1998

A New Approach to Expectation Theory: Rationality Revisited

Tamas Forgacs
Eastern Illinois University

THESIS REPRODUCTION CERTIFICATE

TO: Graduate Degree Candidates (who have written formal theses)

SUBJECT: Permission to Reproduce Theses

The University Library is receiving a number of request from other institutions asking permission to reproduce dissertations for inclusion in their library holdings. Although no copyright laws are involved, we feel that professional courtesy demands that permission be obtained from the author before we allow these to be copied.

PLEASE SIGN ONE OF THE FOLLOWING STATEMENTS:

Booth Library of Eastern Illinois University has my permission to lend my thesis to a reputable college or university or the purpose of copying it for inclusion in that institution's library or research holdings.

7/23/98
Date

I respectfully request Booth Library of Eastern Illinois University **NOT** allow my thesis to be reproduced because:

____

____

____

Author's Signature ______________ Date ______________

thesis4.form

A new approach to expectation theory:

rationality revisited

(TITLE)

BY

Tamas Forgacs

THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF

Master of Arts

IN THE GRADUATE SCHOOL, EASTERN ILLINOIS UNIVERSITY
CHARLESTON, ILLINOIS

1998

YEAR

I HEREBY RECOMMEND THIS THESIS BE ACCEPTED AS FULFILLING
THIS PART OF THE GRADUATE DEGREE CITED ABOVE

7/22/98

DATE

7/21/98

DATE

To my mother and father

Abstract

Expectations play an important role in economics. Traditionally two major branches of expectation theory are distinguished: that of adaptive and rational expectations. This study sets out the goal of investigating inflationary expectations based on real world experiences. The model proposed and tested here abandons the traditional fixed-time-interval-update models for a non-fixed-time-interval-update model. Although the penalty function attached to each error is still subject to debate, it is shown that by reacting with faster updates to errors in expectations economic agents achieve more precise expectations compared to those of a fix time interval update model. We also find the model rational in the weak sense, but we are unable to test the proposed model for strong rationality as of this moment, due to the lack of appropriate econometric tests for non-fixed time interval processes.

The study concludes that time variant adaptive expectations can be regarded as rational in the weak sense, and at the limit they appear to be mathematically identical.

Acknowledgements

First, I would like to thank Dr. Noel Brodsky, my thesis supervisor for his time, dedication and professional help that he has provided me throughout this project. I also thank Dr. Lawrence Bates and Dr. Jim Bruehler, my committee members, for their help and effort to bring the best out of me as well as out of the project. Without the atmosphere they have created for the work, many of the ideas would not have come to life, and the insight I gained in the field could not have been as deep. Their approach and critical remarks showed me what lays in the core of academic work, and how separable academic criticism and personal friendship are. I also owe them a great deal for discussions and advice about my future career. They sacrificed their time and provided help in order to enable me to pursue the goals I set out for myself.

I would like to thank all the professors in the Economics Department at Eastern Illinois University for their help and contributions to my education. The whole faculty presented me how a teacher-student relationship should be handled. It is a valuable experience to a person who is thinking about teaching as a profession.

Last, but not least I would like to thank everybody for his or her support for without friends and family, life and work would be a great deal harder.

I thank you all.

Table of contents

Chapter 1

1.1. Introduction

The theory of expectations in economics has a long history. Scholars have extensively explored the formation of expectations ever since Cagan (1956) committed expectations to a mathematical expression. Two major branches of expectation theory have emerged; the theories of adaptive and rational expectations. Although some tried to bring adaptive expectations closer to rational expectations, a theoretical difference remains between the two: rational expectations asserts that economic agents use all available information to form expectations; adaptive expectations, on the other hand, hypothesizes that agents only use past price information when forming their expectations.

This study is an effort to investigate the formation of expectations based on economic theory, everyday experience and common sense. The study also links rational and adaptive expectations in a way. Thus there is a twin goal of the study – to show that strictly annual updates in the expectation process can be outperformed by a time variant expectation formation process, and to show that this time variant updating process is rational.

The structure of the study is as follows. After the survey of literature we will lay out our hypothesis, which will be followed by the regression and test results. The closing section of the paper includes concluding remarks and areas for future research.

1.2. Survey of Literature

There are numerous reasons why the formation of expectations might be interesting to investigate. Expectations play a key role when *a priori* (are you sure you mean this? Perhaps you mean *ex-ante*?) determining the real interest rate and real wages. It is also a key determinant of economic policy effectiveness. Expectations play a pivotal role in the operation of the economic system.

As early as 1936 expectations began to take an important role in economic literature, when Keynes' The General Theory of Employment Interest and Money was published. Although his approach as non- mathematical, it does deal with the role of expectations in the economy. It also considered how expectations are formed, and what information is relevant in forming those expectations.

> Meanwhile the entrepreneur [including both the producer and the investor in this description] has to form the best expectations he can as to what the consumers will be prepared to pay when he is ready to supply them (directly or indirectly) after the elapse of what may be a lengthy period; and he has no choice but to be guided by these expectations, if he is to produce at all by processes which occupy time. (p. 46)

He also points out later on in the book: "The *actually realized* results of the production and sale of output will only be relevant to employment in so far as they cause a modification of subsequent expectations" (p. 47, italics added). The formation of expectations and the related issues continued to be addressed after The General Theory of Interest and Money. There are expectations about much in the economy. Our interest may lay in exchange rate expectations, interest rate expectations, price expectations or a number of other variables. This study will concentrate on inflationary expectations.

The theory of inflationary expectations relates to the effectiveness of monetary policy, and through monetary policy, to the efficiency of macroeconomic policy. For example, in a frictionless economy with rational expectations, changes in the money supply will not have any effect on output (Madsen 1996, Lucas 1996). On the other hand, as Madsen pointed out, if adjustment of firms' expected inflation to its determinants are slow, there will be an output response to a change in the money supply. In the case of adaptive expectations, a monetary shock to the economy would cause inflation to become persistent because people expect inflation to continue at its present rate. Thus different hypothesized formations of inflationary expectations lead to different hypothesized consequences, both social and economic.

There are a large number of works that have modeled inflationary expectations. The literature can be divided into four different groups: studies of extrapolative, regressive, rational and adaptive expectations.

1.2.1. The extrapolative model

The extrapolative model of expectations (Frenkel, 1975) is probably the simplest. It suggests that individuals form expectations by projecting the present rate of inflation into the future. Due to its simplicity, it cannot account for changes in the expectations other than through the changes in the inflation rate itself. For this reason adjustments only occur if new data about the inflation rate is obtained.

1.2.2. The regressive model

The regressive model of inflationary expectations (Figlewski and Wachtel, 1981) assumes the following pattern of expectations:

$$\pi_t - P_{t-1} = \alpha + \beta(P_{t-1}^N - P_{t-1}) + \varepsilon_t \qquad 0<\beta<1$$

where π_t is the expected inflation rate for the time period t-1:t, P_{t-1} is the rate of price increase over the period t-2:t-1, P_{t-1}^N is a mean rate of price increase over a certain period preceding t-1 (Figlewski and Wachtel used 5 years but it may vary) α and β are constants and ε_t is an error term with a zero mean and constant variance.

This approach essentially asserts that individuals expect the inflation rate itself to adjust or "regress" towards some long run "normal level" represented by P_{t-1}^N. This normal level of inflation can be explained the following way. In every economy there is a historically acceptable rate of inflation that is considered normal by the public. This level varies from country to country. Countries with a stable economic past would probably have a low acceptable level of inflation, as well as those where inflation had done considerable damage in the past. However those with less stable (yet not hyperinflationary) economic history or different political preferences may experience a higher level of acceptable and thus "normal" inflation. Conceptually, regressive expectations is similar to the adaptive expectations. In the case of adaptive expectations however, the adjustment is made towards an inflation rate or price level, which can vary over time instead of the long run normal level represented by P_{t-1}^N.

Since in this model expectations may change due to changes in the long run inflation rate, the past inflation rate or the adjustment coefficient, β, the expected inflation rate may change even if the past inflation rate is constant. Even though the regressive model of expectations is somewhat more sophisticated than the extrapolative model, it is still not perfect. As Figlewski and Wachtel point out: "evidence suggests that a single coefficient time invariant model of inflation expectations cannot be viewed as an adequate representation of a complex process." (p. 9)

1.2.3. The rational expectations model

Rational expectations gave a new angle to the modeling of expectation formations. Muth's early work (1961) served as a theoretical basis for his followers who elaborated and mathematized the concept of rational expectations. (Begg, 1982; Lucas, 1972, 1973, 1996; Mishkin, 1983; Sargent and Wallace, 1975; Sheffrin, 1996)

The rational expectations hypothesis asserts that the market's subjective probability distribution of any variable such as price is identical to the objective probability distribution of that variable, which is conditioned on all available past information (Mishkin, 1983, p. 9). Formally expressed:

$$E_m(X_t|\Phi_{t-1}) = E(X_t|\Phi_{t-1})$$

where

Φ_{t-1} is the set of information available at time t-1

$E_m(\ldots|\Phi_{t-1})$ is the subjective expectations assessed by the market

$E(\ldots|\Phi_{t-1})$ is the objective expectation conditional on Φ_{t-1}

From this definition it follows that price expectations are on average correct and deviations of the expected price level from past prices are influenced only by random errors. Formally:

$$P_t^e = P_t + \varepsilon_t$$

where P_t^e is the expected price level at time t conditioned on the information set, P_t is the actual price level at time t, and ε_t is the error term with zero mean and constant variance. If the expectation operator is applied to both sides of the equation, we obtain the equation we started with.

In this model there is no room for uncertainty that agents could eliminate, because it is assumed that expectations are formed using all available information. This assumption of the rational expectations hypothesis is questioned more than any other (for more detailed critique see Begg, 1982; Blanchard, 1990) because even economists do not know the precise structure of the economy and can not use all available information to form expectations. Other economic agents have less information available, so there is only a minute chance for all available information to be used in forming expectations. Furthermore, as Naish (1993) pointed out, rational expectations requires more information than adaptive expectations and is therefore more costly.

1.2.4. The adaptive expectations model

The fourth approach considered here is the adaptive formation of expectations. The principle of this approach is that agents adjust their expectations every time period by a fraction of the mistake they made in the preceding period; thus expectations adjust by a constant proportion of the previous discrepancy:

$$P^e_{t+1} = P^e_t + \lambda(P_t - P^e_t) \qquad 0<\lambda<1$$

where P^e_t is the expected price level at time t, P_t is the actual price level at time t and λ is a constant. According to this model, agents are never correct concerning their expectations, but they gradually adjust to the actual price level, though not reaching it.

According to Lawson (1980, p. 305) there are three reasons why this particular approach is a good procedure for modeling expectations. As Naish (1993) later argues in accordance with Lawson, in a standard macro model the losses associated with adaptive expectations are shown to be very small, even for quite large monetary disturbances. Thus adaptive expectations may be close to an optimal model as long as the monetary regime is relatively stable. Second, the properties of this model are convenient for econometric work[1]. The third reason is that there is empirical support for adaptive expectations.

Since the rational and adaptive approaches are the two most favored for modeling expectations, it is useful to summarize the major differences between the two. Following Glazer, Stechel and Winer (1990), those differences can be summarized as follows:

[1] Expectation terms along with other variables need to be observable. If we assume adaptive expectations, a simple Koyck transformation will eliminate the expectation term from the regression equation thus solving the problem of unobservability (on the applicability and possible problems of the Koyck transformation see Henry, 1974).

Adaptive Expectations	Rational Expectations
elies totally on the most recent value of he forecasted variable and ignores current xogenous shocks to the system enerating observed values	ny information valuable to the orecaster will be used to form the orecast including current data
orecasters use prior forecasting errors to mprove current predictions	orecasting errors contain no new nformation
mplies a specific model of the xpectations formation process itself, i.e. t is a structural modeling approach	escribes a set of stochastic properties ssociated with the results

Despite the striking differences between the two approaches, according to Glazer et. al (1990) the two hypotheses can co-exist under certain circumstances. As these authors pointed out, forecasts formed adaptively are rational if the underlying process in the forecasted variable is a random walk, that is errors in forecasting are due to only random variations. Restrictive as this condition may be, its importance lies in the fact that it interrelates the two. Nevertheless, we believe that among all the possibilities, inflationary expectations are in fact formed according to the adaptive pattern because the extrapolative approach is too simplistic, and the regressive and rational expectations contain questionable assumptions.

1.3. New approaches in expectations theory

The traditional adaptive expectation model also has problems associated with its ability to react to changes in ongoing economic processes. When inflation is high, uncertainty is also high because economic calculations become uncertain and the risks associated with the loss in the purchasing power of money increase. Numerous studies have dealt with the relationship between the level of inflation and the uncertainty about the level of inflation (Bulkley, 1984; Arnold, 1995; Golob, 1993; Holland, 1993, 1995; Evans, 1989; Ball, 1990; Park, 1995). If inflation increases uncertainty, some authors argue there is a good reason to believe that changes in uncertainty influence the pattern that agents use in forming their expectations. Studies suggest that there is a positive relationship between the level of inflation and the uncertainty about the level of inflation. Some of them even distinguish between uncertainty during high and low levels of inflation. As Park reported (1995): "Our results suggest that there exists a strong positive linear relationship between the trend inflation and uncertainty measures...In general the relationships are negative at low inflation but significantly positive at high inflation." This suggests that uncertainty becomes an important issue only at high inflation; the relationship between inflation and uncertainty is only positive in the high inflation zone. Golob (1993) obtained similar results: "To briefly preview the general results of this survey, there is substantial evidence that increases in inflation are associated with increases in both inflation uncertainty and price dispersion."

Given the way adaptive expectations are formed, high inflation rates will induce higher costs to agents due to greater gaps between the actual and expected price levels. Thus agents may protect themselves by modifying their expectation formation patterns.

Two approaches have been used to model modifications in the expectation formation induced by changing uncertainty form inflation and changing levels of inflation *per se*. The first type (Lawson, 1980; Satchell and Timmermann, 1995) tries to capture growing uncertainty through an adjustment coefficient. The other type (Pami and Subhash, 1992) introduces a separate equation for drifts in the inflation rate. Both approaches have problems.

A single time invariant coefficient cannot sufficiently account for all the changes that might happen over time. This consideration motivated Lawson (1980) in his work on adaptive expectations and uncertainty. As he pointed out, empirical results seem to contradict that the adjustment coefficient (λ) remains constant at different levels of inflation. On the contrary, he says, evidence suggests that λ increases in times of accelerating inflation and general uncertainty. To illustrate his point he constructed the following model:

$$M_t = P_t + \varepsilon_t$$
$$P_t = P_{t-1} + u_t$$

where M_t is the observation of the underlying variable (price) obtained at time t, P_t is the permanent component of it and ε_t is the noise term. Moreover, there is a functional relationship between the permanent components over time, according to the second equation in which u_t is the noise term. Both error terms are assumed to be independently distributed with zero means. The expectation formation pattern then looks like this:

$$P((t+1)/t) = P(t/(t-1)) + \lambda(M_t - P(t/(t-1)))$$
$$0 \le \lambda \le 1$$

where $P((t+1)/t)$ denotes the agents conditional estimate of P_{t+1} formed at time t. Lawson assumes that expectations held at time t-1 about the price level will be revised at time t taking the observation made on it into account:

$$P(t/t) = P(t/(t-1)) + \lambda_t(M_t - P(t/(t-1)))$$

In this setting there are two different types of uncertainty that might influence the adjustment coefficient and make it time variant. The first relates to the agent's degree of belief that the observation is free of noise, that is that ε_t is zero. The second relates to the degree of belief attached to the prior estimate of P_t. Uncertainty is measured by the variance of the error term (ε_t) denoted as W_t. Since he assumes the prior distribution of the expected variable to be normal, the uncertainty about it can also be measured by the variance, denoted here by Σ_t. He shows that in the "revision" equation the adjustment coefficient will look like this:

$$\lambda_t = \frac{\Sigma_t}{W_t + \Sigma_t}$$

Thus as uncertainty rises (ε_t), so does the adjustment coefficient (λ_t) of the revision equation. But because the agents posterior estimate of time t becomes the prior estimate for time t+1, it will make the expected values of the price level relatively less inadequate by increasing λ_t. Following from the works on inflation and uncertainty we can establish that as inflation rises, the degree of belief attached to the agent's prior estimate of the price level will shrink, causing

Σ_t and λ_t to rise. It is not a surprising result, since at higher levels of inflation there is more to lose if one's expectation is incorrect.

Satchell and Timmermann (1995) arrive at essentially the same result except their adjustment coefficient is constructed in a slightly different way. Their final result is:

$$y^e_{t+1} = y_t + \lambda_t (y^e_t - y_t)$$

where

$$\lambda_t = \frac{1}{2 + \varphi - \lambda_{t-1}}$$

and

$$\varphi = \frac{\sigma^2_\varepsilon}{\sigma^2_\eta}.$$

One of the two variances (σ^2_ε) perfectly corresponds to that of Lawson's denoted earlier as W_t. The other (σ^2_η) is the variance of Lawson's u_t error term. It is clear that Satchell and Timmermann also tried to account for changing uncertainty through variances of different error terms.

Both these works are theoretically sound. Nevertheless, Lawson himself points out that there is a problem with setting the error variances (for more detailed discussion see Lawson, 1980, p. 310). Apart from the fact that we do not know what values should be assigned to low uncertainty and high uncertainty, there is also a computational problem as the inflationary process goes on (p. 310). Although these approaches seem to capture reality better than the simple adaptive expectations approach, they fall into the same trap as the rational expectations hypothesis: they fail to recognize that many agents in the economy have limited ability to recognize and to understand the complex economic system. For this reason we will present another model of expectations formation that also reduces the gap between the actual and expected price levels as uncertainty rises, yet in a more intuitive way.

This effort was by Pami and Subhash (1992), who attempted to capture uncertainty in a different way. If inflation varied from its trend line, uncertainty should increase they argued. They introduced a model that accounts for drifts in inflation through a separate equation. If inflation-drift occurs, it changes expectations of the price level. They allow the adjustment coefficient to remain the same, but add an additional term in the expectation pattern that accounts for the change. We believe that this is an important issue. Pami and Subhash do not adjust for uncertainty that results from higher levels of inflation; they only account for drifts in the inflation trend. It is an important contribution, albeit a one-sided one. It seems like that a combination of these two approaches (one that concentrates on changes in the adjustment coefficient and one that concentrates on drifts) would be more accurate than either of the two separately.

There is a gap in the literature concerning time variant expectations. If uncertainty rises, expectations are (or should be) updated more frequently than before in order to avoid losses due to the discrepancy between the actual and the expected level. At low levels of inflation these losses are not significant enough to bother with changing the expectations pattern. As inflation rises however, the losses increase and force agents to revise their expectations more frequently than before.

Chapter 2

2.1. The hypothesis

For reasons argued above this paper will introduce uncertainty in a new way into an expectations model. The model will not capture uncertainty in the adjustment coefficient, nor will it measure uncertainty through variances of error terms. Rather it will introduce a variable that will make expectations time variant, and will introduce this new variable in the time indices.

This is a more accurate approach if adjustments in the expectation formation do not come along at a regular frequency. Although most of the literature on adaptive expectations argues differently (Cagan, 1956; Cashey, 1985; Christiano, 1987; Doran, 1988; Engsted, 1994; Figlewski and Wachtel, 1981; Glazer and Stechel, 1990; Henry, 1974; Just, 1977; Lawson, 1980; Mussa, 1975; Naish, 1993; Pami and Subhash, 1992; Satchell and Timmermann, 1995; Struth, 1984), we believe that there is a non-regularity in the length of expectation updates. That expectations are formed more often than once a year was implicitly suggested by Lawson (1980) who tested his hypothesis with quarterly data, and by Engsted (1994) who worked with monthly observations. Both acknowledged that expectations are updated more often than yearly. The claim, however, has not been previously made that the length of the time period between updates varies over time.

This paper suggests that as uncertainty rises adjustments of expectations are made more frequently at shorter intervals of time. One reason for this is that if prices change at a very rapid pace, agents would lose too much if they waited to update their expectations. In a highly inflationary environment they cannot wait.

They could lose a major fraction of their real wealth if they wait too long irrespective of how big a change they make as expressed by the adjustment coefficient (as it would be the case in Lawson's model) when they finally update.

A time variant adjustment would also mean that adaptive expectations would be close to optimal not only when the monetary regime is relatively stable, but also when it is not (as opposed to Naish, 1993). More frequent updating of expectations leads to a model that approaches rational expectations. The idea of bringing these two types of expectations together is not new (Glazer et al, 1990), but no attempt has been made to bring them together through adjustments in the time indices.

For all these considerations we propose the following model of inflationary expectations:

$$P_t^e = P_{t-\Theta_t}^e + \lambda\left(P_{t-\Theta_t} - P_{t-\Theta_t}^e\right) \qquad 0<\lambda<1$$

where P_t^e is the price level expected at time t, $P_{t-\Theta_t}^e$ is the expected price level at time $t-\Theta$, $P_{t-\Theta_t}$ is the actual price level at time $t-\Theta_t$, and λ is the adjustment coefficient. The variable Θ brings uncertainty into the expectation formation pattern, and needs careful consideration. All the factors that might cause uncertainty in an economy should receive careful consideration and we also have to investigate the question whether those things are measurable. Moreover, factors that influence Θ that are not obvious and observable to the public or which have too high information costs should be excluded.

Therefore we will omit variances of error terms in Θ. Although they measure uncertainty, they are difficult to estimate as Lawson himself admitted (1980, p. 310).

In this model of inflationary expectation formation the adjustment coefficient practically loses its meaning. If it is low, adjustments will be made more often to make up for the possible discrepancies between the expectations and the actual realization of the variable. Also if it is large, adjustments will not be made as often. If Θ picks up all the time adjusting properties of the adjustment coefficient, the adjustment coefficient from this point on may be assumed to be constant.

2.2. Possible determinants of the update length

2.2.1. Inflation

The first determinant of Θ is the actual level of inflation. The literature suggests (Arnold and Hertog, 1995; Ball, 1990; Bulkley, 1984; Davis and Kanago, 1996; Evans, 1989; Golob, 1993; Holland, 1993, 1995; Park, 1995) that there is a positive relationship between the level of inflation and uncertainty. Uncertainty also increases the frequency of expectation formation or revision. Thus the level of inflation will have a negative effect on Θ as well.

2.2.2. Exchange rates

Exchange rates also to play an important role in determining inflation in a country (Dornbusch, 1976). For example if a country devalues its currency, it is highly likely to lead to increases in import prices that contributes to the ongoing inflation in that particular country. Thus theoretically we cannot omit e when gathering all the variables that influence Θ. Also we note, that the exchange rate is likely to have a negative impact on Θ, since a devaluation would mean growing inflation which would show up in a growing Θ. We recognize that in order this variable to have a measurable impact on uncertainty that specific country has to be quite reliant on trade. At the current state of the world economy a lot of countries (including OECD countries) possess this property, so it seems reasonable to assume that changes in the exchange rate would influence uncertainty. We also recognize that inflation and changes in the exchange rate are highly correlated. The reason why we still include the exchange among the influential factors is that changes in the exchange rate can be observed on a daily basis as opposed to inflation. Should circumstances be really uncertain in a given country, agents would still be able to adjust their expectations fast enough by looking at the exchange rate movements.

2.2.3. Losses of real assets

Another variable that needs to be included is one that represents the costs of inflation. Losses in real assets may occur if inflationary expectations are incorrect. To illustrate this point, let us assume an agent who holds money as part of his assets. As inflation rises the real value of this asset drops. However this agent might avoid losses in his real wealth, if his prediction of the change in the prices is accurate, either by lending his money out at an appropriate interest rate, holding foreign currency instead of domestic (also know as capital flight from the country, since foreign currency is not part of the domestic money supply, even though it may be held by domestic residents) or moving into physical assets instead of money. Inaccurate expectations will result in insufficient adjustment and losses in the real assets of that agent. Since these losses might be quite painful, it is reasonable to assume that the agent will change his or her expectation pattern to eliminate future losses. For these reasons Θ should also be a function of losses in real assets.

One might argue that inflation is a good proxy for losses in real assets. However inflation and losses in real assets are two different signaling mechanisms. Agents in a country cannot decide whether a certain level of inflation is really detrimental. But they can see if it does cause substantial losses in real assets. They would be expected to learn through losses in real assets what is detrimental level of inflation. We included both variables thinking that they serve as signaling systems in different stages of inflation, or at different stages of the inflationary history of a country.

2.2.4. Political factors

Political factors in a particular country may also increase general uncertainty. Newly imposed economic measures are hard to foresee, particularly monetary policy. This factor may appear as a dummy variable in the estimating equation of Θ, but Θ is clearly a function of it. (There are other many indicators of political and regime changes such as riots, strikes etc. that might be more observable. For theoretical reasons however it is enough to mention that political factors have an effect on theta). If theta were to be estimated, we would expect this variable to have a negative effect on it. If there is a regime change (the value of the dummy variable is 1) expectation updates should be more frequent, Θ should diminish. This study considers OECD countries only for data availability reasons, where regime changes are not frequent enough to have a measurable effect on the formation of expectations. On the other hand, there are countries where this is not the case. Developing countries, or countries in transition usually experience numerous regime changes; in these cases it would be wise to include this variable in the estimation equation. We will leave the political aspect for future research.

2.2.5. Information costs

Strictly related to changes in uncertainty is information costs. Under growing uncertainty the cost of information goes up relative to stable periods. Agents have to watch more variables which takes more time and resources, thus is more costly. In a rapidly changing environment to be correct requires more frequent collection of information as opposed to a static environment, in which a once obtained piece of information should be of acceptable accuracy for a long period of time. Looking at it from another perspective, the cost of information has decreased over time (lets say over the last 50-80 years) which would tell us that, *ceteris paribus*, agents should update their expectations more often. These considerations tell us that Θ should be a function of the cost of information, and should be a positive function of it - if the costs of information rises, agents will update their expectations less frequently, and Θ will rise. If the cost of information falls, agents update their expectations more often, thus Θ will diminish. Although it may seem that this argument rests on the same basis as the one concerning the inflation rate we have to point out that the cost of information may change independently of inflation. For this reason we should include both variables as determinants of theta.

2.2.6. Length of pay period

Agents in the labor market have to form expectations at least one pay period ahead. That is, if they get paid on a monthly basis, they at least should have forecasts of what is going to happen during that month, so they can act accordingly (and can, for instance, avoid real losses in their assets). As their pay period shrinks, they may have to update their expectations more often. Hence the length of a pay period seems to have a positive effect on Θ.

Formalizing, we can write the following equation:

$$\Theta_t = f(\pi, e, LA_{real}, C_{Info})$$

where π denotes changes in the price level, e denotes changes in the real exchange rate, LA_{real} denotes losses in real assets, and C_{Info} represents costs of gathering information. We take this as a first approximation and do not rule out the possibility that other factors may also have an effect on Θ.

Chapter 3

3.1. Estimation

The first question to focus on is the expectation formation pattern, which must be estimated and tested. It is also necessary to test the significance of the time adjustment parameter, Θ.

Starting with the time adjustment parameter, we already know the variables that should have an effect on it. As we wrote before:

$$\Theta_t = f(\pi, e, LA_{real}, C_{Info})$$

We also established the theoretical effects of each variable on Θ. They are summarized as follows:

Variable	**Effect on Θ**
Inflation	negative
Changes in the exchange rate	negative
Losses in real assets	negative
the cost of information/formation of expectations	Positive

Generally we can write the expectations formation pattern:

$$P^e_{now} = P^e_{previous} + \lambda(P_{previous} - P^e_{previous})$$

When agents form their expectations according to this pattern, they will make mistakes. These mistakes will tell them to update their expectations more often. What is it that picks up the mistakes made in expectations in the equation written above? Clearly, the difference between the actual and the expected price is a mistake that appears in the estimation pattern, but it is a mistake associated with the previous period. There is also another mistake: the one agents make this period. This information appears in the error term, δ. By substituting the previous mistake with the error term of the previous period, we obtain the following:

$$P^e_{now} = P^e_{previous} + \lambda \delta_{previous}$$

From this it can be seen that the error terms are autocorrelated, which is usually a sign of poor model specification. In other words, the error term picks up a piece of information that should appear in a separate variable of the model, and by doing so the error terms of the model become autocorrelated.

A theoretical answer to this problem is to introduce the time varying time index. What we expect to see is that as agents update their expectations more often (or rather as the time index becomes the function of Θ) the size of the errors will fall rapidly; moreover the error terms will be zero on average, and the problem of autocorrelation would go away. In that case, adaptive and rational expectations really would not have different properties. Theoretically, it would only happen if the updates would become continuous. It does not mean however that in reality agents would always watch certain variables, only they would do so frequently enough so the time elapsing between two updates would be statistically insignificant. The proof that time variant adaptive expectations are rational at the limit is included in Appendix A.

From this discussion it follows that theta should be a function of the error term, δ. It does not invalidate our previous discussion of what variables affect Θ because merely we stated that the error term picks up all the information of the ongoing processes in those variables, thus we do not have to specify them explicitly[2]. Formalizing:

$$\Theta = f(\delta)$$

To write a more specific relationship between the two, however, requires the specification of the time indices. The essence of this problem is how to determine what "previous" means in relationship with "now". Simply writing t for "now" and t-1 for "previous" does not capture what we want to say, because the dimension of 1 keeps changing on us. For example if updates follow on an annual basis, the dimension of 1 is years. However if updates follow more often than that, the dimension becomes semi-year, quarter, month and so on. This is what makes the following specification incorrect:

$$\Theta_t = f(\delta_{t-1})$$

[2] Keane and Runkle (1995) have expressed the idea of the error term carrying all relevant information before albeit they were only concerned about rational expectations. In their article they stated that "the main result of our paper is that individual price forecasts are unbiased and rational, conditioned on the forecaster's own past errors" (p. 290)

Problems do not stop there, however. Looking at the actual expectation formation pattern, the same problem arises in more sophisticated form. Let us say we can write the expectation formation pattern the following way:

$$P^e_{t+\Theta} = P^e_t + \lambda(P_t - P^e_t)$$

Moving a period back we can also write how expectations were formed:

$$P^e_t = P^e_{t-\Theta^*} + \lambda(P_{t-\Theta^*} - P^e_{t-\Theta^*})$$

Yet it is strikingly clear that Θ and Θ^* are different. They would be the same if the cost of more frequent updating would exceed the losses associated with erring, or if the agent's expectations were absolutely correct (thus the frequency of our updates do not change). This latter is only a special case of the model and can change any time.

Thus Θ and the price expectation itself are also functions of time, that is they are time variant. For this reason the time index cannot include Θ as a constant, because theta itself is changing with time. How frequently it changes, on the other hand, is a function of Θ itself. When agents update their expectations more frequently, they must also decide more frequently how long to wait to form expectations again. Suppose an agent's current forecast period is 180 days. Thus the agent is forming expectations for 180 days beginning at time t. Also let us assume that the previous forecast period was 360 days. That tells us that we formed our expectation for time t 360 days ago. Because the agent made a mistake he needed to update his expectations and reduced the forecast period to 180 days. ($\Theta = 180$). When these 180 days passed by he will have to decide again how far ahead he wants to forecast, but this is going to be a decision based on another error made in the previous forecast. This "another error" on the other hand depends on the length of the period he was forecasting ahead, and thus depends on Θ. In more formal terms:

$$\Theta_t = f(\delta_{t-\Theta^*})$$

and also

$$\Theta_{t+\Theta_t} = f(\delta_t)$$

Attaching numbers to these equations gives:

$$\begin{aligned} 80 &= f(\delta_{t-360}) \\ \Theta_{t+180} &= f(\delta_t) \end{aligned}$$

This process would imbed Θ in the time index in a way, which would be impossible to track, for it is infinite in nature. Consider the following: If we wanted to write expectations formed for time t some time ago we would write:

$$P_t^e = P_{t-\Theta}^e + \lambda\left(P_{t-\Theta} - P_{t-\Theta}^e\right)$$

Yet we would have to specify what Θ is in the time index for the Θ at time t is a forecast period and is different from the one written in the time index. The one in the time index was decided Θ days ago. So we correct the equation:

$$P_t^e = P_{t-\Theta_{t-\Theta}}^e + \lambda\left(P_{t-\Theta_{t-\Theta}} - P_{t-\Theta_{t-\Theta}}^e\right)$$

It is clear that we now ran into the same problem again only this time in the time index of the time index (Θ). This process has an infinite nature as we pointed out thus we turn to another approach which might avoid this problem.

Rather than focusing on theta over the whole time period under investigation (i.e. thirty some odd years) we will consider only two periods at a time and will write Θ_{Old} and Θ_{New} correspondingly. Since consequent thetas might follow with different intervals (which are determined by the error of the previous forecast), the use of traditional time indices would not be appropriate. This new approach however avoids the difficulties of time embeddedness.

Let:

$$
\begin{aligned}
&(1) P_{t+\Theta_{New}} = P^e_{t+\Theta_{New}} + \delta_{t+\Theta_{New}} \\
&(2) P^e_{t+\Theta_{New}} = P^e_t + \lambda\left(P_t - P^e_t\right) \\
&(3) \Theta_{New} = \Theta_{Old}^{\left(1-\sqrt{\frac{\delta_t^2}{P_t^{e^2}}}\right)}
\end{aligned}
$$

The way we specified the function that determines the new theta is just one among the many possibilities. The main and most important property of equation 3 is that the new Θ is a function of the error term of equation 1. The way we specified it reflects the "real loss" aversion of economic agents because a mistake (δ) decreases the duration of the next forecast exponentially. We had to be careful to take positive and negative errors into account the same way. The rationale behind it is that there are agents who are losing when the expected price level lags behind the actual (consumers, or banks lending money at a fixed interest rate) and there are agents who lose when the actual price lags behind the expected (people borrowing from banks at a fixed rate). So we have to punish positive and negative errors the same way. To achieve this, we squared the error/expected price ratio ($\frac{\delta_t}{P_t^e}$) and took the positive square root of it. Thus the ratio loses sign and becomes always positive, hence always reducing Θ.

It is fairly clear that the specification of equation 3 is crucial to the results of the testing. The more punitive the function is, the faster agents get to daily updates thus the faster they reach rational expectations (here we assume that daily updates can be regarded as continuous). Also the less are the error terms statistically different from zero, as updates become more frequent. If we were to include some kind of a cost of information in the model it should appear in the third equation[3]. It is possible that the error term does not pick up the information costs properly; however, for now we will assume that it does.

The new Θ and the expectations for the time period $t + \Theta_{New}$ get determined simultaneously. This model is forward looking, whereas the traditional adaptive expectation model does not have to have a starting point.[4]

[3] One possible way to do it would be to write equation 3 the following way:

$$\Theta_{New} = \Theta_{Old}^{\left(1 - \sqrt{\frac{\delta_t^2}{P_t^{e2}}} + 0.05\right)}$$

Thus if the error (in percentages) is 5%, the update frequency will not change. In this sense a 5% percent error is acceptable to the agents, anything more than that will decrease the forecasting period while anything less will make it longer.

[4] When writing the model one can go back in time as far as one wishes, which eventually lead to the development of the Koyck transformation that despite of this difficulty enables one to estimate the coefficients of the model. Our model definitely has to have a starting point behind which we can not go, because we would not be able to obtain values of theta. The value of the theta preceding our starting theta would depend on how far we went back in time; thus the number of thetas that could precede the one we chose as a starting point, is infinite. See the discussion of time embeddedness on page 23, above.

As pointed out earlier in the paper, the only case when Θ does not change is when expectations do not differ from the actual price and the error term is zero. This case would then be rational expectations. Thus there is a relationship between adaptive and rational expectations. Rational expectations is an extreme form of adaptive expectations in this formulation. Theoretically there are two possible scenarios that have to be investigated.

1. If the starting value of Θ is such that it results in correct expectation, and the level of prices do not change ever after, it is easy to see that the model would indeed result in rational expectations.

2. If the level of inflation changes, but the value of Θ also changes (i.e. the length of the forecast) the mean of the error of the estimation becomes zero. Here we have to note that if Θ reaches a point where it becomes stable, even if it is not a zero error estimation length, it could be rational if the costs of erring equals the cost of further updating.

Two questions emerge. First, does the time variant update perform better than an annual update? Second, can we show that Θ picks up such values that it ignites on average zero error expectations, or else reaches a stable value, which would mean equal costs of erring and further updating ?

3.2. Setting the initial value of theta

Assume that there is a year when there is no inflation in a given country. Following the traditional approach at the end of that year, agents would decide that the next necessary update comes one year later. For simplicity, assume a 360 day year. Visually Figure 1. shows this model:

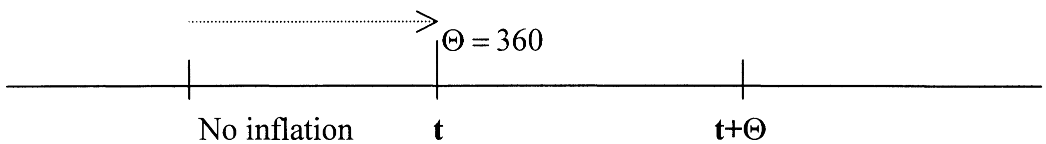


Figure 1.

From the starting point forward the length of the forecast period (Θ) depends on the ongoing economic processes, through the error term of equation 1. Thus given a dataset in which there is a year with no inflation, the model may begin with 360 as starting value for Θ.

Chapter 4

4.1. Test and findings

The formal models to be tested are as follows:

MODEL 1. (Traditional Adaptive Expectations)

$$P_t = P_t^e + \delta_t$$
$$P_t^e = P_{t-1}^e + \lambda\left(P_{t-1} - P_{t-1}^e\right)$$

MODEL 2. (Time Variant Adaptive Expectations)

$$P_{t+\Theta_{New}} = P_{t+\Theta_{New}}^e + \delta_{t+\Theta_{New}}$$
$$P_{t+\Theta_{New}}^e = P_t^e + \lambda\left(P_t - P_t^e\right)$$
$$\Theta_{New} = \Theta_{Old}^{\left(1-\sqrt{\frac{\delta_t^2}{P_t^{e2}}}\right)}$$

In both models we set the adjustment coefficient (λ) equal to 0.5, and held it constant. This way the only difference between the models lies in the periodicity of the updates.

The actual equation that is tested in both cases is as follows:

$$P_t = \alpha_0 + \alpha_1 P_t^e + \delta_t$$

Naturally, the formation of expectations is different in the two models, so the values of P_t^e are also different in the two models.

We seek the answer to two questions. First, is the time variant adaptive expectations more accurate than the traditional adaptive expectations? Second, is time variant adaptive expectations in fact rational? To answer the first question we will compare the results of the two estimated models. To answer the second we will conduct a rationality test following Maddala (1992) on both models hypothesizing that the traditional adaptive expectation model will fail it and the time variant adaptive expectations model will pass.

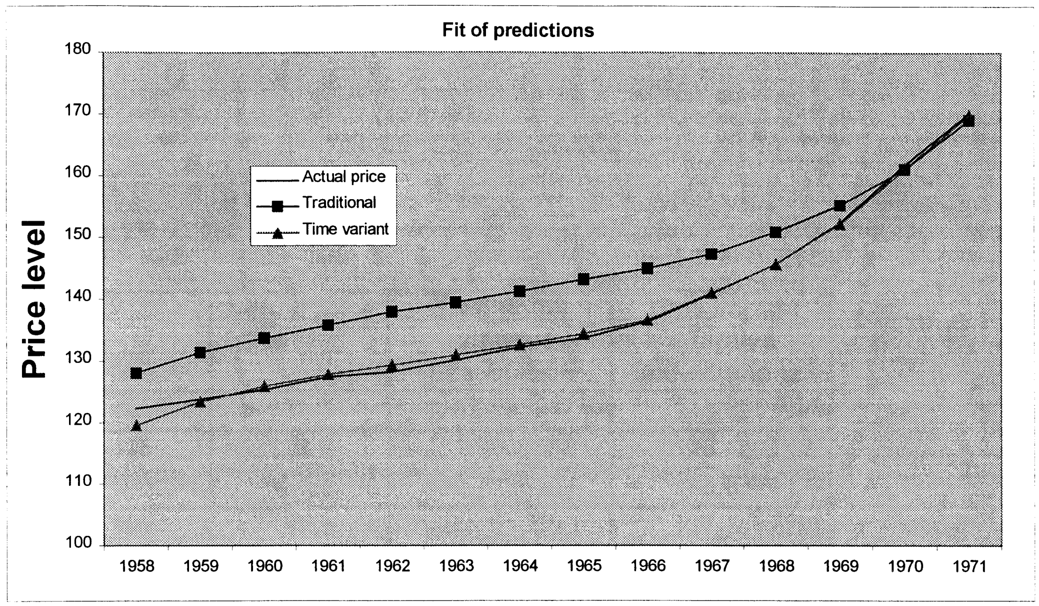
We will focus on the United States because monthly CPI data as it is readily available for the postwar period. According to the discussion above, we chose 1952 to 1953 as a starting point because during that period there was virtually no inflation in the US (the CPI was 1952=113.1, 1953=113.9, base 1947-49 average), set the initial value of Θ at 360 for 1953 and estimated our model.

The regression results can be summarized in the following way:

	Fit of the regression	**Constant**	**Coefficient of P_t^e**
Traditional adaptive expectations	$R^2 = 0.9946$	**2.711** (0.68)	**1.0777** (89.37)
Time variant adaptive expectations	$R^2 = 0.996$	**3.557** (1.44)	**0.9825** (54.65)

Table 1.

The regression results of the time variant adaptive expectations are not based on every observation. In order to be able to compare the two models directly we looked only at the predicted values of January each year. This is an annual update, but the time variant model has updates between the annual ones, so for comparison purposes we omitted those in the middle. Graph 1. depicts the fit of the predicted price levels of the two models:

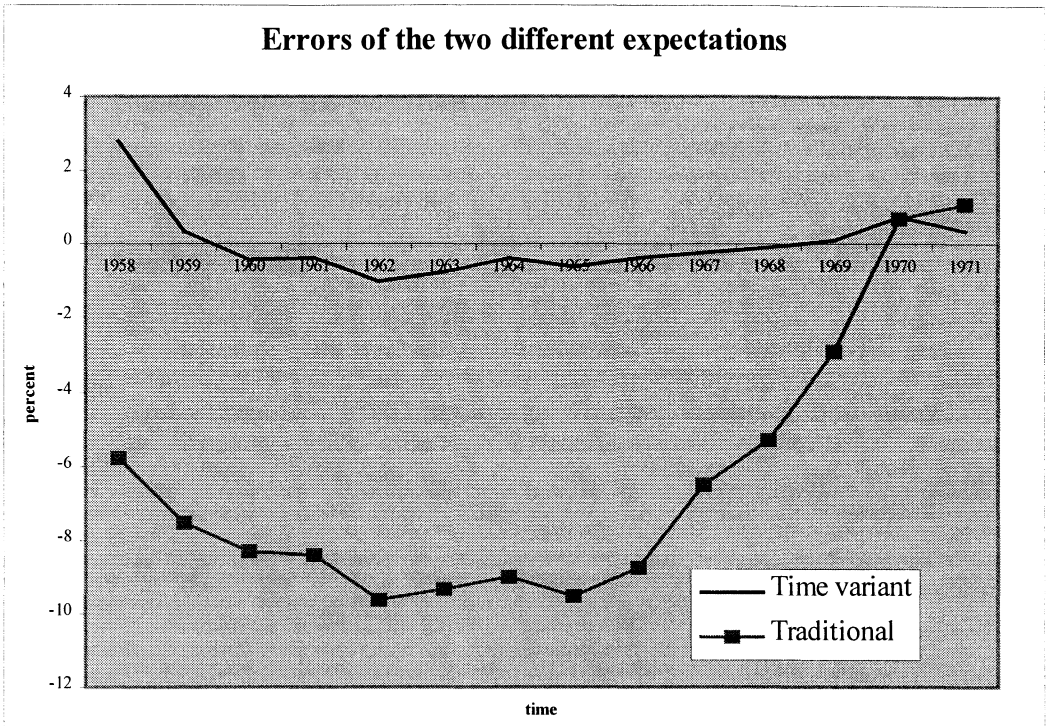


Graph 1.

As can be seen, the time variant adaptive expectations predict the actual price level more accurately then traditional adaptive expectations. This also shows in the data:

	Actual price	**Traditional adaptive expectation**	**Time variant adaptive expectations**
1958	**122.3**	128.0895217	119.5257108
1959	**123.8**	131.3059868	123.4501822
1960	**125.4**	133.7225457	125.8324127
1961	**127.4**	135.7930399	127.7957291
1962	**128.28**	137.9060555	129.3004405
1963	**130.12**	139.4367813	130.8998596
1964	**132.21**	141.1936912	132.5755163
1965	**133.68**	143.1984142	134.3202388
1966	**136.26**	144.9929355	136.6242668
1967	**140.8**	147.2805174	141.0209991
1968	**145.59**	150.8708427	145.6666827
1969	**152.34**	155.2472608	152.2417883
1970	**161.79**	161.0729382	161.0457649
1971	**170.17**	169.0782328	169.8404084

We could not go any further than 1971, as that is when the forecast period dropped below 15 days , which we could not obtain the sufficient data[5].



Graph 2.

Graph 2. Depicts the errors of the predictions of both models for the selected yearly data. Even though econometrically we can not test it (because of the lack of appropriate tests) it looks like that the time variant model's error terms

[5] The method we used to decide how long the forecast period should be was the following. Theta started at 360 days. From then on if it dropped by more than 15 days then we switched to an 11 month long forecast period until theta dropped below 315 days. Generally we used 15 days as the turning point, thus the turning points were: 345, 315, 285, 255, 225, 195, 165, 135, 105, 75, 45, 15. Since we had monthly CPI data, we could obtain observations only until theta dropped below 15 days.

are not autocorrelated as opposed to those of the traditional adaptive expectation prediction's. On the latter we have run an autocorrelation test. The Durbin-Watson statistic was 0.24, which, as expected, shows a strong positive autocorrelation between the error terms.

4.2. Rationality tests

To test rationality we ran the following regression on both models:

$$P_t - P_t^e = \alpha_0 + \alpha_1 P_{t-1} + \varepsilon_t$$

Following Maddala (1992, p. 434) if both the constant and the coefficient of the past price level are zero, rationality holds. We have to point out however, that this is a weak test of rationality[6].

[6]The weak test can only "not disprove" rationality whereas the strong test would prove rationality: thus if the null hypothesis is correct, we can only say that the assumption of rationality cannot be rejected.

Also when running this regression on the time variant adaptive expectation model, the time variant forecast length (Θ) appears in the time index, so the regression changes to:

$$P_t - P_t^e = \alpha_0 + \alpha_1 P_{t-\Theta_{Old}} + \varepsilon_t$$

The results of the regressions are summarized in table 2.

	Constant (α_0)	Coefficient of the previous price level (α_1)
Traditional adaptive expectations	**2.02** (0.525)	**0.077** (6.873)
Time variant adaptive expectations	**1.384** (1.74)	**0.0011** (-0.194)

Table 2.

Taking the results of the regression we tested the null joint hypothesis H_0: $\alpha_0=\alpha_1=0$ in both cases. The appropriate test is the special Wald test (using an F statistic) which tests the overall significance of the regression, hypothesizing that $R^2=0$ (or alternately $\alpha_0=\alpha_1=0$). The test statistic is calculated the following way:

$$F_{theoretical} = \frac{R^2/(K-1)}{(1-R^2)/(T-K)}$$

where K is the number of independent variables (including the constant) and T is the number of observations.

The results are summarized in table 3.

	$F_{empirical}$	$F_{theoretical}$	Decision
Traditional adaptive expectations	47.2445	3.23	**Reject H_0**
Time variant adaptive expectations	0.0376	3.92	**Accept H_0**

Table 3.

Thus we have shown that the traditional adaptive expectation model does not pass the weak rationality test whereas the time variant adaptive expectation model does. This, again, does not mean that the time variant expectations are rational but that we cannot reject that hypothesis.

As we said the above test is a weak rationality test. We also wanted to carry out a strong rationality test but only on the time variant adaptive expectations model, since the traditional failed to pass even the weak test. The strong test of rationality is an autocorrelation test of the error terms. As Maddala (1992) put it "the strong version says that the forecast error is uncorrelated with all the variables known to the forecaster" (p. 434). Because the error of the previous forecast carries all the information apart from the previous price level, this can be translated to no autocorrelation on the error terms.

The problem with using traditional econometric tests for testing autocorrelation (the Durbin-Watson test, for example) is that they are based on the assumption that the error terms follow with regular frequency - an assumption that is violated in the framework of time variant adaptive expectations. Therefore we will forgo this test as we cannot test the strong rationality hypothesis on the time variant adaptive expectations due to the absence of appropriate tests.

Chapter 5

5.1. Concluding remarks and areas for future research

The main findings of this study suggest that the time variant adaptive expectation formation process is rational in a weak statistical sense, and performs better than the traditional adaptive expectation model. We specified a model of a time variant adaptive expectation formation process, which has its roots in both economic theory and everyday life. The idea that economic agents update their expectations faster as inflation rises is strikingly simple. However it led us to a very important conclusion, time variant adaptive expectations are rational in a weak statistical sense. The fact that time varying adaptive expectations perform better than a simple yearly updating process is also reasonable. But it is really the changes in the properties of the error terms of the expectations that would be interesting to firmly establish. Naturally, we could not investigate every problem to the extent it might need to be investigated. There are numerous points of our proposed model that might be subject to future research. To list a few:

1. The idea that the error term and the length of the updating period are functions of each other is basic to our model. The precise relationship however is not clear.

Other specifications of the relationship may well be founded in economic theory and may lead to different results from those of ours.

2. The problem of testing updates faster for periods less than a month is also a challenge. Although exchange rates would carry the appropriate information for economic agents, the problem again would be the form of the specific functional relationship between the exchange rates and Θ.

3. Although we tested our hypothesis for the United States, this model needs to be tested for other countries, possibly with different inflationary histories from that of the United States. It would serve the purpose of finding out whether Θ would also imply zero mean error terms in highly inflationary environments.

4. In countries where inflation is not common, Θ might reach a stable level at which more frequent updating is just as costly as erring in the expectations. It would be of value to find out if this is the case. To find a cost function that would represent the cost of inflation, and moreover, that could be compared to the cost of updating would be of value. This cost could come from information cost, which is connected to the number of transactions on the stock market. The precise functional form would still have to be determined. Moreover there are countries, where the stock market was not (is not) as well developed as it is in the United

States. For these countries some other measure of the cost of information would have to be developed.

We have offered a possible information-cost interpretation, which also needs to be tested. Although it is a step forward, the actual level of the acceptable error is most likely a question of empirical data, which we do not possess. This data needs to be gathered and then put into the proposed model to see whether it gets us closer to reality or not.

5. There is a chance that traditional econometric tests for autocorrelation fail when facing time variant updating processes. This area of the study needs investigation, for until this question is clarified, the hypothesized relationship between rational and adaptive expectations (with regard to the strong rationality test) is unsupported.

We do not feel that we have said everything there is to say in connection with time variant adaptive expectations and its relationship to rational expectations. But we have exposed an idea here, an approach which might induce a debate on, and contributions to the field of expectation theory in the future.

References

Arnold, I. J. M. & den Hertog, R. G. J. (1995). European time-series relationships between inflation and inflation uncertainty: In search of threshold levels, De Economist, 143 (4), 495-519.

Ball, L. (1990). Why does high inflation raise inflation uncertainty?, National Bureau of Economic Research Working Paper: 3224, January

Begg, D. K. (1982). The rational expectations revolution in macroeconomics: theory and evidence, John Hopkins University Press

Blanchard, O.J. (1990). Why does money affect output? In M. Friedman and F. H. Hahn (Eds): Handbook of monetary economics, vol. II.

Bonham, C. & Cohen, R. (1995). Testing the rationality of price forecasts: Comment. American Economic Review, 85 (1), 284-289.

Bulkley, G. (1984). Does inflation uncertainty increase with the level of inflation? European Economic Review, 25, 213-221.

Cagan, P. (1956). The monetary dynamics of hyperinflation. In. Friedman, M. (ed.) Studies in the quantity theory of money, Chicago: University of Chicago Press

Christiano, L. J. (1987). Cagan's model of hyperinflation under rational expectations. International Economic Review, 28 (1), 33-49.

Davis, G. K., & Kanago, B. (1996). The missing link: Intra-country evidence on the relationship between high and uncertain inflation from high-inflation countries. Southern Economic Journal, 63 (1), 205-223.

Doran, H. E. (1988). Specification tests for the partial adjustment and adaptive expectations models. American Journal of Agricultural Economics, 70 (3), 713-723.

Dornbusch, R. (1976). Expectations and exchange rate dynamics. Journal of Political Economy, 84, 1161-1176.

Engsted, T. (1994). The classic European hyperinflations revisited: Testing the Cagan model using a cointegrated VAR approach. Economica, 61, 331-343.

Evans, M. (1989). Inflation rates and inflation uncertainty: Discovering the link and assessing the implications. New York University Salomon Brothers Center Working Paper; S-90-5.

Figlewski, S. & Wachtel, P. (1981). The formation of inflationary expectations. Review of Economics and Statistics, 63 (1), 1-10.

Frenkel, J. (1975). Inflation and the formation of expectations. Journal of Monetary Economics, 1, 403-421.

Glazer, R., Stechel, J. H. & Winer, R. S. (1990). Judgmental forecasts in a competitive environment: rational vs. adaptive expectations. International Journal of Forecasting, 6 (2), 149-162.

Golob, J. E. (1993). Inflation, inflation uncertainty, and relative price variability: A survey. Federal Bank of Kansas City Research Working Paper: 93-15.

Henry, S. G. B. (1974). The Koyck transformation and adaptive expectations: A note. Economica, 41 (161), 79-80.

Holland, A. S. (1993). Uncertain effects of money and the link between the inflation rate and inflation uncertainty. Economic Inquiry, 31 (1), 39-51.

Holland, A. S. (1995). Inflation and uncertainty: Tests for temporal ordering. Journal of Money, Credit, and Banking, 27 (3), 827-837.

Just, R. E. (1977). Estimation of an adaptive expectations model. International Economic Review, 18 (3), 629-646.

Keane, M. P. & Runkle, D. E. (1995). Testing the rationality of price forecasts: Reply. American Economic Review, 85 (1), 290.

Keynes, J. M. (1936). The general theory of employment interest and money. New York: Harcourt, Brace & World, Inc.

Lawson, T. (1980). Adaptive expectations and uncertainty. Review of Economic Studies, 47, 305-320.

Lucas, R. E. Jr. (1972). Expectations and the neutrality of money. Journal of Economic Theory, 4, 103-124.

Lucas, R. E. Jr. (1996). Nobel lecture: monetary neutrality. Journal of Political Economy, 104 (4), 661-682.

Maddala, G. S. (1992). Introduction to econometrics, (2nd ed.). New York: Macmillan Publishing Company.

Madsen, J. B. (1996). Formation of inflation expectations: From the simple to the rational expectations hypothesis. Applied Economics, 28, 1331-1337.

Mishkin, F. S. (1983). A rational expectations approach to macroeconomics, testing policy ineffectiveness and efficient markets models. Chicago: University of Chicago Press.

Mussa, M. (1975). Adaptive and regressive expectations in a rational model of the inflationary process. Journal of Monetary Economics, 1 (4), 423-442.

Muth, J. F. (1961). Rational expectations and the theory of price movements. Econometrica, 29, 315-335.

Naish, H. F. (1993). The near optimality of adaptive expectations. Journal of Economic Behavior and Organization, 20 (1), 3-22.

Pami, D. & Subhash, R.C. (1992). ARIMA models of the price level: An assessment of the multilevel adaptive learning process in the USA. Journal of Forecasting, 11 (6), 507-516.

Park, D. (1995). Inflation and uncertainty: The case of Korea. International Economic Journal, 9 (1), 39-52.

Sargent, T. & Wallace N. (1975). Rational expectations and the theory of economic policy. Minneapolis: Research Department, Federal Reserve Bank of Minneapolis.

Satchell, S. & Timmermann, A. (1995). On the optimality of adaptive expectations: Muth revisited. International Journal of Forecasting, 11 (3), 407-416.

Sheffrin, S. M. (1996). Rational expectations, Cambridge: Cambridge University Press.

Struth, F. K. (1984). Modeling expectations formation with parameter-adaptive filters: An empirical application to the Livingston forecasts. Oxford Bulletin of Economics and Statistics, 46 (3), 211-239.

APPENDIX A

Proof of rational and time variant adaptive expectations being the same

Rational and time variant adaptive expectations are the same, if the time variable theta converges to zero.

Proof: Rational expectations are written the following way:

$$P_t = P_t^e + \varepsilon_t$$

Where the error term has a zero mean and a constant variance, moreover the error terms are not autocorrelated. From MODEL 2. with some substitution we can write the following:

$$\begin{aligned}
P^e_{t+\Theta_{New}} &= P_t^e + \lambda\left(P_t^e + \delta_t - P_t^e\right) \\
P_{t+\Theta_{New}} &= P_t^e + \lambda\left(P_t^e + \delta_t - P_t^e\right) + \delta_{t+\Theta_{New}} \\
P_{t+\Theta_{New}} &= P_t^e + \lambda\delta_t + \delta_{t+\Theta_{New}}
\end{aligned}$$

Applying the expectation operator to both sides of the last equation we obtain the following:

$$\underset{\lim \Theta \to 0}{E}\left(P_{t+\Theta_{New}}\right) = E(P_t) = P_t^e + \lambda E(\delta_t) + E(\delta_t)$$

According to the statistical test we have conducted the mean of delta (thus also its expected value) is zero. Hence we can write the following:

$$\underset{\lim \Theta \to 0}{E}\left(P_{t+\Theta_{New}}\right) = E(P_t) = P_t^e$$

If we apply the expectation operator to the rational expectation equation we obtain the same result, thus we can say that rational and time variant adaptive expectations are the same if the update becomes continuous.

♦

We still have to address the case when the update is not continuous, or when the cost of more frequent updating is equal to the cost of erring thus no further decrease will come about in the length of forecasting (theta). In this case the zero mean property of the error terms disappears in both the rational and the time variant adaptive expectation formation. In other words if information becomes too costly the rational expectations hypothesis has the same defect as the time variant adaptive, i.e. not all relevant information will be gathered and errors will be made in the expectation process. The issue of autocorrelated error terms needs more careful considerations. Rational expectations is *defined* in such a way that the error terms are not autocorrelated. However when writing the model of time variant adaptive expectations (see in proof) we write an equation in which two error terms appear in one expectation which is clear autocorrelation in standard econometrics. Yet we assert that it is not necessarily the case with time variant adaptive expectation formation. The reason for this is the different nature of the error terms over each time period. Since the time periods for which the expectations are formed vary over time, the error terms associated with those expectations are also associated with different time periods, thus differ in nature.

For this consideration we cannot use traditional econometric test to find out if there is autocorrelation between the error terms because the traditional test are designed for error terms that follow each other with a regular frequency.

Writing the case of costly information we get:

$$E(P_t) = P_t^e + E(\varepsilon_t)$$

for rational expectations, and

$$E\left(P_{t+\Theta_{New}}\right) = P_t^e + \lambda E(\delta_t) + E\left(\delta_{t+\Theta_{New}}\right)$$

for the time variant adaptive expectations. To be able to say anything about these two being the same we would have to be able to compare the expected values of the error term which we are quite unable to do. However we can not rule out the possibility that even in this case the two types of expectation formation would be the same.

APPENDIX B

Formation of expectations according to
MODEL 1 and MODEL 2

Traditional adaptive expectations

$$P_t^e = P_{t-1}^e + \lambda\left(P_{t-1} - P_{t-1}^e\right)$$
$$\lambda = 0.5$$

	Pa	Pe t	delta	Pe t+1
1952	113.10			
1953	113.90	113.1	0.80	113.5
1954	115.20	113.5	1.70	114.35
1955	114.3	114.35	-0.05	114.325
1956	114.6	114.325	0.28	114.4625
1957	118.2	114.4625	3.74	116.3313
1958	122.3	116.3313	5.97	119.3156
1959	123.8	119.3156	4.48	121.5578
1960	125.4	121.5578	3.84	123.4789
1961	127.4	123.4789	3.92	125.4395
1962	128.28	125.4395	2.84	126.8597
1963	130.12	126.8597	3.26	128.4899
1964	132.21	128.4899	3.72	130.3499
1965	133.68	130.3499	3.33	132.015
1966	136.26	132.015	4.25	134.1375
1967	140.8	134.1375	6.66	137.4687
1968	145.59	137.4687	8.12	141.5294
1969	152.34	141.5294	10.81	146.9347
1970	161.79	146.9347	14.86	154.3623
1971	170.17	154.3623	15.81	162.2662
1972	175.93	162.2662	13.66	169.0981
1973	182.34	169.0981	13.24	175.719
1974	199.36	175.719	23.64	187.5395
1975	222.77	187.5395	35.23	205.1548
1976	237.88	205.1548	32.73	221.5174
1977	250.17	221.5174	28.65	235.8437
1978	267.21	235.8437	31.37	251.5268
1979	292.3	251.5268	40.77	271.9134
1980	333.04	271.9134	61.13	302.4767
1981	371.94	302.4767	69.46	337.2084
1982	403.24	337.2084	66.03	370.2242
1983	418.66	370.2242	48.44	394.4421
1984	435.93	394.4421	41.49	415.186
1985	451.43	415.186	36.24	433.308
1986	469.05	433.308	35.74	451.179
1987	472.93	451.179	21.75	462.0545
1988	495.13	462.0545	33.08	478.5923
1989	522.41	478.5923	43.82	500.5011
1990	544.96	500.5011	44.46	522.7306
1991	575.8	522.7306	53.07	549.2653

Traditional adaptive expectations

$$P_t^e = P_{t-1}^e + \lambda\left(P_{t-1} - P_{t-1}^e\right)$$
$$\lambda = 0.5$$

	Pa	Pe t	delta	Pe t+1
1992	590.53	549.2653	41.26	569.8976
1993	609.86	569.8976	39.96	589.8788
1994	625.35	589.8788	35.47	607.6144
1995	642.67	607.6144	35.06	625.1422
1996	660.55	625.1422	35.41	642.8461
1997	680.67	642.8461	37.82	661.7581

Time variant adaptive expectations

$$P^e_{t+\Theta_{New}} = P^e_t + \lambda\left(P_t - P^e_t\right)$$

$$\Theta_{New} = \Theta_{Old}^{\left(1-\sqrt{\frac{\delta_t^2}{P_t^{e\,2}}}\right)}$$

$$\lambda = 0.5$$

		Pa	Pe t	Pe t+1	Pa t+1	delta	Theta
							360
1953	1	113.9	113.1	113.5	115.2	0.8	345.3192
1954	1	115.2	113.5	114.35	114.3	1.7	316.3759
1954	12	114.3	114.35	114.325	115	-0.05	315.5805
1955	11	115	114.325	114.6625	117.1	0.675	305.0386
1956	9	117.1	114.6625	115.8813	120.2	2.4375	270.111
1957	6	120.2	115.8813	118.0406	122.3	4.31875	219.241
1958	1	122.3	118.0406	120.1703	123.9	4.259375	180.4895
1958	7	123.9	120.1703	122.0352	123.7	3.729688	153.6099
1958	12	123.7	122.0352	122.8676	124	1.664844	143.4139
1959	5	124	122.8676	123.4338	125.5	1.132422	136.9982
1959	10	125.5	123.4338	124.4669	125.6	2.066211	126.1675
1960	2	125.6	124.4669	125.0334	126.2	1.133105	120.7317
1960	6	126.2	125.0334	125.6167	127.3	1.166553	115.4511
1960	10	127.3	125.6167	126.4584	127.5	1.683276	108.3332
1961	2	127.5	126.4584	126.9792	127.4	1.041638	104.2321
1961	5	127.4	126.9792	127.1896	128.00	0.420819	102.6393
1961	8	128.00	127.1896	127.5948	128.40	0.81041	99.65477
1961	11	128.40	127.5948	127.9974	128.65	0.805205	96.80243
1962	2	128.65	127.9974	128.3237	129.14	0.652602	94.57168
1962	5	129.14	128.3237	128.7318	129.50	0.816301	91.87403
1962	8	129.50	128.7318	129.1159	130.12	0.768151	89.42898
1962	11	130.12	129.1159	129.618	130.24	1.004075	86.35799
1963	2	130.24	129.618	129.929	130.36	0.622038	84.52987
1963	5	130.36	129.929	130.1445	131.47	0.431019	83.29475
1963	8	131.47	130.1445	130.8072	131.84	1.325509	79.62626
1963	11	131.84	130.8072	131.3236	132.08	1.032755	76.92138
1964	2	132.08	131.3236	131.7018	132.33	0.756377	75.02121
1964	5	132.33	131.7018	132.0159	132.94	0.628189	73.49197
1964	7	132.94	132.0159	132.478	133.06	0.924094	71.31427
1964	9	133.06	132.478	132.769	133.43	0.582047	69.98975
1964	11	133.43	132.769	133.0995	133.68	0.661024	68.52491
1965	1	133.68	133.0995	133.3897	133.80	0.580512	67.2731
1965	3	133.80	133.3897	133.5949	134.54	0.410256	66.40789
1965	5	134.54	133.5949	134.0674	135.27	0.945128	64.46563
1965	7	135.27	134.0674	134.6687	135.57	1.202564	62.10104
1965	9	135.57	134.6687	135.1194	135.77	0.901282	60.40855
1965	11	135.77	135.1194	135.4447	136.26	0.650641	59.22729
1966	1	136.26	135.4447	135.8523	137.48	0.81532	57.78991
1966	3	137.48	135.8523	136.6662	138.22	1.62766	55.0482

Time variant adaptive expectations

		Pa	Pe t	Pe t+1	Pa t+1	delta	Theta
1966	5	138.22	136.6662	137.4431	139.08	1.55383	52.59587
1966	7	139.08	137.4431	138.2615	140.06	1.636915	50.17132
1966	9	140.06	138.2615	139.1608	140.68	1.798458	47.68004
1966	11	140.68	139.1608	139.9204	140.80	1.519229	45.7103
1967	1	140.80	139.9204	140.3602	140.92	0.879614	44.62501
	2	140.92	140.3602	140.6405	141.17	0.560648	43.95308
	3	141.17	140.6405	140.9034	141.53	0.525831	43.33577
	4	141.53	140.9034	141.219	141.90	0.631176	42.61027
	5	141.90	141.219	141.5609	142.39	0.683848	41.84305
	6	142.39	141.5609	141.9774	143.01	0.832937	40.93378
	7	143.01	141.9774	142.4925	143.50	1.030235	39.84594
	8	143.50	142.4925	142.9956	143.74	1.006131	38.82253
	9	143.74	142.9956	143.3699	144.24	0.748572	38.08597
	10	144.24	143.3699	143.8025	144.60	0.865299	37.25842
	11	144.60	143.8025	144.203	145.09	0.80091	36.51519
	12	145.09	144.203	144.6487	145.59	0.891468	35.71201
1968	1	145.59	144.6487	145.1171	146.08	0.936748	34.8946
	2	146.08	145.1171	145.5968	146.69	0.959387	34.08465
	3	146.69	145.5968	146.1435	147.18	1.09346	33.19319
	4	147.18	146.1435	146.6624	147.67	1.037744	32.37787
	5	147.67	146.6624	147.1673	148.41	1.009885	31.61179
	6	148.41	147.1673	147.7881	149.15	1.241463	30.70412
	7	149.15	147.7881	148.4667	149.64	1.357251	29.75354
	8	149.64	148.4667	149.0515	150.00	1.169639	28.96876
	9	150.00	149.0515	149.528	150.86	0.95308	28.35188
	10	150.86	149.528	150.196	151.48	1.335813	27.51726
	11	151.48	150.196	150.8368	151.85	1.281673	26.7498
	12	151.85	150.8368	151.3413	152.34	1.009097	26.16808
1969	1	152.34	151.3413	151.8391	152.95	0.995562	25.61211
	2	152.95	151.8391	152.3949	154.18	1.111548	25.01121
	3	154.18	152.3949	153.2865	155.16	1.783307	24.08651
	4	155.16	153.2865	154.2234	155.65	1.87368	23.16776
	5	155.65	154.2234	154.9373	156.63	1.427854	22.50336
	6	156.63	154.9373	155.7853	157.37	1.695954	21.74932
	7	157.37	155.7853	156.5775	157.98	1.584497	21.07863
	8	157.98	156.5775	157.2805	158.72	1.406015	20.50949
	9	158.72	157.2805	158.0003	159.33	1.439528	19.95019
	10	159.33	158.0003	158.6671	160.19	1.333531	19.4525
	11	160.19	158.6671	159.4301	161.18	1.526039	18.90506
	12	161.18	159.4301	160.3026	161.79	1.745046	18.3065
1970	1	161.79	160.3026	161.0458	162.65	1.48629	17.81963
	2	162.65	161.0458	161.847	163.51	1.602418	17.31618
	3	163.51	161.847	162.6772	164.49	1.660483	16.81691
	4	164.49	162.6772	163.5833	165.23	1.812268	16.29638
	5	165.23	163.5833	164.4047	165.96	1.642654	15.846
	6	165.96	164.4047	165.1836	166.58	1.557847	15.43652
	7	166.58	165.1836	165.8799	166.94	1.39269	15.08442
	8	166.94	165.8799	166.4122	167.68	1.064605	14.82398

Time variant adaptive expectations

		Pa	Pe t	Pe t+1	Pa t+1	delta	Theta
	9	167.68	166.4122	167.0467	168.60	1.268823	14.52235
	10	168.60	167.0467	167.822	169.12	1.550701	14.16608
	11	169.12	167.822	168.4715	170.04	1.298945	13.87838
	12	170.04	168.4715	169.2544	170.17	1.565762	13.54323
1971	1	170.17	169.2544	169.7113	170.43	0.91378	13.35402
	2	170.43	169.7113	170.0706	170.95	0.718687	13.20825
	3	170.95	170.0706	170.5121	171.61	0.882938	13.03246
	4	171.61	170.5121	171.06	172.39	1.095962	12.81916
	5	172.39	171.06	171.7267	173.44	1.333372	12.56678
	6	173.44	171.7267	172.5837	173.83	1.713875	12.25331
	7	173.83	172.5837	173.2085	174.49	1.249633	12.033
	8	174.49	173.2085	173.8481	174.75	1.279309	11.81393
	9	174.75	173.8481	174.2989	175.01	0.901452	11.66363
	10	175.01	174.2989	174.6551	175.01	0.712523	11.54709
	11	175.01	174.6551	174.8333	175.67	0.356262	11.48961
	12	175.67	174.8333	175.2496	175.93	0.832624	11.35679
1972	1	175.93	175.2496	175.5886	176.71	0.678109	11.25052
	2	176.71	175.5886	176.1508	176.97	1.124446	11.07748
	3	176.97	176.1508	176.5629	177.50	0.82402	10.95355
	4	177.50	176.5629	177.0307	178.02	0.935604	10.8155
	5	178.02	177.0307	177.5264	178.41	0.991396	10.67224
	6	178.41	177.5264	177.9706	179.20	0.888394	10.54654
	7	179.20	177.9706	178.5854	179.46	1.229588	10.37627
	8	179.46	178.5854	179.0236	180.12	0.876591	10.2578
	9	180.12	179.0236	179.57	180.77	1.092789	10.11306
	10	180.77	179.57	180.1705	181.16	1.200887	9.957773
	11	181.16	180.1705	180.6671	181.69	0.993139	9.832413
	12	181.69	180.6671	181.1771	182.34	1.020164	9.706327
1973	1	182.34	181.1771	181.7594	183.52	1.164575	9.565558
	2	183.52	181.7594	182.6396	185.22	1.760375	9.358623
	3	185.22	182.6396	183.9305	186.53	2.581869	9.067394
	4	186.53	183.9305	185.2305	187.71	2.59992	8.789176
	5	187.71	185.2305	186.4695	188.62	2.478047	8.537286
	6	188.62	186.4695	187.5472	189.41	2.155314	8.328277
	7	189.41	187.5472	188.4787	192.81	1.863048	8.154749
	8	192.81	188.4787	190.6462	193.47	4.334887	7.770496
	9	193.47	190.6462	192.0571	195.04	2.821936	7.538212
	10	195.04	192.0571	193.548	196.48	2.981751	7.305475
	11	196.48	193.548	195.0134	197.66	2.93076	7.088769
	12	197.66	195.0134	196.3351	199.36	2.643467	6.903051
1974	1	199.36	196.3351	197.8468	201.98	3.023415	6.700705
	2	201.98	197.8468	199.9117	204.33	4.129679	6.439865
	3	204.33	199.9117	202.1222	205.51	4.421014	6.180002
	4	205.51	202.1222	203.8165	207.83	3.388594	5.99415
	5	207.83	203.8165	205.8256	209.99	4.018328	5.786213
	6	209.99	205.8256	207.9092	211.65	4.167193	5.584172
	7	211.65	207.9092	209.781	214.31	3.743619	5.413885
	8	214.31	209.781	212.045	216.80	4.527845	5.22008

Time variant adaptive expectations

		Pa	Pe t	Pe t+1	Pa t+1	delta	Theta
	9	216.80	212.045	214.4219	218.62	4.753956	5.030222
	10	218.62	214.4219	216.5234	220.28	4.203002	4.873433
	11	220.28	216.5234	218.4042	221.78	3.761524	4.741172
	12	221.78	218.4042	220.0916	222.77	3.374782	4.628517
1975	1	222.77	220.0916	221.4333	224.44	2.683404	4.542853
	2	224.44	221.4333	222.9341	225.27	3.001724	4.450594
	3	225.27	222.9341	224.0996	226.43	2.330873	4.381658
	4	226.43	224.0996	225.2633	227.42	2.327452	4.314938
	5	227.42	225.2633	226.3432	229.25	2.15974	4.254874
	6	229.25	226.3432	227.7961	231.74	2.905894	4.176503
	7	231.74	227.7961	229.7676	232.40	3.942981	4.074431
	8	232.40	229.7676	231.0854	233.57	2.635499	4.009307
	9	233.57	231.0854	232.3253	234.89	2.479765	3.950007
	10	234.89	232.3253	233.6092	236.39	2.5679	3.890484
	11	236.39	233.6092	234.9982	237.38	2.77797	3.828138
	12	237.38	234.9982	236.1907	237.88	2.384998	3.776338
1976	1	237.88	236.1907	237.0359	238.55	1.690506	3.740593
	2	238.55	237.0359	237.7906	239.04	1.509262	3.709304
	3	239.04	237.7906	238.4169	240.04	1.252638	3.683779
	4	240.04	238.4169	239.2281	241.53	1.622332	3.651238
	5	241.53	239.2281	240.3806	242.86	2.305186	3.605956
	6	242.86	240.3806	241.621	244.19	2.480611	3.558544
	7	244.19	241.621	242.9051	245.35	2.568323	3.510852
	8	245.35	242.9051	244.1282	246.35	2.446177	3.466729
	9	246.35	244.1282	245.2378	247.34	2.219102	3.427774
	10	247.34	245.2378	246.2905	248.01	2.105564	3.391709
	11	248.01	246.2905	247.1489	248.84	1.716791	3.362957
	12	248.84	247.1489	247.9931	250.17	1.688407	3.335208
1977	1	250.17	247.9931	249.0792	252.82	2.172221	3.300204
	2	252.82	249.0792	250.9503	254.32	3.742146	3.241532
	3	254.32	250.9503	252.6329	256.31	3.365093	3.190814
	4	256.31	252.6329	254.4702	257.80	3.674573	3.137416
	5	257.80	254.4702	256.1358	259.46	3.331307	3.090804
	6	259.46	256.1358	257.7986	260.62	3.325676	3.045849
	7	260.62	257.7986	259.2111	261.62	2.824853	3.008902
	8	261.62	259.2111	260.4153	262.62	2.40844	2.978262
	9	262.62	260.4153	261.5154	263.28	2.200233	2.950927
	10	263.28	261.5154	262.3975	264.61	1.764126	2.929465
	11	264.61	262.3975	263.5025	265.60	2.210081	2.903064
	12	265.60	263.5025	264.553	267.21	2.101054	2.878499
1978	1	267.21	264.553	265.8839	269.06	2.66164	2.848042
	2	269.06	265.8839	267.4699	270.90	3.172093	2.812701
	3	270.90	267.4699	269.1836	273.43	3.427319	2.775674
	4	273.43	269.1836	271.3063	275.96	4.245409	2.731341
	5	275.96	271.3063	273.6335	278.95	4.654454	2.684662
	6	278.95	273.6335	276.2931	280.79	5.319295	2.633615
	7	280.79	276.2931	278.5436	282.41	4.50092	2.592396
	8	282.41	278.5436	280.4744	284.48	3.861574	2.558385

Time variant adaptive expectations

		Pa	Pe t	Pe t+1	Pa t+1	delta	Theta
	9	284.48	280.4744	282.4755	286.78	4.002218	2.52432
	10	286.78	282.4755	284.6268	288.39	4.3027	2.488966
	11	288.39	284.6268	286.5081	289.77	3.762463	2.459144
	12	289.77	286.5081	288.1392	292.30	3.262186	2.434078
1979	1	292.30	288.1392	290.2206	295.75	4.162843	2.402996
	2	295.75	290.2206	292.9875	298.52	5.533807	2.363159
	3	298.52	292.9875	295.7519	301.97	5.528813	2.325118
	4	301.97	295.7519	298.8603	305.65	6.216792	2.284243
	5	305.65	298.8603	302.2558	309.33	6.790941	2.241768
	6	309.33	302.2558	305.7948	312.56	7.078016	2.199787
	7	312.56	305.7948	309.1754	315.78	6.761235	2.161775
	8	315.78	309.1754	312.4768	319.00	6.602844	2.126475
	9	319.00	312.4768	315.7386	321.76	6.523649	2.093243
	10	321.76	315.7386	318.7505	324.75	6.023733	2.063949
	11	324.75	318.7505	321.7525	328.21	6.003935	2.03597
	12	328.21	321.7525	324.9797	333.04	6.454353	2.007139
1980	1	333.04	324.9797	329.0099	337.64	8.060517	1.972752
	2	337.64	329.0099	333.3266	342.48	8.63344	1.937892
	3	342.48	333.3266	337.9017	346.16	9.15006	1.903015
	4	346.16	337.9017	342.0305	349.61	8.257575	1.873325
	5	349.61	342.0305	345.821	353.52	7.581174	1.847442
	6	353.52	345.821	349.6727	353.75	7.703291	1.822354
	7	353.75	349.6727	351.7136	356.06	4.081805	1.809632
	8	356.06	351.7136	353.8848	359.28	4.342493	1.796429
	9	359.28	353.8848	356.5816	362.50	5.393473	1.780461
	10	362.50	356.5816	359.5411	365.72	5.918964	1.763494
	11	365.72	359.5411	362.6319	368.94	6.181709	1.746377
	12	368.94	362.6319	365.7884	371.94	6.313081	1.729508
1981	1	371.94	365.7884	368.8627	375.85	6.148609	1.713654
	2	375.85	368.8627	372.3563	378.61	6.987008	1.696259
	3	378.61	372.3563	375.484	380.91	6.255413	1.681268
	4	380.91	375.484	378.1986	384.14	5.429297	1.668685
	5	384.14	378.1986	381.167	387.36	5.936876	1.655326
	6	387.36	381.167	384.2624	391.73	6.190665	1.641831
	7	391.73	384.2624	387.9966	394.72	7.468355	1.626086
	8	394.72	387.9966	391.3597	398.87	6.726245	1.612439
	9	398.87	391.3597	395.1127	399.56	7.505986	1.597732
	10	399.56	395.1127	397.3344	400.71	4.44347	1.589334
	11	400.71	397.3344	399.0207	401.86	3.37253	1.583096
	12	401.86	399.0207	400.4392	403.24	2.837061	1.577934
1982	1	403.24	400.4392	401.8389	404.62	2.799485	1.57291
	2	404.62	401.8389	403.2293	404.16	2.780697	1.567988
	3	404.16	403.2293	403.6943	399.10	0.93003	1.566362
	4	399.10	403.6943	401.3951	409.91	-4.598484	1.558376
	5	409.91	401.3951	405.6542	414.98	8.518234	1.543773
	6	414.98	405.6542	410.3155	417.28	9.322617	1.528444
	7	417.28	410.3155	413.7969	417.97	6.962899	1.517479
	8	417.97	413.7969	415.8829	418.66	4.171927	1.511112

Time variant adaptive expectations

		Pa	Pe t	Pe t+1	Pa t+1	delta	Theta
	9	418.66	415.8829	417.2711	420.07	2.776441	1.506953
	10	420.07	417.2711	418.67	419.36	2.797848	1.502815
	11	419.36	418.67	419.0171	417.60	0.69411	1.5018
	12	417.60	419.0171	418.3096	418.66	-1.414979	1.499739
1983	1	418.66	418.3096	418.4845	418.66	0.349731	1.499231
	2	418.66	418.4845	418.5719	419.01	0.174865	1.498978
	3	419.01	418.5719	418.7918	421.83	0.43984	1.49834
	4	421.83	418.7918	420.3114	424.30	3.039175	1.49395
	5	424.30	420.3114	422.3046	425.71	3.986435	1.488273
	6	425.71	422.3046	424.0061	427.47	3.402845	1.483512
	7	427.47	424.0061	425.7378	428.88	3.463457	1.47874
	8	428.88	425.7378	427.3085	430.99	3.141356	1.474478
	9	430.99	427.3085	429.151	432.05	3.685119	1.469549
	10	432.05	429.151	430.6009	432.76	2.89978	1.465731
	11	432.76	430.6009	431.6783	433.46	2.154704	1.46293
	12	433.46	431.6783	432.5693	435.93	1.782166	1.460634
1984	1	435.93	432.5693	434.2483	437.69	3.357931	1.456344
	2	437.69	434.2483	435.9688	438.75	3.441	1.452012
	3	438.75	435.9688	437.3577	440.86	2.77772	1.448566
	4	440.86	437.3577	439.1093	442.27	3.503301	1.444273
	5	442.27	439.1093	440.69	443.68	3.161278	1.440455
	6	443.68	440.69	442.1851	445.09	2.990266	1.436893
	7	445.09	442.1851	443.6375	446.85	2.904761	1.433475
	8	446.85	443.6375	445.2447	448.97	3.214415	1.42974
	9	448.97	445.2447	447.1055	450.38	3.721648	1.425474
	10	450.38	447.1055	448.7407	450.38	3.270452	1.421783
	11	450.38	448.7407	449.5583	450.38	1.635226	1.41996
	12	450.38	449.5583	449.9672	451.43	0.817613	1.419055
1985	1	451.43	449.9672	450.7002	453.20	1.466027	1.417438
	2	453.20	450.7002	451.9477	455.31	2.495048	1.414703
	3	455.31	451.9477	453.6287	457.07	3.361965	1.411057
	4	457.07	453.6287	455.3502	458.83	3.443017	1.407374
	5	458.83	455.3502	457.092	460.24	3.483543	1.4037
	6	460.24	457.092	458.6677	460.95	3.151399	1.400422
	7	460.95	458.6677	459.8079	462.01	2.280513	1.398079
	8	462.01	459.8079	460.9066	463.42	2.197477	1.395841
	9	463.42	460.9066	462.1608	464.82	2.508366	1.39331
	10	464.82	462.1608	463.4927	466.23	2.66381	1.390649
	11	466.23	463.4927	464.8635	467.64	2.741533	1.387939
	12	467.64	464.8635	466.2537	469.05	2.780394	1.385221
1986	1	469.05	466.2537	467.6536	467.64	2.799824	1.382513
	2	467.64	467.6536	467.6488	465.53	-0.009715	1.382503
	3	465.53	467.6488	466.5891	464.47	-2.119299	1.380476
	4	464.47	466.5891	465.5307	465.88	-2.11687	1.378458
	5	465.88	465.5307	465.7063	468.35	0.351192	1.378124
	6	468.35	465.7063	467.0275	468.35	2.642444	1.375618
	7	468.35	467.0275	467.6881	469.05	1.321222	1.374378
	8	469.05	467.6881	468.3708	471.52	1.365425	1.373102

Time variant adaptive expectations

		Pa	Pe t	Pe t+1	Pa t+1	delta	Theta
	9	471.52	468.3708	469.9456	471.87	3.14956	1.370178
	10	471.87	469.9456	470.9092	472.23	1.927187	1.368409
	11	472.23	470.9092	471.5672	472.93	1.316	1.36721
	12	472.93	471.5672	472.2486	472.93	1.362814	1.365975
1987	1	472.93	472.2486	472.5893	477.51	0.681407	1.365361
	2	477.51	472.5893	475.0503	479.63	4.921993	1.360939
	3	479.63	475.0503	477.338	482.09	4.575437	1.356906
	4	482.09	477.338	479.7153	483.50	4.754567	1.352787
	5	483.50	479.7153	481.6088	485.62	3.786911	1.349564
	6	485.62	481.6088	483.6127	486.67	4.007897	1.346201
	7	486.67	483.6127	485.1433	489.49	3.061169	1.343671
	8	489.49	485.1433	487.3182	491.61	4.349839	1.340116
	9	491.61	487.3182	489.4629	493.02	4.289361	1.336668
	10	493.02	489.4629	491.24	493.72	3.554308	1.333854
	11	493.72	491.24	492.481	493.72	2.481968	1.331914
	12	493.72	492.481	493.1015	495.13	1.240984	1.330952
1988	1	495.13	493.1015	494.1166	496.19	2.030119	1.329387
	2	496.19	494.1166	495.1527	498.30	2.07228	1.3278
	3	498.30	495.1527	496.728	500.77	3.150581	1.325407
	4	500.77	496.728	498.7491	502.53	4.042139	1.322372
	5	502.53	498.7491	500.6406	504.65	3.783104	1.319572
	6	504.65	500.6406	502.6436	506.76	4.005993	1.316647
	7	506.76	502.6436	504.7023	509.06	4.117438	1.313684
	8	509.06	504.7023	506.8824	512.28	4.360086	1.310591
	9	512.28	506.8824	509.5834	514.13	5.401957	1.306819
	10	514.13	509.5834	511.8544	514.59	4.542072	1.303705
	11	514.59	511.8544	513.2201	515.51	2.73131	1.301862
	12	515.51	513.2201	514.3632	522.41	2.286202	1.300333
1989	1	522.41	514.3632	518.3868	520.11	8.047202	1.295001
	2	520.11	518.3868	519.2479	523.33	1.722234	1.293889
	3	523.33	519.2479	521.2894	526.55	4.083031	1.291271
	4	526.55	521.2894	523.9211	529.77	5.26343	1.287942
	5	529.77	523.9211	526.8479	530.70	5.853629	1.284306
	6	530.70	526.8479	528.7716	532.08	3.847361	1.281961
	7	532.08	528.7716	530.4238	533.00	3.304501	1.279973
	8	533.00	530.4238	531.7102	534.84	2.572797	1.278441
	9	534.84	531.7102	533.274	537.14	3.127492	1.276595
	10	537.14	533.274	535.2065	538.52	3.865113	1.274338
	11	538.52	535.2065	536.8632	539.44	3.313377	1.272427
	12	539.44	536.8632	538.1519	544.96	2.577235	1.270956
1990	1	544.96	538.1519	541.5578	547.73	6.811899	1.267105
	2	547.73	541.5578	544.6416	550.49	6.16759	1.263693
	3	550.49	544.6416	547.5643	551.41	5.845436	1.260523
	4	551.41	547.5643	549.486	552.79	3.843265	1.258476
	5	552.79	549.486	551.1372	555.55	3.302453	1.256738
	6	555.55	551.1372	553.3436	557.85	4.412867	1.254441
	7	557.85	553.3436	555.5975	562.91	4.507801	1.252127
	8	562.91	555.5975	559.256	567.52	7.316908	1.248424

Time variant adaptive expectations

		Pa	Pe t	Pe t+1	Pa t+1	delta	Theta
	9	567.52	559.256	563.3866	571.20	8.261189	1.244339
	10	571.20	563.3866	567.2929	572.12	7.812782	1.240573
	11	572.12	567.2929	569.7064	572.12	4.826938	1.238299
	12	572.12	569.7064	570.9132	575.80	2.413469	1.237179
1991	1	575.80	570.9132	573.3576	576.72	4.888922	1.234926
	2	576.72	573.3576	575.0401	577.64	3.365008	1.233397
	3	577.64	575.0401	576.3416	578.56	2.603051	1.232227
	4	578.56	576.3416	577.4527	579.94	2.222072	1.231235
	5	579.94	577.4527	578.6986	581.79	2.491856	1.23013
	6	581.79	578.6986	580.2421	582.71	3.087022	1.228772
	7	582.71	580.2421	581.4741	584.55	2.464058	1.227697
	8	584.55	581.4741	583.0107	586.85	3.073123	1.226367
	9	586.85	583.0107	584.9297	587.77	3.837929	1.224721
	10	587.77	584.9297	586.3494	589.61	2.839511	1.223516
	11	589.61	586.3494	587.9799	590.07	3.260849	1.222144
	12	590.07	587.9799	589.0252	590.53	2.090698	1.221273
1992	1	590.53	589.0252	589.778	592.83	1.505622	1.220649
	2	592.83	589.778	591.3051	596.05	3.054178	1.219389
	3	596.05	591.3051	593.6796	596.51	4.749003	1.217448
	4	596.51	593.6796	595.097	597.43	2.834775	1.216305
	5	597.43	595.097	596.266	599.74	2.337934	1.21537
	6	599.74	596.266	598.0011	601.12	3.470334	1.213991
	7	601.12	598.0011	599.5591	602.50	3.115988	1.212765
	8	602.50	599.5591	601.0285	604.34	2.938814	1.211619
	9	604.34	601.0285	602.6838	606.64	3.310501	1.210338
	10	606.64	602.6838	604.6621	607.56	3.956618	1.208822
	11	607.56	604.6621	606.1115	607.10	2.898856	1.207724
	12	607.10	606.1115	606.6061	609.86	0.989154	1.207352
1993	1	609.86	606.6061	608.2342	612.16	3.256218	1.206131
	2	612.16	608.2342	610.1989	614.47	3.929476	1.204672
	3	614.47	610.1989	612.332	615.85	4.266105	1.203105
	4	615.85	612.332	614.0889	616.77	3.513873	1.201829
	5	616.77	614.0889	615.4277	615.39	2.677483	1.200866
	6	615.39	615.4277	615.4066	617.69	-0.042079	1.200851
	7	617.69	615.4066	616.5468	619.53	2.280328	1.200036
	8	619.53	616.5468	618.0374	620.91	2.981258	1.198979
	9	620.91	618.0374	619.4731	623.21	2.871449	1.197968
	10	623.21	619.4731	621.3417	623.67	3.737092	1.196664
	11	623.67	621.3417	622.5061	623.67	2.328819	1.195859
	12	623.67	622.5061	623.0883	625.35	1.16441	1.195459
1994	1	625.35	623.0883	624.2177	627.58	2.258738	1.194685
	2	627.58	624.2177	625.9001	629.82	3.364747	1.19354
	3	629.82	625.9001	627.8589	630.38	3.917752	1.192219
	4	630.38	627.8589	629.1178	630.94	2.51772	1.191379
	5	630.94	629.1178	630.0266	633.17	1.817705	1.190776
	6	633.17	630.0266	631.5988	634.85	3.144231	1.189739
	7	634.85	631.5988	633.2231	637.08	3.248649	1.188676
	8	637.08	633.2231	635.1529	638.76	3.859703	1.187425

Time variant adaptive expectations

		Pa	Pe t	Pe t+1	Pa t+1	delta	Theta
	9	638.76	635.1529	636.9561	639.32	3.606385	1.186267
	10	639.32	636.9561	638.1371	640.44	2.362037	1.185516
	11	640.44	638.1371	639.2865	640.44	2.298708	1.18479
	12	640.44	639.2865	639.8612	642.67	1.149354	1.184428
1995	1	642.67	639.8612	641.2662	645.47	2.810055	1.183548
	2	645.47	641.2662	643.3658	647.70	4.19925	1.182243
	3	647.70	643.3658	645.5333	649.94	4.335003	1.18091
	4	649.94	645.5333	647.7348	651.05	4.40288	1.179572
	5	651.05	647.7348	649.3943	652.17	3.319129	1.178574
	6	652.17	649.3943	650.783	652.17	2.777254	1.177746
	7	652.17	650.783	651.4773	653.85	1.388627	1.177335
	8	653.85	651.4773	652.6627	655.52	2.370847	1.176635
	9	655.52	652.6627	654.0937	657.20	2.861957	1.175797
	10	657.20	654.0937	655.6474	657.20	3.107512	1.174892
	11	657.20	655.6474	656.4243	656.64	1.553756	1.174444
	12	656.64	656.4243	656.5333	660.55	0.218034	1.174381
1996	1	660.55	656.5333	658.5438	662.79	4.020929	1.173225
	2	662.79	658.5438	660.6667	666.14	4.245842	1.172017
	3	666.14	660.6667	663.4047	668.38	5.475989	1.170477
	4	668.38	663.4047	665.8914	670.05	4.973372	1.169096
	5	670.05	665.8914	667.973	670.05	4.16322	1.167955
	6	670.05	667.973	669.0138	671.73	2.08161	1.16739
	7	671.73	669.0138	670.3725	672.85	2.717339	1.166656
	8	672.85	670.3725	671.6107	675.08	2.476358	1.165992
	9	675.08	671.6107	673.3474	677.32	3.473557	1.165066
	10	677.32	673.3474	675.3335	678.44	3.972157	1.164017
	11	678.44	675.3335	676.8854	678.44	3.103768	1.163205
	12	678.44	676.8854	677.6613	680.67	1.551884	1.162801
1997	1	680.67	677.6613	679.167	682.91	3.01132	1.162022
	2	682.91	679.167	681.0375	684.58	3.741038	1.161062
	3	684.58	681.0375	682.811	685.14	3.547053	1.160159
	4	685.14	682.811	683.9772	684.58	2.332371	1.15957
	5	684.58	683.9772	684.2809	685.70	0.607341	1.159418
	6	685.70	684.2809	684.9916	686.26	1.42136	1.159062
	7	686.26	684.9916	685.6263		1.269524	1.158745

APPENDIX C

Testing rationality of expectations formed according to MODEL 1 and MODEL 2

Testing rationality of MODEL 1 (traditional adaptive expectations)

Regression Statistics	
Multiple R	0.72354447
R Square	0.52351661
Adjusted R S	0.5124356
Standard Er	13.7801768
Observation	45

$$F = \frac{R^2/(K-1)}{(1-R^2)/(T-K)} = 47.244488$$

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	8971.410412	8971.41	47.24449	1.96267E-08
Residual	43	8165.410673	189.8933		
Total	44	17136.82108			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*pper 95%*
Intercept	2.02071602	3.847898183	0.525148	0.602178	-5.7393049	9.780737
X Variable 1	0.07699435	0.011201683	6.873463	1.96E-08	0.054404017	0.099585

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*
1	10.7287768	-9.928776765
2	10.7903722	-9.090372244
3	10.8904649	-10.9404649
4	10.82117	-10.54616998
5	10.8442683	-7.106768287
6	11.1214479	-5.152697939
7	11.4371248	-6.952749766
8	11.5526163	-7.710428787
9	11.6758072	-7.754713494
10	11.8297959	-8.989249065
11	11.897551	-8.637277528
12	12.0392206	-8.319083847
13	12.2001388	-8.870070394
14	12.3133204	-8.068286265
15	12.5119659	-5.849448772
16	12.8615202	-4.740261656
17	13.2303231	-2.419693855
18	13.750035	1.105279661
19	14.4776316	1.330025756
20	15.1228442	-1.459015539
21	15.5663316	-2.324417312
22	16.0598654	7.581091754
23	17.3703092	17.86016937
24	19.1727469	13.5524924
25	20.3361315	8.316488154
26	21.282392	10.0839178
27	22.5943757	18.1787792

Observation	Predicted Y	Residuals
28	24.5261639	36.60041355
29	27.6629136	41.80037509
30	30.6579938	35.3736506
31	33.0679169	15.36790533
32	34.2551697	7.232741394
33	35.5848621	0.659093461
34	36.7782745	-1.036296703
35	38.1349149	-16.383926
36	38.433653	-5.358158512
37	40.1429275	3.674819744
38	42.2433333	2.215540324
39	43.9795558	9.089880974
40	46.3540615	-5.089343117
41	47.4881883	-7.525829062
42	48.976489	-13.50530941
43	50.1691315	-15.11354166
44	51.5026736	-16.09487866
45	52.8793325	-15.05543505

Testing rationality of MODEL 2. (time variant adaptive expectations)

Regression Statistics	
Multiple R	0.02117105
R Square	0.00044821
Adjusted R S	-0.0114512
Standard Err	0.70995182
Observations	86

$$F = \frac{R^2/(K-1)}{(1-R^2)/(T-K)} = 0.037667$$

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.018985253	0.018985	0.037667	0.846582333
Residual	84	42.33865281	0.504032		
Total	85	42.35763806			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*pper 95%*
Intercept	1.38408802	0.794972254	1.741052	0.085336	-0.19680189	2.964978
X Variable 1	-0.0010933	0.005633059	-0.194079	0.846582	-0.012295219	0.010109

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	
1	1.25956565	0.440434353	0.193982
2	1.25814441	-1.308144409	1.711242
3	1.25912834	-0.584128343	0.341206
4	1.25836306	1.179136939	1.390364
5	1.25606721	3.062682786	9.380026
6	1.25267811	3.006696893	9.040226
7	1.25038226	2.47930524	6.146954
8	1.24863304	0.416210707	0.173231
9	1.24885169	-0.11642982	0.013556
10	1.24852372	0.817687221	0.668612
11	1.24688383	-0.113778357	0.012946
12	1.2467745	-0.080221766	0.006436
13	1.24611854	0.437157824	0.191107
14	1.24491596	-0.203277774	0.041322
15	1.24469731	-0.823878213	0.678775
16	1.24480663	-0.434397085	0.188701
17	1.24415067	-0.438945902	0.192674
18	1.24371337	-0.591110984	0.349412
19	1.24344006	-0.427138862	0.182448
20	1.24290436	-0.474753761	0.225391
21	1.24251078	-0.238435486	0.056851
22	1.24183296	-0.619795313	0.384146
23	1.24170177	-0.810682947	0.657207
24	1.24157058	0.083938832	0.007046
25	1.24035706	-0.207602355	0.043099
26	1.23995255	-0.483575202	0.233845
27	1.23969017	-0.611501496	0.373934

Observation	Predicted Y	Residuals	
28	1.23941686	-0.315322519	0.099428
29	1.23874997	-0.656702799	0.431259
30	1.23861878	-0.577595192	0.333616
31	1.23821427	-0.657702478	0.432573
32	1.23794096	-0.827685059	0.685063
33	1.23780976	-0.292681816	0.085663
34	1.23700075	-0.034436777	0.001186
35	1.23620267	-0.334920684	0.112172
36	1.23587469	-0.5852337	0.342498
37	1.23565604	-0.420335544	0.176682
38	1.23512034	0.392539905	0.154088
39	1.23378657	0.320043559	0.102428
40	1.23297755	0.403937509	0.163166
41	1.23203735	0.566420182	0.320832
42	1.23096595	0.288262812	0.083095
43	1.23028813	-0.350673749	0.122972
44	1.23015694	-0.669508449	0.448242
45	1.23002483	-0.704193892	0.495889
46	1.22975643	-0.59858092	0.358299
47	1.22935382	-0.545506031	0.297577
48	1.22895122	-0.396013938	0.156827
49	1.22841441	-0.198179043	0.039275
50	1.22774341	-0.221612337	0.049112
51	1.2272066	-0.478634374	0.229091
52	1.2269382	-0.3616387	0.130783
53	1.22640139	-0.425491605	0.181043
54	1.22599879	-0.33453051	0.111911
55	1.22546198	-0.28871446	0.083356
56	1.22492518	-0.265538031	0.07051
57	1.22438837	-0.130928068	0.017142
58	1.22371736	-0.185973829	0.034586
59	1.22318056	-0.213295407	0.045495
60	1.22264375	0.018818899	0.000354
61	1.22183855	0.135412857	0.018337
62	1.22103334	-0.051394252	0.002641
63	1.22049653	-0.267416951	0.071512
64	1.22009393	0.115719286	0.013391
65	1.21915452	0.062518819	0.003909
66	1.21848351	-0.209386804	0.043843
67	1.21808091	-0.222519169	0.049515
68	1.2175441	-0.105996502	0.011235
69	1.21687309	0.566434166	0.320848
70	1.21553108	0.658149318	0.433161
71	1.21445747	0.213396114	0.045538
72	1.21392066	0.482032896	0.232356
73	1.21284705	0.371649804	0.138124
74	1.21204184	0.193973314	0.037626
75	1.21137084	0.228156818	0.052056
76	1.21056563	0.122964929	0.01512
77	1.20989462	0.316144079	0.099947

Observation	Predicted Y	Residuals	
78	1.20895521	0.536090907	0.287393
79	1.2078816	0.278408188	0.077511
80	1.2072106	0.395207723	0.156189
81	1.20627119	0.454211396	0.206308
82	1.20533178	0.606936284	0.368372
83	1.20425816	0.438395942	0.192191
84	1.20345296	0.354394173	0.125595
85	1.20264775	0.190042547	0.036116
86	1.20197674	-0.137371555	0.018871
			42.33865 =sumsquarederror