Value-At-Risk Forecasting of the CARBS Indices

Coenraad CA Labuschagne, Niel Oberholzer, and Pierre J Venter

Abstract The purpose of this paper is to use calibrated univariate GARCH family models to forecast volatility and value at risk (VaR) of the CARBS indices and a global minimum variance portfolio (GMVP) constructed using the CARBS equity indices. the reliability of the different volatility forecasts are tested using the mean absolute error (MAE) and the mean squared error (MSE). The rolling forecast of VaR is tested using a back-testing procedure. The results indicate that the use of a rolling forecast from a GARCH model when estimating VaR for the CARBS indices and the GMVP is not a reliable method.

1 Introduction

An empirical analysis that gives an indication of the best fitting univariate GARCH model does not give indication of how the models can be used to measure risk. Furthermore, the AIC and BIC show which model is the best fit. However, this does not provide a forward looking estimate of the dependent variable. Therefore, the forecasting ability of three univariate GARCH models is tested in this study.

The optimal parameters of the GARCH models show how volatility reacts after a shock to the index, how significant previous volatility is when modelling current period volatility, and the effect of positive and negative news. It is important to

Coenraad CA Labuschagne
University of Johannesburg, Department of Finance and Investment Management, Auckland Park, Johannesburg, 2006, South Africa, e-mail: ccalabuschagne@uj.ac.za

Niel Oberholzer
University of Johannesburg, Department of Finance and Investment Management, Auckland Park, Johannesburg, 2006, South Africa, e-mail: nielo@uj.ac.za

Pierre J Venter
University of Johannesburg, Department of Finance and Investment Management, Auckland Park, Johannesburg, 2006, South Africa, e-mail: pventer@uj.ac.za

consider how the volatility models can be used in financial risk management and how risk measures can be obtained from the estimated models.

Forecasting from a GARCH model provides an estimate of future volatility. This does not give provide an estimate of possible losses in the future. Hence, using a similar approach to Narsoo (2016) [9], univariate GARCH family models are estimated and used to obtain an out of sample forecast of volatility, the reliability of the different forecasts are tested using the mean absolute error (MAE) and the mean squared error (MSE). Finally, the univariate GARCH models are then used to construct a rolling forecast of VaR, this is tested using a back-testing procedure.

The empirical results in this paper are obtained by making use of the `rugarch` `R` package by Ghalanos (2014) [4]. The remainder of this paper is structured as follows: in the next section the relevant and recent literature is discussed, this is followed by a more formal definition and derivation of VaR and a GARCH model forecast, the empirical results are reported and interpreted, finally the main results and concluding remarks are summarised.

2 Literature review

Many studies have evaluated the forecasting performance of GARCH family models. Alberg et al. (2008) [1] estimated various GARCH type models to forecast the conditional variance of the Tel Aviv Equity Exchange. The AIC indicates that the asymmetric models are a better fit when compared to the symmetric GARCH model. Finally, the forecast performance statistics show that the EGARCH skewed student-t model produces the most reliable forecast.

In a recent study, Braione et al. (2016) [2] tested the VaR forecasting ability of univariate and multivariate GARCH models using different distributional assumptions. The distributional assumptions of three symmetric and three skewed symmetric assumptions. The accuracy of each model was tested using a back-testing procedure. The results showed that the skewed distributions with heavy tails outperform the other distributions. Therefore, it is important to model skewness and kurtosis.

Jimenez-Martin et al. (2009) [7] estimated GARCH, GJR, and EGARCH models, with Gaussian, Student-t and generalised normal distribution errors to forecast VaR before, during and after the GFC of 2008. Furthermore, different VaR forecasting models were also combined. The models were compared based on a back-testing procedure. Jimenez-Martin et al. (2009) [7] finally conclude that taking the supremum of the different forecasts is the most robust and reliable method before, during, and after the GFC.

Similarly, Narsoo (2016) [9] employed three different GARCH models, the standard GARCH, EGARCH and integrated GARCH models to forecast volatility and VaR for the US Dollar/Mauritian Rupee (USD/MUR) exchange rate. In addition, different error distributions were considered for each GARCH family model. The EGARCH model with a student-t error distribution is the best fitting model when modelling volatility of the USD/MUR exchange rate. The standard GARCH model with a fat tailed error distribution is the best performing model when forecasting volatility and VaR of the USD/MUR exchange rate.

3 Methodology

3.1 Value-at-risk (VaR)

The purpose of this section is to discuss the concept of VaR more formally. Particular attention will be given to the derivation of VaR for a portfolio. The following definition from Hull (2012) [6] explains the concept:

When using VaR as a risk measure, the following statement is of interest:

"We are X percent certain that we will not lose more than VaR_X in time T,"

where X is the confidence level ($X \in (0,1)$), and T is the time horizon.

The following derivation of VaR for a portfolio of equity shares follows Embrecths et al. (2005) [8] closely. Consider a portfolio of stocks at time t (Π_t), which consists of ψ_i units of each stock i. Hence, the weighing at time t of each stock i is given by

$$\omega_{i,t} = \frac{\psi_i S_{i,t}}{\Pi_t}. \tag{1}$$

Define the loss factor $Z_{i,t} = \ln(S_{i,t})$. The value of the portfolio at time t in terms of $Z_{i,t} = \ln(S_{i,t})$ is

$$\Pi_t = \sum_{i=1}^{n} \psi_i \exp\{Z_{i,t}\}.$$

Moreover, the gain of the portfolio (G_t) is defined as difference between the portfolio at time $t+1$ and time t,

$$\begin{aligned} G_t &= \Pi_{t+1} - \Pi_t \\ &= \sum_{i=1}^{n} \psi_i S_{i,t} (\exp\{R_{i,t}\} - 1) \end{aligned}$$

where $R_{i,t}$ denotes the log return at time t. The linearised gain (G_t^{Δ}) is in turn given by

$$G_t^{\Delta} = \sum_{i=1}^{n} \psi_i S_{i,t} R_{i,t} \tag{2}$$

Finally, by making use of equation 1 it is possible to write equation 2 as

$$G_t^{\Delta} = \Pi_t \sum_{i=1}^{n} \omega_{i,t} R_{i,t}.$$

In vector notation, the linearised gain is given by $G_t^{\Delta} = \Pi_t \bar{\omega}_t' \bar{R}_t$, where $\bar{\omega}_t'$ is an $(n \times 1)$ vector of portfolio weights and $\bar{R}_t$ is a $(1 \times n)$ vector of log returns which is assumed to follow a distribution with mean vector $\bar{\mu}_t$ and covariance matrix Σ. When the rules regarding the mean and variance of a linear combination of a random vector are applied, the following is obtained

$$\mathbb{E}[G_t^{\Delta}] = \Pi_t \bar{\omega}_t' \bar{\mu}_t = \mu, \text{ and}$$
$$\mathbb{V}ar[G_t^{\Delta}] = \Pi_t^2 \bar{\omega}_t' \Sigma \bar{\omega}_t = \sigma^2$$

If it is assumed that the gain distribution follows a normal distribution, with mean μ ans variance σ^2, VaR can be calculated using the variance-covariance can be computed as follows:

$$\mathrm{VaR}_X = -\mu - \sigma \phi^{-1}(X)$$

where X is the confidence interval, and ϕ^{-1} is the inverse of the standard normal distribution function. The above assumes that the volatility is constant and based on the historical information.

The accuracy of VaR is usually tested using a back-testing procedure. The back-testing parameter takes a value of one when the actual loss is greater than VaR (there is an exception), and zero otherwise. The back-testing parameter is defined by the following indicator function,

$$BT_t = 1_{\{\mathrm{VaR} > R_t\}}.$$

The percentage of exceptions is equal to the sum of BT_t, for t is equal to 0 until the end of the period, divided the number of periods used to back-test. An important factor to consider the accuracy of a forecast of volatility, which can be seen as a forward looking approach which can ultimately be used to compute VaR.

3.2 Forecasting from a GARCH model

To illustrate the concept, the use of forecasting volatility from a GARCH(1,1) model is derived below. A similar approach is used when forecasting from a GJR-GARCH(1,1) and EGARCH(1,1) model, therefore the derivation of the forecast from these models is excluded from this section.

The derivation that follows is based on the work by Hull (2009) [5]. The GARCH(1,1) model takes the following specification,

$$\sigma_t^2 = \gamma + \upsilon\varepsilon_{t-1}^2 + \delta\sigma_{t-1}^2, \tag{3}$$

where $\gamma = (1-\upsilon-\delta)\kappa_L$, and κ_L is the long run average variance rate. Equation 3.2 can be manipulated to obtain

$$\sigma_t^2 - \kappa_L = \upsilon(\varepsilon_{t-1}^2 - \kappa_L) + \delta(\sigma_{t-1}^2 - \kappa_L).$$

Furthermore, the volatility in period $t+\tau$ is

$$\sigma_{t+\tau}^2 - \kappa_L = \upsilon(\varepsilon_{t+\tau-1}^2 - \kappa_L) + \delta(\sigma_{t+\tau-1}^2 - \kappa_L)$$

However, because this information is unknown at time t, it is necessary to take the expectation. In addition, the expected value of $\varepsilon_{t+\tau-1}^2$ is $\sigma_{t+\tau-1}^2$. Hence,

$$\mathbb{E}[\sigma_{t+\tau}^2 - \kappa_L] = (\upsilon+\delta)\mathbb{E}[\sigma_{t+\tau-1}^2 - \kappa_L],$$

where $\mathbb{E}$ denotes the expectation. If the above equation is used repeatedly, the following is an expression for the forecast of volatility, τ periods in future,

$$\mathbb{E}[\sigma_{t+\tau}^2] = \kappa_L + (\upsilon+\delta)^\tau(\sigma_{t+\tau-1}^2 - \kappa_L).$$

This implies that it is possible to forecast future volatility using historical information. The accuracy of this idea is tested when applied to financial risk management in the next section.

3.3 Forecast performance measures

In this study, the MAE and MSE are used to determine the forecasting performance of each model used to forecast volatility. Brooks (2014) [3] explains that the MAE and MSE are obtained by comparing the actual values to an in sample forecast. More specifically, the MAE and MSE consider

$$\text{MAE} = \left|\sigma_{t+\tau}^2 - \mathbb{E}\left[\sigma_{t+\tau}^2\right]\right|, \text{ and}$$
$$\text{MSE} = \left(\sigma_{t+\tau}^2 - \mathbb{E}\left[\sigma_{t+\tau}^2\right]\right)^2$$

respectively. This provides an estimate of the reliability of the forecasting model. Clearly, the model that minimises the MAE and MSE is the most reliable forecasting model.

4 Empirical results

The empirical results section is split into two components, one tests the out of sample forecasting ability of the three GARCH family models, and the other tests the VaR forecasting ability of the GARCH family models.

4.1 Forecasting volatility

In the tables below, the forecast performance measures of the GARCH(1,1), GJR-GARCH(1,1) and EGARCH(1,1) models are reported. In order to test the out of sample forecasting ability of the three GARCH family models included in this study, a 90 day out of sample forecast for volatility was performed. The best performing model for each respective index is the model that minimises the mean squared error (MSE), or the mean absolute error (MAE). The minimum MSE and MAE for each index is shown in bold.

Table 1: GARCH(1,1) forecast performance measures

	Canada	Australia	Russia	Brazil	South Africa	GMVP
MSE	6.47E-05	**6.97E-05**	3.57E-04	1.32E-04	8.10E-05	3.55E-05
MAE	6.12E-03	6.42E-03	1.42E-02	8.32E-03	6.97E-03	4.42E-03

Table 2: GJR-GARCH(1,1) forecast performance measures

	Canada	Australia	Russia	Brazil	South Africa	GMVP
MSE	6.46E-05	6.98E-05	**3.56E-04**	1.32E-04	8.09E-05	3.55E-05
MAE	6.11E-03	**6.41E-03**	**1.42E-02**	8.29E-03	6.97E-03	4.40E-03

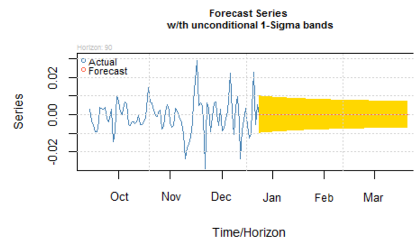
Table 3: EGARCH(1,1) forecast performance measures

	Canada	Australia	Russia	Brazil	South Africa	GMVP
MSE	**6.46E-05**	6.99E-05	3.56E-04	**1.32E-04**	**8.08E-05**	**3.54E-05**
MAE	**6.10E-03**	6.41E-03	1.42E-02	**8.29E-03**	**6.97E-03**	**4.40E-03**

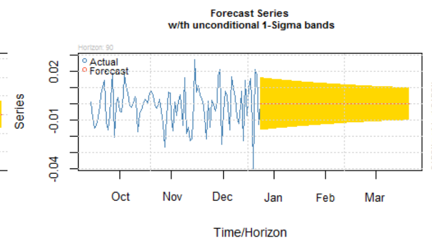
It is evident that the EGARCH(1,1) model produces the best performance measures for most of the indices included in this study, and the GMVP. The results show that the MAE and MSE are consistent, however not for the case of Australia. The GARCH(1,1) model minimises the MSE of the Australian index, but the GJR-GARCH(1,1) minimises the MAE.

Fig. 1: Volatility forecast: Best performing GARCH models

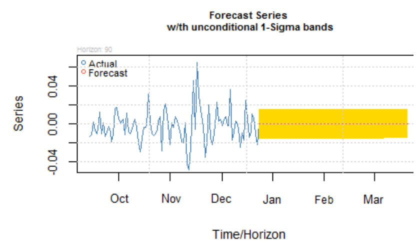
(a) Canada: EGARCH(1,1)



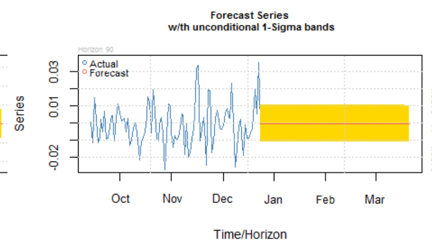
(b) Australia: GJR-GARCH(1,1)



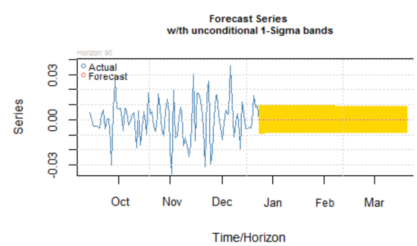
(c) Russia: GJR-GARCH(1,1)



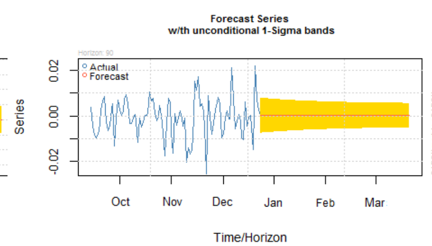
(d) Brazil: EGARCH(1,1)



(e) South Africa: EGARCH(1,1)



(f) GMVP: EGARCH(1,1)



The forecast graphs of the best performing GARCH models according to the MAE are plotted above. The graphs show that the out of sample forecast for the volatility of the indices is close to the mean in each case. The unconditional standard deviation bands also seem fairly small for each out of sample forecast.

4.2 *Forecasting VaR*

The out of sample forecast does provide an estimate of future volatility based on historical information. However, this does not provide an estimate of what the maximum loss is with a certain level of confidence. Therefore, the same GARCH family models were used to obtain a forecast of 99% one day VaR. The performance of each model is assessed using a back-test when using the last 1000 observations of the CARBS indices and the GMVP. A rolling forecast of volatility is used to estimate VaR.

The percentage of exceptions of each model for the CARBS indices and the GMVP are reported below, the lowest percentage of exceptions for each index and the GMVP is shown in bold. From the above, it is evident that the GARCH family

Table 4: GARCH rolling forecast: percentage of exceptions

	Canada	Australia	Russia	Brazil	South Africa	GMVP
GARCH	2.10%	**1.70%**	2.00%	**1.50%**	1.90%	2.60%
GJR-GARCH	2.20%	1.90%	1.90%	1.90%	1.90%	2.30%
EGARCH	**1.80%**	1.90%	**1.80%**	1.90%	**1.70%**	**2.10%**

models used in this study under-forecast VaR. The expected percentage of exceptions is 1%, the back-test of each model yields a greater percentage of exceptions. However, the EGARCH model produces the lowest percentage of exceptions for 4 out of 6 variables. The GARCH(1,1) model produces the lowest percentage of exceptions for Australia and Brazil.

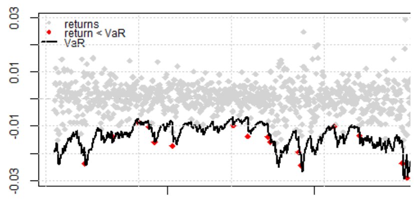
A graphical representation of the back-testing procedure is shown by the figure below. The Y-axis denotes the daily log returns, and the X-axis denotes time. The back-testing procedure of the models that produce the lowest percentage of exceptions are included in the figure. The back-testing graph shows that the rolling forecast VaR estimate for Russia is the highest, and the estimate for the GMVP is the lowest. In addition, it is evident that a rolling forecast VaR from a GARCH model is not a reliable method when estimating VaR. Therefore, a different approach is required.

5 Conclusion

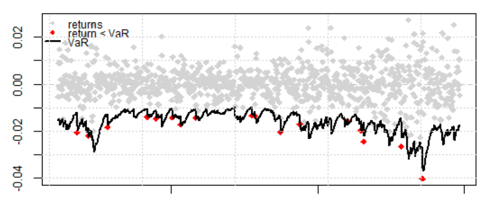
In this study, three univariate GARCH models were estimated and used to forecast volatility, and VaR. Volatility is a measure of the degree of uncertainty in financial markets, and VaR summarises the risk of a financial position in a single number. VaR gives an indication of the maximum loss with a certain level of confidence (Hull (2012) [5]]).

Fig. 2: Back-testing: GARCH rolling forecast

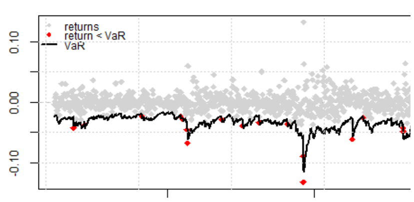
(a) Canada: EGARCH(1,1)



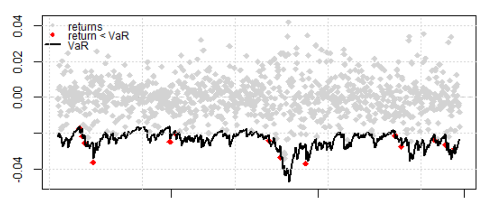
(b) Australia: GARCH(1,1)



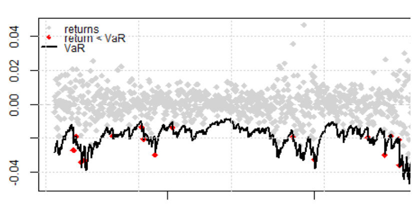
(c) Russia: EGARCH(1,1)



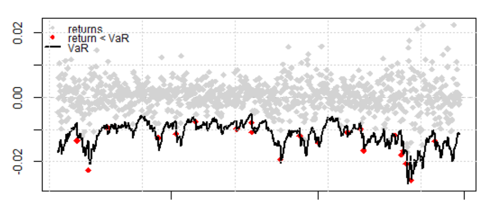
(d) Brazil: GARCH(1,1)



(e) South Africa: EGARCH(1,1)



(f) GMVP: EGARCH(1,1)



Many studies in the literature have focused on the forecasting performance of different volatility models. The empirical analysis of this paper is based on the recent work by Narsoo (2016) [9]. The empirical analysis consists of two parts, firstly the out of sample forecasting ability of the different univariate GARCH models were considered. This was tested using the MAE and MSE, which is the absolute and squared difference respectively, between an in sample forecast based on the model, and the actual value. Secondly, a rolling forecast from the different GARCH models are used to estimate daily value at risk. This process is tested using a back-testing procedure.

Both forecast performance statistics of the out op sample forecast suggest that the EGARCH model produces the most reliable forecast for Cabada, Brazil, South Africa, and the GMVP. Moreover, the MAE and MSE show that the asymmetric GJR-GARCH model produces the most reliable forecast for Russia. Finally, the MSE indicates that the symmetric GARCH model produces the most accurate forecast of volatility for Australia, this is inconsistent with the model suggested by the MAE, which is the asymmetric GJR-GARCH model.

When estimating 99% VaR, the expected number of exceptions when back-testing is 1%. The EGARCH model produces the lowest percentage of exceptions for most of the variables. However, VaR is underestimated in each case when a rolling forecast is used. This implies that the use of a rolling forecast from a GARCH model when estimating VaR for the CARBS indices and the GMVP is not a reliable method. Therefore, a different approach is necessary.

References

1. Alberg, D., Shalit, H. and Yosef, R., (2008) *Estimating stock market volatility using asymmetric GARCH models.* Applied Financial Economics, 18(15), pp.1201-1208.
2. Braione, M. and Scholtes, N.K., (2016) *Forecasting Value-at-Risk under Different Distributional Assumptions.* Econometrics, 4(1), p.3.
3. Brooks, C., 2014. *Introductory econometrics for finance.* Cambridge university press.
4. Ghalanos, A., (2014) *rugarch: Univariate GARCH models,* R package version 1.3-3.
5. Hull, J., (2009) *Options, Futures and Other Derivatives.* Seventh edition. Pearson Education.
6. Hull, J., (2012) *Risk Management and Financial Institutions.* Third edition. John Wiley & Sons.
7. JimenezMartin, J.A., McAleer, M. and PrezAmaral, T., (2009) *The ten commandments for managing value at risk under the Basel II Accord.* Journal of Economic Surveys, 23(5), pp.850-855.
8. Embrechts, P., Frey, R. and McNeil, A., (2005) *Quantitative risk management. Princeton Series in Finance,* Princeton, 10.
9. Narsoo, J., (2016) *Evaluation of GARCH-Type Models in Volatility and Value-At-Risk Forecasting: Evidences from USD/MUR Exchange Rates.* University of Mauritius Research Journal, 22.