-1, 0), the heating substation will be stable at all operating conditions. This is equivalent to that the worst case Nyquist curve doesn't encircle or cross point (-1, 0).

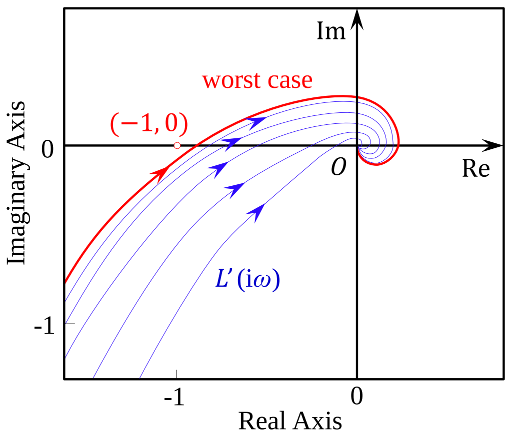


Fig. 9. Nyquist plots of all possible operating conditions (equilibrium points).

With this criterion, the operation stability of heating substation is predictable. Besides, for an unstable operation condition of heating substation, this criterion can also be used to analyze the key factor which leads to the instability of the heating substation and tune the controller to ensure robust stability.

5. Results and discussion

5.1. Validation of the nonlinear model

The nonlinear model of plate heat exchanger described by Eq. (5) and (6) was established in Simulink (Fig. 10). In Ref. [16], Michel and Kugi have tested the dynamic operation of plate heat exchanger, and the measured data were used to validate the dynamic model of plate heat exchanger. If the nonlinear model of plate heat exchanger described by Eq. (5) and (6) is effective, the model should be able to predict the dynamic operation in Ref. [16]. The plate heat exchanger parameters and measured data including the boundary conditions of inlet temperatures and flow rates in both sides given in Ref. [16] have been used to validate the proposed nonlinear model described by Eq. (5) and (6). The nonlinear model was validated using the measured data and plate heat exchanger parameters given in Ref. [16]. The simulated and measured outlet temperatures $T_{h,out}$ and $T_{l,out}$ are shown in Fig. 11, and the relative errors of $T_{h,out}$ and $T_{l,out}$ are shown in Fig.12. Relative errors of the simulated $T_{h,out}$ and $T_{l,out}$ are both varying within $\pm$8%, which is in a satisfied range. Therefore, the nonlinear model of the plate heat exchanger Eq. (5) and (6) can describe the thermal dynamics of the plate heat exchanger with satisfied accuracy.

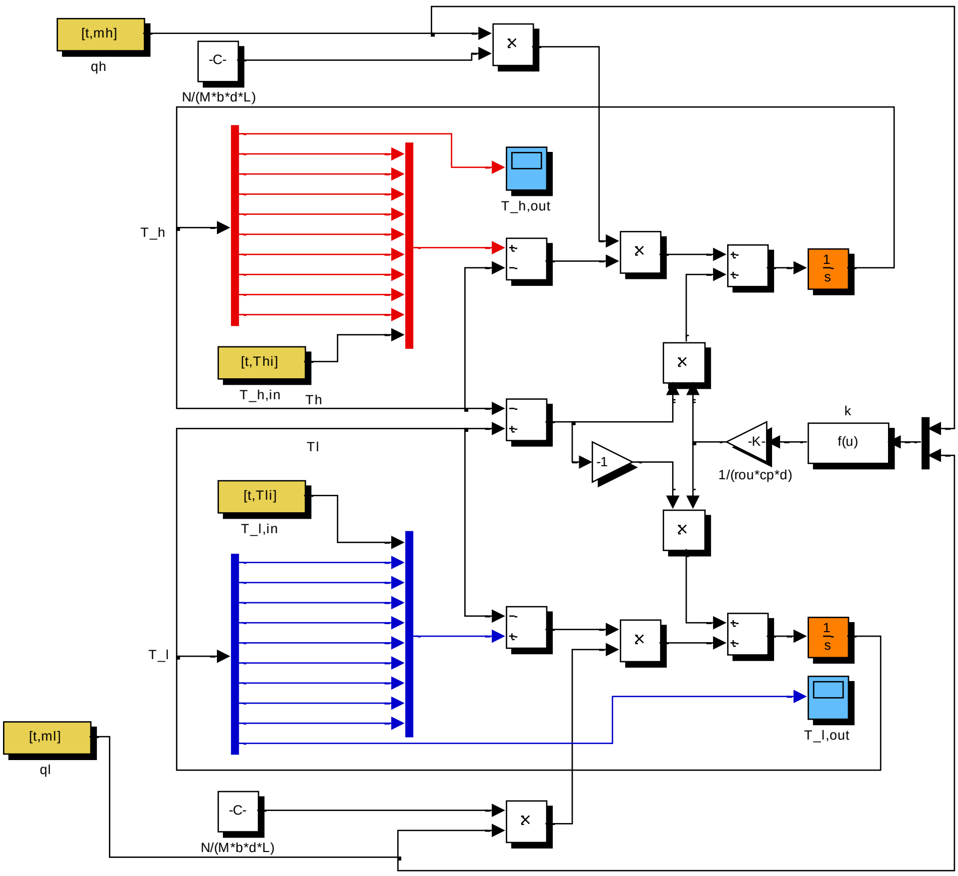
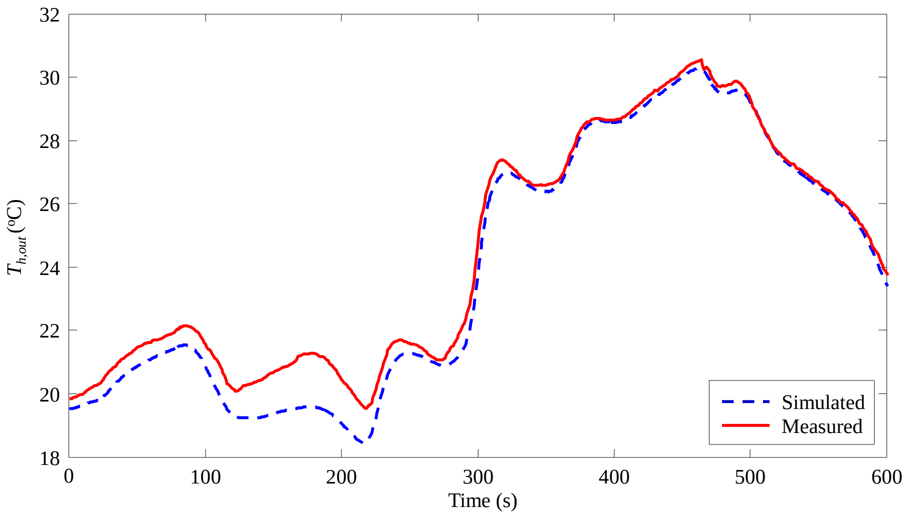
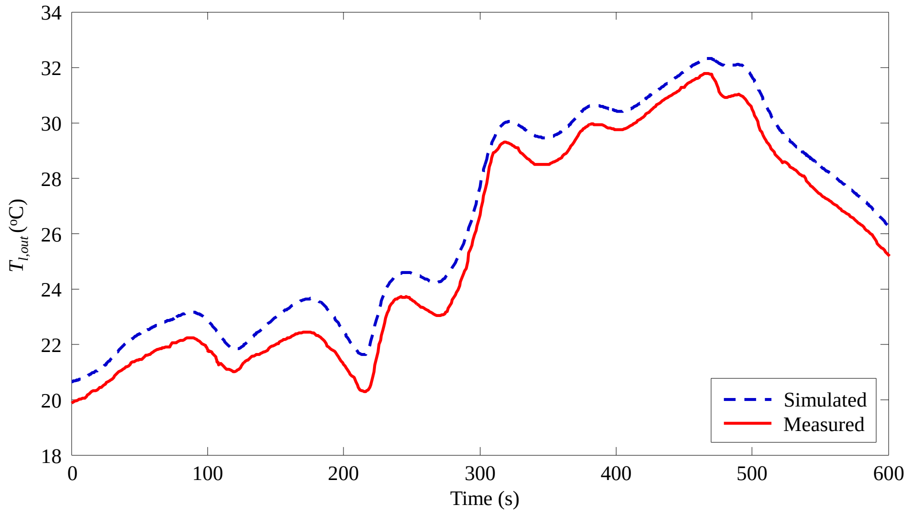


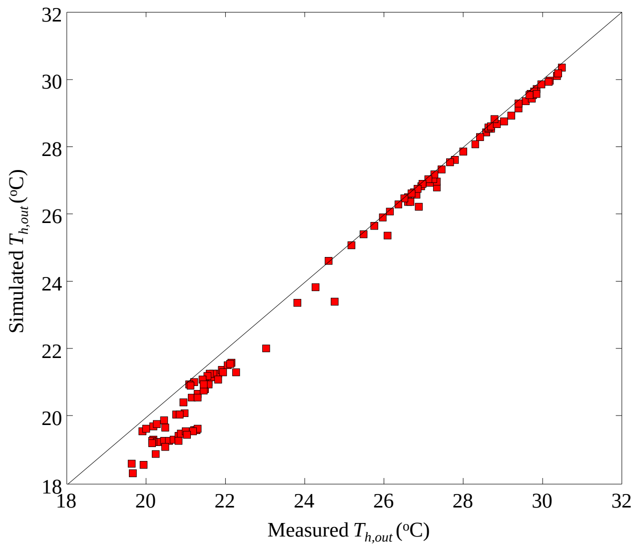
Fig. 10. Simulink model of the nonlinear plate heat exchanger model Eq. (5) and (6).



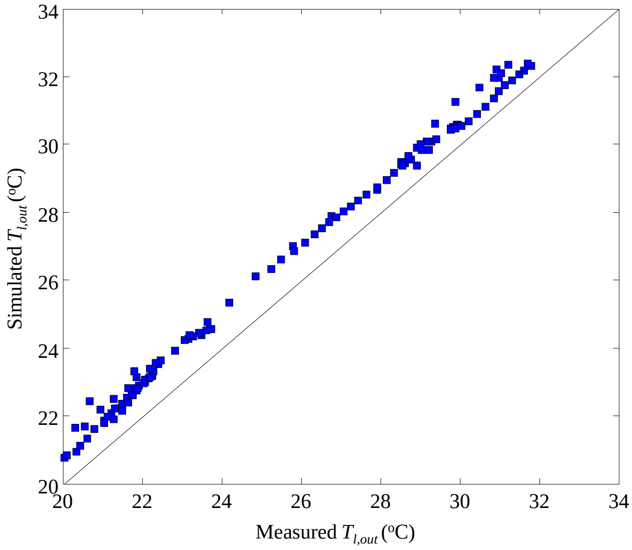
(a)


Simulated
Measured
Time (s)
$T_{l,out}$ (°C)

(b)

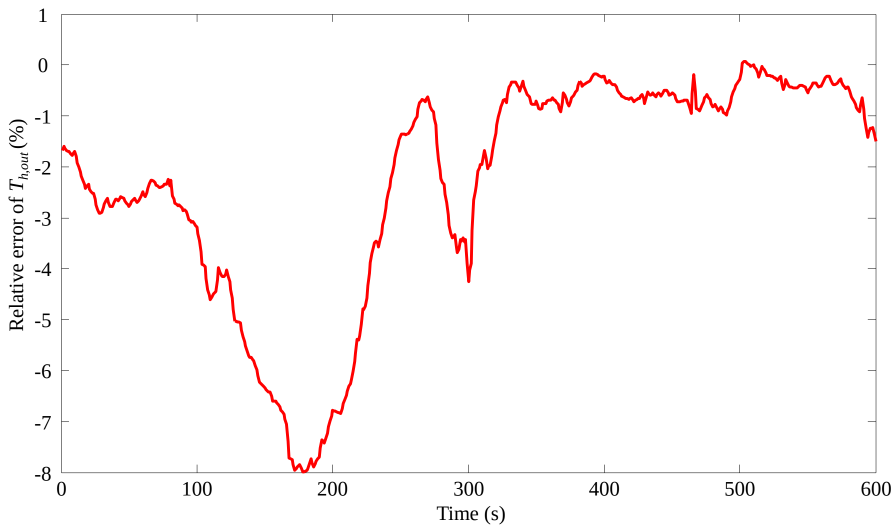

Measured $T_{h,out}$ (°C)
Simulated $T_{h,out}$ (°C)

(c)

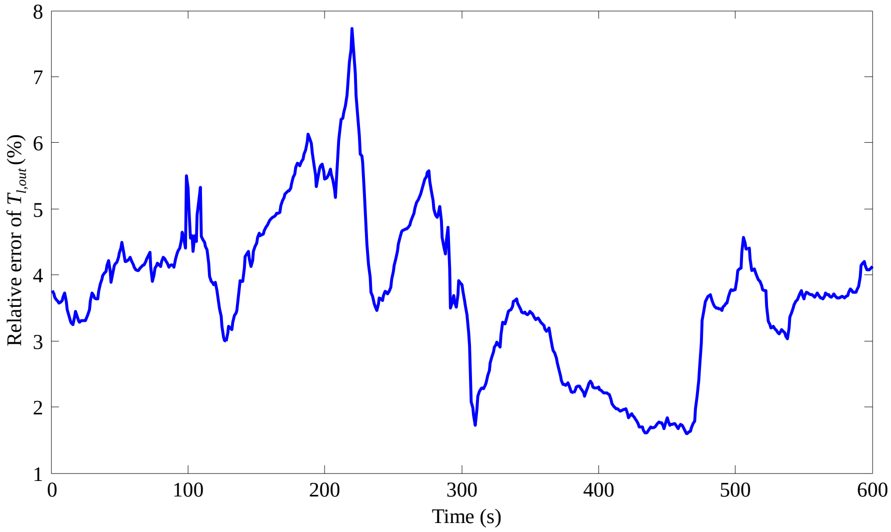


(d)

Fig. 11. Comparison of the measured and simulated outlet temperatures using parameters and measured data provided in Ref. [16]. (a) Outlet temperature of high temperature side. (b) Outlet temperature of low temperature side. (c) Validation of $T_{h,out}$. (d) Validation of $T_{l,out}$.



(a)



(b)

Fig. 12. Relative errors of the nonlinear model Eq. (5) and (6). (a) Relative error of $T_{h,out}$. (b) Relative error of $T_{l,out}$.

5.2. Control and stability of heating substation

The nonlinear heating substation control system composed of plate heat exchanger, equal percentage valve and PI controller was also established in Simulink with the mathematical models Eq. (5), (6), (10) and (11). The dynamic responses of the heating substation control system were calculated with Simulink. The parameters of plate heat exchanger are listed in Table 1. The valve characteristic is shown in Fig. 6. The time delay of temperature sensor is τ = 5s.

Description	Symbol	Value	Unit
channel width	b	0.8	m
channel length	l	1.36	m

water specific heat capacity	c_p	4220	$\mathrm{kJ/(kg \cdot K)}$
empirical parameters	C	0.64	/
distance between neighboring plates	d	4.5	mm
plate thickness	d_p	0.5	mm
number of flow channels in each side	M	137	/
empirical parameters	n_1	0.23	/
empirical parameters	n_1	0.75	/
water density	ρ	970	$\mathrm{kg/m^3}$
thermal conductivity of high temperature side water	λ_h	0.68	$\mathrm{W/(m\ K)}$
thermal conductivity of low temperature side water	λ_l	0.67	$\mathrm{W/(m\ K)}$
thermal conductivity of plate	λ_p	15	$\mathrm{W/(m \cdot K)}$
dynamic viscosity of high temperature side water	μ_h	0.00028	$\mathrm{Pa \cdot m}$
dynamic viscosity of low temperature side water	μ_l	0.00041	$\mathrm{Pa \cdot m}$

Table 1. Parameters of plate heat exchanger

In this section, the operation stability of heating substation was studied. The controller was tuned with the frequency domain approach [12]. In order to illustrate that if the controller is tuned and works well under low primary supply temperature condition, the operation stability may not be ensured at high primary supply temperature, dynamic performances of heating substation under low and high primary supply temperatures with the PI controller tuned at low supply temperature were studied.

5.2.1. Dynamic responses in low primary supply temperature

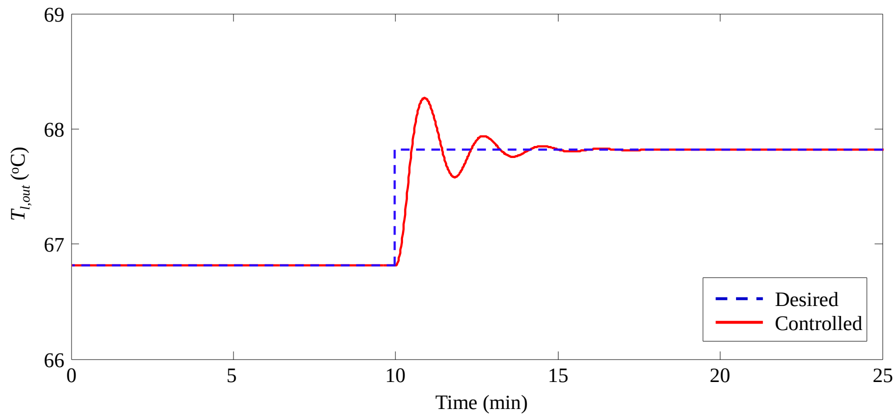
The PI controller is usually tuned under a certain operating condition. For the case study in this subsection, the following low primary supply temperature operating condition is chosen for controller tuning: $T_{l,in} = 40$ °C, $T_{h,in} = 70$ °C, $q_l = 0.03$

m^3/s. The PI controller is tuned by simulating the reference tracking response of $T_{l,out}$ around the operating condition. The PI controller is tuned as:

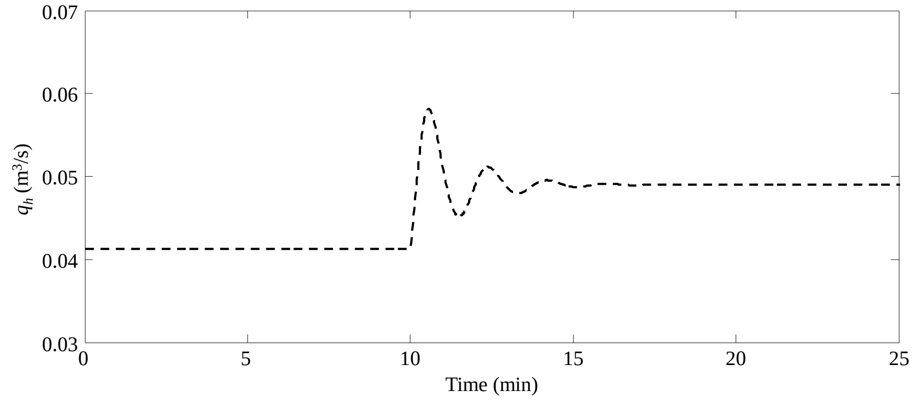
$$K_1 = 0.004\frac{s+1}{s} \quad (14)$$

The dynamic responses of heating substation control system were calculated with the nonlinear Simulink model.

Fig. 13 shows the reference tracking responses under the control of K_1. When the desired secondary supply temperature $T_{l,out}$ changes from 66.8 ^{o}C to 67.8 ^{o}C, the controlled $T_{l,out}$ tracks the new value in about 5 minutes and the overshoot is less than 0.5 ^{o}C. Fig. 14, Fig. 15 and Fig. 16 show the disturbance rejection responses to step variations of $T_{h,in}$, $T_{l,in}$ and q_l, respectively. Fig. 14 shows that when primary supply temperature $T_{h,in}$ changes from 70 ^{o}C to 71 ^{o}C, the deviation of secondary supply temperature $T_{l,out}$ from the desired value can be controlled within 1 ^{o}C. Fig. 15 shows that if secondary return temperature changes from 40 ^{o}C to 35 ^{o}C, the deviation of $T_{l,out}$ from desired value can be restricted within 0.5 ^{o}C. Fig. 16 shows that when the secondary flow rate q_l varies from 0.03 m^3/s to 0.04 m^3/s, the deviation of $T_{l,out}$ from desired value is within 2 ^{o}C.

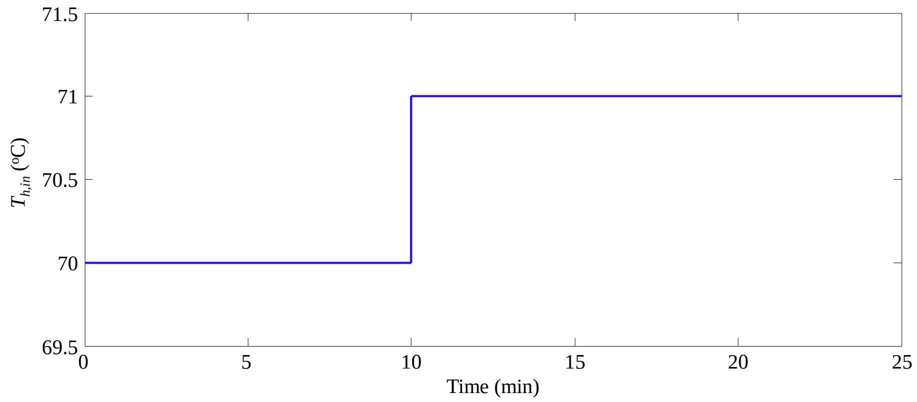


(a)

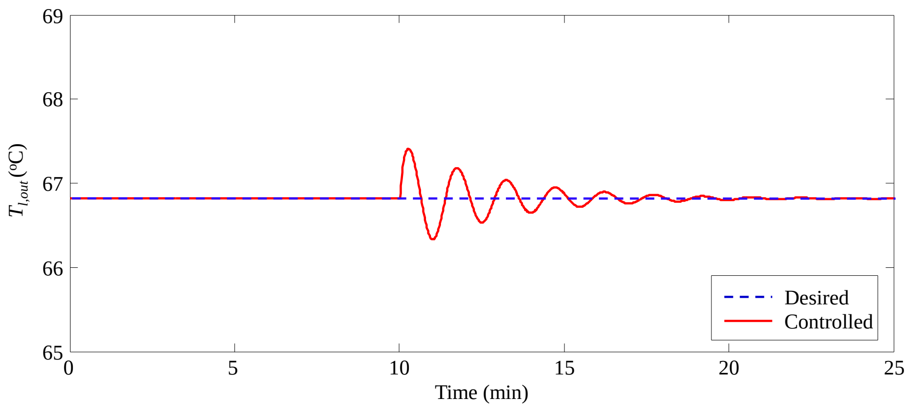


(b)

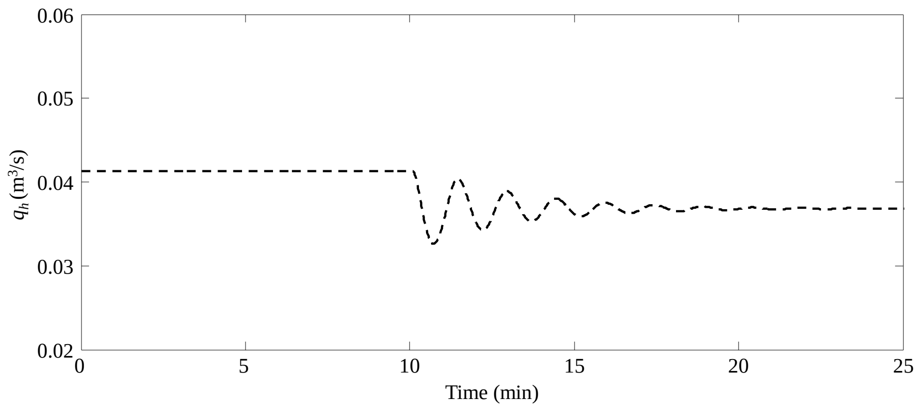
Fig. 13. System responses for tracking desired $T_{l,o}$ under the control of controller K_1 (with $T_{l,in} = 40$ °C, $T_{h,in} = 70$ °C, $q_l = 0.03$ m^3/s.). (a) Tracking response of $T_{l,o}$. (b) Tracking response of q_h.



(a)

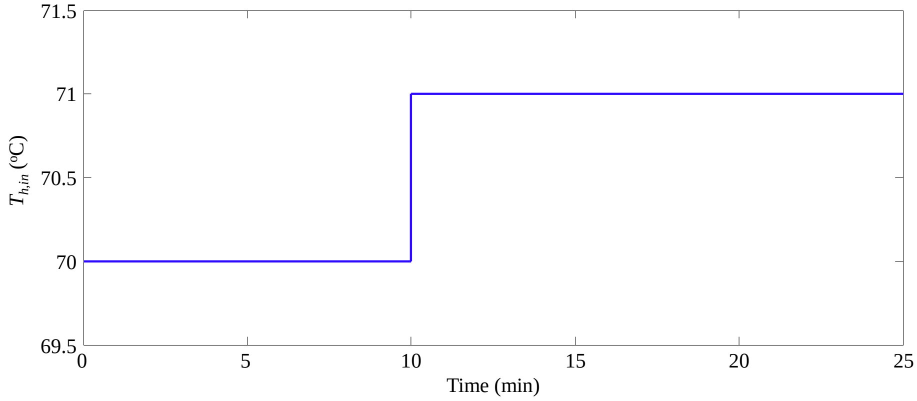


(b)

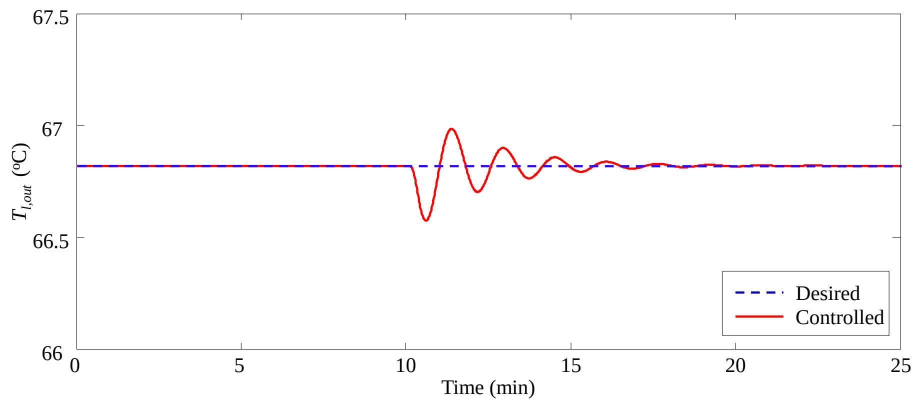


(c)

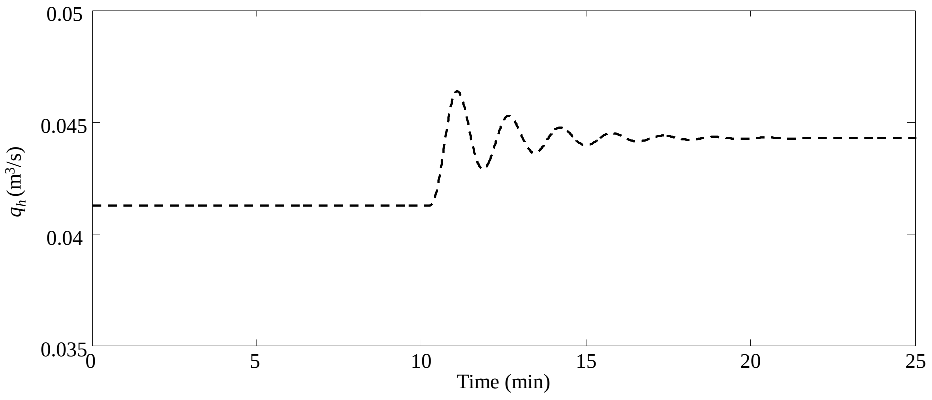
Fig. 14. System responses for rejecting the variation of $T_{h,in}$ under the control of K_1 (with $T_{l,in} = 40$ °C, $q_l = 0.03$ m^3/s and desired $T_{l,out} = 66.8$ °C). (a) Variation of $T_{h,in}$. (b) Response of controlled $T_{l,out}$. (c) Response of q_h.


71.5
71
70.5
70
69.5
$T_{h,in}$ (ºC)
0
5
10
15
20
25
Time (min)

(a)

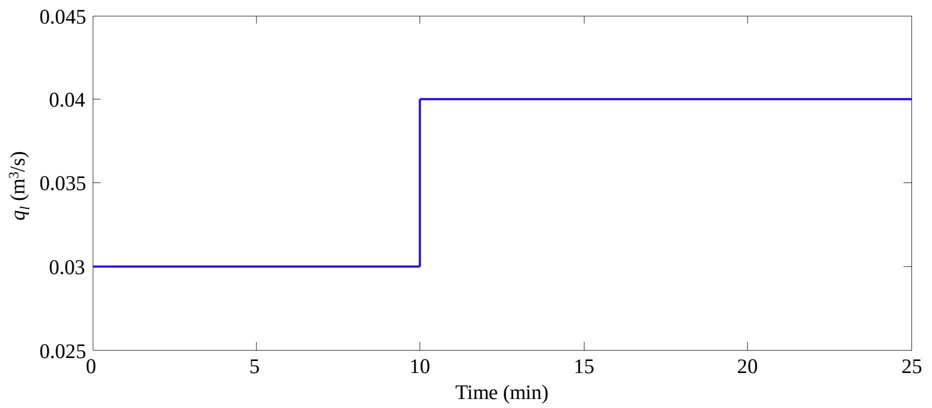

67.5
67
66.5
66
$T_{l,out}$ (ºC)
0
5
10
15
20
25
Time (min)
Desired
Controlled

(b)

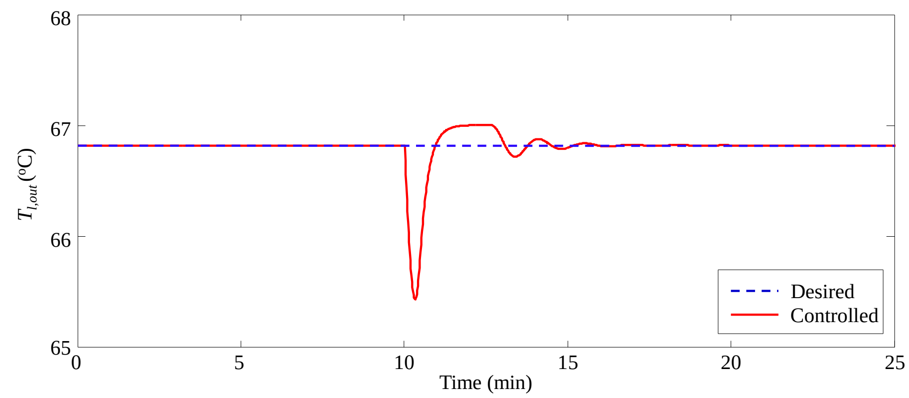


(c)

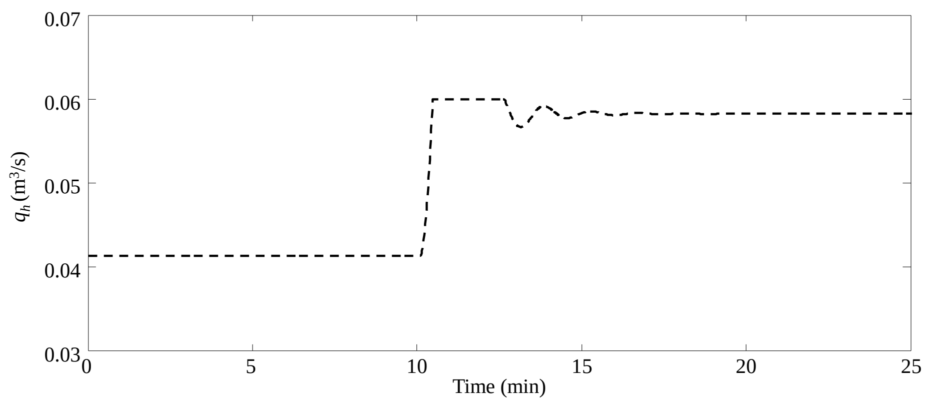
Fig. 15. System responses for rejecting the variation of $T_{l,in}$ under the control of K_1 (with $T_{h,in} = 70$ °C, $q_l = 0.03$ m^3/s and the desired $T_{l,out} = 66.8$ °C). (a) Variation of $T_{l,in}$. (b) Response of controlled $T_{l,out}$. (c) Response of q_l.



(a)



(b)



(c)

Fig. 16. System responses for rejecting the variation of q_l under the control of K_1 (with $T_{h,in} =$ 70 °C, $T_{l,in} = 40$ °C and desired $T_{l,out} = 66.8$ °C). (a) Variation of q_l. (b) Response of controlled $T_{l,out}$. (c) Response of q_h.

These reference tracking and disturbance rejection responses indicate that under the control of K_1, the heating substation system is stable around the operating condition of $T_{h,in}$= 70 °C, $T_{l,in}$ = 40 °C, q_l = 0.03 m^3/s. Hence, the tuned PI controller K_1 seems to be suitable for the control of the heating substation with the proposed parameters. However, the simulation test of responses is only conducted around the operating condition of $T_{h,in}$= 70 °C, $T_{l,in}$ = 40 °C, q_l = 0.03 m^3/s. When the operating condition changes largely, instability may occur as the measured data shown in Fig. 3.

5.2.2. Robust stability test and retuning of the controller

In order to investigate the stability of the heating substation under the control of controller K_1, the stability criterion proposed in subsection 4.3 was adopted. Fig. 17-(a) shows the Nyquist curves of the heating substation under the control of K_1 at all possible operating conditions. The possible operating conditions were defined as conditions that satisfy: 65 °C $\le T_{h,in} \le$ 95 °C, 30 °C $\le T_{l,in} \le$ 50 °C, 0.01 m^3/s $\le q_h \le$ 0.06 m^3/s and 0.03 m^3/s $\le q_h \le$ 0.04 m^3/s. This range can cover most of the operation conditions of the heating substation proposed in this paper. The proposed method in section 4.3 can be used to study the operation stability at these possible operating condition.

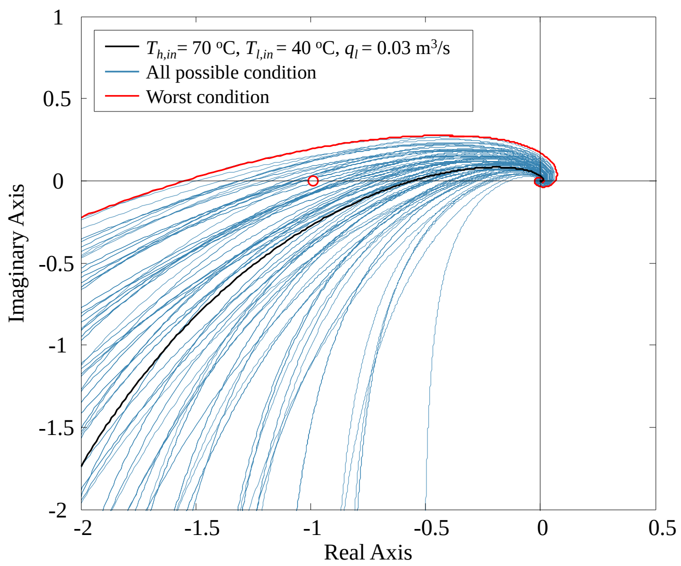
As is shown, the black curve, which denotes the Nyquist curve of the $T_{h,in}$= 70 °C, $T_{l,in}$ = 40 °C, q_l = 0.03 m^3/s condition, does not encircle or cross the point (-1, 0). This is the reason that the heating substation operates stably under the control of K_1 around the condition of $T_{h,in}$= 70 °C, $T_{l,in}$ = 40 °C, q_l = 0.03 m^3/s. However, there are still many cases don't satisfy this criterion. This means that the heating substation will be unstable under the control of K_1 in some operating conditions. Fig. 18-(a) shows the variation of $T_{h,in}$ and desired $T_{l,out}$, which will lead to instability of the heating substation (with $T_{l,in}$ = 40 °C, q_l = 0.03 m^3/s). As is shown in Fig. 18-(b) and 18-(c), the dynamic responses of $T_{l,out}$ and q_h under the control of K_1 (drawn in dark blue) become oscillatory when the primary supply temperature $T_{h,in}$ increases from 70 °C to 85 °C. The oscillation form of the primary flow rate q_h, shown in Fig. 18-(b), is very similar to the measured primary flow rate data shown in Fig. 3-(a). The heating substation becomes unstable because the increase of primary supply temperature $T_{h,in}$ makes the loop gain $|L(\mathrm{i}\omega)|$ larger and causes the curve of $L(\mathrm{i}\omega)$ to encircle point (-1, 0). This phenomenon also indicates that the high primary supply temperature conditions are worse than low supply temperature conditions. This also demonstrates that controllers tuned at a certain operating condition cannot ensure stability for all operating conditions.

In order to stabilize the heating substation, controller K_1 should be tuned again by considering all the possible operating conditions. The red Nyquist curve in Fig. 17-(a) denotes the worst operation condition. If the worst condition Nyquist curve doesn't encircle or cross the point (-1, 0), the heating substation will be stable at all operating condition. Therefore, according to Fig. 17-(a), to make the red curve do not encircle or cross the point (-1, 0), the controller gain k_c should be smaller. Fig. 17-(b) shows the Nyquist curves of the heating substation under the control of K_2 at all possible operating conditions, where

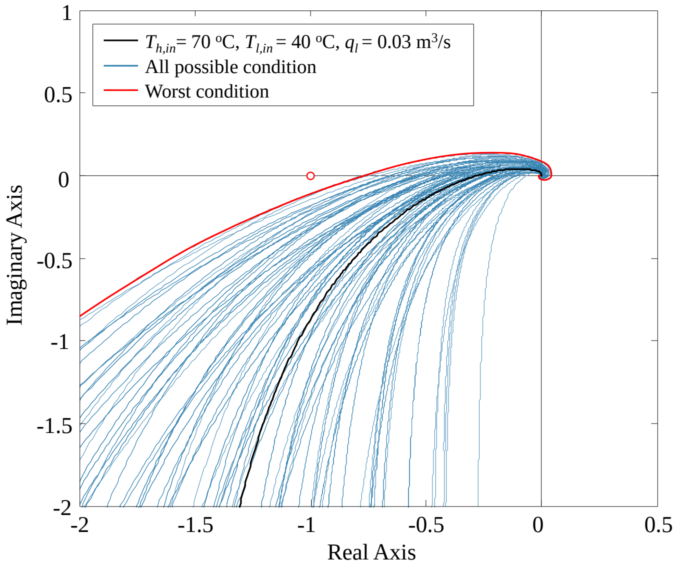
$$K_2 = 0.002\frac{s+1}{s} \quad (18)$$

As is shown in Fig. 17-(b), the Nyquist curves of all the possible conditions do not encircle or cross the point (-1, 0). Therefore the heating substation under the control of controller K_2 will be stable even in worse condition. In Fig. 18-(b) and Fig. 18-(c), the responses drawn in red are under the control of K_2. As is shown, the operation of heating substation remains stable even when primary supply temperature increases to very high.

Hence, the heating substation controller tuned at certain operating conditions may be unstable when operating condition changes in large range. To ensure operation stability of heating substation at all conditions, the operation stability should be tested when operating condition changes, and the proposed method can be used as a tool for analyzing the operation stability of heating substation at all possible operating conditions.

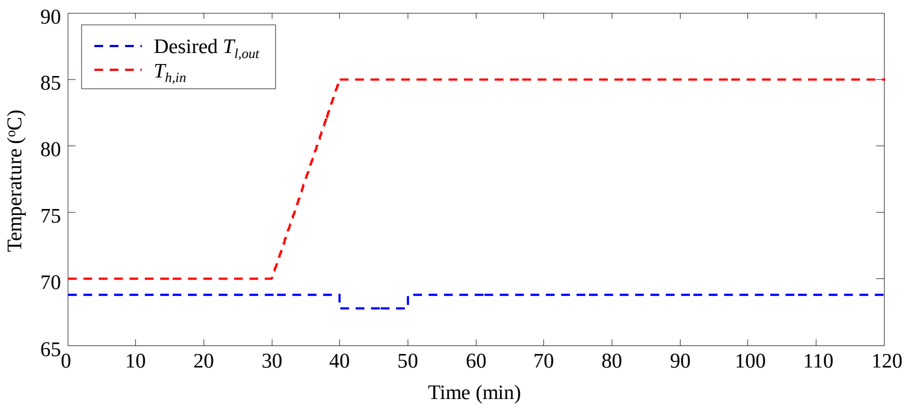


(a)

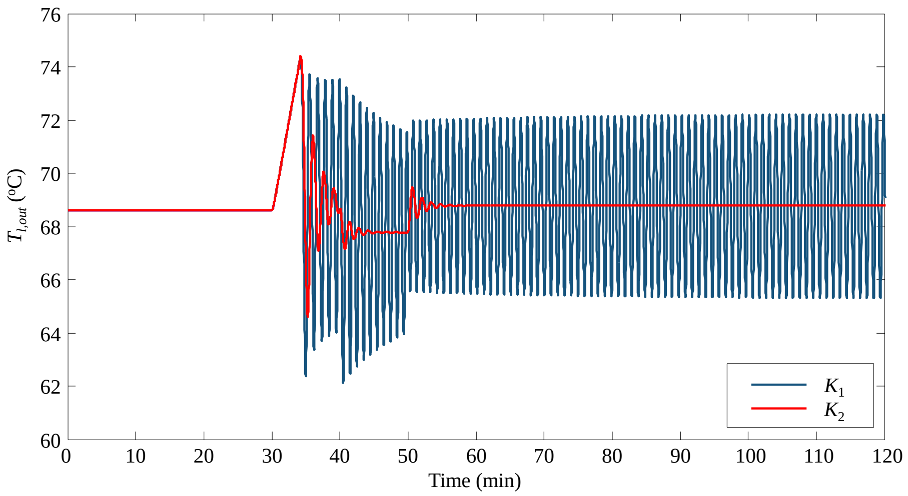


(b)

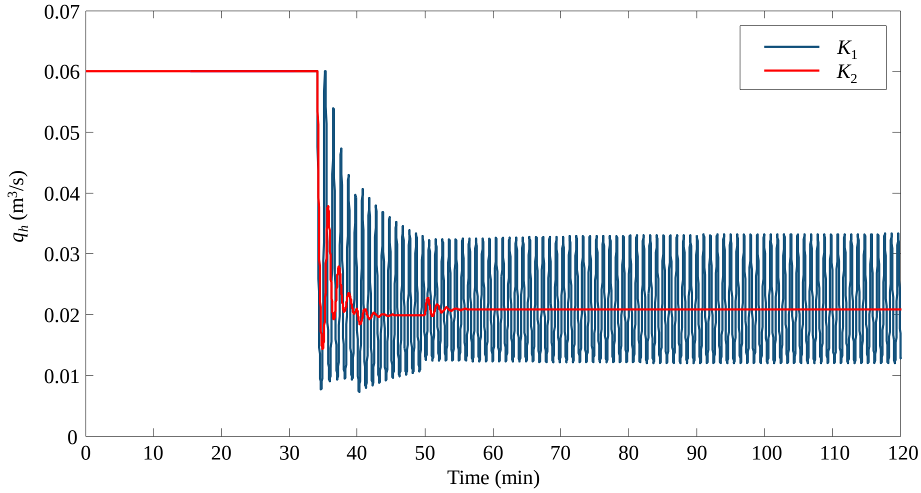
Fig. 17. Nyquist curves of heating substation system at all possible operating conditions. (a) Under the control of K_1. (b) Under the control of K_2.



(a)



(b)



(c)

Fig. 18. Dynamic responses in worse operating condition under the control of K_1 and K_2 (with $T_{l,in}$ = 40 ºC, q_l = 0.03 m³/s). (a) Variation of desired $T_{l,out}$ and $T_{h,in}$. (b) Responses of q_h. (c) Responses of controlled $T_{l,out}$.

6. Conclusions

In this paper, the nonlinear ODE model of plate heat exchanger was developed. Based on the nonlinear ODE model, the linearized model of plate heat exchanger for controller design and stability analysis was derived. The nonlinear ODE model of plate heat exchanger was solved with Simulink. In order to validate the nonlinear plate heat exchanger model, the parameters and measured data provided in Ref. [16] were adopted in simulation. The simulated results were compared with the measured data provided in the literature. Associated with the equal percentage valve model and controller model, the Nyquist stability criterion was proposed for analyzing the operation stability criterion of district heating substation at all operating conditions. The dynamic responses of heating substation under the control of a PI controller tuned at a certain operating condition were analyzed. And the operation stability of heating substation was also studied. And the following conclusions have been drawn:

(1) Comparison of the measured data and simulated results of the plate heat exchanger shows that the proposed nonlinear ODE model has satisfactory accuracy in describing the thermal dynamics of plate heat exchanger. Relative errors of the two outlet temperatures: $T_{h,out}$ and $T_{l,out}$ are both varying within $\pm$8%.

(2) Simulation results of heating substation control system indicate that the controller tuned at a certain operating condition may be unstable, when operating condition changes in large range. For example, the operation instability of district heating substation may occur at the high primary supply temperature, if the controller is tuned at low primary supply temperature.

(3) With the proposed stability criterion for heating substation operation, the controller can be retuned to be stable at all operating conditions.

Since operation stability of heating substation is the basic requirement of the operation and is of great importance to energy conservation of pumping system, reducing the failure rate of control valve and improving the heating quality of the secondary system, the proposed method will be very helpful and applicable to heating substation controller tuning and operation stability analysis for stable operation.

Appendix A. Calculation of heat transfer coefficient k

The overall heat transfer coefficient of plate heat exchanger can be determined by the following formula:

$$k=\left(\frac{1}{k_h}+\frac{1}{k_p}+\frac{1}{k_l}\right)^{-1} \quad \text{(A-1)}$$

where $k_p=\frac{\lambda_p}{d_p}$ is the heat transfer coefficient of the plate. k_h and k_l are determined by the following formulas [16]:

$$k_h=\frac{Nu_h\lambda_h}{D},\ Nu_h=C_{Nu}\cdot Re_h^{n_1}\cdot Pr_h^{n_2},\ Re_h=\frac{\rho D q_h}{\mu_h Mbd},\ Pr_h=\frac{\mu_h c_p}{\lambda_h} \quad \text{(A-2)}$$

$$k_h=\frac{Nu_h\lambda_h}{D},\ Nu_l=C_{Nu}\cdot Re_l^{n_1}\cdot Pr_l^{n_2},\ Re_l=\frac{\rho D q_l}{\mu_l Mbd},\ Pr_l=\frac{\mu_l c_p}{\lambda_l} \quad \text{(A-3)}$$

where $D=2d$ is the hydraulic diameter; λ_h and λ_l are the heat conductivities of the high temperature side water and the low temperature side water, respectively; Nu_h and Nu_l are the Nusselt numbers of the two sides; C_{Nu}, n_1 and n_2 are empirical parameters provided by the manufacturer; Re_h and Re_l are the Reynolds numbers of the two sides; Pr_h and Pr_l are the Prandtl numbers of the two sides; μ_h and μ_l are the dynamic viscosities of the two sides.

Appendix B. Linearized plate heat exchanger model

Do the following parameter replacement:

$$a_1 = -\left(\frac{\bar{k}}{\rho c_p d} + \frac{N\bar{q}_h}{MbdL}\right) \ ,\ a_2 = \frac{N\bar{q}_h}{MbdL} \ ,\ a_3 = \frac{\bar{k}}{\rho c_p d} \ ,\ a_4 = \frac{\bar{k}}{\rho c_p d} \ ,\ a_5 = \frac{N q_l}{MbdL} \ ,$$

$$a_6 = -\left(\frac{N\bar{q}_l}{MbdL} + \frac{\bar{k}}{\rho c_p d}\right) \ ,\ b_{11,j} = \frac{N\left(\bar{T}_{h,j+1} - \bar{T}_{h,j}\right)}{MbdL} + \frac{\left(\bar{T}_{l,j} - \bar{T}_{h,j}\right)}{\rho c_p d} \cdot \left(\frac{\partial k}{\partial q_h}\right)_{\bar{q}_h} \ ,$$

$$b_{12,j} = \frac{\bar{T}_{l,j} - \bar{T}_{h,j}}{\rho c_p d} \cdot \left(\frac{\partial k}{\partial q_l}\right)_{\bar{q}_l} \ ,\ b_{21,j} = \frac{\bar{T}_{h,j} - \bar{T}_{l,j}}{\rho c_p d} \cdot \left(\frac{\partial k}{\partial q_h}\right)_{\bar{q}_h} \ ,$$

$$b_{22,j} = \frac{N\left(\bar{T}_{l,j-1} - \bar{T}_{l,j}\right)}{MbdL} + \frac{\left(\bar{T}_{h,j} - \bar{T}_{l,j}\right)}{\rho c_p d} \cdot \left(\frac{\partial k}{\partial q_l}\right)_{\bar{q}_l}$$

where $\bar{k}$, $\bar{q}_h$, $\bar{q}_l$, $\bar{T}_{l,j}$ and $\bar{T}_{h,j}$ are the equilibrium point values of the nonlinear ODE model of plate heat exchanger Eq. (8) and (9). Then define

$$A_{11} = \begin{pmatrix} a_1 & a_2 & & & & \\ & a_1 & a_2 & & & \\ & & a_1 & a_2 & & \\ & & & \ddots & \ddots & \\ & & & & a_1 & a_2 \\ & & & & & a_1 \end{pmatrix},\ A_{12} = \begin{pmatrix} a_3 & & & & & \\ & a_3 & & & & \\ & & a_3 & & & \\ & & & \ddots & & \\ & & & & a_3 & \\ & & & & & a_3 \end{pmatrix}$$

$$A_{21} = \begin{pmatrix} a_4 & & & & & \\ & a_4 & & & & \\ & & a_4 & & & \\ & & & \ddots & & \\ & & & & a_4 & \\ & & & & & a_4 \end{pmatrix},\ A_{22} = \begin{pmatrix} a_6 & & & & & \\ a_5 & a_4 & & & & \\ & a_5 & a_4 & & & \\ & & a_5 & \ddots & & \\ & & & \ddots & a_4 & \\ & & & & a_5 & a_4 \end{pmatrix}$$

and

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

$$T = \left(T_{h,1} \quad T_{h,2} \quad \cdots \quad T_{h,N} \quad T_{l,1} \quad T_{l,2} \quad \cdots \quad T_{l,N} \right)^{\mathrm{T}}$$

$$B_1 = \left(b_{11,1}, b_{11,2}, \ldots, b_{11,j}, \ldots, b_{11,N}, b_{21,1}, b_{21,2}, \ldots, b_{21,j}, \ldots, b_{21,N} \right)^{\mathrm{T}}$$

$$B_2 = \left(b_{12,1}, b_{12,2}, \ldots, b_{12,j}, \ldots, b_{12,N}, b_{22,1}, b_{22,2}, \ldots, b_{22,j}, \ldots, b_{22,N} \right)^{\mathrm{T}}$$

$$B_3 = \left(O_{1\times(N-1)}, a_2, O_{1\times N} \right)^{\mathrm{T}}$$

$$B_4 = \left(O_{1\times N}, a_5, O_{1\times(N-1)} \right)^{\mathrm{T}}$$

$$C = \left(O_{1\times N}, O_{1\times(N-1)}, 1\right)$$

where A_{11}, A_{12}, A_{21} and A_{22} are $N \times N$ matrices. T, B_1, B_2, B_3 and B_4 are $2N$-dimensional vectors.

The transfer function form of the linearized plate heat exchanger model is:

$$T_{l,out} = G(s)q_h + G_{d,1}(s)q_l + G_{d,2}(s)T_{h,in} + G_{d,3}(s)T_{l,in}$$

where

$$G(s) = C(sI - A)^{-1}B_1$$

$$G_{d,1}(s) = C(sI - A)^{-1}B_2$$

$$G_{d,2}(s) = C(sI - A)^{-1}B_3$$

$$G_{d,3}(s) = C(sI - A)^{-1}B_4$$

Acknowledgement

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Reference

[1] Building energy conservation research center of Tsinghua University, The annual development research report of China building energy conservation, 2015 ed., Building Industry Press, Beijing, 2015. (in Chinese)

[2] J. Gustafsson, J. Delsing, J.V. Deventer, Improved district heating substation efficiency with a new control strategy, Appl. Energy 87(2010) 1996-2004.

[3] J. Gustafsson, J. Delsing, J. V. Deventer, Experimental evaluation of radiator control based on primary supply temperature for district heating substations, Appl. Energy 88(2011) 4945-4951.

[4] H. Gadd, S. Werner, Achieving low return temperatures from district heating substations, Appl. Energy 136(2014) 59–67.

[5] H. Gadd, S. Werner, Fault detection in district heating substations, Appl. Energy 157(2015) 51-59.

[6] M. Brand, J.E. Thorsen, S. Svendsen, Numerical modelling and experimental measurements for a low-temperature district heating substation for instantaneous preparation of DHW with respect to service pipes, Energy 41(2012) 392-400.

[7] M. Brand, A.D. Rosa, S. Svendsen, Energy-efficient and cost-effective in-house substations bypass for improving thermal and DHW (domestic hot water) comfort in bathrooms in low-energy buildings supplied by low-temperature district heating, Energy 67(2014) 256-267.

[8] M. Kuosa, M. Aalto, M.E.H. Assad, et al., Study of a district heating system with the ring network technology and plate heat exchangers in a consumer substation, Energy Build. 80(2014) 276–289.

[9] L. Dobos, J. Abonyi, Controller tuning of district heating networks using experiment design techniques, Energy 36(2011) 4633-4639.

[10] S.K. Al-Dawery, A.M. Alrahawi, K.M. Al-Zobai Dynamic modeling and control of plate heat exchanger, Int. J. Heat Mass Tran., 55(2012) 6873-6880.

[11] A. Michel, A. Kugi, Model based control of compact heat exchangers independent of the heat transfer behavior, J. Process Contr., 24(2014)286-298.

[12] G.F. Franklin, D.J. Powell, A. Emami-Naeini, Feedback Control of Dynamic Systems, seventh ed., Pearson, New York, 2015.

[13] P. Andersen, T.S. Pedersen, J. Stoustrup, et al., Proceedings of the 2000 American Control Conference. ACC (IEEE Cat. No.00CH36334), Chicago, IL, 2000, pp. 135-139 vol.1.

[14] Tahersima F, Stoustrup J, Rasmussen H. An analytical solution for stability-performance dilemma of hydronic radiators, Energy Build. 64(2013) 439-446.

[15] Y. Wang, S. You, X. Zheng, H. Zhang, Accurate model reduction and control of radiator for performance enhancement of room heating system, Energy Build 138(2017) 415-431.

[16] A. Michel, A. Kugi, Accurate low-order dynamic model of a compact plate heat exchanger, Int. J. Heat Mass Tran. 61(2013) 323-331.

[17] S. Skogestad, I. Postlethwaite, Multivariable Feedback Control: Analysis and Design, second ed., Wiley, New York, 2007.

[18] American Society of Heating, Air-Conditioning Engineers, ASHRAE handbook: heating, ventilating, and air-conditioning systems and equipment, SI ed; 2012.