Assisted inflation

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In inflationary scenarios with more than one scalar field, inflation may proceed even if each of the individual fields has a potential too steep for that field to sustain inflation on its own. We show that scalar fields with exponential potentials evolve so as to act cooperatively to assist inflation, by finding solutions in which the energy densities of the different scalar fields evolve in fixed proportion. Such scaling solutions exist for an arbitrary number of scalar fields, with different slopes for the exponential potentials, and we show that these solutions are the unique late-time attractors for the evolution. We determine the density perturbation spectrum produced by such a period of inflation, and show that with multiple scalar fields, the spectrum is closer to the scale-invariant than the spectrum that any of the fields would generate individually. [S0556-2821(98)50118-X]

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I. INTRODUCTION

The idea of cosmological inflation [1,2] is an attractive one, solving a range of otherwise troubling problems. Inflation is normally achieved by a period of the Universe's evolution during which the energy density is dominated by the potential energy of a scalar field. Although quite probably the early Universe contained several scalar fields, it is normally assumed that only one of these fields remained dynamically significant for a long time, with the others rapidly finding their way into the minima of their respective potential energies.

In this paper we consider scalar fields with exponential potentials. These are already known to have interesting properties; for example, if one has a universe containing a perfect fluid and such a scalar field, then for a wide range of parameters the scalar field "mimics" the perfect fluid, adopting its equation of state [3,4]. These scaling solutions are attractors [5] at late times. The behavior of such a field during an inflationary epoch has also been considered [5].

What was not considered in Ref. [5] is the effect of introducing a scalar field with an exponential potential on the other scalar field. The simplest example would be if the other field also possessed an exponential potential. Then the behavior of both fields will be modified, since they feel only their own potential gradient, but experience, via the expansion, the frictional effect of all scalar fields present.

II. DYNAMICS

For simplicity, we begin by considering m scalar fields, ϕ_i, which each have an identical potential

$$V(\phi_i)=V_0\exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi_i}{m_{\rm Pl}}\right), \tag{1}$$

where $m_{\rm Pl}$ is the Planck mass. Note that there is no direct coupling of the fields, which influence each other only via their effect on the expansion. The equations of motion are

$$H^2=\frac{8\pi}{3m_{\rm Pl}^2}\sum_{i=1}^{m}\left[V(\phi_i)+\frac{1}{2}\dot{\phi}_i^2\right]; \tag{2}$$

$$\ddot{\phi}_i=-3H\dot{\phi}_i-\frac{dV(\phi_i)}{d\phi_i}. \tag{3}$$

Our fields are combined additively; this is different from *soft inflation* [6], where an exponential potential multiplies the potential of another scalar field.

If there is only a single scalar field, this leads to the well-known power-law solution [7]

$$a(t)\propto t^p. \tag{4}$$

This is inflationary only if $p>1$, i.e., for sufficiently shallow exponentials. The power-law solution also applies for any p in the range 1/3 to 1, where it is non-inflationary. For $p<1/3$, the asymptotic solution is that of a free scalar field, with $a\propto t^{1/3}$ regardless of the value of p in the range $(0,1/3)$.

We first find a particular solution, where all the scalar fields are equal: $\phi_1=\phi_2=\cdots=\phi_m$. We shall later show it is the unique late-time attractor. With this ansatz the equations become

$$H^2=\frac{8\pi}{3m_{\rm Pl}^2}m\left[V(\phi_1)+\frac{1}{2}\dot{\phi}_1^2\right]; \tag{5}$$

$$\ddot{\phi}_1=-3H\dot{\phi}_1-\frac{dV(\phi_1)}{d\phi_1}. \tag{6}$$

These can be mapped to the equations of a model with a single scalar field $\tilde{\phi}$ by the redefinitions

$$\tilde{\phi}_1^2=m\phi_1^2;\quad \tilde{V}=mV;\quad \tilde{p}=mp, \tag{7}$$

so the expansion rate is $a\propto t^{\tilde{p}}$, provided that $\tilde{p}>1/3$. The expansion becomes quicker, the more scalar fields there are. And in particular, potentials with $p<1$, which for a single field are unable to support inflation, can do so as long as there are enough scalar fields to make $mp>1$. Note also that this solution does not require p to exceed 1/3, only the product mp. If mp is less than one third, then the solution will instead be that of a free scalar field.

Although the solution with all scalar fields equal is a particular one, it is in fact the generic late-time attractor. To see this, keep ϕ_1, but replace the rest with the redefined fields

$$\psi_i = \phi_i - \phi_1, \quad i = 2, \ldots, m. \tag{8}$$

These fields obey the equation

$$\ddot{\psi}_i + 3H\dot{\psi}_i = \frac{V_0}{m_{\rm Pl}}\sqrt{\frac{16\pi}{p}}\exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi_1}{m_{\rm Pl}}\right) \times\left[\exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\psi_i}{m_{\rm Pl}}\right)-1\right], \quad i=2,\ldots,m. \tag{9}$$

This is the equation of a scalar field in an effective potential

$$\ddot{\psi}_i + 3H\dot{\psi}_i = -\frac{\partial V_{\rm eff}(\phi_1,\psi_i)}{\partial\psi_i}, \tag{10}$$

with

$$V_{\rm eff} = V_0\sqrt{\frac{16\pi}{p}}\exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi_1}{m_{\rm Pl}}\right) \times\left[\sqrt{\frac{p}{16\pi}}\exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\psi_i}{m_{\rm Pl}}\right)+\frac{\psi_i}{m_{\rm Pl}}\right]. \tag{11}$$

The minimum in the ψ_i direction is always at $\psi_i = 0$, regardless of the behavior of ϕ_1, so the late-time solution has all the ϕ_i equal. The length of time to reach this attractor will depend on the initial separation (the value of ψ_i) and the extent to which friction, coming from the expansion rate H, is important.

III. DENSITY PERTURBATIONS

It is now well known how to calculate the density perturbation produced in multi-scalar field models. Sasaki and Stewart [8] (see also Ref. [9]) quote the result

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 \frac{\partial N}{\partial\phi_i}\frac{\partial N}{\partial\phi_j}\delta_{ij}, \tag{12}$$

where $\mathcal{P}_{\mathcal{R}}$ is the spectrum of the curvature perturbation $\mathcal{R}$ in the usual units [2], N is the number of e-foldings of inflationary expansion remaining, and there is a summation over i and j. Since $N = -\int H\, dt$, we have

$$\sum_i \frac{\partial N}{\partial\phi_i}\dot{\phi}_i = -H, \tag{13}$$

where in our case each term in the sum is the same, yielding

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 \frac{1}{m}\frac{H^2}{\dot{\phi}_1^2}. \tag{14}$$

Note that this last expression only contains one of the scalar fields, chosen arbitrarily to be ϕ_1. This expression looks as if it is m times *smaller* than the usual formula for a single scalar field (see, e.g., Ref. [2]); however, remember that the presence of multiple fields has modified both H and $\dot{\phi}_1$.

Of particular interest is the spectral index n. This is given by [8]

$$n-1 = 2\frac{\dot{H}}{H^2} - 2\frac{(\partial N/\partial\phi_i)[(8\pi/m_{\rm Pl}^2)(\dot{\phi}_i\dot{\phi}_j/H^2)-(m_{\rm Pl}^2/8\pi)(V_{,i,j}/V)](\partial N/\partial\phi_j)}{\delta_{ij}(\partial N/\partial\phi_i)(\partial N/\partial\phi_j)}, \tag{15}$$

where there is a summation over repeated indices and the commas indicate derivatives with respect to the corresponding field component. Under our assumptions, the complicated second term on the right-hand side of the above equation cancels out, and Eq. (15) reduces to the simple form

$$1-n = -2\frac{\dot{H}}{H^2} = \frac{m_{\rm Pl}^2}{8\pi}\left(\frac{\partial V(\phi_1)/\partial\phi_1}{V(\phi_1)}\right)^2 = \frac{2}{mp}. \tag{16}$$

This result shows that the spectral index also matches that produced by a single scalar field with $\tilde{p} = mp$. The more scalar fields there are, the closer to scale-invariance is the spectrum that they produce. Note, however, that if the fields have such steep potentials as to be individually non-inflationary, $p < 1$, then many fields are needed before the spectrum is flat enough (say $n > 0.7$) to have the possibility of explaining the observed structures. Large numbers of scalar fields are predicted by some theories, for example, the 70 scalar fields, with unknown potentials, of the low-energy compactified superstring effective action [10].

IV. POTENTIALS WITH DIFFERENT SLOPES

We now generalize the above discussion, by considering each potential to have a different slope p_i

$$V_i(\phi_i) = V_0\exp\left(-\sqrt{\frac{16\pi}{p_i}}\frac{\phi_i}{m_{\rm Pl}}\right). \tag{17}$$

Notice that we keep the same V_0 for each field; since changing V_0 is equivalent to shifting the scalar field definition by a constant, this serves to fix the zero values of the fields.

We conjecture that scaling solutions exist, where the energy densities of the different fields attain fixed ratios at late times, relative to an arbitrarily chosen field ϕ_1:

$$\frac{\dot{\phi}_i^2}{\dot{\phi}_1^2}=\frac{V_i(\phi_i)}{V_1(\phi_1)}=C_i. \tag{18}$$

To guess the appropriate form of C_i, we note that the slow-roll approximation gives

$$\dot{\phi}_i^2\simeq\frac{2}{3p_i}\frac{V_i^2(\phi_i)}{\sum_i V_i(\phi_i)}, \tag{19}$$

which suggests

$$C_i=\frac{p_i}{p_1}. \tag{20}$$

We stress though that the slow-roll approximation is not needed in what follows.

Integrating the kinetic part of Eq. (18) then gives

$$\phi_i=\sqrt{\frac{p_i}{p_1}}\phi_1+\alpha_i, \tag{21}$$

where α_i are the integration constants. Ensuring the potentials also scale as in Eq. (18) requires the constants to have values

$$\alpha_i=-\sqrt{\frac{p_i}{16\pi}}\ m_{\rm Pl}\ \ln\frac{p_i}{p_1}. \tag{22}$$

To see that this solution will solve the full dynamical equations, we generalize the scaling argument of Sec. II. Equation (21) reduces us to a single degree of freedom ϕ_1 in a manner consistent with the equations of motion, which become

$$H^2=\frac{8\pi}{3m_{\rm Pl}^2}\frac{\sum_{i=1}^m p_i}{p_1}\left[V_1(\phi_1)+\frac{1}{2}\dot{\phi}_1^2\right]; \tag{23}$$

$$\ddot{\phi}_1=-3H\dot{\phi}_1-\frac{dV_1(\phi_1)}{d\phi_1}. \tag{24}$$

Using the scaling of the potential from Eq. (18), the other scalar wave equations all match the latter of these, confirming consistency of the ansatz. Note in particular that Eq. (21) brings all the exponentials into the same form.

These can then be turned into the equations of a model with a single scalar field via the redefinitions

$$\tilde{\phi}_1^2=\frac{\Sigma p_i}{p_1}\phi_1^2;\quad \tilde{V}_1=\frac{\Sigma p_i}{p_1}V_1;\quad \tilde{p}=\sum_i p_i, \tag{25}$$

of which Eq. (7) is a special case. This result is exact, not requiring a slow-roll approximation, and once more shows that the presence of multiple scalar fields increases the expansion rate. The expansion law is $a\propto t^{\tilde{p}}$, and is valid provided that $\tilde{p}>1/3$.

The scaling construction shows that this solution exactly solves the multi-scalar field equations. One can show that this solution is an attractor by generalizing the argument of Sec. II, via the ansatz

$$\psi_i=\phi_i-\sqrt{\frac{p_i}{p_1}}\phi_1-\alpha_i,\quad i=2,\dots,m, \tag{26}$$

which generalizes Eq. (9) to

$$\ddot{\psi}_i+3H\dot{\psi}_i=\frac{V_0}{m_{\rm Pl}}\sqrt{\frac{16\pi}{p_i}}\exp\left(-\sqrt{\frac{16\pi}{p_1}}\frac{\phi_1}{m_{\rm Pl}}\right)\times\frac{p_i}{p_1}\left[\exp\left(-\sqrt{\frac{16\pi}{p_i}}\frac{\psi_i}{m_{\rm Pl}}\right)-1\right],$$
$$i=2,\dots,m. \tag{27}$$

As before, the effective potentials for the ψ_i fields have a unique minimum at $\psi_i=0$ for all $i=2,\dots,m$. Our scaling solution is therefore the unique late-time attractor.[1]

The calculation of the spectral index follows the same lines as before, yielding

$$1-n=\frac{2}{\tilde{p}}. \tag{28}$$

This reduces to Eq. (16) when the slopes of the potentials are same.

V. CONCLUSION

Although the early Universe is likely to contain many scalar fields, a common assumption when analyzing inflation is that all but one of these fields has become dynamically irrelevant. However, for scalar fields with exponential potentials, the late-time behavior is for the energy densities of the different fields to scale with each other, as had already been noted for the case of a scalar field with an exponential potential plus a barotropic fluid [3–5], even if the fields have no direct coupling to each other and if their potentials have different slopes.

Such multiple scalar fields can act cooperatively to drive a

[1]We have also confirmed these solutions as late-time attractors numerically for a wide range of values of p_i.

period of inflation, even if the individual fields have potentials which are too steep in their own right; the expansion law in the scaling solution is $t^{\tilde{p}}$, where $\tilde{p}=\Sigma p_i$ with the p_i being the power-law expansion rates that the individual fields would drive in isolation. The reason for this behavior is that while each field experiences the ''downhill'' force from its own potential, it feels the friction from all the scalar fields via their contribution to the expansion rate.

We have also studied the density perturbation spectrum produced, which has a spectral index n matching that of power-law inflation driven by a single field at rate $\tilde{p}$. The spectrum is therefore brought closer to scale-invariance, the more fields participate in the inflationary expansion. A perturbation spectrum close to scale invariance is preferred by current observations, and this phenomena may offer assistance to supergravity-based inflation models which often predict spectra which are not all that close to scale invariance [11].

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