Simulation Techniques and Value-At-Risk of the CARBS Indices

Coenraad CA Labuschagne, Niel Oberholzer, and Pierre J Venter

Abstract In this paper, simulation techniques are used to estimate value-at-risk of the CARBS equity indices and a global minimum variance portfolio. The empirical analysis in this paper is divided into two parts, the first part deals with simulating normally distributed returns in order to estimate VaR. In the second part calibrated univariate GARCH models are used to simulate returns series that are consistent with the stylised facts of financial time series. When a normal distribution is assumed, the GARCH model forecast of the returns produces the most reliable result. Finally, when garch processes are simulated, the EGARCH model is superior.

1 Introduction

Monte Carlo simulation has been a valuable computational tool in finance, particularly for the valuation of financial instruments and risk measurement (Boyle et al. (1997) [2]). According to Danielsson (2011) [5] Monte Carlo simulation can be defined as the process of replicating market outcomes by generating computer based random numbers. Furthermore, due of advances in computing power and shortcomings of other VaR methods, Monte Carlo simulation is the preferred method when it comes to financial risk measurement.

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Financial returns series are usually not normally distributed. The results obtained show that the return series considered in this study show signs of leptokurtosis, which is consistent with the stylised facts of financial time series (Embrecths et al. (2005) [7]). Therefore, assuming the normal distribution when simulating returns could lead to unrealistic results. Furthermore, numerous financial models assume a constant volatility, in reality volatility is very volatile.

The problems mentioned above are investigated in this paper. The empirical analysis in this paper is divided into two parts, the first part deals with simulating normally distributed returns in order to estimate VaR. In the second part univariate GARCH models will be used to simulate returns that consist of volatility clustering and are not normally distributed (Alexander (2008) [1]). The accuracy of the different methods is compared using a back-testing procedure. This provides an opportunity to compare the reliability of all the models used in this thesis.

The GARCH simulation component of the empirical analysis requires the use of the `rugarch R` statistical programming package (Ghalanos (2014) [6]). The remainder of this paper is structured as follows: in the next section the relevant literature on GARCH simulation is discussed, thereafter the methods implemented in this study are discussed in detail, this is followed by the empirical results and interpretation.

2 Literature review

In this section, the different studies on GARCH simulation and financial risk management are briefly discussed. Oberholzer et al. (2015b) [9] simulated symmetric GARCH processes using historical data of the JSE returns, which were then compared to the actual returns. They finally conclude that the symmetric GARCH model is not a good fit. Oberholzer et al. (2015b) [9] mention that areas for future research includes the simulation of other GARCH type models.

Labuschagne et al. (2015) [8] simulated GJR-GARCH and EGARCH processes via Monte Carlo to estimate volatility skews for the BRICS equity indices, the volatility skews were then compared to the volatility skews generated via the risk neutral historic distribution (RNHD) model. Labuschagne et al. (2015) [8] conclude that the smoother skew obtained from the GARCH models are more reliable. The essence of the argument is that the simulation obtained from the GARCH models use four calibrated parameters, which is more reliable than the one parameter used in the RNHD framework.

To estimate VaR for Dutch bond portfolios, Vlaar (2000) [10] used historical simulation, variance-covariance, and Monte Carlo Simulation methods. Vlaar (2000) [10] mentions that when implementing Monte Carlo simulation, it is necessary to make certain assumptions regarding the distribution of the assets. Vlaar (2000) [10] assumed a zero mean return, and a GARCH process for the conditional variance. The most reliable VaR estimate was obtained using a combination of the variance-covariance method, and Monte Carlo simulation. The standard GARCH model assumes that positive and negative shocks lead to the same rise in volatility, which is unrealistic in certain cases. Therefore, asymmetric GARCH models will be included in the numerical implementation of this paper.

3 Methodology

3.1 Monte Carlo VaR (standard normal random variates)

If the returns are assumed to be normally distributed with mean μ and standard deviation σ, Chan (2015) [4] argues that a typical model is

$$R_t = \mu + \sigma Y, \text{ where } Y \sim \mathcal{N}(0,1). \tag{1}$$

If the above return dynamics are assumed, it is easy to derive that

$$\text{VaR}_X = y_X \sigma - \mu,$$

where y_X is the X-quantile of the standard normal distribution. In terms of implementation, the following algorithm is a slightly modified version of the algorithm in Chan (2015) [4] in order to include the back-testing procedure.

- *Step 1:* Generate N independent standard normal random variables, i.e.

  $$Y_j \sim \mathcal{N}(0,1)$$

  i.i.d. for $j = 1, ..., N$.
- *Step 2:* Compute $R_j = \mu + \sigma Y_j$.
- *Step 3:* Rank R_j in ascending order to obtain R_j^*.
- *Step 4:* VaR is set equal to the value of $-R_k^*$ where k is equal to $(X \times N)$
- *Step 5:* Create the back-testing vector, which takes a value of one if the actual loss (based on historical data) is greater than VaR, and zero otherwise.

As part of the empirical analysis, the above algorithm will be implemented using the historical mean and standard deviation of each returns series, with the number of simulated variables (N) set equal to 50000. Furthermore, the same algorithm will be implemented using the out of sample forecast of the conditional standard deviation of the returns using three GARCH family models. The different univariate GARCH models will be compared to the use of the historical parameters on the basis of a back-testing procedure.

3.2 Monte Carlo VaR with GARCH models

Financial returns series are usually not normally distributed, there are signs of leptokurtosis. Furthermore, financial returns series exhibit signs of volatility clustering. Therefore, assuming that return dynamics are normally distributed can be considered unreasonable.

According to Carmona (2014) [3], fitted GARCH models provide an efficient way to generate Monte Carlo samples which can be used as possible scenarios of a trading day. Alexander (2008) [1] argues that the main distinguishing feature of the use of GARCH models in Monte Carlo simulation is that it is possible to simulate returns with volatility clustering. The procedure for simulating from a GARCH model is explained by the algorithm below, which is illustrated by Alexander (2008) [1].

- *Step 1:* Fix an initial value for the conditional variance (σ_1^2)
- *Step 2:* Generate $Y_t \sim \mathcal{N}(0,1)$.
- *Step 3:* Compute $\hat{\varepsilon}_t = \sigma_t Y_t$.
- *Step 4:* Find σ_{t+1} from σ_t and $\hat{\varepsilon}_t$ using the optimal parameters of the GARCH model.
- *Step 5:* Return to step 2, $t = t+1$.

After simulating a return series that exhibits volatility clustering, steps (3) to (5) of Monte Carlo VaR algorithm are performed in order to compute VaR.

This procedure is tested numerically. For each return series, 100 sample paths are generated using the three GARCH family models, each sample path is of length 1000, 99% VaR is then computed for each sample path. The average VaR is considered for each respective model. Finally, the reliability of the estimated VaR is then tested using a back-testing procedure. The back-testing procedure is completed using 1000 of the most recent data points.

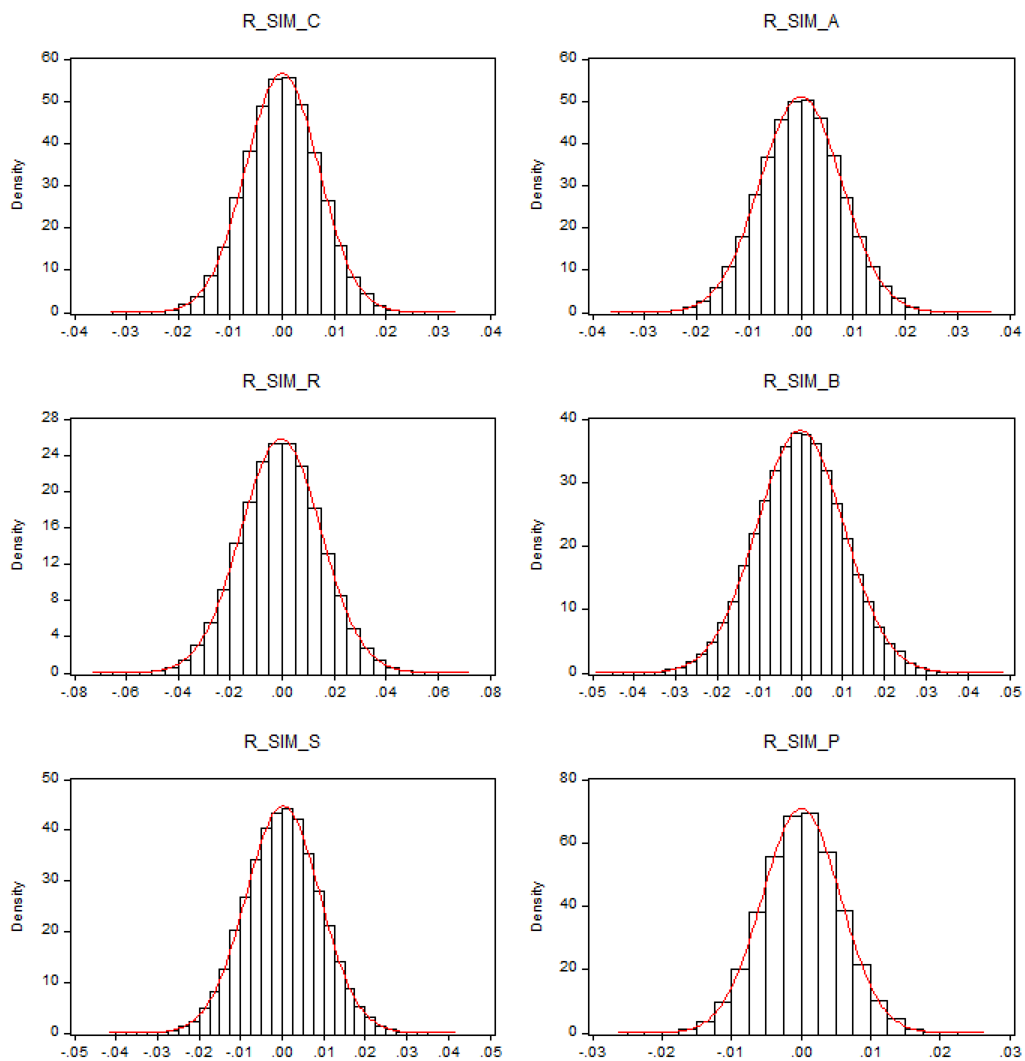
4 Empirical results

In what follows, details of the Monte Carlo VaR and the VaR computed using GARCH simulation are provided. This section is divided into two subsections. For each simulation technique, an example is shown the the simulated return distribution and compared to the normal density.

4.1 Monte Carlo VaR

Figure 1 below illustrates the histograms of the simulated returns using standard normal random variables. When the simulated return distribution is compared to the

Fig. 1: Histograms of simulated returns



normal density, it is evident that the simulated returns seem normally distributed. Furthermore, when the figure above is compared to figure a financial returns distribution, it is clear that the simulated returns do not resemble the same distribution. Most importantly, the simulated returns do not exhibit signs of leptokurtosis, and therefore contradicts one of the stylised facts of financial times series.

VaR was computed using the simulated returns. The mean in all cases is assumed to be equal to the mean of the historical returns. Four different assumptions were tested regarding the variance, namely: the historical variance of returns, and the out of sample (90 day) forecast of the conditional variance using the GARCH family models. The reliability of the computed VaR was tested using a back-testing procedure, the results are reported in the table below. The lowest percentage of exceptions is shown in bold.

Table 1: Monte Carlo VaR: percentage of exceptions

	Canada	Australia	Russia	Brazil	South Africa	GMVP
Historical	1.10%	1.60%	1.30%	1.60%	2.10%	1.70%
GARCH	**0.70%**	**0.30%**	**1.10%**	1.60%	**1.30%**	**0.70%**
GJR-GARCH	1.00%	0.40%	1.30%	1.60%	1.60%	1.40%
EGARCH	1.00%	1.40%	1.30%	**1.50%**	1.90%	1.70%

It is evident that the GARCH model forecast produces the VaR with the lowest number of exceptions. When the returns are simulated using the volatility predicted by the GARCH(1,1) model, VaR is overestimated for Canada, Australia, and the GMVP, VaR is underestimated for the other variables. When the historical variance is used when computing Monte Carlo VaR, risk is underestimated for every variable included in this study. This clearly shows the shortcoming of assuming a mesokurtic distribution, when financial returns are slightly more peaked at the mean and have fat tails.

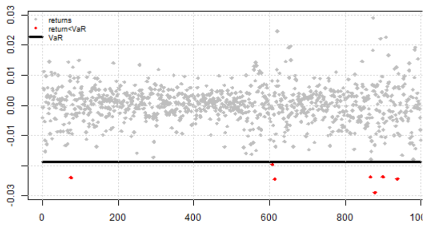
The graphical representation of the back-testing procedure shows the exceptions in red, the figure includes a graphical back-test of the assumption that produces the lowest percentage of exceptions only. The Y-axis denotes the log returns, and the X-axis denotes time. It is evident that the VaR estimated for Russia is the highest, and the VaR estimated for the GMVP is the lowest.

4.2 Monte Carlo VaR with GARCH models

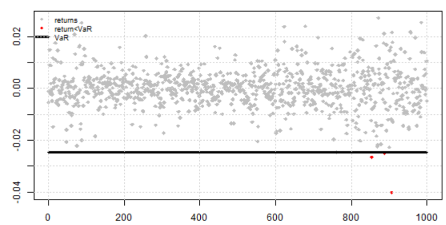
When returns are assumed to follow a GARCH process, it is possible to simulate return series that exhibit signs of volatility clustering. As shown previously, the EGARCH(1,1) model is the best fit for the the return series in this study, therefore sample paths of simulated EGARCH(1,1) return series are plotted below.

Fig. 2: Back-testing: Monte Carlo VaR

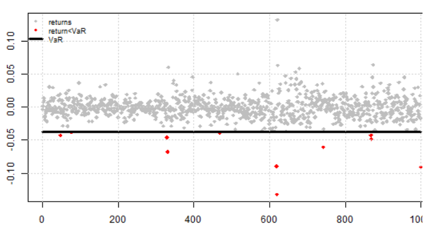
(a) Canada: GARCH(1,1)



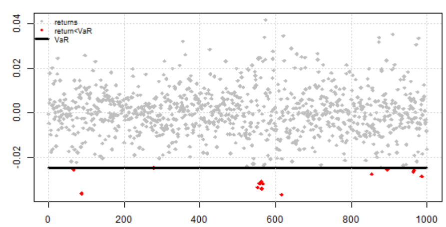
(b) Australia: GARCH(1,1)



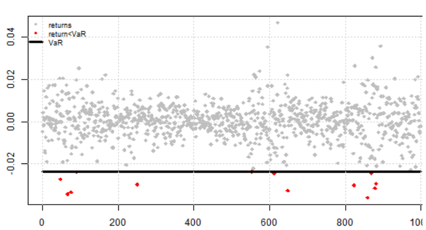
(c) Russia: GARCH(1,1)



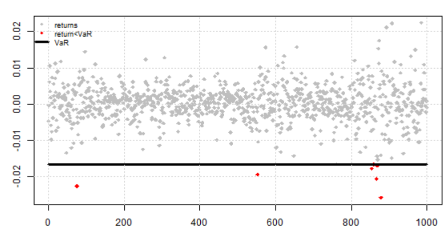
(d) Brazil: EGARCH(1,1)



(e) South Africa: GARCH(1,1)



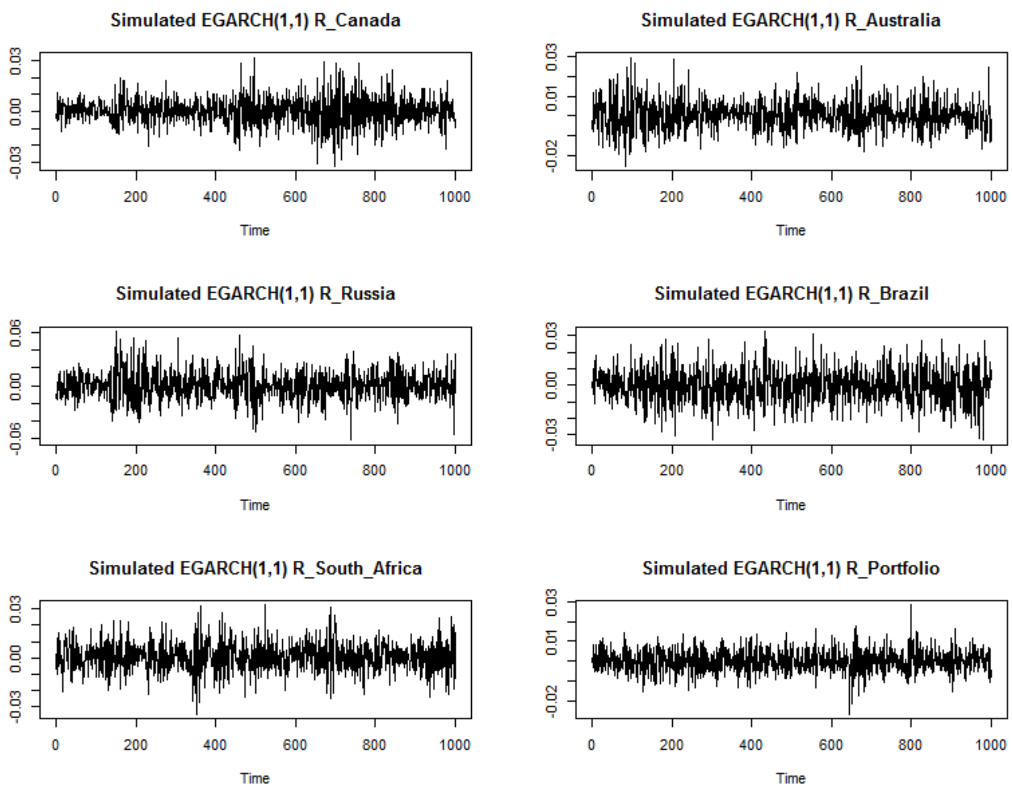
(f) GMVP: GARCH(1,1)



The sample paths above show signs of volatility clustering, this is consistent with the stylised facts of financial time series. Moreover, there also seems to significant asymmetry when the simulated returns are considered. The leverage effect was found to be significant for all the variables included. To give an indication of the distribution, histograms of the simulated sample paths are plotted below.

The histograms exhibit evidence of leptokurtosis and therefore making the assumption of a simulated GARCH process more realistic. Because the distributions above have fat tails, it reduces the chance of underestimating VaR. The results of

Fig. 3: EGARCH simulated returns: sample paths



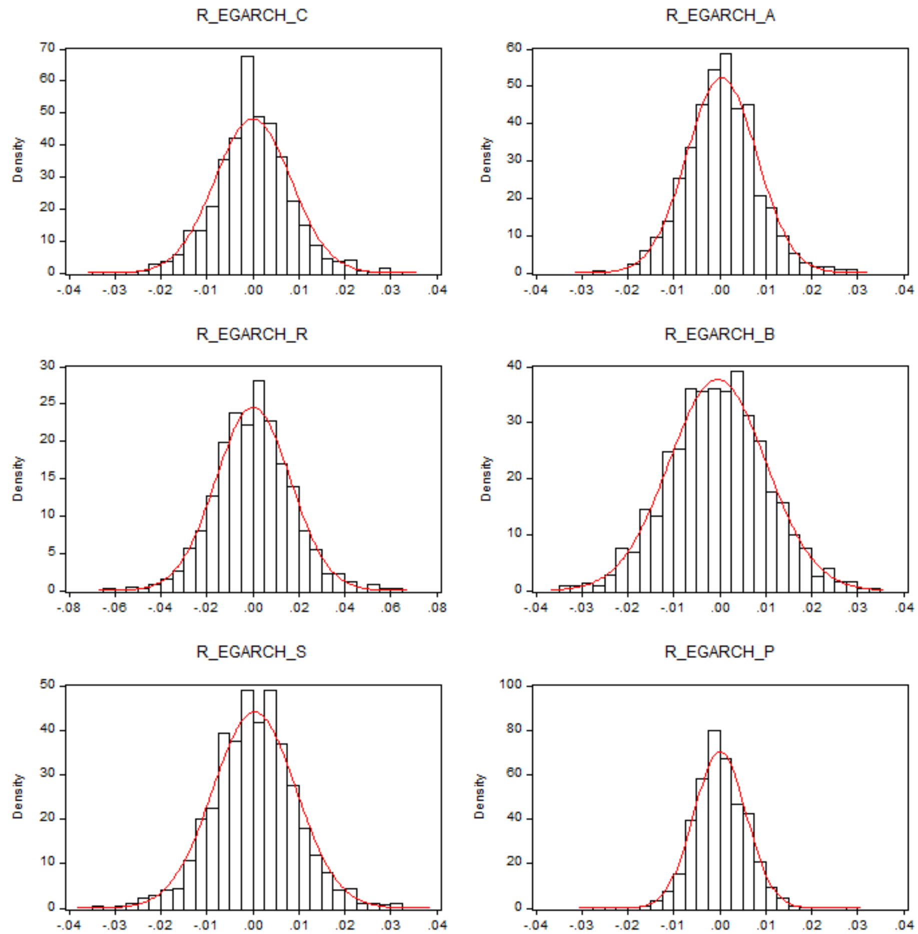
the back-testing procedure of the Monte Carlo VaR using GARCH models are illustrated below.

Table 2: GARCH Monte Carlo VaR: percentage of exceptions

	Canada	Australia	Russia	Brazil	South Africa	GMVP
GARCH	0.90%	1.40%	**0.70%**	1.40%	**0.90%**	1.50%
GJR-GARCH	1.20%	1.60%	0.90%	1.20%	1.30%	1.70%
EGARCH	**0.70%**	**1.20%**	**0.70%**	**1.00%**	**0.90%**	**1.30%**

It is evident that the simulated EGARCH(1,1) process results in the lowest percentage of exceptions for all the return series considered. The simulated GARCH(1,1) process leads to the same percentage of exceptions when compared to the EGARCH process for Russia and South Africa. Of all the assumptions and methods imple-

Fig. 4: Histograms of EGARCH simulated returns

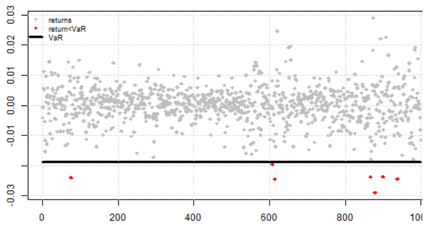


mented in this study, the simulated EGARCH process is the only assumption that reduces the percentage of exceptions of VaR estimated for Brazil to 1%, all the other methods underestimate VaR for Brazil.

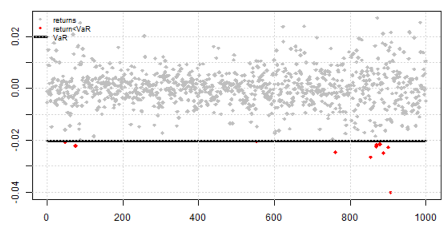
The graphical back-test of the EGARCH Monte Carlo VaR shows that this model performs fairly well when estimating VaR and is reliable when it comes to risk measurement.

Fig. 5: Back-testing: GARCH Monte Carlo VaR

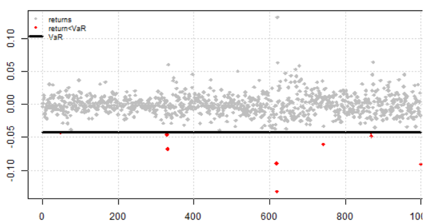
(a) Canada: EGARCH(1,1)



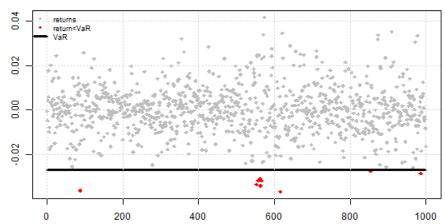
(b) Australia: EGARCH(1,1)



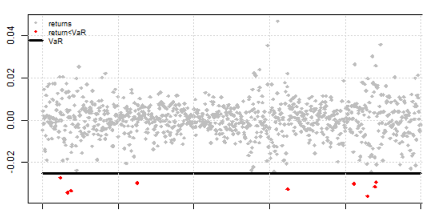
(c) Russia: EGARCH(1,1)



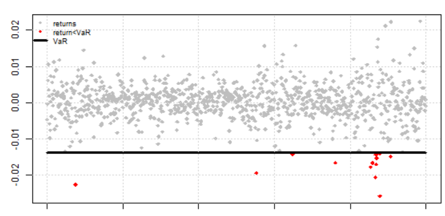
(d) Brazil: EGARCH(1,1)



(e) South Africa: EGARCH(1,1)



(f) GMVP: EGARCH(1,1)



5 Conclusion

Monte Carlo simulation can be defined as the practice of simulating possible scenarios by making use of computer generated random numbers (Danielsson (2011) [5]). It is evident that the use of Monte Carlo methods in finance have been invaluable to the pricing of securities and scenario analysis when it comes to risk measurement.

In this study, two algorithms were implemented to estimate VaR. The Monte Carlo VaR Algorithm makes use of $\mathcal{N}(0,1)$ random variables to generate normally distributed returns, the mean is set equal to the mean of the historical returns, and

4 assumptions are considered for the variance. More specifically, the variance is set equal to the historical variance of the returns, and the 3 forecasts of conditional variance obtained from the GARCH models. The GARCH simulation algorithm makes use of the fitted univariate GARCH models to simulate log returns with volatility clustering.

When the histograms obtained by implementing the Monte Carlo VaR algorithm are compared to the histograms of the historical returns, it is evident that normally distributed returns is an unrealistic assumption. The sample paths of the GARCH simulated log returns seem more realistic when compared to the actual historical log returns.

When the empirical results are considered, the percentage of exceptions produced using simulation techniques is less than the percentage of exceptions using a rolling forecast. When implementing the GARCH simulation algorithm, assuming that the variance is equal to the conditional variance predicted using a GARCH(1,1) model leads to the lowest percentage of exceptions for 5 out of 6 variables. Finally, when the GARCH processes are used to simulate returns, the EGARCH model is the most reliable for the CARBS indices and the GMVP.

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