Univariate and Multivariate GARCH Models Applied to the CARBS Indices

Coenraad CA Labuschagne, Niel Oberholzer, and Pierre J Venter

Abstract The purpose of this paper is to estimate the calibrated parameters of different univariate and multivariate GARCH family models. It is unrealistic to assume that volatility of financial returns is constant. In the empirical analysis, the symmetric GARCH, and asymmetric GJR-GARCH and EGARCH models were estimated for the CARBS indices and a global minimum variance portfolio (GMVP), the best fitting model was determined using the AIC and BIC. The asymmetric terms of the GJR-GARCH and EGARCH models indicate signs of the leverage effect. The information criterion suggest that the EGARCH model is the best fitting model for the CARBS indices and the GMVP.

1 Introduction

Classical methods used to measure risk assume that volatility is constant over time. However, the stylised facts of financial time series indicate that volatility fluctuates over time and that return series show evidence of volatility clustering. Therefore, in order to measure risk using a more realistic approach, it is necessary to use time-varying volatility models.

When it comes to the topic of time varying volatility models in finance, most financial modelling researchers agree that the GARCH model is the most widely used

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and accepted model. Many different GARCH model specifications have been introduced in recent years. In this study, the focus is on the standard GARCH model, and the asymmetric Glosten, Jagannathan, Runkle GARCH (GJR-GARCH) and exponential GARCH (EGARCH) univariate models. This will give an indication of the degree of asymmetry and whether positive and negative shocks have the same effect on volatility. Furthermore, the dynamic conditional correlation GARCH (DCC-GARCH) and the generalised orthogonal GARCH (GO-GARCH) model will be considered in the multivariate GARCH framework.

In order to approximate relaible risk measures using volatility models, it is necessary to use volatility models that are a good fit. The univariate GARCH models are compared using information criterion in this paper, which is based on the study by Oberholzer et al. (2015a) [20]. Furthermore, news impact curves (univariate) and surfaces (multivariate) are approximated. This gives an indication of how the conditional variance or covariance changes after a positive or negative shock. This gives an indication of the degree of asymmetry predicted by the model.

The empirical analysis in this paper relies heavily on the `rugarch` (Ghalanos (2014) [15])and `rmgarch` (Ghalanos (2012) [16]) packages of the `R` programming language. The remainder of this paper is structured as follows: the relevant recent literature is discussed, the different univariate and multivariate GARCH models used in this thesis are specified, the empirical results are reported and interpreted, and finally the problem and results are summarised.

2 Literature review

Literature focussing on the application of GARCH used to model volatility dates back to 1986 (Bollerslev (1986) [7]). Furthermore, the use of univariate symmetric and asymmetric GARCH models are also well-documented. Ahmad et al. (2014) [1] made use of the standard GARCH, GARCH in mean, threshold GARCH, and exponential GARCH (EGARCH) to model volatility of the volatility of Malaysian gold prices, the Akaike (AIC) and Schwarz (SIC) information criterion were used to determine the best fitting model. The EGARCH model was found to be the best model. In addition, the asymmetric GARCH models indicate that positive shocks lead to a greater rise in volatility when compared to negative shocks.

In a similar study, Oberholzer et al. (2015a) [20] used the standard GARCH, GJR-GARCH, and EGARCH to determine the best fitting model for the five indices of the Johannesburg Stock Exchange (JSE) before, during, and after the GFC of 2008. Using a similar approach to Ahmad et al. (2014), the AIC and SIC were used to determine the best fitting GARCH model. The GJR-GARCH model was found to be the best fitting model for the JSE in general. Moreover, the results showed signs of the leverage effect, which according to Black (1976) [6] occurs when negative shocks give rise to a greater volatility when compared to positive shocks.

The use of multivariate GARCH to model conditional covariance and correlation among return series is also well-documented. By making use of the asymmetric

BEKK multivariate GARCH model, Wen et al. (2014) [21] investigated the volatility spillover effect among stock prices of Chinese fossil feul, and energy companies. The findings indicate that negative news about energy and fossil fuel leads to greater volatility in their counter assets. In addition, there is evidence of significant volatility spillovers and asymmetry, which has potential implications for financial risk management and asset allocation.

In a recent study, Basher et al. (2016) [5] used dynamic conditional correlation (DCC) GARCH, Asymmetric DCC-GARCH, and generalised orthogonal (GO) GARCH to estimate optimal cross hedging ratios between emerging market share prices, bond prices, gold prices and oil prices. Basher et al. (2016) [5] add to the literature by comparing the hedge ratios obtained by using different model specifications, and use a one step ahead forecast for the next period hedge, which is usually assumed to be equal to the current hedge. Their results suggest that the asymmetric DCC model is superior when hedging share prices using bonds, gold and oil.

3 Univatiate GARCH models

In this section, the different univariate GARCH models will be defined and specified. The models discussed in this section will implemented in order to determine the best fitting univariate GARCH model for the CARBS indices, and the GMVP. Zivot (2007) [23] explains that univariate GARCH models are concerned with the modelling of the conditional variance of a univariate time series.

3.1 Standard GARCH model

According to Duncan et al. (2009) [11], a typical GARCH(1,1) model is specified as follows:

$$
\begin{aligned}
R_t &= \mu + \varepsilon_t, \qquad\qquad \varepsilon_t|\Omega_{t-1} \sim \mathcal{N}(0,\sigma_t^2) \\
\sigma_t^2 &= \gamma + \upsilon\varepsilon_{t-1}^2 + \delta\sigma_{t-1}^2.
\end{aligned}
$$

Returns are modelled as being dependent on their (zero) mean observation. The error term, ε_t is assumed to be conditioned on past information (Ω_{t-1}) and normally distributed with an expected value of zero and conditional variance σ_t^2.

Because variance can never be negative and cannot be greater than one, it is necessary to impose the following coefficient restrictions:

$$
\begin{aligned}
&\gamma \geq 0, \\
&0 \leq \upsilon, \delta < 1, \text{ and} \\
&\upsilon + \delta \leq 1.
\end{aligned}
$$

In addition to coefficient restrictions, Wennström (2014) [22] explains that another shortcoming of the standard GARCH model is that it does not capture the effect of asymmetries. Hence, the model assumes that positive and negative news will lead to the same rise in volatility.

3.2 GJR-GARCH model

Intuitively, one would would expect a greater rise in volatility after a negative shock, this is referred to as the leverage effect (Brooks 2014 [9]). Therefore, the GJR-GARCH model accounts for negative shocks by including an indicator function, which takes a value of one when the shock is negative, and zero otherwise. According to Asteriou et al. (2015) [4] the specification of the GJR-GARCH(1,1) model takes the following form,

$$\sigma_t^2 = \gamma + \upsilon \varepsilon_{t-1}^2 + \delta \sigma_{t-1}^2 + \eta 1_{\{\varepsilon_t < 0\}} \varepsilon_{t-1}^2,$$

where $1_{\{\varepsilon_t < 0\}}$ is the indicator function.

Asteriou et al. (2015) [4] further explains that good news has an impact equal to υ, while the bad news impact is captured by $\upsilon + \eta$. Clearly, if η is statistically equal to zero, the news impact is symmetric. As mentioned previously, variance cannot take on a negative value, or values greater than one. Therefore Brooks (2014) [9] shows that the following coefficient restrictions are necessary:

$$\begin{aligned} &\gamma > 0, \\ &\upsilon > 0, \\ &\delta \geq 0, \text{ and} \\ &\upsilon + \eta \geq 0. \end{aligned}$$

Although the GJR-GARCH model does capture asymmetric news effects, it does require coefficient restrictions which can be considered a drawback as far as volatility models are concerned.

3.3 The exponential GARCH (EGARCH) model

To overcome the problem of coefficient restrictions, a possible solution a different functional form is utilised (i.e. the natural log function). Consider the following model specification of an EGARCH(1,1) process from Francq et al. (2014) [13]

$$\ln \sigma_t^2 = \gamma + \upsilon \left| \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} \right| + \delta \ln \sigma_{t-1}^2 + \eta \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}$$

In order to solve for σ_t^2, it is necessary to exponentiate the above equation. The exponential function does not take on negative values, this implies that the non-negativity constraints can be dropped. The EGARCH model does capture the effect of asymmetric news, if $\eta < 0$, then it is evidence of a leverage effect.

3.4 Information criterion

In order to compare the goodness of fit of the univariate GARCH models, the AIC and Bayesian (BIC) information criterion are used. According to Asteriou et al. (2015) [4], the AIC and BIC are defined as follows:

$$AIC = \left(\frac{RSS}{k}\right) \exp\left\{\frac{2m}{k}\right\}$$
$$BIC = \left(\frac{RSS}{k}\right) \exp\left\{\frac{m}{k}\right\},$$

where RSS denotes the residual sum of squares, m is the number of explanatory variables, and k is the sample size. In addition, the model that produces the lowest level of AIC or BIC is the best fitting model. These values are usually logged by most statistical software packages.

4 Multivariate GARCH models

The univariate GARCH models considered are used to model the conditional variance. However, these models do not consider how the conditional covariance varies over time. Therefore, multivariate GARCH models are used to model the conditional covariance matrix in this study. According to Alexander (2008) [3] in a multivariate GARCH framework, each return series has a conditional variance (the diagonal of the covariance matrix) modelled by a univariate GARCH model, and each pair of return series have a conditional covariance modelled by a similar equation. Both the generalised orthogonal GARCH (GO-GARCH), and the dynamic conditional correlation GARCH (DCC-GARCH) models are considered below.

4.1 GO-GARCH

Alexander (2001) [2] explains that when using the orthogonal GARCH model, an $(n \times n)$ covariance matrix (Σ_t) can be estimated using m univariate GARCH models, where $m < n$. For this procedure, uncorrelated components are transformed in order to obtain observed data. Van Weide (in Jondeau et al. (2007) [14]) extended this idea, the generalised orthogonal GARCH model includes an invertible matrix which forms a link between innovations.

The model specification that follows is based on the work by Broda (2009) [8]. Consider the following,

$$\bar{R}_t = \bar{\mu}_t + \bar{\varepsilon}_t$$

where $\bar{R}_t$ is the return vector of n assets, $\bar{\mu}_t$ is a vector of constant (zero) mean returns, and $\bar{\varepsilon}_t$ is a vector of disturbances. The disturbances are modelled by a linear combination of n factors $\bar{f}_t$, in matrix form:

$$\bar{\varepsilon}_t = M\bar{f}_t.$$

It is assumed that each factor follows a GARCH(1,1) process, i.e. $\bar{f}_t \sim \mathcal{N}(\bar{0}, \bar{H}_t)$, where

$$\bar{H}_t = \Gamma + \sum_{k=1}^{d} \upsilon_k e_k e_k' \bar{H}_{t-1} e_k e_k' + \sum_{k=1}^{d} \delta_k e_k e_k' \bar{f}_{t-1} \bar{f}_{t-1}' e_k e_k',$$

$\Gamma = \sum_{k=1}^{d} (1 - \upsilon_k - \delta_k) e_k e_k'$, and e_k is a $d \times 1$ vector with the kth element equal to one, and zeros elsewhere. The unconditional covariance is given by $\Sigma = MM'$, and the conditional covariance of the return vector becomes

$$\Sigma_t = M\Gamma M' + \sum_{k=1}^{d} \upsilon_k \nu_k \zeta_k' \Sigma_{t-1} \zeta_k \nu_k' + \sum_{k=1}^{d} \delta_k \nu_k \zeta_k' \varepsilon_{t-1} \varepsilon_{t-1}' \zeta_k \nu_k', \tag{1}$$

where $\nu_k = Me_k$ and $\zeta_k = (M^{-1})' e_k$. In order to simplify the computation, the matrix M can be factorised as follows, using a polar decomposition:

$$M = \Sigma^{\frac{1}{2}} U,$$

where U is an orthogonal matrix, and $\Sigma^{\frac{1}{2}}$ is the square root matrix of the unconditional covariance matrix. Therefore it is necessary to estimate the matrix U and then the matrix M in order to estimate the conditional covariance matrix given by equation 1.

4.2 DCC-GARCH

The DCC-GARCH model provides an effective method for modelling volatility dynamics of time series that have time dependent conditional correlations (Gregoriou (2009) [17]). Hence, the DCC-GARCH model is an extension of the constant conditional correlation GARCH (CCC-GARCH) model which includes a dynamic for the conditional correlation. The assumption of constant conditional correlation can be considered arbitrary, as argued by Francq et al. (2011) [13].

In order to specify the DCC-GARCH model, it is assumed that the vector of disturbances are distributed normally distributed $\bar{\varepsilon}_t \sim \mathcal{N}(0, \Sigma_t)$. The DCC-GARCH model by Engle (2002) [12] is specified by

$$\Sigma_t = D_t P_t D_t, \tag{2}$$

where D_t is a diagonal matrix with the conditional variances of the individual assets along the diagonal, and P_t is the time dependent correlation matrix. Gregoriou (2009) [17] elaborates and shows that by manipulating equation 2, the time dependent correlation matrix is given by

$$P_t = D_t^{-1} \Sigma_t D_t^{-1},$$

which is in turn equal to

$$P_t = diag\left(q_{11,t}^{-1/2} \ldots q_{nn,t}\right) Q_t diag\left(q_{11,t}^{-1/2} \ldots q_{nn,t}\right).$$

The matrix Q_t is positive definite and given by

$$Q_t = (1 - \bar{\upsilon} - \bar{\delta})\tilde{Q} + \bar{\upsilon}\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}_{t-1}' + \bar{\delta} Q_{t-1},$$

where $\tilde{\varepsilon}_t$ are the standardised residuals, and $\tilde{Q}$ is the unconditional variance matrix. Therefore it is necessary estimate the two vectors, $\bar{\upsilon}$ and $\bar{\delta}$.

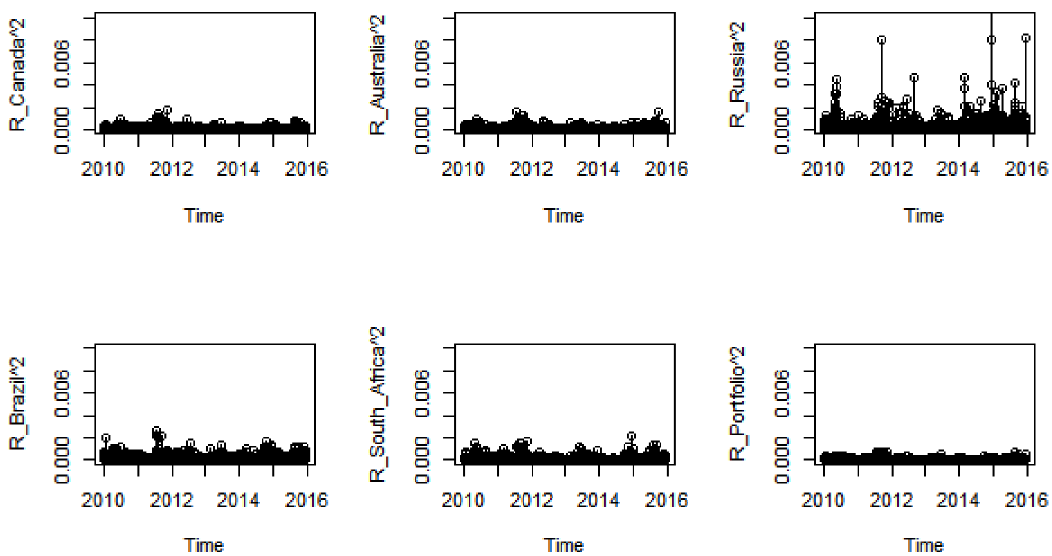
5 Graphical analysis

Using a similar approach to Narsoo (2016) [19] the squared returns are used as a proxy for volatility. The squared returns are plotted below: The *y*-axis limits are the same for each plot, this presents an opportunity to compare the volatility of the different indices. It seems as though the returns on the Russian index is the most volatile. Moreover, the GMVP seems to be the least volatile. This is consistent with our expectations. However, the graphical analysis does not give an indication of how volatility reacts to positive and negative news.

6 Empirical results

The purpose of this section is to report and interpret the calibrated parameters of the different univariate and multivariate GARCH models outlined in previous sections. By making use of a similar approach to Oberhozer et al. (2015a) [20] the AIC and

Fig. 1 Volatility of the CARBS indices and the GMVP



BIC information criterion will be used to determine which GARCH model is the best fit for the indices included in this study, and for the GMVP.

6.1 Univariate GARCH models

In order to estimate the optimal parameters of the different GARCH models in this study, it is necessary to establish the presence of autoregressive conditional heteroskedasticity (ARCH) effects. This is done by perform the ARCH Lagrange Multiplier (ARCH LM) test. This is shown in the table below. Both the F-statistic and and the Obs*R^2 show evidence of the presence of ARCH effects at a 1% level. Hence there is volatility clustering, and it is possible to estimate the optimal parameters of different GARCH family models.

The table below illustrates the optimal parameters of the GARCH(1,1) model for the CARBS indices and the GMVP:

It is evident from the table that the coefficients do not violate any of the constraints, which implies that the models are admissible. Furthermore, according to Koop (2006) [18] if the sum of υ and δ is close to one, it suggests that shocks to the index (positive or negative) will be persistent. This is the case for all the CARBS

Table 1 ARCH LM test

	F-statistic	Obs*R^2
Canada	59.6887***	58.1492***
Australia	129.6562***	122.4807***
Russia	869.9389***	622.1316***
Brazil	58.4749***	56.9977***
South Africa	72.2573***	70.0003***
GMVP	161.7760***	150.7241***

*(**) [***]: Statistically significant at a 10(5)[1] % level

Table 2 GARCH(1,1) optimal parameters

	Canada	Australia	Russia	Brazil	SA	GMVP
γ	1.0E-06	1.0E-06**	6.0E-06	4.0E-06***	2.0E-06**	0
υ	0.0748	0.0685***	0.1122***	0.0622***	0.1200	0.0583
δ	0.9087***	0.9150***	0.8686***	0.9034***	0.8593***	0.929***
AIC	-7.2544	-7.0382	-5.7269	-6.3749	-6.7718	-7.7090
BIC	-7.2440	-7.0278	-5.7165	-6.3644	-6.7614	-7.6986

*(**) [***]: Statistically significant at a 10(5)[1] % level

indices, and the GMVP. The optimal parameters of the GJR-GARCH(1,1) model are shown in the table below.

Table 3 GJR-GARCH(1,1) optimal parameters

	Canada	Australia	Russia	Brazil	SA	GMVP
γ	1.00E-06	1.00E-06	4.00E-06	3.00E-06***	2.00E-06**	0
υ	0	0.0006	0.0439***	0	0.0079***	0
δ	0.9267***	0.9261***	0.9031***	0.9284***	0.8835***	0.9248***
η	0.1084***	0.1068***	0.0748***	0.0947***	0.1681***	0.1122***
AIC	-7.2807	-7.0661	-5.7383	-6.4002	-6.8033	-7.7562
BIC	-7.2676	-7.0531	-5.7253	-6.3871	-6.7903	-7.7432

*(**) [***]: Statistically significant at a 10(5)[1] % level

From the above, it is clear that the volatility of the CARBS indices and the GMVP show a certain degree of asymmetry. The asymmetry terms of all the returns series are statistically significant at a one percent level. Moreover, the estimated parameters do not violate any of the constraints discussed in the previous section. In addition, the AIC and BIC indicate that the asymmetric GJR-GARCH model is a better fit when compared to the symmetric GARCH model. The optimal parameters and information criterion of the EGARCH model are illustrated by the table below.

As stated, if the asymmetry term of the EGARCH model is statistically significant and less than zero, it implies the existence of the leverage effect. This is the case for all the returns series used in this study. This suggests that negative news

Table 4 EGARCH(1,1) optimal parameters

	Canada	Australia	Russia	Brazil	SA	GMVP
γ	-0.1903***	-0.2707***	-0.2599***	-0.2408***	-0.2650***	-0.2303***
υ	-0.1177***	-0.0966***	-0.0702***	-0.0834***	-0.1210***	-0.1163***
δ	0.9810***	0.9723***	0.9686***	0.9735***	0.9720***	0.9782***
η	-0.0582***	-0.1081***	-0.1852***	-0.0792***	-0.1351***	-0.0475***
AIC	**-7.3024**	**-7.0783**	**-5.7482**	**-6.4020**	**-6.8124**	**-7.7748**
BIC	**-7.2893**	**-7.0653**	**-5.7352**	**-6.3890**	**-6.7993**	**-7.7618**

*(**) [***]: Statistically significant at a 10(5)[1] % level

will lead to a greater rise in volatility, when compared to the rise in volatility after a positive shock. Finally, the information criterion show that the EGARCH model is the best fit for all the variables included in this study.

The analysis shows that asymmetric GARCH models perform better when explaining the time varying volatility of the returns series. News impact curves can be defines as a graphical representation of the degree of asymmetry of volatility to positive and negative shocks (Brooks (2014) [9]). Hence, the news impact curve plots the value of the conditional variance (σ_t^2) that would arise from various values of lagged shocks (ε_{t-1}). The news impact curves of the univariate models estimated are plotted below.

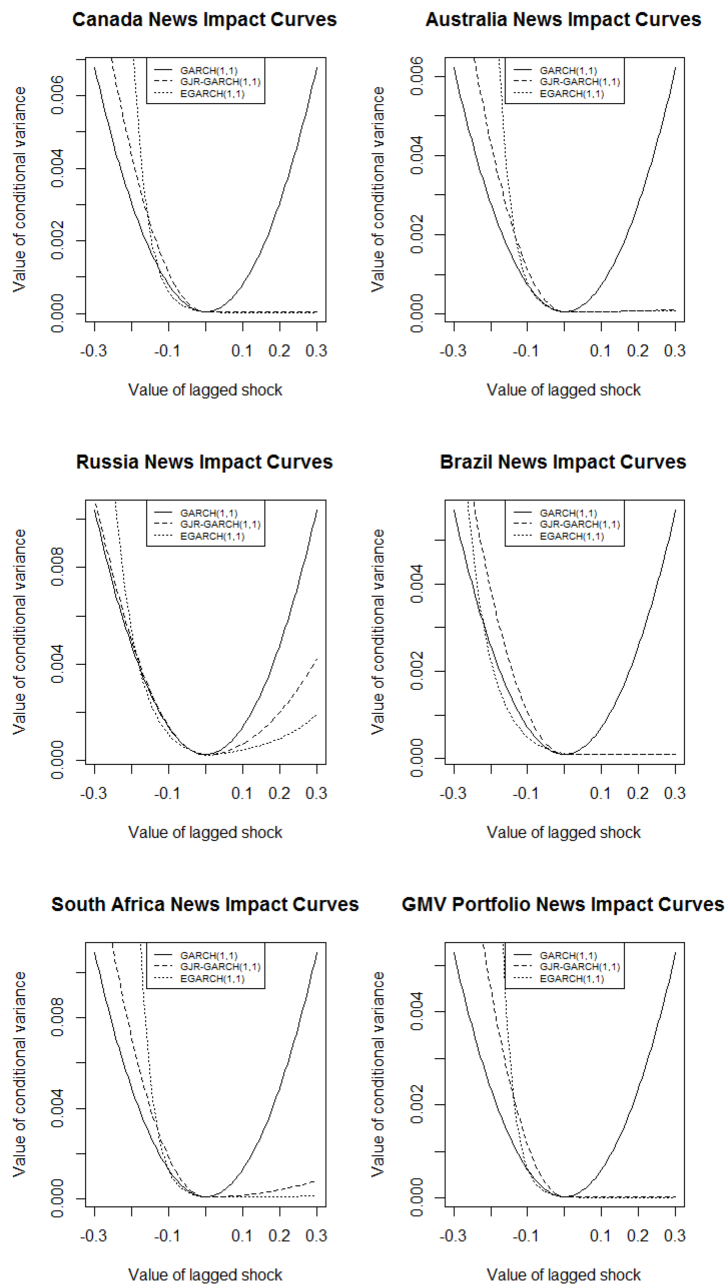
As expected, the news impact curves derived from the estimated GARCH(1,1) parameters are symmetric. There is some degree of asymmetry when the GJR-GARCH and EGARCH news impact curves are considered. When the value of a lagged shock is negative, it is clear that the value conditional variance, as modelled by the GJR-GARCH and EGARCH models, increases at a faster rate when compared to the conditional variance modelled by the GARCH model. Furthermore, when the value of a lagged shock is positive, the conditional variance increases at a slower rate when modelled by the GJR-GARCH and EGARCH models.

6.2 Multivariate GARCH models

In this section, the coefficients and estimated matrices of both the GO-GARCH and DCC-GARCH models are reported. The data used to estimate the models includes the 5 indices of the CARBS countries. The GMVP is not included in this analysis. Given the covariance matrix of the five indices used to construct the GMVP, it is easy to estimate the variance of the GMVP. In addition to the coefficients and estimated matrices, the news impact surfaces of the estimated models are considered.

When the GO-GARCH model is applied to the data, the following matrices are obtained:

Fig. 2 News Impact curves derived from univariate GARCH models



$$U = \begin{bmatrix} -0.6337 & 0.6751 & -0.3560 & -0.0996 & 0.0798 \\ 0.6051 & 0.5814 & -0.1140 & 0.5303 & 0.0408 \\ -0.0393 & -0.3900 & -0.6140 & 0.2931 & 0.6197 \\ 0.1416 & 0.2238 & 0.4490 & -0.3698 & 0.7692 \\ 0.4591 & 0.0632 & -0.5320 & -0.6973 & -0.1278 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.0067 & -0.0004 & -0.0010 & -0.0016 & -0.0005 \\ 0.0019 & -0.0006 & 0.0033 & 0 & -0.0067 \\ 0.0061 & -0.0135 & 0.0040 & -0.0002 & -0.0008 \\ 0.0076 & -0.0013 & 0.0006 & 0.0070 & 0.0002 \\ 0.0049 & -0.0014 & 0.0071 & -0.0008 & 0.0014 \end{bmatrix}.$$

Moreover, when the DCC-GARCH model is applied to the CARBS index data, the following parameters are obtained:

Table 5 DCC-GARCH(1,1) optimal parameters

	Canada	Australia	Russia	Brazil	South Africa
υ	0.0748	0.0685	0.1122***	0.0622***	0.1200
δ	0.9087	0.9150***	0.8686***	0.9034***	0.8593*

*(**) [***]: Statistically significant at a 10(5)[1] % level

Table 6 DCC-GARCH(1,1) optimal parameters

	Canada	Australia	Russia	Brazil	South Africa
υ	0.0748	0.0685	0.1122***	0.0622***	0.1200
δ	0.9087	0.9150***	0.8686***	0.9034***	0.8593*

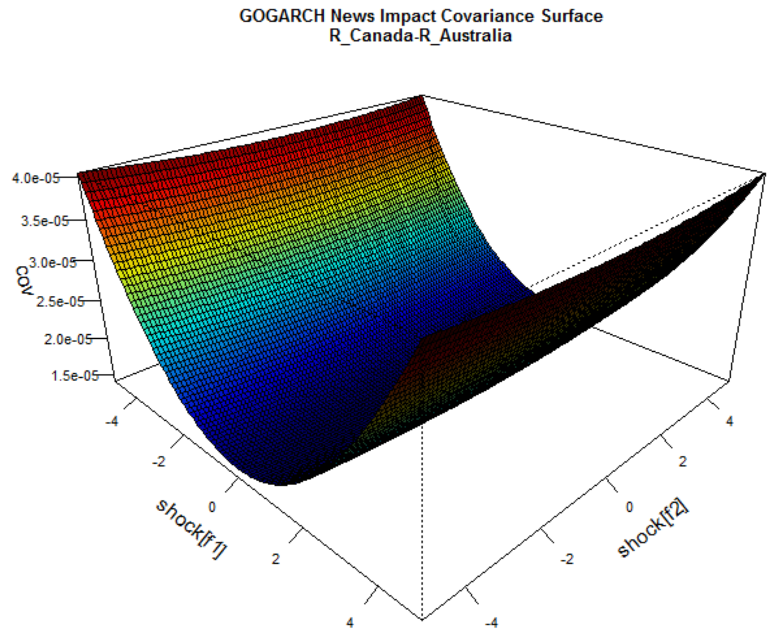
*(**) [***]: Statistically significant at a 10(5)[1] % level

The above coefficients are statistically significant when used to model the conditional covariance of the CARBS indices. However, this does not show how the conditional covariance changes after a shock to a specific index.

Caporin et al. (2011) [10] explain that a news impact surface is a multivariate extension of the news impact curve. Hence, it shows how the conditional covariance reacts after a shock to a specific index. The news impact surfaces of the conditional covariance between Canada and Australia, when modelled by GO-GARCH and DCC-GARCH models, are plotted below:

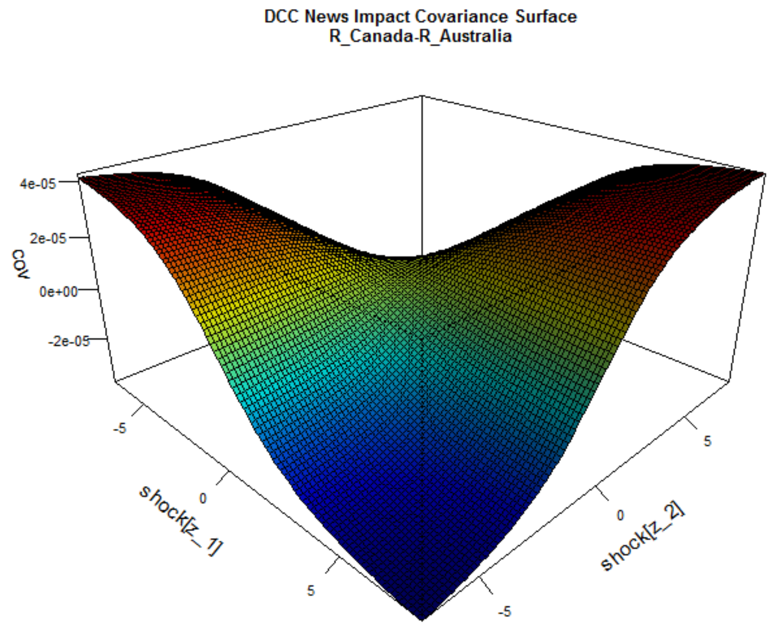
It is important to note that shock[f1] and shock[f2] in figure 3 denote specific shocks to Canada and Australia in the GO-GARCH framework respectively. It is evident that when the conditional covariance is modelled by a GO-GARCH model, the conditional covariance is close to zero when shocks to Canada are approximately equal to zero, and increases as shocks to Canada approach -4 and also increase as shocks to Canada increase to 4.

Fig. 3 Multivariate GARCH model news impact surfaces



Similarly, shock[z_1] and shock[z_2] in figure 3.3b denote specific shocks to Canada and Australia in the DCC-GARCH framework. It seems as though the conditional covariance in figure increases as shocks to the two indices are of the same sign, and decrease when shocks are of opposite signs when modelled using the DCC-GARCH model. Put differently, the conditional covariance is negative when the magnitude of a shock to Canada is 5, and the magnitude of a shock to Australia is -5. The conditional covariance is significantly positive when both shocks have a magnitude equal to 5 or -5.

The above models show how the optimal parameters when a GARCH process is used to model conditional variance, or the conditional covariance matrix. However, the use of the estimated models to measure risk is not considered in this study.



7 Conclusion

The purpose of this paper is to estimate the calibrated parameters of different univariate and multivariate GARCH family models. It is unrealistic to assume that volatility of financial returns is constant. Therefore it is necessary to estimate the parameters of time varying volatility models. The ARCH LM test showed that evidence of volatility clustering, and therefore the GARCH family models could be estimated.

Using a similar approach to Oberholzer et al. (2015) [20], the symmetric GARCH, and asymmetric GJR-GARCH and EGARCH models were estimated for the CARBS indices and the GMVP, the best fitting model was determined using the AIC and BIC. The asymmetric terms of the GJR-GARCH and EGARCH models indicate signs of the leverage effect, which suggests that negative news leads to a greater rise in volatility when compared to the rise in volatility after a positive shock. In addition, the AIC and BIC indicate that the EGARCH model is the best fitting model for all the indices and the GMVP.

The news impact curves derived from the GARCH family models show the degree of asymmetry, which shows that there is a greater rise in volatility after a negative shock. In terms of multivariate GARCH models, the optimal parameters of the GO-GARCH and the DCC-GARCH models were used to approximate a news impact surface. When the GO-GARCH model is used to model the conditional covariance matrix, the covariance seems to increase if the magnitude of the shock to a specific index increases. However, when the DCC-GARCH model is used, the covariance increases when the shocks are of the same sign and magnitude.

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