

Manuscript Details

Manuscript number	SAFETY_2017_122
Title	Resiliency Assessment of Urban Rail Transit Networks: Shanghai Metro as an Example
Article type	Research Paper

Abstract

This paper presents a general framework to assess the resilience of large and complex metro networks by quantitatively analyzing its vulnerability and recovery rapidity within unifying metrics and models. The connectivity performance of network is indicated by the network efficiency. The resilience of a metro network can be associated to the network performance loss triangle over the relevant timeline from the occurrence of a random or intentional disruption to full recovery. The proposed resilience model is applied to the Shanghai metro network with its 303 stations and 350 links as an example. The quantitative vulnerability analysis shows that the Shanghai metro with its L-space type of topology has a strong robustness regarding connectivity under random disruption but severe vulnerability under intentional disruption. This result is typical for small-world and scale-free networks such as the Shanghai metro system, as can be shown by a basic topological analysis. Considering the case of one disrupted metro station, both the vulnerability and resilience of the network depend not only on the node degree of the disrupted station but also on its contribution to connectivity of the whole network. Analyzing the performance loss triangle and the associated cost from loss of operational income and repair measures, an appropriate recovery strategy in terms of the optimum recovery sequence of stations and the optimum duration can be identified in a structured manner, which is informative and helpful to decision makers.

Keywords	Network Resilience; Shanghai Metro; Node Connectivity; Topology; Robustness; Vulnerability.
Taxonomy	Metro Tunnels, Resilience, Transport Infrastructure
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Research Data Related to this Submission

There are no linked research data sets for this submission. The following reason is given:
Data will be made available on request

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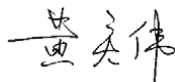
Aug 21, 2017

Dear Editor,

Thank you very much for your letter dated June 14 2017 with regard to our manuscript entitled “**Resiliency Assessment of Urban Rail Transit Networks: Shanghai Metro as an Example (SAFETY_2017_122)**”. Authors’ special thanks also goes to the reviewers. All the comments and recommendations are quite insightful and valuable to improve the quality of our manuscript. According to the comments, we have checked the manuscript thoroughly and revised it carefully. Revised portion are marked in **yellow** in the revised manuscript. Detail responses to the comments can be found in the document named by “response to reviewers”.

Should you need to contact me, please use the above address or call me at (86)-21-6598-9273. You may also contact me by fax at (86)-21-6598-5017 or via e-mail at huanghw@tongji.edu.cn

Sincerely,



Hongwei Huang

Ref: SAFETY_2017_122

Resiliency Assessment of Urban Rail Transit Networks: Shanghai Metro as an Example

Dear Editor,

We would like to thank the reviewers and editor for their time and their thoughtful comments. All the comments and recommendations are quite insightful and valuable to improve the quality of our manuscript. According to these comments, the manuscript has been thoroughly revised. The revised portion is marked in **yellow** in the revised manuscript. Detailed responses to the comments can be found below.

Reviewer #1:

1. Recommend restructuring sections or rewording the introduction to include a stronger framework with more succinct transitions between sections.

Reply:

Following the reviewer's very good suggestion, **the authors have revised the introduction to add transitions between sections and an explicit framework in the last paragraph of the introduction.**

Line 63 – 65: *“Quite a number of studies have been carried out by analyzing networks for the safety of metro systems in particular with the topographical mapping modelling (Crucitti et al., 2003; Derrible and Kennedy, 2010; Watts and Strogatz, 1998; Zhang et al., 2013; Zhang et al., 2011).”*

Line 78 – 81: *“The above network assessment models, however, are mainly qualitative with conceptualized measures that do not offer a rigorous comparison of safety levels among networks. In this respect, network analysis with quantitative measures of robustness and vulnerability is thus helpful for examining the safety level of a specific network.”*

Line 89 – 91: *“Additionally, disrupting a large-scale metro network by an accident affects not only the robustness but also the subsequent recovery. Recovery profiles over time greatly affect the economic and social wellbeing outcomes that are of great concern to metro owners.”*

Line 94 – 97: *“In the context of recovery, robustness and vulnerability analyses are still insufficient to offer a rational recovery strategy in terms of recovery sequencing and duration for stations and tunnels in a metro network.”*

Line 98 – 101: *“A comprehensive safety assessment model includes both the network robustness and the recovery profile as provided by Ayyub (2014). In this context, the concept of “resilience” would provide an appropriate solution to cover both the robustness and recovery in a single model for safety evaluation of a network system (Ayyub, 2015; Bruneau et al., 2003).”*

Line 124 – line 127: *“This paper provides a general framework for the resilience analysis of large-scale metro network systems that offers an immediate basis for identifying both the best recovery sequences to minimize the performance loss and the best repair duration to minimize the costs associated with disruption and recovery.”*

Line 135 – 141: *“The main body of the paper is provided in two subsequent primary sections. The first primary section provides the detailed description of the resilience model according to the above four items. The second primary section provides an application of the above resilience model to the Shanghai metro. Recovery strategies are discussed for (a) a single multi-line station, and (b) multiple stations in terms of repair sequences that minimize the resilience loss triangle. Then, a brief application example focusing on cost analysis during disruption and recovery is used to illustrate the concepts introduced. Finally, concluding remarks are presented.”*

2. While I do believe this study is fascinating and "worthy of investigation" I urge you to consider making a more substantive claim earlier in the paper demonstrating how findings in this study may have implications and applications outside of Shanghai.

Reply:

The reviewer's comment is quite appreciated. **Yes, the proposed resilience analysis model is suitable not only for the Shanghai case, but also for complex metro network systems of other big cities**, such as London, Beijing, New York, Paris, and etc. The Shanghai metro is only one example used in this paper for the purpose of demonstrating the application of the proposed model to such a complex metro network. **The authors have added the potential use of the proposed model explicitly in the last paragraph of introduction in line 141 – 144.**

“Although the illustrative examples in this paper are based on the Shanghai metro, the proposed resiliency model is applicable for safety analysis of other complex metro networks, such as in London, New York, etc., with the emphasis on network robustness and recovery.”

-Reviewer 2

Review comments

Manuscript number SAFETY_2017_122

Title: “Resiliency Assessment Model of Urban Rail Transit Networks: Shanghai Metro as an Example”

1. Title: Ok
2. Abstract: Clear and logical. Nicely point out the key findings of the results.
3. Keywords: Add “vulnerability”

Reply:

We have added the key word “Vulnerability” in the revised manuscript. Thanks for the good suggestion.

4. Introduction

(i) At p-5, the sentence “cost related to recovery stage should also be taken into account for a full assessment of the resiliency of the metro network” is mentioned. But this paper has not addressed the cost related recovery analysis.

Reply:

Yes, this is very important. We have revised the manuscript extensively to address this issue. Apart from proposing the general framework for resilience analysis, we now discuss the cost during disruption and recovery stage, however omit the cost-related analysis for the safe operation in lifetime of the metro system due to less relevance to the scope of the paper. For an optimized recovery strategy, when the best recovery sequence is determined by the proposed resilience model (i.e., maximizing the index Re), the key question for the decision maker is the best repair duration to be determined. Obviously, it should be related to the cost during the disruption and repair works. **Hence, the cost in this paper includes the disruption cost $C_{disruption}$ and the repair cost C_{repair} . Each component of the costs, i.e., $C_{disruption}$ and C_{repair} , is newly discussed extensively in the following:**

For disruption cost $C_{disruption}$, two major categories are analyzed in the conceptualized form, including: 1) disfunction of equipment and abandonment of facility in metro station; and 2) income and stocks loss due to close of metro operation. Taking the flood induced failure of metro station for an example, once the station is assumed to be disrupted by the severe flood, the equipment such as track barriers, escalator, signal board and etc., could be partially or totally disabled. Later on, the control rooms and power plant rooms might also be affected due to the floods. Furthermore, when the metro station is completely closed, the incomes including the tickets, the retails and other business around the metro stations might be lost. In addition, the stocks of the metro company will be further affected due to the disruptions. **All those cost factors are listed in Table 1 (newly added in the revised manuscript).** However, to incorporating all these factors in this study would be too complicated and straightforward for the decision-making. In this case, for ease of discussion, the cost analysis for $C_{disruption}$ in this manuscript is mainly based on the income loss of the tickets C_{ticket} ($C_{disruption} = C_{ticket}$ in this paper) which can be quantitatively evaluated. Those other cost factors in Table 1 could be related to the ticket income qualitatively with a coefficient ($C_{others} = \lambda \times C_{ticket}$) if interested in the next study.

For the repair cost C_{repair} , it includes the direct cost and indirect cost. The direct cost is related to the salary of workers, consumption of engineering materials, rental of the engineering machinery etc. Some typical direct costs are also shown in Table 1. For a certain repair project, shortening the repair duration t can be achieved by an increase of salary, the use of high quality materials, and more machinery. Hence, the direct cost is believed to be negatively correlated with the repair time duration t . The indirect cost is caused by social related factors (also shown in Table 1), which is believed to be positively correlated with the time duration t . **Hence, the repair cost is a typical time-cost trade-off problem which can be solved by the LP/IP hybrid method proposed by Liu et al. (1995).**

Table 1 Cost produced due to disruption of metro station

Cost Category	Content	
Disruption cost	Disfunction of equipment	track track barrier escalator signal board ticket barrier air-conditioning system ventilation system
	Abandonment of facility	managing control room power plant room ventilation room signal system room escape shaft
	Income loss	metro ticket retail mall rental real estate
	Stocks	stocks
Repair Cost	Direct cost	salary of workers monitoring and inspection before repair site investigation construction materials rental of engineering machinery purchase of power, ventilation and signal system
	Indirect cost	time value social impact

The above detailed discussion on the cost and Table 1 are newly added in the revised manuscript.

Line 292 – line 298: “*The recovery of network connectivity is a primary goal for decision makers; however, disruption and repair costs associated with recovery profiles a key criterion in decision making. Once the recovery sequence is selected, the particulars of recovery plans, i.e., repair methods, equipment, etc., affect the repair duration and subsequently the recovery cost. Generally, advanced techniques for repair work require less time to complete recovery task, but at a greater cost. Hence, a recovery cost analysis in terms of repair time duration helps to determine the optimized cost of repair.*”

Line 303 – line 322: “*Following the basic model proposed by Henry and Ramirez-Marquez (2012), the total cost C_{total} during the disruption and repair stages is generally composed of two parts, namely the cost $C_{\text{disruption}}$ and the cost C_{repairs} , represented as:*

$$C_{\text{total}} = C_{\text{disruption}} + C_{\text{repair}} \quad (11)$$

where the $C_{\text{disruption}}$ is referred to the cost or income loss due to the disruption of network connectivity and the C_{repairs} means the cost or investment for the repair works. Taking the flood-induced failure of metro station as an example, the $C_{\text{disruption}}$ might include two major categories, i.e., damaged but repairable equipment and damaged but non-repairable equipment, and income loss due to closing of metro operations. Once a station is set as disrupted by flooding, the equipment such as track barriers, escalator, signal board and etc., could be partially or totally disabled. Later on, the control rooms and power plant rooms might also be affected due to flooding. Furthermore, when the metro station is completely closed, loss in income from ticket sales, the retails and other

business around the metro stations comes on top. In addition, the stocks or reputation of the metro company could be affected due to the disruptions. Details of the above disruption cost are shown in Table 1. However, incorporating all the factors shown in Table 1 for disruption cost $C_{disruption}$ requires data that are not available in some cases. For ease of discussion, the cost analysis for $C_{disruption}$ in this paper includes mainly the income loss in the ticket sales C_{ticket} ($C_{disruption} = C_{ticket}$ in this paper) which can be quantitatively evaluated. Some other cost factors in Table 1 could be related qualitatively to the ticket income using perhaps multiplication coefficients ($C_{others} = \lambda \times C_{ticket}$) as an example.”

Line 406 – line 409: “After determining the optimal repair sequence by the resilience analysis model as shown in Figure 4, the optimal repair duration for each station could be obtained from the above cost analysis. Therefore, the best recovery strategy due to the disruption can be achieved both with the best recovery sequence and duration.”

Overall, the philosophy behind the recovery strategy for disrupted metro system is first to select a best recovery sequence and second to select a best repair duration considering both the disruption and repair cost. **In order to show the application of the above cost analysis, an example of cost analysis for recovering a completely disrupted metro station by rebuilding a new station is discussed in the revised manuscript:**

Line 531 – line 539: “Furthermore, repair duration should be optimized on the basis of minimizing the disruption and repair costs. The philosophy behind the recovery strategy for a disrupted metro system is first to select the best recovery sequence either for an exchange station or for multiple stations and second to select the best repair duration considering both the disruption and repair cost. In this regard, the following section shows an example of three typical cases of the resilience analysis of the Shanghai metro network: Case 1 is to obtain the best recovery sequence for a disrupted exchange station; Case 2 is to obtain the best recovery sequence for multiple disrupted stations; and Case 3 is to obtain the best repair duration for rebuilding a disrupted metro station. ”

Line 621 – line 665:

“Case 3: Recovery cost to rebuild a disrupted metro station

In this hypothetical example, a metro station is disrupted completely at time t_0 and requires rebuilding of the damaged station. For ease of discussion, t_0 is assumed to be zero. In addition, time-value of money is not considered in this example for simplicity. During the repair procedure, the metro station is closed to the public. As mentioned previously, the disruption cost in the recovery stage is assumed to only include the income loss related to metro tickets. From the statistics of the Shanghai metro, the daily passenger flow volume is more than 10 million. Hence, the disruption of one station among the 303 stations of the whole network is assumed to lead in a loss of about 40 thousand passengers flow volume every day. The ticket price of Shanghai metro is about 4 RMB on average. For this case, from Eq. 13b, the disruption cost during the recovery procedure can be calculated as below:

$$C_{disruption} = \frac{1}{2} \times 4 \times 40000 (t_1 - t_0) \quad (15)$$

where t_1 is the recovered time moment that is unknown and needs to be optimized and t_0 is equal to zero that is assumed in this example.

The rebuilding of a metro station requires several stages in sequence, including the site investigation, construction of a pile foundation of underground structures, construction of a soil retaining system, soil excavation with construction of supports and dewatering, and construction of underground structures of the metro station. From personal communication with the managers in charge of design, construction and operation, Table 8 shows a typical example of repair options associated with the direct cost and duration for rebuilding one typical metro station in Shanghai. It is quite obvious from direct cost perspectives that the shorter the repair duration, the higher the cost. Solving such a typical time-cost tradeoff requires the use of linear and integer programming (LP/IP) models, such as Liu et al. (1995). Table 9 shows the calculated results of repair duration and associated direct cost of

construction. The indirect cost is assumed, in this example, to be equal to 100 thousands RMB per day. Hence, the total indirect cost can also be calculated based on the calculated duration, as shown in Table 9. Figure 14a shows the calculated direct cost, indirect cost and the repair cost against the repair duration, respectively (solid circles for C_{repair} , hollow triangles for direct cost and hollow rectangles for indirect cost). It is observed from Figure 14a that the optimized repair duration based on repair cost is around 455 days. Corresponding to this calculated repair duration, the disruption cost in terms of the ticket income loss can be derived from Eq. 15, as displayed in Table 9. Hence, the total cost C_{total} both including $C_{disruption}$ and C_{repair} can be obtained as shown in Figure 14b (solid circles for C_{total} , hollow circles for C_{repair} and hollow rectangles for $C_{disruption}$). The optimized repair duration after incorporating the income loss is about 450 days, which is less than the optimized duration considering only repair cost. This finding is reasonable, as the income loss will continue to increase when the repair work is not finished. In this example, the disruption cost only includes the ticket income loss, resulting in a five-day decrease of the optimized duration. Although the number of days in this case is small, the best repair duration might be reduced further as other cost elements of disruption are considered. If the other disruption cost as shown in Table 1 is considered to be proportional to the ticket income loss, i.e., $\lambda = (C_{disruption} - C_{ticket})/C_{ticket}$, Figure 14c has illustrated the effect of parameter λ on the optimized repair duration days. It is clear that as the λ becomes larger, i.e., that is to say, the other disruption cost increases, the optimized repair duration days would be greatly reduced. It is consistent with the engineering practice that if the disruption cost is invaluable or social effect of the metro accident is unacceptable, the best repair strategy should be as fast as possible.”

Table 8 Repair options for rebuilding a metro station in Shanghai

Procedure	Options	Cost (million RMB)	Duration (day)
Site investigation	crew1+equipment1	6	50
	crew2+equipment2	8	40
Pile foundation	crew1+equipment1	6	30
Retaining wall	crew1+equipment1	29	80
	crew2+equipment2	33	70
Excavation /support /dewatering	crew1+equipment1	15	180
	crew2+equipment2	20	165
Underground structure	crew1+equipment1	30	120
	crew2+equipment2	35	110

Table 9 Integrated repair duration and cost for rebuilding a metro station in Shanghai

Duration (day)	C_{repair}			$C_{disruption}$	C_{total}
	Direct Cost	Indirect cost	sum		
415	102.0	41.5	143.5	33.2	176.7
420	98.5	42.0	140.5	33.6	174.1
425	95.1	42.5	137.6	34.0	171.6
430	92.5	43.0	135.5	34.4	169.9
435	90.9	43.5	134.4	34.8	169.2
440	89.5	44.0	133.5	35.2	168.7
445	88.0	44.5	132.5	35.6	168.1
450	87.0	45.0	132.0	36.0	168.0
455	86.4	45.5	131.9	36.4	168.3
460	86.0	46.0	132.0	36.8	168.8

Note: cost unit is million RMB.

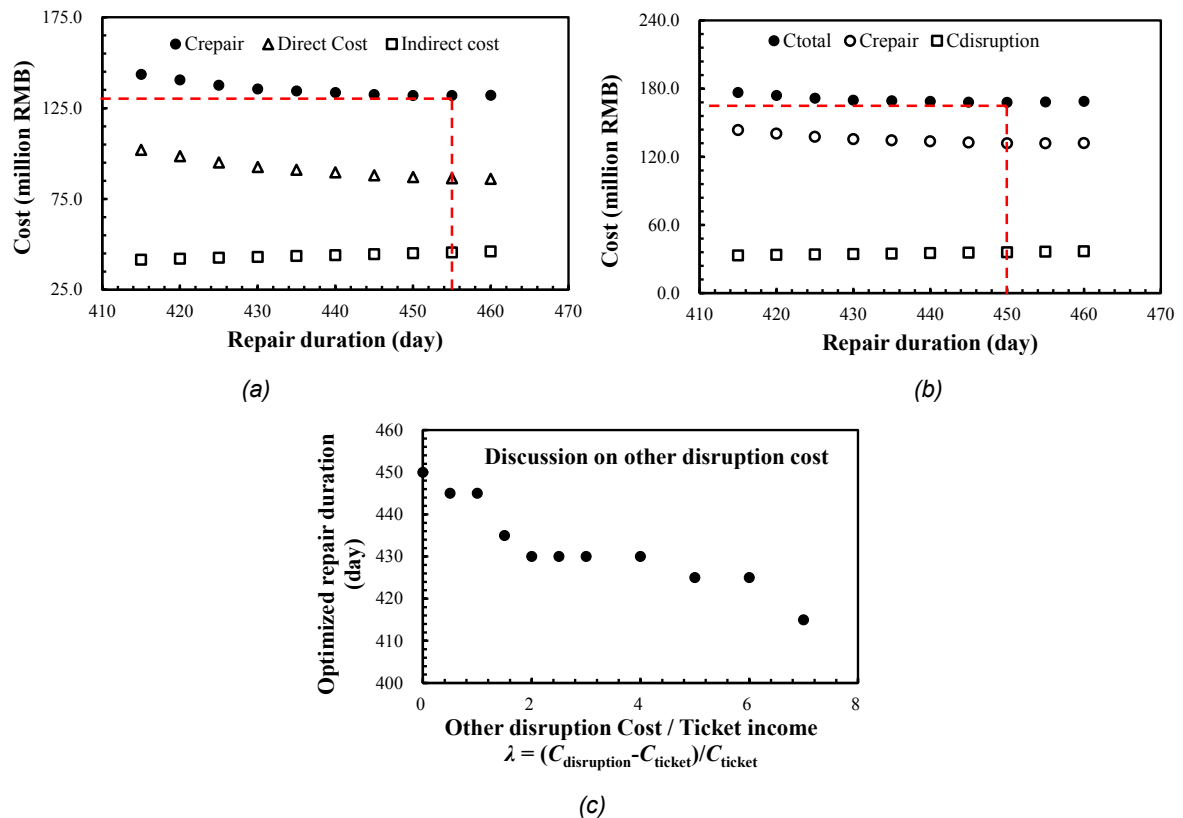


Figure 14 Time-cost trade-off analysis for rebuilding a metro station: a) time-cost for repair cost; b) time-cost for total cost including disruption cost and repair cost; and c) effect of other disruption cost on the optimized repair duration days ”

(ii) The paper has discussed on the cost related to disruption and recovery stage; but has not contributed in the cost-related analysis.

Reply:

We thank the reviewer’s understanding of the cost analysis discussed in the manuscript that is mainly for the disruption and recovery stage. The detailed responses could be referred to the response to the comment (i) in 4. introduction.

(iii) At p-5, in the second paragraph, it is not clear why repair measures are mentioned along with recovery measure? It is not discussed at all.

Reply:

Sorry for the wording mistake in the original version of the manuscript. **We have deleted the sentence to avoid the confusion of miswording.**

5. General framework

(i) Citation error in “Von Ferber et al. (2009)”. It is not matched with the one in reference section.

Reply:

Thanks for the careful check of the citation. We have double checked with the original reference. The correct citation should be von Ferber et al. (2009). Hence, the name for author von Ferber is corrected in the revised manuscript.

(ii) In this and subsequent sections, there should be Figure 1 or 2 or 3, etc.; not Fig. 1, 2, or 3.

Reply:

We have revised all the figure captions in the text following the reviewer’s suggestion.

(iii) In citation, it is shown that authors used “Zhang, et al., 2011”, which should be “Zhang et al., 2011”. Extra comma comes up after the first author’s surname, which should not be there as per APA style.

Reply:

Thanks for the reviewer’s kindly reminder. We have read through the formatting requirement for the citation for the journal “Safety Science”. We revised all the citations with et al. in the text.

(iv) At p-8, explain briefly power curve.

(v) Provide reference for the value range in .

Reply:

For a scaled-free network, the distribution $P(k)$ of node degree k , i.e., the relative frequency of node degree k , would have a power law with the node degree k . The general power law could be represented by Eq. 5 in the manuscript. **The parameter α and γ are the regression coefficients to obtain a best-fitted pow curve between node degree k and its distribution $P(k)$ for a specific network.** Literatures have indicated the exponent parameter γ usually ranges from 2.1 to 4 (Barabasi and Albert,1999). Hence, we have revised the description of Eq. 5 in the revised text:

Line 193 – line 197: “The scale-free network is a network that has a distribution of node degrees $P(k)$, i.e., the relative frequency of node degree k , following a power distribution:

$$P(k)=\alpha k^{-\gamma} \tag{5}$$

where the parameters α and γ are the regressed coefficients for the power curve of a specific network. The value of γ usually ranges from 2.1 to 4.0 (Barabasi and Albert, 1999).”

The corresponding reference is added after the typical range.

Reference:

Barabasi, A.L., Albert, R., 1999. Emergence of scaling in random networks. Science 286, 509-512.

(vi) At p-10, citation error is noticed in “Bruneau et al (2003)”, where full stop should come up after “al”.

Reply:

We have revised the citation. Thanks for the careful check.

(vii) At p-10, in the line 216, the sentence “Hence, for an original metro network without failure.....” is not complete.

Reply:

Sorry to misleading the sentence by inappropriate separation. We have re-worded the sentences for the readability in the revised text:

Line 244 – line 246: “Hence, the magnitudes of robustness and vulnerability of a network depend on the node that is disrupted. By examining all the nodes, one at a time, robustness and vulnerability could indicate the order of importance of these metro stations for the network.”

(viii) At p-12, in the second line from the top, the sentence “... some of the metro stations are interchanges....” has grammatical mistake.

Reply:

We have reworded this sentence as the reviewer suggested.

(ix) At p-12, Figure 3 should be explained clearly with the proper sequence of a to d, which is not maintained.

Reply:

Thanks for the good suggestion. The authors have added the description of the sequence for recovery in Figure 3. The figures are re-ordered accordingly.

Line 282 – line 284: “A typical sequence is shown in Figure 3 with the first stage to recover Line 2 (Figure 3c), the second to recover Line 1 (Figure 3d) and the last stage to recover Line 3 to its original state (Figure 3a).”

(x) The sub-section 2.4 should have the sub heading “Cost during disruption & recovery stage for a full Decision Making”, not for recovery stage only.

Reply:

We have replaced the sub-heading following the reviewer’s suggestion to “**2.4 Cost during Disruption and Repair Stage for a Full Decision Making**”

(xi) Typo is found at p-13 in the serial paper line number 282, “ticket prize” is written, which should be “ticket price”.

Reply:

We are sorry for the typo error. The revised manuscript has been carefully checked to avoid such kind of error again.

(xii) At p-14 in the paper line number 294, recovery function was taken as linear, which in real cases is hardly observed.

Reply:

The authors have to admit that such kind of recovery of passenger flow volume is hardly observed neither in linear form nor in other non-linear form. It is difficult to evaluate precisely the recovery procedure for passenger flow even after the re-open of the recovered station. However, **for the recovery of multiple stations, it should make sense that the recovery of the passenger flow is a step function which can be approximately modelled by the linear function for simplicity in calculation.** Overall, for the purpose of the calculation of the disruption cost, the recovery curve of passenger flow volume is assumed to be linear as none of other nonlinear curves is evident.

(xiii) At p-16 at paper line number 345, full form of “PV” is not mentioned.

Reply:

$C_{total, PV}$ means the present value (PV) equal to the total cost C_{total} at time of full recovery ($t = t_1$). It should be less than the value of money exactly at present (C_{total}). The text in page 16 has been revised and the notation $C_{total, PV}$ is explained in detail in notations section.

Line 400 – line 401: “...(present value equal to the total cost at time t_1 , i.e., $C_{total, PV}$)...”

Line 734: “ $C_{total, PV}$ Present value equal to the total cost at time of recovery”

6. Application of the Resilience Model...

(i) At p-19, paper line number 412, calculation of is not shown. Consequently, it makes it unclear for the readers.

Reply:

We are sorry to make the reviewer unclear about the calculation of network connectivity. Basically, the network connectivity E_f is calculated based Eq. 6. The node number N is 303 for an initial network without failure. The path length d_{ij} for all pair nodes in the network is calculated by using the Floyd algorithm (Floyd, 1962) from the correlation matrix A . Since the calculation requires repeated searches for shortest path length, a Matlab-based code is used. The authors could share the Matlab function codes if the reviewer is interested.

In addition, the text in the manuscript has been newly revised in:

Line 174 – line 177: “There are quite a number of classical algorithms to calculate the path length d_{ij} efficiently from the correlation matrix A , e.g., Dijkstra’s algorithm, Bellman-Ford algorithm, Floyd algorithm (Zhan and Noon, 1998).”

Line 462 – line 466: “The initial network connectivity E_f without any failure is calculated by using Eq. 6, where the node number N is equal to 303 and the path length d_{ij} for all pair nodes in network is calculated by using the Floyd algorithm (Floyd, 1962). The calculated E_f is equal to 0.0994. After the disrupted node is removed, the similar procedure of calculation could be repeated for a disrupted network as described above.”

Reference:

Floyd, R.W., 1962. Algorithm 97 (Shortest Path). *Communications of the Acm* 5, 345-345.

Zhan, F.B., Noon, C.E., 1998. Shortest path algorithms: an evaluation using real road networks. *Transportation science* 32: 65-73.

(ii) At the same page in paper line number 420, inflection point is mentioned as 0.3. It is not clearly mentioned in the figure. In text, its significance is not discussed, as well how it is obtained?

Reply:

We appreciate the reviewer’s good suggestion on the elaboration of inflection point in Figure 10. By drawing two extension lines for the two distinct stage of network efficiency shown in Figure 10 (the variation of network efficiency approximately appears to be linear for each stage), the intersection point could be regarded as the inflection point. The inflection point for intentional attack is about 0.10, while for random attack is about 0.24. Obviously, the inflection point for random attack is greatly moving to the right side of the inflection point for intentional attack. It clearly shows the robustness of the metro network under the condition of random attack. **Correspondingly, the text has been revised in the manuscript in:**

Line 478 – line 480: “By drawing two extension lines (black dash line in Figure 10) along these two distinct stages, the intersection of these two lines could be regarded as the inflection point. ”

Line 485 – line 488: “The inflection point for random attack is greatly moving to the right side of the inflection point for intentional attack. It clearly shows that the robustness of the metro network under the condition of random attack is much better than the robustness under the condition of intentional attack.”

In addition, Figure 10 is also re-drawn to explicitly show the inflection point.

(iii) At p-20 in the paper line number 442, calculation of original connectivity is not shown.

Reply:

Thanks for the reviewer’s comment. Details of the calculation for original connectivity could be referred to the response to comment (i) in 6. Application of the Resilience Model.

(iv) At p-22 in the paper line number 485, one small calculation of should be shown for better understanding by the readers.

Reply:

Thanks for the reviewer’s rigorous point of view on the calculation of resilience index R_e . Basically, the calculation is carried out using Eq. 10. **A typical calculation example is newly added in the revised manuscript to explicitly show the calculation procedure for ease of understanding.**

Line 557 – line 563: “Considering the recovery sequence 2-6-9-4 as an example, the calculated initial network connectivity is equal to 0.0994, and the network connectivity after the disruption of the station is 0.0932. After recovery according to the sequence 2-6-9-4, the network connectivity is gradually increased from 0.0932 to 0.0953 (Line 2 is recovered), to 0.0973 (Lines 2 and 6 are recovered), to 0.0985 (Lines 2, 6 and 9 are recovered), and to 0.0994 (fully recovered for Lines 2, 6, 9 and 4). Hence, by using Eq. 10, the resilience index R_e is calculated to be 0.974, i.e., $0.974 = 0.5 \times (0.0932 + 2 \times 0.0953 + 2 \times 0.0973 + 2 \times 0.0985 + 2 \times 0.0994) \times 4 / (4 \times 0.0994)$.”

(v) At p-22 in the paper line number 492, the significance of “Stage zero” has not been discussed.

Reply:

Thanks for the reviewer’s good suggestion on the description of stage zero in the text. **We have revised the text in line 570 – line 572:**

“Stage zero stands for a metro network in its normal state without any disruption. The initial network efficiency as calculated in the robustness analysis is about 0.0994.”

(vi) For both the example cases, cost has not been taken whereas the authors admit that cost factors should be considered for a full assessment of the resiliency of the metro networks (see p-5 at paper line numbers 99-101) which is the prime focus of this current study.

Reply:

Thanks for the reviewer's suggestion. The potential contribution of this manuscript, in the authors' opinion, should be twofold, i.e., the first and most important one is the resilience model and its associated metric for determining the recovery sequence, and the second one is the additional cost analysis to assist the recovery strategy after selecting a best recovery sequence. With this idea in mind, the authors have added one more example for cost analysis in the revised manuscript. Details of the example could be referred to the response to the comment (i) in 4. Introduction.

7. Reference

(i) The only reference mentioned at line number 643 needs to be changed to the same style used for other referencing.

(ii) The reference mentioned at the page line number 673, check which one is correct whether "Von..." or "von...".

Reply:

The formatting of the reference has been double checked and revised following the suggestion.

8. Overall:

The paper though presents a nice contribution towards bridging the gap in resiliency assessment model for urban rail transit networks; however, it has some serious limitations that should be discussed appropriately. For example, why the cost factors are not considered needs to be explained clearly. In addition, the logic behind the assumptions made in this paper has to be provided. Further, many typos, referencing, grammar, etc. need to be corrected.

Reply:

First of all, the authors are greatly appreciated for the reviewer's insightful comments, which definitely has improved the quality and rigorousness of the manuscript. For each comment, we have made a thorough revision and detailed responses. We hope the manuscript now is clear and meet the requirement of the journal.

Highlights:

1. A general framework to quantitatively assess resilience of complex metro network by analyzing its vulnerability and recovery rapidity is proposed in this paper.
2. The best recovery strategy in terms of optimum recovery sequence of stations and optimum duration can be identified in a structured manner.
3. Shanghai metro system is a typical small-world and scale-free network with strong robustness of connectivity under random disruption but severe vulnerability under intentional disruption.

Resiliency Assessment of Urban Rail Transit Networks: Shanghai Metro as an Example

Abstract: This paper presents a general framework to assess the resilience of large and complex metro networks by quantitatively analyzing its vulnerability and recovery rapidity within unifying metrics and models. The connectivity performance of network is indicated by the network efficiency. The resilience of a metro network can be associated to the network performance loss triangle over the relevant timeline from the occurrence of a random or intentional disruption to full recovery. The proposed resilience model is applied to the Shanghai metro network with its 303 stations and 350 links as an example. The quantitative vulnerability analysis shows that the Shanghai metro with its L-space type of topology has a strong robustness regarding connectivity under random disruption but severe vulnerability under intentional disruption. This result is typical for small-world and scale-free networks such as the Shanghai metro system, as can be shown by a basic topological analysis. Considering the case of one disrupted metro station, both the vulnerability and resilience of the network depend not only on the node degree of the disrupted station but also on its contribution to connectivity of the whole network. Analyzing the performance loss triangle and the associated cost from loss of operational income and repair measures, an appropriate recovery strategy in terms of the optimum recovery sequence of stations and the optimum duration can be identified in a structured manner, which is informative and helpful to decision makers.

Keywords: Network Resilience; Shanghai Metro; Node Connectivity; Topology; Robustness; Vulnerability.

1 **Resiliency Assessment of Urban Rail Transit Networks: Shanghai Metro as an**
2 **Example**

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Abstract: This paper presents a general framework to assess the resilience of large and complex metro networks by quantitatively analyzing its vulnerability and recovery rapidity within unifying metrics and models. The connectivity performance of network is indicated by the network efficiency. The resilience of a metro network can be associated to the network performance loss triangle over the relevant timeline from the occurrence of a random or intentional disruption to full recovery. The proposed resilience model is applied to the Shanghai metro network with its 303 stations and 350 links as an example. The quantitative vulnerability analysis shows that the Shanghai metro with its L-space type of topology has a strong robustness regarding connectivity under random disruption but severe vulnerability under intentional disruption. This result is typical for small-world and scale-free networks such as the Shanghai metro system, as can be shown by a basic topological analysis. Considering the case of one disrupted metro station, both the vulnerability and resilience of the network depend not only on the node degree of the disrupted station but also on its contribution to connectivity of the whole network. Analyzing the performance loss triangle and the associated cost from loss of operational income and repair measures, an appropriate recovery strategy in terms of the optimum recovery sequence of stations and the optimum duration can be identified in a structured manner, which is informative and helpful to decision makers.

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25 1. Introduction

26 Urban rail transit systems (named subsequently as metro systems) offer an effective solution
27 for addressing transportation problems in cities by significantly increasing the capacities of
28 public transportation. This benefit has driven an expeditious development in the construction
29 and operation of metro systems in many metropolitan cities. As the number of metro lines
30 increases, metro systems often grow to a large and complex network scale. For example,
31 Shanghai has a metro system with 303 stations and 350 tunnels over 617km. Although a large-
32 scale metro network makes public transportation attractive and convenient, any accident
33 impacting this mega system would greatly affect not only the serviceability of this critical
34 infrastructure but also the safety of passengers. For example, in September of 2011, a signal
35 failure occurred in a station of metro line 10 in Shanghai, China. Two metro trains crashed in a
36 tunnel due to the loss of signal causing 271 injured passengers and 30-hour halt of the whole
37 metro line (Mu, 2011). In view of these circumstances, the safety of metro networks is a key
38 concern that requires an enhanced understanding of these networks through extensive research.

39 Quite a number of studies have been carried out by analyzing networks for the safety of metro
40 systems in particular with the topographical mapping modelling (Crucitti et al., 2003; Derrible
41 and Kennedy, 2010; Watts and Strogatz, 1998; Zhang et al., 2013; Zhang et al., 2011).
42 Essentially, a metro network can be mapped into a topological graph with the simplification of
43 metro tunnels and metro stations, respectively, as links and nodes used in topology (Zhang et al.,
44 2013). Topological analysis, i.e., consisting of nodes and links, the path length and cluster
45 coefficient of a network provides an effective and logical basis to characterize the safety of a
46 transportation network (Derrible and Kennedy, 2010). Watts and Strogatz (1998) proposed a
47 model termed *small-world network* for complex network analysis. The small-world network

48 system is typically highly clustered and yet have small characteristic path lengths, which
49 conceptually shows a robust connectivity of the nodes for a network subjected to any disruption.
50 [Barabasi and Albert \(1999\)](#) investigated large-scale networks and found that the node
51 connectivity in those networks, i.e., network connectivity, often follows a scale-free power-law
52 distribution. This result suggests that the connectivity of the network is robust under a random
53 failure yet is vulnerable under an intentional attack.

54 **The above network assessment models, however, are mainly qualitative with conceptualized**
55 **measures that do not offer a rigorous comparison of safety levels among networks. In this**
56 **respect, network analysis with quantitative measures of robustness and vulnerability is thus**
57 **helpful for examining the safety level of a specific network.** [Albert et al. \(2000\)](#) quantitatively
58 analyzed the robustness of metro networks in the event of an accident. The robustness of a metro
59 network here is expressed in terms of the residual network connectivity after the disruption of
60 nodes in the network. [Crucitti et al. \(2004\)](#) also studied the robustness of two types of large-
61 scale networks, i.e., a random network and scale-free network, for the performance of network
62 connectivity. These two studies revealed that scale-free networks have a high robustness index
63 under random attacks but a low robustness index under intentional attacks. Recently, a similar
64 analysis of the robustness of metro networks was also reported by [Zhang et al. \(2011\)](#) and [Yang](#)
65 [et al. \(2015\)](#). **Additionally, disrupting a large-scale metro network by an accident affects not**
66 **only the robustness but also the subsequent recovery. Recovery profiles over time greatly affect**
67 **the economic and social wellbeing outcomes that are of great concern to metro owners.** The
68 rapid recovery of a network's connectivity from a disrupted state to the normal state is a key
69 concern among engineers ([Francis and Bekera, 2014](#)). However, a robustness assessment
70 focuses only on the network safety in the event of an accident without considering recovery. **In**

71 the context of recovery, robustness and vulnerability analyses are still insufficient to offer a
72 rational recovery strategy in terms of recovery sequencing and duration for stations and tunnels
73 in a metro network.

74 A comprehensive safety assessment model includes both the network robustness and the
75 recovery profile as provided by Ayyub (2014). In this context, the concept of “*resilience*” would
76 provide an appropriate solution to cover both the robustness and recovery in a single model for
77 safety evaluation of a network system (Ayyub, 2015; Bruneau et al., 2003). Resilience,
78 according to U. S. Presidential Policy Directive (PPD-21, 2013), means “*the ability to prepare*
79 *for and adapt to changing conditions and withstand and recover rapidly from disruptions.*”
80 Fundamentally, the resilience of a system is often quantified by relating it to a resilience loss
81 triangle represented by the difference between a normal performance evolution curve and a
82 disrupted performance curve along with the time duration of disruption and recovery stage
83 (Frangopol and Soliman, 2015). A resilient system can be quantitatively defined by the system
84 with a minimized performance loss triangle, also termed resilience loss triangle. The optimized
85 recovery sequences and duration necessary to minimize the resilience triangle thus offer a basis
86 for defining recovery strategies after disruptions (Zhang and Wang, 2016).

87 The concept of resilience as used herein was initially and formally introduced by Holling
88 (1973) for ecologic systems. Later on, a broader interest in resilience was triggered by the 2001
89 World Trade Center attack in the United States. Typical uses of resilience analysis has covered
90 mostly water resource systems (Hashimoto et al., 1982), power networks (Henry and Ramirez-
91 Marquez, 2012) and the seismic hazards for bridges (Dong and Frangopol, 2015). Resilience
92 analysis for urban rail transit systems, however, has been quite limited. On the other hand,
93 resilience analysis is of great necessity and importance in order to identify optimized recovery

94 sequences after network disruptions. One might realize that a metro network, to some degree, is
95 similar to the power network but with a different topological space of nodes and links and a
96 different recovery philosophy in terms of sequence and timing. These similarities can be
97 exploited in the development of respective approaches. In addition, the current practice of
98 recovery is purely based on empirical judgement without a rational model to obtain an optimum
99 repair duration and costs during the recovery stage (Huang and Zhang, 2016).

100 This paper provides a general framework for the resilience analysis of large-scale metro
101 network systems that offers an immediate basis for identifying both the best recovery sequences
102 to minimize the performance loss and the best repair duration to minimize the costs associated
103 with disruption and recovery. The performance of a metro network in this paper refers to the
104 connectivity of stations in an integrated metro network (hereafter termed network connectivity).
105 The development of this framework requires the introduction of several concepts and models in
106 the context of metro networks as follows:

- 107 1. Basic mapping of a metro network into a topological graph;
- 108 2. Defining and measuring vulnerability and robustness of the topological metro network;
- 109 3. Developing resiliency metrics based on the topological metro network;
- 110 4. Accounting for costs during the disruption and recovery stage.

111 The main body of the paper is provided in two subsequent primary sections. The first primary
112 section provides the detailed description of the resilience model according to the above four
113 items. The second primary section provides an application of the above resilience model to the
114 Shanghai metro. Recovery strategies are discussed for (a) a single multi-line station, and (b)
115 multiple stations in terms of repair sequences that minimize the resilience loss triangle. Then, a
116 brief application example focusing on cost analysis during disruption and recovery is used to

117 illustrate the concepts introduced. Finally, concluding remarks are presented. Although the
118 illustrative examples in this paper are based on the Shanghai metro, the proposed resiliency
119 model is applicable for safety analysis of other complex metro networks, such as in London,
120 New York, etc., with the emphasis on network robustness and recovery.

121 **2. Proposed General Framework: Resilience Models for Large Metro Networks**

122 **2.1 Topological Mapping of Metro Networks**

123 Several types of topological graphs are suitable to model transportation networks and systems
124 based on the use of nodes and links among nodes. von Ferber et al. (2009) summarized four
125 types of topological graphs, specifically, L-space type, B-space type, P-space type and C-space
126 type, for representing a typical bus transportation network. Among all these models, preference
127 is given to the L-space type graph for metro networks since it provides clear definitions in
128 relation to transportation networks. Nodes stand for bus stops and links among nodes stand for
129 connections between two successive bus stops of a bus line. A similar treatment can be adopted
130 for a metro system as graphically illustrated in Figure 1. Figure 1a shows an example of a metro
131 network. Each node represents a metro station, while a link between nodes stands for a metro
132 tunnel. Different metro lines are marked with different colors. Figure 1b is a typical L-space
133 topological graph for the metro network shown in Figure 1a.

134 A topological graph for a metro network can be expressed by a vector G :

$$135 \quad G = [S, E] \quad (1)$$

136 where S is the node set for all the stations in a metro network, denoted as $S = \{s_i | i = 1, 2, 3, \dots, N\}$
137 (N is the total number of stations), and E is the set of links for all the tunnels in the metro
138 network, denoted as $E = \{e_{ij} | i, j \in S\}$. The state of connection between any two nodes in a network
139 can be represented by a correlation matrix A , denoted as $A = [a_{ij}]_{N \times N}$. If nodes s_i and s_j are not

140 connected directly, then a_{ij} is equal to infinity. If there is a link between them, then a_{ij} is equal to
 141 one. If i is equal to j , then a_{ij} represents the connection of a node with itself, which is set to zero.
 142 In the topological analysis of the metro system in this paper, factors such as upper and lower
 143 lines, link distance between stations, departure frequency of metro trains, passenger capacity and
 144 the political reasons are not weighted in the network. The model used essentially results in an
 145 undirected and unweighted network system, which is frequently used to represent the
 146 transportation network system.

147 For the topological network of a metro system, the node degree k_i stands for the number of
 148 nodes that have a direct connection with the node s_i . The average degree k^* is defined as the
 149 average of the node degrees k_i for all nodes in the network. The minimum number of nodes that
 150 need to be passed through from node s_i to node s_j is named path length d_{ij} . There are quite a
 151 number of classical algorithms to calculate the path length d_{ij} efficiently from the correlation
 152 matrix A , e.g., Dijkstra's algorithm, Bellman-Ford algorithm, Floyd algorithm (Zhan and Noon,
 153 1998). The characteristic path length L is defined as the average over all path lengths d_{ij} for all
 154 pairs of nodes (s_i, s_j) in a network:

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij} \quad (2)$$

156 Correspondingly, the diameter of a network D is defined as the maximum path length d_{ij}
 157 among all pairs of nodes (s_i, s_j) . The clustering coefficient C_i is defined as the ratio of the
 158 number of links e_i between node s_i and its adjacent total of k_i nodes with respect to the maximum
 159 possible number of links $k_i(k_i-1)/2$:

$$C_i = \frac{e_i}{k_i(k_i-1)/2} \quad (3)$$

161 The network-clustering coefficient C is the mean of all clustering coefficients C_i for all the
162 nodes.

163 By using the above basic notations, two typical properties of a topological network can be
164 characterized; the small-world property and the scale-free property. A small-world network is a
165 network that has a high network clustering coefficient C but is characterized by a small path
166 length L . A network is called a small-world network if

$$167 \quad L > \frac{\ln(N)}{\ln(k^*)} \quad (4a)$$

$$168 \quad C < \frac{k^*}{N} \quad (4b)$$

169 From a physical point of view, a small-world network possesses a reasonable local connectivity
170 (Barabasi and Albert, 1999). The scale-free network is a network that has a distribution of node
171 degrees $P(k)$, i.e., the relative frequency of node degree k , following a power distribution:

$$172 \quad P(k) = \alpha k^{-\gamma} \quad (5)$$

173 where the parameters α and γ are the regressed coefficients for the power curve of a specific
174 network. The value of γ usually ranges from 2.1 to 4.0 (Barabasi and Albert, 1999). Generally
175 speaking, the power distribution of node degree means that the network has a few nodes with
176 large node degrees, and most of the nodes possess small node degrees.

177 **2.2 Robustness and Vulnerability of Metro Networks**

178 For a large-scale metro network, the connectivity from one station to another should be a key
179 criterion for metro operation. Once a metro network experiences a disrupting event, whether due
180 to degradation, human errors or intentional attack, the node connectivity would inevitably
181 decrease. The robustness of a metro network is its ability to resist and maintain residual node
182 connectivity after such an event. Hence, before evaluating network robustness, the node

183 connectivity needs to be characterized first. The network efficiency E_f is usually defined as an
184 indicator to quantify the node connectivity of a topological network (Latora and Marchiori,
185 2001):

$$E_f = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}} \quad (6)$$

186
187 where d_{ij} is the path length between nodes s_i and s_j . The inverse of path length d_{ij} essentially
188 means the connectivity efficiency between nodes s_i and s_j in a network. Hence, it is clear from
189 Eq. 6 that the meaning of network efficiency E_f is the average of node efficiency for all nodes in
190 a network. The value of E_f could range from zero to one, i.e., zero means no connectivity
191 between any two nodes in the network, whereas one means any two nodes are connected. This
192 specifies the lower and upper bounds for network connectivity.

193 Theoretically speaking, the connectivity of a topographical metro network could be affected
194 by failures both from metro stations (nodes) and from metro tunnels (links). In this paper, the
195 failure of metro stations is specifically considered but not the links. The key reason is that the
196 metro station is open to the public and thus vulnerable to attacks or other hazards. In addition,
197 the failure of a network node is often considered in L-space type topological graphs for
198 transportation networks (Yang et al., 2015; Zhang et al., 2011). The failure of a metro station
199 can be classified into two types. The first failure type occurs randomly and is caused possibly by
200 natural hazards, power and signal malfunction or even human errors. The probability of
201 occurrence of such a failure can be assumed to be the same for all stations in a network. This
202 assumption can be changed and varied values used without affecting the overall approach
203 although the computationally complexity increases. The second type of failure is of an
204 intentional nature as caused mainly on purpose by arson or terrorism. Usually, the most
205 important station in a large-scale network, e.g., a multi-line interchange, could be intentionally

206 targeted and disrupted. For topological analysis, the failure of a station can be modeled by
 207 removing the node from the network. As for the random failure, the node is removed randomly
 208 following a specific probability distribution function (Crucitti et al., 2004). As for the intentional
 209 failure, assuming that attacks occur in the order of importance of the nodes, the nodes are
 210 removed sequentially following a descending order of magnitude of the node degree k (Crucitti
 211 et al., 2004).

212 Hence, the metric of robustness of a topological metro network can be quantitatively
 213 described by the changed connectivity after the removal of a network node:

$$E'_f = \frac{1}{N'(N'-1)} \sum_{i \neq j} \frac{1}{d'_{ij}} \quad (7)$$

214 where N' is the number of nodes after the removal of failed nodes, and d'_{ij} is the newly calculated
 215 path length between nodes s_i and s_j . Correspondingly, the vulnerability of the topological metro
 216 network can be calculated by the decrease of connectivity due to the failure of nodes:

$$V = \Delta E_f = E_f - E'_f \quad (8)$$

217 As for a node removal strategy, only one node is removed at a time. Then the changed
 218 connectivity E'_f and vulnerability V are calculated as the robustness and vulnerability of the
 219 network, respectively. Hence, the magnitudes of robustness and vulnerability of a network
 220 depend on the node that is disrupted. By examining all the nodes, one at a time, robustness and
 221 vulnerability could indicate the order of importance of these metro stations for the network.

222 **2.3 Resilience of Metro Networks**

223 The resilience of a metro network as discussed in this paper is defined as the connectivity level
 224 after the node disruption and the ability of rapid recovery of the connectivity to an acceptable
 225 level with appropriate repair measures. The concept of the “resilience triangle” is often used for
 226
 227

228 infrastructure networks as proposed by [Bruneau et al. \(2003\)](#) to quantify the loss of resilience
 229 after performance disruption. Figure 2 shows an example of such a resilience triangle shaded in
 230 grey. The vertical axis stands for the performance index $Q(t)$ of the network, while the
 231 horizontal axis is the time t . The system performance is disrupted at time t_0 . With particular
 232 recovery measures applied in a time interval of t_h , the performance can be recovered to the
 233 original level at time t_l (i.e., $t_l=t_0+t_h$). Then, the resilience triangle is illustrated by the difference
 234 between the area covered by constant Q_0 for the undisrupted system and the area covered by
 235 varied $Q(t)$ for the disrupted system within time t_h . Furthermore, the area covered by $Q(t)$ for the
 236 disrupted system in this period (yellow shaded in Figure 2) is an indicator for resilience of the
 237 system subjected to the disruption at time t_0 . Hence, on the basis of the resilience triangle,
 238 [Bocchini and Frangopol \(2012\)](#) proposed a general metric of resilience for bridge network
 239 infrastructure as:

$$R_e = \frac{\int_{t_0}^{t_0+t_h} Q(t) dt}{t_h Q_0} \quad (9)$$

241 The resilience index R_e is essentially the ratio of the area covered by the disrupted performance
 242 curve $Q(t)$ (yellow shaded area in Figure 2) with respect to the area covered by the undisrupted
 243 curve (constant Q_0 in the example of Figure 2) within the period of t_h . Hence, the metric of
 244 resilience for a metro network can be represented by Eq. 9 with the performance $Q(t)$ referring to
 245 the network efficiency E_f . Hence, the resilience index R_e can be expressed as:

$$R_e = \frac{\int_{t_0}^{t_0+t_h} [E_f(t)] dt}{t_h E_{f0}} \quad (10)$$

247 where $E_f(t)$ stands for the network efficiency at time t , and E_{f0} is the initial network efficiency for
 248 the original metro network where no disruption has occurred.

249 When the connectivity of the network after removal of nodes needs to be recovered, a
250 sequential recovery strategy is usually assumed in the resilience analysis (Henry and Ramirez-
251 Marquez, 2012), i.e., only one metro station for one metro line could be recovered at a time. In a
252 large-scale metro network such as the Shanghai metro system, some of the metro stations are
253 multi-line exchange stations as shown in Figure 3a. When losing the functionality of such
254 stations, the recovery of connectivity for different metro lines in this station also needs to be
255 implemented sequentially. Specifically, if the exchange station connects m lines, there are m
256 stages in sequence to fully recover this station. Hence, in total, the number of optional sequences
257 is equal to $P(m,m)$ (i.e., m -permutations of m). As for the example shown in Figure 3b, in total,
258 there are six optional sequences to fully recover the three-line exchange station A (i.e., $6=3\times 2\times 1$).
259 A typical sequence is shown in Figure 3 with the first stage to recover Line 2 (Figure 3c), the
260 second to recover Line 1 (Figure 3d) and the last stage to recover Line 3 to its original state
261 (Figure 3a). Each recovery sequence corresponds to a specific resilience index R_e . Hence, an
262 optimal decision for a recovery strategy is to find the best sequence for recovery that maximizes
263 the value of R_e . Take the example of recovery of node A shown in Figure 3, the performance
264 recovery curves corresponding to the six optional recovery sequences are plotted in Figure 4.
265 Engineers thus could select sequence 5 as the optimal recovery strategy because it is associated
266 with the largest resilience index R_e . For the case of several damaged nodes the procedure applies
267 accordingly.

268 **2.4 Disruption and Repair Costs for Decision Making**

269 The recovery of network connectivity is a primary goal for decision makers; however, disruption
270 and repair costs associated with recovery profiles a key criterion in decision making. Once the
271 recovery sequence is selected, the particulars of recovery plans, i.e., repair methods, equipment,

272 etc., affect the repair duration and subsequently the recovery cost. Generally, advanced
273 techniques for repair work require less time to complete recovery task, but at a greater cost.
274 Hence, a recovery cost analysis in terms of repair time duration helps to determine the optimized
275 cost of repair. Similar to other published resilience models, such as (Francis and Bekera, 2014),
276 the proposed model for the resilience of metro networks so far does not incorporate the cost-
277 benefit analysis directly in the decision-making process of selecting an appropriate repair
278 sequence. As an extension, this section proposes a general cost analysis model in conceptual
279 terms. Note that the cost analysis here is discussed based on the selected optimum repair
280 sequence by using the above resilience model. Following the basic model proposed by Henry
281 and Ramirez-Marquez (2012), the total cost C_{total} during the disruption and repair stages is
282 generally composed of two parts, namely the cost $C_{disruption}$ and the cost $C_{repairs}$, represented as:

$$C_{total} = C_{disruption} + C_{repair} \quad (11)$$

284 where the $C_{disruption}$ is referred to the cost or income loss due to the disruption of network
285 connectivity and the $C_{repairs}$ means the cost or investment for the repair works. Taking the flood-
286 induced failure of metro station as an example, the $C_{disruption}$ might include two major categories,
287 i.e., damaged but repairable equipment and damaged but non-repairable equipment, and income
288 loss due to closing of metro operations. Once a station is set as disrupted by flooding, the
289 equipment such as track barriers, escalator, signal board and etc., could be partially or totally
290 disabled. Later on, the control rooms and power plant rooms might also be affected due to
291 flooding. Furthermore, when the metro station is completely closed, loss in income from ticket
292 sales, the retails and other business around the metro stations comes on top. In addition, the
293 stocks or reputation of the metro company could be affected due to the disruptions. Details of
294 the above disruption cost are shown in Table 1. However, incorporating all the factors shown in

295 Table 1 for disruption cost $C_{\text{disruption}}$ requires data that are not available in some cases. For ease
 296 of discussion, the cost analysis for $C_{\text{disruption}}$ in this paper includes mainly the income loss in the
 297 ticket sales C_{ticket} ($C_{\text{disruption}} = C_{\text{ticket}}$ in this paper) which can be quantitatively evaluated. Some
 298 other cost factors in Table 1 could be related qualitatively to the ticket income using perhaps
 299 multiplication coefficients ($C_{\text{others}} = \lambda \times C_{\text{ticket}}$) as an example.

300 It is reasonable to assume that the income loss in metro ticket sales C_{ticket} is closely related to
 301 the loss of network connectivity. In other words, the loss of system performance in terms of
 302 network connectivity directly affects the passenger volume (Vol) of the network, in particular in
 303 and around the disrupted stations. The passenger volume would further affect the income from
 304 the ticket sales I . In that case, the ticket income cost C_{ticket} is assumed to have a positive
 305 correlation with the loss of passenger volume by a linear model. As mentioned previously, the
 306 disruption cost $C_{\text{disruption}}$ in this paper is assumed to be equal to ticket income loss C_{ticket} . Hence,
 307 the $C_{\text{disruption}}$ is assumed as below:

$$308 \quad C_{\text{disruption}} = C_{\text{ticket}} = \beta \cdot Vol_{\text{Loss}} \quad (12)$$

309 where Vol_{Loss} is the total loss of the passenger volume during the whole disruption period, and β
 310 is the average metro ticket price of the network. Similar to the network connectivity, the
 311 passenger volume Vol will experience a sharp reduction once a metro station is attacked or
 312 removed from the network at time t_0 . This reduction from the initial state of Vol_n to a disrupted
 313 state Vol_d will be gradually recovered during the repair measures to the final recovered state at
 314 time t_1 . Hence the passenger volume $Vol(t)$ is also a function of time t between the failure time t_0
 315 and the final recovered time t_1 . Figure 5a shows two examples of linear and step functions of
 316 recovery of passenger volume Vol . Corresponding to the passenger volume recovery, the
 317 operation income is also recovered due to the increase of passenger volume, from the income I_d

318 under disruption to the fully recovered value of income I_n . This effect is captured by a
 319 multiplication factor corresponding to the average ticket price β , see Eq. 12. Figure 5b shows the
 320 two types of variation of operation income from I_d to I_n . In Figure 5, the shaded area
 321 corresponds to the total passenger volume loss Vol_{Loss} (Figure 5a) and the total cost of operation
 322 income loss $C_{disruption}$ (Figure 5b), respectively. If the recovery function of passenger volume Vol
 323 is assumed to be linear, then the passenger volume loss can be represented as:

$$324 \quad Vol_{Loss} = \frac{1}{2}(Vol_n - Vol_d)(t_1 - t_0) \quad (13a)$$

325 and the total cost of disruption of system performance $C_{disruption}$ is:

$$326 \quad C_{disruption} = \frac{1}{2}\beta(Vol_n - Vol_d)(t_1 - t_0) \quad (13b)$$

327 This cost model does include indirect losses such as costs associated with longer travel times,
 328 costs associated with increased demands on other transportation systems, costs attributed to
 329 impacts on reputation and confidence in the system, etc. A complete accounting of all of these
 330 effects requires simulating the dynamics of the system with the passengers and other
 331 transportation modes.

332 Given an existing large-scale metro network, the average ticket price β and the initial
 333 passenger volume Vol_n are known. After a node failure at time t_0 , the disrupted passenger
 334 volume Vol_d can also be predetermined for the disrupted system. Hence, the only variable in Eq.
 335 13b that governs the cost $C_{disruption}$ is the time t_1 that is needed to recover the system to its initial
 336 state, i.e., removed nodes are all repaired. Note that the time t_1 is positively correlated with the
 337 cost $C_{disruption}$, e.g., a positive linear relationship could be found in the above example of linear
 338 model (Eq. 13). It is obvious that the longer the repair works last, the greater the cost is.

339 The cost for the implementation of repair measures C_{repair} is determined by solving a typical
340 time-cost trade-off problem within the discipline of engineering economics (Golzarpoor, 2012;
341 Hegazy, 1999; Liu et al., 1995). The total cost of repair works consists of the direct cost and the
342 indirect cost, as shown in Figure 6. The direct cost is related to the salary of workers,
343 consumption of engineering materials, rental of the engineering machinery etc. Some typical
344 direct cost is shown in Table 1. For a certain repair project, shortening the repair duration t can
345 be achieved by an increase of salary, the use of high quality materials, and more machinery.
346 Hence, the direct cost is believed to have a negative correlation with the repair time duration t as
347 shown by the red dashed line in Figure 6. The indirect cost is caused by social related factors
348 (also shown in Table 1), which is believed to have a positive correlation with the time duration t ,
349 as indicated by the blue dashed line in Figure 6. It is widely recognized that the detailed
350 evaluation of direct and indirect cost is quite complicated and specifically case-oriented
351 (Golzarpoor, 2012). Hence, the cost C_{repair} is represented by a non-monotonic and nonlinear
352 function of repair duration t_I without loss of generality, as shown in Figure 13, marked as black
353 solid line, from which an optimum value of t_I that minimizes the C_{repair} can be found (point A in
354 Figure 6).

355 Hence, by substituting the above expressions for $C_{disruption}$ and C_{repair} into Eq. 11, the total cost
356 C_{total} becomes:

$$C_{total} = \frac{1}{2} \beta (Vol_n - Vol_d) (t_1 - t_0) + C_{repair}(t_1) \quad (14a)$$

358 Figure 7 shows a graphical representation of Eq. 14a by plotting the total cost C_{total} against the
359 repair duration t_I . Again, for a particular large-scale metro network under a specific case of
360 disruption and recovery, the dominant variable in Eq. 14a is the repair duration t_I . Similar to the
361 non-monotonic function of C_{repair} , the total cost C_{total} is also a non-monotonic function of time t_I .

362 To minimize C_{total} is again a typical time-cost trade-off problem. Interestingly, it is not always
 363 the case that a shorter recovery is associated with a smaller total cost. The optimum value of t_I
 364 that minimizes the total cost of recovery needs to be determined by solving the time-cost trade-
 365 off problem illustrated in Figure 7 with point B being the solution. Since the total cost includes
 366 the operation income loss (first term on the right-hand side of Eq. 14a), the optimum value of t_I
 367 is smaller than the value of time t that minimizes C_{repair} , i.e., $t_{I, B}$ in Figure 7 is smaller than $t_{I, A}$
 368 in Figure 6.

369 If the disruption is quite serious, the duration of full recovery can take several years or even
 370 longer. For example, the reported disruption of metro lines in Shanghai due to deformational
 371 performance took almost six years to be fully recovered (Huang and Zhang, 2016) and the
 372 disruption of metro lines in St. Petersburg due to seepage in tunnels takes almost eight years of
 373 recovery (Ryumin, 2004). In those cases, the factor of time-value of money should be included
 374 in the cost analysis at the present time. In the simplest model for time-value of money a constant
 375 discount rate i is assumed (Brigham and Ehrhardt, 2002), and Eq. 14a is modified to:

$$C_{total, PV} = \left[\frac{1}{2} \beta (Vol_n - Vol_d) (t_1 - t_0) + C_{repair}(t_1) \right] \cdot \frac{1}{(1+i)^{t_1}} \quad (14b)$$

376
 377 It is clear from Eq. 14b that the total cost with full recovery at time t_I (present value equal to
 378 the total cost at time t_I , i.e., $C_{total, PV}$) is less than the value of money at present (C_{total}). A
 379 generalized illustration of the comparison between $C_{total, PV}$ and C_{total} is also plotted in Figure 7.
 380 The corresponding optimum value of t_I might move forward due to the effect of time value of
 381 money as the point D shown in Figure 7. Owner or practitioner with the responsibility for the
 382 repair and maintenance works of large-scale metro networks should undertake such cost analysis.
 383 After determining the optimal repair sequence by the resilience analysis model as shown in

384 Figure 4, the optimal repair duration for each station could be obtained from the above cost
385 analysis. Therefore, the best recovery strategy due to the disruption can be achieved both with
386 the best recovery sequence and duration. At the present stage, it should be noted that the detailed
387 evaluation for each parameter in Eq. 14b is limited to typical scenario where both the data from
388 disruption and recovery could be collected.

389 **3. Application: Shanghai Metro System**

390 The Shanghai metro system now is the longest metro network system in the world by the
391 route length. Up to December 31, 2015, the total length of the 14 Shanghai metro lines in
392 operation was 617km with 303 metro stations. These numbers exclude the operating magnetic
393 train line that only has a departure station and an arrival station. The planned and being-
394 constructed metro lines and stations are also excluded from these values. For planning reasons,
395 Metro lines 3 and 4 in Shanghai partially share the same line and metro stations along the line.
396 Those shared metro stations and lines are considered only once in building the network as
397 described in this paper. The necessity of a detailed resilience analysis for such a huge-scaled
398 network is quite obvious from this perspective.

399 **3.1 Topological Characteristics of the Shanghai Metro**

400 A typical L-space topological network representing the Shanghai metro system is built and
401 plotted in Figure 8. The nodes in Figure 8 represent the metro stations, and the links between
402 nodes are the metro lines. The correlation matrix A is built for all 303 stations in this network,
403 $A=[a_{ij}]_{303 \times 303}$. Based on the correlation matrix A , the characteristics of the metro network, i.e.,
404 average degree k^* , characteristic path length L and network diameter D , are calculated by using
405 the algorithm proposed by [Floyd \(1962\)](#). Table 2 displays these typical characteristics for the
406 Shanghai metro network.

407 The average node degree k^* is equal to 2.31, which indicates that every station is connected to
408 2.31 other stations on average. Note that a node degree k equal to one refers to an endpoint of a
409 single line. A k equal to two characterizes an intermediate normal station where only one metro
410 line passes through. A value of k larger than two indicates an exchange station for at least two
411 lines in a metro system. Figure 9a shows the histogram for the distribution of node degrees for
412 the 303 stations, represented by the solid black column. In total, about 76% of the metro stations
413 are normal stations passed by one metro line. The remaining 24%, i.e., 52 stations, are multi-line
414 exchange stations. For comparison, the distribution of node degrees for the Shanghai metro
415 system from 2010 is plotted in the grey column in Figure 9a based on data extracted from [Zhang
416 et al. \(2011\)](#). It can be seen that the relative frequency for the node degree equal to 2 is smaller
417 in 2015, while the values for node degree equal to 4, 5 or 6 are larger. Furthermore, the largest
418 value of node degree k is equal to 8, which means that four metro lines pass through this station.
419 The largest k in the year of 2010 is equal to 7. This indicates the fast development of the
420 Shanghai metro network during these five years from 2010 to 2015.

421 Figure 9b shows a plot with logarithmically scaled axes for both the node degree k and the
422 relative frequency $p(k)$, corresponding to Figure 9a for the case of 2015. The plot provides some
423 indication that the relationship between the logarithm of relative frequency $p(k)$ and the
424 logarithm of node degree k can be approximated in a linear manner. This corresponds to a power
425 curve for the distribution of the node degree k . Linear regression yields the parameters $\alpha=3.858$
426 and $\gamma=3.525$. Hence, it is indicated that the Shanghai metro network is a scale-free network,
427 which corresponds to robustness of the network for random failure but vulnerability of the
428 network for an intentional failure ([Albert et al., 2000](#); [Crucitti et al., 2003](#); [Crucitti et al., 2004](#)).

429 The characteristic path length L shown in Table 2 is equal to 14.87, which indicates that the
430 shortest path between any two stations in the network needs to pass 14.87 stations from an
431 average perspective. The network diameter D is equal to 41, which stands for the longest metro
432 line with 41 metro stations. The calculated network cluster coefficient C is 0.0082. Table 2 also
433 displays the calculated path length limit of 6.82 ($\ln N / \ln k^*$) and the limit for the network cluster
434 coefficient C as 0.0076 for concluding whether the network is a small-world network or not, see
435 Eq. 4. Both criteria indicate that the Shanghai metro network is a small-world network. From
436 the definition of small-world given by [Milgram \(1967\)](#), the Shanghai metro network is proved to
437 have an intensive connectivity locally and a good quality of operation for the whole network.

438 **3.2 Robustness and Vulnerability Evaluation of Shanghai Metro**

439 The initial network connectivity E_f without any failure is calculated by using Eq. 6, where the
440 node number N is equal to 303 and the path length d_{ij} for all pair nodes in network is calculated
441 by using the Floyd algorithm ([Floyd, 1962](#)). The calculated E_f is equal to 0.0994. After the
442 disrupted node is removed, the similar procedure of calculation could be repeated for a disrupted
443 network as described above. The robustness of the Shanghai metro is calculated for both types
444 of failure scenarios, random failure and intentional failure, corresponding to the respective node
445 removal strategies. For random failure, stations are removed randomly following a discrete
446 uniform distribution over the stations. The network connectivity $E_{f,random}$ is calculated using Eqs.
447 6 and 7 after each station removal. For intentional failure, the stations are removed in the order
448 of descending node degree. The network connectivity $E_{f,intention}$ is calculated after each station
449 removal.

450 Figure 10 shows the calculated $E_{f,random}$ (green line with solid triangle) and $E_{f,intention}$ (blue line
451 with solid rectangle) against the fraction of removed stations in the network. The constant

452 dashed line stands for the calculated initial network connectivity E_f (i.e., 0.0994) without failure
453 of any station. It is observed that the decrease of $E_{f,intention}$ with the growing fraction of removed
454 nodes experiences two distinct stages. First there is a sharp decrease at the beginning followed by
455 a more gentle decrease once a number of stations have already failed. **By drawing two extension**
456 **lines (black dash line in Figure 10) along these two distinct stages, the intersection of these two**
457 **lines could be regarded as the inflection point.** The inflection point shown in Figure 10 is around
458 the fraction of removed nodes equal to 0.10. This fraction of removed nodes is approximately
459 the fraction of nodes in the network with node degree greater than 3, see Figure 9a. This is a
460 clear indication for the importance of those multi-line exchange stations for the connectivity of
461 the metro network. Similarly, the decrease of $E_{f,random}$ also shows these two stages, as well, with
462 the inflection point at around 0.24. **The inflection point for random attack is greatly moving to**
463 **the right side of the inflection point for intentional attack.** It clearly shows that the robustness of
464 **the metro network under the condition of random attack is much better than the robustness under**
465 **the condition of intentional attack.** But the curve for $E_{f,random}$ is less sharp compared to the curve
466 for $E_{f,intention}$ due to the large value at the inflection point for random failure. Hence, the
467 Shanghai metro network when subjected to the random failure of stations is more robust than the
468 network when subjected to intentional attacks. This result is consistent with the general
469 conclusions for scale-free networks and supports our hypothesis that the Shanghai metro is a
470 scale-free network. Further, when the fraction of removed stations is greater than 35%, $E_{f,intention}$
471 is slightly larger than $E_{f,random}$ since the random removal leaves some exchange stations for
472 removal in later stages causing a slightly stronger decrease of connectivity in those stages.

473 For comparison, the results based on the data of the network in 2010 are also represented in
474 Figure 10. The grey line with hollow triangles illustrates the random removal strategy, while the

475 red line with hollow rectangles indicates the intentional removal strategy. By comparing the
476 results for 2015 and 2010, both the $E_{f,random}$ and $E_{f,intention}$ for 2015 are slightly larger than those
477 for 2010, especially when the fraction of removed nodes is small. This suggests that the
478 robustness, in particular at the beginning of removal stages, has been increased by the five-year
479 development of the metro network from 2010 to 2015. Furthermore, the network connectivity in
480 2010 under intentional removal would be close to zero when 40% of stations are removed, which
481 shows high vulnerability of the network. This issue has been resolved by 2015.

482 The vulnerability of the initial network subjected to only one node removal is calculated by
483 using Eq. 8 for all the 303 nodes. The ten most critical stations, identified by the largest V
484 calculated from Eq. 8, are displayed in Table 3. The *Caoyang Road station* is ranked most
485 critical station in the network. If this station is removed, the connectivity would be reduced by
486 0.0073, which is about 7.4% of the value of original connectivity $E_f(0.0994)$. Inspecting the ten
487 most critical stations in Table 3 it appears that these stations do not necessarily have a large node
488 degree k (last column in Table 3). The stations with node degree equal to six are not ranked in
489 the top ten most critical stations. Apart from the *Century Ave station* with k equal to 8, all the
490 other stations have a node degree k of 5 or less. This result shows that the vulnerability of a
491 network due to one-node removal is not significantly correlated with the node degree k (Hu,
492 2007).

493 The specific locations of the ten most critical stations are marked in the network map in
494 Figure 8. Analyzing this map it becomes obvious that most of the critical nodes are located in a
495 position where the node is the only connection between the suburban and the downtown area.
496 Furthermore, the suburb metro lines still have a significant number of stations beyond the critical
497 stations. Once such a critical node is removed, the entire connection between downtown and the

498 respective suburb would be completely disrupted. This is the key reason for the vulnerability of
499 the Shanghai metro. As a conclusion, a new circle line should be implemented into the network
500 to remove this criticality. On the other hand, the nodes with a node degree k equal to 6 are
501 mostly located in the downtown area. Since the network in the downtown area is much denser
502 than that in the suburban area, even removal of such node with large k does not affect the
503 network connectivity badly. Once such node is removed, there are still sufficient options for
504 connecting through alternative lines.

505 **3.3 Resilience Evaluation of Shanghai Metro**

506 It is clear from Figure 4 that the resilience of a network highly depends on the recovery
507 strategy, e.g., sequence of recovery for multiple disrupted nodes and sequence of recovery for
508 multiple lines in an exchange station. Furthermore, repair duration should be optimized on the
509 basis of minimizing the disruption and repair costs. The philosophy behind the recovery strategy
510 for a disrupted metro system is first to select the best recovery sequence either for an exchange
511 station or for multiple stations and second to select the best repair duration considering both the
512 disruption and repair cost. In this regard, the following section shows an example of three
513 typical cases of the resilience analysis of the Shanghai metro network: Case 1 is to obtain the
514 best recovery sequence for a disrupted exchange station; Case 2 is to obtain the best recovery
515 sequence for multiple disrupted stations; and Case 3 is to obtain the best repair duration for
516 rebuilding a disrupted metro station.

517 ***Case 1: Recovery sequence given disruption of one multi-line exchange station***

518 This analysis concentrates on the station with the largest node degree in the network, i.e.
519 *Century Ave station* connecting four metro lines, namely line 2, 4, 6 and 9, leading to a node
520 degree of 8. The robustness analysis indicated that the vulnerability of the network is

521 significantly determined by possible disruption of this station. That is, this station plays an
522 important role for the connectivity of the network, see Figure 11. High resilience is thus required
523 for this station. The key question is the decision for the best recovery strategy. There are 24
524 possible recovery sequences to fully recover all four lines in this station, i.e., $24 = P(4, 4) =$
525 $4 \times 3 \times 2 \times 1$. Each sequence corresponds to a specific resilience index R_e representing the recovery
526 of network connectivity. A decision has to be made for that recovery sequence that maximizes
527 the value of R_e among the 24 choices. For carrying out the resilience analysis, we assume that
528 the recovery time for each line in this station is the same no matter what kind of recovery
529 measures are applied. On this basis, the difference of R_e between the possible sequences is
530 solely affected by the sequence of recovering metro lines in this station and not by any other
531 factors. Subsequently, the recovery sequences are denoted using the metro line numbers, such as
532 2-6-9-4 indicating that first line 2 is recovered, then line 6 and so on.

533 The resilience index R_e is calculated, using Eq. 10, for the 24 possible recovery sequences.
534 Considering the recovery sequence 2-6-9-4 as an example, the calculated initial network
535 connectivity is equal to 0.0994, and the network connectivity after the disruption of the station is
536 0.0932. After recovery according to the sequence 2-6-9-4, the network connectivity is gradually
537 increased from 0.0932 to 0.0953 (Line 2 is recovered), to 0.0973 (Lines 2 and 6 are recovered),
538 to 0.0985 (Lines 2, 6 and 9 are recovered), and to 0.0994 (fully recovered for Lines 2, 6, 9 and 4).
539 Hence, by using Eq. 10, the resilience index R_e is calculated to be 0.974, i.e., $0.974 = 0.5 \times$
540 $(0.0932 + 2 \times 0.0953 + 2 \times 0.0973 + 2 \times 0.0985 + 2 \times 0.0994) \times 4 / (4 \times 0.0994)$. The results are
541 listed in Table 4. The largest R_e shown in Table 4 is equal to 0.974, which is produced by the
542 recovery sequence 2-6-9-4. It indicates that if *Century Ave station* is totally disrupted, the first
543 metro line to be recovered is line 2, followed by line 6, then line 9, and line 4. On the other hand,

544 the smallest R_e is obtained as 0.962 for the recovery sequence of 9-4-6-2. The curves illustrating
545 the change of the network efficiency over these two recovery sequences are plotted in Figure 12.
546 The black line with solid squares reflects the recovery sequence 2-6-9-4, while the grey line with
547 hollow circles represents the recovery sequence 9-4-6-2. Stage zero stands for a metro network
548 in its normal state without any disruption. The initial network efficiency as calculated in the
549 robustness analysis is about 0.0994. Stage 1 stands for the removal of the station causing the
550 sharp reduction of network efficiency to 0.0932. After four stages, the station is fully recovered
551 with a network efficiency restored to 0.0994 corresponding to the initial or normal condition. It
552 is quite obvious from Figure 12 that after each intermediate recovery stage, i.e., stage 2-4, the
553 network efficiency for sequence 2-6-9-4 is larger than that for sequence 9-4-6-2. Hence, the
554 resilience loss triangle for sequence 2-6-9-4 is much smaller than that for sequence 9-4-6-2. The
555 difference between the resilience triangles is shaded in red color in Figure 12. It indicates the
556 extent to which resilience of this station is influenced by decision margins regarding the recovery
557 sequence and that the sequence 2-6-9-4 is the best choice. Following this principle, the best
558 recovery strategy for each of the 52 exchange stations can be determined. Table 5 shows the best
559 recovery strategy for each exchange station in Shanghai based on the resilience analysis. This
560 result may be helpful for decision making when any of the stations is affected and waiting for
561 recovery.

562 ***Case 2: Recovery sequence given disruption of multiple stations***

563 For the case of multi-station disruption, the removal of four nodes at once is taken as an
564 example. Two typical sub-cases are discussed: (I) with different node degree k values for the
565 four nodes, and (II) with the same node degree k for the four nodes. For multiple stations, in
566 order to show the effect of recovery sequence of different stations clearly, the recovery sequence

567 of metro lines for an exchange station discussed previously is not considered. In other words,
568 any of the removed stations is fully recovered only by one stage without considering the
569 sequence of recovering different lines.

570 For Sub-case I, the *Century Ave station* (node 43), *Peoples' Square station* (node 13),
571 *Oriental Sports Center station* (node 133) and *Shanghai railway station* (node 16) are selected
572 with their corresponding node degree k equal to 8, 6, 5 and 4, respectively. Again, there are 24
573 possible recovery sequences, i.e., $24 = P(4, 4)$. The recovery sequences are denoted using the
574 station numbers, i.e. 43-13-133-16 for recovery in the order of stations as named above. The
575 resilience index R_e for each of the recovery sequences is calculated by Eq. 10. Hence, there are
576 24 calculated index values R_e as shown in Table 6. The maximum value of R_e is calculated based
577 on the sequence 16-43-133-13 which is the best choice according to the resilience measure. It is
578 quite interesting that the best recovery sequence is not based on the descending order of the node
579 degree k as 43-13-133-16. Instead, the best recovery sequence reflects the order of nodes ranked
580 according to their criticality for network vulnerability based on the robustness analysis. The key
581 criterion is the contribution of node to the network connectivity, and hence to the network
582 efficiency E_f , rather than the node degree k . The minimum value of the R_e is obtained as 0.885
583 for sequence 13-133-43-16, which is 5% less than the maximum value of R_e under the sequence
584 16-43-133-13. The difference of the index R_e between these two sequences corresponds to the
585 difference in the resilience loss triangle produced by the curves of recovered network efficiency
586 along the specific recovery sequence, as shown in Figure 13.

587 For Sub-case II with four removed nodes having the same node degree k value, *Xu Jia Hui*
588 *station* (node 8), *Peoples' Square station* (node 13), *Han Zhong Road station* (node 15) and *West*
589 *Nanjing Road* (node 39) are selected for analysis. In this example node degree k is equal to 6 for

590 each of these stations. Again, 24 possible recovery sequences are evaluated in the resilience
591 analysis. The respective values R_e calculated from Eq. 13 are listed in Table 7. It is interesting
592 that the values of R_e for different recovery sequences are quite close to one another. Hence, in
593 the present case the effect of the sequence of recovery for the nodes considered is not significant
594 for the resilience of the metro network. However, it should not be concluded, in general, that
595 this effect is caused by the same node degree. The reason is rather the similar contribution of
596 these nodes to the network connectivity, as the calculated robustness of the network connectivity
597 after the disruption of each node is almost the same among these four nodes.

598 ***Case 3: Recovery cost to rebuild a disrupted metro station***

599 In this hypothetical example, a metro station is disrupted completely at time t_0 and requires
600 rebuilding of the damaged station. For ease of discussion, t_0 is assumed to be zero. In addition,
601 time-value of money is not considered in this example for simplicity. During the repair
602 procedure, the metro station is closed to the public. As mentioned previously, the disruption cost
603 in the recovery stage is assumed to only include the income loss related to metro tickets. From
604 the statistics of the Shanghai metro, the daily passenger flow volume is more than 10 million.
605 Hence, the disruption of one station among the 303 stations of the whole network is assumed to
606 lead in a loss of about 40 thousand passengers flow volume every day. The ticket price of
607 Shanghai metro is about 4 RMB on average. For this case, from Eq. 13b, the disruption cost
608 during the recovery procedure can be calculated as below:

$$609 \quad C_{disruption} = \frac{1}{2} \times 4 \times 40000 (t_1 - t_0) \quad (15)$$

610 where t_1 is the recovered time moment that is unknown and needs to be optimized and t_0 is equal
611 to zero that is assumed in this example.

612 The rebuilding of a metro station requires several stages in sequence, including the site
613 investigation, construction of a pile foundation of underground structures, construction of a soil
614 retaining system, soil excavation with construction of supports and dewatering, and construction
615 of underground structures of the metro station. From personal communication with the managers
616 in charge of design, construction and operation, Table 8 shows a typical example of repair
617 options associated with the direct cost and duration for rebuilding one typical metro station in
618 Shanghai. It is quite obvious from direct cost perspectives that the shorter the repair duration,
619 the higher the cost. Solving such a typical time-cost tradeoff requires the use of linear and integer
620 programming (LP/IP) models, such as Liu et al. (1995). Table 9 shows the calculated results of
621 repair duration and associated direct cost of construction. The indirect cost is assumed, in this
622 example, to be equal to 100 thousands RMB per day. Hence, the total indirect cost can also be
623 calculated based on the calculated duration, as shown in Table 9. Figure 14a shows the
624 calculated direct cost, indirect cost and the repair cost against the repair duration, respectively
625 (solid circles for C_{repair} , hollow triangles for direct cost and hollow rectangles for indirect cost).
626 It is observed from Figure 14a that the optimized repair duration based on repair cost is around
627 455 days. Corresponding to this calculated repair duration, the disruption cost in terms of the
628 ticket income loss can be derived from Eq. 15, as displayed in Table 9. Hence, the total cost
629 C_{total} both including $C_{\text{disruption}}$ and C_{repair} can be obtained as shown in Figure 14b (solid circles for
630 C_{total} , hollow circles for C_{repair} and hollow rectangles for $C_{\text{disruption}}$). The optimized repair duration
631 after incorporating the income loss is about 450 days, which is less than the optimized duration
632 considering only repair cost. This finding is reasonable, as the income loss will continue to
633 increase when the repair work is not finished. In this example, the disruption cost only includes
634 the ticket income loss, resulting in a five-day decrease of the optimized duration. Although the

635 number of days in this case is small, the best repair duration might be reduced further as other
636 cost elements of disruption are considered. If the other disruption cost as shown in Table 1 is
637 considered to be proportional to the ticket income loss, i.e., $\lambda = (C_{\text{disruption}} - C_{\text{ticket}}) / C_{\text{ticket}}$, Figure
638 14c has illustrated the effect of parameter λ on the optimized repair duration days. It is clear that
639 as the λ becomes larger, i.e., that is to say, the other disruption cost increases, the optimized
640 repair duration days would be greatly reduced. It is consistent with the engineering practice that
641 if the disruption cost is invaluable or social effect of the metro accident is unacceptable, the best
642 repair strategy should be as fast as possible.

643 **4. Conclusions**

644 In view of the ever-increasing concerns on the safety of metro systems, a general framework
645 for resilience analysis of large-scale metro networks is presented in this paper. This framework
646 includes the topological mapping of a metro network and the quantitative evaluation of
647 robustness and vulnerability as a basis to quantify resilience and to derive decisions regarding the
648 most resilient recovery strategy. The analysis framework captures both random failure and
649 intentional attacks. The node connectivity, defined by the network efficiency E_f , is regarded to
650 be the key performance criterion of a metro network. The proposed resilience assessment
651 framework is applied to the Shanghai metro network, which is one of the largest metro networks
652 in the world. The following conclusions are drawn from analysis.

653 1) The Shanghai metro system is a typical small-world and scale-free network under the
654 topological L-space. It has strong connectivity locally and a good quality of operation for the
655 whole network. The characteristics of a scale-free network, i.e., strong robustness of network
656 connectivity under random attacks; but serious vulnerability under intentional attacks, has

657 been quantitatively validated by the proposed analysis model for evaluation of robustness and
658 vulnerability for the Shanghai metro network.

659 2) The vulnerability of the metro network is quantified by the network global efficiency when
660 removing disrupted nodes. However, the magnitude of vulnerability when removing a
661 disrupted node does not only depend on the degree of this node, but also on the contribution
662 of this node to the global network efficiency. That is, the most critical node for vulnerability
663 of a metro network, i.e., *Caoyang Road station* in the Shanghai case, might not be the node
664 with the largest node degree k but is the node with the largest contribution to the network
665 connectivity.

666 3) The recovery strategy in terms of recovery sequence for the case of recovering a multi-line
667 exchange station and the case of recovering multi stations can be derived with the concept of
668 minimizing the area of the resilience triangle based on the performance evolution curves.
669 Similar to the vulnerability analysis, the optimum recovery sequence depends not only on the
670 degree of the recovered nodes but also on the contribution of the recovered nodes to the
671 network connectivity. Given an optimized recovery sequence, the recovery time duration can
672 be optimized by minimizing the cost including the disruption and repair cost for the disrupted
673 metro system. The total cost is a non-monotonic function of repair duration time t_I , with its
674 local minimum of cost indicating the best choice for recovery measures at the associated
675 optimum time t_I .

676 It should be specifically noted that the topology of the metro network discussed in this paper
677 is under the assumption of an undirected and unweighted network. Although this simplification
678 is widely accepted for basic understandings of a large-scale infrastructure network, some of the
679 practical factors are not included in this model, such as the physical length of tunnels between

680 stations, the time and route for exchanging metro lines, the volume for different metro lines and
 681 also political factors. Those weights on the importance of nodes and links in the metro network
 682 could be explored in future studies.

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 687 data.

688 **6. Notations**

689	G	Topological vector for metro network
690	S	Node set of a metro network
691	s_i	Node in a metro network
692	N	Node number in a metro network
693	E	Link set of a metro network
694	e_{ij}	Link between node s_i and s_j in a network
695	A	Correlation matrix for a network
696	a_{ij}	Correlation between node s_i and node s_j
697	d_{ij}	Path length between node s_i and s_j
698	L	Characteristic path length of a network
699	D	Diameter of a network
700	C_i	Clustering coefficient for a node in a network
701	C	Clustering coefficient of a network
702	$P(k)$	Distribution of node degree for a network
703	E_f	Network efficiency
704	V	Vulnerability of topological metro network
705	$Q(t)$	System performance of a metro network at time t
706	t_0	Time moment when failure or attacks occur to the metro network
707	t_l	Time moment when the metro network is fully recovered from a disruption

708	t_h	Time duration from the disruption moment t_0 to the recovered moment t_f
709	R_e	Resilience index for metro network
710	C_{total}	Cost during the system disruption
711	$C_{total, PV}$	Present value equal to the total cost at time of recovery
712	$C_{disruption}$	Cost referred to the loss of system performance
713	C_{ticket}	Cost referred to the income loss of metro ticket
714	Vol	Passenger volume of the metro network
715	Vol_n	Initial passenger volume of a metro network in normal condition
716	Vol_d	Reduced passenger volume of a metro network after disruption
717	Vol_{Loss}	Total loss of the passenger volume during the disruption
718	C_{repair}	Cost related to the implementation of repair measures

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Table and Figure Captions

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833 b) time-cost for total cost including disruption cost and repair cost; and c) effect of other
834 disruption cost on the optimized repair duration days.
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838 Table 1 Cost produced due to disruption of metro station

Cost Category	Content
Damaged equipment	track
	track barrier
	escalator
	signal board
	ticket barrier
Disruption cost	Abandonment of facility
	air-conditioning system
	ventilation system
	managing control room
	power plant room
Income loss	ventilation room
	signal system room
	escape shaft
Stocks	metro ticket
	retail
	mall
Repair Cost	rental
	real estate
	stocks
	salary of workers
	monitoring and inspection before repair
Indirect cost	site investigation
	construction materials
	rental of engineering machinery
purchase of power, ventilation and signal system	
time value	
social impact	

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842 Table 2 Characteristic indicators of Shanghai metro network

Characteristic of Network	Calculated value for Shanghai metro
Node N	303
Link	350
Average node degree k^*	2.31
Characteristic path length L	14.87
Diameter of network D	41
Network cluster coefficient C	0.0082
Limit state of L ($\ln N / \ln k^*$)	6.82
Limit state of C (k^* / N)	0.0076

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847 Table 3 Ranking of node vulnerability

No.	Removed node	Vulnerability V	$V/E_f(\%)$	Node degree k
1	Caoyang Rd. Stn.	0.0073	7.37	4
2	Shanghai Railway Stn.	0.0066	6.69	4
3	Siping Rd. Stn	0.0065	6.57	4
4	Zhenping Rd. Stn.	0.0065	6.55	4
5	Longyang Rd. Stn.	0.0062	6.24	5
6	Yishan Rd. Stn.	0.0062	6.22	5
7	Century Ave Stn.	0.0062	6.22	8
8	Hongkou Football Stadium Stn.	0.0052	5.18	4
9	Luoshan Rd. Stn.	0.0050	5.08	4
10	Original Sports Center Stn.	0.0044	4.46	5

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852 Table 4 R_e for different line recovery sequences

Recovery sequence ^(*)	R_e	Recovery sequence	R_e
2-6-9-4	0.974	6-4-2-9	0.968
2-6-4-9	0.973	4-2-9-6	0.967
6-2-9-4	0.973	4-6-2-9	0.967
6-2-4-9	0.972	9-2-4-6	0.966
2-9-6-4	0.971	6-9-4-2	0.965
2-4-6-9	0.971	4-9-2-6	0.964
2-4-9-6	0.970	6-4-9-2	0.964
6-9-2-4	0.969	9-4-2-6	0.964
2-9-4-6	0.969	9-6-4-2	0.963
4-2-6-9	0.969	4-6-9-2	0.963
9-2-6-4	0.969	4-9-6-2	0.962
9-6-2-4	0.968	9-4-6-2	0.962

853 Note: * Sequence 2-6-9-4 stands for the recovery sequence for metro line 2 as the first, line 6 as
854 the second, line 9 as the third and line 4 as the last.
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859 Table 5 Best recovery sequence of metro lines for all exchange stations in Shanghai by
860 minimizing the resilience triangle.

Metro Station	Best metro line recovery sequence
Xujiahui Stn.	11-1-9
Shanxi Rd.(S) Stn.	12-10-1
People's Square Stn.	2-8-1
Hanzhong Rd. Stn.	1-12-13
Nanjing Rd.(W) Stn.	12-2-13
Longyang Rd. Stn.	2-16-7
Yishan Rd. Stn.	9-3-4
Oriental Sports Center Stn.	8-11-6
Caobao Rd. Stn.	12-1
Shanghai Sport Center Stn.	1-4
Changshu Rd. Stn.	7-1
Shanghai Railway Stn.	1-3
Zhongshan Park Stn.	2-3
Jiangsu Rd. Stn.	11-2
Jing'an Temple Stn.	2-7
Nanjing Rd.(E) Stn.	2-10
Longcao Rd. Stn.	12-3
Hongqiao Rd. Stn.	10-3
Jinshajiang Rd. Stn.	13-3
Caoyang Rd. Stn.	11-3
Zhenping Rd. Stn.	7-3
Hongkou Football Stadium Stn.	3-8
Hailun Rd. Stn.	4-10
Dalian Rd. Stn.	12-4
Lancun Rd. Stn.	4-6
Xizang Rd.(S) Stn.	8-4
Damuqiao Rd. Stn.	12-4
Dong'an Rd. Stn.	4-7
Jufeng Rd. Stn.	6-12
Gaoke Rd.(W) Stn.	7-6
Yaohua Rd. Stn.	7-8
Longhua Rd.(M) Stn.	7-12
Zhaojiabang Rd. Stn.	9-7
Changshou Rd. Stn.	7-13
Siping Rd. Stn.	10-8
Qufu Rd. Stn.	8-12
Laoximen Stn.	8-10
Lujiabang Rd. Stn.	9-8

Jiashan Rd. Stn.	12-9
Madang Rd. Stn.	13-9
Jiaotong University Stn.	11-10
Xintiandi Stn.	10-13
Tiantong Rd. Stn.	12-10
Longde Rd. Stn.	11-13
Longhua Stn.	11-12
Luoshan Rd. Stn.	16-11
Shanghai South Railway Stn.	1-3
Hongqiao Airport Terminal 2 Stn.	2-10
Baoshan Rd. Stn.	3-4

863
 864 Table 6 Resilience index R_e for different recovery sequence of metro stations with different node
 865 degrees.

Recovery sequence (*)	Re	Recovery sequence	Re
16-43-133-13	0.927	43-13-16-133	0.906
43-16-133-13	0.926	133-43-13-16	0.904
16-133-43-13	0.923	16-13-133-43	0.903
43-133-16-13	0.921	43-13-133-16	0.901
16-43-13-133	0.920	13-16-43-133	0.895
43-16-13-133	0.919	13-43-16-133	0.894
133-16-43-13	0.919	133-13-16-43	0.894
133-43-16-13	0.917	133-13-43-16	0.893
16-133-13-43	0.911	13-16-133-43	0.890
43-133-13-16	0.908	13-43-133-16	0.889
16-13-43-133	0.907	13-133-16-43	0.886
133-16-13-43	0.907	13-133-43-16	0.885

866 Note: * Recovery sequence 16-43-133-13 means the recovery for node 16 (Shanghai Railway
 867 station) as the first, node 43 (Century Ave station) as the second, node 133 (Oriental Sports
 868 Center station) as the third and node 13 (People's Square station) as the last.
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873 Table 7 Resilience index R_e for different recovery sequence of stations with the same node
874 degrees.

Recovery sequence	Re	Recovery sequence	Re
15-39-8-13	0.969	13-8-39-15	0.965
39-15-8-13	0.968	39-8-13-15	0.965
15-8-39-13	0.967	13-8-15-39	0.965
15-13-8-39	0.966	13-39-15-8	0.965
15-39-13-8	0.966	39-13-15-8	0.965
13-39-8-15	0.966	13-15-39-8	0.964
39-13-8-15	0.966	8-15-39-13	0.964
15-8-13-39	0.966	8-39-15-13	0.963
39-8-15-13	0.966	8-15-13-39	0.962
13-15-8-39	0.966	8-13-39-15	0.962
39-15-13-8	0.966	8-39-13-15	0.962
15-13-39-8	0.965	8-13-15-39	0.962

875

877 Table 8 Repair options for rebuilding a metro station in Shanghai

Procedure	Options	Cost (million RMB)	Duration (day)
Site investigation	crew1+equipment1	6	50
	crew2+equipment2	8	40
Pile foundation	crew1+equipment1	6	30
	crew2+equipment2	6	30
Retaining wall	crew1+equipment1	29	80
	crew2+equipment2	33	70
Excavation /support /dewatering	crew1+equipment1	15	180
	crew2+equipment2	20	165
Underground structure	crew1+equipment1	30	120
	crew2+equipment2	35	110

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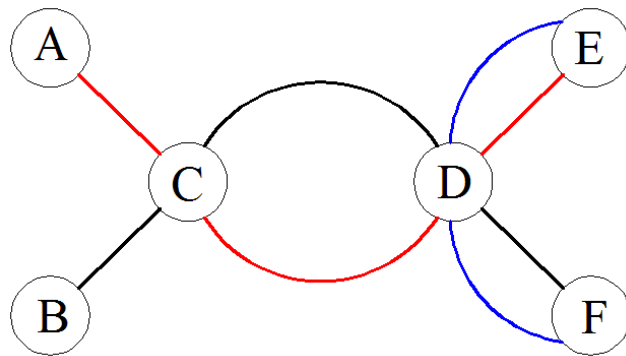
880 Table 9 Integrated repair duration and cost for rebuilding a metro station in Shanghai

Duration (day)	Direct Cost	C_{repair} Indirect cost	sum	$C_{\text{disruption}}$	C_{total}
415	102.0	41.5	143.5	33.2	176.7
420	98.5	42.0	140.5	33.6	174.1
425	95.1	42.5	137.6	34.0	171.6
430	92.5	43.0	135.5	34.4	169.9
435	90.9	43.5	134.4	34.8	169.2
440	89.5	44.0	133.5	35.2	168.7
445	88.0	44.5	132.5	35.6	168.1
450	87.0	45.0	132.0	36.0	168.0
455	86.4	45.5	131.9	36.4	168.3
460	86.0	46.0	132.0	36.8	168.8

881 Note: cost unit is million RMB.

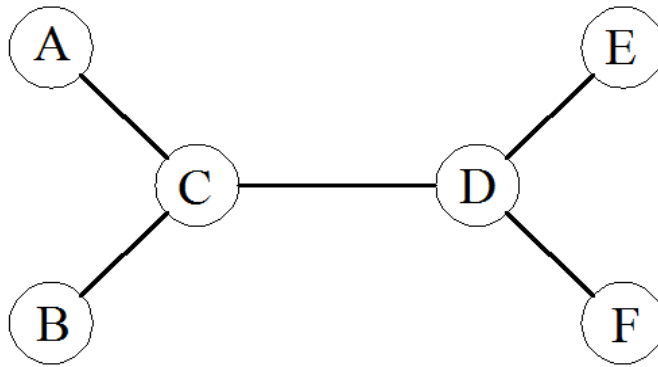
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(a)

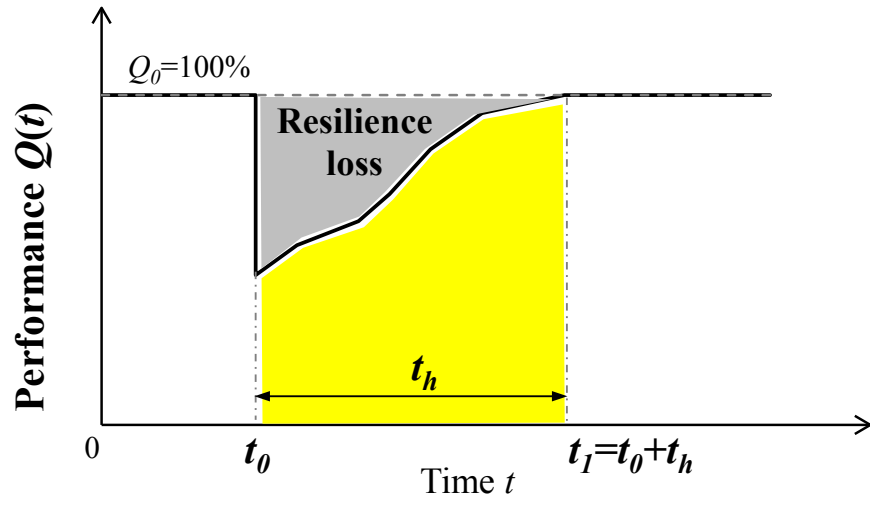


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(b)

890 Figure 1. L-space type of topological graph for typical metro network: a) example of metro
891 network; b) topological graph corresponding to the metro network

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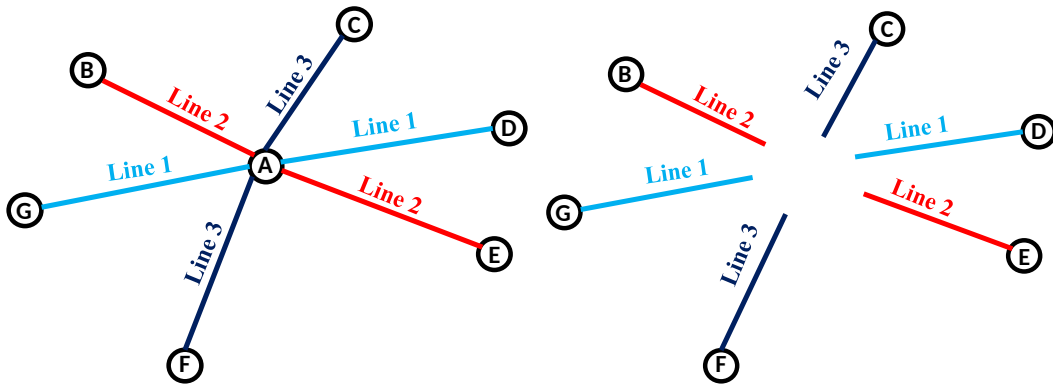


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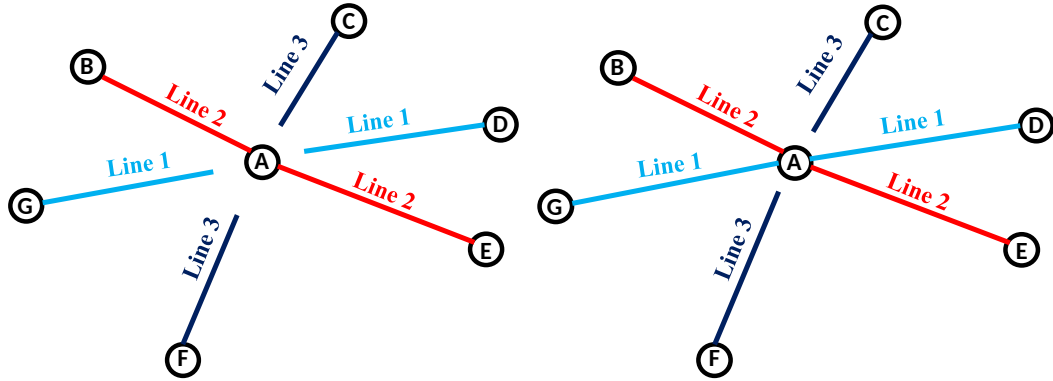
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Figure. 2 Graphical measure of resilience including resilience loss triangle

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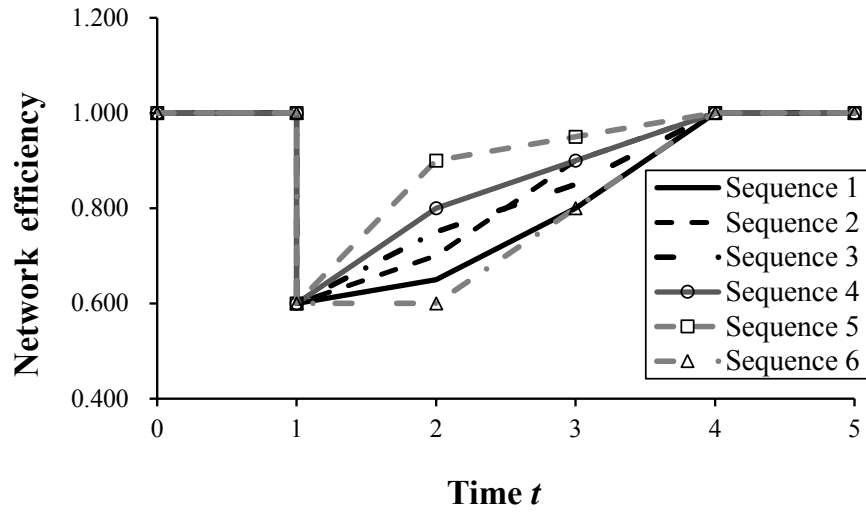
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903 Figure. 3 Recovery strategy for a multi-line exchange station (three-line exchange as an
904 example): a) original state; b) disconnected state; c) first stage of recovery; d) second stage of
905 recovery;

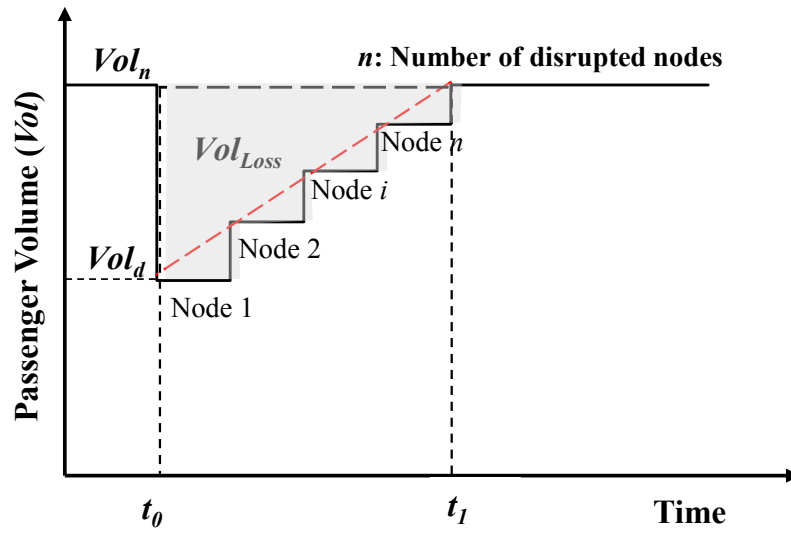
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909 Figure. 4 Performance recovery curves corresponding to specific recovery sequence for node A
910 in Figure 3.

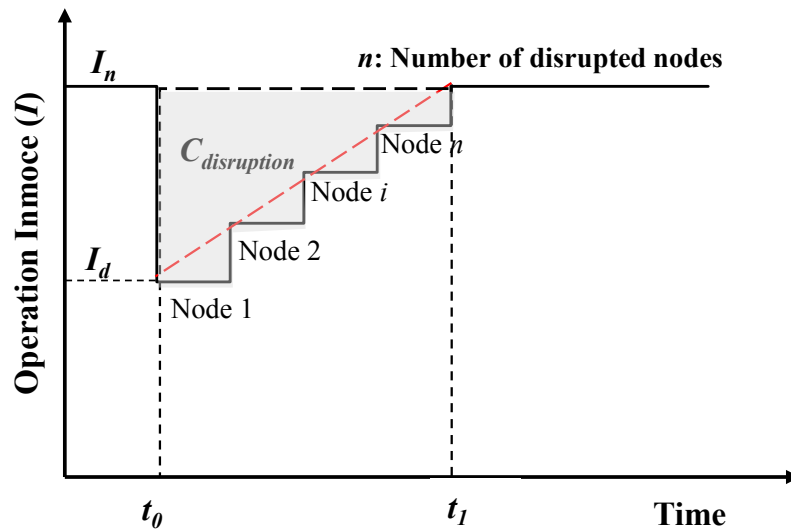
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(a)



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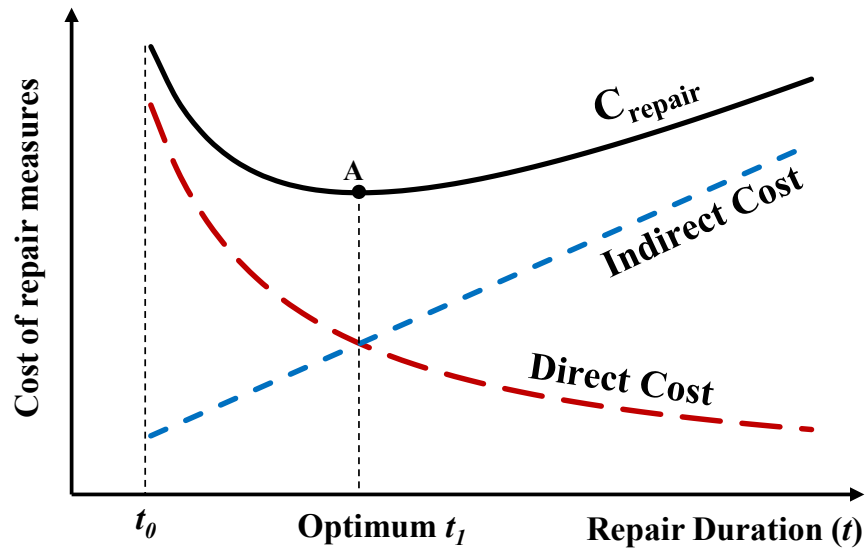
(b)

917 Figure 5 The time-dependent recovery curve of passenger volume and operational income: a)

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passenger volume; b) operational income;

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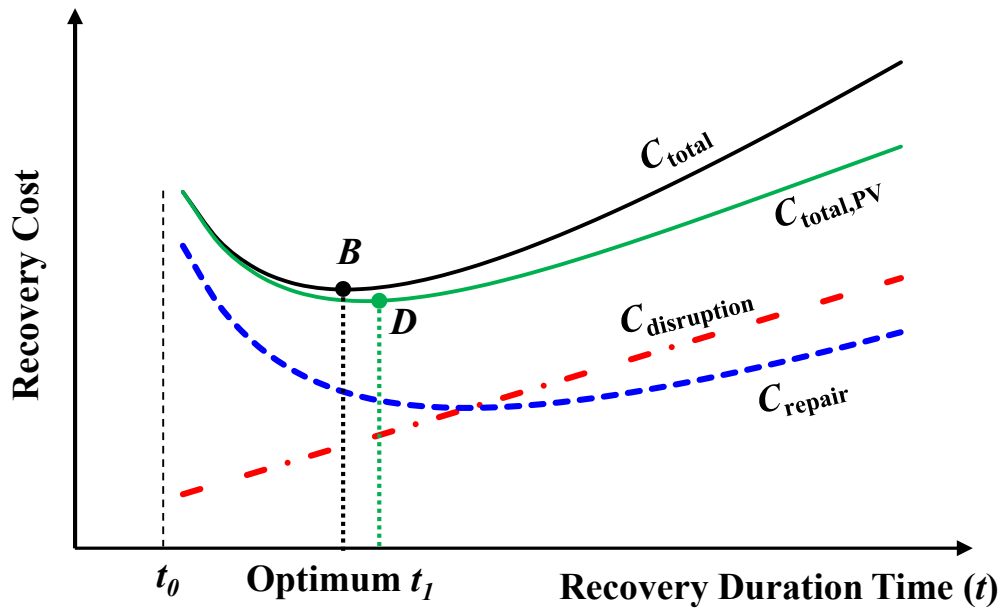


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Figure 6 Cost of implementation of repair works including the direct cost and indirect cost.

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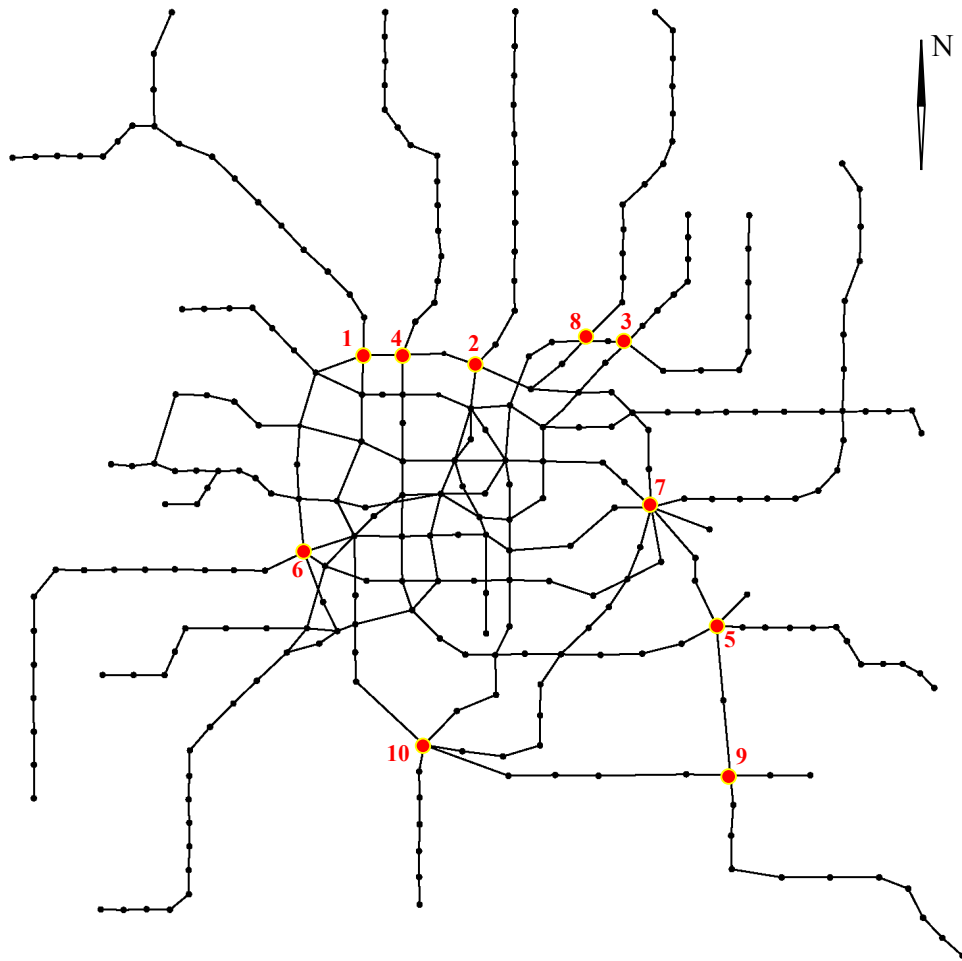


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Figure 7 Time-cost curve for the recovery of a disrupted metro network

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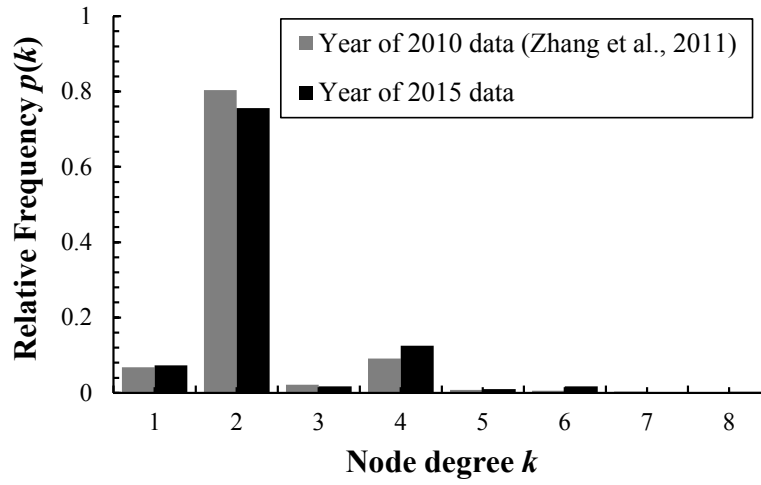


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Figure 8 Typical topological network for the Shanghai metro system

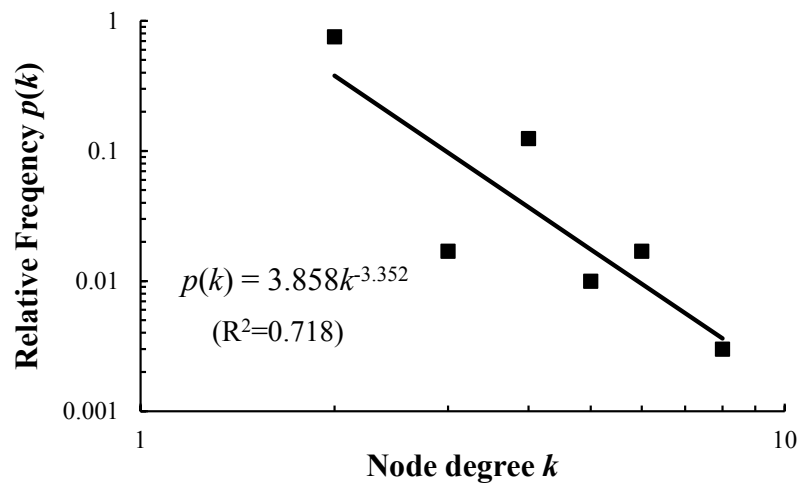
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(a)



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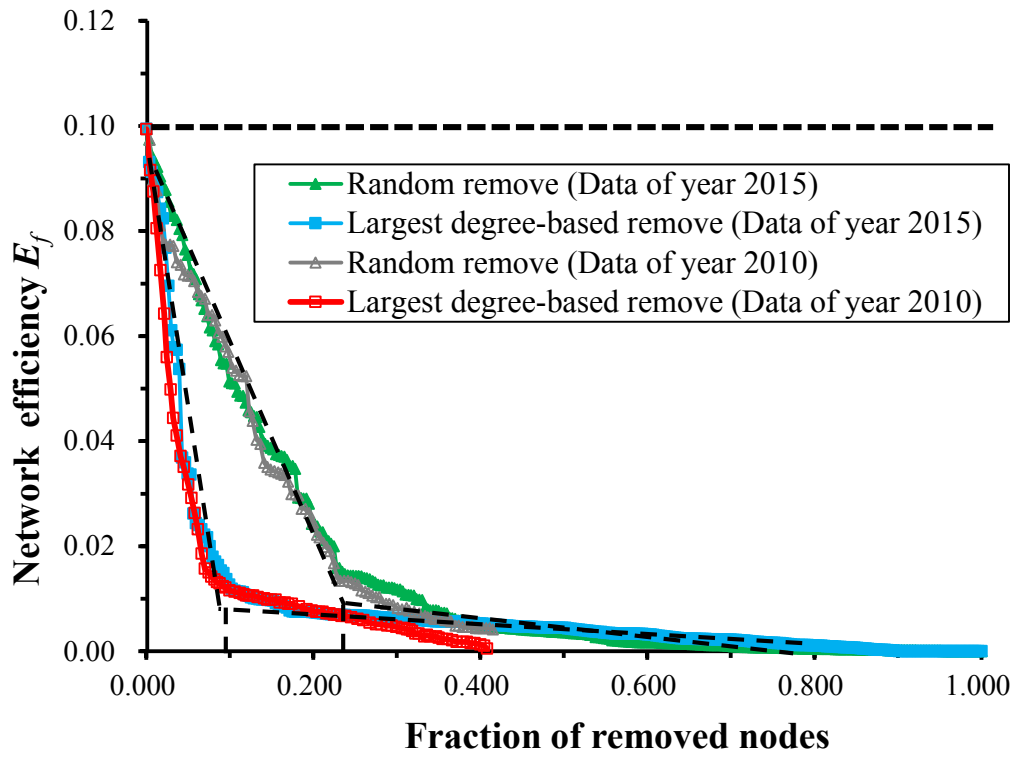
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(b)

937 Figure 9 Distribution of node degrees for 303 metro stations in the Shanghai metro system: a)

938 histogram of k ; b) linear fitting of the relative frequency of $p(k)$ with k for data from year of 2015

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941

942 Figure 10 Robustness of the Shanghai metro network under different node removal type (Data

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for year of 2010 is extracted from Zhang et al., (2011))

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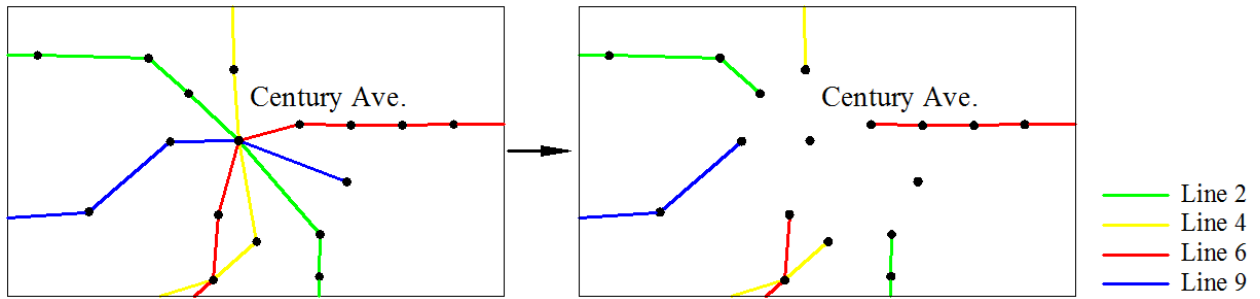
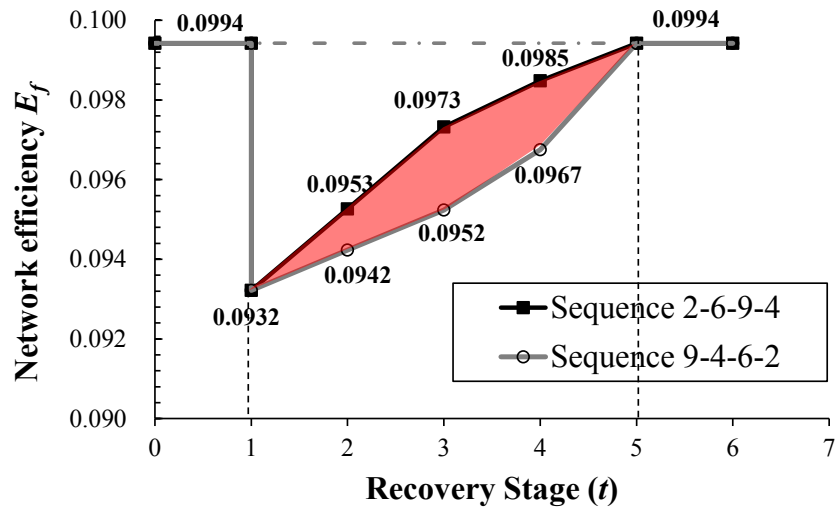


Figure 11 The effect of failure of Century Ave station on the surrounding network

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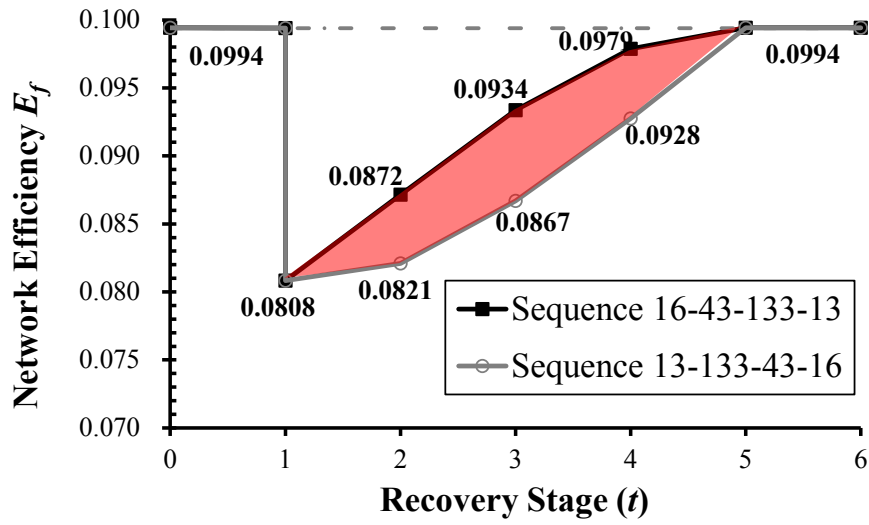


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951 Figure 12 Comparison of change of network efficiency under two different recovery sequences

952 for recovering a multi-line exchange station.

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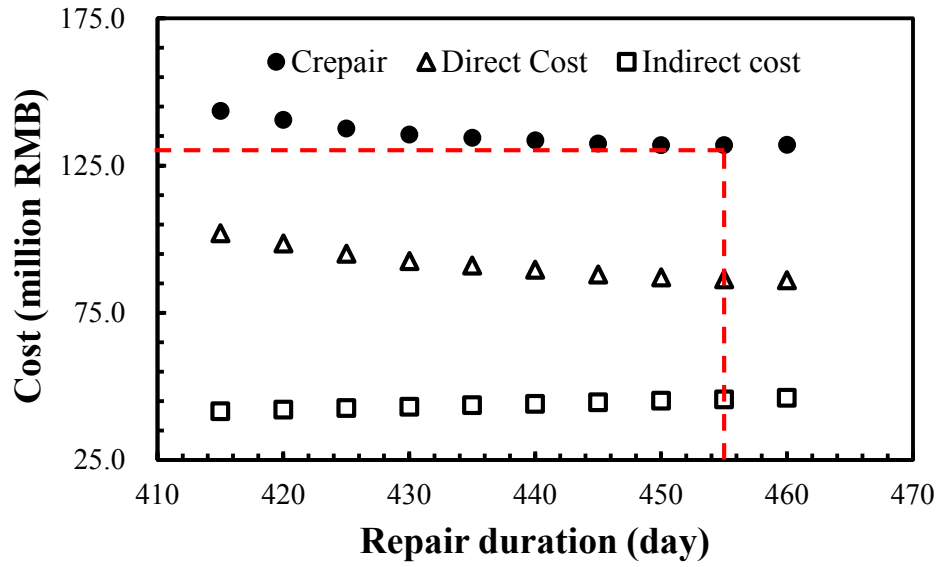


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956 Figure 13 Comparison of change of network efficiency under two different recovery sequences

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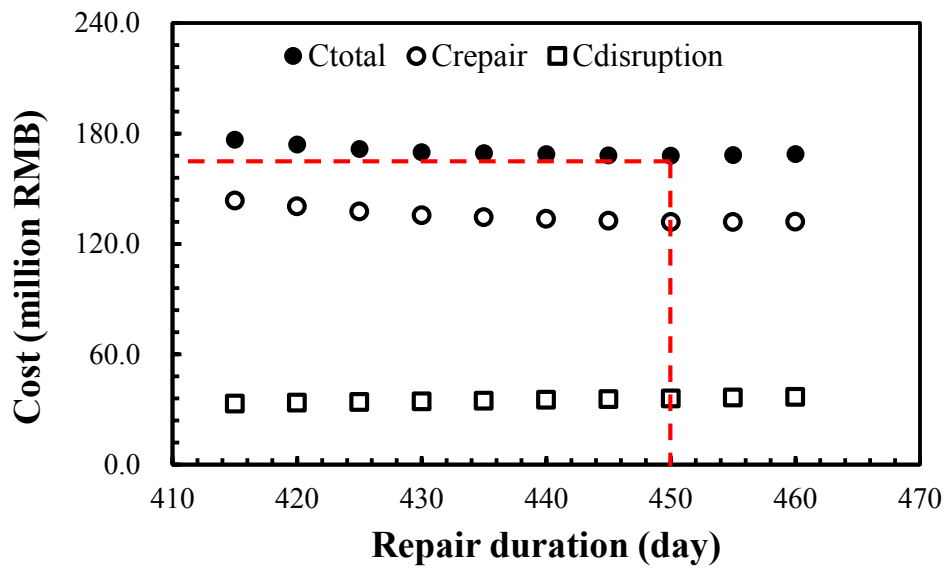
for recovering a multi-line exchange station.



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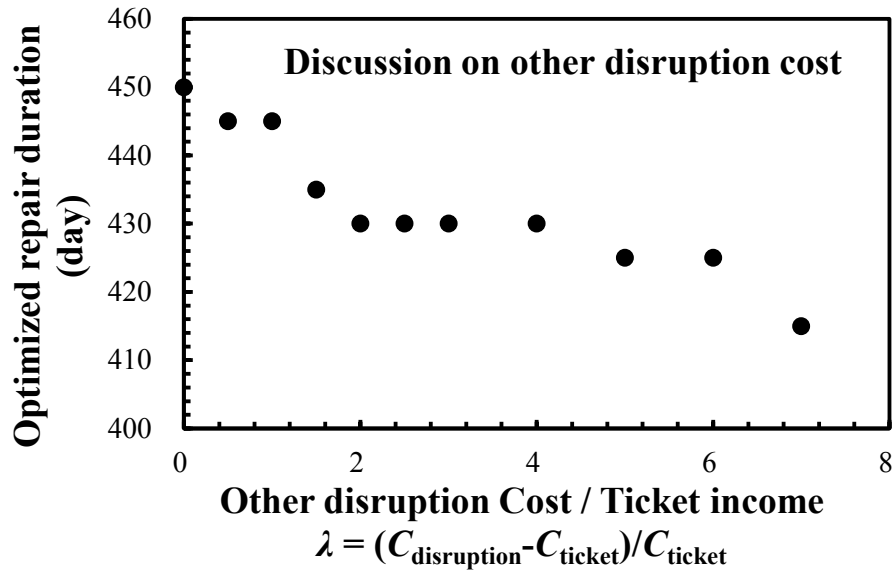
(a)



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(b)



(c)

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Figure 14 Time-cost trade-off analysis for rebuilding a metro station: a) time-cost for repair cost; b) time-cost for total cost including disruption cost and repair cost; and c) effect of other disruption cost on the optimized repair duration days