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Actions of $SL(n, \mathbb{Z})$ on homology spheres

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Abstract

Any continuous action of $SL(n, \mathbb{Z})$, where $n > 2$, on a r-dimensional mod 2 homology sphere factors through a finite group action if $r < n - 1$. In particular, any continuous action of $SL(n + 2, \mathbb{Z})$ on the n-dimensional sphere factors through a finite group action.

1   Introduction

In this paper, we study the actions of $SL(n, \mathbb{Z})$ on spheres and, more generally, actions on homology spheres. The group $SL(n, \mathbb{Z})$ acts on the $(n-1)$-dimensional sphere via the linear action on vectors in $\mathbb{R}^n$. This action is minimal in the following sense.

Theorem 1.1. *Any continuous action of $SL(n, \mathbb{Z})$, where $n > 2$, on a r-dimensional mod 2 homology sphere factors through a finite group action if $r < n - 1$.*

Since all spheres are mod 2 homology spheres, any continuous action of $SL(n+2, \mathbb{Z})$ on a n-dimensional sphere factors through a finite group action. This result supports the following conjecture of Farb and Shalen (see [FS]).

Conjecture. *Any smooth action of a finite-index subgroup of $SL(n, \mathbb{Z})$, where $n > 2$, on a r-dimensional compact manifold factors through a finite group action if $r < n - 1$.*

This conjecture is an analogue of a special case of one of the central conjectures in the Zimmer program (see [Zi]). Theorem 1.1 may also be viewed as a (partial) generalization of Witte's theorem in [Wi].

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Theorem 1.2 (Witte). *If Γ is a subgroup of finite index in $SL(n, \mathbb{Z})$ with $n \geq 3$, then every continuous action of Γ on the circle factors through a finite group action.*

We cannot obtain our result for finite-index subgroups because we rely heavily on the existence of finite order elements in $SL(n, \mathbb{Z})$, and there are subgroups of finite index in $SL(n, \mathbb{Z})$ that have no elements of finite order (see Corollary 6.13 in [Rg]).

The paper is organized in the following manner. In section 2, we prove the existence of certain desirable finite-order elements in $SL(n, \mathbb{Z})$ and reduce the problem to a problem of a finite group action. In section 3, we use some classical results from the theory of compact transformation groups to prove that certain groups cannot act effectively (faithfully) on homology spheres and show that these results imply Theorem 1.1.

In section 4 we observe that the action of $SL(n, \mathbb{Z})$ is trivial on low-dimensional spheres. This result is analogous to the following theorem by Weinberger (see [We]).

Theorem 1.3 (Weinberger). *The discrete group $SL(n, \mathbb{Z})$, with $n \geq 3$, can act smoothly on the torus T^m, $m < n$, only trivially.*

Definitions and Notation

For us a n-dimensional mod 2 homology sphere is a locally compact, finite-dimensional Hausdorff space X such that $H^*(X; \mathbb{Z}_2) = H^*(S^n; \mathbb{Z}_2)$, where H^* means cohomology with compact support and S^n is the n-dimensional sphere. Furthermore, the n-dimensional mod 2 homology sphere is also a n-dimensional generalized manifold—a finite-dimensional metric **ANR** with $H^n(X, X - x; \mathbb{Z}) = \mathbb{Z}$ and $H^i(X, X - x; \mathbb{Z}) = 0$ for all $i \neq n$, for all points $x \in X$ (see [Bd2] for equivalent conditions).

In section 2, we will prove the existence of certain finite groups in $SL(n, \mathbb{Z})$; for this we need manageable notation to deal with matrices. Consider the standard basis of vectors in $\mathbb{R}^n$ and let $(1) = (1, 0, 0, .., 0)$, $(2) = (0, 1, 0, ..., 0)$,..., $(n) = (0, 0, 0, ..., 1)$, $(-1) = (-1, 0, 0, .., 0)$, $(-2) = (0, -1, 0, ..., 0)$,..., and $(-n) = (0, 0, 0, ..., -1)$. The permutation (i, j), with $1 \leq i, j \leq n$, is the matrix that takes the vector (i) to the vector (j) and the vector (j) to (i).

For example, we have the following matrices in $SL(4, \mathbb{Z})$.

$$(1, -1)(2, -2) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, (1, 2)(3, 4) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

This notation is unconventional, but it will simplify matrix multiplication by reducing it to the standard procedure of multiplying permutations. Also, let I and $-I$ be the identity matrix and the negative identity matrix respectively.

2   Almost simple groups and $SL(n, \mathbb{Z})$

Definition 2.1. An element g in a group G is *central* if g commutes with every element in G. A subgroup H of the group G is central if every element of H is central. Also let Z denote the *center*, the subgroup of all central elements.

It will be clear from the context what is meant by Z. For example, G_1/Z and G_2/Z are factor groups obtained when the groups G_1 and G_2 are quotiented by their respective centers.

Definition 2.2. A group G is *almost simple* if every normal subgroup is either finite and central, or has finite index in G.

The Margulis normal subgroups theorem (see [Mg]) asserts that an irreducible lattice in a semi-simple Lie group with $\mathbb{R}$-rank ≥ 2 is almost simple. In particular, $SL(n, \mathbb{Z})$ is almost simple for $n \geq 3$.

The following lemma follows easily from Margulis' Theorem and the definition of an almost simple group.

Lemma 2.3. *Let $\phi : SL(n, \mathbb{Z}) \to H$ be a homomorphism where $n \geq 3$. If $\phi(g) = 1$ for some non-central element g, then the kernel of ϕ is a finite-index subgroup, and therefore, ϕ factors through a homomorphism of a finite group.*

So to prove Theorem 1.1, it suffices to show that a finite-order, non-central element acts trivially. In the next section we prove that there always exists an involution which acts trivially. Now we show that subgroups containing non-central involutions always exist.

Theorem 2.4. *If n is odd, there exists a subgroup of $SL(n, \mathbb{Z})/Z$ isomorphic to $(\mathbb{Z}_2)^{n-1}$. If n is even, there exists a subgroup of $SL(n, \mathbb{Z})/Z$ isomorphic to $(\mathbb{Z}_2)^{n-2}$.*

Proof. First note that Z is just the identity matrix when n is odd and only contains the identity and the negative identity matrices when n is even. The theorem is very easy to prove when n is odd. Simply count the elements in the subgroup of diagonal matrices that have only ± 1 in the diagonal entries. This number is given by the following formula (which is true for even and odd n).

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots = 2^{n-1}$$

So we have an abelian subgroup containing 2^{n-1} elements, all of order 2. By the classification of finite abelian groups, this group is isomorphic to $(\mathbb{Z}_2)^{n-1}$. This proves that there is always exists a subgroup of $SL(n, \mathbb{Z})$ isomorphic to $(\mathbb{Z}_2)^{n-1}$.

When n is even, there are 2^{n-2} cosets of diagonal matrices. So we have a subgroup isomorphic to $(\mathbb{Z}_2)^{n-2}$ in $SL(n, \mathbb{Z})/Z$. $\square$

Lemma 2.5. *There exists a subgroup of $SL(4, \mathbb{Z})/Z$ isomorphic to $(\mathbb{Z}_2)^3$.*

Proof. The above theorem states that there are 4 cosets of diagonal matrices. We observe that the equivalence class of $(1, 2)(3, 4)$ commutes with the equivalence classes of diagonal matrices in $SL(4, \mathbb{Z})/Z$.

$(1, -1)(2, -2)(1, 2)(3, 4) = (1, -2)(3, 4) = (1, 2)(3, 4)(1, -1)(2, -2)$

$$(1, -1)(3, -3)(1, 2)(3, 4) = (1, 2, -1, -2)(3, 4, -3, -4) =$$
$$- I(1, -2, -1, 2)(3, -4, -3, 4) = (1, 2)(3, 4)(1, -1)(3, -3)$$

All other cases are similar or easier. So we have a subgroup isomorphic to $(\mathbb{Z}_2)^3$ in $SL(4, \mathbb{Z})/Z$. $\square$

Note that for any action of $SL(n, \mathbb{Z})$ or $SL(n, \mathbb{Z})/Z$ on an orientable, generalized manifold M, the induced action of the subgroups in Theorem 2.4 and Lemma 2.5 on M is orientation-preserving. This is because $SL(n, \mathbb{Z})$ is a perfect group, that is, it is equal to its commutator subgroup. Any homomorphism from a perfect group to an abelian group must be trivial.

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In particular, any homomorphism from $SL(n, \mathbb{Z})$ to $\mathbb{Z}_2$ must be trivial, and so, any action of $SL(n, \mathbb{Z})$ on an orientable, generalized manifold must be orientation-preserving. Then the same result holds for subgroups of $SL(n, \mathbb{Z})$ and $SL(n, \mathbb{Z})/Z$.

We now introduce new notation which will greatly simplify the proof and the statement of the next lemma. In $SL(n, \mathbb{Z})$, let $-I_i^j = (i, -i)(i+1, -i-1)...(j, -j)$, where $1 \leq i < j \leq n$ and $j - i$ is odd. So when n is even, $-I = -I_1^n$ and the center $Z = \langle -I_1^n \rangle$.

Lemma 2.6. *Consider the action of $SL(n, \mathbb{Z})/\langle -I_1^n \rangle$, for even $n \geq 4$, on an orientable generalized manifold M and suppose that $-I_{n-1}^n$ acts on M with a non-empty fixed point set. Then there is an induced action of a subgroup isomorphic to $SL(n-2, \mathbb{Z})/\langle -I_1^{n-2} \rangle$ on the fixed point set of $-I_{n-1}^n$.*

Proof. Let F be the fixed point set of $-I_{n-1}^n$. Now all elements of $SL(n, \mathbb{Z})$ that leave the vectors $(n-1)$ and (n) fixed commute with $-I_{n-1}^n$. This implies that there is a subgroup isomorphic to $SL(n-2, \mathbb{Z})$ in the centralizer of $-I_{n-1}^n$, and therefore, it acts on F modulo the action of $-I_1^n$. The action of $-I_{n-1}^n$ of F is trivial, and so, the action of $-I_1^n$ on F is equivalent to the action of $-I_1^{n-2}$ on F. This proves that there is an action of $SL(n-2, \mathbb{Z})$ on F modulo the action of $-I_1^{n-2}$; in other words, there is an induced action of a subgroup isomorphic to $SL(n-2, \mathbb{Z})/\langle -I_1^{n-2} \rangle$ on the fixed point set of $-I_{n-1}^n$. $\square$

3   Actions on homology spheres

We begin this section by recalling some very famous theorems from the theory of compact transformation groups. Most of the results stated below are true for all primes but we only consider the case when the prime is two. Good references for these theorems are [Bd1], [Bo], [S1], and [S2].

Theorem 3.1 (Smith). *The group $\mathbb{Z}_2 \times \mathbb{Z}_2$ cannot act semifreely and effectively on any mod 2 homology sphere, that is, no two elements in the group $\mathbb{Z}_2 \times \mathbb{Z}_2$ have the same fixed-point set (which may be empty) when the action is effective.*

As a corollary, we get the result that the group $\mathbb{Z}_2 \times \mathbb{Z}_2$ cannot act freely on any mod 2 homology sphere.

Theorem 3.2 (Smith). *When $\mathbb{Z}_2$ acts effectively on a n-dimensional mod 2 homology sphere X, the fixed set (pointwise) is a m-dimensional mod 2 homology sphere with $m < n$.*

Furthermore since X is a generalized manifold, one can distinguish between actions that preserve orientation and actions that reverse orientation. If orientation is preserved, $n - m$ is even (see [Bd2]).

The following theorem is due to Smith and is well known. In [S2] Smith proves that any action of $(\mathbb{Z}_2)^{n+2}$ on a n-dimensional mod 2 homology sphere cannot be effective. Theorem 3.3 follows from his proof and other arguments in [S1]. We provide a simple proof below.

Theorem 3.3. *Any orientation-preserving action of $(\mathbb{Z}_2)^{n+1}$ on a n-dimensional mod 2 homology sphere cannot be effective.*

Proof. We prove this theorem by induction. The result easily follows for the 0-dimensional sphere—S^0. Every orientation-preserving action of $\mathbb{Z}_2$ on S^0 is trivial. Now assume that the theorem is true for all $k \le n - 1$, that is, assume that any orientation-preserving action of $(\mathbb{Z}_2)^{k+1}$ on a k-dimensional homology sphere is not effective for all $k \le n - 1$.

Suppose that an effective, orientation-preserving action of $(\mathbb{Z}_2)^{n+1}$ on a n dimensional mod 2 homology sphere exists. Theorem 3.1 asserts that this action cannot be free and so there is some element acting with a non-empty fixed-point set. By theorem 3.2, we know that this fixed set is always a m-dimensional mod 2 homology sphere with $m < n$. Let h be the element with the property that the fixed-point set of h, $F(h)$, is maximal among all other fixed sets of elements in $(\mathbb{Z}_2)^{n+1}$, that is, $F(h)$ is not properly contained in the fixed set of any other element in $(\mathbb{Z}_2)^{n+1}$. Let m be the dimension of $F(h)$; note that $m \le n - 2$ since the action is orientation-preserving.

Let G be the subgroup isomorphic to $(\mathbb{Z}_2)^n$ such that $h \notin G$. Now because the action is abelian, there is an induced action of G on $F(h)$, the m-dimensional mod 2 homology sphere. This action of G on $F(h)$ may not be orientation-preserving, but in any case, there is an index 2 subgroup isomorphic to $(\mathbb{Z}_2)^{n-1}$ with an orientation-preseving action on $F(h)$. The induction hypothesis implies that this action cannot be effective and so there exists a $g \in G$ such that the fixed set of g contains $F(h)$. The maximality of $F(h)$ implies that h and g have the same fixed-point set and this contradicts Theorem 3.1. So any orientation-preserving action of $(\mathbb{Z}_2)^{n+1}$ on a n-dimensional mod 2 homology sphere cannot be effective. $\square$

Note that Theorem 3.3 is sharp, that is, there are effective actions of $(\mathbb{Z}_2)^n$ on n-dimensional spheres that preserve orientation. Theorem 2.4 shows that there exists a subgroup of $SL(n+1, \mathbb{Z})$ isomorphic to $(\mathbb{Z}_2)^n$ and $SL(n+1, \mathbb{Z})$ acts effectively on the n-dimensional sphere via the linear action on vectors in $\mathbb{R}^{n+1}$. This effective action of $SL(n+1, \mathbb{Z})$ is orientation-preserving and so the action of its subgroup isomorphic to $(\mathbb{Z}_2)^n$ is also effective and preserves orientation.

We are now ready to prove the main theorem.

Proof of Theorem 1.1. The theorem is very easy to prove in the case when $r = 0$ because the 0-dimensional mod 2 homology sphere is just the set of two points—S^0. Now because the group $SL(n, \mathbb{Z})$ is perfect, it has an orientation-preserving action on S^0, and all orientation-preserving actions on S^0 are trivial.

So assume that $r \geq 1$. There are two cases to consider, $-I$ does not act trivially and when $-I$ acts trivially.

CASE 1: $-I$ does not act trivially.

In this case, Theorem 2.4 asserts that there is a subgroup of $SL(n, \mathbb{Z})$ isomorphic to $(\mathbb{Z}_2)^{n-1}$, and then it follows from general principles that the action of this subgroup is orientation-preserving. Theorem 3.3 implies that $(\mathbb{Z}_2)^{n-1}$ cannot act effectively on any r-dimensional mod 2 homology sphere, where $r < n-1$. So a non-central involution acts trivially and by the remark made after Lemma 2.3, this is sufficient to prove that the action of $SL(n, \mathbb{Z})$ on any r-dimensional mod 2 homology sphere, where $r < n - 1$, factors through a finite group action. Note that if n is odd, $-I$ does not belong to $SL(n, \mathbb{Z})$, and so, the theorem has been established for odd integers greater than 2.

CASE 2: $-I$ acts trivially.

When $-I$ acts trivially, n must be even and the action factors through an action of $SL(n, \mathbb{Z})/Z$. In this case, rather than showing only that some non-central element of $SL(n, \mathbb{Z})$ acts trivially, we will prove the more precise conclusion that some diagonal element conjugate to $-I_1^2$ acts trivially.

For $n = 4$, Lemma 2.5 states that there is a subgroup isomorphic to $(\mathbb{Z}_2)^3$ in $SL(4, \mathbb{Z})/Z$ and the result follows in a similar fashion. So every action of $SL(4, \mathbb{Z})$ on a r-dimensional mod 2 homology sphere, with $r < 3$, factors through an action of a finite group. In the next section, we will prove that the action of $SL(4, \mathbb{Z})$ on low-dimensional mod 2 homology spheres—dimension

less than 3—is trivial. This implies that the action of the subgroup isomorphic to $(\mathbb{Z}_2)^3$ in $SL(4, \mathbb{Z})/Z$ on mod 2 homology spheres with dimension less than 3 must be trivial. So the equivalence classes of diagonal elements conjugate to $-I_1^2$ act trivially; call these elements diagonal elements of length 2. In particular, for every action of $SL(4, \mathbb{Z})/Z$ on a mod 2 homology spheres with dimension less than 3, there exists an equivalence class of a diagonal element of length 2 which acts trivially.

Now suppose that for every action of $SL(2k - 2, \mathbb{Z})/Z$ on a mod 2 homology spheres with dimension less than $2k - 3$, there exists an equivalence class of a diagonal element of length 2 which acts trivially. Consider the action of $SL(2k, \mathbb{Z})/Z$, for $k > 2$, on a r-dimensional mod 2 homology sphere, where $r < 2k - 1$, and suppose that no class of diagonal elements of length 2 acts trivially. The action of the equivalence classes of a diagonal elements of length 2 cannot be free, by the corollary to Theorem 3.1, and so, without loss of generality assume that the fixed point set of the class represented by $-I_{n-1}^n$ is maximal among all the fixed point sets of equivalence classes of a diagonal elements of length 2. This fixed point set must be a mod 2 homology sphere with dimension m, where $m < 2k - 3$, since the action of $-I_{n-1}^n$ is orientation-preserving. Lemma 2.6 implies that we have an action of $SL(2k - 2, \mathbb{Z})/Z$ on this m-dimensional mod 2 homology sphere and it now follows that there is an equivalence class of a diagonal element of length 2 which acts trivially on this m-dimensional mod 2 homology sphere. The maximality of the fixed point set of the class of $-I_{n-1}^n$ implies that two elements have identical fixed point sets and this contradicts Theorem 3.1. Therefore, there exists an equivalence class of a diagonal element of length 2 in $SL(n, \mathbb{Z})/Z$ that acts trivially, and it follows that some diagonal element conjugate to $-I_1^2$ in $SL(n, \mathbb{Z})$ acts trivially.

So when n is even and greater than 3, there exists a non-central involution which acts trivially. By the remark made after Lemma 2.3, this is sufficient to prove that the action of $SL(n, \mathbb{Z})$ on a r-dimensional mod 2 homology sphere, where $r < n - 1$, factors through an action of a finite group. $\square$

4   Actions on low-dimensional spheres

In this section we prove that any continuous action of $SL(n, \mathbb{Z})$, with $n > 2$, on the circle and any continuous action of $SL(n, \mathbb{Z})$, with $n > 3$, on S^2 must be trivial. Since $SL(n, \mathbb{Z})$ is perfect, actions on S^0 are trivial.

It follows from the proof of Theorem 1.1 that the actions of $SL(n, \mathbb{Z})$, with $n > 2$, on the circle and the actions of $SL(n, \mathbb{Z})$, with $n > 3$, on S^2 must factor through actions of finite groups, and it is well known that finite group actions on S^1 and S^2 are conjugate to linear actions of finite subgroups in $O(2)$ and $O(3)$ respectively (see [Ed]). Therefore, it suffices to prove the following theorem.

Theorem 4.1. *Every homomorphism from $SL(3 + m, \mathbb{Z})$ to $O(2)$ and every homomorphism from $SL(4 + m, \mathbb{Z})$ to $O(3)$, with $m \geq 0$, is trivial.*

Proof. First note that Theorem 1.1 implies that the images of the homomorphisms must necessarily be finite. By a famous result of Milnor, Bass, and Serre, every homomorphism of $SL(n, \mathbb{Z})$ with finite image factors through $SL(n, \mathbb{Z}_k)$ for some integer $k > 1$ (see [BMS]). It is well known that the group $SL(n, \mathbb{Z}_k)$ has no abelian quotients. It is also known, perhaps not well known, that the smallest non-trivial quotient group of $SL(n, \mathbb{Z}_k)$ has order at least $2^{n(n-1)/2} \Pi_{i=2}^{n-1}(2^i - 1)$ (see [FM]).

Now consider a homomorphism $h : SL(3 + m, \mathbb{Z}) \to O(2)$. Compose h with the determinant map from $O(2)$ to $\mathbb{Z}_2$. Every homomorphism from $SL(3+m, \mathbb{Z})$ to $\mathbb{Z}_2$ is trivial because the congruence subgroups $SL(3+m, \mathbb{Z}_k)$ have no abelian quotients. This implies that h factors through $SO(2)$. So h must be trivial because all finite subgroups of $SO(2)$ are abelian, and again, congruence subgroups $SL(3 + m, \mathbb{Z}_k)$ have no abelian quotients.

Now consider a homomorphism $g : SL(4+m, \mathbb{Z}) \to O(3)$. The arguments above show that it suffices to prove that homomorphisms from $SL(4+m, \mathbb{Z})$ to $SO(3)$ are trivial. All finite subgroups of $SO(3)$ have been classified—they are all cyclic groups, all dihedral groups, and all subgroups of the polyhedral groups that have order 12, 24, or 60. The smallest non-trivial quotient group of $SL(4+m, \mathbb{Z}_k)$ has order at least 1344 and so subgroups of the polyhedral groups cannot be the images of g. Since the congruence subgroups $SL(4 + m, \mathbb{Z}_k)$ have no abelian quotients, it follows that they have no solvable quotients, and therefore, the cyclic and dihedral groups cannot be the images of g also. So every homomorphism from $SL(4 + m, \mathbb{Z})$ to $O(3)$ must be trivial. $\square$

In general, continuous actions on higher-dimensional spheres are not conjugate to linear actions. In [Bg] Bing constructed a continuous $\mathbb{Z}_2$ action on S^3 with Alexander's horned sphere as a fixed-point set. This is impossible for a linear $\mathbb{Z}_2$ action and so Bing's action is not conjugate to any linear action.

It is conjectured that every smooth action of a compact Lie group on S^3 is conjugate to a linear action. However, non-linear smooth actions exist on S^n for every $n > 3$ (see [Gd]).

It is known that every homomorphism from $SL(n, \mathbb{Z})$ to $GL(r + 1, \mathbb{R})$, with $r < n - 1$, is trivial. This fact is not well known and the author could not find a reference. A nice proof of this result was shown to the author by Dave Morris, formerly Dave Witte. In particular, this implies that every linear action of $SL(n, \mathbb{Z})$ on S^r, with $r < n-1$, must be trivial. We conjecture that every continuous action of $SL(n, \mathbb{Z})$ on a r-dimensional mod 2 homology sphere, with $r < n - 1$, must be trivial.

Acknowledgments

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