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Of music, mathematics, and magic: Why math is all made up and why it works so well

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Of Music, Mathematics, and Magic:

Why Math is all Made up and why it Works so Well

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BY

Gregory A. Leach

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Of Music, Mathematics, and Magic

Abstract

The purpose of my study was to answer the philosophical question of whether mathematics is invented or discovered. This question is an age-old and important one with many implications regarding the understanding of mathematics, logic, science, and truth. Answering this question correctly can inform us not only about the nature of what is known, but even what is possible to know. In the field of mathematics itself, the answer to this question could change the way that mathematics is taught, learned, and implemented in the future. The problem was addressed through research, comparison, and analysis of books and articles where the question of whether math is created or discovered was either dealt with explicitly or implicitly. The answer to this question was found to be that mathematics is invented. Proof of this result is shown through debunking the myth of the objective world, and showing instead, how perception and thought give us a world of objects through creative use of metaphors. Mathematics is dependent on metaphorical, and not truly objective, thinking. The conclusions suggest that reality is not ultimately made of mathematics, and as such, there is a growing dissonance between the highly mathematical human-made world and much more fluid natural world. The world, not being the sum (or algorithm) of its parts, should be encountered in new ways (that are sometimes actually quite old), if one wants to gain fresh perspectives on the nature of reality and the meaning of truth. With these results in mind, subjective domains such as dreaming, creating art, and perceiving have much more truth to tell than could probably be imagined under the old taken for granted paradigm that science and math are the primary and most important tools that humans use in understanding the world and universe correctly.

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"Take all of your expertise and trade it for wonder." (Siddhartha Buddha from Tunneshende, 2004, p. 6). Imagine a vivid dream that you are completely involved in and ask yourself how often you actually know, in those dreams that are so vivid and accurate to normal life that they seem real, that you are dreaming. How often do you stop and ask yourself, "am I dreaming?" I would guess that the answer for most of us is that when we are dreaming we most often take it as reality, or we aren't lucid (aware that we are dreaming.) Our minds can construct entire worlds that are so real (we can taste, see, smell, hear, and touch in our dreams) that we can live in our dreams just like we live in the everyday world, and while it is happening we believe that it is real. In our dreams, we can crawl through a tunnel and discover a hidden lake and mountains on the other side. Now the question is, "are we truly discovering that hidden lake and mountain, or is it all created in our mind's eye?" Scientists have much to unravel as far as what is really happening when we dream. Many people would probably agree that dreams are completely a product of the imagination. People who are better at lucid dreaming, or basically have "stronger" dreams, are probably the same ones who would argue that there is something about our dreams which is not made up. The better of a dreamer you are, the more real the dream becomes, and the better your ability to distinguish between the dreaming state and the waking state would be.

The same thing could be said for scientists and mathematicians. To many a non-academic layman, the notion that scientific and mathematical principles are created in the minds of people might not be so hard to accept. However, those who are the brightest and best in the scientific and mathematical communities might tend to argue that they are discovering theories and axioms which have a deeper reality. Part of the reason for this is that for a mathematician, mathematics is created in their minds to a much greater extent than that of the average person. Their "dream of math" is more vivid, and being so detailed and well constructed, they take it as a separate reality that they are discovering, or as actually discovering the truth about reality absolute. The benefit for their own ego in this case is tremendous, as the physicists and mathematicians become detectives on all of the ultimate and transcendent questions about existence. Discovering the "truth" is an important job, and one that often has consequences which change technology and society in many profound ways. I am hypothesizing that all of math is indeed created by human imagination, and that the very people who have invented and continue to define it, are the most apt to believe they are discovering math and not creating it, because math in their minds is vast, deep, and detailed, and also because it suits their egos to believe so.

There are other interesting parallels we can draw between one who is dreaming and one who is deeply involved in critical thought. In dreaming we are normally unaware of the sensuous world around us to a great extent, and while lost in thought, we can also lose touch with the physical world around us. In the following pages, I will show, not only how and why math is created by human beings, but also I will tell the more human story of how mathematical or objective thinking has displaced an ancient and vital element of human perception, which has had important consequences for human wonderment, the environment, quality of life, morality, music, and more. Simply put, the world is more dynamic than our words. It has meanings and mysteries that can never be calculated or rationally understood.

Purpose of Study

The purpose of this study was to determine whether mathematics is created, discovered, or both. This was accomplished through literature review, talking with professors and anyone with an opinion, and many hours of thought and self reflection.

Limitations of Study

The limitations of this study are similar to the limitations of knowledge in general. In many ways, this study is precisely about what the limitations of knowledge are and exactly what knowledge is. Mathematical knowledge has always had the advantage of being testable and provable. However, since this study involves asking the question of "precisely what this mathematical knowledge is and where it comes from," it is limited by the limits of reason itself so to speak. This is a philosophical question that can neither be proved with mathematical equations and proofs, nor from the collection of scientific data. That is because we are asking questions here about what the *content* of scientific data and mathematical analysis *actually is.* Ultimately, the question of whether or not there is an actual mathematical dimension to reality in the Platonic sense cannot be proven or disproven either. These were the most significant limitation in this study.

Review of Literature

The Philosophy of Mathematics

Greek ideas remained dominant in the philosophy of mathematics Europe until the 17th century. At this time, and beginning with Gottfried Leibniz, the focus shifted strongly to the relationship between mathematics and logic. This perspective dominated the philosophy of mathematics through the time of Gottlob Frege and of Bertrand Russell, but was brought into question by developments in the late 19th and early 20th century.

At the start of the 20th century, philosophers of mathematics were already beginning to divide into various schools of thought about all these questions, broadly distinguished by their pictures of mathematical epistemology and ontology. Three leading schools of thought, formalism, intuitionism, and logicism, emerged at this time (Kleene, 1971).

Surprising developments in formal logic and set theory early in the 20^{th} century led to new questions concerning what was traditionally called the foundations of mathematics. The axiomatic approach, which had been taken for granted since the time of Euclid around 300 B.C. as the natural basis for mathematics, began to be openly explored. The Zermelo-Fraenkel axioms for set theory were formulated in the early 20^{th} century, which provided a conceptual framework in which much mathematical discourse would be interpreted. With Gödel numbering, propositions could be interpreted as referring to themselves or other propositions, enabling inquiry into the consistency of mathematical theories. This reflective critique in which the theory under review "becomes itself the object of a mathematical study" led David Hilbert to call such study metamathematics or proof theory (Kleene, 1971, p.5).

At the middle of the century, a new mathematical theory was created by Samuel Eilenberg and Saunders Mac Lane, known as category theory, and it became a new contender for the natural language of mathematical thinking (Maziers, 1969). As the 20th century progressed, however, philosophical opinions diverged as to just how well-founded were the questions about foundations that were raised at its opening. Hilary Putnam summed up one common view of the situation in the last third of the century by saying:

When philosophy discovers something wrong with science, sometimes science

> has to be changed- Russell's paradox comes to mind, as does Berkeley's attack on the actual infinitesimal- but more often it is philosophy that has to be changed. I do not think that the difficulties that philosophy finds with classical mathematics today are genuine difficulties; and I think that the philosophical interpretations of mathematics that we are being offered on every hand are wrong, and that "philosophical interpretation" is just what mathematics doesn't need (Putnam, 1975, p.176).

Philosophy of mathematics today proceeds along several different lines of inquiry, by philosophers of mathematics, logicians, and mathematicians, and there are many schools of thought on the subject. The terms *philosophy of mathematics* and *mathematical philosophy* are frequently used as synonyms (Maziars, 1969). The latter, however, may be used to refer to several other areas of study. One refers to a project of formalizing a philosophical subject matter like, aesthetics, metaphysics, or theology in a purportedly more exact and rigorous form. Another refers to the working philosophy of an individual practitioner or a like-minded community of practicing mathematicians, philosophers, or logicians. Additionally, some understand the term "mathematical philosophy" to be an allusion to the approach taken by Bertrand Russell in his books *The Principles of Mathematics* and *Introduction to Mathematical Philosophy* (Russell, 1993 & 2010).

Contemporary Schools of Thought

It is often said that most mathematicians are Platonist at heart, but there are many problems one might observe with Platonism such as: precisely where and how do the mathematical entities exist, and how do we know about them? Is there a realm, completely separate from our physical one, which is occupied by these mathematical entities? If there is such a world, then how can we gain access to this separate reality and discover truths about the entities? Twentieth century mathematician Kurt Gödel's Platonism postulates a special kind of mathematical intuition that lets us perceive mathematical objects directly, which bears resemblance to Immanuel Kant's idea that mathematics is synthetic a priori (Tegmark, 2008).

Empiricism is a school of mathematical philosophy traceable back to Aristotle, denying that mathematics can be known a priori at all. While not Platonist, technically, empiricism could be said to be a realist philosophy. Empiricism purports that we discover mathematical facts by empirical research, just like facts in any of the other sciences. One problem for Aristotelian realism is what account to give of higher infinities, which may not be realizable in the physical world (Franklin, 2009).

Another contemporary school of thought, mathematical realism, holds that mathematical entities exist independently of the human mind. Humans do not invent mathematics, but rather discover it, and any other intelligent beings in the universe would presumably do the same, meaning there is really one sort of mathematics that can be discovered: Squares and threes, for example, are real entities, not creations of the human mind (Franklin, 2009).

Logicism is the thesis that mathematics is reducible to logic, and just a part of logic Logicists philosophy holds that mathematics can be known a priori, but suggests that our knowledge of mathematics is just part of our knowledge of logic in general, not requiring any special faculty of mathematical intuition. In this view, logic is the proper foundation of mathematics, and all mathematical statements are necessary logical truths

(Carnap, 1931).

The most popular contemporary school of mathematical thought is the formalist school. Formalism holds that mathematical statements may be thought of as statements about the consequences of certain string manipulation rules. For example, in the "game" of Euclidean geometry one can prove that the Pythagorean Theorem holds. According to Formalism, mathematical truths are not about numbers and sets and triangles and the like - in fact, they aren't "about" anything at all. Another version of formalism is often known as deductivism. In deductivism, the Pythagorean Theorem is not an absolute truth, but a relative one. The same is held to be true for all other mathematical statements (Hilbert, 1999).

In mathematics, intuitionism is a program of methodological reform whose motto is that "there are no non-experienced mathematical truths." L.E.J. Brouwer, the founder of the movement, held that mathematical objects arise from the *a priori* forms of the volitions that inform the perception of empirical objects. Another intuitionist, Leopold Kronecker, said: "The natural numbers come from God, everything else is man's work" (Hawking, 2007).

Like intuitionism, constructivism involves the regulative principle that only mathematical entities which can be explicitly constructed in a certain sense should be admitted to mathematical discourse. In this view, mathematics is an exercise of the human intuition, not a game played with meaningless symbols. Instead, it is about entities that we can create directly through mental activity. In addition, some adherents of these schools reject non-constructive proofs, such as a proof by contradiction (Hawking, 2007).

Embodied mind theories hold that mathematical thought is a natural outgrowth of the human mind which finds itself in our physical, embodied, universe. For example, the abstract concept of number springs from the experience of counting discrete objects. It is held that mathematics is not universal and does not exist in any real sense, other than in human brains. Humans construct and create, but do not discover, mathematics (Lakoff & Nunez, 2000).

Fictionalism is another school of mathematical philosophy asserting that math is created. Fictionalism says a mathematical statement like "2+2=4 is just as false as Sherlock Holmes lived at 221B Baker Street"- but both are true according to the relevant fictions. By this account, there are no metaphysical or epistemological problems special to mathematics. The only worries left are the general worries about non-mathematical physics and about fiction in general. Social constructivism or social realism theories see mathematics primarily as a social construct, as a product of culture, subject to correction and change (Field, 1980).

Methodology

This study was conducted through reading and analyzing available literary sources pertaining to the question, either directly or implicitly, of whether mathematics is created or discovered. Being a very philosophical topic, the study also involved much introspection and contemplation based on the writer's own worldview, opinions, knowledge, and past experiences.

Results

The Evolution of Mathematics

"The so-called Pythagoreans, who were the first to take up mathematics, not only advanced this subject, but saturated with it, they fancied that the principles of mathematics were the principles of all things" (Aristotle, 350 B.C., pgs. 1-5). The origin of mathematics is subject to much scholastic debate. Whether the birth of mathematics was a random happening or induced by necessity duly contingent of other subjects like physics is topic of much contemporary and historical interest. All available evidence suggests that the human species has had a recognizable concept of abstract numbers for at most 8,000 years. Formal, symbolic mathematics with equations proofs and theorems only dates back about 2,500 years. Calculus wasn't developed until the 17th century, while negative numbers were not in wide-spread use until the 18th century, and modern abstract algebra, where symbols like x, y, and z denote arbitrary entities, is just over 150 years old. However, the anthropological record suggests that humans have possessed relatively the same brain structure for over 50,000 years. The size current size of the human brain was reached even earlier, about half of a million years ago. Einstein's brain could have hypothetically existed even in the Iron Age, but nothing that we would call mathematics existed in the Iron Age. This means that whatever features of our brains enable us to do mathematics must have evolved thousands of years before we had any mathematics. All evidence suggests that this is because the same features that enable humans to use language are the ones which are operational in mathematical thought (Devlin, 2000).

In order to get a feel for just how deeply embedded the possibility for mathematical thought runs in terms of evolutionary history, it is useful to look at the numerical abilities of animals. Recent studies have suggested that there is something like a *number sense*, which is shared by human infants and many kinds of animals. Number sense, defined by Keith Devlin, is simply the ability to distinguish and compare small numerosities and does not require a concept of numbers as abstract entities or even an ability to count. Studies have shown that this ability is shared by humans with chimpanzees, gorillas, rats, lions, pigeons, and more (Devlin, 2000, p. 18).

One of the first to realize animals had such a sense was the German psychologist Otto Koehler during the 1940's and 1950's. Koehler was able to demonstrate that ravens had the ability to compare the sizes of two collections presented simultaneously and also the ability to remember numbers of objects presented successively in time. In one case a raven was presented with two boxes, one containing food. The lids of the boxes had a certain number of randomly arranged spots. A card placed next to the two boxes had the same number of spots (in a different spatial arrangement) as the box with the food in it. Through many repetitions the raven eventually learned that in order to get the food, it had to open the box whose lid had the same number of spots as the card. Eventually, the raven learned to distinguish between two, three, four, five, and six spots. Another example comes from Irene Pepperberg, who trained her African Gray parrot to say how many objects it saw on a tray (Devlin, 2000, p. 19-20).

We can also see a basic number sense and even a small amount of numerical ability when looking at our closest human relative, the chimpanzee. Some have been trained, although painstakingly, to correctly use the Arabic numerals 1 to 9 to give numbers of objects in a collection with up to 95 percent accuracy, and also to put those numbers in order, to count. Another interesting facet is that response times increased significantly when the numbers were greater than three, suggesting numbers 1 to 3 can be almost perceived directly, where counting comes into play with larger collections.

Rhesus monkeys have been shown to have a similar ability. If the evolutionary timeline is correct, this would suggest our ancestors have possessed these abilities for over 25 million years (Devlin, 2000, p. 26-27).

Another example of just how deeply this number sense is embedded in the human mind comes from studies of infants less than a year old. In one experiment by Karen Wynn, it was demonstrated that if a baby sees two puppets disappear behind a screen, it shows little surprise when the screen is lowered to reveal two red balls (surprise being measured by how long the baby stares at the balls). However, the baby appears troubled when the screen is lowered to reveal one puppet or one red ball. Apparently the idea that one object can change into another is less confusing than a change in the numbers of objects. The arithmetic abilities of babies are limited to simple additions and subtractions involving the numbers 1, 2, and 3. Babies younger than one year old seem unable to distinguish between four, five, and six objects. They have not yet learned how to count (Devlin, 2000, p. 36-37).

Taking into account the evidence suggested by studies on infant humans and animals, there is a basis at least for numbers that goes back millions of years in our evolutionary history. However, as any contemporary mathematician knows, mathematics is about so much more than numbers. This wasn't always the case. Ancient Egyptian, Babylonian, and Chinese mathematics consisted almost solely of arithmetic, and was more of the utilitarian cookbook variety (Devlin, 2000, p. 6). Our present number system was developed over 2,000 years ago by the Hindus, and essentially reached its present form in the sixth century (Devlin, p. 50). Between 500 B.C. and A.D. 300, mathematics expanded beyond the study of numbers. This door was perhaps pushed open by the

Pythagoreans who elevated the study of mathematics to mystical levels.

Pythagoras, an ancient Greek mathematician reported to have lived in the 6^{th} century B.C., suggested that the highest purification of a life is in pure contemplation and that the philosopher who contemplates science and mathematics is released from the cycles of reincarnation. The pure mathematician's life therefore, is life at the highest plane of existence. It is this contemplation about the world that forms the greatest virtue in Pythagorean philosophy (Burnet, 1892; Russell, 1967). Pythagorean thought was dominated by mathematics, but it was also profoundly mystical. The Pythagorean account actually begins with Anaximander's teaching that the ultimate substance of things is "the boundless," or what Anaximander called the "apeiron" (Burnet, 1892). The Pythagorean account holds that it is only through the notion of the "limit" that the "boundless" takes form, or "peiron." When the apeiron is inhaled by the peiron it causes separation, mathematically meaning that form "separates and distinguishes the successive terms in a series." Instead of an undifferentiated whole we have a living whole of inter-connected parts, separated by *void* between them. This inhalation of the apeiron is also what makes the world mathematical since it shows numbers and reality to be upheld by the same principle. Both the continuum of numbers and the field of reality, the cosmos - both are a play of emptiness and form, apeiron and peiron. What really sets the Pythagoreans ideas apart from Anaximander's original ideas is that this play of apeiron and peiron must take place according to harmony. Both music and math, for Pythagoras, were a necessary attempt by human beings, to harmonize and balance the peiron, this world of forms.

Whether or not Pythagoras' disciples believed that everything was related to mathematics and that numbers were the ultimate reality is unknown. It was said that Pythagoras was the first man to call himself a philosopher, or lover of wisdom, (Cicero, 45 B.C.) and Pythagorean ideas exercised a marked influence on Plato, and through him, all of Western philosophy. There is evidence that Plato possibly took from Pythagoras the idea that mathematics and, generally speaking, abstract thinking is a secure basis for philosophical thinking as well as for substantial theses in science and morals (Russell, 1967). Plato and Pythagoras also shared a mystical approach to the soul and its place in the material world. Bertrand Russell, in his *A History of Western Philosophy*, contended that the influence of Pythagoras on Plato and others was so great that Pythagoras should be considered the most influential of all Western philosophers (Russell, 1967).

In general, Greek philosophy on mathematics after Pythagoras was strongly influenced by their study of geometry. For example, ancient Greeks once held the opinion that 1 was not a number, but rather a unit of arbitrary length. A similar argument was made that 2 was not a number but a fundamental notion of a pair. Earlier Greek ideas about numbers were changed by the discovery of the irrationality of the square root of two. Hippasus, a disciple of Pythagoras, showed that the diagonal of a unit square was incommensurable with its unit-length edge: in other words he proved there was no existing, or rational, number that accurately depicts the proportion of the diagonal of the unit square to its edge. This caused a significant re-evaluation of Greek philosophy of mathematics. According to legend, fellow Pythagoreans were so traumatized by this discovery that they murdered Hippasus to stop him from spreading his heretical idea (Russell, 1967).

The notion of the square root of two is also a challenge to a very fundamental

Platonic idea: the existence of a realm of eternal numerical forms. What is the form for the square root of two? *Platonism* is a type of realism that suggests that mathematical entities are abstract, have no spatiotemporal or causal properties, and are eternal and unchanging. This is often claimed to be the view most people have of numbers. *Platonism* has meaningful, not just superficial connections for the study of mathematical philosophy, because Plato's ideas were preceded and probably influenced by the hugely popular Pythagoreans of ancient Greece, who believed that the world was, quite literally, generated by numbers.

This line of ancient Greek thought was continued and built upon right on through the Renaissance and was exemplified especially in the thoughts of the prominent 15th century astronomer and scientist, Galileo Galilei who asserted that only properties of matter that are directly amenable to mathematical measurement are real. Galileo wrote, "This grand book the universe...is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth" (Jones, 1989, p. 22). Rene Descartes would later update and establish the Platonic view even more with his book *Meditations* in which he separates the thinking mind, or subject, from the material world of things, or objects. That material reality came to be commonly spoken of as a strictly mechanical realm, as a determinate structure whose laws of operation could be discerned only via mathematical analysis (Abram, 1996, p. 32). Phenomenologist David Abram writes:

> Galileo asserted that only those properties of matter that are directly amenable to mathematical measurement (such as size, shape, and weight) are real; the other,

more "subjective" qualities such as sound, taste, and color are merely illusory impressions, since the "book of nature" is written in the language of mathematics alone. In Galileo's own words, 'This grand book of the universe...is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth...It was only after the publication of Descartes' *Meditations*, in 1641, that material reality came to be commonly spoken of as a strictly mechanical realm, as a determinate structure whose laws of operation could only be discerned via mathematical analysis. By apparently purging material reality of subjective experience, Galileo cleared the ground and Descartes laid the foundation for the construction of the objective or "disinterested" sciences (Abram, 1996, p. 32).

The Case that Math is Discovered

Mathematics is the language of much of science. This statement has a double meaning. The normal meaning is that the natural world contains patterns or regularities that we call scientific laws and mathematics is a convenient language in which to express these laws. This would give mathematics a descriptive and predictive role. And yet, to many, there seems to be something deeper going on with respect to what has been called, 'the unreasonable effectiveness of mathematics in the natural sciences' (Byers, 2010: 15).

There seem to be cases in physics where we cannot see any deeper than the mathematics, such as the case where the electron *is* its mathematical description via the Schrodinger equation. Phenomena such as this lead many to believe that there then must exists a mathematical, Platonic substratum to the real world. If this were indeed the case then we might infer that we cannot get closer to reality than mathematics because the mathematical level *is* the deepest level of the real. (Byers, 2010: 15-16).

Princeton Mathematics Professor Andrew Wiles is the mathematician who gave the final resolution to what was perhaps the most famous mathematical problem of all time- the Fermat conjecture. In an interview Wiles reflects on the process of doing mathematical research:

> Perhaps I can best describe my experience of doing mathematics in terms of a journey through a dark unexplored mansion. You stumble around bumping into the furniture, but gradually you learn where each piece of furniture is. Finally after six months or so, you find the light switch, you turn it on, and suddenly it's all illuminated. You can see exactly where you were. Then you move into the next room and spend another six months in the dark. So each of these breakthroughs, while sometimes they're momentary, sometimes over a period of a day or two, they are the culmination of -and couldn't exist without- the many months of stumbling around in the dark that precede them (Byers, 2010, p. 1).

This mathematical exploration and parable of illumination described by Wiles surely seems to lend credence to the notion that mathematics is discovered. In fact, many professional mathematicians can probably relate to the feeling of adventure, unknown, darkness, and finally illumination that Wiles speaks of. This idea of objective discovery in the domains of math and science is a quite natural one for a variety of reasons. Mathematics is obviously the driving force behind modern technology, science, and to a great extent philosophy and perception, or culture. The level of effectiveness or usefulness of mathematics varies greatly in these related areas though. Mathematics does quite well in the objective or hard sciences such as physics, chemistry, and biology, and is valuable and predictive to a less extent in humanistic sciences like philosophy, anthropology, and sociology. As far as technology is concerned, math is generally indispensable in many of the reason innovations in computer and digital technology. It would be hard to imagine how the world would be different without all of the algorithms and equations used to drive our cell phones, bank security systems, cars, televisions, and just about any other modern technology one cares to list. I will not dispute the importance of math in terms of scientific and technological innovations and discoveries.

Rooted in the interaction between human biology and the natural world, human beings have developed mathematics as a vast cultural project that spans the ages and all civilizations. Many people feel that mathematics is capable of revealing absolute, objective truth. Mathematical truth exists, but is not to be found absolutely in the world nor in the content of any particular theorem or set of theorems.

> The intuition that mathematics accesses the truth is correct, but not in the manner that it is usually understood. The truth is to be found more in the fact than in the content of mathematics. Thus it is consistent, in my view, to talk simultaneously about the truth of mathematics and about its contingency. The truth of mathematics is to be found in its human dimension...the impossible is rendered possible through acts of genius- this is the very definition of an act of genius, and mathematics boasts genius in abundance. In the aftermath of these acts of genius, what was once considered impossible is now so simple and obvious that we teach it to children in school (Byers, 2010, p. 15-16).

One reason for the effectiveness and apparent objective truth of mathematics is that the properties of mathematics are, in many ways, properties that one would expect from our folk theories of external objects. Mathematical properties are based on our experience of external objects and experiences. One property that we find in external objects and inside mathematics is universality. External objects tend to be the similar for everyone; a banana is a banana regardless to a chimpanzee and to a two year old girl. We could say the same for basic mathematical objects; two is two regardless of who you are. There are in fact many aspects of the human conceptual system that are universal across culture, and important consequences follow for mathematics (Lakoff & Nunez, 2000, p. 351). One such universal conceptual system gives human beings an inference-preserving mechanism such as conceptual metaphor, which means that mathematics has inferential stability, meaning mathematical proof and computations are cognitively stable. Proofs made using inference-preserving mechanisms remain valid and correct computations remain correct.

Another facet of mathematics and external objects is stability. In the real world, we must isolate a physical fact in time and space and then it won't change. That is, particular occurrences at a given time and place don't change and are stable over time. If there was a monkey on your desk at 10 A.M. this morning, it will always be the case that on this day in history there was a monkey on your desk at 10 A.M. Likewise, once established firmly within a community of mathematicians, mathematical inferences and calculations for a given subject matter tend not to change over time, space, or culture. This apparent stability is a consequence of the fact that normal human beings all share the same relevant aspects of brain and body structure and the same relevant relations to their environment that enter into mathematics.

> Another source of stability are spatial-relations concepts. The primitive spatial-relations schemas used in mathematics are universal across human languages- for example, the concept of containment, a path, or a center. What counts as a bounded region of space or a path in the spatial-relations system of any language in any culture is the same. Moreover, conceptual metaphors. Such as Numbers Are Points on a Line, have been shown to have a property extremely important for mathematics: namely, that they preserve inferences; that is, the inferential structure of one domain (say, geometry) can be used by another (say, arithmetic). Once a metaphorical mapping is established for a mathematical community, the inferences are the same for anyone in that community, no matter what culture they come from. If that mathematical community extends over generations or longer, the inferences are stable over those generations (Lakoff & Nunez, 2000, p. 352).

Abstraction is another powerful cognitive tool that aids the effectiveness of mathematics. Mathematics has general conceptual categories, such as integral, prime, square. "Proofs about such categories can hold for all members of those categories, whether they ever have been, or ever could be, thought of by any real human beings or not. Proofs about all prime numbers, for example, are proofs about all members of a category; they hold even for prime numbers so large we could never conceptualize them as individual numbers" (Lakoff & Nunez, 2000, p. 353). Such mathematical models across branches of mathematics, once established, can remain stable indefinitely. Since conceptual metaphors preserve inference, the consequences of such a metaphorical model can be drawn out systematically by mathematicians over the course of long periods of time as generation after generation of mathematicians draw out the consequences of assumptions and models established by previous mathematicians. "Once results are established, they are stable and take on a seemingly 'timeless' quality" (Lakoff & Nunez, 2000, p. 354).

Other important aspects of mathematics are consistency, generalizability, and precision. The physical world as we normally experience it is consistent in the sense that a bird cannot be in and outside of a cage at the same time. The world is subject to generality- there are basic properties of trees that generalize to new trees we have never encountered and properties of fish that generalize to fish yet unborn. One of the most impressive qualities of mathematics is precision. Given a treasure chest full of emeralds there is a precise answer as to how many emeralds are in the chest. Precision is made possible because human beings can make very clear, if not arbitrary, distinctions among objects and categories. We can then fix these categories in our minds and consistently recall abstract entities like numbers and shapes. Precision is greatly enhanced by the human capacity to symbolize and continue to define more subtle mathematical objects, objects able to ever increase the accuracy of mathematics in modeling, manipulating, or describing physical reality (Lakoff & Nunez, 2000, p. 351-354).

Facts about the physical world can also be discovered. You might even discover buried treasure in your own backyard! Such treasure hunts are also possible in mathematics, however in a different way- the things we discover do not exist until we invent them with mathematical thought, even if many other minds may later discover the same "truths." Once mathematical concepts and assumptions are established within a mathematical community, it is possible to make discoveries by reasoning alone, without recourse to empirical evidence. Once we have the definition of prime number in place, we can discover all kinds of prime numbers through attempting to factor any given number. However, these discoveries are contingent upon cognitive creations. Without the underlying mathematical definitions and assumptions they are meaningless. We certainly aren't going to find any prime numbers without our notion of what a prime number is. The type of discovery that happens in mathematics begins in our minds, intersects the world, and ends up back in our minds.

> Mathematics is a mental creation that evolved to study objects in the world. Given that objects in the world have these properties, it is no surprise that mathematical entities should inherit them. The view that mathematics is a product of embodied cognition- mind as it arises through interaction with the world- explains why mathematics has these properties...In short, mathematics is not a reflection of a mathematics existing external to human beings; it is neither transcendent nor part of the physical universe. Mathematics is effective in characterizing and making predictions about certain aspects of the real world as we experience it. We have evolved so that everyday cognition can, by and large, fit the world as we experience it. Mathematics is a systematic extension of the mechanisms of everyday cognition. Any fit between mathematics and the world is mediated by, and made possible by, human cognitive capacities. Any such "fit" occurs in the human mind, where we cognize both the world and mathematics (Lakoff & Nunez, 2000, p. 349-353).

In looking for objective or absolute truth, the area mathematics is least able to inform is the very area that is most important if we are to have a real discussion about whether mathematics is discovered or invented, and that is philosophy. If mathematics is truly discoverable, then it must exist in some natural dimension or as the very fabric of nature and reality. However, whether or not reality is truly mathematical is a philosophical question! It is left to philosophy since we have no way of devising scientific studies to get at the answer. If we attempt to answer the question using mathematics, such as by stating that $e = mc^2$ or pointing to examples of the golden ratio in nature, we are begging, and side-stepping the question. Of course reality will appear to be mathematical if we are using math to describe it! We can hardly reconcile logic and philosophy in order to find our answer here either. Logic utilizes linear and algorithmic pathways of thought, but philosophy is informed by abstract and intuitive thinking and spatiotemporal relationships that are often anything but linear. Then there is the question of whether or not there is "any world of objects to begin with?" If there are no physical objects then mathematical objects get tossed out with the bathwater. The short answer to this question is that there are not really objects in the world, but more like open ended processes and various energetic fluxes which are categorized by human and animal minds (and perhaps vegetables and all kinds of other perceivers) as an evolutionary adaptation, as a way of perceiving the world and making the best of their situation in it. We shall investigate this matter in much more detail soon.

Number Magic: Mysticism and Mathematics

The Tao gives birth to One.

One gives birth to Two.

Two gives birth to Three.

Three gives birth to all things.

All things have their backs to the female
and stand facing the male.
When male and female combine,
all things achieve harmony.

Ordinary men hate solitude.
But the Master makes use of it,
embracing his aloneness, realizing
he is one with the whole universe.

(*The I-Ching*, Number 42: Lao-tzu, *trans. S. Mitchell*).

While many have argued that mathematics underlies physical reality, others have gone even farther into believing that numbers and mathematical entities also possessed mystical, transcendental, even magical qualities. The Chinese Book of Numbers, *The I-Ching*, is an example of a divination technique based on arbitrary qualities ascribed to certain positive integers. Such use of mathematics and various forms of numerology abound in the modern world and have their roots in ancient times. It is even likely that in the beginning of the development of mathematics, many people were equally interested in the magical qualities of math as they were in its practical value. One needs look no farther than the Pythagoreans to see this phenomenon. Mathematician Underwood Dudley writes variously, "What Pythagoras did was to turn number mysticism loose on the world. Number mysticism, from the unluckiness of 13 on up, has been spread far and wide, and it is still spreading. It is possible that Pythagoras absorbed number-mystical ideas from the East. Classifying numbers as deficient or abundant is mathematical, but when the Pythagoreans classified them as male or female- odd numbers are male and even numbers are female- they were being number-mystical" (Dudley, 1997, p. 5-13).

The logical plausibility of numbers having transcendental value to many a modern mathematician may seem quite laughable, but yet it is there, a stark reminder of the interaction between the creative human mind and mathematics.

Mystical numbers are also vividly present in the Hebrew tradition that is foundational for so much of Western culture and thought. David Abram writes:

> Since the letters of the *aleph-beth* also at times served as numbers for the Hebrew people, written words and phrases could also be compared by calculating the total numerical value of the letters that comprise them- a Kabbalistic technique called *gematria*. Through both permutating the letters and calculating their numerical values, mystics were able to demonstrate hidden equivalences and correspondences between various words and names contained in the scripture. Elohim, for instance, one of the most sacred names of God in the Hebrew Bible, could be shown to have the same numerical value as the Hebrew word for nature, *hateva-* evidence of the hidden unity of God and nature (Abram, 1996, p. 246).

The Kabbalah also contains ideas about transcendent mathematical ideas, such as duality and infinity. There it is written:

> Do not attribute duality to God. If you suppose that infinity emanates until a certain point, and that from that point on is outside of it, you have dualized. Realize that infinity exists in each existent. However, anything visible, and anything that can be grasped by thought, is bounded. Anything bounded is finite. Anything finite is not undifferentiated. Conversely, the boundless is called

Infinity. It is absolute un-differentiation in perfect, changeless, oneness. Since it is boundless, there is nothing outside of it. Since it transcends and conceals itself, it is the essence of everything hidden and revealed. The philosophers acknowledge that we comprehend it only by way of no (Matt, 1995, p. 24).

This idea that infinity transcends and conceals itself seems to imply that by the very nature of the concept, infinity, like God, cannot be grasped or bounded by thought. We can't say exactly what infinity is without using an endless stream of inductions, always n+1, and one more than that, and one more than that. By studying the concept of infinity, we can easily see that there is no correlate in the physical world or in any world of ideals, since the concept itself is unbounded, and therefore, not able to be well defined.

The Meanings of Infinity

Infinity as a religious or philosophical concept can only be approached by the mind by way of negation, a sort of knowing that you can't know. Being endless and unbounded, infinity is not a "thing" and therefore can't be a legitimate "object" of scientific analysis. Nunez and Lakoff argue that this concept of infinity does not allow us to evaluate it properly, because it does not give us any idea about infinite *things*. Aristotle made a distinction between potential infinity (such as the square root of 2, or an n + 1 polygon) and actual infinity, or infinity as a thing, like points at infinity. Lakoff and Nunez (2000) also say that we need a positive notion of infinity, a notion of infinity as an entity-in-itself (p. 155).

Lakoff and Nunez argue that by looking at what linguist call the *aspectual system* we can get a positive concept of infinity. The aspectual system is said to characterize the structure of events as we conceptualize them. Actions like breathing and tapping for instance, are inherently iterative, where something like moving about is inherently continuous. So mathematics aside, they argue that the literal concept of infinity outside of mathematics is used whenever one thinks about perpetual motion. Linguists refer to a process conceptualized as not having an end as an imperfective process, because the process is not perfect in the sense that it has no completion. Two subtypes of imperfective processes are known as iterative and continuative. Continuative processes, like flying or swimming, differ from iterative processes, like jumping or tapping, in that continuative processes lack even intermediate endpoints or results. Due to the constraints of human language, continuous processes are often conceptualized as if they were iterative processes (Lakoff & Nunez, 2000, p.155-156). The reason for this is because indefinitely continuous motion over long periods is impossible to visualize so what we do with our minds is visualize short motions and then repeat them, thereby conceptualizing indefinitely continuous motion as repeated motion. Another reason for doing this is that everyday continuous actions, such as walking, require iterative action, such as taking steps. This conflation of continuous action and repeated actions gives rise to the metaphor by which continuous actions are conceptualized in terms of repeated actions.

This metaphor is also used in the conceptualization of mathematics in order to break down continuous processes into infinitely iterating step-by-step processes, in which each step is discrete and minimal. So the metaphorical magic employed in understanding and using the concept of infinity in mathematics makes use of the metaphorical result of a process without an end- by imagining it has an end. Lakoff and Nunez (2000) describe this as the basic metaphor of infinity (BMI). Anytime that infinity comes up in mathematics, the BMI is used to add to the target domain the completion of the process and its resulting state (p. 158). The fact that it is the *final* state of the process means that there is no earlier final state and there is no later final state. Therefore, the uniqueness of the final state of a complete process is a product of human cognition, and not a fact about the external world. Existence of degrees of infinity (such as transfinite numbers) requires multiple applications of the BMI (Lakoff & Nunez, 2000, p. 160).

> Thus the paradox of the infinite series of positive terms points to an actual mathematical difficulty, one that the Greeks could not have solved for a number of reasons. They had no concept of the real number system as a whole; in fact, the real number system is vastly larger than the number systems with which they worked. The other problem was of course the potential/actual infinity paradox...by identifying this question as problematic the Greeks were displaying a legitimate mathematical intuition. There *was* a problem here. The solution to the problem was not mathematical so much as conceptual. The problem is not so much solved as defined away by *assuming* a new axiom, something like 'every convergent sequence of rational numbers *defines* a real number.' In other words, the problem is not solved in the conventional way; rather, somehow a new mathematics is built on top of the old by taking the process/object paradox as the new *definition* of number, so that now a number is entirely defined by an approximating sequence. The side effect of this definition is that you open the door for a vastly enriched concept of number. This makes possible the calculus, analysis, and, in a way, the modern world. With the proposition about infinite sets we can now define an infinite set to be one with the same cardinality as a proper subset. In this case it is not clear what is gained by doing so (Byers, 2010,

p.137).

Boundaries: Edge and Essence, Self and Other

> Reason is a trickster because it seems to take something away from reality, to extract some truth, but it actually adds to reality. Reason sings its riddle to us endlessly. "I am a piece of you, but I seem to be missing. You believe that I am separation, the opposite of existence and connection, but that is only because you indulge in me. Stand back from me and laugh at me. Only then will you gain perspective and see that really, I am just a piece of your own imagination, painted black (Leach, unpublished, 2000).

It may seem blasphemous to say that logic, so often opposed to imagination, is the product of it, but even Hilbert suggested that a powerful imagination is required to be a great mathematician when one of his students switched to poetry and Hilbert remarked that his student had lacked the imagination required to be a mathematician anyhow.

Math employs mentally created boundaries in all of its discoveries, uses, and theories. By imagining some complete boundary we mentally create the perception or idea of "one" and by induction that leads to another, and several, and many, and on towards infinity. Once these created quantities are established, many relationships and symmetries can be explored between them. However, no matter how useful, beautiful, complex, or elegant mathematics is or becomes, none of it would be possible without a completely imagined and creative concept, the idea of boundary. Ask the famous philosopher Nietzsche and he might say that it is human thought, and not physical reality that has boundaries. In *Thus Spake Zarathrustra*, the hero-sage-prophet of the story Zarathrustra says, "Everything parts, everything greets every other thing again; eternally the ring of being remains faithful to itself. In every Now, being begins; round every Here rolls the sphere There. The center is everywhere. Bent is the path of eternity" (Nietzsche, 1977).

Perhaps no other mathematician or philosopher did as much in illuminating the elusiveness of the concept of boundaries as the great 20th century mathematician, Bertrand Russell. Russell's paradox highlights the subtle magic involved in creating the notion of a set. Basically, either sets are members of themselves or they are not. For example, the set of all sets is a thought, but the set of all women is not a woman. So if we were to represent the set of all sets, which are members of themselves by *A* and denote the set of all sets that are not members of themselves by *Z*, we will see a contradiction. That's because if *Z* is a member of itself, then it is actually a member of *A*, but *Z* is not a member of itself. But if *Z* is not a member of itself, then it is a member of itself, which also leads to a contradiction (Eves, 1990, p. 625-626). The contradiction comes from the mental process of creating boundaries around what we call a set. We could ask the question about what the boundary is. Is the boundary a part of the set or a part of the compliment of the set?

Russell used the idea of the Vicious Circle Principle to resolve the paradox, stating, "No set *S* is allowed to contain members *m*, only definable in terms of *S*" (Eves, 1990, p. 627-628). Although this principle allows for the ideas of sets to be freely used it does not answer the original question about the boundary problem, which creates the paradox in the first place. It does admit that any set is only definable in terms of negation, in terms of something outside the set. In a roundabout way this shows how boundaries are imaginatively and creatively employed in the elementary mathematical notion of a set.

The boundary itself cannot be said to be a part of the set or the compliment of the set. This ultimately stems from the fact that no known boundaries exist in an absolute sense, and neither do sets or their compliments, because without boundaries, sets are part of their compliments, and vice-versa. Boundaries are an essential part of all linguistic and mathematical thought, but the boundaries themselves are like the quantity zero, empty, invented, and nonexistent. It's like trying to look at what we have when we put a boundary around zero. From a number line perspective this seems quite possible that we might have some actual object, but from a perspective of quantity, we don't have anything at all, and the boundaries around that emptiness are devoid of quantity as well, to say non-existent. On the other hand, when we try to understand boundaries in relation to infinity, instead of disappearing, the boundary becomes infinite too, always one more than infinity, which is to say an endless boundary, which we can never find, which really isn't a boundary when you think about it. The very idea of boundary diverges under a critical analysis. The reason for these mathematical and philosophical paradoxes has to do with the nature of thought more so than some objective reality. Nature is boundless, ever-changing, and endlessly linked. All boundaries are imposed by our mind as a way to objectify and simplify our world. In the Buddhist view, this leads to ignorance, and ignorance to suffering. Buddhists try to transcend suffering by non-attachment to objects because objectification of the boundless, which is ignorance, leads to hate and lust, attraction and repulsion to "things." This overvaluation and misperception of reality is what leads to suffering (Lama, 2009).

Objectification and the Elusiveness of Truth

I think people get it upside down when they say the unambiguous is the reality

> and the ambiguous merely uncertainty about what is really unambiguous. Let's turn it around the other way: the ambiguous is the reality and the unambiguous is merely a special case of it, where we finally manage to pin down some very special aspect (David Brohm, from Byers, 2010: 25).

One of the most important writers of the 20th century was Jerome David Salinger, whom recently passed away. Despite the fact that Salinger was never a mathematician, in his short story, *Teddy,* Salinger writes a passage involving a dialogue between two characters, Teddy and Nicholson, which gets at the heart of the question of just what logic and mathematics are, and how they might be related to ancient religious ideas. In summary, Teddy is a young boy wise beyond his years in spiritual advancement, while Nicholson is an aspiring writer who is interested in how Teddy "gets out of the finite dimensions." Teddy responds to Nicholson's question saying, "Everybody just *thinks* things keep stopping off somewhere. They don't...The only reason things *seem* to stop off somewhere is because that's the only way most people know how to look at things" (Salinger, 1981, p. 189-190). Teddy goes on to further elucidate his point:

> You asked me how I get out of the finite dimensions when I feel like it. I certainly don't use logic to do it. Logic's the first thing you have to get rid of...You know that apple Adam ate in the Garden of Eden...You know what was in that apple? Logic. Logic and intellectual stuff. That was all that was in it. So- this is my point- what you have to do is vomit it up if you want to see things as they really are. I mean if you vomit it up, then you won't have any more trouble with blocks of wood and stuff. You won't see everything stopping *off* all the time (Salinger, 1981, p. 190-191).

What Salinger is getting at here is the mystical idea of transcending the trappings of logic by shutting off the internal dialogue, which constantly tells us what is what. The idea is that everything that is a thing, is only such because the human mind has words and ideas that tell us that they are, but in reality, there is no inherent existence to anything as an isolated object. Everything is part of a greater web and we use our mind to isolate and extract things out of a continuous and perhaps infinite flux and flow. Objectivity can't crack the equation of what reality is because there is no equation! And hence, if there are no objects as such, there certainly aren't any mathematical objects that exist inherently. They exist only in the human mind or perhaps the mind of some yet to be encountered aliens.

In Genesis, it is written, "God commanded man. You are free to eat from any tree in the garden, but you must not eat from the tree of knowledge of good and evil." Of course Adam and Eve end up defying God and eating from the Tree of Knowledge and so the Bible goes on, "Then the eyes of both of them were opened, and they realized they were naked, so they sewed fig leaves together and made coverings for themselves." Adam and Eve go on to hide from God in the garden and when God asks them why they have hidden, Adam replies, "I heard you in the garden, and I was afraid because I was naked, so I hid."

The recurrent theme from this passage is that of boundaries: clothes, separation, duality; and the necessity of these boundaries for a new way of thinking – specifically, ideas of duality applied to morality. Although duality or multiplicity is a natural result of objectification, the kind of duality referred to here is a much more extreme type than perhaps humans had ever achieved before this. The boundaries have grown thicker. The result is that even morality becomes bounded, dualized, and polarized by the concepts of absolute good and evil. The controversial 19th century philosopher Friedrich Nietzsche had many ideas on the topic – and one of his books was even titled, *Beyond Good and Evil.* In it, he wrote, "What is done out of love always occurs beyond good and evil" (Nietzsche, 1977, p. 153). In a later work, Nietzsche writes "What is good and evil *no one knows yet*, unless it be he who creates. He, however, creates man's goals and gives the earth its meaning and its future. That anything at all is good and evil- that is his creation" (Nietzsche, 1977, p. 308).

For Nietzsche, Knowledge of good and evil comes from saying "No" mentally, imposing a boundary – According to Friedrich Nietzsche the creative we create an outside, objective world through the use of negation or negative thought. Negating is necessary to even form any word or concept at all in the first place. In order to have a word for something in the real world, we must negate the fact that there are subtle differences between every other thing we use the exact same word for! However, if we named every single thing uniquely language never could have developed as this would be an absolutely endless and impossible task. Nietzsche expresses the paradox of the search for truth in his book, *On Truth and Lie in an Extra-Moral Sense.* There he writes:

> Every concept originates through our equating what is unequal. No leaf ever wholly equals another, and the concept "leaf" is formed through an arbitrary abstraction from these individual differences, through forgetting the distinctions; and now it gives rise to the idea that in nature there might be something besides the leaves which would be "leaf"--some kind of original form which all leaves have been woven, marked, copied, colored, curled, and painted, but by unskilled

> hands, so that no copy turned out to be a correct, reliable, and faithful image of the original form (Nietzsche, 1977, p. 46).

In *Thus Spake Zarathrustra*, Nietzsche's overman hero-sage Zarathrustra states:

> How lovely it is that there are words and sounds rainbows and illusive bridges between things which are eternally apart? Precisely between what is most similar, illusion lies most beautifully; for the smallest cleft is the hardest to bridge. Speaking is a beautiful folly: with that man dances over all things (Nietzsche, 1977, p. 322).

Perhaps this generalizing quality of thought is easier to grasp when we think about a leaf, since it isn't hard to believe that no two leaves are exactly the same. When words represent things that are a part of the real natural world such as legs, leaves, blocks of wood, stars, snowflakes and such, we can easily prove to ourselves that we will never find two items exactly the same, although we might find many that are very similar. Things are a little bit different when our words represent more abstract ideas; words like one, infinity, equality, morality, freedom, will, right, wrong, and the like. Here we find an interesting crossroads. When the abstract ideas represent moral, political, or religious ideas, like right, wrong, freedom, destiny and such, we are apt to find a great divergence in people's ideas about exactly what those things are and mean. Yet when we are talking about mathematical words like two, infinitesimal, and multiplication, these ideas appear to be the same across time and space for all people. Part of the reason for this is that math, once created, doesn't have a complex relationship to the world it came from, like the word freedom does. Mathematical objects are more completely in the mind and don't need to refer back to the world they came from to prove themselves. Math is not the essence of reality, but objectification is essential to thought and an important part of human perception in general. Mathematical thought strips down the world to pure objects, which opens the door to logical creations and accomplishments of all kinds.

It is this very distance of objective thought from the actual world that is a defining feature of our current cultural milieu and this distance seems to have grown over time with every advance in science and technology. This objectification of the world was not only philosophically erroneous, but the greatest sin according to Nietzsche's sage Zarathrustra who preached:

> To sin against the earth is now the most dreadful thing, and to esteem the entrails of the unknowable higher than the meaning of the earth...no longer bury one's head in the sand of heavenly things, but bear it freely, an earthly head, which creates a meaning for the earth...It was the sick and decaying who despised body and earth and invented the heavenly realm...They wanted to escape their own misery...Ungrateful, these people deemed themselves transported from their bodies and this earth. But to whom did they owe the convulsions and raptures of their transport? To their bodies and this earth...There is more reason in your body than in your best wisdom...you are no longer able to create beyond yourselves. And that is why you are angry with life and the earth. An unconscious envy speaks out of the squint-eyed glance of your contempt (Nietzsche, 1977, p. 125, 144-147, 164).

For Nietzsche, the possibility of "discovering" absolute objective truth was about as likely as the possibility of a heavenly realm or a realm of perfect Platonic prototypes. It seems truthful when he writes that, "Never has truth hung on the arm of the unconditional. It is only in the market place that one is assaulted with Yes? Or No?" (Nietzsche, 1977, p. 172). The key to the progression of objectification in human society lies in this relationship. The market place creates a *need* for not only naming and objectifying, but also a need for quantifying, for mathematics, reason, the building up of the objective lens in human perception. How much is an item *worth*? All haggling aside, eventually we have to decide on some arbitrary price in the market place in order for a transaction to happen. As our societies have become more economic, the world has obviously been treated as more and more of an object. This way of perceiving the world as object is what sets us on our quest for *truth* in the belief that we can actually capture it with mathematics and language, with science and reason. If it is not a world of objects and numbers, math and reason will never be able to discover any absolute truths because they aren't there to begin with. However, because the world we have created is more mathematical all of the time, it seems more and more all the time like math is a fundamental part of reality.

There is a creative current to all of thought and perception that becomes less and less recognizable the more we rationalize and objectify the world as individuals and as a society. This creative current and impulse in the final analysis cannot be ignored if it is indeed *truth* that we are searching for. *Truth* cannot be purely discoverable because of the way the human mind works to create it. Zarathrustra describes the limitations of our *truth* search in the following ways:

> And what you have called world, that shall be created only by you...All the permanent- that is only parable...'Will to truth,' you who are wisest call that which impels you and fills you with lust? A will to the thinkability of all beings: this I

> call your will. You want to *make* all being thinkable, for you doubt with well-founded suspicion that it is already thinkable. But it shall yield and bend for you. Thus your will wants it. It shall become smooth and serve the spirit as its mirror and reflection......I am not, like scholars, trained to pursue knowledge as if it were nutcracking...All that is straight lies, all truth is crooked...In everything one thing is impossible: rationality. A *little* reason to be sure...A little wisdom is possible indeed...but there is no eternal spider or spider web of reason. The world is deep-and deeper than day had ever been aware. Not everything may be put into words in the presence of the day (Nietzsche, 1977, p.198, 225, 237, 270, 278).

A more recent scholar who wrote extensively about the limitations of rationality was controversial anthropologist Carlos Castaneda. Castaneda wrote over a dozen story length biographical books based on his lifelong initiation into the shamanic realm of the Toltec Mayan tradition, a realm that Castaneda describes as a completely different way of cognition than the way in which average Westerners perceived the world. Castaneda's stories largely relate experiences of hidden dimensions of reality that most people never access, and often Castaneda was aided in altering his perception by psychedelic plants and other times it was purely on the suggestions and manipulations of his shaman Yaqui informant, Don Juan. Because of these fantastic realities Castaneda describes, many scholars have debated the legitimacy of Castaneda's writings and even whether or not such a Don Juan existed. Fantastic tales or not, what cannot be debated is that Don Juan's teachings largely directed Castaneda to reevaluate the importance of reason in the way he saw the world, and in doing this, question all that he held as real. In Don Juan's world, reason is not only limited, but limiting, an intoxicant and poison in a very real way.

Variously, he tells Castaneda:

> The greatest flaw of human beings is to remain glued to the inventory of reason. Human beings are perceivers, but the world that they perceive is an illusion: an illusion created by the description that was told to them from the moment they were born...in essence, the world that their reason wants to sustain is the world created by a description and its dogmatic and inviolable rules, which their reason learns to accept and defend...Their reason makes them forget that the description is only a description, and before they realize it, human beings have entrapped the totality of themselves in a vicious circle from which they rarely emerge in their lifetimes (Castaneda, 1998, p.135-137).

For Don Juan, as for Zarathrustra, the most damaging aspect of reason is that it leaves us to perceive ourselves and the world as objects, which is a gross underestimation that limits our awareness and creativity. Don Juan says:

> We think there is a world of objects out there only because our awareness. But what's really out there are the Eagle's emanations, fluid, forever in motion, and yet unchanged, eternal... Reason doesn't deal with man as energy. Reason deals with instruments that create energy, but it has never seriously occurred to reason that we are better than instruments: we are organisms that create energy. We are bubbles of energy...Human beings are not objects; they have no solidity. They are round, luminous beings; they are boundless. The world of objects and solidity is only a description that was created to help them, to make their passage on earth convenient (Castaneda, 1998, p. 135-137, 164, 236, 241).

From this perspective, the value of logic, science, math, and reason is much less than we generally give it in our mainstream culture. The value of scholarship and *objective truth* becomes subordinate and dependent upon the search for *subjective meaning.* Don Juan regularly made fun of Castaneda's scholastic efforts. At one point, he tells Castaneda, "After arranging the world in a most beautiful and enlightened manner, the scholar goes back home at five o'clock in the afternoon in order to forget about his beautiful arrangement" (Castaneda, 1998, p. 164).

Castaneda was not the only scholar of the psychedelic sixties to reevaluate the place of reason and objectivity in the search for truth. The reevaluation of objective truth was a part of the fabric of the counter-culture movement of the times. Few scholars that took part in this reevaluation were more influential and respected than Aldous Huxley. Huxley distrusted the "artificial piety" of symbol-manipulating religions that focus on creeds, dogmas, and beliefs and he was a self-proclaimed seeker of *understanding* as opposed to *knowledge*.

> To him, knowledge was acquired when one could merely fit a new experience into the system of concepts based upon old experiences. Understanding, on the other hand, 'comes when we liberate ourselves from the old and so make possible a direct, unmediated contact with the new... Understanding is not conceptual and therefore cannot be passed on.' Huxley often equated 'understanding' with 'truth' (Marty, 1971, p. 1).

According to Huxley, science deals with the more public of human experiences, and a scientist does their best to ignore the world of private experiences and so is not concerned with the concreteness of some unique event, but with the abstract generalizations, in terms of which all events of a given class *make sense*. Huxley said that sciences "seek to establish explanatory laws, especially when they deal with relationships between the invisibles and intangibles underlying appearances." Because of this, scientists inhabit a "radically different universe -- not the universe of given appearances, but the world of inferred fine structures, not the experienced world of unique events and diverse qualities, but the world of quantified regularities." In science, "the world's enormous multiplicity is reduced to something like unity, and the endless succession of unique events of a great many kinds get tidied and simplified into a single rational order" (Marty, 1971, p. 2).

It is this creation through reduction that is so inherent in rational thought which is incompatible with "discovering" any *truth.* Only by reducing our perceptual input from the world can we think about it at all. True objectivity may not be even a possibility. As phenomenologist David Abram writes, perhaps what we are really doing when we are doing science and attempting to be objective is that we are being *intersubjective*. Abram writes:

> That tree bending in the wind, this cliff wall, the cloud drifting overhead: these are not merely subjective; they are *intersubjective* phenomena- phenomena experienced by a multiplicity of sensing subjects...The striving for objectivity is thus understood, phenomenologically, as striving to achieve greater consensus, greater agreement or consonance among a plurality of subjects, rather than an attempt to avoid subjectivity altogether...The living pulse of subjective experience cannot finally be stripped from the things that we study (in order to expose the pure unadulterated "objects") without the things themselves losing all existence for us...The sensuous, breathing body is, as we have seen, a dynamic, ever-

> unfolding form, more a process than a fixed or unchanging object. As such, it cannot readily appropriate inert "facts" or "data."...The everyday world in which we hunger and make love is hardly the mathematically determined "object" toward which the sciences direct themselves. Despite all the mechanical artifacts that now surround us, the world in which we find ourselves before we set out to calculate and measure it is not an inert or mechanical object but a living field, an open and dynamic landscape subject to its own moods and metamorphoses (Abram, 1996, p. 32, 34, 38, 120).

Whether or not there is a world of objects our there or if any objectivity is possible, it is still certainly the case that categorization and objectification are deep rooted and important parts of human perception and apparently to many other beings as well. We know that most animals can categorize, or treat new and unique objects as being in the same category or of the same kind as some other "objects." The problem with science and math is not that they objectify and categorize in order to "discover." In fact there would be no other way to "progress." The problem is when people mistake these "discoveries" for being absolutes (Levitin, 2006, p. 140-142, 147). When that happens we miss out on the complexity, interdependence and subtlety of the real world. We miss out on the creativity inherent in the scientific process. As musician turned scientist and author, Daniel Levitin writes, there are more similarities between the artistic and scientific processes than one might assume at first glance:

> The Oxford historian Martin Kemp points out a similarity between artists and scientists. Most artists describe their work as experiments- part of a series of efforts designed to explore a common concern or to establish a

> viewpoint...William Thompson adds that the work of both scientists and artists involves similar stages of development: a creative and exploratory "brainstorming" stage, followed by testing and refining stages that typically involve the application of set procedures, but are often formed by additional creative problem-solving...Both require specialized tools, and the results are open to interpretation. What artists and scientists have in common is the ability to live in an open-ended state of interpretation and reinterpretation of the products of our work (Levitin, 2006, p. 4-5).

It is this art-in-science that must not be acknowledged if we are to get an accurate picture of the potential for mathematics and science to "discover truth." Because of this necessary creative element, truth absolute is not even discoverable to begin with, just as objectivity is not possible, but the very notion springs from living subjects. Math and science do generate many facts, but neither these facts nor their sum can find *the truth.* I think a poem by Alan Marty, published in 1971 states it nicely:

On Not Confusing the Facts With the Truth

Facts are guesses
We make them
They are labels
for a stage in the life
of an idea

Facts are things
we can't help

but believe in
We need them
to establish certainty

But even "hard facts"
have soft cores
since we choose them.
Sometimes, unknowingly,
we fudge them –
to preserve "truth."

Knowing the truth
at each halt
on our errant way,
we no longer notice
the accidents of things –
only the substance of the universe.

Knowing this truth
(that exists a priori)
we regain omnipotence
lost in childhood.
(Marty, 1971).

Mathematics as Metaphor Manipulation

Sonnet- To Science

Science! True daughter of Old Time thou art!
Who alterest all things with thy peering eyes.
Why preyest thou thus upon the poet's heart,
Vulture, whose wings are dull realities?
How should he love thee? Or how deem thee wise,
Who wouldst not leave him in his wandering
To seek for treasure in the jewelled skies,
Albeit he soared with an undaunted wing?
Has thou not dragged Diana from her car,
And driven the Hamadryad from the wood
To seek a shelter in some happier star?
Hast thou not torn the Naiad from her flood
The Elfin from the green grass, and from me
The summer dream beneath the tamarind tree?

(Edgar Allan Poe, 1829).

In mathematics, there is a formalization of the notion of "sameness," called equivalence. The notion of sameness is used to divide up the universe (here the set) into larger objects- it is a classification or naming scheme. Mathematics takes this original metaphor of equivalence and makes up in symbols a new universe of discourse whose units are not the original elements but the classifications. Many times in math, the new universe inherits structures from the old one. "The essence of what is going on involves multiple representations of mathematical objects- a process (equivalence) that becomes an object (equivalence class)" (Byers, 2010, p. 216). It is this very pattern of process becoming object which is central to objectification, logical thinking, and mathematics. Without creating and using the metaphor of sameness, we cannot even begin.

> When the notion of equivalence is applied to whole categories of mathematical objects, groups, rings, topological spaces, and so on, we have the variety of sameness that is called *isomorphism.* Isomorphism is fundamental to any mathematical subject- in a sense it defines the subject. Each category of mathematical objects carries with it the appropriate notion of isomorphism, which is a formal way of saying that two objects are identical from the point of view of that particular subject. For example, two sets are isomorphic from the point of view of Cantor's theory if they have the same cardinality. If you are studying metric spaces, where the abstract notion of distance is defined, then the isomorphism will be called *isometry*, a distance preserving mapping (Byers, 2010, p. 216).

"Tautologies in the form of logical equivalences are ambiguous. They are ambiguous because what they do is to compare two frames of reference and show that they are really both referring to the same situation" (Byers, 2010, p. 212). This ambiguity that mathematician William Byers speaks of here is important in the understanding the creative underbelly of mathematics and logic. We have here an essential element to all mathematical doing, which is at odds with the nature of the universe. The universe says, "no two situations are ever exactly the same-all is unique." Math says, "here we have a 2, by definition, any other 2 is equivalent or the same as the original 2 we had." The essential creative act is in equating.

Elaborating, Byers writes:

> One has to differentiate between the 'formal' point of view in which a tautological statement is merely restating the same fact in different language and the 'ambiguous' point of view in which these equivalences may say very important things indeed. A question that has often been asked about mathematics is the following, 'If mathematics is merely tautological, how do we ever do anything that is new? Why is mathematics so successful in so many ways?' The answer to this question is frankly obscure from a formalist perspective. From my perspective...certain tautologies are valuable precisely because they are ambiguous and so contain a multiple perspective that expands our understanding of the mathematical situation in question (Byers, 2010, p. 212).

This way of looking at things has implications for how we view the scientific enterprise as a whole. These implications extend to the most fundamental of questions, such as "What is (mathematical) truth?" and "What is knowledge?" To be sure, there is a certain metaphoric quality that is inherent in even the simplest mathematical situations. We look at "1 + 1 = 2" and we glance over many layers of meaning and say to ourselves, "true." We feel that we understand it completely and that there is nothing further to be known, but since knowledge is created, there are as many meanings as we are creative enough to perceive. We might think about the fact that "one" and "two" are deep and important ideas that are important to all of science, religion, perception, and cognition. Think back to the Garden of Eden and the separation from God. What was "one" became "two." Duality led to knowledge of good and evil, a polarizing aspect to perception.

Equality is another idea whose meaning has many social implications. Then we have the equation itself, stating that the unity and duality have a relationship with one another that we represent by "equality." This is like saying that there is unity in duality and duality in unity. This deeper structure that is implicit in the equation is typical of ambiguous situations from which creativity can arise. Looking at things this way, we could even say that the most elementary mathematical expressions have a profundity that may not be apparent on the surface level. This profundity is due to the endless possible meanings of each situation. Arthur Koestler said that creativity arises in a situation where "a single situation or idea is perceived in two self-consistent but mutually incompatible frames of reference" (Byers, 2010, p. 28). These two frames of reference must be *mutually incompatible*, even though they are individually self-consistent. It is in spite of this incompatibility, where there exists an over-riding unitary idea. Incompatibility is unacceptable in mathematics and it is this need to resolve incompatibility that makes the situation of ambiguity so dynamic, so potentially creative. Creating mathematical truth happens when we see there are two (or more) perfectly harmonious ways of looking at a situation, yet they are in opposition to one another- they are different symbolically. The restoration of equilibrium can only come at a level that is higher categorically than either of the original frames of reference. The equilibrium condition may not yet exist and may only come into existence as a result of the need to reconcile the incompatibility of the original situation. This makes every situation, mathematical or otherwise, ambiguous, and full of creative possibilities (Byers, 2010, p. 27-30).

For Byers, meditating on Zen Buddhist thought has helped him to understand the subtle creative part of mathematics. He writes:

> Zen demonstrates that there is a way to work with situations of conflict, situations that are problematic from the normal, rational point of view. The rational, for Zen, is just another point of view. Paradox, in Zen, is used constructively as a way to direct the mind to subverbal levels out of which acts of creativity arise...For me this means that ambiguity, contradiction, and paradox are an essential part of mathematics- they are things that keep changing and developing. They are the motor of its endless creativity...the ambiguous always has a component of the problematic about it (Byers, 2010, p. 18).

By looking at the concept of equivalence, we can see mathematical creativity at work because there are many equivalent ways to define the continuity of a function of one variable, $y = f(x)$. One way to look at continuity is how the function treats converging sequences and generalizes to metric spaces, situations in which distance is defined. Another involves the way the function treats open sets and generalizes to topological spaces. Each different original definition gives us another way of thinking about what continuity means, which adds flexibility to our understanding of the concept. Another form in which "equality" arises in mathematics is the notion of equivalence. We can for instance say that two fractions are equivalent if they represent the same number: 3/5, 6/10, 9/15 are all equivalent fractions. All equivalent fractions have the same number representation or value, so when we refer to a fraction we may be referring either to their common value or to one specific numerical example such as 6/10. Thus there is an ambiguity here too (Byers, 2010, p. 213-215).

> There is a 'discrepancy between the actual work and activity of the mathematician and his own perception of his work and activity.'...The most pervasive myth about

> mathematics is that the logical structures of mathematics are definitive- that logic captures the essence of the subject... When pressed, many mathematicians retreat back to a formalist position. However, most practicing mathematicians are not formalists. What they want is understanding...a difficult thing to talk about. For one thing, it contains a subjective element, whereas drawing logical inferences appears to be an objective task that even sophisticated machines might be capable of making. Nevertheless, if one wants to come close to plumbing the depths of mathematical practice, it will be necessary to begin by seeing beyond a formalist approach of equating mathematics with the trinity of definition-theorem-proof. Logic is indispensable to mathematics. For one thing, logic stabilizes the world of mathematical results so that it presents itself to our minds in the conventional manner- as a body of permanent or absolute truths. However, logic is not the essence of mathematics nor can mathematics be reduced to logic. Mathematics transcends logic. Mathematics is one of the most profound areas of human creativity (Byers, 2010, p. 25-26).

The way people usually divide up the art from science is by seeing art as ambiguous and science as certain, perhaps nothing more certain than mathematics. However, mathematics is also a human, creative activity. Many creative insights of mathematics arise out of ambiguity, contradiction, and paradox. Math proceeds beyond these limitations through the creative use of objective metaphor, of process becoming object (Byers, 2010, p. 11). Because of the creative and metaphorical dimension of math it is necessary to reexamine the role of logic and rigor in mathematics. It is the formal, logical dimension of mathematics that gives it its timeless quality. The other dimension is creative and developmental and the two work in tandem to advance the subject (Byers, 2010, p. 3).

In the final analysis, mathematics is fully and completely, from start to finish, a product of human beings.

> It uses very limited and constrained resources of human biology and is shaped by the nature of our brains, our bodies, our conceptual systems, and the concerns of human societies and cultures. The parts of human cognition that generate advanced mathematics as an enterprise are normal adult cognitive capacities- for example, the capacity for conceptual metaphor. Such cognitive capacities are common to all human beings. The subject matters of mathematics arise from human concerns and activities: counting and measuring, architecture, gambling, motion and change, grouping, manipulating written symbols, playing games, stretching and bending objects (Lakoff & Nunez, 2000, p. 351-352).

Another way of examining the creativity inherent in mathematics is to take note that mathematics is not monolithic in its general subject matter. There is no such thing as *the* geometry or *the* set theory or *the* formal logic. Rather, there are mutually inconsistent versions of geometry, set theory, logic, and so on. Each version forms a distinct and internally consistent subject matter (Lakoff & Nunez, 2000, p. 351-352). The claim that transcendent mathematics exists is untenable and can be demonstrated even by looking at perhaps the simplest mathematical concept, that of number. "Two plus two is four," the layman might say in response, but upon deeper examination, we can see that even in modern mathematics, numbers are characterized in ontologically inconsistent ways. Consider that on the number line, all numbers are points on a line, zero-dimensional geometric objects. Therefore if numbers are literally and objectively points on a line, then they too- as abstract transcendent entities- must be zero-dimensional geometric objects. Now consider numbers from the point of view of set theory, in which numbers are described as sets. Here, zero is the empty set. One is the set containing the empty set. Two is the set containing the empty set and also containing the set which contains the empty set, one. Therefore, we can see that from the perspective of set theory, numbers are not zero-dimensional geometric entities; numbers have size. In set theory, infinity is inconceivably large, whereas in number theory it is completely dimensionless. The difference between the two ways of conceptualizing infinity are well, infinite. But wait, there are even more ways mathematicians have created to objectify "number." In one of the latest ways, combinatorial game theory, numbers are values of positions in combinatorial games. In this theory, numbers are neither points on a line nor sets, but values of positions in combinatorial games (Lakoff & Nunez, 2000, p. 342-343). As we can see, even the foundations of mathematics aren't as solid as Plato and Pythagoras may have dreamt.

We can see the same foundational problem when we closely examine another idea fundamental to modern mathematics, that of equality. Although we might think the symbol "=" is absolutely basic and indispensable to mathematics, it is actually no more than a few hundred years old. Up until the 16th century, mathematicians actually got by without such a symbol. Instead of a symbol, mathematicians used language to express what they meant, statements such as, "Which numbers can be decomposed into the sum of three cubes?" or "This number, subtracted from 42, yields 24." Now look at the statement "4 + 3 = 7." This is usually understood to mean that the operation of adding 3 to 4 yields 7 as a result. Here 7 is the result of an addition process and "=" establishes the relationship between the process and the result. But "7 = 4 + 3" is usually understood in a completely different way- that the number 7 can be decomposed into the sum of 4 plus 3. The difference is subtle but real nonetheless. We also have the case where "4 + 3 = 6 + 1," which is usually understood in an equal result frame, stating that the result of the process of adding 4 and 3 yields the same number as the process of adding 4 and 1. The "=" in these instances represents three different concepts. What hides the cognitive and linguistic nuances is the use of the same symbol in all three cases (Lakoff & Nunez, p. 376-377). This doesn't mean "=" is not an extremely useful concept in mathematics. It is just the way that abstraction, metaphor, reduction, and logic work.

Math, like language, and even more-so in many ways, is rooted in metaphoric thought. Mathematical thinking is a way of conceptualizing abstract concepts in concrete terms, using ideas and modes of reasoning grounded in the sensory-motor system. Mathematics uses conceptual metaphor to comprehend the abstract in terms of the concrete, as when we conceptualize numbers as points on a line. In fact, mathematics is made of layers of metaphor upon metaphor. Mathematics is a sort of culmination of the perceptual-thought process of abstracting the sensual world. Far from having a concrete reality of its own, it represents the pinnacle of abstract thought- a metaphorical masterpiece. In fact, it appears that mathematical abstraction led the way to the abstract achievement of written language. This is because the 8,000 year old Sumerian accounting tables are the earliest known writing system, meaning that the use of markings to denote numbers preceded the use of markings to denote words (Devlin, 2000, p. 49).

Discussion

Perception on the way to Language and Math

> In Twilight *of the Idols*, Friedrich Nietzsche writes about formal science: a doctrine of signs, such as logic and that applied logic which is called mathematics. In them reality is not encountered at all, not even as a problem- no more than the question of the value of such a sign-convention as logic (Nietzsche, 1977, p. 3).

Innovations in the philosophy of language during the 20th century renewed interest in whether mathematics is, as is often said, the *language* of science. Although most mathematicians and physicists, and many philosophers, would accept the statement "mathematics is a language," linguists believe that the implications of such a statement must be thoroughly considered. For example, the tools of linguistics are not generally applied to the symbol systems of mathematics, that is, mathematics is studied in a markedly different way than other languages. If mathematics is a language, it is a different type of language than natural languages. Indeed, because of the need for clarity and specificity, the language of mathematics is far more constrained than natural languages studied by linguists. However, the methods developed by Frege and Tarski for the study of mathematical language have been extended greatly by Tarski's student Richard Montague and other linguists working in formal semantics to show that the distinction between mathematical language and natural language may not be as great as it seems. In fact, as our society and worldview has become more mathematical, so have our language and perception.

The fluid realm of direct experience has come to be seen as a secondary, derivative dimension, a mere consequence of events unfolding in the *realer* world of quantifiable and measurable scientific *facts*. It is a curious inversion of the actual, demonstrable state of affairs. Subatomic quanta are now taken to be more primordial and *real* than the world we experience with our unaided senses. The living organism is assumed to derive from a mechanical body whose reflexes and "systems" have been measured and mapped (Abram, 1996, p. 34). However, there is a fundamental mistake in this worldview:

> We conceptually immobilize or objectify the phenomenon only by mentally absenting ourselves from this relation, by forgetting or repressing our sensuous involvement...By linguistically defining the surrounding world as a determinate set of objects, we cut our conscious, speaking selves off from the spontaneous life of our sensing bodies (Abram, 1996, p. 56).

If mathematics and the objective world created by humans has become such a fundamental part of life, perception, and thought we may wonder why and how. History of Math textbook writer Howard Eves suggests that early peoples were too busy staying alive to develop scientific and mathematical traditions, and did not develop math and science very far. Eves suggests that science and math were the products of increased leisure time by some members of agricultural societies (Eves, 1990, p. 2-20). However, much evidence suggests that more important factors in the growth of math were written language and social stratification, and a deep underlying basic change in the way people perceived the world.

The idea that hunter-gatherers would not have had enough leisure time to develop math and science cannot be well founded when we consider mid-20th century studies of the !Kung San tribe of Africa's Kalahari desert, one of the last functional hunter gatherer tribes greatly studied by anthropology. Anthropologists found that the average work day for a San person was 2-3 hours, most of this being devoted to hunting animals or gathering plants, activities considered leisure by many people today. And the San were making a living on a sparse and hostile desert environment– without modern-day population pressures, our hunter-gatherer ancestors presumably enjoyed plenty of "leisure" time, and they were probably much less "busy" than people are today, even if they didn't live as long in most cases (Gowdy, 1997).

But if our ancestors had so much free time, why didn't they create great math and science traditions? The answer is that they simply didn't need to. They probably used words and concepts in a very different way than we do today. Their entire perception of things might have been much different. Where does language come from anyway? Math is a language. The archeological record seems to support that a more complex vocal structure correlated with a relatively rapid increase in the brain size of our ancestors around one million years ago, around the same time that evidence begins showing up that our ancestors were using fire to their advantage. Perhaps words became a way to communicate when we were sitting around fires for long hours in the dark, and couldn't see each other's gestures very well.

Fast forward to ten thousand years ago and language goes through a big transformation- with agriculture comes power, wealth, and the written word. The development of writing, which went hand in handing with increased specialization and stratification in developing agricultural society, enabled the development of the fields of science and mathematics, which have been subsets of language all along. With writing the stage was set for the formal developments of math and science. Anthropologist and

Phenomenologist David Abram (1996) in his book *The Spell of the Sensuous:*

> Learning to read and write with the alphabet engenders a new, profoundly reflexive, sense of self. The capacity to view or dialogue with one's own words after writing them down enables a new sense of autonomy and independence from others, and even from the sensuous surroundings" (p. 153).

Writing, in essence, seems to forge a more absolute and permanent boundary, weight, and power to thoughts and ideas. Of the many scholars who have written about the fundamental differences between oral and literate societies, Walter Ong is perhaps the most widely read theorist on this topic, with the core of his argument being that the technology of communication a society possesses affects the way its people think. Ong observes that the word has great power in oral traditions, even generative power to make things come into being. He also states that in oral societies the separation between subjective and objective categories is not so marked. The creation of the subjective and objective world becomes substantially more elaborate when language is written down, and this process of writing and abstracting also leads to a greater sense of individuality and objectivity (Ong, 1982).

In *Spell of the Sensuous*, David Abram explores the contrast between written and oral in another light. Abram tries to understand the disconnection from nature of the modern world by examining the fact that, for oral peoples, language is more closely tied to the land than it is in literate cultures. He writes, "The ancestral wisdom of the community resides...in the stories, but the stories- and even the ancestors themselves- reside in the land" (Abram, 1996, p. 160). Many tribes, like the Swampy Cree of Manitoba and the Inuit Eskimos say that they were given spoken language by the animals.

According to various oral mythologies around the world, humans and animals all originally spoke the same language (Abram, 1996, p. 87). After spending many months living and studying with an oral culture, Abram reflects:

> I began to wonder if my culture's assumptions regarding the lack of awareness in other animals and in the land itself was less a product of careful and judicious reasoning than of a strange inability to clearly perceive other animals- a real inability to clearly see, or focus upon, anything outside the realm of human technology, or to hear as meaningful anything other than human speech (Abram, 1996, p. 27).

The overall point that Abram makes is that for literate cultures the animate Earth provides much less meaning than it does for oral peoples. A myth from the Dogon, an oral society from West Africa, corroborates this idea. French anthropologist Marcel Griaule, wrote down the myth:

> The jackal...laid hands on the fibers in which language was embodied, that is to say, on his mother's skirt. His mother, the earth, resisted this incestuous action...The incestuous action was of great consequence. In the first place it endowed the jackal with the gift of speech so that ever afterwards he was able to reveal to diviners the designs of God (Griaule, 1965, p. 21-22).

That speech was taken from the Earth implies that language for the Dogon is something more encompassing than human speech. What I mean by more encompassing can be understood by examining Dogon ideas about the creator, which they refer to as Amma. Although the Dogon do have altars to Amma, and invoke the name at the beginning of prayers, Amma's altar is said to be beyond speech. Amma's words are not human words at all, but the real events that happen in the world we live in (Calame-Griaule, 1986, p. 118-119). The conception of language in religions of the book, such as Christianity seems to be much different. In the Bible, God actually speaks creation into being, therefore creating an intimate connection between God, creation, reality, language, and man. In the oral culture, words come from the Earth, while in Christianity, Islam, and Judaism, language comes from God, and existence comes from that language. The story of the Tower of Babel is an example of a defense for the hypothesis that words came from God, since language should be the same for everyone if it came from an absolutely perfect God.

Language in oral cultures however, comes from the Earth. In these cultures spoken language seems to give voice to, and thus enhance and accentuate, the sensorial affinity between humans and the environing earth. In other words, language seems to encourage and augment the participatory life of the senses, while in Western civilization language seems to deny or deaden that life, promoting massive distrust of sensorial experience while valorizing an abstract realm of ideas hidden behind or beyond the sensory appearances. The most prevalent view of language:

> ...at least since the scientific revolution, considers any language to be a set of arbitrary but conveniently agreed upon words, or 'signs,' linked by a purely formal system of syntactic and grammatical rules. Language, in this view, is rather like a *code*; it is a way of *representing* actual things and events in the perceived world, but it has no internal, nonarbitrary connections to that world, and hence is readily separable from it (Abram, 1996, p. 71-72, 77).

In the final analysis, it is impossible to definitively say what language is because the only medium with which we can define language is language itself. We are unable to circumscribe the whole of language within our definition. It is analogous to trying to see our own face- we must use a mirror. Language can't "see" itself either. And the only mirror it can use is made of language (Abram, 1996, p. 73). It is because of this that language can't be "out there" in some perfect realm that has been revealed to us. Language is not and product of human discovery and linguistic meaning is not some ideal and bodiless essence that we arbitrarily assign to a physical sound or word and then toss out into the "external" world.

> Rather, meaning sprouts in the very depths of the sensory world, in the heat of meeting, encounter, and participation. We do not, as children, first enter into language by consciously studying the formalities of syntax and grammar or by memorizing the dictionary definitions of words, but rather by actively making sounds- by crying in pain and laughing in joy, by squealing and babbling and playfully mimicking the surrounding soundscape, gradually entering through such mimicry into the specific melodies of the local language, our resonant bodies slowly coming to echo the inflections and accents common to our locale and community. *We thus learn our native language not mentally but bodily* (Abram, 1996, p. 75).

European civilization's neglect of the natural world and its needs has clearly been encouraged by a style of awareness that disparages sensorial reality, denigrating the visible, audible, and tangible order of things on behalf of some absolute source assumed to exist entirely beyond, or outside, the bodily world. Some historians and philosophers have concluded that the Jewish and Christian traditions, with their other-worldly God, are primarily responsible for civilization's attitude toward the earth, while other thinkers have turned toward the Greek origins of our philosophical tradition, in the Athens of Socrates and Plato.

> A long line of recent philosophers have attempted to demonstrate that Plato's philosophical derogation of the sensible and changing forms of the world- his claim that these are mere simulacra of eternal and pure ideas existing in a non-sensorial realm beyond the apparent world- contributed profoundly to civilization's distrust of bodily and sensorial experience, and to our consequent estrangement from the earthly world around us" (Abram, 1996, p. 94).

It seems that each of these two ancient cultures have contributed to our contemporary estrangement- one seeming to establish the spiritual supremacy of humankind over nature, the other effecting a rational dissociation of the human intellect from the organic world. In many other respects these two traditions were vastly different. However the common ground here is that they were both, from the start, profoundly informed by writing. Both cultures made use of the creative and potent technology which we have come to call "the alphabet" (Abram, 1996, p. 95).

> With the advent of the *aleph-beth*, a new distance opens between human culture and the rest of nature...*A direct association is established between the pictorial sign and the vocal gesture, for the first time completely bypassing the thing pictured.* The evocative phenomena- the entities imaged- are no longer a necessary part of the equation. Human utterances are now elicited, directly, by human-made signs; *the larger, more-than-human life-world is no longer part of the semiotic, no longer a necessary part of the system*" (Abram, 1996, p. 100-101).

Writing, however, was often seen as strange, magical, and even dangerous to the majority of people and it wasn't really until after Plato and his mostly non-literate teacher Socrates (469?-399 B.C.E.) that the sensuous, mimetic, profoundly embodied style of consciousness proper to orality gave way to the more detached, abstract mode of thinking engendered by alphabetic literacy on a large scale. It was Plato who carefully developed and brought to term the collective thought-structures appropriate to the new technology (Abram, 1996, p. 108-109).

For Plato the psyche became that aspect of oneself that is refined and strengthened by turning away from the ordinary sensory world in order to contemplate the intelligible Ideas, the pure and eternal forms that, alone, truly exist. This cognitive stance perpetuated by Plato, in other words, *is* the literate intellect, that part of the self that is born and strengthened in relation to written letters (Abram, 1996, p. 113). This literacy has opened up many doors of perception, but it has seemed to shut the one that connects us to our immediate environment.

> If we no longer experience the enveloping earth as expressive and alive, this can only mean that the animating interplay of the senses has been transferred to another medium, another locus of participation. It is the written text that provides this new locus. For to read is to enter into a profound participation, with the inked marks upon the page. In learning to read we must break the spontaneous participation of our eyes and our ears in the surrounding terrain in order to re-couple those senses upon the flat surface of the page. As nonhuman animals, plants, and even "inanimate" rivers once spoke to our tribal ancestors, so the "inert" letters on the page now speak to us! *This is a form of animism that we*

> *take for granted, but it is animism nonetheless- as mysterious as a talking stone.* And indeed, it is only when a culture shifts its participation to these printed letters that the stones fall silent. Only as our senses transfer their animating magic to the written word do the trees become mute, the other animals dumb…
>
> That alphabetic reading and writing was itself experienced as a form of magic is evident from the reactions of cultures suddenly coming into contact with phonetic writing. Much of the Kabbalah is centered around the conviction that each of the twenty-two letters of the Hebrew aleph-beth is a magic gateway or guide into an entire sphere of existence. Perhaps the most succinct evidence for the potent magic of written letters is to be found in the ambiguous meaning of our common English word "spell." As the roman alphabet spread through oral Europe, the Old English word "spell," which had meant simply to recite a story or tale, took on the new double meaning: on the one hand, it now meant to arrange, in the proper order, the written letters that constitute the name of a thing or a person; on the other, it signified a magical formula (Abram, 1996, p. 131-133).

How did we come so far so fast? If perception, in its depths, is wholly participatory, how could we ever have broken out of those depths into the mechanical and determinate world we now commonly perceive? Language and mathematics, although rooted in perception, nevertheless have a profound capacity to turn back upon, and influence, our sensorial experience. While the reciprocity of perception engenders the more explicit reciprocity of mathematics and language, perception always remains vulnerable to the decisive influence of language and mathematics, as a mother remains especially sensitive to the actions of her child. "It was this influence that led the

American linguist Edward Sapir to formulate his hypothesis of linguistic determination, suggesting that one's perception is largely determined by the language one speaks" (Abram, 1996, p. 90-91).

> Many mathematicians usually don't talk about mathematics because talking is not their thing- their thing is the "doing" of mathematics. Educators talk about teaching mathematics but rarely about mathematics itself...there is a great need to think about the nature of mathematics...mathematics is important...in fact to everyone who is touched in one way or another by the "mathematization" of modern culture. Mathematics is one of the primary ways in which modern technologically based culture understand itself and the world around it...not only are these new technologies reshaping the world, but they are also reshaping the way in which we understand the world...all these new technologies stand on a mathematical foundation...Mathematization involves more than just the practical uses of arithmetic, geometry, statistics...it involves what can only be called culture, a way of looking at the world. Mathematics has had a major influence on what is meant by "truth," for example, or on the question, "what is thought?" Mathematics provides a good part of the cultural context for the worlds of science and technology. Much of that context lies not only in the explicit mathematics that is used, but also in the assumptions and worldview that mathematics brings along with it (Byers, 2010, p. 7).

Of Music and Mathematics

On the surface it may seem surprising to make any connection between music and mathematics. We often think of musicians as the epitome of everything we think a mathematician is not: socially cool, perceptually sensitive, aesthetically creative and so on as opposed to the stereotype of the mathematical or "nerdy" type who we might describe with words such as: socially awkward, often unaware of their sensuous surroundings, and ultra-rational- not creative. However, if we look hard enough, we can see the place where these divergent roads are connected. In fact, we can start right at the beginnings of much of Western mathematical thought and the so-called Pythagoreans. Certainly for them, the connection between music and mathematics was not a casual one; it was an important one, with even mystical implications. Perhaps the reason for this is that both fields are ultimately about pattern creation and recognition. Anthropologically speaking, there are good reasons to believe that music is a much deeper part of human heritage than mathematics and what we now recognize as human language. Math is in this sense a certain rhythm and harmony of thought, as opposed to a rhythm and harmony of sounds. Pythagoras turned the truth on its head though. Pythagoras believed that music and the universe itself were generated by mathematics. The consequences of this are far reaching and perhaps impossible to enumerate or explain completely. On the surface we can at least see that the Pythagoreans mathematized music, giving us musical scales and scores that are abstract and ultimately mathematical.

This connection between music and mathematics has in recent times been substantiated by modern imaging techniques that show which parts of the brain are active while carrying out various tasks. As it turns out the image patterns produced when professional musicians listen to music are extremely similar to those images produced when professional mathematicians work on a mathematical problem. Apparently, expert musicians and mathematicians use the same brain circuits in their respective professions (Devlin, 2000, p. 78). While initially this may seem like an interesting detail, but in light of the writings of certain anthropologists and modern-day shamans this little piece of information leads to some very important observations. While it is important that the brain activity of musicians and mathematicians is so similar, the question of why this is the case is even more fascinating. Perhaps the best reason for the similar brain activity is that both of these activities require some sort of pattern recognition. The difference is in where the patterns are coming from! In the case of the musician *listening* to music, the pattern is coming directly from the senses, namely the ears and body. However, in the case of the mathematician, these patterns are coming from inside of the brain itself. Even when the mathematician is working something out on paper, he or she is using their sense of sight only as a way of keeping track of their mental operations. The real work happens inside of their heads. So while a mathematician is working with patterns, they might be almost oblivious to the sensuous patterns of the world around them, but they are extremely attuned to the "noise" (read thoughts) going on inside of their own mind. The musician on the other hand, is using the same brain circuits in a completely different, almost opposite way. For the musician must focus on the patterns coming from outside of their minds, and thus, instead of tuning out the world like a mathematician, they must tune out the "noise" or voice in their own mind and tune into the sounds outside of them in the real world.

Malidoma writes that in his community in West Africa, "when a person cannot drum, that person has, among other things, a hearing problem. It is hard to create a rhythmical space with this kind of person" (Some, 1993, p. 67).

> Some of the oldest physical artifacts found in human and proto-human excavation sites are musical instruments: bone flutes and animal skins stretched over tree stumps to make drums. Whenever humans come together for any reason, music is there: weddings, funerals...more so in non-industrialized cultures than in modern Western societies, music is and was part of the fabric of everyday life. Only relatively recently in our own culture, five hundred years or so ago, did a distinction arise that cut society in two, forming separate classes of music performers and music listeners. Throughout most of the world and for most of human history, music making was as natural an activity as breathing and walking, and everyone participated. Concert halls, dedicated to the performance of music, arose only in the last several centuries (Levitin, 2006, p. 6).

Mathematics, Magic, and the Creative Mind

Believe it or not, this may be the key that Salinger's character Teddy was talking about, when he speaks of getting out of the finite dimensions. For further elaboration on this idea, we can look to the science of anthropology and the words of mystics and shamans from other cultures. Malidoma Some for one, a West African Dagara medicine man and Western-trained scholar, has much to say about the issue. In his book *The Healing Wisdom of Ancient Africa*, Some discusses his early troubles with learning indigenous knowledge, and blames much of his difficulties on the highly mathematical and logical training he had received since his youth in seminary school. Malidoma explains that in order to enter into a true ritual space, we have to let go of our internal dialogue or description of the world. In essence, our attention must shift from the noise of our thoughts to the sounds of the world around us.

The patterns on the stream's surface as it ripples over the rocks, or on the bark of an elm tree, or in a cluster of weeds, are all composed of repetitive figures that *never exactly repeat themselves*, of iterated shapes to which our senses may attune themselves even while the gradual drift and metamorphosis of those shapes draws our awareness in unexpected and unpredictable directions. In contrast, the mass-produced artifacts of civilization, from milk cartons to washing machines to computers, draw our senses into a dance that endlessly reiterates itself *without variation*. To the sensing body these artifacts are, like all phenomena, animate and even alive, but their life is profoundly constrained by the specific "functions" for which they were built. Once our bodies master these functions, the machine-made objects commonly teach our senses nothing further; they are unable to surprise us, and so we must continually acquire *new* built objects, new technologies, the latest model of this or that if we wish to stimulate ourselves further- all these still carry, like our bodies, the textures and rhythms of a pattern that we ourselves did not devise, and their quiet dynamism responds directly to our senses. Too often, however, this dynamism is stifled with mass-produced structures closed off from the rest of the earth, imprisoned within technologies that plunder the living land. The super-straight lines and right angles of our office architecture, for instance, make our animal senses wither even as they support the abstract intellect; the wild, earth-born nature of the materials- the woods, clays, metals, and stones that went into the building- are readily forgotten behind the abstract and calculable form. It is thus that so much of our built environment, and so many of the artifacts that populate it, seem sadly superfluous and dull when we identify with our bodies and taste the world with our animal senses. (Of course, this is not to say that these artifacts are innocuous: many of them are exceedingly loud, even blaring, for what they lack in variation and nuance they must make up in clamorous insistence, monopolizing the perceptual field.) Whenever we assume the position and poise of the human animal then the entire material world itself seems to come awake and to speak, yet organic, earth-born entities speak far more eloquently than the rest" (Abram, 1996, p. 64-65).

For this reason some says that for a Dagara ritual to take place there must be music going on all the time, from start to finish. Presumably, it is the sensuous nature of music that pulls awareness out of the abstract thought realm and into the sensuous world, and once there, the possibility of ecstatic consciousness (getting out of the finite dimensions) becomes a real possibility (Some, 1999). "Magic" and "song"--especially song like that of birds- are frequently expressed by the same term. The Germanic word for magic formula is *galdr*, derived from the verb *galan*, "to sing," a term applied especially to bird calls" (Eliade, 1964, p. 96-98).

Logical thought on the other hand, makes such awareness all the more difficult, and to most modern people, which is why mystical experience is so often written off as superstition and fantasy. That is because most people have developed such an abstract way of perceiving the world that anything else seems impossible. Perceptions that don't align with our preconceived ideas of the world are disregarded or thought to be mere hallucinations. The problem is, the more logical or mathematical ones way of thinking is, the less likely they are to experience anything like a meaningful ecstatic state, or a state of enhanced or alternate sensory awareness. But what if Salinger is right? What if this place where music and math diverge is the very crossroads where the majority of humans lose something of vital importance and meaning.

The controversial 20th century anthropologist Carlos Castaneda has also been written off by mainstream scientists for his fantastic portrayals of his experiences with his shaman informant, Don Juan, but the logic he elucidates is eerily consistent with the question at hand. Through a series of over ten story length books, Castaneda relates his supposed entry into the shaman's cognitive world. A cognitive world he claims is very different from that of most modern people. The key to entry into the shaman's world according to Castaneda is stopping the internal dialogue and in order to become free of our internal chatter, we must first unseat logic from its place of supremacy in perceiving the world. One of the most important techniques that Castaneda learns is that of "listening to the world." Not even listening to just human made music, but the "music" of the world in general. So the two keys to shamanic awareness correlate exactly with the hypothesis that music and math are two divergent pathways of the same cognitive apparatus. Since it is the same parts of the brain involved in both processes, going one way inhibits going the other way, due to the conservation of energy as much as anything else. The irony perhaps, is that many people (especially mathematicians and scientists) believe that logic and reason give us the most accurate picture of the real world, when in actuality, logic and reason take our direct awareness away from the real world, while music and sound pull our awareness out into it (Casteneda, 1998).

Castaneda's informant, real or imaginary, had much to say about the connections between knowledge, rationality, and awareness. Castaneda explains that it is our the quality of our awareness and energy that determines the type of knowledge one has or can achieve, and that the type of knowledge achieved through thinking and objectifying is only one way of knowledge and can actually hook our attention from gaining knowledge directly, through feats of awareness that might well be called magical or supernatural by mainstream culture. Stopping the internal chatter is the key to accessing the majority of available knowledge, and when this dialogue shuts off, we become aware, through direct perception, that we do not inhabit a world of things. Castaneda (1998) writes, "There is a world of happiness where there is no difference between things because there is no one there to ask about the difference" (p. 20). Castaneda contrasts this world of thought-free awareness with the way that we commonly operate:

> We talk to ourselves incessantly about our world. In fact we maintain our world with our internal talk. And whenever we finish talking to ourselves about ourselves and our world, the world is always as it should be. We renew it, we rekindle it with life, we uphold it with our internal talk. Not only that, but we also choose our paths as we talk to ourselves. Thus we repeat the same choices over and over until the day we die, because we keep on repeating the same internal talk over and over until the day we die. A warrior is aware of this and strives to stop his internal talk...The *internal dialogue* is what grounds people in the daily world. The world is such and such and so and so, only because we talk to ourselves about its being such and such or so and so. The passageway into the world of shamans opens up after the warrior has learned to shut off his internal dialogue…The flaw with words is that they always make us feel enlightened, but when we turn around to face the world they always fail us and we end up facing the world as we always have, without enlightenment. For this reason, a warrior seeks to act rather than to

talk, and to this effect, he gets a new description of the world- a new description where talking is not that important, and where new acts have new reflections. (Castaneda, 1998, p. 61, 117, 124).

For Malidoma, there is a different sort of rhythm that happens in a ritual space. He writes, "*Ritual is not compatible with the rapid rhythm that industrialism has injected into life*. So whenever ritual happens in a place commanded or dominated by a machine, ritual becomes a statement against the very rhythm that feeds the needs of that machine. It makes no difference whether it is a political machine or otherwise" (Some, 1993, p. 19). He later writes, "Wherever there is technology, there is a general degeneration of the spiritual. This is because the Machine is the specter of the Spirit, and in such a state, it does not serve because it can't serve. It needs servants" (Some, 1993, p. 66).

Some talks about the great difficulties he initially encountered at his initiation due to the fact that he was unable to *see*, or enter what is often referred to as the state of trance or ecstasy. Malidoma said that his problems stemmed from his Western way of perceiving the world, engrained through 20 years of forced formal schooling in mathematics, reading, and writing. He writes that he had to forget his logical way of seeing the world in order to *see*. He added that once he learned how to see, or perceive in the traditional fashion, he found an overwhelming love, ecstasy, and sense of meaning that was all beyond logic and words. He writes, "Human words cannot encode meaning because human language has access only to the shadow of meaning" (Some, 1999, p. 223).

Some explains the difference between the perception and knowledge he learned in his Western education versus the training he received in his traditional initiation. He expresses in his book, *Of Water and the Spirit that* many of his ancestors feared his traditional initiation would kill him because the literate teachings he picked up in school had ruined his memory. Malidoma explains the difference between the knowledge of his traditionally oral culture, and that which he learned at seminary school as being like the difference between a liquid and a solid:

> The contrast between this state of mind and what I had been accustomed to at the seminary was the same as the difference between liquid and solid...What I was learning made sense only in terms of relationship. It was not fixed, even when it appeared to be so...By contrast, I could see that the Western knowledge I had been given had the nature of a solid because it is wrapped in logical rhetoric to such a degree that it is stiff and inflexible (Some, 1995, p. 201).

Another question might be, "what is the importance of being aware of the world around us versus being more aware of our own thoughts." For traditional shamans, the answer is easy. In almost all cultures where "shamanic type" practitioners have been found, their main function is to act as a go-between or mediator between the human and more-than-human (say spiritual or natural) worlds. The shaman must be acutely attuned to the sensuous surroundings for this very reason. Without this awareness, they wouldn't make any kind of good mediator because they wouldn't even be very aware of the more-than-human world. In contrast, their job is to be hyper aware of the more-than-human world since they must balance what has become unbalanced in order to heal their patients and to keep their societies as a whole in balance with the natural world. In many human societies, the traditional magician or medicine person functions primarily as a go between for the human and natural world, a person who transcends conventional borders. Anthropologist David Abram (1996) writes:

> Only by temporarily shedding the accepted perceptual logic of his culture can the sorcerer hope to enter into relation with other species on their own terms; only by altering the common organization of his senses will he be able to enter into a rapport with the multiple nonhuman sensibilities that animate the local landscape. It is this, we might say, that defines a shaman: the ability to readily slip out of the perceptual boundaries that demarcate his or her particular culture- boundaries reinforced by social customs, taboos, and most importantly, the common speech or language- in order to make contact with, and learn from, the other powers in the land. His magic is precisely this heightened receptivity to the meaningful solicitations- songs, cries, gestures- of the larger, more-than-human field (p. 8-9).

As a contrast, we can see in modern cultures, perhaps with the loss of the shaman as a religious leader, there is a great imbalance going on between human beings and the natural world. The societies of the world are by contrast largely led by people who are good at grammar and mathematics, meaning people who are very aware of the abstractions in their own minds, and as a consequence of the law of conservation of energy, probably not nearly so aware of the sensuous world around them. The fate of the world now seems to be in the hands and minds of people who are mainly aware of their ideas about the world and not the world itself. The further science and technology progress, the more humans are able to make the world fit their logical expectations, which only reinforces this distanced perceptual stance from our direct surroundings.

The idea of feeling separate from creative divine forces comes from thinking itself, because rational thought uses boundaries imaginatively to create symmetries and tangents of all kinds. The philosophical question: "Is mathematics created or discovered?" is actually intimately related to the mystical question: "Are humans and reality created, and if so by what?" These two questions are intimately related because the same perceptive, cognitive processes which create for us an objective and logical world, the world of language and mathematics; this very same perceptive stance gives us the feeling of isolation, separation from what religious people might call God, or scientific thinkers might call nature.

Conclusions: Creating a Proof

Proving that math is invented is possible by a creative use of set theory. If we can show that all of mathematics is a subset of something else which we can know to be invented by humans, then we can deduce that mathematics has been invented by humans as well. Let perception be the largest set of knowledge, containing all that is felt, dreamt, sensed, and thought, and each of these (feelings, dreams, sensations, and thoughts) being subsets of perception, intersecting each other in various places, but none of them completely containing the other. Mathematics we could say is a proper subset, not only of human perception, but also of human thought. This means that there is no way for our bodies to know mathematics purely through dreams, emotions, or sensations.

In Mathematics, thought must also always be involved. Even the so-called *number sense* needs a thought to claim existence. We can easily prove that all of thought is created by humans since thoughts happen in the human mind. Even human perception is a creative, imaginative act, as evidenced by different human cultures and languages throughout time and space. The categorizing and objective nature to perception is a creative act used by beings to improve their lot in life. This act does not actually render reality into categories or objects. This creative act of categorizing is the essence of mathematics, logic, and rational thought. Since these categories can exist only in a mind capable of such generalizations, such as that of the human being, we can safely say that these categories are made up by the minds which they inhabit. The creative use of such categories is the foundation of all mathematics, language, and science. In this sense, we do *discover* a world of objects and reason, including mathematical ones, but this world of object and reason is not discovered in the world around us, but inside of our own heads and by way of our own thoughts. The fact that these thoughts can be shared and shown to have certain relationships with each other and the physically "measured" world which can be agreed upon by almost everyone does not make mathematics any more real or discovered. It only points to math's metaphoric distance from reality, and therefore its ease of mental manipulation. Math in this sense is a field of "perfect objects," and since any object exists only in the mind, math exists especially in the mind and not in the world.

> Before all else, we are perceivers, but to perceive everything would quickly kill us. Infinity is more blinding than the sun. So we see by not looking. We hear by not listening. Faced with the mystery of infinity, we think using the shield and sword of our minds: zero and one, boundary and essence. From this original mental objectification comes all duality: heaven and hell, good and evil, object and subject, science and art. But all thoughts and fields of study- from abstract algebra to anthropology- are products of our imaginations and creations of our minds. We did not create the sun. We created the idea of the sun. We did not

create our creator either, but we've created the idea of a creator. We have also created the ideas of creation and discovery. Discoveries in math and science, no matter how complex, beautiful, or useful are really mental creations obtained by applying formal rigorous methodologies to agreed-upon ideas. All of this type of truth is relative. If we discover truth at all as human beings, it is more in the way that we perceive and dream the boundless, fluid energy of the universe. Thought always diminishes the absolute to something tangible and relative. In reality, no boundaries exist, only a fluid flowing and perhaps infinite field of energy. Even if we draw a boundary around ourselves, it would be like saying we are not taking part in the creation of all that we think, perceive, and feel. It is the same as denying that we exist in the very flux of reality that we are speaking of. It is the same as denying that we are alive at all. And we are alive, after all, aren't we? Don't think about it. Just breathe. (Leach, unpublished, 2011).

References

Abram, David. (1996). *The spell of the sensuous.* New York: Pantheon Books

Aristotle. (2008[350 B.C.]). *Metaphysics.* Cosimo Classics.

Burnet, J. (2010[1892]). *Early Greek philosophy.* Kessinger Publishing Company.

Byers, W. (2010). *How mathematicians think: Using ambiguity, contradiction, and paradox to create mathematics.* Princeton University Press.

Calame-Griaule, G. (1986[1965]). *Words and the dogon world.* Philadelphia: The Institute for the Study of Human Issues.

Carnap, R. (1931). The Logicist Foundation of Mathematics. *Philosophy of mathematics, 2nd Edition.*

Castaneda, C. (1998.) *The wheel of time: The shamans of ancient Mexico, their thoughts about life, death and the universe.* Los Angeles, California: LA Eidolona Press.

Cicero. (2007[45 B.C.]). *Tusculan disputations.* Neeland Media.

Devlin, K. (2000). *The math gene: How mathematical thinking evolved and why numbers are like gossip.* Great Britain: Basic Books.

Devlin, K. (2005). *The math instinct: Why you're a mathematical genius (along with lobsters, birds, cats, and dogs).* New York: Thunder's Mouth Press.

Dudley, U. (1997). *Numerology, or what Pythagoras wrought.* USA: The Mathematical Association of America.

Eliade, M. (1964). *Shamanism: Archaic techniques of ecstasy.* Princeton University Press.

Ernest, P. (2008). Is mathematics created or discovered?

http://people.exeter.ac.uk/PErnest/pome12/article2.htm

Eves, H. (1990). *An introduction to the history of mathematics*. Brooks Cole.

Field, H. H. (1980). *Science without numbers: A defense of nominalism*. Princeton University Press.

Franklin, J. (2009). Aristotelean Realism. *Handbook of the philosophy of science*. 101-153.

Gowdy, J. (1997). *Limited wants, unlimited means: A reader on hunter-gatherer economies and the environment*. Washington, D.C.: Island Press.

Griaule, M. (1965). *Conversations with Ogotemmeli: An introduction to dogon religious ideas*. London: Oxford University Press.

Hawking, S. (2007). *God created the integers: The mathematical breakthroughs that changed history*. Philadelphia: Running Press Book Publishers.

Hersh, R. (1997). *What kind of thing is a number?* http://www.edge.org/3rd_culture/hersh/hersh_p1.html

Hilbert, D. (1999). *Principles of mathematical logic*. American Mathematical Society.

Jones, E. (1989). *Reading the Book of Nature: Phenom. The Study of Creative Expression.* Ohio University Press.

Kleene, S. (1971). *Introduction to metamathematics.* Netherlands: North-Holland Publishing Company.

Lakoff, G., & Nunez R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.

Lama, D. (2009). *The art of happiness: A handbook for living*. Riverhead Hardcover.

Levitin, D. J. (2006). *This is your brain on music: The science of a human obsession.*

New York: Plume Printing.

Marty, A. T. (1971). Essay on Aldous Huxley.

Marty, A. T. (1971). On not confusing the facts with truth. *New England Journal of Medicine.*

Matt, D.C. (1995). *The essential kabbalah.* New York: Quality Paperback Book Club.

Maziars, E. (1969). Problems in the philosophy of mathematics. *Philosophy of Science, 36.*

Merleau-Ponty, M. (1962). *Phenomenology of perception.* London: Routledge & Kegan Paul.

Mill, J. S. (1973). *The collected works of John Stuart Mill.* New York: Penguin Books.

Nietzsche, F. (1977). *The portable Nietzsche.* New York: Penguin Books.

Ong, W. (1982). *Orality and literacy: The technologizing of the word.* London and New York: Methuen.

Poe, E. A. (1989). *The complete tales and poems of Edgar Allan Poe.* New York: Penguin Books.

Putnam, H. (1975). *Mathematics, matter and method. Philosophical papers, vol.1.* Cambridge: Cambridge University Press.

Russell, B. (1967). *The history of western philosophy.* New York: Simon & Schuster.

Russell, B. (1993). *Introduction to mathematical philosophy.* New York: Dover Publications.

Russell, B. (2010). *The principals of mathematics.* New York: Nabu Press.

Salinger, J. D. (1981). *Nine Stories.* Boston: Little, Brown and Company.

Some, M. P. (1993). *Ritual: Power, healing, and community.* New York: Penguin.

Some, M. P. (1995). *Of water and the spirit: Ritual, magic, and initiation in the life of an african shaman.* Arkana: Penguin.

Tegmark, M. (2008). The mathematical universe. *Foundations of Physics, 38.* (pgs. 101-150)

Tunneshende, M. (2004). *Twilight language of the nagual: The spiritual power of shamanic dreaming.* Inner Traditions/Bear & Company.

Whitehead, A. N., & Russel, B. (2009). *Principia mathematica.* New York: Merchant Books.