UNBALANCED MANUAL FLOW LINE OPERATING CHARACTERISTICS

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A B S T R A C T

UNBALANCED STEADY AND NON-STEADY STATES OPERATING CHARACTERISTICS FOR MANUAL UNPACED PRODUCTION LINES

This thesis studies the operating behaviour of the manual unpaced lines, which are the most important of the flow lines' systems. The lines examined are unbalanced and six types of imbalance are considered, namely, the imbalances of mean service times, coefficients of variation (Covars), buffers' capacities, means and Covars, means and buffers, and Covars and buffers. It is argued that the deep understanding of the behavioural characteristics of such lines, contributes towards the achievement of practical solutions to many of their problems. The lines are simulated under both steady and non-steady states conditions, with positively skewed Weibull work times distributions, different values of line length (N), buffer capacity (B), degree of imbalance (DI), and pattern of imbalance, utilizing full factorial designs. The data are subjected to the analysis of variance, multiple regression, multiple comparisons with control, pairwise comparisons, canonical correlation, and utility analysis. A simple utility approach is also explored briefly.

Some of the important conclusions for all the unbalanced lines' investigations are:

(1) At least one unbalanced pattern generates superior idle time (I) and/or mean buffer level (ABL), over those of a balanced line. The superiority in I decreases as DI rises, whereas the advantage in ABL reduces as DI is decreased.

(2) The DI of the best unbalanced pattern can substantially or moderately be increased and still yields approximately equal I to that of a balanced configuration.

(3) If a line is unbalanced in the wrong direction, significantly inferior performance to that of a balanced design will result.

(4) The unbalanced patterns' I tends to decrease, when N and DI reduce and B increases, while ABL falls directly with B.

(5) The I's transient size increases as N and B become higher and DI increases, while the ABL's transient size rises whenever B reduces.

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PREFACE

This thesis is divided into four parts:

Part One	-	The Literature Review
Part Two	-	Research Design
Part Three	-	Research Investigations
Part Four	-	Discussions and Conclusions

The body of the literature review and the methodological and design aspects, along with the detailed reports of the investigations, are contained in Volume I. The appendices corresponding to the body of Volume I are contained in Volume II. Appendices are classified by the chapter to which they relate, not by their serial order. Thus, Appendix 7.1 is the first appendix corresponding to chapter seven. All the tables, figures and pages which an appendix consists of, are prefixed by the letter A.

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PART ONE

THE LITERATURE REVIEW

The Literature Review comprises four chapters.

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CHAPTER ONE

BALANCED UNPACED MANUAL LINES UNDER STEADY-STATE CONDITIONS

INTRODUCTION

The great amount of capital expenditure on high-volume production and the widespread application of sequential manufacturing methods, have stimulated an increasing amount of research into the attitudinal and general aspects of flow lines, the aim being to supply production management with effective principles and rules so that production lines' output is increased and unit production cost is reduced. Flow lines are, in this instance, systems of serially connected manufacturing facilities or work stations, with the dominant part of the work at stations and the inter-station transportation means being of a manual nature. Thus, the completely automated 'transfer line' kind of production system is not included.

Prior to studying the research on the manual unpaced flow line behaviour, the different kinds of lines need to be recognised. Wild's (176) classification is primarily used here, i.e. 'moving belt paced lines' and 'unpaced series queue lines'. The former utilize conveyor belts and are of two types; 'fixed item', where work units cannot be taken away from the line and work is carried out on them while they move on the belt, or 'removable item', in which case work units are removed from the line to be processed at the work stations and then returned back to the conveyor line. Unpaced lines are queuing systems in series with a provision for storage space between stations. Such lines can be either 'machine dominated', with the variation in production being caused by the breakdown and repair characteristics of each station, or 'manual', with the changes in production being mainly due to operator work time variability. Production lines, whether paced or unpaced, may be further subdivided into 'single model', i.e. only one model being processed, 'multi model', i.e. at least two models being manufactured in batches, and 'mixed model', i.e. at least two models being produced at the same time on the line.

In order to concentrate on a manageable area of research, out of many such areas in production lines, the single model unpaced manual flow lines will be the only ones considered in this thesis. Four reasons may be put forward as to why unpaced manual lines are sufficiently important to merit an inclusive investigation:

(1) Of all line types the manual unpaced lines are the most widespread in practice. Lehman (104), in a survey of the United States production industry, discovered that the largest category was definitely the unpaced manual lines which accounted for 34% of all assembly methods, whereas moving belt lines constituted 22%.

Wild's (175) more recent and more applicable study into methods of assembly in various British industries, found that moving belt lines, both removable and fixed, covered 43.1% of production methods, while manual unpaced lines represented 55%.

(2) There is some evidence that lack of pacing affects attitudinal factors. Walker and Guest (172) found that higher than median absenteeism was related to lower than median job score, the latter being indicative of, among other things, the repetitiveness and pacing of the job. Globerson and Crossman (64) reported that turnover rate substantially dropped when the pacing degree was reduced. Manenica (109) discovered that operators seemed to be prepared in advance to their work when the line was unpaced, and this represents a psychological advantage in favour of the unpaced lines. Buxey (20) argued that the unpaced line is more robust and flexible with regard to absenteeism and turnover than the paced line, since it is relatively easy for the former to function with a decreased manning level if there is an adequate interstation storage space, whilst under-manning for the latter requires hasty line rebalancing and conveyor speed resetting. Recognition of the human element problem has put an increasing pressure on management to improve

job design and quality of working life and to introduce the so-called 'job enlargement' schemes such that the pacing factor is likely to be removed and cycle times increased, which represents an element against the moving belt lines' continued use.

(3) The description of manual unpaced lines ~~by means~~ as ~~of~~ a queuing system in series can also be conveniently employed to portray larger production lines, with each station having several operators or even a whole production department within it.

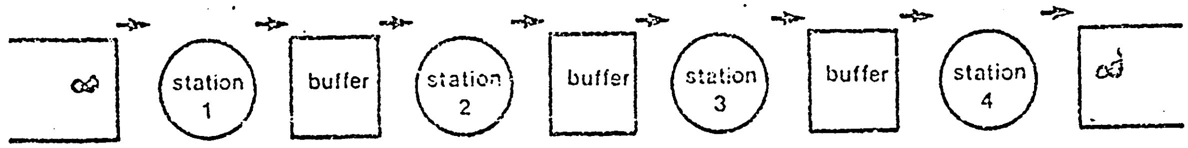
(4) Several researchers, such as Davis (38), Sury (165), Conrad (32), (33), Murrell (125), and Dudley (41), found that the unpaced lines are superior to the paced moving belt lines in terms of the number of errors resulting from excessive fatigue, and the number of missed items due to the allowance of incomplete or defective items in the moving belt lines.

MANUAL UNPACED LINES - SYSTEM DESCRIPTION

Most of the research conducted on flow line behaviour has been directed on unpaced manual lines, since this line type is the most common in practice, as was mentioned earlier. The major characteristic of this system of production is that no form of mechanical pacing by a conveyor or machine exists. It is possible for a worker to work freely at his own pace without the presence of any constraint on the amount of time available to him for finishing his work task. For this reason no unfinished items will be made in such a line type.

The line is made up of several interlinked work stations, each of which is manned by one or more operators and possibly equipped with some tools. Each operator performs a given amount of work on the part or work piece whose time is referred to as 'operation' or 'service' time. The stations are connected in a serial order in such a way that each part enters the line from the same first station and moves down the line from one station to its successor, where each operator adds his share of work to it, until it leaves the end station in the shape of the desired final product.

Usually, provision is made for keeping partly finished work-in-progress inventories between the stations so that when work is completed at one station, the item is transferred to a storage location. These inventory places are generally termed 'buffer stores' or simply 'buffers', and the capacity of each of them represents the maximum number of items that can be stored between stations. The mechanical pacing absence does not mean that some form of material handling device between stations is precluded. Inter-station 'roller' conveyor is commonly used, and in this case the buffer capacity is the maximum number of units which can be held by the conveyor's length before its space reaches a congestion point. The figure below illustrates the manual unpaced line system.



The internal inventories' function in flow lines is like their role elsewhere in as much as they behave as a decoupling agent where there is an inequality between supply and demand. The variability of operation times causes short-term inequality between the production rates of the stations in the line. Even though two stations that are side by side can have the same mean production rate (mean service time), their individual service times may differ in the short-run, the reason being the fact that operator work times are stochastic in nature which causes them to scatter around their mean value from one unit to the next. When the minimum (zero) or maximum interstation buffer stores' capacity limit is exceeded, as a result of this short-horizon operation times' unbalance, idle time will occur rendering the line's operation inefficient. If supply is exceeded by demand, i.e. when an operator works faster than his predecessor temporarily, such that buffer stocks are depleted, the succeeding station will suffer from 'starving' delays when it completes its work and finds no unit waiting for service in its buffer store, and at the same time the preceding station has not released the item it is processing. When demand is exceeded by supply, i.e. if an operator temporarily works faster than his successor, such that the buffer capacity limit is reached, the preceding station will suffer from 'blocking' delays when it finishes an item of work but cannot release it, since the following buffer is full and the succeeding station is still engaged in its work and has not withdrawn another item from its buffer. In this manner delays are transferred to and fro along the line because if a station is blocked, the probability of the preceding stations being also blocked is increased and when a station is starved, the likelihood that the succeeding stations will also starve rises.

Unless buffer stores are provided between stations in the unpaced non-mechanical lines, the random variations in service times will not be smoothed out and, consequently, a great deal of idle time caused by both starving and blocking will take place, decreasing the rates of output and utilization. In order that a beneficial decoupling action occurs, the buffer stores' level has to fluctuate throughout the production run between zero and the full capacity. Note that blocking and starving idle times resulting from the short-term mutual dependence of the stations (and referred to as 'system loss'), should be differentiated from those that are caused by a permanent, long-term, imbalance between stations' work times characteristics.

PERFORMANCE MEASURES

Various measures of performance or efficiency can be used to quantify and evaluate manual unpaced lines' behaviour.

These performance measures are classified as follows:

(1) Activeness Measures: the most important of these measures are:

(a) Idleness (I): this is the percentage or proportion of the line's inactive time to the total working time period.

(b) Delay (D): delay is the inactive time to the active time ratio or percentage during the period of line's operation, and is related to (I) by the formula:

$$D = \frac{I}{1-I}$$

Both I and D are totals consisting of starving and blocking concomittant portions.

(c) Utilization (U): which is defined as the proportion (percentage) of the time the operator is busy working and is expressed as $U = 1 - I$.

(d) Production (Output) Rate (PR): PR is the mean number of items released by the line per unit time. It measures the mean efficiency of the operator in comparison with that of a hypothetical perfect line in which all the stations have constant work times. PR is equal to 1/service rate.

(e) Number of Units Produced: this is the total number of items outputed at the end of the production period.

(2) Stockholding Measures: the most significant of these measures are:

(a) The line's total number of units (L): which identifies the mean number of items held in the line as a whole.

(b) The total number of units at the stations (LS) = N(1 - I) = NU, where N = the number of stations in the line.

(c) The total number of units in the buffers (LB) = L - LS.

(d) Average buffer level in the line (ABL) = $\frac{LB}{(N-1)}$

i.e. the proportion of the buffers' content to the number of buffers in the line.

(e) Buffer utilization (BU) = $\frac{ABL}{B}$, where B = the buffer store's capacity.

(f) Space utilization (SU) = $\frac{L}{N+B(N-1)}$

i.e. the total number of units held in the line as a proportion of the line's total physical space.

(3) Queuing Characteristics Measures: these measures are of some interest from a queuing theory viewpoint and include:

(a) Mean waiting time of the units in the buffers (MWT): which is regarded as a measure of delay in the system.

(b) Total time spent in the line (TT): which is

The total time required to produce a unit. This measure is important only when the temporary entities (the units) are of individual significance, which is not the case in most manual unpaced lines.

A tentative relationship between the queuing and activeness performance measures is that when I decreases both MWT and TT are reduced. Throughout this thesis the same above-mentioned effectiveness measures' notations will be used as appropriate, in order to avoid confusion and duplication.

In the following, the 'balanced' unpaced manual flow lines operating under 'steady-state' conditions will be presented. In a balanced production line each station has the same mean amount of work to perform. Because of the variability of operators' work times, perfect balance (equilibrium) is impossible and so, balance here means that every station has the same work time probability distribution with equal means and variability (as measured by the co-efficient of variation (Covar), which is defined as the standard deviation of the service times (σ) divided by their mean (μ)), and where each buffer store has exactly the same capacity. Such a balanced line is often called 'nominally' balanced.

It will also be assumed that the production line has reached a stable working condition after passing through a transient period of instability in the pattern of work. During the steady-state period the effectiveness measures will exhibit stable mean values over a given period, although their actual individual values might fluctuate from one point in time to another, due to their inherent variability, no matter how many of them are measured.

DETERMINISTIC LINE BALANCING METHODS

Unless the total time necessary to complete an item on a production line is equally assigned among the stations, a state of imbalance will exist, resulting in idle time which is caused by balancing loss. Therefore, deciding on the way of distributing work to each station is very important, if the line is to operate efficiently. Two objectives were cited in the literature (see for example Wild (176)) for balancing the line, namely:

(1) Given a certain output, minimise the number of operators (stations) and, therefore, minimise the idle time resulting from imbalance.

(2) Given the number of operators, maximise output and therefore, minimise the cycle time (the largest mean service time).

The first objective seems to be more appropriate for the unpaced manual production lines.

The line balancing problem was considered so important that numerous techniques have been developed in an attempt to solve the very difficult imbalance problems often encountered in practice. Most of the important balancing procedures and algorithms have been reviewed and evaluated by several authors, for instance, Kilbridge and Wester (89) and Ignall (80), the latter, in an estimate of the balancing problem's magnitude, stated that there are approximately $N^1/2^r$ feasible work element sequences available, where N signifies the number of work elements in the task and r is the number of arrows depicting the technological constraints which restrict the distribution of work elements to various stations in a 'precedence' diagram.

Three approaches to the line balancing problem have been advanced, viz:

(1) The analytical approach, which makes use of rigorous mathematical techniques, mainly linear, integer and dynamic programming models. This approach is usually so involved that a mathematician is needed to cope with it, especially as the size of the balancing problem is increased.

(2) The heuristic approach which provides good though suboptimal solutions for the line balancing effort.

(3) The empinical approach, which tries to allot work elements to stations using a non-systematic but practical way that suits the existing conditions in the industrial plants.

If the sum of work element times allocated to a station (or if the largest element time) exceeds the cycle time, the line balance will be more difficult and the resultant balancing loss increases. Rather than operating the line at a high imbalance degree, it may be better to either parallel (duplicate) the station(s), or man it by more than one operator (multiple manning), in order to obtain a good balance. All the previous line balancing approaches dealt with lines having one operator per station (single - operator lines). Paralleling was first considered by Nanda and Scher (130) who tried to balance lines that allow for two or more operators working simultaneously on the same copy of a product. The authors stated that their model offers advantage over the single operator counterpart, in terms of an increased output for the same cycle time.

PROBABILISTIC LINE BALANCING METHODS

A major assumption in all the foregoing line balancing methods is that of deterministic element and operation times, ignoring the fact that manual operation times are naturally probabilistic. The service times' variability implies that line balance is merely a notional concept. Wild (176) argued that it is virtually impossible to achieve a perfect balance, in the real-world operation, for any manual flow line even if the stations service times' means are balanced (equated), since it is extremely improbable that all operation times, at any one time, are exactly the same.

Recognition of the stochastic nature of service times has led to the development of several nominal line balancing procedures that take into account nonconstant work element times. Among such procedures are those of Moodie and Young

(122), Mansoor and Ben Tuvia (112), Brennecke (14), and Ramsing and Downing (141). The basic approach in these heuristic methods is first, to assume, for a given cycle time and line length, that element times are random variables with normal or Poisson distributions, known means and variances, and second, to assign work elements to stations, using one of the deterministic balancing techniques available, so that all the stations will have, as much as possible, equal cycle times (equal mean service times) and at the same time ensuring that the probability that the sum of element times' means (operation times' mean, assuming that the element times' means are additive) being higher than the cycle time, does not exceed a specific proportion, such as 0.95, i.e. the probability or confidence level of not exceeding the cycle time is 0.95. For manual unpaced lines this probability can be taken to mean the likelihood of having unequal service times' means with their consequent line imbalance. A shortcoming in all these methods is that they deliberately include balancing loss in the line, however small is the possibility of its occurrance. Mansoor and Ben Tuvia (112) tried to overcome this problem by introducing an incentive payment scheme which motivates workers to finish their entire assignment within the cycle time. Reeve and Thomas (145) compared several line balancing heuristics to increase the chances of completing the tasks without exceeding the service times' mean.

The stochastic line balancing techniques reported so far attempted to minimise the number of workers' cost, disregarding the cost of the deliberate imbalance of work which is introduced in the model. Kottas and Lau (98) developed a heuristic procedure for minimising the total of the aforementioned two costs, and found it to be more efficient than that of Moodie and Young (122) with regard to the total cost. Vrat and Virani (171) extended Kottas and Lau's model to permit the operation times to be greater than cycle times, by providing parallel stations. In this model the work times' means, u_i, for all parallel stations were equivalent to the integer multiple of the cycle time, and the number of parallel stations, P_i, required for task i is given by:

$$P_i = \frac{\mu_i}{CT} + F$$

where CT = the cycle time

F = a fraction added to make P_i integer.

Vrat and Virani assumed that if the paralleled station has a slack time, it can be assigned some more tasks. The introduction of slack time in this lodel, however, is a drawback which needs to be rectified since it represents an under-utilization of manpower in the line.

One limitation in all the previous line balancing methods is the assumption that no inventories may accumulate between stations; a situation which is unrepresentative of a real production line situation. Taylor and Davis (167) created a technique which can maintain inventories. By manipulating, in a gradual fashion, two variables; the output of the line and line length, inventories are gradually decreased and maintained at a desired level. Line balance is achieved when the inventories' level does not change, indicating that all stations operate simultaneously at a uniform service rate. The advantages of line balancing should be weighed against the estimated time required to find such balance. It might be a difficult and uneconomical task to find a good balance. The time penalty of such exercise is a function of the balancing method employed, the number of stations, the number of work elements, and the constraints on their ordering.

ELEMENT TIMES' VARIABILITY

Little is known about the nature of element times' distribution and in what way the parameters of this distribution, namely, its mean and variance, will affect the parameters of the whole task which is made up of these element times. The use of normal or Poisson distribution to describe work element times in the line balancing procedures, and the additive characteristic of element times' means, reported earlier, are only simplified assumptions to fascilitate development of techniques without a real-world research support.

The investigation of Brady and Drury (13) is the only one so far that attempts to shed some light on work element times' variability. They examined the relationship between the co-efficient of variation (Covar) of a work task and the Covars of the work elements which constitute the task. Their conclusions are as follows:

(1) The addition of a new element to a group of

elements will cause the Covar of the new group to be smaller than that of the old one, on the condition that the $Covar^2$ of the new element is smaller than (1 + 2r) times the $Covar^2$ of the old group, where r refers to the proportion of the old group's mean to the new group's mean.

(2) The Covar of a group of N identical elements is always less than that of the individual elements constituting the group.

(3) The Covar of a group of N non-identical elements is less than the greatest Covar of the individual elements constituting the group.

QUEUING AND SIMULATION APPROACHES

Most of the research work on investigating the operating characteristics of the unpaced manual flow lines has made use of either queuing theory approach (both analytical and numerical), or computer simulation approach. An analytical queuing solution utilises mathematical analysis, comprising differential and integral calculus, and provides a formula which holds for any value of the variables of the system. A numerical queuing solution, on the other hand, substitutes numbers for the variables and manipulates these numbers. Generally, the earlier studies used a queuing theoritic approach which relies on describing the production line's system by means of the different states it passes through. Assuming each station service times being exponentially distributed, the line's states can be described by a Markovian process whose state probabilities satisfy a set of linear equations in the limiting case of statistical equilibrium. These system equilibrium state probabilities can be obtained by solving the linear equations. By taking into account all states of the last station of the line in which it is working, the production rate (and hence line's efficiency) can be determined.

Such a queuing approach is acceptable from a mathematical viewpoint since it is possible to prove all the solutions, however, the following limitations restrict its value in its present status:

(1) A major difficulty is that a lengthy computation time is involved in order to obtain exact solutions, in the case of relatively short lines, and becomes computationally infeasible for long lines. The reason for that is the fact that the number of system states for an N-station and B-buffer capacity lines increases rapidly with slight increases in N and B. Muth (127) explained that, for example, a line with three stations and no buffers has 8 system states, whereas a line with 10 stations and zero buffer capacity has 6765 states. Therefore, the queuing model is practically insoluble, even when using high speed computers, if the number of states becomes very large.

(2) As the system size grows up, an adequate description and identification of its states, in order to develop the steady-state linear equations,

become complex and difficult.

(3) Operator work times distribution is restricted, in the queuing approach, to the exponential or Erlang distributions which permit mathematical manipulation. Neither distribution is of a real representation of manual tasks' operation times. Moreover, if the Erlangian distribution, with parameter k and Covar of $1/\sqrt{k}$, is used to approximate a normal distribution with Covar = 0.3, k has to be at least $\frac{1}{\sqrt{0.3}} = 11$, if the first two moments of the fitted Erlangian distribution are to approximate those of the normal distribution. This approximation will enlarge the problem by a factor of k since the system states' enumeration has to be extended, so that phase (1,2,....,k) of the working state in each station can be included. According to El-Rayah (44), in the case of k = 11, the largest system one can hope to solve with a B = 1, say, employing the most efficient computerised convergence techniques available to solve a system of linear equations, would be N = 3. In addition, when the solution is determined, it will merely be an approximation since the operation time distribution is, itself, approximated.

(4) The queuing approach can handle, using limit theorems, situations where steady state operating conditions exist, but cannot deal easily with

the transient behaviour of the line, since the mathematical queuing theory assumes all system's parameters are in equilibrium state and finds it very difficult to analytically determine the length of the transient period before all nonsteady-state effects die down.

(5) As Barten (8) pointed out, the queuing technique requires that the arrival rate to a station is independent of its service rate. This condition is not met in the more practical situation of a finite B which allows blocking to occur and hence, the arrival rate will depend on the service rate and can never exceed it. Another problem in the queuing approach is that, except for exponential service times, the interdeparture times between the stations are strictly dependent, as will be explained shortly.

As yet, there is no indication of a major breakthrough in the queuing theory using mathematical analysis alone, and for more practical purposes one should be prepared to sacrifice generality and exactitude in exchange for some form of reliable empirical relationships. Therefore, computer simulation is the alternative to queuing analysis. In this technique the original real-life model is substituted by a flow diagram setting the logical rules which govern system's behaviour, and operators' work times are obtained by the generation of random samples from any representative probability distribution. Unlike the queuing approach, the simulation one imposes virtually no constraints on the kind of system that can be examined. Although no proven relationships may emerge from the simulation approach, it is useful in providing insights into the operating characteristics of the line and in giving guidelines for line design, especially where the line cannot be sufficiently described or solved by a mathematical queuing model. In some instances, queuing models based on simulation results were used to approximate the optimal behaviour of production lines.

QUEUING THEORETIC RESEARCH

(a) Queuing Characteristics Measures: these measures are mainly of a theoretical interest and will not be used in the investigative part of this thesis. A short review of the literature in this area is presented here. Jackson (82) was a pioneer in the study and visualisation of a production line as a series queues system, where each unit of work must pass through a serial sequence of service operations. Prior to his work, all queuing theory studies were concerned with a system in which each unit receives one service operation, although two or more channels of service may be used. He analysed a two-station line with infinite queue size, Poisson input, and exponential service times, and found that the queue lengths (buffer levels) in the buffer stores are independent random variables in the steady state. Burke (19) initiated analytical work with regard to interdeparture times. He examined an N-station line with Poisson arrivals, exponential operation times, and infinite buffer capacity, and found that the steady-state distribution of the departure intervals from one station (and hence the arrival intervals to the next station) was Poisson. A corollary of this result, as explained by Magazine and Silver (106), is that the output rate from this line is equal to the minimum of the input and service rates.

Reich (146), investigating a line similar to that examined by Burke, added to Burke's work the finding that the departure process is independent of queue sizes and that the times spent by items of work in successive stations are independent. Reich also stated that if waiting times are defined as only including the time spent in queue, excluding the service times, then the independence of such waiting times is an open problem. Finch (48) found that Burke's Poisson departures hold only when infinite queues are allowed between stations. In addition, he proved that the successive interdeparture times are independent random variables only when in the case of exponential service times and unbounded queue lengths.

Burke (16) further studied a 2-exponential station line with Poisson arrival pattern and reached the conclusion that the waiting times in the buffers, at the equilibrium state, are dependent. In addition, Burke (17), (18) obtained some further results which showed that some of the waiting times for lines with multiple operators in parallel are dependent. The presence of statistical dependency in stations' arrival and departure processes violates one fundamental assumption of queuing theory, and complicates further analytical work on series queues systems.

Avi Itzhak (5) considered an N-station line with random arrivals, constant service times, and limited buffer capacity. He showed that the time spent by an item in the line is independent of both the order of stations and of the buffer sizes, and that this is true whether each station has one operator or r-operators in parallel, each with the same service times.

Avi Itzhak and Yadin (6) dealt with a line consisting of two stations with either zero or finite buffer capacity, random Poisson arrivals at the first station, and service times which are either deterministic or exponential. They showed that the probability density function of the time spent by an item in the system, as well as the buffer level, are independent of stations' sequence. They also showed that increasing the size of the buffers decreases the total time spent in the line by a work unit.

Fraker (57) developed approximate expressions for finding the steady state mean and variance of waiting times, along with the covariance of the departure process, for a single-server line having infinite buffer capacity and Erlangian operation times. Rosenshine and Chandra (149) extended Fraker's work by developing approximate formulas for the waiting times' mean and variance in multiple-operators' lines in parallel. A common feature in the above queuing characteristic research is the use of work times distributions which do not reflect reality, whether being constant, exponential, or Erlangian ones.

(b) <u>Activeness and Stockholding Measures</u>: these measures are more important and practical than their queuing features predecessors and, therefore, will be used in this research investigation. Hunt's work (78) formed the basis of most of the studies that followed on in this area. The relevant manual unpaced line cases that Hunt has treated are:-

(1) Each station is allowed infinite queues before it.

(2) No queues are permitted, except for station 1 which has an infinite queue in front of it.

(3) Each station is preceded by a finite queue, except for the first with an unbounded queue preceding it.

In each of the above cases Hunt assumed that the input to the first station is Poisson, the service times for each station are exponential, the queue discipline is first come first served without any defection, as soon as an empty station received a unit service starts instantaneously, and the moment the operation time ends there is an instantaneous transfer of units from one station to its successor.

The maximum possible utilization was calculated for cases 1 through 3 for the purpose of showing the impact of buffer stores, where utilization is defined as the ratio of mean arrival rate to mean service rate. In order to avoid the system states' dimensional problem, Hunt dealt with a maximum of 4-station line for cases 1 and 2, and 2-station line in case 3 (3-station line when the buffer capacity equalled only one unit). In addition to utilization, Hunt calculated the average total number of units in the line for cases 1 and 2 but did not obtain it for the more important and practical case, 3.

The significance of Hunt's work lies in its being the first one that mathematically proved and quantified the influence of two line's variables, even though for short lines only. The two main results of this study are:

(1) As the inter-station buffer capacity is increased, blocking is decreased and thus, the efficiency of the line increases, but the marginal increase in efficiency reduces with increased buffer capacity, given a fixed number of stations in the line. Furthermore, only a relatively small amount of buffer capacity is needed to reach a very high level of efficiency.

(2) Given the buffer capacity, increasing the line's length increases starving and blocking idle times which results in a decreased level of line efficiency, but the marginal decrease in efficiency decreases as the number of stations in the line is increased. The implication of this result, as Hillier and Boling (75) have reasoned, is that if the total amount of work remains the same,

the overall production rate of two 2-station lines will be higher than that for a line of four stations, and a higher overall production rate can be achieved using four one-station lines which means that when operation times are variable, 'individual assembly' production system may be preferable to the flow line system of manufacturing. However, the inherent advantages of the production line may outweigh the benefits of the individual assembly in that labour specialisation rises the efficiency of the production line to a great extent, learning time is reduced, and the amount of space, inventory, tools and other equipments is usually less in flow lines as compared to individual assembly (see Buxey (20) for a complete discussion).

Hunt's study can be criticised on the grounds that the results are too few, and arguably too specific, to be generalised, the values of line length (all cases) and buffer capacity (cases 1 and 2) are unrepresentative of those in more practical lines, and in case 3 the total number of units in the system was not considered.

The work of Hillier and Boling (72) marked the next major step. They developed a technique for obtaining a numerical solution to the set of equations representing system's state for longer lines. They offered an exact procedure which provides the output rate and the mean number of units in the system for lines having exponential or Erlangian operation times and any buffer capacity. Despite its relative efficiency, the exact procedure is still time consuming even for short lines. As a result, a more feasible approximate numerical procedure was devised which can obtain the average production rate for lines with exponentially distributed work times, and whose computational time is nearly proportional to line length. Hillier and Boling recognised that their approximate method considerably overestimates the utilization of the line, when the line length is relatively long ($N>6$), and buffer capacity is small ($B<3$). Another shortcoming of the approximate procedure is that it was solely concerned with the mean production rate, ignoring the number of items in the line which is of an important economic value, considering the amount of investment in the work-in-progress.

Basu (9) devised formulae for utilization, delay and average number of units in the system, employing a mathematical model which provides approximate solutions for exponentially distributed service times. These formulae are:

$$D = \frac{2(N-1)}{(B+3)N-2}$$

$$U = \frac{(B+1)N}{(B+3)N-2} \qquad \text{and}$$

$$L = \frac{(B+1)N}{2}$$

Comparing his results with those of Hillier and Boling (72), Basu found that for U they were very close for high values of B, and for L they were somewhat lower than Hillier and Boling's at lower values of N, and higher at higher B. Basu has also noticed that the effect of increasing B on raising U (and thus decreasing D) is more significant than that of reducing N, i.e. B is more important than N as regards their impact on line's efficiency.

Recently, Panwalker and Smith (134) offered the following equation to predict the output rate of an N-station line with finite B and exponential service times:

$$PR = A - C_N Y \quad \text{where}$$

$$A = 1 \text{ for all the values of N and B}$$

$$C_N = \text{constant for a given N}$$

$$Y = 1/(B+3)$$

Comparing the output rates of Hillier and Boling's approximate procedure with those of their equation, the authors demonstrated that they were remarkably close for higher B values, but for smaller B their values were less than those given by the approximate procedure, the difference increases with N,for a given B. Panwalker and Smith argued that this can be expected since Hillier and Boling have indicated that their approximations tend to overestimate the production rate for larger N and smaller B. On the other hand, when the authors' equation results were compared with the exact procedure of Hillier and Boling, it has been found that they were lower than those of the exact procedure for small N and B. This demonstrates that, as yet, no formula exists that can provide completely reliable results for lines whose N and B are large, but there are only workable approximations to fascilitate the analysis of such lines. The assumption of exponential service times limits the value and range of application of Panwalker and Smith's work.

Rao (143), in a study of a two-station production line, showed that the production rate is significantly affected by the type and shape of the operation times distribution, if the Covar value is large. For example, the normal service times' output rate is significantly higher than that of the Erlangian times when each distribution has a Covar of $\geqslant 0.7$.

OPTIMAL BUFFER CAPACITY

It has been mentioned earlier in this chapter that the higher the buffer capacity, the greater the degree of uncoupling which reduces the amount of idle time and increases utilization and production rates. Whenever an infinite storage capacity is permitted, the efficiency of the line is maximised since it enables each operator to work independently of every other one. But infinite buffer spaces never exist in reality and the cost of even providing large B is not often justified. In the other extreme, allowing no B between the stations will render each operator fully dependent on the others, causing massive idle time and maximum inefficiency. Between these two extremes the provision of a finite B value is the most common practice in manual unpaced lines.

As was stated earlier, the buffer stores decouple stations' work. However, a point will be reached beyond which the uncoupling function of the buffers no longer earns enough returns to justify the inventory and storage space costs incurred as a result of providing buffers between stations. At this point the least cost optimal buffer capacity level is attained. Therefore, a valid and essential design factor for production lines that production management desires to control, is the capacity of each of the individual buffers.

The buffer capacity provision cost is divided into three major elements:

(1) Idle Cost: which refers to the cost of operators' idle time with its consequent decrease in production rate.

(2) Buffer Storage Capacity Cost: which includes the cost associated with providing the actual floor space required to store the work-in-progress inventory, together with the cost of storage equipment's maintenance and investment.

(3) Stockholding Cost: which is the cost of the tied-up capital resulting from keeping semi-finished items in the buffers and stations along the line, as well as the costs of handling the inventory into and out of the line by special handling machinery, taxes, insurance and stock damage.

These three cost elements are all assumed significant, quantifiable, and dependent on, other things being equal, the inter-station buffer capacity. While the idle cost reduces when B is increased, the buffer space and inventory holding costs go up with the increase in B. Consequently, the operations manager has to solve the problem of balancing the three cost elements, in order to establish the least cost, optimum B which combines a desirable mixture of high output rate and low cost of production. The usual procedure is to obtain the partial derivative of the total cost function with respect to B,and then equating the first differential to zero so as to get the optimal B.

Young (184) and Basu (9) attempted to determine such a minimal B for production lines having identical exponential service times in all stations. The latter's expression is more complete and accurate because it takes account of all the three cost parameters and is derived from a better idle cost formula. This expression is given by:

$$B^* = \frac{2}{N}\left(1 + \sqrt{\left[\frac{C_1 N(N-1)}{C_2 N + 2C_3(N-1)}\right]}\right) - 3 \text{ for } N>1$$

where

B^* = optimum B

C_1 = idle cost per unit time

C_2 = stockholding cost per unit per unit time

C_3 = storage space cost per unit per unit time

This expression is especially relevant to production lines with 'cyclic queues',where the item is affixed to a fixture or jig. Basu demonstrated that the results of cyclic queues are identical, under the same conditions, to those of Hunt's 'open-ending' queues.

Two criticisms might be pertinent to Basu's optimal B model. First, the irrepresentative nature of the exponential working times which were used in deriving his equation. Second, as the author admits, his results are slightly but consistently higner than those given by Anderson and Moodie's simulations, which will be mentioned later on in this chapter. Nevertheless, the chief merit of Basu's equation lies in its simple and general form which addresses itself to lines having any number of stations.

INVESTIGATIONS THROUGH SIMULATION

Perhaps one of the most widely simulated systems is that of a sequence of servers ordered in series. Generally speaking, unpaced manual line's simulation studies describe and represent more aspects of real-life production lines than their queuing theory counterparts. As a tool for investigation, simulation is advantageous in two main respects. Firstly, distributions other than the exponential or Erlangian can be used to describe workers operation times, and secondly, much longer lines may be examined through computer simulation.

The first manual unpaced lines simulation results were published by Barten (8) who used the normal distribution to represent workers' operation times. He simulated lines with 2, 4, 6 and 10 stations, 0-6 buffer capacity units and 0.30, and 0.333 Covars. In addition to confirming the effects of N and B on lines' operating efficiency, which were provided by Hunt's queuing approach, Barten found that the Covar itself influences the idle time of the line in that, given N and B, as the variability of the service times distribution increases, the probability of different work times among successive operators also rises and hence, idle time becomes higher.

Anderson (1) simulated lines whose lengths varied between 2 and 5 stations, with B's of 0, 4, 8, 16 and 20 units, normal distribution of service times, and a Covar of 0.3. He found that the exponential service times' output rate (which he defined as the total number of units produced at the end of a production period of 100 time units), fluctuates more often than that when the operation times are normally distributed. He also found that the normal service times' model exhibited more production rate than that of the exponential service times' counterpart.

El-Rayah (44) conducted an investigation of lines having 3, 4 and 12 stations, 0, 2 and 4 buffer capacity units, and service times that are normal, lognormal, and exponential, with Covars of 0.3, 0.3, 1.0, respectively. His major contributions are as follows:

(1) The decrease in the output rate (the increase in idle time), as a result of increasing N, diminishes as B goes up and as the Covar is reduced.

(2) The rise in production rate due to increasing B, is higher the higher the Covar is, while the marginal improvement in PR from raising the level of B decreases with lower Covar and N values, other things being equal.

(3) Increasing N and B will increase the mean number of units in the system, L, but the rise in L will marginally reduce as B increases, and remains relatively constant as N goes up.

(4) L becomes lower the greater the Covar is, provided that B and N are small ($B \leq 2$ and $N \leq 4$). This relationship

disappears when B increases, while N remains small and then gets reversed, i.e. as the Covar becomes greater, L tends to increase when B rises and N stays relatively small.

(5) The effect of the Covar on L is much more important than that of the shape of the operation times distribution. Normal and lognormal service times, which differ slightly in shape, result in practically equal L values when they have the same Covar, especially as B increases.

(6) The output rate is influenced mainly by B, followed by the Covar, and finally by N, whereas L is affected by N, B, and the Covar respectively.

Moberly and Wyman (121) used simulation to study 'double' production line's performance in comparison with that of the 'single' line. In the former, the line has two identical operators in parallel at each station, with each double station having a buffer capacity equivalent to one-half that of the single station. This means that a single line has twice the total buffer capacity of a double line. For both single and double lines the authors utilized normal service times with N = 6, 10 and a Covar of 0.3, and B values of 4, 8. The results indicated that the double line is significantly better than the single line counterpart in terms of the output rate when B is relatively high (B = 8), and/or N is relatively small (N = 6). In addition, the effect of increasing B is much more significant than that of decreasing N on improving the performance of the production line, be it single or double. This study highlighted the importance of the double line as a legitimate design factor that may increase the efficiency of the line. However, the use of a total double line's buffer capacity which is only half that of the single lines, i.e. comparing one single line's performance to that of two double lines, is unfortunate and the opposite comparison should have been made in order to obtain meaningful and practical results.

Slack (160) simulated lines consisting of 5, 10 and 15 stations with buffer capacities of 1, 2, 3, 4, 6 and 8 units. The service times were described by the Weibull distribution with its positively skewed probability function. He found that this distribution is more representative of the real operation times than the normal distribution, after examining various published work time distributions. The main findings of Slack are summarised below:

(1) The proportion of starving and, therefore, blocking idle time is influenced by buffer capacity, i.e. as B increases, so does starving, but blocking is reduced. In addition, no marked effect on starving seems to result from the rise in line length.

(2) The relationship between L and N, B is expressed by:-

$$L = 0.444BN - 0.244B + 0.993N - 0.239$$

(3) Both N and B affect space utilization, SU. When B or N increases, SU tends to decrease. The functional

form of this relationship is:

$$SU = \frac{0.444BN - 0.244B + 0.993N - 0.239}{BN - B + N}$$

(4) Buffer utilization, BU, behaves in a similar manner to that of SU with respect to the impact of B and N on it.

(5) The idle time and stockholding results of the Weibull service times are of the same form as those of the normal operation times, with the idle time of the former being higher than that of the latter, but the difference between them is insignificant.

INDIVIDUAL STATIONS AND BUFFERS RESULTS

All the foregoing results were pertinent to the line as a whole. Anderson (1) was the first to investigate the idle time and inventory behaviour of the individual stations and buffers along the line. He simulated a 4-station line with a B of 10 units for both normal and exponential service times. Anderson's salient results are:

(1) The position of a station in the line influences its mean blocking idle time amount, with the first station experiencing the greatest blocking proportion, the second having the next biggest blocking, and so on, i.e. blocking decreases along the line. The proportion of starving idle time is also a function of where the station is positioned in the line, i.e. starving increases when moving from the first to the last station. Both blocking and starving for the exponentially distributed service times are higher than those achieved when the

service times are normal.

(2) The average buffer level, ABL, is dependent on the position of the buffer along the line, with the early buffers accumulating high ABL which causes a great amount of blocking in the early stations, whereas the ABL is lower at the later buffers resulting in a large proportion of starving in the later stations, i.e. ABL decreases down the line. The exponential operation times exhibit a higher degree of buffer level fluctuation between the empty and full capacity than that demonstrated by the normal work times.

(3) As B increases, the total idle time at any station tends to become less, with diminishing returns as B continues to increase.

(4) Blocking and starving durations for any station are very close to being exponentially distributed with a mean that is equal to the standard deviation of the service time distribution. This is true for both normal and exponential operation times.

In his study Slack (160) obtained the following results:

(1) The rate of increase in starving idle time along the line is not uniform but appears, in most cases, to be sharper towards both ends and relatively more gradual towards the middle stations.

(2) The decrease in ABL along the line seems less sharp in the middle buffer stores.

(3) The ABL is affected by N and B. As N or B increases, ABL tends to decrease. The relationship between the

average buffer size (b) in front of station n was found to be:

$$b = m\left[n - \frac{(N-1)}{2}\right] + ABL \quad \text{where}$$

$$m = 0.005BN - 0.018 - 0.001N - 0.086B$$

$$ABL = \text{the mean buffer level for the line as a whole}$$

Slack reasoned that the implication of ABL's reduction from one station to the next is that no uniform optimal buffer capacity value will be adequate, since if the value of B is the same throughout the line, excessive blocking will take place toward the end buffers, and excessive starving toward the early buffers. Another implication is that any economic buffer capacity formula will be influenced by the unequal distribution of the mean buffer level. Slack demonstrated that when using the optimal buffer size expression of Anderson and Moodie (2) (see the next section), the stockholding cost is overestimated by a quantity equivalent to $\frac{mN^2}{12}$ and the economic buffer capacity is increased by approximately 10% of L/2.

Rao (144) showed that in a 2-station line having any service time distribution, both stations will have approximately the same total idle time amount in the steady-state. He argued that this is generally true for line lengths of $N>2$.

OPTIMAL BUFFER CAPACITY VIA SIMULATION

Attempts to derive optimum B formulae were made by Barten (8), Anderson (1), Slack (160), and El-Rayah (44). Of all these formulae, the one provided by El-Rayah can be considered as the most general and complete. So as to obtain this formula, the author simulated lines having N = 2,3,4,8, 12 stations, B = 0,2,4,6,10, 30 units, and Covars of 0.075, 0.15, 0.30. In all these cases the service times were lognormally distributed. Utilizing regression analysis, the following expressions were obtained for the mean idle time and mean number of units in the line:

$$I = -0.019+0.001B+0.705\ Covar/B+1+0.0.604\ N.Covar/B+1$$

$$L = 3.7+1.980N+0.554\ B.Covar+0.518\ N.B.Covar\ -0.044\ N.B.-[10/(B+1)].$$

Based on the above two expressions, the following minimal cost buffer capacity (B^*) was obtained:

$$B^* = \sqrt{\frac{(0.705\ Covar + 0.064\ N\ Covar)K_1 - 10K_3}{(N-1)K_2+(1.846N + 0.554\ Covar + 0.518\ N\ Covar)K_3}} - 1$$

where K_1 = idle cost/unit time

K_2 = storage space cost/unit/unit time

K_3 = stockholding cost/unit/unit time

Note that this expression is of the same general form offered by Anderson (1) and Slack (160), but the three expressions differ in three respects. First, El-Rayah's expression takes the Covar into consideration, whereas the other two expressions assumed that it is fixed. Second, the empirically derived constants in the three formulae are not the same. Third, Anderson's optimum buffer capacity expression for lines having exponential service times provides higher B^* than the lognormal and

Weibull process times' formulae of El-Rayah and Slack, which, in turn, supply greater B^* than the normal operation times' expression of Anderson, the reason being that the service times distribution with the higher variability tends to give greater idle times, which lead to a higher B^* value than that provided by a less variable distribution. Young (184) mentioned that the optimum buffer capacity for paralleled lines are slightly higher than that computed for single operator lines, and for each additional parallel station, B^* increases by about 10%.

Despite being compact and useful, all B^* formulae suffer from two drawbacks. These are:

(1) All the expressions are difficult to compute and their constants are pertinent only to the specific simulated situations and, therefore, may not safely be used outside the range of the parameters' levels of the simulation experiment, although they may give a rough indication of the expected reasonable buffer capacity which should be provided in the line.

(2) The expressions assume that all the work units in the line have a unified unit inventory holding cost structure. This assumption is not always justified since the degree of completion of the work units is not the same, but increases as the units progress down the line and, consequently, the stockholding cost per unit may also increase along the line. As yet, this fact has not been reflected by the research.

A GENERALISED LINE'S BEHAVIOUR APPROACH

So far, all the simulation research studies mentioned were concerned with the operating charactistics of unpaced manual lines having specific ranges of the N, B, and Covar parameters, and a particular type of work times distribution. No basis has been provided for extrapolation from such investigations. In order to solve this specifity problem, an investigation was conducted by Knott (96) with the objective of finding the similarities between systems having different parameters and distributions, so that a general theory of queuing systems in series may be developed. Such an approach cannot be conveniently classified as a queuing theoretical or a simulation approach, but can be regarded as containing the essential elements of both these approaches.

Knott pointed out to the existence of a consistent mathematical structure in the numerical calculations of delay for the unpaced manual lines, and went on to devise several formulae that predict the efficiency of line's operation, alongside a theoretically based reasoning which can give credit to extrapolation. Where results were unobtainable by an overt mathematical analysis, computer simulation was resorted to. The procedure adopted by Knott was to study the delay experienced by a two-station production line with a buffer capacity of zero for various distributions of operation times, as the Covar is increased. From this the influences of increasing N and B are examined and included in a formula which describes the general 'inactivity' of the line; inactivity being idleness (I) for lines with service times described by the exponential or Erlangian distributions, and delay (D) for the other distributions used, namely:

$$\text{I or D} = \frac{\text{Covar}^2 V}{(B + J)} \quad \text{where}$$

$V = f(N) = 2(N - 1)/N$

J = a constant which depends on the type of work times distribution used and may be obtained from the following formula when $N = 2$ and $B = 0$:

$J = \text{Covar}/(\text{I or D}/\text{Covar})$

For instance, $J = 1.773$ Covar if the distribution is normal

$J = 3$ if the distribution is exponential

So as to ascertain the validity of his inactivity expression, Knott compared the expression's results with those obtained from his own and other authors' simulations and queuing studies. The comparison showed that the expression gives results that fall within 4% of the exact ones and, consequently, can be taken to provide reasonably good approximations. Knott tried also to improve the efficiency of his expression by developing a somewhat more accurate formula through the introduction of a constant, Y, such that:

$$I(B = 0, N)/\text{Covar} = wV + Y$$

where $W = I(B = 0, N = 2)/\text{Covar} - Y$

Y = a constant determined empirically

Knott's work is valuable with respect to its generality and simplicity. However, a major shortcoming is its lack of an expression for the mean total number of units in the line (although this can be estimated).

SUMMARY

In this chapter various types of production systems were identified and the reasons for selecting the manual unpaced line for an inclusive investigation were elaborated. A description of this kind of line as a series queues system and the important function of the buffer store as a decoupling agent, which eases bottlenecks when supply and demand are unequal, were then offered.

Three main measures of performance and their elements were introduced. These are the activeness, stockholding, and queuing characteristics measures. It has been stated that the first two are more important, practically, than the third. Next, the line balancing problem was explained, together with the analytical, heuristic, and empirical approaches to deal with it. Several stochastic line-balancing techniques were then reviewed as well as the drawbacks which render them not entirely suitable for practical lines.

Presented also were the queuing theory and simulation approaches, and their inherent advantages and disadvantages were compared, emphasising the utility of simulation in dealing with real-life lines. The chapter went on to survey the important queuing and simulation investigations and their findings for the balanced manual unpaced lines working under steady-state conditions. Among the most significant results in this area are:-

Whole Line Results

(1) The efficiency of the line is a function of the buffer capacity, B, increasing directly with the increase in B, but the continuing rise in B will generate diminishing returns.

(2) The efficiency of the line is dependent on the line length, N, decreasing as N increases. However, the marginal decrease in efficiency reduces when N continues to rise.

(3) The line's efficiency is influenced by the co-efficient of variation, Covar, decreasing when the Covar increases.

(4) The efficiency of the line is affected by the shape of the work times distribution, only if the Covar's value is high. In this case efficiency increases as the distribution becomes less variable.

(5) The mean total number of units in the line, L, is influenced by both N and B and rises directly with them. However, the marginal rise in L diminishes as a result of the increase in B, but remains relatively unchanged as N increases.

(6) The effect of the Covar on L is not clear. On one hand, L decreases when Covar increases, if N and B are relatively small. On the other hand, L rises directly with the increase in Covar, if B goes up and N remains relatively small.

(7) The efficiency of the line is mainly affected by B,

Covar, then N, respectively, while L is affected respectively, by N, B, and Covar.

(8) The shape of operation times distribution is less important in its impace on L than the Covar.

(9) Starving and blocking idle times are both functions of B, the former increases but the latter decreases, as B is increased. N, on the other hand, seems not to have a significant influence on both starving and blocking.

(10) Buffer utilization, BU, and space utilization, SU, are affected by both N and B. As N and/or B rises, both BU and SU decrease.

Individual Buffers and Stations Results

(1) The steady-state total idle time is approximately equal for all the stations in the line.

(2) Individual stations' blocking and starving are influenced by their positions in the line. As one moves up the line, blocking proportion reduces and starving proportion increases, the increase being sharper at the beginning and end stations.

(3) The position of the buffer along the line affects the mean buffer level, ABL, in that ABL drops from one station to the next, but the decrease is more gradual towards the middle buffer stores.

(4) B influences both stations' total idle time and ABL. When B increases, both the idle time and ABL decline. A similar relationship exists between N and ABL.

Furthermore, the chapter discussed the determination of an optimal buffer capacity formula and the current shortcomings which render it often inappropriate for more practical lines. Finally, the chapter is concluded by reviewing a novel procedure (attributed to Knott) which tries to avoid the problem of specifity of the simulation results, which makes them relevant only to lines having a particular type of service times distribution and certain N, B, and Covar values. This procedure attempts to free the analysis from this limitation so that the results become more general.

Having reviewed the research efforts into the balanced manual, unpaced lines which have reached a steady-state operational mode, the next logical step is to survey the literature on the nonsteady-state operating characteristics for such lines. This will be the aim of the next chapter.

C H A P T E R   T W O

BALANCED UNPACED MANUAL LINES UNDER NON-STEADY STATE CONDITIONS

INTRODUCTION

The research work reported so far has been concerned with the operating characteristics of lines working under stable state conditions. The steady-state phase of a production line's operation occurs as soon as nearly all the transient effects have died down. During the transient period the mean values of the performance measures are not stable but continue to change, and can be quite different from those of the steady state. Gradually, but ultimately, the line will converge to a steady state mode.

This chapter will review the research conducted into the behaviour of lines having unstable operational patterns during the non-steady state period. The majority of the unpaced manual flow lines investigations have focused on analysing their steady state (SS) behaviour, in the belief that the non-steady state (NSS) behaviour represents an unfortunate feature of line's operations and, consequently, is of little value. However, according to some authors, such as Wild (176) and Slack (160), there are sound grounds for believing that, in practice, a large segment of lines' working time is being spent under NSS conditions. This implies that much importance should be given to the design of such lines so as to obtain efficient NSS results.

CAUSES OF TRANSIENT CONDITIONS

There are several reasons why an unpaced manual line experiences non-equilibruim working conditions:

(1) Start-up of the line: the line usually starts to operate at an 'idle and empty' state, where all the stations are idle and all the buffers are empty. Under this condition, the line passes through an initial transient period before it settles down. This line's start-up takes place on such occassions as the beginning of the working day.

(2) Depletion of raw materials' supply to the first station: if, for any reason, the stock of raw materials feeding the first station is exhausted and not replenished, the station will stop working throughout the period of the cessation of the external supply, which leads to a series of chain stoppages in all the following stations down the line. When the supply is resumed and the line starts working, a NSS situation occurs.

(3) Stoppage of the line as a whole: the whole line may stop at certain intervals, such as tea or lunch breaks, shift changes, power supply failure, routine maintenance checks, line rebalance, etc. (assuming that such disturbances do affect the operation of the line). As soon as work is restarted a start-up period will result through the synchronisation of work along the line.

the production rate of a station may drop in the short term from its normal level, due to human factors, e.g. contingency and personal needs, which increases blocking idle times at the preceding stations. A station(s) can, in the extreme instance, stop working altogether for a temporary period, because of the above or other factors, such as minor breakdowns. This stoppage may last for a short time, but its effects can influence line's effectiveness for a considerable duration. If a middle station is forced down for any cause, after a relatively short period all the buffers in front of it will be full and all the buffers succeeding it will be empty. As a result, throughout the stoppage period the preceding stations will be fully isolated from the succeeding stations and, therefore, the line can be viewed as composed of two separate lines. As the stoppage ends and work resumes, these two independent lines will initially behave as if they have full and empty buffers, respectively.

(5) Learning: where a new product or process is introduced, workers will experience a NSS period before reaching a steady level of productivity. Since the learning element is important in estimating the transient period, a review of its significant aspects is presented below.

LEARNING CURVES AND FACTORS

Learning curve (start-up or manufacturing progress curves) theory is based on three assumptions:

(1) The completion time of a given repetitive task will be reduced each time the task is repeated, as a result of skill acquisition which is a function of the

number of task's repetitions. This decrease in unit production time is associated with increased productivity and decreased unit cost.

(2) The decrease in task's completion time will continue, but at a reduced rate.

(3) The reduction in task's completion time follows a certain predictable function, which is called 'learning curve' function.

Many such functions were claimed to provide good fit to learning times data, however, the most famous and simple is of an exponential form, namely:

$Yx = ax^b$ where

Yx = cumulative average man hours required to produce unit number x.

x = number of finished units or number of repetitions.

a = man hours of the first unit (the initial performance time).

b = learning improvement factor which is the slope of the learning curve and = In R/In 2, where R = learning rate and is determined empirically, depending on the particular industry and product.

Nadler and Smith (129) found that each work task has its own individual learning curve. Furthermore, they found that the learning curve of the whole product is a time-weighted combination of its individual tasks' learning curves. This complicates the theory of learning curves and, as yet, there exist no mathematical formulae for the single tasks' learning curves or their weighted combination. Another difficulty surrounding the learning curve theory, as was mentioned by Towill and Bevis (169), is the fact that each of the work elements which constitute an individual task also has its own associated learning time function. Globerson (63), moreover, stated that the learning curve of a task is a function of the individual learning curves of its elements and that the values of its parameters differ from those of its work elements.

Kaloo and Towill (85) discovered the existence of a 'post learning' or 'drift' phase, during which very small and slow improvements in productivity, which can be adequately described by a quadratic function, take place. This means that tiny improvements in performance may still be expected even during the SS. However, it can be argued that such insignificant improvements which, in a sense, signify the continuity of the NSS period, may be safely ignored and, for all practical purposes, the SS condition is taken as being attained since, as will be discussed in Chapter five, the SS is only approached but is never completely realised.

The learning curve model has found many applications in various industries, especially with respect to training, placement, and production planning. The importance of the learning curve as a tool which can be used in predicting workers' performance during task learning is derived from the fact that the learning period represents a cost to the firm since, during this period, task's completion times of trainee workers are higher than those achievable when they finish training, the difference being regarded as an inefficiency cost. The learning curve helps in estimating this cost so that the necessary control measures may be taken.

The significance of the learning period is reflected in a survey, reported by Buxey (20), into USA industries, which revealed mean learning times of up to 83.6 hours for assembly line tasks. In addition, Baloff (7) declared that in some industrial situations learning times of between ½ - 2 years were experienced. Under these circumstances the cost of inefficiency becomes considerable.

Several factors affect the learning of tasks, the most important quantifiable ones are:

(1) Task length (cycle time): Kilbridge (87) identified the following three aspects which are affected by task length:

(a) The initial learning period: which is the period from start-up until a steady performance is attained. Kilbridge stated that longer tasks require more repetitions and hence, longer initial learning period because lengthier tasks contain more to be learned.

(b) The pace ultimately achievable: this is the performance or working speed attainable at the ultimate case of a relatively stabilized production level. The author suggested that the pace achievable is a function of task length, decreasing as task length is below or above a certain optimal range.

(c) The recurring learning period: which represents the initial learning period experienced by new workers, due to turnover. The cost of this period is influenced by task length, the longer the latter the higher this cost is.

(2) Task complexity (difficulty): Hancock (68) stated that the initial learning period is more affected by task complexity than by task length, and that a relationship exists between task length and complexity. He also indicated that the complexity of the task influences the learning duration in the same way as the length of the task does, i.e. the more complex the task, the longer the necessary learning period is.

Kvalseth (101) tried to determine the effect of task difficulty on the learning curve function by considering three levels of difficulty, namely, low, medium, and high. His results showed that increasing the complexity significantly raised both mean service time (average man hours) and number of repetitions required.

(3) Task similarity: which signifies the extent to which a task, that is being taught to an operator, bears a similarity to another being, partially or fully, learnt by the same operator in the past. When the learning of an old task assists in the learning of a new task, a positive transfer of learning due to task similarity is said to occur. The

field of task similarity is still largely undeveloped since no attempt has yet been made to quantify it, probably because of the difficulty encountered in measuring such an intricate concept as similarity.

In trying to examine the transient behaviour of the balanced unpaced manual line, the only practical and satisfactory tool available to researchers is the simulation technique. The queuing theory approach uses the limit theorem to investigate the SS behaviour and it is a well known fact that transient solutions to queuing systems are either extremely protracted, or unobtainable. The problem arises from the complexity encountered in solving the birth-death differential-difference system's equations which renders the solution mathematically intractable. Therefore, the queuing approach is unsuitable for examining the NSS phase of the line's operation.

SIMULATION INVESTIGATIONS

Comparatively little research into the NSS conditions was reported, because most studies have concentrated on the SS conditions. The NSS research may be divided into two parts; the first being concerned with the whole line, whereas the second considers the individual facilities (stations and buffers) during the start-up period.

Whole Line Investigations

Several studies concerning the transient behaviour of the line as a whole have appeared in the literature. The first author to touch on the transient behaviour was Barton (8), who in the course of reporting his SS results, specified that seven hundred product cycles were discarded as making up the transient period, before collecting data on the SS effectiveness measures. Although this period provides an indication of the significance of the start-up duration, Barten viewed it as an unavoidable disadvantage of the simulation procedure, rather than depicting a real line's feature when it commences its operations.

Anderson (1) showed that during the early part of the simulation run the mean number of units in the line, starting the simulation with empty buffers, increases at a high initial rate. As time elapses, the rate of buffer build-up decreases until the buffer levels approach those of the SS. Anderson's work was also concerned with testing the hypotheses that by allowing the buffer capacity to be a time-dependent variable, total costs can be reduced during the NSS phase. To achieve that, he initially chose a buffer capacity value of zero, then increased B from time to time, as the mean buffer level, ABL, grew up, until ABL reaches the optimum SS value of B. The increase in B was controlled by what Anderson termed as the 'control rate', R, which is defined as the number of product cycles elapsed before B is enlarged by one unit. As soon as the SS optimum buffer levels were arrived at, B's sizes were kept unchanged throughout the rest of the simulation run. The author assumed that the change from NSS to SS occurs R product cycles after reaching the SS optimal buffer period, T_t, is:

$$T_t = R(B^* + 1)$$

Anderson simulated a 4-station line having normal service times, buffer capacities in the range of 0 - 28 in steps of 4, and control rates of 50, 100, 150 and 200. The simulation results were subjected to an analysis of variance which verified that neither the transient idle time nor transient mean number of units in the line are significantly affected by the control rate. However, the length of the transient period has been found to be significantly influenced by the control rate. Anderson demonstrated that in order to minimise the SS total cost function, the value of the control rate should be zero. "Therefore, total cost .... is minimised by initially setting the buffers at the steady state level and not attempting to control the in-process inventory during the transient period". The benefit of this study lies only in its elimination of a dubious technique, from the outset, of improving efficiency, since in a real-life production line it is inconvenient, if not impracticable, to make continuous adjustments on the capacity of buffer stores.

Kala and Hitchings (84) simulated a line with four stations, an infinite buffer size in each buffer, and Covars ranging from zero to 0.24 in 30 different steps. Such unlimited B line has been shown by Hunt (78) to reach SS after an infinite time period elapses (i.e. there are no SS conditions in such lines), which is an unrealistic duration in any simulation investigation and, thus, it can be safely assumed that such lines operate under NSS conditions throughout the simulation run. The authors' findings, however, contradicted with those of Barten (8) and El-Rayah (44) with regard to the effect of the Covar and the shape of the service times distribution on the resultant amount of idle time. Kala and Hitching's results may be regarded as either inaccurate or unprecisely inferred from.

To date, the most important NSS study is that of Slack (160) who simulated lines with N's of 5, 10, 15, and B's of 1, 2, 3, 4, 6, and 8. The transient values of idle time and mean buffer level were measured for both empty and full start initial conditions, the aim being to determine the length and magnitude (size) of the transient state, where the NSS length is defined as the time (in product cycles) between the start of line's operations and the point when the state of the line is considered to be adequately close to SS. The transient's magnitude, on the other hand, is defined as the size of deviation of the line's transient values from those of the SS.

As far as transient length is concerned, a testing procedure using the z statistic was employed to determine if there are significant differences between the values of idle time and mean buffer level during each transient period, and their SS counterparts values (the simulation run was divided into 10 transient periods of 50 product cycles each). The results of this test seemed to indicate the following for empty as well as full start initial conditions:

(1) The line length affects the transient length, longer lines having longer transient periods.

(2) The buffer capacity influences the length of the transient period, lines with higher buffer capacities will have longer transient periods.

(3) The idle time's transient length is shorter than that for mean buffer level.

(4) The transient length for both idle time and average buffer level is a function of the 'system capacity' expression, $N + B(N - 1)$, which represents the total number of spaces available in the line. A higher system capacity induces longer transient length.

The following transient length (T) expressions were obtained for the range of parameters' values used in the simulations:

Empty start, idle time transient:-

$$T = 4.149((N-1)B+N) - 26.966$$

Full start, idle time transient:-

$$T = 4.136((N-1)B+N) - 17.601$$

Empty start, mean buffer level transient:-

$$T = 9.169((N-1)B+N) - 79.232$$

Full start, average buffer level transient:-

$$T = 10.70((N-1)B+N) - 80.629$$

Slack's testing procedure may be criticised on the grounds that it assumed that the SS takes place at the start of the first period whose mean idle time and mean buffer level valuesdid not significantly differ from that of the SS, even if a succeeding period is significantly different in value from the corresponding SS value.

With respect to the magnitude of the transient, Slack considered it as the ratio of the idle time and average buffer level values over the first part of the simulation run (50 product cycles) to those of SS. All the transient size results showed much less variability than those of the transient length. The general conclusions may be summarised as follows:

(1) The idle time's transient size is a function of both B and N, increasing N and/or B raises the transient size. This is true for the empty and full start conditions.

(2) The average buffer level's transient size is a function of B as well as N. For the empty start conditions, the transient size tends to decrease as N and B go up, whereas for the full start conditions the transient size rises dirently with the increase in N and B.

Regression analysis provided linear functions for idle time and hyperbolic functions for mean buffer level's transient magnitudes. Table (2.1) shows these functions for both empty and full-start conditions.

Individual Stations and Buffers Investigations

Moreno (123) attempted to develop expressions which estimate the transient mean queues lengths in the individual buffers. He simulated a six-station line with a buffer capacity of 90 and a Covar of 0.3. The work time

TRANSIENT SIZE CURVES - FROM SLACK (160)

N.B. S = transient size (average value in first 50 product cycles/steady state value)
B = Buffer capacity

STARTING CONDITIONS	TRANSIENT TYPE	LINE LENGTH 5	LINE LENGTH 10	LINE LENGTH 15
Empty	Idle Time	$S = 0.592 + 0.335B$ ($R^2 = 0.99$)	$S = 0.388 + 0.583B$ ($R^2 = 0.99$)	$S = 0.484 + 0.654B$ ($R^2 = 0.99$)
Empty	Buffer Level	$S = \frac{1}{0.48 + 0.562B}$ ($R^2 = 0.99$)	$S = \frac{1}{0.505 + 0.782B}$ ($R^2 = 0.99$)	$S = \frac{1}{0.538 + 1.052B}$ ($R^2 = 0.99$)
Full	Idle Time	$S = 0.486 + 0.412B$ ($R^2 = 0.99$)	$S = 0.360 + 0.624B$ ($R^2 = 0.99$)	$S = 0.464 + 0.639B$ ($R^2 = 0.99$)
Full	Buffer Level	$S = \frac{1}{0.408 + 0.511B}$ ($R^2 = 0.97$)	$S = \frac{1}{0.368 + 0.477B}$ ($R^2 = 0.99$)	$S = \frac{1}{0.319 + 0.464B}$ ($R^2 = 0.99$)

distribution was described by an unusual function whose inverse transformation (defined by $X(R) = R^A(1-R^C)^{-B}$ for $C>0$) and direct $F(X)$ cannot be determined analytically, but its parameters (Covar, skewness and kurtosis) can be obtained by utilizing Bessel functions. The large buffer capacity size used practically implied that it is unbounded, as it is extremely unlikely that any of the buffers will be full during the simulation run and, therefore, an unstable operating behaviour occurs.

The main observations reached by Moreno are:

(1) The mean buffer levels for all the buffers have simular, though significantly different, patterns of behaviour.

(2) The mean buffer level at each buffer will increase continuously, when the buffer size is infinite, at a rate of growth of a T^b, where T = the simulation run time.

(3) The Covar significantly affects the buffer level's build up, but the influence of skewness and kurtosis may be safely neglected.

Moreno's work suffers from two shortcomings. First, it only dealt with the case of unlimited B, which may be interesting theoretically, but not practically. Second, as admitted by the author, the regression procedure contained some degree of bias which overestimated inventory build-up to some extent.

Payne et al (136) simulated a 20-station line having Covars of 0.1, 0.2, 0.3, infinite buffer capacity, and normally distributed operation times. The following results emerged:

(1) As the Covar increases, both the idle time at each station and the maximum queue level at each buffer follow suit. This result is similar to that for lines under SS conditions.

(2) The individual stations' idle times increase along the line, as a function of their location. This behaviour is attributable to the NSS operational conditions for such lines since, in the SS, the expected idle time amount at each station is approximately the same.

(3) The maximum queue length at each buffer is higher for the buffer at the beginning of the line than that for the end buffers.

Much of the merit of this research is reduced by the use of unlimited B's.

Part of the simulation study of Wild and Slack (179) investigates the stations' behaviour for the double and the single lines. Basically the same whole line conclusions regarding the superiority of the double lines over the single lines, with respect to idle time, were obtained in the case of the individual stations. In addition, the individual buffers and stations' NSS results resembled those of the SS. Furthermore, the authors found that the finding of Payne et al as to the functional relationship between idle time at a station and the position of that station, along the line, is also applicable to the limited buffer capacity case.

Optimum Buffer Capacity

No expression for determining the transient economic buffer capacity has been reported in the literature. Young (184) mentioned that he conducted several short-run simulations for a line with various buffer sizes, in order to prevent it from converging to a SS mode of operation. The results clearly demonstrated that the optimum transient buffer capacity is between 25% and 100% of that representing the SS. This result is expected to some degree since during the transient period (especially its early sections), the mean buffer level is relatively low and that reduces the stockholding cost and, therefore, the optimal B value becomes lower.

SUMMARY

This chapter is concerned with reviewing the research into the transient behaviour of balanced manual unpaced production lines. It started with identifying the non-steady state characteristics as differentiated from those of the steady state. Five likely causes for the occurrence of transient periods were given as line's start-up, first station's raw materials exhaustion, stoppage of the whole line, service times' temporary increase at a particular station, and learning effects.

Various aspects of learning were discussed, including the concept and assumptions of learning curves theory, the likelihood of having different learning curves for each work task and each element which goes into this task, and the factors affecting learning, viz, the length, complexity and similarity of the task.

The chapter went on to discuss the difficulties encountered when attempting to handle the transient behaviour by means of a queuing theoretic approach and suggested that, as the state of knowledge currently stands, the simulation approach is the only available tool for examining the transient behavioural pattern.

Several simulation investigations were presented whose major findings may be listed as follows:

(1) <u>Whole Line Results</u>

(a) The effects of N, B and Covar on the operating efficiency of the line in the NSS are, in general, similar to those representing the SS conditions; the two sets of effects differ only in terms of their absolute magnitude.

(b) The length of the transient period depends on N and B. Lines with longer N and/or higher B experience longer transient length. This is true for both idle time and mean buffer level.

(c) The transient period for the ABL is longer than that for the idle time.

(d) Both (b) and (c) are correct for empty and full buffers initial conditions.

(e) Both I's and ABL's transient sizes are influenced by B and N, the former increases as N and/or B rises, whether the line starts to operate with empty or full buffers, whereas the latter decreases with the rise in N and B for the empty startcase, but increases directly with N and B for the full start case.

(2) Individual Facilities Results

(a) The distribution of starving, blocking, and mean buffer level along the line in the NSS, and the impact of the increase in individual stations' Covar on I and ABL, are similar to those of the SS.

(b) The total idle time increases from one station to the next during the NSS phase, whereas in the SS it is expected to be nearly equal for all the stations.

The double line arrangements, other things being equal, demonstrated their superiority over the single lines counterpart, in terms of reducing the amount of idle time, for both the line as a whole and the single stations, especially for lines having large N and Covar, and small B. The chapter was then ended with the statement that the NSS optimal B is usually lower than that of the SS.

Having surveyed the steady and non-steady states' literature on the balanced unpaced lines, the next two chapters will review what is already known about the transient and equilibrium states for the unbalanced unpaced lines.

CHAPTER THREE

UNBALANCED STEADY-STATE MANUAL UNPACED LINES

INTRODUCTION

The aim of this chapter is to review the research into the behaviour of lines that are unbalanced and operating under SS conditions as a prelude to the investigative part of this thesis. The majority of the early studies on unpaced manual lines' characteristics have been exclusively concerned with lines that are balanced in terms of having equal operation times' means and Covars for all the stations, and equal buffer capacities for all the buffers. The main reason behind this direction in research efforts probably lies in the assumption that the efficiency of the balanced line is higher than that achievable by the unbalanced line. However, several authors, such as Slack (160), Carnall and Wild (26), and De La Wyche and Wild (39), argued that the unbalanced line, whether being unbalanced with respect to its process times' means, or Covars, or buffer capacities, is of great interest and can in fact improve the performance of the line. They put forward the following reasons to justify unbalancing the line:

(1) In practice some degree of imbalance is unavoidable since, in most cases, the precedence and technological constraints prevent the allocation of equal amounts of work to stations, giving rise to balancing loss. Experimental studies conducted by Kilbridge and Wester (92)

revealed that, on the average, between 5% - 10% of operators working time is being wasted by industry in the form of imbalance delay. Clearly, this loss is costly and disruptive to the production system.

(2) The degree of imbalance, which represents the manner of unevenly alloting work to the stations, is considered to be as much a valid parameter for line's design as line length and buffer capacity.

(3) Even if a notional mean service times' balance is achieved, there is no guarantee that the line will operate at its maximum possible efficiency. Therefore, some form or another of imbalance may result in a superior performance to that of the balanced line.

(4) There is ample evidence which points to the fact that individual operators, performing even simple tasks, have different mean service times as a result of having different mean speed capabilities, i.e. workers may be classified as fast, medium, and slow, and the slowest operator (the one with the largest mean work time) will delay all those preceding and succeeding him.

(5) The workers may differ in their operation times' variability, as measured by the Covar, as a consequence of inherent differences in their service times' variation patterns and in task's nature, because the work elements which make up each individual task differ in their complexity and specifity.

(6) The amount of space available in the line is often restricted by some technical considerations which

may make it impossible to distribute the total buffer capacity evenly among the individual buffers.

Recently the unbalanced line's operating characteristics have gained more popularity among researchers and, consequently, several investigations have started to appear in the literature. These investigations, as was the case when reviewing the balanced lines' studies in Chapter 1, are classified as queuing theoretic and simulation approaches, and a third broad approach which does not conveniently fit in any of these two approaches. The same criticisms which were directed at the use by researchers of deterministic, exponential, or Erlangian service times, buffers capacities of zero, short line lengths ($N<5$), and queuing characteristics' performance measures (refer to Chapter 1), are valid in this and the next chapters and, therefore, will not be repeated again.

Four types of line's imbalance were examined by researchers, namely, the operation times' means unbalance, the variability of service times imbalance, the unbalanced distribution of total buffer capacity, and service times' means and Covars joint imbalance.

UNBALANCED SERVICE TIMES' MEANS

In this case the means of operation times are unequal, but all the Covars and buffer capacities are equal for all the stations and buffers respectively.

(a) <u>The Queuing Theoretic Approach</u>

Hunt (78) touched on the case of means imbalance where not all the stations have the same service rate. He obtained an expression for the maximum possible utilization (Pmax) for a 2-station line which is given by:

$$Pmax = m_2(m_1^{B+2} - m_2^{B+2}) / (m_1^{B+3} - m_2^{B+3})$$

where m_i = mean service rate for station i, i = 1,2

The corresponding formulae for three and more stations become more complex because of the disproportionate rise in the number of state probabilities that need to be identified and computed.

Hunt, further, derived the following formula for the mean number of units in the line, L:

$$L = \sum_{i=1}^{N} P_i/(1 - P_i)$$

where

$$P_i = \frac{\text{mean arrival rate for station } i}{\text{mean service rate for station } i} = \text{utilization of station } i, \quad i = 2,3,4$$

The author stated that increasing P_i increases L and that the marginal increase in L goes up as P_i rises.

Makino (108) suggested that it is possible to increase the utilization of a 3-station line having exponential service times and zero buffer capacity, by allocating a lower mean operation time to the middle station.

Patterson (135) studied several lines with the objective of minimising the interdeparture time for a given interarrival rate. For a slow interarrival rate and exponential service

times, he found that a monotone ordering of the stations' mean service rates is preferable, while for a very fast interarrival rate (which is the limiting case and refers to infinite supply of raw materials to the first station) he favoured the very fast stations to be separated by very slow ones, i.e. the service rates will alternate between high and low along the stations.
Avi Itzhak (5) proved that for a line with unequal and constant service times, the total time spent in the system is independent of the order of the service times.

The same suggestion of Makino was studied by Hillier and Boling (75) who used their exact procedure to investigate the impact of the deliberate imbalance of the operation times' means, for lines with up to four stations. For a 2-exponential station line they reached the conclusion that the production rate decreases when unbalancing the line, the decrease becomes larger, given the imbalance degree, the larger the buffer capacity is. The authors indicated that the reason for this reduction in PR is due to the fact that as B increases, the blocking is reduced, and the efficiency of the line, therefore, becomes more sensitive to the slowest station, i.e. PR is affected by the imbalance which exists in the line. Hence, the best policy, the authors argued, is to balance the two-station line in order to achieve higher output rates.

For a 3-station line having exponential service times, Hillier and Boling's analysis was fașcilitated by the finding that the production rate function of the line is symmetrical with regards to the mean service times of stations 1 and 3, in that they can be interchanged without affecting the line's production rate. Thus, given the mean service time of station 2, the production rate rises as the difference between the mean service times of stations 1 and 3 decreases, and is maximised if equal operation times' means are sssigned to both these stations. This finding was referred to as the 'symmetry' property.

The authors also found that the service times means' sequences of 1, 2, 3 and 3, 2, 1 give the same output rate. This ability to reverse the stations' order without influencing PR was termed as the 'reversibility' property. Furthermore, the authors discovered a third property, the so-called 'bowl phenomemon' property, which signifies that the middle station should be assigned a lower mean operation time than the end stations, i.e. it was found that the production rate can reach a maximum by shifting some proportion of the work load from station 2 to stations 1 and 3. As buffer capacity goes up, the authors noticed that the potential improvement in PR of the bowl phenomenon arrangement over that of the balanced line reduces, and that the mean service time of station 2 starts to increase towards that of the end stations, i.e. the bowl phenomemon becomes less pronounced when B rises.

For a line with four exponential stations the authors' work revealed that the symmetry, reversibility, and bowl phenomenon properties are still valid. They showed that so as to maximise the mean production rate of the line, stations 1 and 4 and stations 2 and 3 respectively must have the same mean service time, with stations 2 and 3 having slightly lower mean service time than that of stations 1 and 4.

The authors stated that the improvement in output rate, by adopting the bowl pattern, over that of the balanced pattern, increases from nearly 0.55% for a 3-station line to about 0.94% for a 4-station line, both lines with a zero buffer capacity. Even though the percentage improvement in efficiency by utilizing the bowl arrangement is relatively low, it can result in substantial costs' saving over the lifetime of a production line, considering the very large total line's operating costs, especially if the saving continues to rise as the line length is increased.

Moreover, the authors indicated that a line unbalanced in accordance with the bowl phenomenon exhibited some degree of robustness, in that it maintained its relative efficiency in comparison with that of the balanced line even where a high imbalance degree exists, e.g. it was possible to unbalance three and four-station lines having a B of zero by 17% and 28% respectively and still their output rates were approximately equal to that obtainable by a balanced line.

Hillier and Boling admitted that they have no intuitive reasoning to explain the bowl phenomenon except that the middle stations may be more important than the end stations, since they influence the stations before and after them and thus, it could be that speeding up these middle stations will be beneficial.

Table 3.1 shows the maximum improvement in the output rates of the bowl phenomenon over those of the balanced line counterpart, together with the maximum degree of imbalance that can be tolerated before the improvement achieved by the bowl arrangement disappears, for N = 3, 4 and B of up to 4 units.

Hatcher (69) dealt with lines of 2 and 3 stations with exponentially distributed operation times and developed the following mean output rate's (PR) expression for a two-station line:

$$PR = PR_1 \; \frac{1 - P^{S+1}}{1 - P^{S+2}}$$

where PR_1 = mean production rate of the first station

PR_2 = mean production rate of the second station

S = capacity of the buffer store and second station combined

P = ratio of PR_1/PR_2

Dividing both sides by PR_2 gives the following more useful formula:

$$PR/PR_2 = \frac{P\left(1 - P^{S+1}\right)}{\left(1 - P^{S+2}\right)}$$

TABLE 3.1

OPTIMAL AND MAXIMAL PRODUCTION RATES OF THE BOWL PHENOMENON FOR EXPONENTIAL SERVICE TIMES - ADAPTED FROM HILLIER AND BOLING (75)

N	B	COVAR	PR_b	PR^*	$BL\%^*$	$\left(\frac{PR^*-PR_b}{PR_b}\right)$x100%	MAX BL%	PR(MAX BL%)
3	0	1.00	0.564	0.567	8.6	0.55	17	NA
3	1	1.00	0.671	0.674	5.9	0.49	12	0.670
3	2	1.00	0.734	0.737	4.4	0.40	9	0.734
3	3	1.00	0.777	0.779	3.5	0.32	7	0.777
3	4	1.00	0.808	0.810	2.8	0.27	6	0.807
4	0	1.00	0.515	0.520	13.7	0.94	28	NA

where

PR_b = production rate of the balanced line

PR^* = optimal production rate

$BL\%^*$ = optimal degree of imbalance (%)

Max BL% = maximal degree of imbalance (%)

N = line length

B = buffer capacity

Covar = co-efficient of variation

Hatcher found that as the value of P decreases, i.e. as PR_1 becomes smaller, giving rise to a monotone decreasing order of mean service times, the effect of increasing the buffer capacity on PR becomes less, since the second station will be faster than the first such that the buffer between them is quickly exhausted and station 2 will be starved. Increasing the buffer capacity in this situation will have little effect on PR. For example, when the first station has $\frac{1}{5}$ the speed of the second, increasing B from 0 to infinity causes an increase in PR of nearly 12%. On the other hand, when P becomes extremely large, leading to a monotone increasing mean service times' order, the second station will always have units waiting in its preceding buffer and, as a result, increasing B will not affect its output rate. The limit of PR/PR_2, as B goes up, depends on the value of P. When $P<1$ the limit is P, whereas if $P \geqslant 1$ the limit is unity. When $P = 1$, i.e. the line is notionally balanced, raising the buffer capacity has more effect on PR/PR_2 than the cases of $P <$ or > 1.

When extending his analysis to a line with three stations, Hatcher derived a very lengthy PR/PR_3 formula. As is the case for the 2-station line, he found that for a line having 3 stations with large differences between their mean production rates, the addition of more B will only slightly influence PR/PR_3. The limiting values of PR/PR_3

were found to be as follows:

Condition	Pattern of Mean Service Rates' Arrangement	Limit of PR/PR_3
$P_1 \leqslant 1, P_2 < 1$	(\) and (‾\)	P_2
$P_2 \geqslant 1$, all P_1 values	(_/), (✓), (/) and (V)	1
$P_1 > 1, P_2 < 1$	(^) and (∧)	$1-\frac{(1-P_2)}{P_1}-\frac{(P_1-1)(1-P_2)(P_1+P_2)}{P_1(P_1+P_2-P_1P_2)}$

These limits were seemingly converged to when the buffer capacity was 10, i.e. a value of $B > 10$ has very little impact on P_1 and P_2.

Kraemer and Love (100) derived the following expression for the expected number of units in the system, L, for a 2-station, exponentially distributed operation times line:-

$$L = \frac{P}{1-P} - \frac{(B+3)P^{B+3}}{1-P^{B+3}} \qquad \text{for } P \neq 1$$

where $P = \frac{\text{mean output rate of station 1}}{\text{mean output rate of station 2}}$

The authors' main objective was to develop an optimum buffer capacity model. They tried to derive an expression which minimises cost or maximises profit with respect to three factors, viz, the output rate of the line, stockholding cost, and space provision cost. The profit function for

$P \neq 1$ is given by:

$$T = \text{[illegible]}\left(1 - \frac{\text{[illegible]}}{1-P^{B+3}} \right) \text{[illegible]}_1 \left(\frac{P}{1-P} - \frac{(B+3)P^{B+3}}{1-P^{B+3}} \right) - c_2.B$$

where g = gain from releasing one unit

c_1 = stock-carrying cost/unit

c_2 = storage space cost/unit

It was shown that this profit function is always integer concave for $P \geqslant 1$, but its shape for $P<1$ is a function of the relationships between the mean operation time of station 2, the gain from producing an item, and the unit stockholding cost, however, an upper limit for the optimal buffer size can be determined. The novelty of the formula above is in its inclusion of a profit rather than a total cost function and in the replacement of the idle time cost by the gain per unit. However, the authors' protracted method of analysis effectively prevents the speedy derivation of the optimum B even for this two-station line.

In his study, Buzacott (25) suggested that if a buffer, due to unbalanced mean service times, is always blocked or starved, it has no beneficial effect or useful purpose. In order to redress the detrimental impact of the imbalance and improve the production rate, he favoured duplicating a station if it is occupied by a slow operator, even if no buffers were provided between the stations.

Hillier and Boling (74) established the validity of their previous conjectures of symmetry, reversibility and bowl phenomenon, using their exact numerical procedure, by extending their existence to the following two lines' cases:-

(1) $N \leq 6$, exponential service times and small values of B.

(2) $N=3$, Erlangian service times with a shape parameter, k, of $k \leq 7$, and small B.

A subsequent analytical study by Muth and Mehta (128) supported the validity of the reversibility property for $k = 1$ (exponential service times), as well as for $k>1$.

In another paper, Hillier and Boling (76) put forward the following expressions for their three conjectures:

(1) Symmetry: $m_i^* = m_{N-i+1}^*$ where

m_i^* = optimal mean service time for station i, $i = 1,2...,N$

(2) Reversibility: $PR(m_1, m_2, ..., m_N) = PR(m_N, m_{N-1}, ..., m_1)$

(3) Bowl phenomenon: $m_i^* > m_{i+1}^*$ for $1 \leq i \leq \frac{N-1}{2}$

$m_i^* < m_{i+1}^*$ for $\frac{N+1}{2} \leq i \leq N-1$

i.e. the stations are assigned a decreasing mean service times' sequence as they get closer to the centre of the line, and an increasing mean operation times' order as they move away from the centre. Properties 1 and 3 above have not been proven yet and the authors argued that the proven reversibility conjecture implies that a unique optimal solution must satisfy the symmetry property. They further indicated that both the analytical and simulation studies supported the contention that an assymetrical optimum allocation of mean service times is implausible and cannot take place.

The authors reasoned that the rationale behind the symmetry property is that the production rate is likely to be influenced by a blocked station towards the beginning of the line in the same way as that by a starved station near the end of the line. Furthermore, the bowl phenomenon may be attributed to the fact that a particular station's starving and blocking effect is highest on those stations which are very close to it, whereas this influence is reduced as the stations get further away. Since the middle stations affect stations in two directions and both the early and end stations influence stations in a single direction only, the middle stations are more important and, therefore, should be alloted smaller mean service times (higher mean service rates).

Tembe and Wolff (168) investigated two different orderings in a 2-station line with exponential service times and found that if the station with the larger mean service time is placed first, the total time spent by a unit in the system decreases, i.e. pattern (\) is better than pattern (/) in terms of the above performance measure.

Rao (144) examined a line with two stations and both Erlangian and normal operation times. He showed that when both the stations have identical variability, the symmetry and reversibility properties will maximise the line's output rate if the two stations are balanced with respect to their mean operation times. This validates the conclusion of Hillier and Boling (75) for two exponential stations and extends its relevance to the Erlangian and normal stations.

Meister (117) examined a series queuing system with infinite buffer capacity and increasing service time's order, and provided a complex formula, based on the equivalence theorems, for the waiting time distribution of the units in the buffers. It is doubtful if this type of work is useful practically even though it may be of some theoretical interest.

Magazine and Silver (106) used heuristics based on the Fibonacci numbers (defined by the difference equation ($F_{n+1} = F_n + F_{n-1}$, $F_o \equiv F_1 \equiv 1$, $n = 2,3,\ldots$) to estimate both the production rate and stations' mean service times of the bowl phenomenon pattern for any number of stations. They compared their heuristic's output rates and mean service times' values with those obtained from Hillier and Boling's (72) exact procedure for line lengths of three to six stations having $B = 0$. The comparison showed that the percentage of error resulting from using the heuristic was very low and, consequently, the heuristic's PR estimates seem very good and it may be effectively employed as a predictive tool, especially since it is inefficient to use the exact procedure to obtain the values of PR for more than six stations.

The authors showed that for $B = 0$, N up to 100 and exponential operation times, it is advantageous to imbalance the service times' means of the line according to the bowl pattern, since it produced improvements in the output rate over that of the balanced line in all the cases, with the greatest gain being achieved for $N = 10$. In general, when $B = 0$ and N between 5 and 20 the heuristic produces good, though not optimal, results which give an increase in PR in the range of 1% - 1.65%.

Comparing their results with those of the exact procedure for $N = 3,4$, and $B = 1,2,3,4$, the authors reached the conclusion that the heuristic overestimates the end stations' mean operation times, underestimates the middle stations' mean (i.e. it magnifies the bowl), and somewhat underestimates PR which meant that under such conditions the heuristic is less useful. Studying situations where Erlangian service times exist, the authors stated that for N between 5 and 10, small B and small k, unbalancing the line is effective. When the values of N, B, k are outside the abovementioned ranges, searching for imbalanced patterns that are superior to the balanced arrangement is only warranted if it is computationally inexpensive, otherwise balancing the line will not result in much loss.

The merit of this work rests in the fact that it aids in finding reasonable mean service times' allocations for values of line length that were not previously considered. However, the shortcomings of this research are:

(1) As the authors admitted, it does not provide good PR's estimates for relatively large N, small B, and non-exponential operation times.

(2) The heuristic itself assumes that fractional parts of workers can be assigned to any station, e.g.

half an operator is one who spends half his working time at a particular station and the other half at another one doing a different task. This clearly violates the universal assumption made by all previous researchers of assigning an operator to one station only.

Dattatreya (37) proved that the conjecture of reversibility is valid also in the case of multi-operator stations if their operation times are completely independent. He showed that this property holds for the total time spent by n units in an N-station line with exponentially distributed service times, i.e. the total time needed to serve n units in one direction is identical to that required to produce n units in the reverse direction.

In their latest useful paper, Hillier and Boling (73) made a further study of the bowl phenomenon for line lengths of up to 6 stations, buffer capacities of up to 4 units, and Erlangian service times with a shape parameter, k, ranging from 1 to 7. The main conclusions of this study may be summarised as follows:

(1) As N ($N>2$) increases, the mean amount of unbalance in the optimal bowl pattern of mean service times remains nearly the same.

(2) m_2^*, m_3^*, ..., m_{N-1}^* are approximately equal, while m_1^* and m_N^* are much larger, when N goes up, especially as B becomes higher, where m_i^* = the mean operation time for station i in the optimal bowl phenomenon arrangement.

(3) Increasing N, for a given B and k, will increase the percentage improvement in PR of the bowl pattern over that of the balanced line, but at a decreasing rate.

(4) The advantage, in terms of improved PR percentage, of the bowl phenomenon over the balanced line counterpart decreases rapidly with the increase in B. In addition, increasing B reduces the optimal degree of line imbalance, but even for large B values it is still worthwhile to unbalance the line in the appropriate direction (the direction of the bowl phenomenon), in which case a slightly higher PR than that achievable by a balanced line will result. This is an indication of the robustness of the bowl pattern with respect to raising the value of B.

(5) Given B, as k is increased (as Covar is decreased), the improvement in PR as a result of adopting the bowl arrangement, over the balanced counterpart, is diminished. However, the effect of increasing B on reducing the improvement in PR is much greater for $k>1$ than for $k = 1$.

(6) The simultaneous effect of B and k is far greater than that of either one separately, i.e. increasing both k and B is likely to more rapidly decrease the bowl pattern's advantage, as well as the optimum degree of imbalance, than that obtained when increasing B and k on a separate basis.

(7) The function of PR for the bowl configuration is almost flat near its maximum peak value, and it seems likely that the imbalance degree can be doubled and still PR is slightly better than that of the balanced line.

Furthermore, any other unbalanced mean service times' arrangement which substantially differ from that of the bowl phenomenon, will lead to a quick drop in PR. Therefore, it is quite important to assign the operation times' means according to the bowl shape, or at least in its direction.

Table 3.2 exhibits the effects of N, B, k on the bowl phenomenon's performance.

(b) <u>The Simulation Approach</u>

Anderson (1) was the first to report an investigation of unbalanced production line's behaviour through simulation. He simulated a four-station line having normal service times with a Covar of 0.3 and buffer capacity of six units per buffer. The service times' means of the four stations were 1.0, 0.9, 1.1, 1.0 units respectively, i.e. their pattern is $\surd\!\wedge$. The general observations of this research are:

(1) The slowest station (station 3) exhibits the least idle time, whereas the fastest station (station 2) demonstrates the highest idle time.

(2) The average inventory levels for the two buffers in front of the slowest station are near their capacity, whereas the mean buffer level of the succeeding buffer is very low that this buffer is nearly empty.

The same production line was then simulated with exponential service times and the resultant observations are generally similar to those obtained for normal operation times, however,

TABLE 3.2

OPTIMAL PRODUCTION RATES OF THE BOWL PHENOMENON AS COMPARED TO THOSE OF THE BALANCED LINE - DERIVED FROM HILLIER AND BOLING (73)

N	B	COVAR $(=1/\sqrt{k})$	K	PR_b	PR^*	$BL\%^*$	$\left(\frac{PR^*-PR_b}{PR_b}\right)\times100\%$
3	0	0.71	2	0.640	0.644	6.7	0.50
3	1	0.71	2	0.761	0.764	3.8	0.35
3	2	0.71	2	0.822	0.824	2.6	0.24
3	3	0.71	2	0.858	0.860	1.9	0.18
3	4	0.71	2	0.882	0.883	1.5	0.14
3	0	0.58	3	0.684	0.687	5.6	0.45
3	1	0.58	3	0.810	0.812	2.8	0.26
3	2	0.58	3	0.865	0.866	1.8	0.17
3	3	0.58	3	0.895	0.896	1.3	0.12
3	4	0.58	3	0.914	0.915	1.0	0.10
3	0	0.50	4	0.714	0.717	5.0	0.41
3	1	0.50	4	0.840	0.842	2.2	0.21
3	2	0.50	4	0.890	0.891	1.4	0.13
3	0	0.45	5	0.736	0.739	4.5	0.38
3	1	0.45	5	0.862	0.863	1.9	0.18
3	0	0.41	6	0.753	0.756	4.1	0.36
3	0	0.38	7	0.768	0.770	3.9	0.34
4	1	1.00	1	0.631	0.636	9.3	0.80

....Cont

TABLE 3.2 - (CONTINUED)

OPTIMAL PRODUCTION RATES OF THE BOWL PHENOMENON AS COMPARED TO THOSE OF THE BALANCED LINE - DERIVED FROM HILLIER AND BOLING (73)

N	B	COVAR ($=1/\sqrt{k}$)	K	PR_b	PR^*	$BL\%^*$	$\frac{PR^* - PR_b}{PR_b} \times 100\%$
4	2	1.00	1	0.701	0.705	6.9	0.63
4	3	1.00	1	0.748	0.752	5.5	0.51
4	4	1.00	1	0.782	0.785	4.5	0.42
4	0	0.71	2	0.598	0.603	10.6	0.83
4	0	0.58	3	0.646	0.650	8.9	0.75
4	1	0.71	2	0.731	0.735	6.0	0.56
4	2	0.71	2	0.798	0.801	4.1	0.39
5	0	1.00	1	0.486	0.492	16.9	1.20
5	1	1.00	1	0.608	0.614	11.4	0.97
5	2	1.00	1	0.681	0.686	8.5	0.76
5	0	0.71	2	0.572	0.578	13.1	1.04
6	0	1.00	1	0.467	0.473	19.2	1.37
6	1	1.00	1	0.592	0.598	13.0	1.07

where

PR_b = production rate of the balanced line

PR^* = optimal production rate

$BL\%^*$ = optimal degree of imbalance (%)

Max BL% = maximal degree of imbalance (%)

k = shape parameter of the Erlangian distribution

N = line length

B = buffer capacity

Covar = co-efficient of variation

the exponential distribution, with its high variability, caused station 3 to experience much more idle time than that experienced when its service times were normally distributed. Moreover, in the exponential distribution case, the first and second buffers, though showing relatively high mean buffer levels, were not as near their capacity as in the case of normal service times. The pioneering effort of Anderson has the shortcoming of completely concentrating on studying the behaviour of the individual stations and buffers for unbalanced lines, which is though of some importance, not as significant as investigating the operating characteristics of the unbalanced line in totality.

To date, the most important simulation work concerning lines unbalanced with respect to their average service times is that of El-Rayah (46) who simulated lines of 3, 4, 12 stations having service times described by the normal, lognormal (positively skewed), and exponential distributions, with buffer capacities of 0, 2, 4 units, Covars of 0.15, 0.3, 1.0, and degrees of unbalance ranging from 0 (a balanced line) to 0.20. A search procedure was employed to help in finding the optimal degree of imbalance which provides the maximum production rate in comparison with the balanced line's PR and further, to explore the neighbourhood of this optimal degree in order to locate the maximal permitted degree which coincides with the achievement of an equivalent, or nearly equivalent PR, to that of the balanced line (a breakeven point). El-Rayah also used the multiple comparisonsand multiple ranking procedures, as well as the analysis of variance in his investigations to test various configurations of mean service times imbalance.

For a 3-station line having zero buffer capacity, exponential operation times, and different imbalance degrees, the patterns of means imbalance were:

(1) low - medium - high means arrangement (a monotone increasing order), which has been suggested by Davis (38) whose work will be presented in the next chapter.

(2) High - medium - low means configuration (a monotone decreasing order).

(3) High - low - high (the bowl phenomenon).

(4) Low - high - low (the reverse of the bowl phenomenon).

The author verified that the first pattern is always better than the second with respect to PR and also established that, in terms of PR, the fourth pattern is the worst and consequently, decided not to use patterns 2 and 4 in any subsequent experiments. Confirming Hillier and Boling's (75) results, the optimum design was the third, i.e. the bowl phenomenon, which resulted in a 0.54% increase in output rate over that obtained by a balanced line.

For a 4-station line having exponential, normal, and lognormal operation times, Covars of 0.15, 0.3, and a maximum B of 4 units, four unbalanced means patterns were considered, viz,

(1) Low - high - low - high (Patterson's (135) arrangement). El-Rayah has initially verified that this pattern is superior to its opposite counterpart, i.e. high - low - high - low.

(2) High - low - low - high (the bowl phenomenon).

(3) Low - low - high - high.

(4) A monotone increasing order of mean operation times (attributable to Davis (38)).

Again, the bowl phenomenon pattern showed itself superior to both the balanced and unbalanced counterparts with regard to PR.

For lines consisting of 12 stations only the normal and exponential distributions were utilized and the configurations of imbalance were as follows:

(1) A monotone increasing order such that stations 1-4, 5-8, and 9-12 have low, medium, and high mean operation times respectively (i.e.).

(2) Low - high - low - high - .... - high (Patterson's design).

(3) The bowl phenomenon: of all the possible alternatives, only the following three were considered:-

(a) Stations 1-3, 10-12 have the same high mean work times, while stations 4-9 have the same low mean service times.

(b) Stations 1-4, 9-12 have equivalent high means values, whereas stations 5-9 are alloted equal low means values.

(c) The two stations in the middle are assigned the lowest means values, whilst the remaining stations are successively allocated higher means values in equal increments as one moves away from the centre of the line.

Again, the simulation results indicated that the three bowl phenomenon varieties are better, in terms of PR, than the balanced and unbalanced configurations, with pattern c being the best, achieving a maximum improvement of 1%, followed by a then b. This means that if c is impractical for certain lines, designs a and b, which closely resemble it, may be advantageously adopted.

The broad conclusions of El-Rayah's work may be summarised as follows:

(1) The bowl phenomenon is the most efficient pattern, on the conditions that B is relatively small ($B \leqslant 2$) and/or the Covar is relatively high ($Covar \geqslant 0.15$), since the potential for improving PR as well as raising the optimal and breakeven degrees of imbalance from adopting the bowl phenomenon strategy, increases as the Covar is increased and B is reduced.

(2) The balanced pattern is optimal, as concerns PR, when a high B ($B > 4$) is provided, or if a relatively moderate B ($2 \leqslant B \leqslant 4$) is associated with a relatively small Covar ($Covar \leqslant 0.15$), since, as was stated in (1), increasing B and reducing the Covar have the impact of decreasing the bowl phenomenon efficiency.

(3) In all the situations considered the Patterson's method of line unbalancing results in a very tiny

improvement in PR over that of the balanced line, provided that the balancing loss degree is<0.04 and the value of B is zero, but this method is inferior to that of the balanced line outside this range and, therefore, is regarded as unreliable. On the other hand, the monotonically increasing operation times' means pattern, as favoured by Davis, is consistently the worst tested; it almost always produced significantly lower PR than that achievable by a balanced line even when the unbalance degree is very low and, as a consequence, is considered inefficient.

(4) This investigation confirms the robustness of the bowl phenomenon that has been advanced by Hillier and Boling (75), in that the degree of imbalance can reach a high magnitude and still the yielded PR is equal to, or slightly lower, than that obtained from the balanced line.

(5) The imbalance degree affects the mean PR of the unbalanced line, no matter which pattern of means imbalance is employed. For the configurations which failed to generate PR's higher than that of the balanced line arrangement, increasing the degree of imbalance significantly reduces PR, whereas for the patterns that provided superior PR values to those obtainable from the balanced line, increasing the imbalance degree initially increases PR until an optimal PR value is arrived at, beyond which any further increase in DI will reduce PR, and after reaching a breakeven point, the PR will deteriorate below the level of the balanced line counterpart.

(6) The normal and skewed shapes of the operation times distribution have little influence on the optimum and breakeven degrees of imbalance. In general, PR is lower for a line with lognormal service times than that for one with normal operation times. The Covar, however, was shown to be more important than the distribution's shape in influencing the optimal and breakeven degrees of line imbalance, the greater the Covar the higher such degrees are. On the other hand, using a less variable distribution, such as the normal, supplies results which are strongly related to those of a more variable distribution, such as the exponential.

(7) The maximal potential increase in PR over that of the balanced line, when the bowl shape is used, tends to become higher as N goes up. This is in line with Hillier and Boling's finding.

The author stated that, based on the above conclusions, the bowl configuration is preferable to the balanced line, the higher the Covar, the smaller the B, and the larger the N. The reason being that under such conditions the probability of blocking and starving idle times rises, and the bowl phenomenon alleviates the detrimental effects of these conditions.

The author went on to advance an explanation of the bowl phenomenon considering a 7-station line, with station 4 being in the middle and having the fastest mean operation time, while stations 1-3 forming a monotone decreasing order of mean service times and stations 5-7 forming a monotone increasing order. The author explained that the likelihood of blocking and chain blocking, in a balanced line, goes up the closer a station is to the beginning of the line. Likewise, the probability of starving and chain starving increases for stations towards the end of the line. These two components of idle time offset each other in the steady state's operational mode so that the utilization of all stations is approximately equal. The superiority of the bowl design lies in its being capable of attacking these idle times' components where they take place more often. The decreasing order of mean operation times from stations 1 to 3 diminishes the chance of them being blocked when finishing their work. On the other hand, the monotone increasing order for stations 5 through 7 decreases the probability of starving them. The middle station behaves as a buffer between the two conflicting kinds of idle time. When the Covar is reduced and/or B is increased, the dependence between stations decreases and, as a result, the bowl design (and any other unbalanced means design) will become less efficient since the existing line's imbalance starts to play its influence.

El-Rayah concluded that the direction of imbalance is so important that an incorrect manner of imbalance renders its PR much less than that produced by a balanced line. The percentage improvement in PR by deliberately unbalancing the line in the right direction can be large if the alternative is any pattern which comes closest to a notionally

balanced line. He showed that in a 4-stations line, the line's designer may prefer an inverted bowl configuration with a degree of imbalance of 0.02, over a bowl arrangement with a balancing loss degree of 0.04, because the former is nearer to the balanced line. When simulating both patterns with normal service times, B = 0, and Covar = 0.3, he found that the PR's of the inverted bowl and the bowl designs are, respectively, lower and higher than that of the balanced line. Therefore, in view of the fact that the nominal balance is often unlikely in practice, the selection of a correct pattern of imbalance is an important factor contributing to the efficiency of the line.
Table 3.3 provides data on the robustness of the bowl phenomenon along with its maximum achievable PR.

A shortcoming of this work, despite its merits, is that the stockholding properties of the means imbalance were not investigated. Clearly, the stockholding measures constitute an important part of any production line's research, in addition to the activeness measures.

(c) <u>The Third Approach</u>

Van Beek (170) stated that the unequal mean operation times' imbalance leads to long-term delays which differ from the short-term system's loss delays. Both the balancing and system losses influence the efficiency of the unbalanced production line, and are always concomitant in their contribution to the total idle time of the line. The

TABLE 3.3

THE BOWL PHENOMENON'S OPTIMALITY AND FLEXIBILITY - ADAPTED FROM EL-RAYAH (46)

N	B	COVAR	OPERATION TIMES DISTRIBUTION	PR_b	PR^*	$BL\%^*$	$\frac{(PR^*-PR_b)}{PR_b} \times 100\%$	MAX BL%	PR(MAX B
3	0	0.30	Normal	0.802	0.804	4	0.27	7	0.801
3	2	0.30	Normal	0.945	0.946	1	0.14	2	0.945
3	4	0.30	Normal	0.966	0.967	1	0.02	2	0.963
3	0	0.30	Lognormal	0.796	0.798	4	0.31	7	0.795
3	2	0.30	Lognormal	0.941	0.942	1	0.09	2	0.940
3	4	0.30	Lognormal	0.963	0.963	0 (balanced)	0.0	1	0.963
3	0	0.15	Lognormal	0.887	0.889	2	0.16	4	0.886
3	2	0.15	Lognormal	0.982	0.982	0 (balanced)	0.0	1	0.980
4	2	1.00	Exponential	0.701	0.705	8	0.67	14	0.700
4	4	1.00	Exponential	0.782	0.786	6	0.52	10	0.781
4	0	0.30	Normal	0.780	0.784	5	0.48	10	0.780
4	2	0.30	Normal	0.939	0.941	2	0.23	3	0.939
4	4	0.30	Normal	0.961	0.963	1	0.20	2	0.961
4	0	0.30	Lognormal	0.770	0.774	5	0.50	10	0.770
4	2	0.30	Lognormal	0.935	0.936	2	0.12	3	0.934
4	4	0.30	Lognormal	0.957	0.958	1	0.12	2	0.956
4	0	0.15	Lognormal	0.873	0.875	3	0.27	5	0.873
4	2	0.15	Lognormal	0.979	0.979	0 (balanced)	0.0	1	0.978
12	0	1.00	Exponential	0.417	0.422	10	1.00	20	0.417
12	0	0.30	Normal	0.731	0.738	5	0.87	10	0.730
12	2	1.00	Exponential	0.623	0.625	5	0.32	10	0.623
12	4	1.00	Exponential	0.715	0.716	5	0.22	NA	NA

where

PR_b = production rate of the balanced line
PR^* = optimal production rate
$BL\%^*$ = optimal degree of imbalance (%)
Max BL% = maximal degree of imbalance (%)
N = line length
B = buffer capacity
Covar = co-efficient of variation

author indicated that the line length affects the balancing loss in that the higher the former, the greater the latter is, since the likelihood of a larger difference in mean service times between the fastest and the slowest operators is increased as N becomes higher.

Kilbridge and Wester (90), in an attempt to determine the optimum extent of the division of labour, identified the means imbalance as an inherent cost. This cost results from the fact that the industrial tasks in most cases are not perfectly divisible, since the elements making up the task are usually not discrete in nature, which will ensure an unequal distribution of the total work content among stations, resulting in unequal mean service times.

The magnitude of the balancing loss cost depends on the range and the distribution of work element times, their sizes in comparison to that of the cycle time, the constraints imposed on the specific order of their assembly, and the method of line balancing used. For a given cycle time a relatively low degree of imbalance results when the task is made up of many small elements rather than a few large ones. Moreover, the imbalance degree is reduced if there is no or few restrictions on the arrangement of work elements, since the job designer will have more freedom in assigning the work elements to the tasks. Furthermore, other things being equal, paralleling the stations can reduce the imbalance, especially if one or more of the element times is larger than the cycle time.

Kilbridge and Wester indicated that there is a functional relationship between the cycle time and balancing loss. As the cycle time decreases and becomes closer to the element times, the degree of imbalance goes up, until a further sub-division of the task becomes technically infeasible. The authors obtained the following general expression for the imbalance experienced in various assembly settings:

$$\% \text{ imbalance} = a/CT^{b}$$

where CT = cycle time

a and b = constants which depend on the task's nature and the restrictions on line balancing.

Another formula for the percentage of balancing loss (B%) was suggested by Wild (177), and is given by:-

$$B\% = a/b$$

where a = average number of workelements assigned to a station.

b = constant, usually around 20.

Muth (127) developed a novel approach to examine the behaviour of production lines. Rather than trying to identify and list all the system's states, as is done in the queuing approach, or to find an approximate general formula like that of Knott (96) (see Chapter 1), he employed the order statistics' techniques to describe the durations of starving and blocking, and derived analytical solutions for lines with two and three stations, as well as developing procedures for calculating the upper and lower bounds of the production rate for any number of stations and any service time distribution. In an attempt to free his examination from any direct reference to buffer capacity, he argued that B buffers can be viewed as B serial work stations, each with a service time of zero. In this manner a line having N stations and N-1 buffers, each with a capacity of B, may be conceptualised as a k-station line where $k = N + B(N - 1)$, and $B(N - 1)$ of the stations have process times of zero time units.

Muth then proved that the starving and blocking idle times are jointly distributed random variables with a strong relationship of dependence. He gave the following expression for the mean production rate, PR, as related to blocking, starving, and service times:

$$PR = \frac{1}{E(S_i) + E(B_i) + E(I_i)} \qquad \text{where}$$

$E(S_i)$ = expected service time of station i

$E(B_i)$ = expected blocking time of station i

$E(I_i)$ = expected starving time of station i

Assuming that the production rate of the slowest station in the line determines the upper limit of the production rate of the line as a whole, PR_u, this upper bound for an N-station line may be given as:

$$PRu = \frac{1}{\max\left(E(S_1), E(S_2), \ldots, E(S_n)\right)}$$

where

$E(S_i)$ = expected service time for station i

The author stated that PR_u is only achieved if the operation times are constant,or if the buffer capacity is unlimited. Likewise, the lower bound on the output rate, PR_L, for a line with N stations is given by:

$$PR_L = \frac{1}{E\left(\max(S_1, S_2, S_3, \ldots, S_k)\right)} = \frac{1}{T}$$

where

S_i = service time at station i

$$T = \int_0^\infty (1 - F(t)\ \ dt$$

and

$$F(t) = F_1(t)\ F_2(t)\ F_3(t)\ \ldots\ F_k(t)$$

$F_i(t)$ = cumulative probability function of operation times for station i.

Muth showed PR_L/PR_u to change with k for lines having various Covars and operation times distributions, in that when the Covar increases, so does the difference between the upper and lower bounds of mean production rates. He next argued that because a realistic Covar value is <0.1, the difference between PR_u and PR_L will be small, and therefore, a reasonably accurate estimate of PR can be obtained by this method. Having examined Hillier and Boling's (72) results for a line with N stations, zero buffer capacity, and Erlangian operation times, the author concluded that the real output rate's value is nearer to PR_L than PR_u.

Two main objections may be raised against this study. Firstly, the real manual work times' data suggest that, on the average, a Covar of 0.1 or less is unrepresentative and very small. Thus, for a realistic mean Covar value of around 0.274 (as will be mentioned later), the difference between the lower and upper bounds of PR will be wider, and the actual PR will not necessarily be closer to PR_L than PR_u. Secondly, the results of Hillier and Boling correspond to lines with buffer capacity of zero, which decreases the PR towards the lower bound, while Muth's PR_u, PR_L expressions are entirely insensitive to the line's buffer capacity and, consequently, do not provide realistic output rates. The paper remains worthwhile, however, because of its employment of order statistics as a tool for determining the characteristics of the general series queues lines, and due to its setting of upper and lower limits for the efficiency of the line.

Jacobson and Sadowski (83), recognising the viability of the unbalanced line design, presented procedures for obtaining feasible fractional and integer task assignments, using linear and integer programming, which can be employed to schedule workers on an unbalanced line, with the objective of minimising the total inventory cost in the line so as to maximise the line's output. However, the assumption of deterministic unbalanced operation times, as well as the assumption of full operator's utilization with no blocking or starving idle times, make these procedures inappropriate for the real life lines. Nevertheless, they can be viewed as a step in the right direction to determine algorithms for the unbalanced assignment of tasks to operators.

Shimshak and Sphicas (157) considered a 2-station line where each item of work is composed of n distinct tasks, and k_1 of those tasks are allocated to the first station and $k_2 = n-k_1$ are assigned to the second station, with the objective of distributing the n tasks to the two stations in order to minimise the total waiting time. They assumed Poisson arrivals and exponential task times such that when more than one task is alloted to a station, its service times distribution becomes Erlangian. The interdeparture times of such lines are statistically dependent and since the output from station 1 is an input to station 2, the interarrival times at the second station are also dependent.

As a consequence of the complete dependence of the second station's arrival and departure processes, an exact formula for its mean waiting time is unobtainable from a statistical viewpoint, however, it is possible to compute it through mathematical approximations or simulation. For this purpose some approximate expressions, including that of Fraker (57), were used and their outcomes were compared to those obtained from simulations. In these simulations, the chosen values of the utilization, P, were 0.1, 0.5, 0.9, 1.3, and those of n were 2,3,4,5,6,10,20. For each value of n (except for n = 20) all the possible alternative k_1 and k_2 assignments were taken into consideration, including balanced ($k_1 = k_2$) and unbalanced ($k_1 \neq k_2$) allocations, with the latter having either increasing or decreasing mean task times' order, i.e. (/), (\\), for various degrees of imbalance. The comparison demonstrated that all the approximate formulae did reasonably well relative to the simulation results.

The authors concluded that an equal (balanced) work assignment can be regarded as the best design, since all the unbalanced assignments were inferior in terms of the mean waiting time. Furthermore, when n is even, a balanced line is possible, but if n is odd, a balanced line is impossible and the two slightly unbalanced alternatives, whereby $k_1 = (n-1)/2$ and $k_1 = (n+1)/2$, give nearly the same results and no one is superior to the other, but both are preferable to other unbalanced arrangements. The optimality of the balanced (or approximately balanced) design was also found to be true with respect to the idle time and the mean buffer level.

Although interesting, this study suffers from the drawback that the model did not provide for blocking, as admitted by the authors, in addition to the use of a very short line and exponential task times, which renders its findings inapplicable in practice.

UNEQUAL COVARS' IMBALANCE

In this type of line's imbalance the stations have unequal variability of operation times but equal means, and the total buffer capacity is allocated equally between the various buffers.

(a) The Queuing Theoretic Approach

Tembe and Wolff (168) examined the sequencing of a two-station line with one station having a constant service time and the other having variable operation times, with a specific distribution function. They found that placing the deterministic station first in order is favourable, as far as the total waiting time is concerned.

Rao (144) stated that the effect of the change in the values of Covar and buffer capacity on the performance of an unbalanced two-station line having different Covars, is similar to that for a balanced line counterpart, i.e. increasing the Covar and/or reducing the size of the buffers, decreases the efficiency of the line. He also found that the reversal of the order of the two stations with unequal Covars, has no effect on PR which means that the reversibility property is valid for the Covars, as well as the means imbalance.

(b) The Simulation Approach

Anderson (1) simulated a line with 4 stations, notionally balanced mean service times, and a buffer capacity of six units per buffer. The individual stations' Covars and type of operation times distribution were as follows:

Station	Covar	Distribution
1	0.1	Normal
2	0.3	Normal
3	1.0	Exponential
4	0.2	Normal
Average Covar	0.4	

The findings of this study are summarised below:

(1) Since the mean Covar for the line as a whole is greater than that for a balanced line with all the stations having normal service times and a Covar of 0.3 each, the idle time for this line will be higher than that for the balanced normal line. In addition, since this line's average Covar is less than that for a balanced exponential line, its idle time is expected to be less than that for the exponental line.

(2) The individual stations with greater Covars experience higher idle times than those with smaller Covars.

(3) There appears to be no effect on the mean buffer level from having an unbalanced Covars arrangement.

In a subsequent paper, Anderson et al (3) simulated 2, 3 and 4-station lines with 0,2,4,6,8 buffer capacities and normally distributed operation times. The individual stations' Covars were allowed to differ and the overall Covar of the line ($\overline{\text{Covar}}$) was varied from 0.01 to 0.30 in increments of 0.01, and was calculated from the following formula:

$$\overline{\text{Covar}} = \sqrt{\frac{(\text{Covar}_1)^2 + (\text{Covar}_2)^2 + \ldots\ldots + (\text{Covar}_N)^2}{N}}$$

where Covar i = Covar of station i, i = 1, N

For each overall Covar's value two patterns of Covars imbalance were used, for example, in a 3-station line with an overall Covar of 0.20 the two patterns were:-

Pattern	Covar 1	Covar 2	Covar 3	Overall Covar	Shape
1	0.20	0.05	0.28	0.2007	VSV
2	0.24	0.24	0.08	0.2013	VVS

where

V = relatively variable Covar

S = relatively steady Covar

The authors' results indicated that the idle time of the line is a function of buffer capacity, the overall Covar, and line length, in a similar fashion to that when the line is balanced (see Chapter 1). The results also showed that the pattern of Covars imbalance has no significant influence on the line's idle time. This is in contrast with El-Rayah's finding, which will be reported later on in this section.

With regard to the mean total number of units in the line, L, it appeared that L is affected by both B and N, as was the case for the balanced lines, but with the addition that the increase in L diminishes then stops as B rises further. Additionally, the overall Covar seems to exert no significant impact on L. Moreover, the results on the individual stations and buffers' behaviour for this unbalanced Covars investigation are in general agreement with those of the balanced line (Chapter 1), with the new finding that the overall Covar appears to have little effect on the individual buffers' mean inventory level.

Anderson et al developed the following expressions for I and L:

$$I = \frac{1}{B+0.453}\left(0.134 + 0.131N^{0.028} + 0.111\ \overline{Covar}^{0.870} + 0.052N\ \overline{Covar}\right)$$

$$L = 0.08 - 0.27B + 0.93N + 0.41NB$$

The authors went on to derive an approximation to the optimal buffer capacity (B^*) as:

$$B = \sqrt{\frac{a_1 k}{-0.27L_1 + 0.41NL_1 + (N-1)L_2}} - a_2$$

where $a_1 = -0.134 + 0.131N^{0.028} + 0.111\ \overline{Covar}^{0.870} + 0.052N\ \overline{Covar}$

$a_2 = 0.453$

k = idle time cost/unit time

L_1 = stockholding cost/unit/unit time

L_2 = space provision cost/unit/unit time

El-Rayah (45) simulated 3,4,12-station lines having notionally balanced normal service times and zero buffer capacity for all the buffers, and unbalanced Covar values, utilizing the analysis of variance and multiple comparison for the data analysis. The experimental design, output rate, and the percentage change in PR over that of the balanced line from using the various unbalanced Covars patterns are shown in Table 3.4.

It should be noted that in this table the second, third, and fourth patterns in B3 are, respectively, the high - low - low- high, low - high - low - high, and low - medium - medium - high configurations, which are similar in shape to those investigated by the author in (46) with respect to operation times' means imbalance. Note also that the four patterns of both C1 and C2 are similar in form to their counterparts in A and B3. In order to reduce the number of patterns in C1 and C2 to a small subset out of an enormous number of possible combinations, the restriction has been made that a station having a low Covar should not be separated by another one having a high Covar.

The main conclusions of this study are that the differences in L between the various patterns in designs A through C2 are slight and not statistically significant, indicating that the inequality of Covars has little impact on L. On the other hand, the Covars imbalance does have a significant effect on PR. It was found that the best Covars unbalance pattern is the high - low - low - high which resulted in a significantly superior PR over those obtained by the other unbalanced patterns and the balanced

TABLE 3.4

THE EFFECTS OF UNBALANCING STATIONS' COVARS FOR LINES WITH NORMAL SERVICE TIMES AND ZERO BUFFER CAPACITY - ADAPTED FROM EL-RAYAH(45)

		STATION i COVAR'S VALUE, i = 1,2,....,12													
N	DESIGN	1	2	3	4	5	6	7	8	9	10	11	12	PR	
3	A	0.30	0.30	0.30	-	-	-	-	-	-	-	-	-	0.8021	d
3	A	0.15	0.30	0.30	-	-	-	-	-	-	-	-	-	0.8223	2
3	A	0.30	0.15	0.30	-	-	-	-	-	-	-	-	-	0.8279	3
3	A	0.30	0.30	0.15	-	-	-	-	-	-	-	-	-	0.8226	2
4	B1	0.30	0.30	0.30	0.30	-	-	-	-	-	-	-	-	0.7803	d
4	B1	0.30	0.30	0.30	0.15	-	-	-	-	-	-	-	-	0.7940	1
4	B1	0.30	0.30	0.15	0.30	-	-	-	-	-	-	-	-	0.8009	2
4	B1	0.30	0.15	0.30	0.30	-	-	-	-	-	-	-	-	0.8008	2
4	B1	0.15	0.30	0.30	0.30	-	-	-	-	-	-	-	-	0.7941	1
4	B2	0.30	0.30	0.15	0.15	-	-	-	-	-	-	-	-	0.8170	4
4	B2	0.30	0.15	0.30	0.15	-	-	-	-	-	-	-	-	0.8187	4
4	B2	0.30	0.15	0.15	0.30	-	-	-	-	-	-	-	-	0.8251	5
4	B2	0.15	0.30	0.30	0.15	-	-	-	-	-	-	-	-	0.8119	3
4	B2	0.15	0.30	0.15	0.30	-	-	-	-	-	-	-	-	0.8190	4
4	B2	0.15	0.15	0.30	0.30	-	-	-	-	-	-	-	-	0.8168	4
4	B3	0.225	0.225	0.225	0.225	-	-	-	-	-	-	-	-	0.8258	d
4	B3	0.25	0.20	0.20	0.25	-	-	-	-	-	-	-	-	0.8272	0

.... Cont

TABLE 3.4 (CONTINUED)

THE EFFECTS OF UNBALANCING STATIONS' COVARS FOR LINES WITH NORMAL SERVICE TIMES AND ZERO BUFFER CAPACITY - ADAPTED FROM EL-RAYAH(45)

		STATION i COVAR'S VALUE, i = 1,2,...,12													
N	DESIGN	1	2	3	4	5	6	7	8	9	10	11	12	PR	M
4	B3	0.20	0.25	0.20	0.25	-	-	-	-	-	-	-	-	0.8249	-0.11
4	B3	0.20	0.225	0.225	0.25	-	-	-	-	-	-	-	-	0.8253	-0.06
12	C1	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.7313	datum
12	C1	0.15	0.15	0.15	0.15	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.7419	1.43
12	C1	0.30	0.30	0.30	0.30	0.15	0.15	0.15	0.15	0.30	0.30	0.30	0.30	0.7609	3.89
12	C1	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.15	0.15	0.15	0.15	0.7425	1.51
12	C2	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.7879	datum
12	C2	0.25	0.25	0.25	0.20	0.20	0.20	0.20	0.20	0.20	0.25	0.25	0.25	0.7879	0.41
12	C2	0.20	0.25	0.20	0.25	0.20	0.25	0.20	0.25	0.20	0.25	0.20	0.25	0.7835	-0.15
12	C2	0.20	0.20	0.20	0.20	0.225	0.225	0.225	0.225	0.25	0.25	0.25	0.25	0.7814	-0.42

where

N = line length

PR = production rate of the unbalanced line

M = % change in PR over that of the balanced line

Datum = production rate of the balanced line as a basis for comparisons line. This points out the existence of a bowl phenomenon with regard to the service times' Covars imbalance, in addition to that which exists in the case of means unbalance. Furthermore, the superiority of the Covars bowl phenomenon, over the other unbalanced patterns, increases as N is increased. The tentative rationale of such a phenomenon, as argued by El-Rayah, is that the allocation of low Covars to the middle stations produces the same effect of speeding them up as when they have low mean operation times, which was discussed earlier in this chapter.

Apart from the major conclusions, some other findings related to each particular design are:

(1) In A, assigning the lower Covar to either the beginning or the end station will result in almost identical PR.

(2) In B1, the first and fourth configurations are significantly worse than the second and third, as far as PR is concerned. Thus, the low Covar should be alloted to either stations 1 or 3.

(3) In B2, the fourth pattern, which is the opposite of the third (the bowl phenomenon pattern), is significantly the worst. No significant differences in PR were found among the other patterns (with the exception of the bowl phenomenon arrangement).

(4) In B3, the balanced configuration is significantly superior to the low - high - low - high pattern, but not significantly better than the low - medium - high pattern. The results of B3 resemble those of the unequal

service times' means, with the single difference that the pattern representing an increasing order of Covars is better than the low - high - low - high pattern.

(5) In C1, there is no significant difference in PR between the first and third patterns, which confirms the result of A.

(6) In C2, the first configuration is superior (significantly) to the fourth, but not so to the third, lending support to the result of B3.

Carnall and Wild (26) examined the effect of having constant and variable stations (Weinbul distributed) in the line. The authors' experimental design is exhibited below:

Experiment	N	B	Covar of the Variable Station	Pattern of Covars Imbalance
1	4	1,2,3	0.10,0.27,0.50	V,C,C,V (The bowl phenomenon) C,V,V,C (an inverted bowl)
2	10	1,2,3	0.27,0.50	V,V,V,C,C,C,C,V,V,V (the bowl phenomenon) C,V,C,V,V,V,V,C,C,V (random) C,C,V,V,V,V,V,V,C,C (an inverted bowl)

where

V = relatively variable station (Covar = 0.1 or 0.27 or 0.5)

C = constant station (Covar = 0.0)

The results were subjected to the analysis of variance and pairwise comparisons. The main findings of this research are:

(1) Placing the constant service times at the middle stations and the variable service times at both ends of the line (the bowl pattern) is the best policy, reducing the mean idle time and increasing the output rate by up to 4% over that of the other unbalanced

designs. But raising B and reducing the Covar will decrease the advantage of the bowl pattern.

(2) B and Covar significantly affect the efficiency of the line in the same manner as that of the balanced line case.

(3) The gain in efficiency, from arranging the Covars according to the bowl phenomenon, is likely to be significantly higher than that obtainable from assigning the means according to the bowl configuration.

UNEQUAL BUFFER CAPACITIES' IMBALANCE

In this third type of imbalance, all the stations in the line have exactly the same service times' means and Covars, while the individual buffers have unequal capacities.

(a) The Queuing Approach

Hatcher's previously reviewed study (69) was the only one to touch the unbalanced buffer capacities situation from a queuing perspective. Two cases were investigated for a 3-station line:

(1) When one buffer has a considerably larger capacity than the other, increasing the size of the smaller buffer, rather than the larger one, will yield the highest rise in PR.

(2) If both buffers are equal in capacity, increasing the size of the second buffer, rather than the first one, is more beneficial. The difference in benefit (D) depends on B and is given by:

$$D = \frac{1 - (\frac{1}{2})^{B + 1}}{B^2 + 5N + 9/2}$$

(b) The Simulation Approach

El-Rayah (45) simulated 3 and 4-station lines having normally distributed operation times and Covar of 0.3 for each station. The simulations' data were then subjected to the multiple comparisons and analysis of variance procedures. Initial runs established that when the total buffer capacity of the line is $< 4(N - 1)$, i.e. the mean capacity per buffer is < 4, unbalancing the buffer capacities for short lines has the consequence of reducing PR and increasing L, therefore, the optimal design, in terms of both PR and L, is an equal distribution of the total available B. The author chose total buffer capacities of 8, 24 for 3-station lines, (i.e. 4, 12 units on average per buffer) and 12, 36 for 4-station lines, (i.e. a mean capacity of 4, 12 units per buffer). The adopted method of unbalancing the buffer sizes was to reduce the capacity of a single buffer by one or two units and assign it (them) to another buffer. All the permutations of the unbalanced buffer capacities were considered.

Table 3.5 shows the patterns of imbalanced buffer capacities, together with their results. These results indicate the following:

(1) Unbalancing the capacities of the individual buffers will not lead to a significant improvement in PR over that of the balanced buffer sizes design, since the latter results in a very high PR which means that it is virtually impossible to significantly improve upon it. In general, the effect of the inequality of buffer sizes on PR is very low.

THE EFFECTS OF UNBALANCING BUFFER CAPACITIES FOR SHORT LINES WITH NORMAL SERVICE TIMES AND A COVAR OF 0.3 - ADAPTED FROM EL-RAYAH (45)

N	TB	B1	B2	B3	PR	L	M-PR	M-L
3	8	4	4	-	0.9665	7.016	datum	datum
	8	3	5	-	0.9633	6.349	-0.32	-9.51
	8	5	3	-	0.9645	7.637	-0.20	+8.85
	24	12	12	-	0.9901	14.144	datum	datum
	24	11	13	-	0.9889	13.104	-0.13	-7.35
	24	13	11	-	0.9905	15.001	+0.03	+6.06
	24	10	14	-	0.9875	12.469	-0.26	-11.84
	24	14	10	-	0.9901	15.582	0.0	+10.17
4	12	4	4	4	0.9609	10.165	datum	datum
	12	3	4	5	0.9065	9.309	-5.67	-8.42
	12	3	5	4	0.9612	9.782	+0.02	-3.77
	12	4	3	5	0.9590	9.680	-0.20	-4.77
	12	4	5	3	0.9615	10.734	+0.06	+5.60
	12	5	3	4	0.9584	10.535	-0.26	+3.64
	12	5	4	3	0.9594	11.098	-0.16	+9.18
	36	12	12	12	0.9887	24.005	datum	datum
	36	11	12	13	0.9881	23.162	-0.06	-3.51
	36	11	13	12	0.9890	23.762	+0.03	-1.01
	36	12	11	13	0.9878	23.358	-0.09	-2.70
	36	12	13	11	0.9878	24.323	-0.09	+1.33
	36	13	11	12	0.9882	23.084	-0.05	-3.84
	36	13	10	13	0.9867	23.519	-0.21	-2.03
	36	12	14	10	0.9877	25.142	-0.11	+4.74
	36	11	14	11	0.9890	23.762	+0.03	-1.01

where

N = line length

PR = mean production rate

L = mean total number of units in the line

Bi = buffer capacity of buffer i

TB = total buffer capacity

M = % change in PR or L over that of the balanced pattern

Datum = balanced line's PR or L as bases for comparisons

(2) Wherever imbalance of buffer capacities is unavoidable, allocating higher capacities to the middle buffers and lower capacities to the early and end buffers is more efficient, with respect to PR, than other unbalanced configurations. Therefore, a bowl phenomenon with regard to buffers contents' imbalance does not distinctly exist and, for high TB and low imbalance, it results in a similar PR to that of the balanced line. This implies, in essence, that the most efficient and reliable pattern is a balanced one.

(3) If the aim is to maximise PR, an increasing order of B's will generate a lower PR than that obtainable by the balanced and any other unbalanced arrangements and, therefore, should be discouraged, whereas if the objective is to decrease L, it should be encouraged since it yields significant reduction in L (up to 12%) over that of the balanced configuration. Furthermore, in almost all the cases tested the savings in L, from adopting an increasing order of B, far exceeds the resultant decrease in PR. However, the absence of an objective measure of the relative importance of L and PR precludes any firm recommendation in this direction.

(4) A decreasing order of B increases L, as compared to that of the other unbalanced and balanced patterns, and gives a PR which is lower than that achievable by the balanced line, but not always higher than that of the other unbalanced configurations.

(5) The sensitivity of short lines to the unbalanced allocation of buffer capacities is much less than that

of the service times' means or Covars imbalance.
Though interesting and advantageous, this work has two main shortcomings (in addition to that of utilizing a short N value). Firstly, the selected TB (and hence MB) values were so high that it was not possible to obtain significant contrasts between the various patterns, since they initially produced high PR's. Secondly, no attempt has been made to determine the influence of having higher degrees of imbalance on PR, when allocating the total B among the buffers.

(c) The Third Approach

The research study of Knott (96) showed that the highest achievable efficiency, for a given TB value, takes place where all the buffers have the same amount of storage capacity. This supports El-Rayah's finding.

Soyster et al (164) used separable programming to determine the optimal distribution of the total buffer capacity and then constructed a simulation model to evaluate the mathematically obtained results, for N = 4,6,7,8 and different TB allotments. They found that a modest imbalance in buffer capacities' assignment appears not to affect PR, however, an extreme unbalance can significantly reduce PR.

(4) OPERATION TIMES' MEANS AND COVARS IMBALANCE

This type of imbalance is characterised by the fact that both the means and Covars of stations' service times are jointly unbalanced, while the line is otherwise balanced insomuch as the allocation of total buffer capacity is concerned.

(a) The Queuing Approach

The research into the steady state behaviour of lines unbalanced with reference to both their means and Covars is solely confined to the queuing theoretic approach and, as yet, no simulation study of such lines has been reported.

Rao (143) examined a two-station production line with one station having exponential service times (Covar = 1), and the other having deterministic service times (Covar = 0). Any order of the two stations is allowed. He showed that a small percentage increase in PR of nearly 0.26% is achieved when the exponential station is slightly faster, i.e. when its mean operation time is slightly lower. Increasing B, however, quickly diminishes the improvement of this pattern, e.g. for a B of zero, the optimum ratio of the constant to the exponential stations' mean service times is 1.08:1, for B = 1 the optimum ratio becomes 1.01:1, and for B = 2 a balanced line (a ratio of 1:1) is the best. In addition, Rao demonstrated the robustness of unbalancing such lines in the correct manner by showing that no appreciable reduction in PR results, even if the mean service time of the constant station is 40% higher than that of the exponential one (a means ratio of 1.2:1).

In a second paper (144) Rao examined a 2-station line with Erlangian and normal process times and proved that if the two stations are unidentical in terms of their variability, a balanced arrangement of mean service times is either detrimental or at least not optimal, as far as PR is concerned. In this case the PR can be optimised when a slightly higher mean is assigned to the less variable station. As the difference between the Covars of the two stations increases, so does the optimum degree of means imbalance, until it reaches to a limiting value of 0.07 when the difference in Covars becomes 1.0, i.e. when one station is constant and the other is exponential. The author further indicated that unbalancing the line in the wrong direction reduces its PR. He argued that the deliberate unbalance of the stations' mean service times tends to correct the already existing imbalance in their Covars. Moreover, increasing the value of B plays a corrective role on the existing Covars imbalance by decreasing the need to unbalance service times' means.

For a line having exponential first station and Erlangian second station with B = 0, the author found that alloting slightly higher mean to the less variable (Erlangian) second station will slightly increase PR, and that the higher the Covar of the Erlangian station, the lower the improvement in PR over that of the balanced mean service times. When the Covar of the Erlangian station becomes 0.8, the difference in variability between the two stations narrows close enough to warrant balancing the line. Similar results were obtained when the second station has normal, instead of Erlangian service times.

In a third paper, Rao (142) established the validity of the concept of assigning a slightly lower mean operation time to the more variable station regardless of its position in the line, so as to increase PR, for a three station line with each station having a non-identical Covar. He examined six different patterns of means imbalance for the cases of having one deterministic and two exponential stations, and two constant and one exponential station. In each case the maximum PR was achieved when the exponential station(s) was made faster.

Rao explained that in a 3-station line having identical exponential stations, the bowl phenomenon results in a maximum improvement in PR of 0.54% over that of the balanced counterpart. However, with non-identical stations (i.e. stations with different Covars), the maximum improvement in PR rises sharply to 6.79%. Remembering that the maximum PR's increase for a 2-station line with non-identical Covars was 0.26%, the sharp increase to 6.79% for a 3-station line encourages the speculation that longer lines may yield even larger improvements.

The bowl phenomenon's effect was then compared to that of the non-identical variabilities of the individual stations, and it was found that sometimes the latter outweighs the former. For example, when the individual stations are, respectively, exponential, deterministic, and exponential, the optimal pattern of their mean service times is an inverted bowl, because the variability imbalance requires that the less variable station (station 2) should be assigned the highest mean, whereas the bowl phenomenon demands the

the mean of this station to be the lowest, but it was less prominent and, therefore, was outweighed by the variability imbalance.

Studying a line having a uniformly distributed middle station and exponential end stations, the author found that when the Covar of the central station is 0.5, the bowl phenomenon and variability imbalance effects are exactly equal and cancel each other, therefore, the optimal PR is obtained when the line is balanced. Furthermore, when the difference in Covars between the uniform station and the exponential ones is <0.5, the bowl phenomenon effect is more predominant, while if the difference is >0.5, the variability imbalance effect is more prominent.
Figure 3.1 depicts Rao's results.

Mishra et al (119) investigated the same three-station line of Rao, with the single exception that the middle station being Erlangian with parameter k, rather than being uniform. They found the bowl phenomenon to predominate when the central station's Covar is in the range $1/\sqrt{3} < \text{Covar} \leq 1.0$, whereas the variability imbalance prevails if the Covar of the middle station is in the range $0.0 < \text{Covar} < 1/\sqrt{3}$. The two effects neutralise each other, resulting in an optimum balanced line, when the middle station's Covar is exactly $1/\sqrt{3}$.
Table 3.6 shows the abovementioned findings. From this table it seems that the robustness of the inverted bowl configuration increases as the Covar is decreased, which is opposite to that of the bowl phenomenon, i.e. the

FIGURE 3.1

THE EFFECT OF THE BOWL PHENOMENON VERSUS THAT OF THE VARIABILITY IMBALANCE - FROM RAO (142)

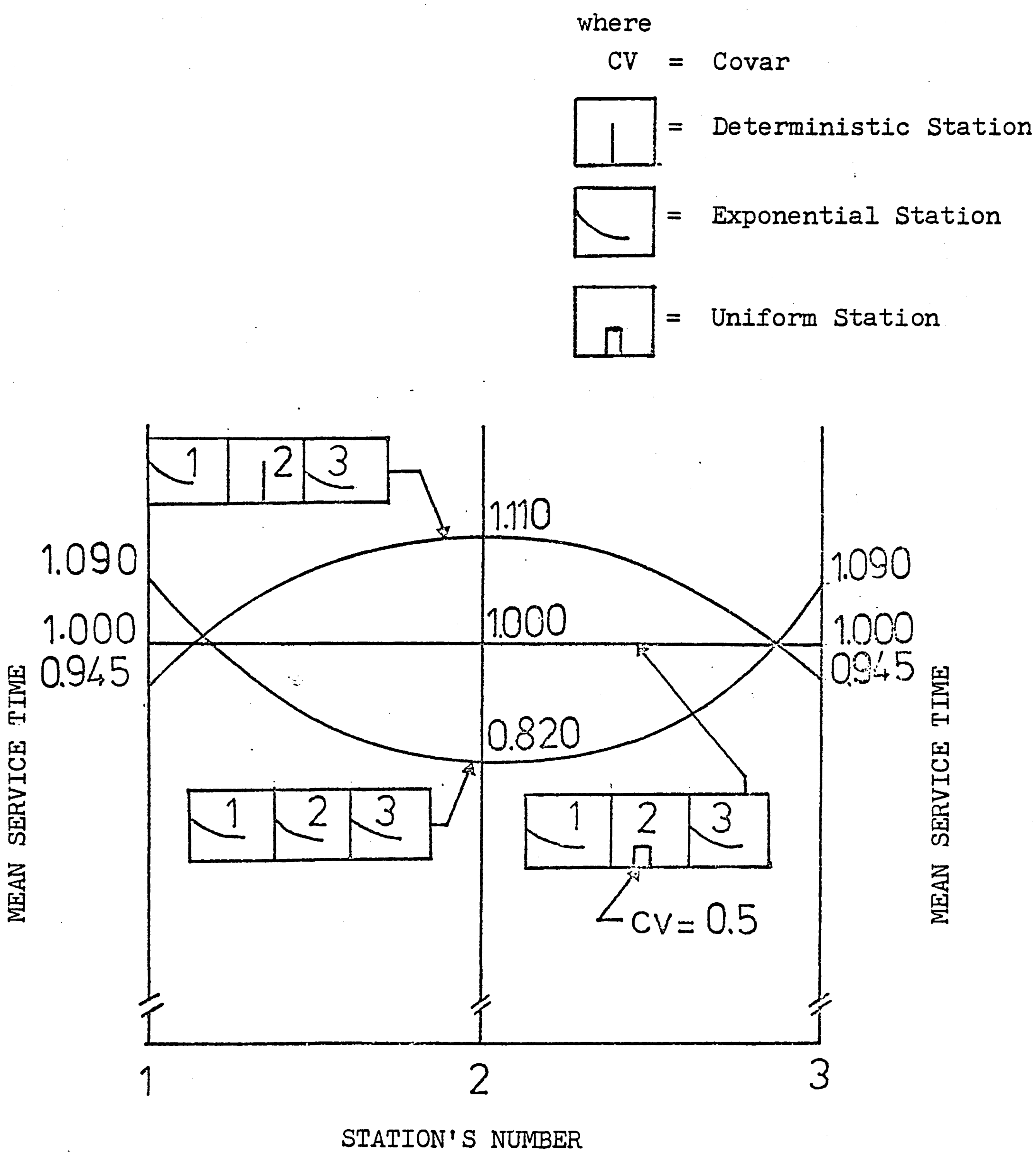


TABLE 3.6

OPTIMALITY AND ROBUSTNESS OF THE BOWL AND INVERTED BOWL PHENOMENA FOR A 3-STATION LINE WITH ERLANGIAN MIDDLE STATION AND EXPONENTIAL END STATIONS - ADAPTED FROM MISHRA ET AL (119)

2ND STATIONS COVAR	PHENOMENON	PR_b	PR^*	$\frac{(PR^*-PR_b)}{PR_b} \times 100\%$	$BL\%^*$	MAX BL%	PR(MAX I
1.0	bowl	0.5641	0.5672	0.55	9	NA	NA
0.9	bowl	0.5712	0.5730	0.32	6	NA	NA
0.8	bowl	0.5784	0.5793	0.16	4	NA	NA
0.7	bowl	0.5855	0.5858	0.05	2	5	0.5854
0.6	bowl	0.592415	0.592417	0.0003	1	2	0.592[illegible]
0.5	Inverted bowl	0.59894	0.59902	0.01	1	2	0.598[illegible]
0.4	Inverted bowl	0.6048	0.6053	0.08	3	5	0.604[illegible]
0.3	Inverted bowl	0.6098	0.6108	0.16	4	8	0.6097
0.2	Inverted bowl	0.6136	0.6151	0.25	5	10	0.6134
0.0	Inverted bowl	0.6167	0.6189	0.36	6	NA	NA

where

PR_b = production rate of the balanced line
PR^* = optimal production rate
$BL\%^*$ = optimal degree of imbalance (%)
NA = not available

flexibility of the bowl pattern rises with the increase in the Covar.

Summary

This chapter was concerned with reviewing what is currently known in the area of unbalanced manual unpaced lines operating under stable working conditions. It started by enlisting the main reasons why an unbalanced line, in one way or another, is worthy of investigation. Four imbalance types were identified, viz, unequal operation times' means, imbalanced Covars, unequal buffer capacities, and simultaneous means and Covars unbalance. Following that, the research efforts into each of these kinds of imbalance were surveyed in turn, while subdividing them into queuing theoretic, simulation approaches, and a third one which cannot be fitted into either. The most important of the research findings may be summarised as follows:

(1) Means Imbalance

(a) The output rate for stations with identical Covars is not affected if the order of mean service times is reversed, and if the early means are interchanged with their corresponding end means (the so-called 'reversibility' and 'symmetry' properties) and for $N>2$, PR is maximised if the pattern of means resembles that of a bowl (the 'bowl phenomenon' property). For $N = 2$ a balanced design is optimal.

(b) The bowl pattern is more advantageous and efficient than any other unbalanced configuration, in terms of PR. It is also superior to the balanced line and its advantage over the balanced design decreases as N, Covar are reduced, and as B is increased, other things being equal. The marginal decrease in the bowl phenomenon's advantage rises with the continuing increase in B, the reduction in N, and for Covar<1.

(c) A line unbalanced in the bowl phenomenon's direction is robust insomuch as it can tolerate a relatively high degree of imbalance (referred to as a 'breakeven' degree) and still has a PR which is nearly the same as that for the balanced line.

(d) Increasing B tends to decrease the optimal and breakeven degrees of imbalance.

(e) The joint effort of B and Covar on the maximal PR's improvement as well as the optimal imbalance degree, is much larger than their separate effects.

(f) For unbalanced patterns other than the bowl arrangement, as the imbalance degree goes up, the line's PR and any potential improvement over the balanced line will become lower. However, for the bowl phenomenon pattern, the value of PR initially increases as the degree of imbalance rises and, beyond a breakeven point, it starts to drop.

(g) The influence of the Covar on the optimal and breakeven imbalance degrees is far greater than that of the shape of the service times distribution, but a more symmetrical distribution will usually yield a higher PR than that of a skewed one.

(2) Covars Imbalance

(a) An unbalanced Covars' situation has little effect on the mean total number of units in the line, L.

(b) The pattern of Covars imbalance exercises little influence on L, but significantly affects PR.

(c) A bowl phenomenon with respect to the Covars imbalance exists (i.e. a high - low - low - high arrangement) and its resultant PR is significantly higher than those of the balanced and unbalanced configurations. The superiority of the bowl pattern increases when N rises. An inverted bowl design is the worst with regards to PR.

(3) Buffer Capacities Imbalance

(a) For a total buffer capacity (TB) less than 4(N - 1) and short lines, it does not pay to unbalance the sizes of the buffers. On the other hand, for a TB$\geqslant$4(N - 1), the allocation of unequal capacities along the buffers cannot signicicantly improve the PR performance of the line over that of a balanced line.

(b) When it is necessary to imbalance the buffer sizes, the most efficient configuration is whereby high capacities are assigned to the middle buffers, and lower capacities are alloted to the end buffers.

(c) An increasing order of buffer capacities' design is favourable if the aim is to achieve a significant reduction in L, but it is detrimental from a PR viewpoint.

(d) Short lines are less sensitive to an imbalance in buffer sizes than to either means or Covars imbalance.

(4) Means and Covars Simultaneous Imbalance

(a) For a line with non-identical stations in terms of their Covars, a balanced mean service times' pattern is detrimental. A configuration whereby the station with the smaller Covar is assigned a slightly higher mean service time, irrespective of its location in the line, is optimal regarding PR. Any other unbalanced pattern is unfavourable. The PR of this optimal pattern, however, declines as B is increased and N is reduced.

(b) This optimum Covars and means imbalance pattern is robust in that the degree of means imbalance can be relatively high and the line's PR does not significantly fall below that of a balanced line.

(c) The optimal degree of means imbalance goes up as the differences in stations' Covars are increased.

(d) The percentage improvement in PR over that of the balanced line, for the best pattern of means and Covars imbalance, is substantially higher than that achieved by the bowl arrangement for the means imbalance.

(e) When both the means and Covars are unbalanced, two different effects come into play; the bowl phenomenon effect for means imbalance and the variability effect (resulting in an inverted bowl pattern) for Covars imbalance. Which of these two dominates the other depends on the stations' Covars and their particular service times distribution.

C H A P T E R   F O U R

UNBALANCED NON-STEADY STATE UNPACED MANUAL LINES

INTRODUCTION

This chapter has the objective of presenting a review of the extent of knowledge gained from several research invėstigations into the operating characteristics of production lines which are unbalanced, and whose performance mode is still unsteady. Most of the studies on such lines were not initially intended to reflect the non-stable behavioural conditions, and some of them were claimed to represent the equilibrium state for such unbalanced lines, but due to technical reasons, including insufficient simulation runs' lengths, it was felt that they, more accurately, reflect the non-steady state conditions, and it was decided, therefore, to survey them in this chapter.

Except for one investigation, the mainstream of emphasis in the rest is the simulation approach. This is mainly because of the astronomical difficulty in handling the non-steady state operational phase through a queuing theoretical approach, due to the dimensional problem being faced when trying to solve the differential - difference equations which portray the transient phase of line's behaviour.

As was the case in the previous chapter, the drawbacks of using unrepresentative distributions to describe operation times, such as the exponential distribution, short line lengths, zero and infinite buffer capacities, and the queuing characteristics measures of performance which are of mere theoretical interests, are still valid and, as a result, will not be repeated in this chapter. The types of imbalance covered by the research studies are five, viz, imbalances in terms of service times' means, Covars, means and Covars, buffer capacities, and Covars and buffer capacities. Below is a review of each of them.

(a) INEQUALITY OF OPERATION TIMES' MEANS

One cause for unbalanced mean service times is the presence of learning in the line. In addition to the non-steady state impact of learning, it may lead to work imbalance. This occurs when one or more trainee workers, who have not reached the speed level of the skilled workers, are utilized in an existing line to compensate for turnover. In this situation the operators service times' means are unequal and an out of balance case arises. The learning curves of the individual trainees are valuable tools in predicting the gradual decrease in their mean operation times, i.e. the decrease in line's imbalance, until they achieve their ultimate paces where they acquire the experienced workers' speed.

According to Kilbridge (87), the line's imbalance cost which results from learning may be substantial when the turnover rate is high, and is a function of the number and timing of replacements. Clearly, the introduction of a new worker into a comparatively new line which consists of workers still undergoing learning, is likely to be less costly than adding a new operator to a line having fully experienced workers.

Bohlen and Barany (11) showed that, in general, the various trainee workers may have different learning abilities and that the tasks may differ in their complexities along the line, giving rise to learning curves (and imbalance degrees) which depend on the characteristics of both the operator and the task.

<u>The Queuing Approach</u>

Sastri (151) and Wilhelm and Sastri (180) analysed a 2-station line having exponential operation times and buffer capacities ranging from 0 to 20, during a start-up period which immediately follows the introduction of a new product into the line, and is characterised by excitement about the programme of the new product, leading to an enthusiastic learning that is referred to as 'incentive manufacturing progress'.

A continuous time Markovian model has been developed to provide a simultaneous set of first-order differential equations which describe the transient behaviour and was solved by a modified version of Runge-Kutta integration. Moreover, a special learning curve's function was derived to overcome the apparent shortcoming of the traditional learning curve model of the assumption that learning will continue without limit as the cumulative output is increased in volume. The new learning model considers a steady-state phase with a predetermined speed rate and a transitional phase with different cutoff points. As soon as the output rate of both stations becomes at least 97.5% of the maximum desired level of 99.8%, the transient phase comes to an end.

The time-dependent learning curve model is given as:

$$a_{(t)} = a_o + (a - a_o)(1 - \exp(- CE_{(t)}))$$

where

$a_{(t)}$ = production rate at time t

a_o = initial production rate which may be measured on the line or estimated through experience

a = steady-state production rate which is fixed by management or by technological considerations

$E(t)$ = the learning period from time 0 to t

C = a parameter determined by fitting the learning curve to historical data, or by relating it to a second parameter, S', which is defined as the learning factor of the station and is given by:

$$S' = \frac{\text{processing rate at time (2t)}}{\text{processing rate at time (t)}} = a(2t)/a(t)$$

The initial value of S' is S and if learning exists, both S' and S are >1. The relationship between S and C is indicated by:

$$C = -\ln\left(\frac{S}{2} \pm \frac{1}{2}(S^2 - (4(S-1)a/a_o)/(a/a_o) - 1))^{\frac{1}{2}}\right)$$

And if $(a/a_o)\min \geqslant S^2/(S-2)^2$, then C is real.

The authors considered only the positive roots of C, given S, which corresponds to an 'incentive learning' situation which is typified by a rapid improvement in the processing rate shortly after the start of the transient period, followed by a reduced rate of improvement when the initial incentive abates, until no more learning takes place, signifying the start of the steady-state stage. The modified learning model also assumes that the learning process discontinues during starving and blocking periods, i.e. the learning takes place only when an operator is busy. The authors then used numerical analysis to investigate the same line with $a_{o1}=a_{o2} = 30$, $a_1=a_2 = 135$, $S = 1.2, 1.3$, and $P = 0.5, 1.0, 1.5$ (i.e. (\\), balanced, (/) patterns of means).

where

a_{oi} = initial production rate for station i, i = 1,2

$$P = \frac{\text{service rate of the first station}}{\text{service rate of the second station}}$$

The following results emerged from this study:

(1) When the buffer capacity is zero, increasing the manufacturing progress factor, S, increases the output rate and reduces the duration of the transient. However, the use of zero B greatly decreases the utilization of the line, given P, and negates the influence of increasing S

on utilization. Another result is that the highest reduction in the start-up duration was achieved when the line was balanced.

(2) When the buffer capacity is > 0, the following points are concluded:

(a) The rise in the production rate, as a result of increasing S, is higher than that of raising the buffer capacity, i.e. the buffer size seems to have little impact on the resultant increase in the amount of the PR from increasing S.

(b) A P's value of 1.5 (i.e. an increasing order of operation times' means) generates higher PR than that of either P = 1 or P = 0.5 (i.e. a balanced line and a decreasing means' order respectively), even if the learning factor, S, is large. The best result, in terms of PR, is achieved when a large value of P is coupled with a high B. This is not in line with the results of Hillier and Boling (75) for a 2-station line under steady-state conditions, where a balanced line's arrangement was found to be optimal. However, this contradiction may be due to the presence of learning in Sastri's study.

(c) For P = 1.5 the mean buffer level is a nondecreasing function of the start-up time, whereas for P = 0.5 it is a convex function.

(d) The initial processing rate, a_o, has a considerable effect on the value of the mean buffer level.

(e) Increasing P and B will tend to reduce the transient period, but the marginal decrease in the start-up period diminishes as B rises, especially when B is $\gg 15$.

(-) the value of B needed for the unbalanced lines under NSS conditions, given S, is smaller than that required in the SS. In general, the value of B necessary for an efficient SS performance is also adequate for an effective NSS operation.

Sastri's work is credited for attempting, for the first time, to find numerical solutions to the transient state conditions, which are usually protracted and untraceable. Another merit of this work is the development of a new exponential learning curve which can accommodate a distinctly finite transient learning period. However, apart from the unrealistically very short line used, a major drawback is the comparatively short start-up period obtained from the incentice learning function, which may not reflect that of the real life.

<u>The Simulation Approach</u>

Davis (38) examined a 3-station line having infinite buffer capacity which meant that a steady-state situation cannot be reached during the limited simulation period. The service times distributions were positively skewed (approximated by a Pearson's type III distributions) and normal. The line was unbalanced in terms of having slow, medium, and fast service times' means, and all the six possible permutations of speeds were considered.

The resulte showed that the configuration fast - slow - medium, i.e. (/) means sequence, exhibited lower idle time than the other unbalanced configurations. In addition, interarrival time, was nearly equal for all the patterns of means imbalance and closely related to the mean operation time of the slowest station. Furthermore, it was found that increasing the interarrival time increases both the idle and inter-departure times for all the patterns.

Payne et al (136) simulated a 20-station line having a Covar of 0.2, normal service times, and unlimited buffer capacity. Two patterns of means imbalance were considered; the first was of a (/) form, while the form of the second was (\), i.e. increasing and decreasing means orders. The degree of imbalance for both patterns was nearly 0.005. The results showed that the first pattern is superior to both the second and the balanced line arrangement, in terms of reducing the idle time of each station in the line. However, the first pattern is inferior to the balanced line configuration with regard to the number of units outputed and the maximum inventory level, the former being slightly less and the latter being higher, for all stations, than that of the balanced line.

Slack (160) investigated a 5-station line with a Weibull service times, a Covar of 0.27, imbalance degrees of 0.01 - 0.06 in increments of 0.01, and a B of 6. Both the empty and full buffer levels' starting conditions were examined which, respectively, represent patterns (/), (\) of mean service times. A summary of the resultant findings of Slack's study is given below:

(1) Since in pattern (/) the service rates of the stations towards the beginning of the line are higher

than those of the end stations, the build up of the buffer levels will be accelerated and therefore, the buffer levels of the balanced steady-state lines will be reached sooner for pattern (/) than that of an equivalent balanced line. However, the process of buffers' build-up goes on beyond the level of the balanced steady-state line, causing greater amount of blocking than that of the balanced line. The higher the degree of imbalance, the quicker the arrival at the balanced steady-state buffer levels and the greater the overshooting of these levels.

(2) In pattern (\) the reverse situation occurs, such that the buffers are depleted faster in the direction of the balanced steady-state levels (especially at the early part of the transient period), until these levels are surpassed, which increases the starving idle time. Again, as the degree of imbalance increases, the balanced SS buffer levels are reached sooner and overshot more.

(3) Within the range considered, there is an optimal degree of imbalance which minimises the cumulative idle time, as compared to that of a balanced line, for any elapsed transient period value. This is true for both the full and the empty start initial conditions. Therefore, unbalancing the line during the start-up period is seen as advantageous.

(4) The initial amount of idle time is high and it decreases as time elapses, but the marginal decrease diminishes as time progresses until the steady-state buffer level is approached (which is similar to the

notionally balanced line's results). At this time the balancing loss does not contribute to total idle time. Additionally, raising the degree of imbalance tends to increase the idle time's rate of reduction. However, when the minimal SS idle time point is reached, the influence of the balancing loss increases the idle time, the higher the imbalance degree, the greater the rise in I is.

(5) The minimum periodic idle time is achieved when the buffer level becomes nearly equal to that of the balanced steady-state line.

(6) For any of the ten simulated transient periods, unbalancing the line leads to a cumulative idle time that is lower than that for a balanced line. This is true for both the empty and the full start conditions, except for the last two periods of the full start condition. The improvements in cumulative idle time from the two unbalanced configurations are significant at the earlier periods, but insignificant at the later periods.

Globerson and Tamir (65) used simulation to investigate the NSS behaviour of an unbalanced 'service industry' line. They argued that an assembly line is used in the manufacturing as well as the service industries and that a service line differs from a manufacturing one in two major respects. First, the buffer capacity is practically infinite in the service line since paper (the common material) does not require much space. Second, in the service line there is the service system. A simulation model has been developed which integrates 'technological' variables (i.e. the imbalance of work and the number of paralleled lines) and 'human behaviour' variables (i.e. learning, operator's service times, absenteeism, and turnover).

Regarding the imbalance of work, the authors made use of Kilbridge and Wester's (90) formula: $PIM = aCT^{-b}$ where, PIM = percent of imbalance, CT = cycle time, a and b = parameters, estimated as 105 and 0.86 respectively. The authors considered a single line having 12 stations, as well as 2,3,4,6,12 paralleled lines having, respectively, 6,4,3,2,1 stations each, with all the lines having the same total amount of work to perform. Note that the 12 paralleled stations' arrangement represents the case of individual assembly and, therefore, a line does not really exist. With respect to learning, they used a learning curve model attributed to De Jong and given as:

$$T(S) = T(1)\left[M + (1 - M)/S^{m}\right]$$

where

T(S) = time required for unit s

T(1) = time required for the first unit

M = that fraction of the first service time which cannot be eliminated with practice, $(0 \leqslant M \leqslant 1)$

m = human progress factor or exponent of reduction and is = 0.32

The values of S in Globerson and Tamir's simulations were 20,50, 70,100 units, and the values of the initial skill level of the workers at the start of the simulation runs were 4,20,50,100 repetitions (units). The authors, further, assumed the operation times were normally distributed with a Covar of 0.10. Moreover, the investigators assumed that two parallel lines are, intuitively, absence of an operator will only disturb half of the system. The following expression was used for determining the percent of absenteeism (AB):

$$AB = MAB + k/ST^c$$

where k and c are parameters with the estimated values of 13.5 and 1.43 respectively, and MAB is the minimum (unavoidable) absenteeism level due to such reasons as real sickness, and is estimated to be 7%, and ST is the task's standard time and is defined as the mean operation time for a completely trained and skilled operator working at normal pace. This standard time is obtained from predetermined time standards, such as MTM. Depending on an analysis by De Jong which showed that approximately 1000 units are enough to attain the MTM standard time for many operations, this figure was adopted by the authors.

Globerson and Tamir used the following formula to calculate the percentage of turnover (TO):

$$TO = MTO + e/ST^f$$

where

MTO = minimum turnover rate

ST = task's standard time

e and f = parameters

The performance measures of the simulation runs were the percent of work accomplished (PWA) which, in a sense, reflects the production rate, and the response time (RT) which is the total time spent by a job in the system.

The results of this study demonstrated the following:

(1) The relationship between PWA and the number of paralleled lines (NPL) is convex, with the maximum PWA being achieved when NPL = 3 for S = 20,50,70, and NPL = 4 for S = 100. This is

only true if the initial skill level is low, which implies that the optimal strategy can be chosen only from tasks whose skill levels are low.

(2) PWA is less sensitive to NPL for larger S, for NPL < the optimal one, and for tasks having very high initial skill levels.

(3) In general, PWA is higher the greater S and the initial skill level are.

(4) There is a concave functional relationship between RT and NPL with the minimum RT being obtained at NPL=2 for S=50, 100, and NPL=3 for S=20,70. This is true only for low initial skill levels. For considerably high skill level, RT as a function of NPL, decreases monotonically.

(5) The two performance criteria indicate that a strategy comprising few lines in parallel is preferrable to either of the extreme strategies, i.e. the single line and the individual assembly.

This work is, as yet, the first of its kind in terms of considering absenteeism, turnover, learning, and line paralleling within an integrated simulation model, and it opens a new frontier to examine aspects of production lines that were not investigated previously. However, it may be criticised on the following grounds:

(a) The quantification of the human behaviour parameters is difficult and, therefore, the values used by the authors to estimate the learning curve, absenteeism, and turnover paramerts, were somewhat arbitrary and may not reflect the real life values.

(b) The selected Covar value of (0.1) is unrepresentative of the normal mean value which is around 0.274 (see the next chapter).

The Third Approach

Shimshack (155) studied a 2-station line having unlimited buffer capacity and Erlangian service times with parameter k, in order to determine an optimal sequence of stations, such that the sequence having smaller mean and variance of the total waiting times in the system is regarded as optimal. Two sequences were investigated, namely, A and B. In sequence A all the units are serviced first by station 1, then by station 2, whereas in sequence B the opposite order is true.

The author's approach is a mixture of simulation and numerical queuing analysis. Initially simulation experiments were conducted to examine the impact of the two stations having unequal utilization (mean service) rates on their ordering, while equating and fixing their variances. The utilization rate ranged from 0.30 to 0.90 in six experiments, with the first station's utilization being always less than that of the second for both sequences A and B. This may be shown as follows:

Sequence	Station's i Service Rate	<or>	Station's i Service Rate	Pattern of Service Times' Means
A	1	$<$	2	decreasing order (\)
B	2	$>$	1	increasing order (/)

The first four experiments displayed the optimality of sequence B in terms of having significantly smaller mean and variance of waiting times than A. In this case a first degree 'stochastic dominance' is said to pervail. The last two experiments showed that sequence B's mean waiting time is lower than that of A, but not significantly, and that the waiting times' variance is higher for sequence B. Therefore, it appears that there exists an overlapping in the waiting times distributions of the two sequences in that neither can be considered as optimal in accordance with the first degree stochastic dominance, because B dominates in terms of the mean waiting time, while A dominates in respect to the variance.

When the author separately examined the utilization rate (P) and the variance of the service times distribution (σ^2), it emerged that there is some relationship between them which affects the optimality of stations' sequences. This observation motivated the development of mathematical approximations, using some approximate formulae, such as that of Fraker (57) for the mean waiting time, so as to fascilitate the determination of which sequence is optimal. Defining $\sigma_1{}^2/P_1 = a_1$ and $\sigma_2{}^2/P_2 = a_2$, the author advanced and tested the validity of the hypothesis that if $a_1 < f(a_2)$, then sequence A dominates sequence B, while if $a_1 > f(a_2)$, then B dominates A, whereas if $a_1 = f(a_2)$, then the waiting times are indifferent to the ordering of the stations, where $f(a_2)$ is some function of a_2.

Shimshak attempted to mathematically find the indifference equation reflecting the relationship between a_1 and a_2, which gives the same mean waiting time for both sequences. When the indifference equation was determined, the actual a_1 value obtained from the simulation experiment was compared to the predicted value of a_1, derived from the indifference curve. Given a_2, if the actual a_1 is lower than the predicted a_1, then sequence A will yield the smaller waiting time. On the other hand, if the simulated a_1 is greater than the predicted a_1, sequence B will result in the shorter waiting time.

Following that, the author extended his findings to the study of stochastic dominance. The data showed that, given a_2, if a_1 is near the indifference curve, an overlapping in the distribution functions of sequences A and B will occur, giving rise to a second degree stochastic dominance, whereas if a_1 is much larger or smaller than that of the indifference curve, it leads to a first degree stochastic dominance. On the basis of that, confidence intervals were set up around the indifference curve in order to determine if the differences in the waiting times between the two sequences are due to first or second degree dominance.

The author went on to consider the situation where both stations have the same service times distribution, and after plotting a graph of the relationship between a_1 and a_2 and constructing the indifference curve and confidence bounds, it was concluded that when both stations have exponential service times, the mean waiting time is indifferent to their ordering, however, when the two stations have the same, but non-exponential operation times, e.g. Erlangian times with parameter 2, their ordering influences the mean waiting time.

The work of Shimshak is stimulating, since it employed the stochastic dominance rules, for the first time, to investigate production lines' behaviour, but it needs to be expanded such that more realistic N, B, and service times distribution values are dealt with.

(b) INEQUALITY OF COVARS

Since no queuing approach study relating to Covars imbalance has appeared in the literature, only the simulation and the third approaches will be reviewed.

The Simulation Approach

The first simulation investigation is that of Payne et al (136) which examined the effects of two patterns of Covars imbalance; the first is whereby Covars of 0.1, 0.2, 0.3 are assigned to stations 1 to 7, 8 to 13, and 14 to 20 respectively, and the second is the allotment of Covars of 0.3, 0.2, 0.1 to the same stations of the first pattern, i.e. increasing and decreasing sequences of Covars along the stations. The results revealed that the first pattern reduces the individual stations' idle time as compared to that of the balanced Covars situation, but this improvement in the idle time is associated with a slight reduction in the output, and a higher maximum buffer level for the last 10 stations.

The next investigation was conducted by Kala and Hitchings (84) who simulated lines having 4 stations and unlimited buffer capacity. The Covars of three stations were the same and equivalent to 0.063, while the Covar of the remaining station was permitted to increase progressively. This station was also allowed to take the position of the first through the fourth stations. The results of this study showed that:

(1) When the station with a higher Covar is positioned towards the end of the line, the PR is slightly increased, the idle time is minimised, and the maximum buffer content rises considerably, whereas if the higher Covar is allocated to the middle stations, lower PR and maximum buffer level, and higher idle time will result. Therefore, the middle of the line should have lower (steadier) Covars.

(2) The maximum buffer level in front of the stations with the same Covar is constant, irrespective of their location on the line, but the maximum buffer level, for a particular buffer, rises when the Covar increases. Moreover, the maximum buffer content increases if the higher Covar is assigned to the middle of the line.

De La Wyche and Wild (39) simulated 3, 4, 12-station lines having normal service times, Covars of 0.1 - 0.3, and 0.1 buffer capacities. Four policies for arranging the Covars along the individual stations were employed. They are:

(1) The steadier stations are concentrated towards the end of the line.

(2) The steadier stations are concentrated at the middle (the Covars' bowl phenomenon).

(3) The more variable stations are separated from each other by less variable (steadier) stations.

(4) The steadiest stations are assigned to the centre of the line, while the most variable stations are allocated to the beginning of the line.

The analysis of variance and comparisons with control (i.e. with the balanced design) were applied to the results. Table 4.1 exhibits the various patterns and their relative efficiencies. Note that due to the use of insufficient transient length and number of repetitions (by Slacks (160) criteria and four replications respectively), the authors' results are considered as referring to the NSS conditions.

TABLE 4.1

THE EFFECTS OF DIFFERENT PATTERNS OF COVARS IMBALANCE ON THE IDLE TIME - ADAPTED FROM DE LA WYCHE AND WILD (39)

N	COVARS IMBALANCE PATTERN	B = 0	B = 1
4	V,S,S,V	15.78	4.55**
	V,S,V,S	16.36	5.28
	S,S,V,V	16.78	5.70
	S,V,S,V	16.81	5.89
	V,V,S,S	17.08	6.07
	S,V,V,S	17.24	6.52
	(Datum)	(16.63)	(5.57)
12	S,V,S,V,S,V,S,V,S,V,S,V	20.66	6.33
	V,S,V,S,V,S,V,S,V,S,V,S	20.89	6.65
	V,V,V,S,S,S,S,S,S,V,V,V	21.22*	7.89**
	S,S,S,V,V,V,S,S,S,V,V,V	21.71**	8.03**
	V,V,V,S,S,S,V,V,V,S,S,S	21.82**	7.79**
	S,S,S,V,V,V,V,V,V,S,S,S	23.51**	8.43**
	S,S,S,S,S,S,V,V,V,V,V,V	23.53**	8.93**
	V,V,V,V,V,V,S,S,S,S,S,S	23.68**	9.40**
	(Datum)	(20.52)	(6.57)
3	V,S,M	14.17	3.77
	M,S,V	14.18	4.13
	M,V,S	14.27	4.41
	S,M,V	14.27	4.58
	V,M,S	14.28	4.46
	S,V,M	14.33	4.75
	(Datum)	(14.39)	(4.27)

....Cont

TABLE 4.1 (CONTINUED)

N	COVARS IMBALANCE PATTERN	B = 0	B = 1
12	V,M,S,V,M,S,V,M,S,V,M,S	19.94	7.01
	M,S,V,M,S,V,M,S,V,M,S,V	20.05	6.68
	V,S,M,V,S,M,V,S,M,V,S,M	20.18	6.58
	M,V,S,M,V,S,M,V,S,M,V,S	20.23	6.16
	S,M,V,S,M,V,S,M,V,S,M,V	20.27	6.26
	S,V,M,S,V,M,S,V,M,S,V,M	20.40	7.22**
12	V,V,V,V,S,S,S,S,M,M,M,M	21.67**	8.37**
	M,M,M,M,S,S,S,S,V,V,V,V	21.73**	8.10**
	S,S,S,S,V,V,V,V,M,M,M,M	22.15**	8.10**
	V,V,V,V,M,M,M,M,S,S,S,S	22.17**	7.83**
	M,M,M,M,V,V,V,V,S,S,S,S	22.33**	7.64**
	S,S,S,S,M,M,M,M,V,V,V,V	22.39**	7.95**
	(Datum)	(19.91)	(6.20)

where

N = line length

B = buffer capacity

Covar = co-efficient of variation

V = relatively variable station (Covar = 0.3)

S = relatively stable station (Covar = 0.1)

M = relatively medium station with variability between S and V (Covar = 0.2)

Datum = balanced line

* = significantly different from the datum at the 97.5% level

** = significantly different from the datum at the 99% level

The results demonstrated the following:

(1) For short lines (N = 3) policy 2, given B, results in some reduction in the idle time in comparison with that of the balanced line. However, for longer lines (N = 12) this policy gives rise to a significantly higher idle time than that of the datum (balanced arrangement), but results in a lower idle time than that of all the patterns below it in the table. Therefore, a Covars' bowl phenomenon exists only for very short lines, for the particular patterns examined. This does not conform to the finding of Carnall and Wild, but the contradiction may be attributed to the fact that these authors used deterministic stations in their study.

(2) Policy 3 generates relatively close results to those provided by the datum, irrespective of N and B. In addition, the results of this policy are superior to those of the bowl arrangement for long lines, but not for shorter lines. The authors suggested that a good compromise may be a design comprising both policies 2 and 3.

(3) Policies 1 and 4 are detrimental for longer lines but indifferent for shorter lines.

The Third Approach

In another part of his work, Shimshak (155) examined the same 2-station line with unlimited B and Erlangian operation times. He initially simulated the line to determine the influence of having unequal variances of operation times on the optimal sequence of the two stations, while fixing and equating their utilization rate at 0.75, which meant that their service rates are equal. In both sequences A and B the variance of the first station was always larger than that of the second, i.e. pattern V,S for A and pattern S,V for B.

Four experiments, with varying variances for stations 1 and 2, were conducted. The results showed that the optimal ordering of stations is always one whereby the smaller (steadier) variance is placed first (i.e. pattern S,V). Therefore, sequence B dominates sequence A according to the rules of the first degree of dominance. The author went on to develop the indifference equation for the situation where both stations 1 and 2 have Erlangian service times with parameters 2 and 3 respectively. Plotting the relationships between a_1 and a_2, the regions of the first and second degrees of dominance were determined. As a consequence, it was found that for lines with different Erlangian stations, no general expression can be developed.

(c) <u>INEQUALITY OF BUFFER CAPACITIES</u>

The sole study of the effects of unbalanced buffer capacities on the operating characteristics of production lines was that of De La Wyche and Wild (39) who, in another part, simulated 5 and 9-station lines with total buffer capacities, TB, of up to 16 units, and Covars of 0.3, 0.5. Two hypotheses were tested:

(1) Placing most of TB at the end of the line will reduce the idle time.

(2) A decrease in the idle time will result from placing the bulk of the TB at the central part of the line.

Table 4.2 provides data on the various patterns of allocating TB among the different buffers.

The major conclusions of this work are:-

(1) Given N, TB, Covar, a zero buffer capacity allocation for any individual buffer should be avoided, otherwise significantly very high idle times will result.

(2) Given N, Covar, the best and most consistent policy is to divide the available TB as equally as possible among the buffers. If imbalance is unavoidable, then concentrating more B towards the centre of the line, while avoiding any zero assignment, is advantageous and leads, at best, to a line's efficiency nearly equal to that for a balanced line. This NSS conclusion supports that of El-Rayah (46) which represented the SS conditions.

(3) Assigning the largest portion of TB to the rear of the line is detrimental in terms of the idle time incurred.

(d) <u>INEQUALITY OF BOTH SERVICE TIMES' MEANS AND COVARS</u>

<u>The Simulation Approach</u>

McGee and Webster (115) simulated a 2-station line having infinite buffer capacity, normal service times, and normal inter-arrival times, with Covar = 0.125. The operation times' means ranged from 2.0 to 3.9 in increments of 0.1,

TABLE 4.2

THE EFFECT OF VARIOUS BUFFER CAPACITIES' IMBALANCE PATTERNS ON THE IDLE TIME - FROM DE LA WYCHE & WILD (39)

N	TB	BUFFER CAPACITIES' IMBALANCE PATTERN	COVAR = 0.3	COVAR = 0.5
5	1	0,0,1,0	19.42	29.39
	1	0,1,0,0	19.64	29.33
	1	0,0,0,1	21.26	30.96
	1	1,0,0,0	21.44	31.05
	2	0,1,1,0	16.09	25.36
	2	0,1,0,1	16.67	26.26
	2	1,0,1,0	16.92	25.42
	2	0,0,1,1	19.05	28.49
	2	1,0,0,1	19.20	28.40
	2	1,1,0,0	19.37	28.68
	3	1,1,1,0	14.26	22.82
	3	0,1,1,1	14.57	23.16
	3	1,0,1,1	15.24	23.32
	3	1,1,0,1	15.26	24.07
	4	1,1,1,1 (balanced)	(8.21)	(17.23)
	4	0,1,1,2	14.76**	22.26**
	4	2,1,1,0	14.85**	22.95**
	4	1,0,3,0	15.16**	23.44**
	4	0,2,2,0	15.26**	23.11**
	4	0,3,0,1	15.67**	24.04**
	4	0,2,0,2	16.04**	24.42**
	4	2,0,2,0	16.36**	25.02**
	4	0,1,0,3	16.62**	26.15**
	4	3,0,1,0	16.92**	25.73**
	4	2,0,0,2	19.25**	28.16**
	8	2,2,2,2 (balanced)	(5.41)	(11.87)
	8	1,3,2,2	5.65	12.39
	8	1,3,3,1	5.98	12.86
	8	2,2,1,3	6.10	13.54

....Cont

TABLE 4.2 (CONTINUED)

N	TB	BUFFER CAPACITIES' IMBALANCE PATTERN	COVAR = 0.3	COVAR = 0.5
5	8	2,2,3,1	6.25	13.33
	8	3,1,1,3	6.44	14.91**
	8	3,2,2,1	6.53*	13.40
	8	1,2,2,3	6.67*	12.81
	8	1,3,1,3	6.78*	14.00**
	8	3,1,3,1	6.83**	13.66*
	8	3,1,2,2	7.02**	13.72*
9	8	1,1,1,1,1,1,1,1 (balanced)	(10.02)	(19.05)
	8	2,0,2,0,2,0,2,0	16.79**	25.86**
	8	0,2,0,2,0,2,0,2	16.88**	26.03**
	8	2,2,1,1,1,1,0,0	18.61**	28.17**
	8	0,0,1,1,1,1,2,2	19.04**	27.95**
	8	0,0,2,2,2,2,0,0	19.32**	28.61**
	8	2,2,0,0,0,0,2,2	22.54**	32.21**
	16	2,2,2,2,2,2,2,2 (balanced)	(6.30)	(13.15)
	16	1,2,2,3,3,2,2,1	6.19	13.65
	16	3,2,2,1,1,2,2,3	7.10	15.57**
	16	3,1,3,1,3,1,3,1	7.10	14.52**
	16	1,1,2,2,2,2,3,3	7.27	15.54**
	16	1,3,1,3,1,3,1,3	7.34*	14.92*
	16	3,3,2,2,2,2,1,1	7.47	14.76*

where

N = line length
TB = total available buffer capacity
Covar = co-efficient of variation
* = significantly different from the datum at the 97.5% level
** = significantly different from the datum at the 99% level

with the restriction that P_1 and P_2 are <1.0, where $P_1 = a/m_1$, $P_2 = a/m_2$, a = mean arrival rate, and m_i = mean service time of station i. Four patterns of variances' arrangement were considered. These are:

Pattern	Station's 1 Variance	Station's 2 Variance
1	0.16	0.16
2	0.16	1.00
3	1.00	0.16
4	1.00	1.00

The measures of performance used by the authors were the departure times of the first station, the total time spent in the system per arrival, and the expected buffer level for buffers 1 (in front of the first station) and 2. The results of each of these measures are reviewed below.

(1) First Station's Interdeparture Times

The authors found that the analysis of a production line is greatly fascilitated if each station is independent of its preceding and succeeding counterparts, so that the relationship between the arrival and the departure processes can be established from a queuing theory viewpoint. Therefore, the inderdeparture intervals for the first station were studied by means of their autocorrelation function, to check whether or not they are independent (an explanation of this function will be presented in the next chapter).

The study showed that the first station's interdeparture times are negatively autocorrelated, for lag 1, over all the P_1 values examined, and consequently, the conventional statistical methods cannot be used to determine the distribution of the inter-departure intervals. Neverhteless, the empirical distribution of the simulated departure process takes the form of normal or other similar symmetrical distributions.

In addition, the author found that the mean interdeparture time rises generally when P_1 is increased and therefore, the departure and the arrival processes of the first station are non-identical, however, the variance of the interdeparture times decreases as a result of the increase in P_1. Moreover, the autocorrelation between two successive departure intervals becomes less negative when P_1 rises.

(2) <u>Mean Total Time in the System (TE)</u>

As m_1 or m_2 goes up, TE increases smoothly, but TE rises sharply, when P_1 or P_2 becomes > 0.95, since this will make one of the stations permanently busy, rendering the system unstable. The regression analysis has been applied to determine the relationship between TE and the mean inter-arrival time, a_o, and m_1, m_2. The following equations were obtained for the four patterns of variances' imbalance:

Pattern	Regression Expression
1	$TE = 1.474\,{}^{m}1 + 1.496\,{}^{m}2 + 1.20\,{}^{a}o - 2.487$
2	$TE = 1.593\,{}^{m}1 + 1.568\,{}^{m}2 + 1.20\,{}^{a}o - 3.270$
3	$TE = 1.470\,{}^{m}1 + 1.616\,{}^{m}2 + 1.20\,{}^{a}o - 3.062$
4	$TE = 1.598\,{}^{m}1 + 1.652\,{}^{m}2 + 1.20\,{}^{a}o - 3.689$

These equations indicate that increasing the service times' variance for both stations raises the value of the constants, and hence TE will go up.

(3) Mean Individual Buffers' Levels (Q_1,Q_2)

It has been noticed that the mean level of buffer 1 is a function of the arrival rate and the service time of the first station. When the first station's variance is 0.16, Q_1 is greatly less than 1.0, and when it is 1.0, the value of Q_1 rises appreciably, but cannot reach 1.0 even when P_1 = 0.95. As regards to Q_2, it has been observed that it is dependent on a_o, m_1, and m_2, increasing directly with m_2, but decreasing with the rise in m_1, because the increase in m_1 tends to reduce the number of departures from the first station, and hence Q_2 goes down. Furthermore, the authors found that Q_2 is always <1.0 for both values of the variance, even if P_1 and P_2 are = 0.95, which demonstrates that Q_1 and Q_2 behave in a similar manner. Moreover, the data showed that Q_1 and Q_2 are exponentially related to m_1 and m_2.

McGee and Webster's work has the disadvantages of concentrating on performance measures which are of no practical importance, besides the drawbacks of investigating short lines with infinite buffer capacity. As a consequence, no significant results have emerged concerning the effect of the joint imbalance of both the means and the variances.

The Third Approach

Another part of the work of Shimshak (155) has been concerned with the examination of the impact of unbalancing a 2-station line having unlimited B, in terms of both its operation times' means and Covars. The analysis of the results of various simulations and mathematical approximations proved that if a station has an exponential service time (with Covar = 1), then sequence B, with the exponential station being placed last in the order, is always optimal with respect to the mean waiting time. In order to determine the degree of stochastic dominance of sequence B over sequence A, the simulation experiments showed that when P_1 and P_2 are small, a second degree dominance takes place.

(e) INEQUALITY OF BOTH COVARS AND BUFFER CAPACITIES

In this situation, all the operation times' means are nominally equal (balanced), but the Covars and the buffer capacities are jointly unbalanced.

The Simulation Approach

Smith and Brumbaugh (163) simulated a 3-station line with normal service times, Covars of 0.06, 0.12,0.17 and TB's of 4,8,12 units. The authors considered all the six possible permutations of assigning the three Covars' values among the stations, as well as three configurations of distributing the available TB among the two buffers, namely, 25-75%, 50-50%, and 75-25%.

Table 4.3 shows the authors' experimental design, together with the throughput and idle time results for TB = 4. It should be noted that this study was regarded as representing the NSS conditions because the start-up period and the number of replications used were not enough (1500 time units and 6 replications). The main conclusions of this work are as follows:

(1) When TB is small (TB = 4), the effect of the patterns of the Covars and buffer capacities imbalance, as well as the impact of their interaction on throughput, are highly significant. Increasing TB to 8 makes the effect of the Covars pattern disappear. At TB = 12, only the influence of the interaction remains significant.

(2) The lowest mean throughput occurs when the highest Covar is positioned at the middle station.

(3) The balanced buffer capacity arrangement (i.e. 50-50%) is superior to any other arrangement, as far as throughput is concerned. The effect of the departure from an equal TB allocation on reducing throughput, is greater the less TB is.

(4) Alloting relatively higher buffer capacity around the stations with the larger Covars, and lower capacity around the less variable stations, is beneficial in terms of throughput. The greater the differences in Covars between the stations, the more the advantage gained from this allocation policy.

(5) If TB is unequally distributed along the buffers, the cost of an improper arrangement of TB is higher than the benefit of a proper allocation of TB.

TABLE 4.3

THE EFFECT OF JOINTLY UNBALANCING THE COVARS AND BUFFER CAPACITIES ON THROUGHPUT AND IDLE TIME FOR A 3-STATION LINE WITH TB OF 4 - FROM SMITH & BRUMBAUGH (163)

PATTERN OF TB ALLOCATION	MEASURE OF PERFORMANCE	PATTERN OF COVARS IMBALANCE					
		S,M,L	S,L,M	M,S,L	M,L,S	L,S,M	L,M,S
25-75% (unbalanced)	Throughput	1786	1779	1785	1773	1781	1777
	Mean % idle time	1.03	1.35	0.97	1.70	1.28	1.55
	σ % idle time	0.16	0.09	0.32	0.12	0.22	0.37
50- % (balanced)	Throughput	1783	1785	1788	1786	1783	1787
	Mean % idle time	1.27	0.98	0.87	1.00	1.08	0.93
	σ % idle time	0.27	0.04	0.17	0.22	0.28	0.04
75-25% (unbalanced)	Throughput	1775	1772	1777	1778	1785	1788
	Mean % idle time	1.59	1.72	1.38	1.40	0.97	0.89
	σ % idle time	0.21	0.24	0.11	0.13	0.07	0.02

where

TB = total buffer capacity

S = relatively small Covar (= 0.06)

M = relatively medium Covar (= 0.12)

L = relatively larger Covar (= 0.17)

(6) Since the values of the Covars used by the authors are relatively small, the improvement in throughput from adopting a correct pattern of Covars and buffer capacities imbalance, is significant, though it is only about 1%.

(7) The improvement in throughput always coincides with a lower percentage of mean idle time, but does not always correspond with a lower variance of idle time, implying that the maximum throughput does not necessarily entail equal idle time% and utilization for each individual station.

(8) The mean time to complete an item goes up as TB is increased, all other things being equal. For TB = 4 the effects of the Covars and TB assignment patterns, together with the interaction's effect, are all significant. For TB = 12, the interaction's impact vanishes. This is not in line with the results obtained for throughput.

(9) The expected completion time is much more sensitive than throughput to the patterns of allocating the Covars and TB.

(10) The longest, the second longest, and the lowest average times to finish a unit, occur respectively, when the station with the highest Covar is placed at the rear of the line, in the middle, and at the beginning of the line.

(11) A strategy whereby the highly utilized buffers are increased in size at the expense of the less highly utilized buffers, until an equal buffer utilization is achieved, has little influence on increasing the throughput,

i.e. the largest throughput will not necessarily result from an equal buffer utilization.

(12) It appears that the mean buffer level is a function of the capacity of each buffer. The three TB allocation patterns (i.e. the 25-75%, 50-50% and 75-25%) resulted in, respectively, higher, slightly higher, and considerably higher mean buffer levels in the second, first, and first buffers respectively.

(13) It is necessary to specify the adopted performance measure before attempting to distribute TB among the buffers, since the different performances criteria may produce different responses and therefore, if one measure improves effectiveness, another might decrease it.

Though interesting and compact, a shortcoming of this research, besides the employment of very short lines and queuing characteristics performance measure, is the use of a short Covars' range, i.e. 0.06, 0.12, and 0.17. Consequently, the differentiation between the variable and steady stations becomes relatively unclear and, thus, some of the findings of this study may be imprecise. Another shortcoming is the utilization of the throughput, rather than the idle time, or the output rate, which have more practical significance than the total number of units produced at the end of the simulation period (the throughput).

Another simulation study is the one performed by De La Wyche and Wild (39) who examined a 5-station line having four steady stations (all with a Covar of 0.1) and one relatively variable station (with a Covar of 0.3). Additionally, the TB values of 0,1,2 were used. The patterns of the simultaneous imbalance of the Covars and buffer capacities were:

(1) Positioning higher B around the high Covars' stations than around the steadier Covars' stations.

(2) Assigning an even amount of B around the variable station.

(3) Allocating B after the variable station, rather than in front of it.

(4) Placing more B between the station with the high Covar and the line's centre, rather than between this station and either the beginning or the rear of the line.

Table 4.4 exhibits the patterns of the joint Covars and buffer capacities imbalance, along with their resultant idle times. From the table it seems that, in terms of the idle time, pattern 2 is the best followed, respectively, by patterns 1, 4 and 3.

A major criticism of this investigation is the use of very small TB values, resulting inevitably in the assignment of zero B to at least two buffers, which makes the patterns of buffer capacities imbalance rather unclear and difficult to distinguish, and causes a great deal of inefficiency in the operation of the line. Another criticism, as the authors declared, is the "limited range

TABLE 4.4

THE EFFECT OF SIMULTANEOUSLY UNBALANCING THE COVARS AND BUFFER CAPACITIES ON IDLE TIME FOR A 5-STATION LINE - FROM DE LA WYCHE AND WILD (39)

TB	PATTERN OF TB ALLOCATION	PATTERN OF COVARS IMBALANCE		
		S,V,S,S,S	S,S,V,S,S	S,S,S,V,S
0	0,0,0,0	13.98**	13.96**	13.60**
1	0,1,0,0	10.92	13.05	-
1	1,0,0,0	13.44	-	-
1	0,0,1,0	-	12.33	11.71
1	0,0,0,1	-	-	12.89
2	1,1,0,0	8.06	-	-
2	0,2,0,0	10.04	12.20	-
2	2,0,0,0	13.17	-	-
2	0,0,1,1	-	-	7.71
2	0,0,2,0	12.33	11.02	-
2	0,0,0,2	-	-	12.89
2	0,1,1,0	-	6.57	-

where

TB = total buffer capacity

** = significantly different from the datum at the 99% level

S = relatively small Covar

V = relatively large Covar

of situations simulated, from which only tentative conclusions may be drawn". The merit of this study is the consideration of some novel imbalance policies for reasonable N and Covars' values.

SUMMARY

The purpose of this chapter was to present a survey of the current state of knowledge on the behaviour of unbalanced production lines which operate under transient conditions. The sources of imbalance in the line were divided into five types, each of which was reviewed separately and, if possible, under three major headings: the queuing theoretic, simulation approaches, and a third miscellaneous one. The line's imbalance kinds were those of work times' means, Covars, buffer capacities, means and Covars, and Covars and buffers.

Below is a summary of some of the most important research findings:

(1) Means Imbalance

(a) The build up of the buffer levels is higher in pattern (/) than that of a balanced line, such that the balanced SS line's levels are reached sooner and then overshot, causing an increased amount of I as compared to that of a balanced line, especially for higher DI. The reverse is true for pattern (\).

(b) There is an optimal degree of imbalance which minimises the cumulative I relative to that of a balanced line, for any elapsed transient duration.

This is valid for both empty and full start initial conditions.

(c) The initial I is high, but it reduces as time passes. The marginal decrease in I diminishes as time elapses, until the SS buffer level is converged at. The rate of reduction in I tends to rise as DI goes up. When the minimal SS idle time is reached, I starts to increase again, due to the influence of DI, especially when DI becomes higher.

(d) The minimum periodic I coincides with a buffer level which is approximately equal to that of a balanced SS line.

(e) The cumulative I, for any of the transient periods, is lower than that of a balanced line.

(2) Covars Imbalance

(a) The bowl phenomenon policy of Covars imbalance is the best for short lines, resulting in a decreased I in comparison with that of a balanced configuration. Increasing B and reducing the Covar, decreases the advantage of the bowl phenomenon. However, for longer lines this policy gives rise to significantly higher I than the I achievable by a balanced line.

(b) The improvement in I from arranging the Covars according to the bowl phenomenon is significantly higher than that obtainable by the bowl configuration of mean service times.

(c) A policy whereby the variable stations are separated from each other by steadier stations provides close I's results to those of a balanced arrangement.

(3) Buffer Capacities Imbalance

(a) A zero buffer capacity assignment should be avoided, since it generates significantly high I.

(b) The best and most consistent policy is to apportion TB as equally as possible among the buffers.

(c) If imbalance is unavoidable, concentrating more toward the line's centre is advantageous.

(d) The policy of allocating most of TB to the rear of the line is detrimental.

(4) Means and Covars Joint Imbalance

(a) The mean buffer level, for a two-station line, is a function of the arrival rate and the service times of the preceding and succeeding stations.

(b) ABL is <1.0, for all the utilization rates and the variances' values considered.

(5) Covars and Buffer Capacities Combined Imbalance

(a) Positioning the highest Covar at the middle station results in the lowest throughput and highest mean I.

(b) A balanced buffer capacity pattern is superior to any other arrangement, especially when TB is low.

(c) Alloting higher B around the variable stations, and lower B around the steadier stations, is beneficial, especially as the difference between the variable and steady stations increases.

(d) A strategy of increasing the size of the highly utilized buffers, at the expense of the underutilized ones, will not increase throughput.

(e) The ABL of any buffer is a function of its B.

(f) Assigning an even amount of B around the variable station is a beneficial pattern.

It seems that further studies, especially simulation ones, are needed on each of the five sources of imbalance, as well as a sixth type (the means and buffer capacities joint imbalance), in order to close some existing gaps and extend the range of the parameters and the patterns of imbalance experimented with. The next chapter will deal with these matters.

P A R T   T W O

RESEARCH DESIGN

Part Two is comprised of one chapter:

CHAPTER FIVE - METHODOLOGY AND EXPERIMENTAL DESIGN

C H A P T E R   F I V E

METHODOLOGY AND EXPERIMENTAL DESIGN

INTRODUCTION

Chapters 1 to 4 discussed the published research into various aspects of the unpaced manual flow line's design and operation. This chapter states the objective of the investigative part of this thesis and indicates its methodological, statistical design, modelling,and programming considerations.

OBJECTIVE

This thesis has the overall objective of obtaining results which, hopefully,will shed lights on some important features of the unbalanced and unpaced manual line's behaviour. Due to the significance attached to the unpaced manual production lines,it was decided to concentrate all investigations on this line type. A decision has also been made to focus solely on the unbalanced manual line, mainly because of the growing awareness among researchers of the need to explore this line's behaviour as a viable design consideration.

Since up to the present time no mathematically supported procedure can be found which is capable of handling the unbalanced steady and non-steady states' characteristics of such a stochastic production line, simulation was resorted to as the most suitable technique for this kind of exploratory study. Recall that proven relationships between the various variables are impossible to determine when employing a simulation methodology, but nonetheless, their form can be usefully supplied by regression analysis.

It was decided to investigate the relative efficiencies of the following types of unbalanced production lines for both the steady state and transient conditions, in a more comprehensive, systematic and methodological way than hitherto attempted. Specifically examining:

(a) Line unbalance resulting from the fact that individual workers may have different mean work performance times.

(b) Line imbalance emerging from the variability of the work times at the individual stations, as expressed in terms of different co-efficients of variation (Covars) of stations' work times distribution.

(c) Imbalance due to the allocation of an unequal amount of buffer storage capacity to the various buffers.

(d) Imbalance emanating from the combined effect of imbalance types (a) and (b) above.

(e) Imbalance resulting from the joint effect of (a) and (c) above.

(f) Imbalance achieved as a consequence of the simultaneous effect of (b) and (c) above.

STRATEGIC EXPERIMENTAL DESIGN

The purpose of strategic experimental design is to construct a set of sound experiments which will yield the desired information from carrying out the simulation runs, so that valid statistical conclusions may be drawn. There are some questions of a strategic nature which need to be elaborated and discussed at this stage. Fishman and Kiviat (55) distinguished two experimental design objectives, namely, the comparison of the outcomes of different alternative designs, and the determination of either the relative importance and effect of input variables on the output variables (analysis of variance), and/or discerning the general functional relationships between those variables over some levels of interest (regression analysis). These two aims do not conflict with each other and therefore, will both be sought when analysing the outcomes of this research. Another objective, due to Naylor et al (132), is the search for the optimal levels of the input variables, which usually requires search techniques. This goal will not be pursued in this thesis since it would necessitate a separate and exhaustive work on its own. The experimental design should also specify the types of the variables used in the simulation experimentations, together with their levels. These are discussed below.

SIMULATION VARIABLES

It may be convenient to categorise the variables utilized in simulation models into exogenous variables, status variables, and endogenous variables (Naylor et al (132)). These variables will be defined in turn:

(1) The exogenous variables are defined as the independent input variables which influence the system but not being

affected by it. In simulation studies these variables are determined, a priori, by the experimenter. If these exogenous variables assume discrete values, then there is a finite number of alternatives, but when they take on continuous values, there is virtually an infinite number of possibilities. The discrete exogenous variables can be, further, quantitative if their levels have numerical values, or they might be qualitative. In this thesis all the exogenous variables used are of the discrete kind, but both quantitative (e.g. N) and qualitative (e.g. service times means' pattern) discrete exogenous variables are employed. Furthermore, the exogenous variables can be divided into two classes; controllable and uncontrollable. Uncontrollable exogenous variables are those being generated by the environment surrounding the system, but not by the system itself. Controllable variables, on the other hand, are subject to the control of the decision maker.

The controllable exogenous variables in the context of the particular unpaced unbalanced manual lines being studied are as follows:

- the total number of stations in the line, N.
- the total amount of buffer capacity for the line, TB.
- the capacity/mean capacity of each buffer, B/MB.
- the range of Covars values.
- The pattern of mean work times' imbalance.
- the pattern of Covars imbalance.
- the pattern of buffer capacities imbalance.
- the joint patterns of the combination of means and Covars, means and buffers, and Covars and buffers imbalances.

- the degree of unbalance of service times' means for the line which, in turn, determines the magnitude of individual stations' means.

The type of distribution of each operator's work times, as described by its skewness and shape, may be viewed as an uncontrollable exogenous variable. The identification of the type of distribution which is representative of a wide range of practical service times will be specified later in the chapter.

Although the aforementioned exogenous variables are all reasonable and viable factors, it is clear that a very large number of simulation runs will be needed if the whole population of factor levels, or even a representative large sample, was examined; a task constrained by the limited avvilability of computer running time. The decision has been made therefore, to experiment only with a few reasonable factor levels.

Insofar as the length of the line, N, to be used throughout the experimental phase of this thesis, is concerned, N's values of 5 (odd) and 8 (even) were decided upon. Wherever the need arises to gain more insight into a particular design's conduct, an additional N level, i.e. N = 10 was selected. The desire to examine lines having more than a maximum of 10 stations was hampered by the resulting increase in the required simulation time. It has been stated that computer time is either proportional to N, and approximately doubles as N doubles (Slack (160)), or is exponential with N (Barten (8)). As a consequence, lines in excess of 10 operators were regarded as extremely expensive to simulate.

It has been reported by Wild (175) that a substantial number of the manual unpaced lines investigated have N's values of 2-10. This may be viewed as lending some support to the contention that the selected N's range in this thesis is representative of a large proportion of the line lenghts met in practice. Furthermore, previous research (see e.g. Slack (160)) has demonstrated that even if the value of N lies outside the practical range, extrapolation can prove to be extremely accurate.

The selected range of B's values of 1, 2, 3, and 6 is, on the other hand, similar to that adopted by other researchers, e.g. Slack (160). B's values higher than 6 were not experimented with since it is well known from previously conducted research (see e.g. Barten (8) and Anderson & Moodie (2)) that the buffer capacity need not be large in order to attain high production rates. Moreover, a B's value of zero has not been used in all the experiments because this value was always being found inefficient (see Sastri (151) and De La Wyche & Wild (39) who indicated that sizable benefits were gained when avoiding the zero B's value).

With respect to deciding on the appropriate values of the imbalance degree, DI, the research of Kilbridge and Wester (92) had shown that normally between 5 - 10% balancing loss prevails in industry. Therefore, %DI values of 2, 5 and 12% were chosen, with the value of 2% being selected so as to see the effect of a slight imbalance on line's operation. In addition, the decision has been taken to explore the influence of the presence of a high unbalance degree, wherever it was deemed essential to do so.

(2) The status or intermediate variables portray the system's state or that of one of its components during the operation of the line. These variables are of interest only when it is desired to gain deeper understanding of the internal working of the simulation model and the interactions within it, and include the amount of time each station being idle waiting for a particular item, and the time each item spends at each buffer store. Some more status variables will be mentioned later on.

(3) The endogenous variables are the dependent or output variables of the system and are generated from the interaction of the system's exogenous and status variables. The endogenous variables of the unpaced manual lines are the various line's efficiency measures which were described in Chapter one.

Of the activeness measures, the idle time, I, rather than delay or production rate was decided upon, the resons being that I is practically more important and understandable than delay, and that it is easier to compute and derive I's starving and blocking data than those for the production rate.

Additionally, in line with most other researchers, the ultra theoretical queuing characteristics' effectiveness measures are regarded as not important and as a result, were not calculated by the simulator.

Thus, for the purpose of this thesis, the following main performance measures were adopted:

(1) % total idle time of the whole line, I.

(2) Mean buffer level for the line as a whole, ABL.

(3) % idle time due to blocking and starving, BL and ST.

From these directly calculated measures, the following useful characteristics may be obtained (refer to Chapter 1):

(a) Mean total number of units in the line, L.

(b) Buffer utilization, BU.

(c) Space utilization, SU.

In addition to the above-mentioned three types of variables that appear in the simulation model, certain quantities (constants) which influence the endogenous variables may also be included. These constants which are termed as 'parameters' include, for example, the mean service time when the line is balanced.

FACTORIAL DESIGN

The proper design of experiments is an efficient tool to provide a methodological structure for any investigation so as to improve the interpretation of its outcomes. There are many types of experimental designs, the most efficient, powerful, and elegant of which, for examining the effects of changing the discrete exogenous variables' values, is the complete factorial design which has the following advantages:

(1) In this design it is possible to determine whether the exogenous variables interact in their effect on the endogenous variables.

(2) This design enables the researcher to exert greater experimental control and, consequently, more sensitive statistical tests (i.e. more reliable estimate of factors' effect through the reduction of the error term) will result.

(3) It is possible in the factorial design to test the separate as well as the combined effects of several variables.

(4) In factorial experiments the effect of an exogenous variable is studied across different values of other exogenous variables. Consequently, generalisations from such experiments are broader than those from any other experiment.

Due to its efficiency, the decision has been made to utilize the factorial design throughout this thesis.

In factorial experiments all the desired levels of a given factor are combined and simultaneously considered with all the levels of every other factor. Therefore, the required number of data points (cells) in this design is the product of the number of levels of each factor. It has been established through trial experiments that each cell takes a relatively long time to simulate, for example, a 5-station line (which is the shortest used in this research) needed about 1.5 minutes of computer time to simulate. Obviously, this calls for a reduction in the number of cells, if the restricted availability of computer time is to be taken into consideration. Furthermore, in order to get a manageable experiment's size, it is necessary to reduce the number of factors and/or factor levels in a full factorial experiment with six factors having three levels each, 3^6 or 729 cells will be required for a single replication. Therefore, it was decided to restrict the number of factors to a maximum of five, and the number of factor levels to a maximum of four for all the investigations. In addition, an equal number of observations was generated for all the cells, since this will minimize the effect of the inequality of the error terms' variances, if present, on the cells' outcomes.

In a factorial arrangement each simulation run is a combination of various exogenous variables which define one cell (alternative or course of action). The set of all the cells constitutes an investigation. A total of six such investigations were conducted for the steady and non-steady state conditions.

STATISTICAL TOOLS

The strategic experimental design should also include the statistical techniques to be utilized for the analysis of the simulation data. For each investigation the following statistical techniques were chosen:

(1) Analysis of Variance (ANOVA): which is a technique that provides information about the sources of significant variations in the behaviour of the model, when qualitative factors are present along with quantitative ones. The ANOVA determines the main effects (the effects of each exogenous variable alone on the endogenous variable(s)) and the interactions (the combined effects of two or more variables), and whether or not any of them is significant.

(2) Multiple Regression Analysis: which compliments the ANOVA through the examination of the nature of the relationship between the endogenous and exogenous variables, by fitting the best line to the data, i.e. it provides the values of the co-efficients in the hypothesized equation of the line by the method of least squares, which attempts to minimise the sum of the squared deviations of the observed data from those predicted by the fitted equation. This technique should replace ANOVA when all the factors are quantitative.

In the traditional simple regression analysis the relationship between one exogenous variable and the endogenous variable, while fixing the levels of all other exogenous variables is determined, the relationship between another exogenous variable and the endogenous one is then studied, and so on. However, relationships cannot be understood and explained in this lengthy way since it prevents the discovery of the more complete and sophisticated inter-relationships between the variables, taken collectively or separately. This may force the experimenter to choose very few exogenous variables in order to reduce his experiment to a controllable size. The formulae provided by some researchers, such as Slack (160) and Anderson (1), can be criticised on the grounds that they employed the less efficient simple regression.

Multiple regression is the proper method which avoids the aforementioned shortcoming of the simple regression. In this technique the magnitude of the relationship between the exogenous and endogenous variables is specified by the co-efficient of multiple determination (or correlation), R^2, which refers to the proportion of the endogenous variable's variation that is explained or accounted for by the exogenous variables in combination.

(3) Multiple Comparisons' Methods: these methods are of two types; the first compares the output of a design with that of any other alternative design and is known as the 'pairwise comparisons' method, and the second compares the outcome of any of several alternatives with that of a standard design, and is referred to as the method of 'comparisons with control'. These two methods are most suitable when one or more of the factors are qualitative, otherwise regression analysis will be favoured. Below is a brief presentation of such methods.

Multiple Pairwise Comparisons

A significant overall F value in ANOVA, on a main effect, indicates that at least one of a multitude of possible comparisons among the means of alternative designs is significant, but it does not necessarily signify that all the means are significantly different from each other. It further, does not tell which particular comparison is significant. Therefore, a researcher cannot stop his analysis after getting a significant F, and the cause of this significance ought to be located. To do so, a follow-up analysis, known as pairwise comparisons, must be performed.

Several such analyses have been developed, including Fisher's LSD, Duncan's multiple range test, Newman-Keuls, Tukey's HSD, and Scheffé's test. All these statistical procedures take any possible pair of means, called 'contrasts', and compare them under exactly the same operating conditions, to determine if they differ significantly. These procedures are classified into two kinds, viz, liberal and conservative procedures. A liberal test will find a significant difference between two means that are relatively close together, whereas a conservative test will indicate that two means are significantly different only when they are far apart. Of the above-stated five methods, Fisher's LSD is the most liberal and Sheffé's is the most conservative, followed by Tukey's (the latter was employed by El-Rayah (44), who is the only user of the multiple comparisons' techniques in production lines' simulation). So as to exercise some degree of caution in interpreting this research's outcomes, it was decided to utilize the Sheffé's test to avoid a higher risk of Type 1 error. (For a complete discussion of the various pairwise comparisons' methods, see Winer (183)).

The Sheffé's Test

This procedure has received widespread use, and has the important property that the probability of a Type 1 error for any comparison does not exceed the level of significance specified in ANOVA. The Sheffé's formula is as follows:

$$F = \frac{\left(M_1 - M_2\right)^2}{MS_e \left(\frac{1}{n_1} + \frac{1}{n_2}\right)(k-1)}$$

where Mi = mean outcome for design i

MSe = mean square error

n_i = number of observations for design i

k = number of designs to be compared

When all the n_i's are of equal size, this test may be employed in obtaining a critical difference between the means, d, which is given by:

$$d = \sqrt{\frac{2(k-1)(\text{tabled } F)(MSe)}{n}}$$

The value of d is the minimum difference between any two means which is necessary for a significant contrast to result.

Multiple Comparisons with Control - The Dunnett's Test

This statistical technique tests the significance of the difference between each of alternative systems and a control system, while maintaining a certain significance level for the whole set of possible comparisons. The Dunnett's test has been chosen to compare the mean performance of different unbalanced lines' patterns with that of a balanced line counterpart. Two-sided tests with 0.95 and 0.99 simultaneous confidence intervals were utilized. The Dunnett's test is calculated as follows:

$$t = \frac{M_1 - M_2}{\sqrt{MSe\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where M_i, MSe, n_i are the same as those defined in the Sheffé's formula.

Like the Sheffé's test, the Dunnett's test is conservative, and the likelihood of a Type 1 error in any comparison does not exceed the level of significance specified in ANOVA. A critical difference, d, may be used when the n_i's are of equal size:

$$d = t\sqrt{\frac{2MSe}{n}}$$

Note that if the value of d or MSe is relatively high, fewer significant contrasts will result for both the Sheffé's and Dunnett's procedures. Note also that, in essence, all the multiple comparisons are techniques of ANOVA. Accordingly, they all make the standard assumptions of independence, normality, and equality of the variances of the experimental errors. The independence assumption is generally satisfied in computer simulation experiments when either the subruns or replication methods (a discussion of which will be presented soon) are used. The normality assumption was met in the design of this research (as will be discussed shortly), but the third assumption may not be completely satisfied. However, the departure from this assumption has been shown by Kleijnen (93) to have little effect on inferences about means if the number of observations per experimental cell is equal. This requirement was heeded to in all the experiments by selecting equal subrun's sizes.

(4) Canonical Correlation: canonical correlation is the generalisation of multiple regression analysis to any number of endogenous variables and, therefore, the latter can be considered as a special case of the former. This technique seeks to determine the inter-relationships between two sets of variables with k exogenous and m endogenous variables. Through the method of least squares, two linear composites are formed, one for the exogenous and another for the endogenous variables, and the correlation between these two composites represents the canonical correlation.

In addition to the aforementioned statistical techniques, graphs and tables will be presented to show the general patterns of relationships among the variables of interest.

It should be noted that one of the multiple ranking (ordering) procedures, such as that attributable to Bechhofer (see Kleijnen (93)), may be utilized for determining the number of observations (sample means) that should be taken from each of $k(\geq 2)$ populations (designs), in order to select the best, second best, ..., and the worst populations. Such procedures guarantee, with a probability of at least P^*, that the best population's mean is at least δ^* better than the next best mean, and so on. However, it was decided against using such procedures because of the following reasons:

(a) There is no objective rule for choosing δ^* and, therefore, the value of δ^* is usually selected arbitrarily by the experimenter. As a result, when making an arbitrary selection of δ^* for each of the factor's levels, the probability of incorrect ranking of the patterns increases.

(b) The number of observations per population may be fixed when conducting a pilot study, or if the available computer time is limited. It is obvious that the application of multiple ranking techniques to all the experimental cells in this exploratory research is impracticable and, in fact, requires a separate study on its own.

(c) The multiple comparisons' tests are often employed to obtain an indirect ranking of the designs. Consequently, the various unbalanced lines' patterns utilized in this study will be ranked in accordance with this indirect way.

THE MULTIPLE RESPONSE PROBLEM

According to Hunter and Naylor (79), the multiple response problem forms an underdeveloped area of experimental design and analysis. This problem arises when more than one endogenous variables (outcomes) are generated by the simulator. Naylor et al (132) suggested that it can be bypassed by treating an experiment with many outcomes as several experiments, each with a single outcome. Another likely way to overcome this problem is through the use of utility theory to combine the different outcomes (e.g. by addition) into one single outcome. The utility approach attempts to find a utility function which can be used to assign weights to the different responses and, in this manner, they are reduced to only one response which lends itself to statistical analysis. A third approach to tackle the multi-outcomes problem is that of Kleijnen (93) who indicated that this problem may need a multivariate analysis. One such analysis is the canonical correlation which has been explained earlier.

TACTICAL EXPERIMENTAL DESIGN

The aim of the tactical design of experiments is to determine how the strategically designed experiments are to be executed. Several tactical considerations are often encountered when designing computer simulation models, the manner of tackling them may affect the utility of the simulation results and sometimes, even the feasibility of the whole investigation. In this context, the type of work times distribution to be used, the values of its parameters, the parameters of the simulation run, the development of the simulation model, and its validation will be discussed. Some more detailed tactical considerations and decisions pertinent to each of the particular unbalanced lines' investigations are to be explained in Chapter Six.

WORK TIMES DISTRIBUTIONS

In manually unpaced production systems a major source of inefficiency is the fact that the work time for each operator is stochastic in nature, and may be described by a probability distribution. As Conrad (32) has put it "this variability is a perfectly normal human characteristic and it cannot be prevented by an individual however hard he tries". A work times (operation, service, or process times) distribution is defined by Slack (158) as "the probability distribution based on the frequency with which an operator takes a particular time to perform his work task".

The likely shape of the distribution of operator work times has been studied by several researchers, such as Dudley (42), Conrad and Hille (33), and Murrell (126). From these studies it emerged that the distribution of operation times for experienced workers performing paced tasks was nearly normally distributed, even when the task was paced at a rate which is equal to that of the unpaced mean operation time. This normality of process times was largely due to the mechanical restriction of pacing. The same normal distribution was found to approximate the performance times of trainees who were manufacturing satisfactory work units but could not match the high speed set by the experienced operators.

In the case of experienced workers doing unpaced tasks, it is now generally established that the distribution of their operation times tends to be positively skewed. Conrad (32) stated that as a result of this skewness of service times, approximately 66% of total service times will tend to be less than or equal to the mean service time. On the other hand, according to Dudley (40), the impact of experience (which is gained through training) is to maintain the same range of work times as that for trainees, while increasing the frequency of performing the tasks in shorter times.

A detailed study of the published histograms of work times which are met in practice was made by Slack (160). He concluded that a probability distribution describing work times has the following attributes:

(1) It has a lower limit which no operator is able to perform any faster.

(2) It is positively skewed.

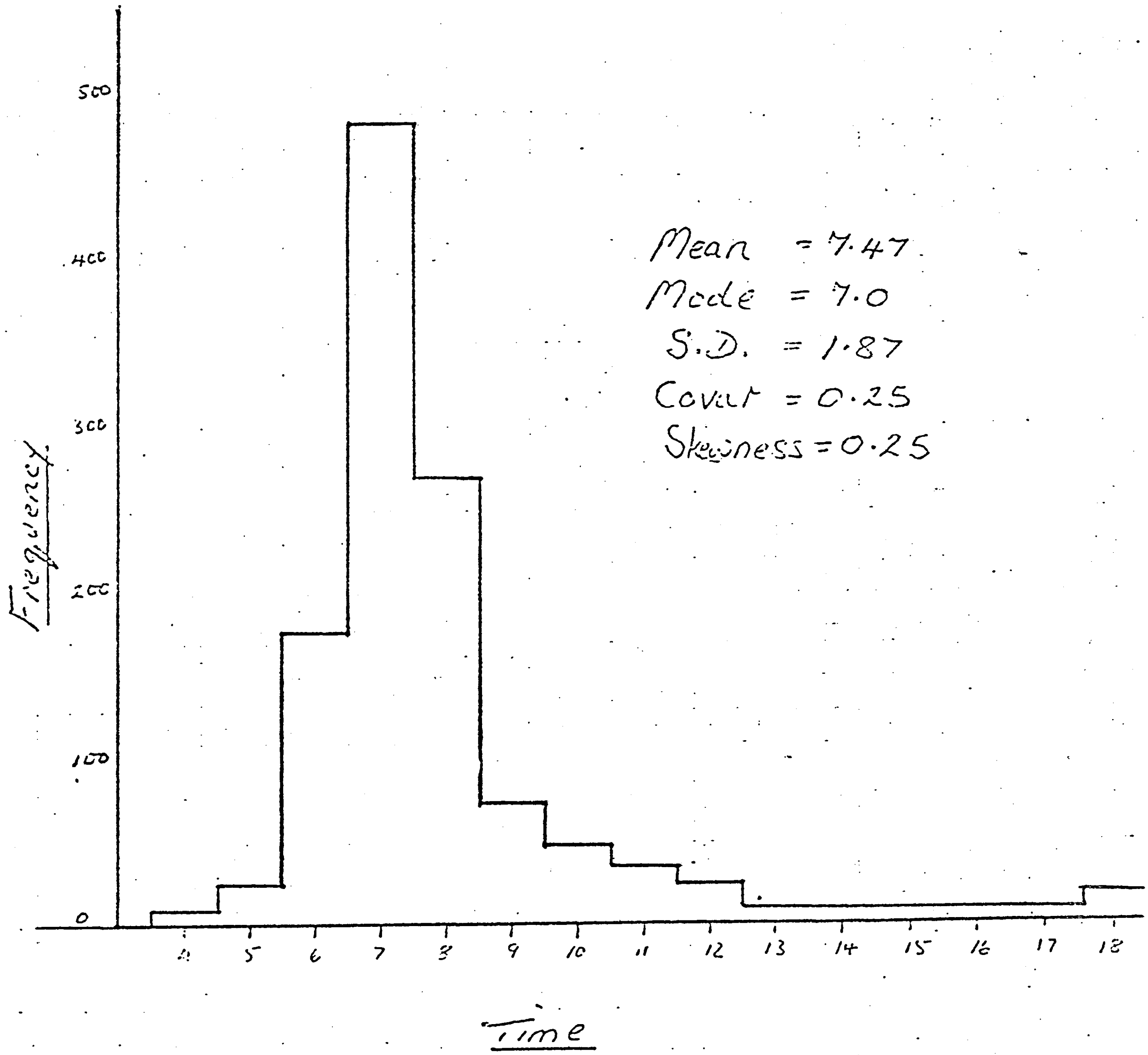
(3) Its skewness value, as measured by Pearson's 1st co-efficient of skewness, $\left(\frac{\text{mean-mode}}{\text{standard deviation}}\right)$, is approximately between 0.10 and 0.89.

(4) Its Covar's value varies in the range of 0.08 to 0.5, with a likely average of around 0.274.

Figure 5.1 shows a typical work times distribution adapted from Conrad and Hille (33) by Slack (160). The simulation investigations of this thesis utilize the Weibull distribution to describe the work times function. A summary of this distribution is presented below.

FIGURE 5.1

A TYPICAL WORK TIMES DISTRIBUTION - FROM CONRAD (33)



THE WEIBULL DISTRIBUTION

This distribution was discovered by W. Weibull in 1951 and since then it has received a widespread application, particularly in the areas of reliability, life testing, and inventory control. Three parameters define the Weibull function and determine its location, range and shape. These are 'a' the location parameter, 'b' the range parameter and 'c' the shape parameter. The Weibull probability density function (P.D.F.) is:

$$p(x) = \frac{c}{b} \left(\frac{x - a}{b}\right)^{c - 1} \exp\left[- \left(\frac{x - a}{b}\right)^{c} \right]$$

for $x \geqslant a$, $b > o$, $c > o$

The Weibull cumulative distribution (C.D.F.) is:

$$F(x) = 1 - \exp\left[- \left(\frac{x - a}{b}\right)^{c} \right]$$

$p(x)$ is unimodal for $c > 1$ and has a reversed J shape for $o < c \leqslant 1$.

Tables A5.1 and A5.2 show the normalised Weibull's P.D.F. and C.D.F. respectively. In both tables the values of a and b are, respectively, zero and unity, while the value of c ranges from 0.1 to 10.0 in the case of P.D.F., and 0.1 to 4.0 for C.D.F.

A comparison between the observed and theoretical work times distributions has been undertaken by Slack (160) in order to determine the values of the parameters of the Weibull distribution which can result in a distribution that is reasonably reflective of the published histograms of observed unpaced manual process times. The following may be concluded from this comparison:-

(1) The shape parameter, c, cannot be indicated by only one value, but a mean c value of 1.6 will give rise to a Weibull distribution which is representative of the published empirical work times distributions.

(2) Assuming that a balanced line with a Covar of 0.274, a mean of 10, and a skewness of 0.62 is required, the values of the Weibull parameters are as follows:

$$a = 5.780$$

$$b = 4.702$$

$$c = 1.6$$

It was decided to fix the value of the shape parameter, c, at 1.6 for all the simulation experiments because it has been shown by El-Rayah (44) and Rao (143) that no significant differences in efficiency were obtained between symmetrical and skewed distributions, as long as the difference in their Covars is not large. It is clear that the difference in effectiveness is expected to be lower between two skewed Weibull distributions.

SIMULATION RUN'S PARAMETERS - STEADY STATE UNBALANCED LINES

There are several parameters which have to be taken into consideration before running the simulator:-

(1) The initial conditions at the beginning of the start-up period.

(2) The length of the transient period.

(3) The length of the simulation run.

(4) The number of observations of line's effectiveness measures to be taken.

Each of these parameters will be discussed in turn.

INITIAL CONDITIONS

The initial conditions of the simulation model may produce transient components in the steady state simulated output and thus, an inadvertent selection of these conditions can result in a biased and unrepresentative pattern of system's behaviour. There are generally two main initial conditions' selection rules:

(1) Start the system in an empty and idle status - a condition whereby no temporary entity (work unit) exists in the system and all the permanent entities (e.g. operators) are idle. This rule prolongs the approach to the SS.

(2) Make the initial conditions as close to those of the SS as possible to cut down or minimize the length of the stabilisation period and accelerate the convergence to the SS. Conway (34) suggested starting the system at a non-empty state, making use of any a priori information about the system. This second approach is being used in this research to select better initial conditions than the idle and empty state. Previous experience and knowledge about the operation of the unpaced manual system (see Slack (160) and Wild (177)) have indicated that the approximate SS buffer utilization is about 0.5 for various buffer capacities and line lengths. It was decided, therefore, to start the system with all the buffers being half-full.

Moreover, as Hillier and Lieberman (77) have advised, the same initial conditions were used for all the research experiments because using different starting conditions for different configurations will bias the outcome of any comparison between them. Also, the same random numbers' seed was used in all the experiments so as to reproduce an identical sequence of events for all the designs, which sharpens the contrast between the alternatives by reducing the variation in their performance's differences and therefore, much smaller run lengths will be required to identify any significant contrast.

TRANSIENT LENGTH

The problem of determining the length of the transient is complicated by the difficulty encountered in defining the statistical equilibrium or SS itself. The system approaches SS when the successive observations of its output are statistically insignificant, whereby a new observation does not supply a fresh knowledge about the future conduct of the system. Before discussing the practical determination of the NSS interval, it is necessary to explain the statistical concept of SS.

From a statistical viewpoint the system is considered as being in the steady state if there is no variation over time in its probability of being in some state. This does not imply that no action takes place in the system such that it cannot leave its current state, but it signifies that the system moves from one state to another at different intervals in accordance with a fixed probability distribution that is independent of the state of the system. In the theory of Markovian processes this fixed distribution is termed as 'stationary' distribution (Kleijnen (93)). Therefore, throughout the steady state operational mode the observed values of the measures of activeness and stockholding for each station and buffer, are regarded as random variables which are being drawn from 'limiting' probability distributions having certain mean and variance values which are independent of time under SS conditions. Conway (34) stated that such limiting distributions are assumed to exist in any simulation study, with the aim of empirically estimating these distributions or their moments.

In a theoretical sense, the SS condition is a limit that is converged at after the elapse of a long time horizon, but never attained. As a result, no one point in time exists which signals the decisive change in system's state from the transient to the stable mode. A point exists, however, which for all practical purposes, can be viewed as being close enough to justify the assumption that the SS is being realized. At this point there is a small enough difference between the temporal and the stationary state probability distributions (which continuously decreases through time), to discard the resulting error. The rate of approach to the stationary distribution for a single server exponential queue is, as suggested by Conway, not proportional to time, but rather geometric in the shape of a negative exponential function. However, the rate of convergence to the equilibrium state for more complex systems, such as the series queues production line system, can be appreciably slower than that attainable by the single server queuing system, although it may take on the same form.

The importance of determining the NSS period is highlighted by the fact that if the transient length is insufficient, such that the sample means were computed before the realization of the SS conditions, they will be biased estimators of the true population mean and thus, incorrect conclusions will be derived (Emshoff and Sisson (47)).

Two methods have been suggested to remove the effects of the transitional phase. The first is to use long simulation runs so as to render the data from the transient period inisgnificant, relative to that of the steady state. This method is extremely costly in terms of computer running time. The second method, and the one commonly used, is to choose a fixed NSS duration and run the simulator until the end of the simulation period, then discarding all statistics accumulated during the transient period, and keeping only those representing the SS. The main problem here is the decision on the length of the stabilisation period and the point in time where, practically, the transient results are indistinguishable from those of the steady state.

Three approaches to deal with this issue have evolved; the application of time series analysis techniques, the study of queuing models, and the development of heuristic rules of thumb. Although the results derived from time series analysis and queuing theory are rigorous and precise, they have a rather limited applicability. For this reason, many heuristic rules were formulated in order to specify how to minimise the transient period and identify the truncation point, beyond which data are not significantly distorted by the initial transient. Gafarion et al (61) evaluated five such heuristic rules and found that all performed poorly and are not suitable for their proposed aim. Likewise, Wilson and Pritsker (181), (182) developed an evaluation procedure through which seven rules were examined and shown to result in excessive truncation. They reached the conclusion that the judicious selection of initial conditions is more effective than truncation in improving the performance of the sample mean. This judicious selection has been done earlier (refer to the initial conditions' discussion).

With respect to the NSS interval, Conway (34) suggested that the most satisfactory method for determining the effective end of the transient is by statistically comparing the mean transient values with their corresponding SS values. Being aware of this proposal, Slack (160) used a comparison procedure and found that between 50 and 450 product cycles were sufficient for the steady state operation to be reasonably approximated for the percentage idle time and mean buffer level, depending on the line length and the buffer capacity. On the basis of this conclusion, the unbalanced steady state lines' findings, to be reported in Chapter 7, do not take any results from the first 500 product cycles of simulated time in order to avoid the detrimental effects of the transient.

RUN'S LENGTH AND NUMBER OF OBSERVATIONS

The length of the simulation run (sample size) affects the statistical reliability of the system's mean outcome. The choice of a suitable run's length increases confidence in the simulation results and reduces the computational cost. The decision on the number of observations, given a fixed simulation duration, is affected by the choice of the length of each individual abservation. Each observation should be sufficiently long to avoid the occurrence of autocorrelation, but not so lengthy that prohibitive computer time will be needed and therefore, a tradeoff has to be made between these two requirements.

Autocorrelation or serial correlation means that a future output value is directly influenced by, and dependent on, a present output value, e.g. the output in the second of two adjacent periods depends, in part, on what has happened during the first one. It is generally known that successive observations from a simulation sample tend to be highly correlated and, as a result, each extra observation will provide little new information and sizable redundant ones (Emshoff and Sisson (47)). As the separation between the observations increases, the autocorrelation diminishes, and beyond some interval size it may reasonably be ignored.

The autocorrelation function estimates the correlation between an observation at any simulated time, t, and an observation at time t + s, and is given by:-

$$P(s) = \frac{1}{n-s} \frac{\sum_{s=1}^{n-s} \left[(X_t - \bar{X}) (X_{t+s} - \bar{X}) \right]}{\left[\hat{\sigma}^2 (n - s) \frac{1}{n} \sum_{s=1}^{n} (X_t - X)^2 \right]}$$

where

$P(s)$ = serial correlation co-efficient for lag s.

X_t = an individual observation of output at time t.

n = total number of observations (sample size).

$\bar{X}$ = average output over the simulation run and

$$= \sum_{s=1}^{n} \frac{X_t}{n}$$

$\hat{\sigma}^2$ = variance of the data over the whole simulation run.

Failure to consider autocorrelation is unacceptable since the reliability of the simulation output is overestimated, as a consequence of the fact that the variance or the standard error of the autocorrelated output data is underestimated, and unless a term is added to account for autocorrelation, an excessively optimistic (short) confidence interval for the true mean, as estimated from the sample mean, will be calculated and therefore, differences between alternative configurations may appear significant when in fact they are not.

A drawback suffered by much of the research works reported in Chapters 1 to 4, excluding those which have used the method of replicated runs, is that the experimenters have approached the statistical analysis of the simulation output in one of two ways; either they computed the output means and variances, ignoring the presence of autocorrelation, or they accounted for it by dividing the run's length into arbitrary intervals which would, hopefully, exclude major autocorrelation, without scientifically attempting to check the absence of autocorrelation, and then dealt with the observations on these supposedly independent intervals. As a result of this arbitrary choice of intervals, no high confidence can be placed on these results since it is highly likely that they contain serial correlations.

Four approaches have been suggested to deal with the problem of autocorrelated simulated data, namely:

(1) Regeneration points.

(2) Serial correlation estimation and spectral analysis.

(3) Replicated runs.

(4) Independent subruns.

These approaches will be reviewed in turn.

REGENERATION POINTS

In some systems, such as the simple queuing system, the system returns back to the point of being empty and idle a number of times, i.e. the system can be viewed as a renewal process, where it regenerates itself at this regeneration (tour or cycle) point. These regeneration points are random variables which depend upon the system's evolution and may be considered as independent i.e. there is no autocorrelation between them. However, this approach requires the system to return to its empty state a number of times in order to have enough independent regeneration points within the simulation run; a property not shared by many more complicated systems, including the series queues flow line system, because in this system the probability that all the operators are idle and the system is in an empty state is small. Therefore, this method requires a prohibitive run length which renders it inappropriate.

Fishman (49) tried to overcome this shortcoming by attempting to generalise the concept of regeneration points to include the starting of the simulated system with its most frequently entered state as a representative of the regeneration point, using the Markovian property of certain states of the model. However, as Kleijnen (93) argued, the experimenter may have problems in identifying the Markovian structure of the complex queuing systems.

SERIAL CORRELATION ESTIMATION AND SPECTRAL ANALYSIS

In this approach the simulation process is carried out as a single long run with the initial transient bias being removed and the individual observations being treated as data taken from a time series. Then, the autocorrelation function is precisely estimated by observing the outcomes at short and regular intervals, and its impacts are included in refining the variance of the data. This entails computing the autocorrelation co-efficient and determining the maximum lag for which this co-efficient is appreciably higher than zero (Shannon (154)). In this case the observations are not independent and if the standard statistical techniques are used, the assumption of independence stipulated by these techniques will be violated.

Consequently, spectral analysis has been suggested to analyse the correlated simulation data, where the spectrum is the Fourier transform of the autocorrelation. This analysis decomposes a time series into basic sine and cosine components and the interpretation of the series becomes easier when the periodic effects are revealed and removed. Although promising, spectral analysis has been criticised for the following reasons:

(1) According to Fishman (52) it requires the experimenter to have a deep knowledge of complicated time series techniques and consequently, a heavy burden is placed on him.

(2) Fishman (54) also argued that it is excessively expensive with respect to the computer time needed. In addition, it requires a considerable amount of data, especially autocovariances, to be stored and analysed which is time consuming (Gordon (67)).

(3) Duket and Pritsker (43) found that it produced unreliable estimates of the sample mean's variance.

(4) It cannot be used to study the NSS conditions since, as Fishman and Kiviat (55) stated, it requires the simulated system to be 'convariance stationary', i.e. in the SS.

(5) Where there are fairly subtle cycles in the data, the spectral analysis, in its current form, cannot detect them (Emshoff and Sisson (47)). Although the theory underlying this technique and its application in the physical sciences have reached a reasonable state, its utilization in simulation experiments is, as yet, limited and not well defined.

(6) The traditional statistical methods are necessary to examine the simulation data and to compare several alternative designs. Spectral analysis is mainly used to study the behaviour of a single stochastic process or alternative, but it becomes increasingly impractical when multiple comparisons of several configurations, or power spectra, are involved, especially when their number is substantial.

REPLICATED RUNS

In this approach a number of separate and independent simulation runs of equal lengths are executed using a new stream of random numbers for each run. In this situation the simulation outcomes will vary from one replication (or re-run) to another due to the presence of some random variations caused by the use of a different random numbers' sequênce which generates a different succession of events for each replication.

In this method each replication, excluding the transient period, gives one mean output which is regarded as one independent observation (x_i). The mean of all observations represents the output grand mean, $\bar{x}$. According to the central limit theorem, each x_i is approximately normally distributed. Hence the t statistic can be used to construct a confidence interval of the form $\bar{x} \pm t_{\alpha/2}\sigma_{\bar{x}}/\sqrt{n}$, where $\sigma_{\bar{x}}$ is the standard deviation of the x_i $(i = 1,2,\ldots,n)$ observations.

The replication method is most suitable when it is required to investigate the non-steady state operating characteristics, and will be used for this purpose in Chapter 8. However, for the steady-state conditions it has the following two disadvantages:

(1) Each replication requires an initial stabilisation period before approaching the steady state, which means that much of the simulation time is unproductive as a result of throwing out the start-up segment of each replication.

(2) It was shown by Fishman (52) that each replication contributes one degree of freedom to the variance of the mean output, regardless of how long the replication is. On the other hand, Cheng (28) proved that the mean

square error of the mean output for the replication method is greater than that for the subruns' method, which will follow immediately. Additionally, Law (102) has empirically shown that the mean of the subruns' method is superior to that obtained from the replication method.

INDEPENDENT SUBRUNS

This method is superior to the other three methods and is adopted for the SS investigations. In this approach a continuous simulation run having a length, N, consisting of n autocorrelated observations, denoted $X_1, X_2, \ldots X_n$, is divided into k consecutive subruns (blocks or batches) of length m each, such that N = km (after discarding the transient observations). In effect, this amounts to a run of length m being repeated k times, with its end state becoming the starting state of the succeeding run. This is more representative than starting each run from the same initial conditions, as was the case in the replication method, and shortens the stabilisation period.

Following that, an observation is redefined as being the average of the m observations within a block. Denoting these new observations by $\bar{X}_1, \bar{X}_2, , , , \bar{X}_k$,
where

$$\bar{X}_1 = \frac{X_1 + X_2 + \ldots + X_m}{m}$$

$$\vdots$$

$$\bar{X}_k = \frac{X_{m(k-1)+1} + X_{m(k-1)+2} + \ldots + X_n}{m}$$

The assumption is then made that the autocorrelation between the observations is positive (a reasonable assumption in most simulations (Kleijnen (93)), and decreases as the 'lag', i.e. the distance between the individual observations increases. Consequently, assuming there exist no periodicities in the data, only the first few observations of a subrun, i, will be correlated with the last observations of subrun i - 1 and, therefore, the averages $\bar{x}_i$ and $\bar{x}_{i-1}$ will show only small correlations. If the subruns are long enough, the correlations among their averages may be neglectes for practical purposes, that is, $P(s) \triangleq o$. The overall mean and the variance of the subruns' output can be estimated as follows:

$$\hat{\mu} = \sum_{i=1}^{k} \frac{\bar{X}i}{k}$$

$$\hat{\sigma}_{\mu}^{2} = \sigma_{\bar{x}}^{2}/k$$

It has been shown by Fishman (52) that if the autocorrelation co-efficient for lag 1 is nearly zero, the $\bar{x}_i$'s are considered to be independent, and if the subrun size (m) is long enough, the $\bar{x}_i$'s will then be normally distributed, according to the central limit theorem, and the statistic C_k has mean zero, variance $(k - 2)(k^2 - 1)$, and distribution that is remarkably close to the normal for k as small as 8, i.e. the required number of subruns is $k \geqslant 8$

where

$$C_k = \hat{\rho}_{1,m} + \left[(\bar{X}_{1,m} - \hat{u})^2 + (\bar{X}_{k,m} - \hat{u})^2\right] /2k\hat{R}o,m$$

where

$\hat{\rho}_{1,m}$ = the sample autocorrelation co-efficient for lag 1 and subrun length m

$\bar{X}_{i,m}$ = the mean of subrun i with length m, i=1,2...,k

$\hat{R}o,m$ = the sample variance of the subruns' means for subrun length m

Several heuristic procedures have been suggested to estimate the necessary subrun length which reduces the autocorrelation to a negligible value. Fishman (51) verified that they all require a very large number of subruns, which renders the run length needed immense in terms of the computer time. Fishman concluded that "it remains a matter of judgement to choose a suitable batch (subrun) size. The only safe procedure is to use test runs in which to try several batch sizes and test for the presence of correlation in the results".

In accordance with Fishman's suggestion, and in order to determine the best subrun length that reduces the serial correlation to zero approximately, a trial procedure which takes sequentially longer subruns was employed for an 8-station line having a B of 6 and a Covar of 0.274. For a SS period of 3000 product cycles (p.c.), i.e. 3000 times the mean service time of 10 time units (t.u.s), subrun lengths of 10,50,100,150,200,250p.c. were used, giving rise to 300,60,30,20,15,12 subruns respectively, for five different random numbers' seeds.

The subroutine (AUTO) has been used to calculate the autocorrelations between the subruns' means for both I and ABL. This subroutine computes the autocovariances according to the following formula:

$$\gamma_s = \int_{-\infty}^{\infty} (X_t - E(X_t))\,(X_{t+s} - E(X_t))\,F(X_t)dt$$

where

γ_s = the autocovariance of lag s

$$E(X_t) = \int_{-\infty}^{\infty} X_t F(X_t)$$

The autocorrelations are then obtained by dividing the autocovariances by the variances of the subruns' means, i.e. $P_s = \gamma_s / V(X_t)$.

For a fixed run length of 35000 time units (3500 product cycles), including 5000 time units for the transient period, it was found that the minimum autocorrelations of nearly 0.001 for I and 0.0 for ABL, were achieved when the subrun length was 250 product cycles (p.c.), i.e. the required number of subruns is 12. It was felt, therefore, that a simulation run of 35000 time units represents a reasonable sample size in order to generate reliable means' outputs, taking into account the constraint of computer time's availability, the large number of experiments needed in this research, and the values used by previous researchers.

Summarising, it was decided to use the following parameters' values for all the SS simulation investigations:

(1) Start the simulation run with all the buffers nearly half full.

(2) Discard all the accumulated statistics from a 500p.c. start-up period.

(3) Adopt a SS run length of 3000p.c. divided into 12 subruns of 250p.c. each, i.e. the mean values of the endogenous variables are recorded every 250p.c., and the grand mean, representing the average of these 12 mean values, is then obtained.

It should be noted that all the idle time, mean buffer level, and other results of Chapter 7 represent grand means.

SIMULATION RUN PARAMETERS - NON-STEADY STATE UNBALANCED LINES

The NSS simulation run is defined by the following four parameters, which should be considered before attempting to explore the system's characteristics during its unstable phase:

(1) The initial conditions: since the system is being investigated from the start of its operations until it approaches the SS, it seems that the most reflective state of its initial conditions is to start it with all the operators being idle and all the buffers being empty.

(2) The run length: the results of Slack (160) for the NSS balanced lines indicated that approximately 450p.c. are needed before the values of I and ABL become statistically indistinguishable from those of the SS. It was decided to set the NSS run length for the unbalanced lines' investigations at 500p.c. The reason behind this decision is

two fold: firstly, no previous knowledge of the required transient duration for the unbalanced lines, is available and, therefore, a NSS interval of the same size as that of the balanced line looks an obvious choice. Secondly, one of the objectives of the research into the line's transient characteristics is the determination of the NSS length and consequently, a transient period had to be initially chosen in order to find whether its results deviate significantly from those representing the SS.

(3) The total number of observations: in order to avoid the occurrence of autocorrelation and obtain independent observations, the decision has been taken to divide the NSS run length into two subruns of 250p.c. each. The disadvantage of using such a relatively large length of subrun, is that the first subrun will include much of the initial NSS period, which may cause the loss of some information, through aggregation, on the behaviour of the line during the early part of the transient. However, if independence is to be maintained so as to use the conventional statistical tools, the benefit of choosing a wide subrun interval far outweighs any resultant loss.

(4) The number of replications: although it was possible to utilize as little as 2 subruns and 4 replications, i.e. a total of 8 observations, in order to ascertain their normality and independence (see Fishman (51)), it was felt that 20 observations (2 subruns x 10 replications) would best represent the necessary sample size to generate valid results. It was further decided to use the same

set of 10 different random seed numbers, each of which accounts for one replication, in all the experimental cells, in order to obtain identical sequences of events for them.

THE SIMULATION MODEL - AN OVERVIEW

The simulation model used for the investigative part of this thesis has the following characteristics:

(1) It is general, in that it is not being intended to handle a specific real life system, since it is unlikely that any two real production lines will be identical in every design detail. The model, therefore, portrays the attributes of an ideal system, while incorporating all the main and important features of the basic manual line system.

(2) It is dynamic, in the sense that the system is examined in the context of successive streams of events.

(3) It is stochastic and open, in that random numbers are utilized to infer the system's behaviour during the following intervals, given a certain initial conditions' setting.

(4) It is discrete, in the sense that its variables change in a discrete fashion at some discrete points in time.

(5) It is mainly physical, portraying a physical system. Nevertheless, it is partly behavioural since the endegenous variables are generated through the interaction between a behavioural factor, i.e. operation times' variation, and physical entities,such as the line length.

(6) It is not aggregate, because only a small degree of aggregation is necessary in this model of an unsophisticated and general production system.

TIME ADVANCE

The sort of time advance mechanism to be adopted is an important decision to take, prior to the consideration of the detailed logic of any discrete simulation model. Two methods of time progression exist: event advance and unit-time advance. In the former, the line's state is examined and updated whenever an event takes place, then the 'clock' of the simulator proceeds to the next event, and so on. On the other hand, in the uniform unit-time advance approach, the states of all the entities in the line are checked and updated at regular and constant time intervals and, therefore, it is unnecessary to make a continuous check on the time of the next event. The following points were made against the use of the unit-time advance method:

(1) There are occasions when no event takes place during a unit time period, resulting in an unnecessary check being made on the system's states (Kleijnen (93)). In addition, it has been demonstrated by Slack (160) that if the mean service time is 10t.u.s and the maximum allowable unit time advance is every 0.1t.u., then the approximate time between events = the mean operation time/N, and will be in the range of 2 to 0.66t.u.s. for N = 5,10,15 respectively. Therefore, in the type of system being simulated in this thesis, where the

chosen values of N are 5,8,10 (see the next chapter), adopting the uniform time advance method will result, at best, in 6 out of 7 update checks, and at worst, 9 out of 10 such checks being redundant.

(2) It has been demonstrated by Gafarian and Ancker (60) that information is always lost, sometimes entirely, when using the unit-time advance, especially if several events occur between the recording points. This might happen, for instance, if a departure immediately follows an arrival, leaving the system in the same state. In this situation, the abovementioned method fails to find anything taking place in the system during the unit-time advance and, consequently, it may be viewed as depicting only an approximate picture of the system's state as it evolves through time. Attempts to overcome this difficulty by making the unit time interval very small are only effective for relatively simple models and at the expense of excessive computer time.

Conway et al (35) argued that this method becomes favourable when the time between events is small relative to the unit time interval, and the number of state variables is large. However, since there is only a small number of state variables in the relatively uncomplex system being employed in this thesis, it would seem that an event advance mechanism is preferable.

THE FLOWCHART AND COMPUTER PROGRAMME

Figure 5.2 exhibits a summary of the detailed flow-chart describing the basic logic of the simulation model. A listing of the computer programme, employed in executing the simulation experiments, is included in Appendix 5.2. The programme was based on one originally developed by Slack (160). However, several adaptations and amendments were made, viz, amending the random numbers' generation subroutine to increase its efficiency, creating the START statement and the major NASH loop in order to execute a batch of more than one run, amending the DIMENSION, ANTVAL, and ENTIME statements, adding the starving and blocking idle times' computation formulae, and amending the I and ABL calculation and printing statements.

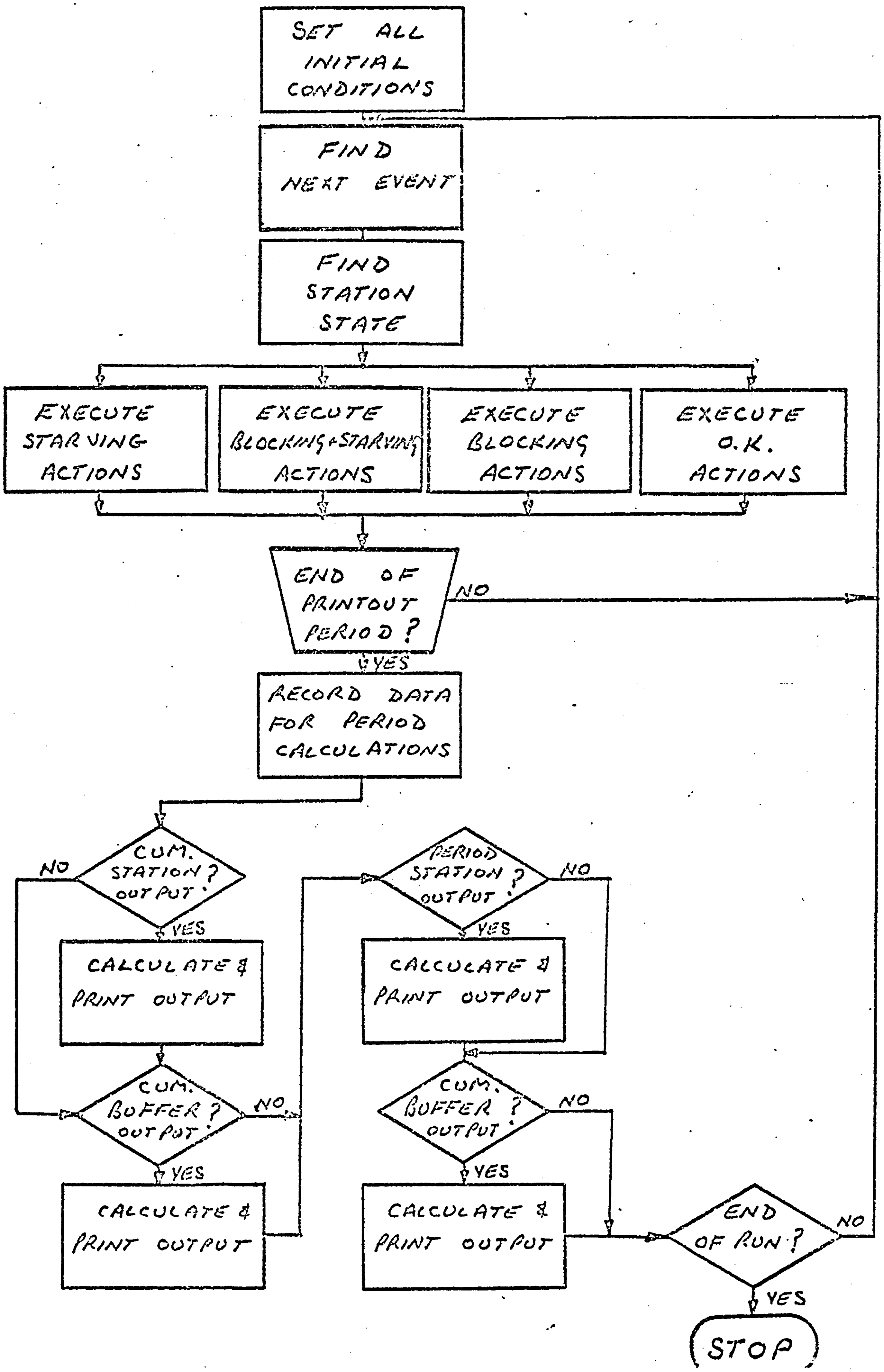
The computer programme was written in FORTRAN, to be run on an IBM 2741 computer at Sheffield City Polytechnic Computer Centre. It can be divided into three major parts: the input data, supplied directly by the experimenter to the simulator, the core which simulates and measures the behaviour of the system, and the output data which supplies an extensive amount of useful information on the endogenous variables of the system. Details of the two main 2-dimensional data storage arrays (A,(I,J) and Q(I,J)) are shown in Appendix 5.3.

In order that the stochastic operation times of the individual stations are sampled from a Weibull distribution, it is necessary to generate pseudo-random numbers, RN,

FIGURE 5.2

THE FLOWCHART OF THE SIMULATION MODEL -

FROM SLACK (160)



by applying a deterministic formula which results in numbers that, for all practical purposes, are considered to behave as true random numbers, i.e. to be also real numbers, uniformly distributed between 0.0 and 1.0, and mutually independent. This independence implies a long enough cycle length, i.e. a long sequence of pseudo RN will be generated before repeating it again. The utilized pseudo RN generator subroutine is G05 BAF, which uses the implicit multiplicative congruential method.

The pseudo RN are then transformed into random variables, representing the individual stations' service times, by the method of inverse transformation which is as follows:

The Weibull cumulative probability function = Fx

$$\text{and } Fx = 1 - \exp -\left(\frac{x-a}{b}\right)^{c}$$

$$\exp - ((x-a)/b)^{c} = 1 - Fx$$

$$\exp ((x-a)/b)^{c} = \frac{1}{1-Fx}$$

Taking the log of both sides gives:-

$$((x-a)/b)^{c} = \log \left(\frac{1}{1-Fx}\right)$$

$$(x-a)/b = \left[\log \left(\frac{1}{1-Fx}\right)\right] 1/c$$

$$\text{Therefore, } X = a + b \left[\log \left(\frac{1}{1-Fx}\right)\right] 1/c$$

MODEL ASSUMPTIONS

The following assumptions are fundamental to the understanding of the simulation model:

(1) All the parts to be processed are identical and flow in the same serial sequence through each of the stations in the line, before leaving the system altogether.

(2) Each station services only one part at a time, i.e. there is only one operator at each station.

(3) The Weibull service times of the stations are independent fron one another and from the state of the system, and they include the time to receive, position, work on, and transport the part to the succeeding buffer.

(4) There is always enough supply of parts to the first station, as well as sufficient space after the end station, i.e. the first station cannot be starved and the last one cannot be blocked. Consequently, the line as a whole cannot be stopped.

(5) Service on a new part starts immediately after becoming available at the preceding buffer. Furthermore, a part cannot be serviced until the previous part has been moved to the next buffer.

(6) Individual stations' breakdowns and maintenance works, be it major or minor, do not occur during the simulation run.

(7) No defective items are produced and, therefore, no reprocessing takes place in the line's type being simulated.

(8) A blocked station will eject the part it holds as soon as a space becomes available at the succeeding buffer. Similarly, a starved station will unstarve immediately after its preceding station releases a part to the next buffer.

(9) No labour absenteeism occurs and no interline or interstation labour transfers are allowed during the simulation run.

(10) Since all parts are identical, no service discipline for their selection is necessary.

Because the production line system being investigated is not complex, the foregoing assumptions are general and have been adopted by other researchers to fascilitate model's development. Therefore, it may be contended that the simulation model does not represent a gross departure from the real line's structure, but depicts most of its salient features.

MODEL VALIDATION

The validation of a simulation model refers to the building of an acceptable level of confidence in it and its results. To date, there is no agreement among researchers on a universal standard validation procedure and, therefore, the validation process is still very much an art. Slack (160) argued that the validation of the manual unpaced line's model is much easier than that of other models, because no real life situation is being simulated. Consequently, it is only required to, firstly, check the method of generating the random numbers, in order to make sure that reasonable service times are being generated, and, secondly, to ascertain that the model produces results which can be verified either mathematically or intuitively.

The first validation requirement can be considered as being met, since the multiplicative congruential and the inverse transformation methods are well tested and established procedures (see Naylor et al (132) for generating uniformly distributed independent pseudo RN in the interval (0.0,1.0) and then transforming them into independent random variables drawn from a particular probability density function.

In order to meet the second validation requirement, the simulation model has been executed using both deterministic and exponential balanced operation times for a 3-station line having B = 1,2 and 3 units. As expected, running the simulator with constant operation times resulted in zero I, while running it with exponential service times provided very close I's results (within 1%) to those published by Hillier and Boling (72).

Moreover, a third validation test, suggested by Mihram (118), was made in this research and showed that the simulation outcomes were repeatable for identical or similar conditions, and that when small changes were made on the exogenous variables, the endogenous variables changed only slightly.

Furthermore, a fourth validation test, suggested by Emshoff and Sisson (47), was performed and demonstrated that replicating the same conditions with different seed numbers, results in a low variance of the mean output.

The four aforementioned validation processes can be taken to consolidate the consistence, reasonability, and overall face validity of the simulation model.

PART THREE

RESEARCH INVESTIGATIONS

Part Three comprises three chapters which describe the simulation investigations together with their results, relationships, and conclusions:

CHAPTER SIX	-	UNBALANCED STEADY STATE LINES - INVESTIGATIONS
CHAPTER SEVEN	-	UNBALANCED STEADY STATE LINES - RESULTS
CHAPTER EIGHT	-	UNBALANCED NON-STEADY STATE LINES - INVESTIGATIONS AND RESULTS

CHAPTER SIX

UNBALANCED STEADY STATE LINES' - INVESTIGATIONS

INTRODUCTION

This chapter discusses the detailed tactical experimental designs for six types of investigations into the operating characteristics of the unbalanced and unpaced manual lines, working under SS conditions. These investigations may be divided into two broad categories which correspond to the presence of one or two sources of imbalance in the line, and are listed below:

(1) Investigation of one source of imbalance: which comprises three separate investigations:

- (a) The imbalance of mean service times.
- (b) The Covars' imbalance.
- (c) The buffer capacities' imbalance.

(2) Investigation of two causes of imbalance: which is composed of three distinct investigations:

- (a) The joint imbalance of both the means and Covars of the service times.
- (b) The combined unbalance in terms of both the mean service times and buffer capacities.
- (c) The simultaneous imbalance with respect to both the Covars and buffer capacities.

In addition, two more types of imbalance may be studied, viz, an imbalance whereby the skewnesses of the individual stations' Weibull operation times are unequal, and the joint imbalance of the means, Covars and buffer capacities. However, since previous results have indicated (see El-Rayah (44)) that the symmetrical and the skewed service times distributions produced similar results (not significantly different), it is expected that skewed distributions with different degrees of skewness will give rise to even closer results and, therefore, it was decided not to investigate the skewness imbalance. On the other hand, since the investigation of three sources of imbalance requires substantially more experimental cells (at least twice) than those needed to investigate two sources of imbalance, it was decided to leave this investigation out, because it would have necessitated a computer time equivalent to roughly half the total time needed to conduct all the six major unbalanced lines' studies.

OBJECTIVES

The main objectives of the aforementioned investigations are:

(1) To determine whether these investigations will lead to a general pattern of results. Unlike previous research, the present one aims at relating the major findings of each investigation with those of the other ones, in order to see if a general mode of behaviour for different types of unbalanced lines will emerge. This may constitute a significant step in the direction of obtaining more complete understanding of the unpaced manual lines' operating characteristics.

(2) To generate results which can be relied upon, from a statistical perspective, through the construction of full factorial experiments. Almost all the findings reported in the foregoing chapters were not derived from such a design methodology and, consequently, they do not maintain high degrees of statistical reliability.

(3) To examine the effects of having one and two causes of imbalance in the line, to evaluate the relative merits of various patterns of imbalance, and to find if the so called 'bowl phenomenon' leads to an improved efficiency, especially when two types of imbalance co-exist.

(4) To determine the stockholding properties for the different kinds of unbalanced lines, which were ignored by nearly all the preceding studies, and furthermore, to find if the best pattern in terms of ABL corresponds to that with respect to I.

(5) To study the previously unreported behaviour of the two I's components, namely, starving, ST, and blocking, BL, for the unbalanced lines.

(6) To test the hypothesis that I, ST, BL, ABL, and the other stockholding criteria are functions of the major design parameters N, B, DI, and the pattern of imbalance.

To achieve the objectives of this research, a series of unbalanced lines' investigations were planned, executed, and data on them were collected and analysed, making use of several statistical tools. Basically, because of the computer time limitation and the employment of the complete factorial design, which requires a larger number of cells than that of any other design, it was decided to decrease the levels of N and B, for all the investigations, to two and three respectively, so as to reduce each experiment to a manageable size, rather than decrease the levels of the other factors, such as the degree of imbalance and the Covars imbalance pattern. However, the decision has been made to obtain additional data on B and N for the best emerging pattern, in terms of I, in each investigation, so that more points on the graphs are generated.

The decision to obtain extra data for the best pattern with respect to I rather than ABL, was essentially based on two factors. First, getting more data on both I and ABL will considerably increase the number of cells, which is constrained by the limited availability of computer time. Second, I is generally regarded as more important than ABL in its impact on the line's performance. Furthermore, the limitation of computer time tempered the desire to get an extensive coverage of alternative patterns of imbalance in each of the investigations and therefore, only the obvious and rather important patterns were experimented with.

INVESTIGATION OF ONE SOURCE OF IMBALANCE

As was mentioned earlier,this first category comprises three investigations: those of the unbalanced lines in terms of their means, Covars, and buffer capacities. These investigations will be discussed in turn.

MEAN SERVICE TIMES' IMBALANCE

The aims of this investigation are two fold. First, to provide a basis for performing the later investigations on the joint effect of the mean service times' imbalance and each of the Covars and buffer capacities' imbalances. Second, to compliment the work of El-Rayah (46) who has not provided any results relevant to ST, BL, ABL, and the other stockholding measures. Clearly, these measures, especially ABL, are important enough to be generated and it is the objective of this investigation to extend the study of El-Rayah.

Each station in this investigation has the same Covar of 0.274, and all the buffers are balanced with regard to their capacities. The particular values of the four design variables are as follows:

- Line length : 5,8 (and 10 for the best pattern).
- Buffer capacity : 1,2,6 (and 3 for the best pattern).
- Balancing loss % : 2,5,12,18.
- Means' imbalance pattern :
  - (1) Monotone increasing order (/).
  - (2) Monotone decreasing order (\).
  - (3) The bowl phenomenon (V).
  - (4) An inverted bowl (Λ).
  - (5) Random arrangement (for N=5 and for N=8).

Number of experimental cells: 2 x 3 x 4 x 5 = 120 basic + 24 additional = 144.

Figure 6.1 shows the experimental design.

FIGURE 6.1

FULL FACTORIAL DESIGN - SERVICE TIMES MEANS' IMBALANCE

	LINE LENGTH	5												8											
	BUFFER CAPACITY	1				2				6				1				2				6			
	DEGREE OF MEANS IMBALANCE (%)	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18
PATTERN OF MEANS IMBALANCE	MONOTONE INCREASING ORDER (/)																								
	MONOTONE DECREASING ORDER (\)																								
	RANDOM ORDER																								
	AN INVERTED BOWL (Λ)																								
	THE BOWL PHENOMENON (V)																								

Of the many possible monotone increasing and decreasing mean service times' patterns, such as , , and , only those depicting strictly straight lines, i.e. (/),(\) were chosen, because the exhaustive exploration of all the possible designs is obviously impracticable and therefore, beyond the capacity of this thesis. Moreover, experimenting with a bowl pattern having the shape of (V) rather than (✓), (–✓), or any other alternative version, is preferable according to the symmetry property which states that the efficiency of the line is maximised if pattern (V) is used. By the same token, only an inverted bowl pattern of the shape (Λ) rather than (^\), (–Λ), or any other possible version, has been considered.

At this stage, it is appropriate to explain the methods used to calculate the individual stations' mean operation times for the abovementioned modes of imbalance. In order to determine the values of the Weibull parameters a and b, the assumption has been made that the value of parameter c is constant and = 1.6. The standard deviation and the mean of the standardised Weibull distribution are:

$$\text{standard deviation} = d = \sqrt{\Gamma(1 + 2/c) - \Gamma^{2}(1 + 1/c)}$$

$$\text{mean} = m = \Gamma\left(1 + \frac{1}{c}\right)$$

When a and b are introduced:

$$\text{standard deviation} = bd$$

$$\text{mean} = a + bm$$

$$\text{Covar} = K = \frac{\text{[illegible]}}{\text{mean}}$$

$$K = \frac{bd}{a + bm} = \frac{d}{\frac{a}{b} + m}$$

$$\frac{a}{b} = \frac{d - Km}{K} = Q \ldots\ldots\ldots\ldots\ldots (1)$$

To determine the values of a and b, another equation should be introduced. If the mean service time is 10t.u.s then:

$$a + bm = 10 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From equations (1) and (2):

$$\frac{am}{10 - a} = Q$$

$$am = 10Q - 10a$$

$$a = \frac{10Q}{m + Q} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

From equations (2) and (3):

$$b = \left(10 - \frac{10Q}{m + Q}\right)\frac{1}{m}$$

$$b = \frac{10}{m + Q} \ldots\ldots\ldots\ldots\ldots (4)$$

If c = 1.6, then:

$$m = 0.8966 \text{ and } d = 0.5738$$

And if $K = 0.27$:

$$Q = \frac{0.5738 - (0.27 \times 0.8966)}{0.27} = 1.2285$$

Therefore, from equations (3) and (4): $a = 5.780 = 10 \times 0.5780$

$b = 4.702 = 10 \times 0.4702$

In the same manner, it is possible to obtain the values of a and b when the mean service time $\neq 10$ and the line is unbalanced, i.e. $a = \text{mean} \times 0.5780$

$b = \text{mean} \times 0.4702$

Since the first pattern of imbalance reflects an ascending order of mean service times, the most convenient function to describe it will be linear.

Figure 6.2 shows such a function. The overall mean service time, M, for the line as a whole, and the 'imbalance parameter', X, define the mean service time, Sn, at station n in the following way:

The general straight line's equation is:

$$\frac{S - S_1}{n - n_1} = \frac{S_2 - S_1}{n_2 - n_1}$$

Substituting in this equation gives:

$$\frac{Sn - M}{n - \frac{N+1}{2}} = \frac{M + XM - M}{N - \frac{N+1}{2}}$$

$$\frac{S_n - M}{\frac{2n - N - 1}{2}} = \frac{XM}{\frac{2N - N - 1}{2}}$$

$$\frac{2(S_n - M)}{2n - N - 1} = \frac{2\ XM}{N - 1}$$

$$2S_n - 2M = \frac{2XM(2n - N - 1)}{N - 1}$$

$$2S_n = \frac{2XM(2n - N - 1)}{N - 1} + 2M$$

$$= \frac{2XM(2n - N - 1) + 2M(N - 1)}{N - 1}$$

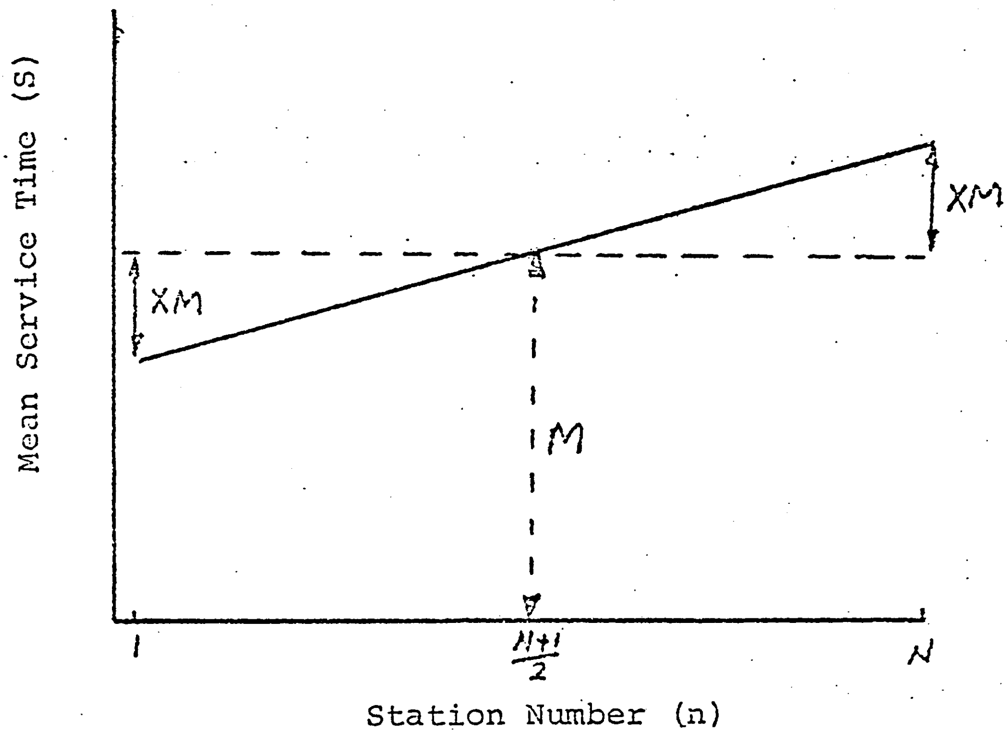
$$= \frac{2\left[XM(2n - N - 1) + M(N - 1)\right]}{N - 1}$$

$$S_n = \frac{XM(2n - N - 1) + M(N - 1)}{N - 1}$$

$$= \frac{M\left[X(2n - N - 1) + (N - 1)\right]}{N - 1}$$

FIGURE 6.2

THE MONOTONE INCREASING ORDER OF
MEAN SERVICE TIMES



M = overall mean service time
X = Imbalance parameter

$$= M\left[X\left(\frac{2n - N - 1}{N - 1}\right) + 1\right]$$

$$= M\left[X\left(\frac{2n - 2 - N + 1}{N - 1}\right) + 1\right]$$

$$= M\left[X\left(\frac{2n - 2}{N - 1} - 1\right) + 1\right]$$

$$\therefore S_n = M\left[X\left(\frac{2}{N - 1}(n - 1) - 1\right) + 1\right]$$

Note that since the value of M is a function of both the relative weight (location) of each station and the total number of stations, therefore, it does not depend on whether the end station's number is even or odd. Hence, the equation of S_n is general, irrespective of N. Furthermore, if N is odd, the middle of the line will be taken by station $\frac{N + 1}{2}$, whereas if N is even, the middle of the line is represented by stations $\frac{N}{2}$, $\frac{N}{2} + 1$, and will lie in between them.

The manner in which the balancing loss is related to X was derived by Slack (160) as follows:

$$\text{balancing loss} = \frac{NS_m - S_n}{NS_m}$$

where

S_m = maximum mean service time

S_n = mean service time at station n.

$$\text{balancing loss} = \frac{NS_m - NM}{NS_m}$$

$$= \frac{S_m - M}{S_m}$$

$$\text{Since } S_m = M(1 + X)$$

$$\therefore \text{ balancing loss} = \frac{M(1 + X) - M}{M(1 + x)} = \frac{(1 + X) - 1}{(1 + X)}$$

$$\text{balancing loss} = \frac{X}{1 + X}$$

Table 6.1 shows the values of the individual stations' mean service times, together with their corresponding Weibull parameters a and b, for pattern (/), N = 5,8, c = 1.6, and X = 0.02, 0.53, 0.136, 0.220 (i.e. for balancing losses of 0.02, 0.05, 0.12, 0.18). These values can be obtained for pattern (\) by reversing their order in pattern (/), or by the substitution of -X in place of X in the S_n equation.

The third imbalance pattern representing the bowl phenomenon is shown in Figure 6.3. This pattern is viewed as being composed of two straight lines' segments, namely, L_1 and L_2. The mean service times of the individual stations were determined as follows:

The equation of the straight line is: $\dfrac{S - S_1}{n - n_1} = \dfrac{S_2 - S_1}{n_2 - n_1}$

The line L_1 is identified by the two points A and B. Substituting, we get:

$$\frac{S_1 - M(1 + X)}{n - 1} = \frac{M(1 - X) - M(1 + X)}{\frac{N + 1}{2} - 1} = \frac{M - XM - XM - M}{\frac{N + 1 - 2}{2}}$$

$$S_1 - M(1 + X) = \frac{-4XM}{N - 1}(n - 1)$$

$$\therefore S_1 = \frac{-4XM}{N - 1}(n - 1) + M(1 + X) \quad \ldots\ldots.(1)$$

MEAN SERVICE TIMES AND PARAMETERS a AND b

FOR PATTERN (/) WITH MEDIUM COVARS

Line length = 5, X = 0.020, c = 1.60

STATION NUMBER	MEAN OPT	a	b
1	9.8000	5.6644	4.6080
2	9.9000	5.7222	4.6550
3	10.0000	5.7800	4.7020
4	10.1000	5.8378	4.7490
5	10.2000	5.8956	4.7960

Line length = 5, X = 0.053, c = 1.60

1	9.4700	5.4737	4.4528
2	9.7350	5.6268	4.5774
3	10.0000	5.7800	4.7020
4	10.2650	5.9332	4.8266
5	10.5300	6.0863	4.9512

Line length = 5, X = 0.136, c = 1.60

1	8.6400	4.9939	4.0625
2	9.3200	5.3870	4.3823
3	10.0000	5.7800	4.7020
4	10.6800	6.1730	5.0217
5	11.3600	6.5661	5.3415

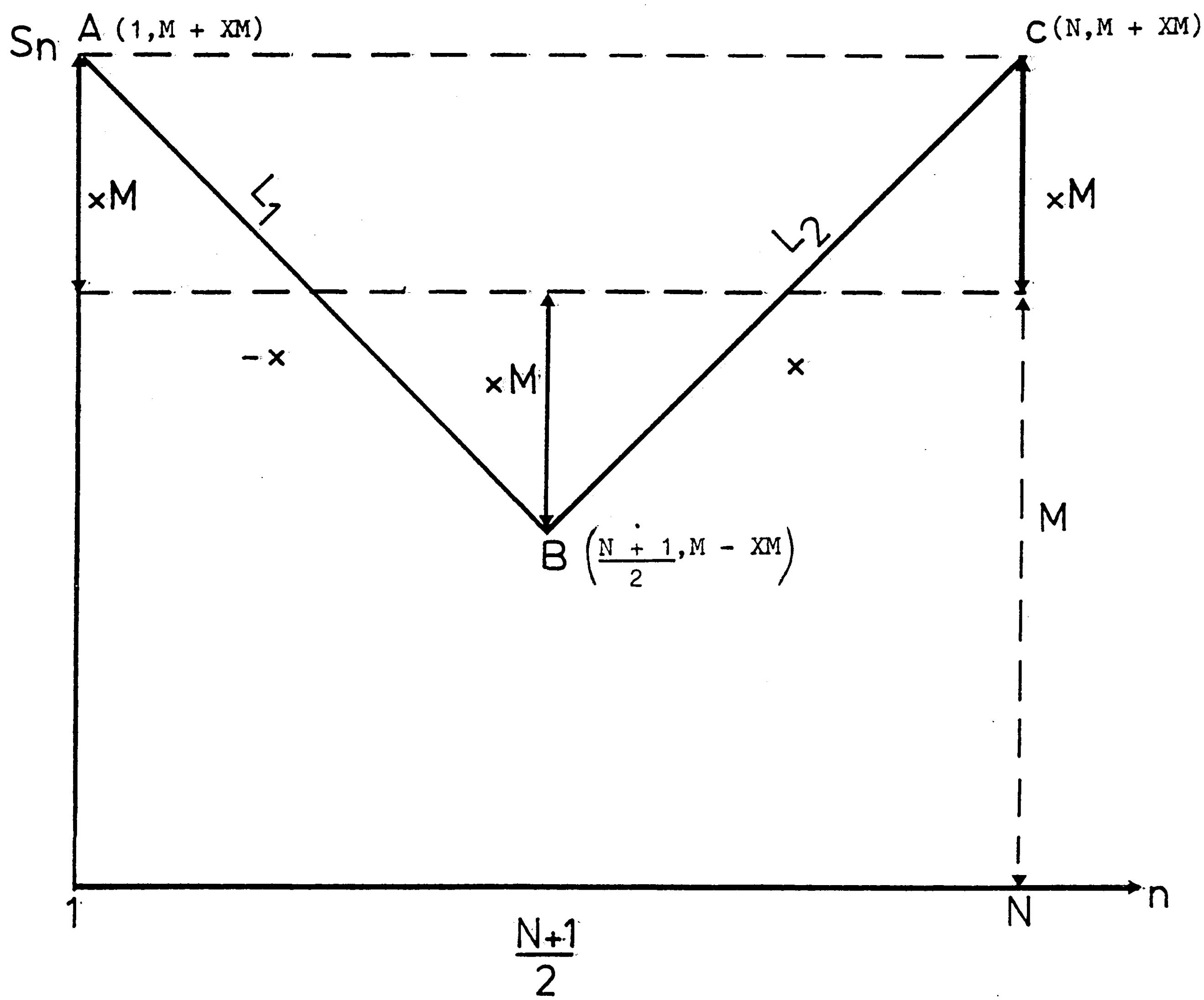
Line length = 5, X = 0.220, c = 1.60

1	7.8000	4.5084	3.6676
2	8.9000	5.1442	4.1848
3	10.0000	5.7800	4.7020
4	11.1000	6.4158	5.2192
5	12.2000	7.0516	5.7364

STATION NUMBER	MEAN OPT	a	b
L = 8,	X = 0.020,	c = 1.60	
1	9.8000	5.6644	4.6080
2	9.8571	5.6974	4.6348
3	9.9143	5.7305	4.6617
4	9.9714	5.7635	4.6886
5	10.0285	5.7965	4.7154
6	10.0856	5.8295	4.7423
7	10.1427	5.8625	4.7691
8	10.1998	5.8955	4.7960
L = 8,	X = 0.053,	c = 1.60	
1	9.4700	5.4737	4.4528
2	9.6214	5.5612	4.5240
3	9.7728	5.6487	4.5952
4	9.9242	5.7362	4.6664
5	10.0756	5.8237	4.7376
6	10.2270	5.9112	4.8087
7	10.3784	5.9987	4.8799
8	10.5298	6.0862	4.9511
L = 8,	X = 0.136,	c = 1.60	
1	8.6400	4.9939	4.0625
2	9.0286	5.2185	4.2453
3	9.4172	5.4431	4.4280
4	9.8058	5.6678	4.6107
5	10.1944	5.8924	4.7934
6	10.5830	6.1170	4.9761
7	10.9716	6.3416	5.1589
8	11.3602	6.5662	5.3416
L = 8,	X = 0.220,	c = 1.60	
1	7.8000	4.5084	3.6676
2	8.4286	4.8717	3.9631
3	9.0572	5.2351	4.2587
4	9.6858	5.5984	4.5543
5	10.3144	5.9617	4.8498
6	10.943	6.3251	5.1454
7	11.5716	6.6884	5.4410
8	12.2002	7.0517	5.7365

FIGURE 6.3

THE BOWL PHENOMENON



where

M = overall mean service time

X = imbalance parameter

Equation (1) may be used to obtain the mean service times for the first half of the line, i.e. for stations 1 to $\frac{N+1}{2}$.

The line L_2, on the other hand, is defined by points B and C. Substituting, we get:

$$\frac{S_2 - M(1 - X)}{n - \left(\frac{N+1}{2}\right)} = \frac{M(1 + X) - M(1 - X)}{N - \left(\frac{N+1}{2}\right)}$$

$$\frac{S_2 - M(1 - X)}{\frac{2n - N - 1}{2}} = \frac{M + XM - M + XM}{\frac{2N - N - 1}{2}}$$

$$S_2 - M(1 - X) = \frac{2XM}{N - 1}(2n - N - 1)$$

$$\therefore\ S_2 = \frac{2XM(2n - N - 1)}{N - 1} + M(1 - X) \ \ldots\ldots\ldots\ldots (2)$$

Equation (2) may be used to determine the service times' means for the second half of the line, i.e. for stations $\frac{N+1}{2}$ to N.

At $n = \frac{N+1}{2}$, substituting in equations (1) and (2) gives:

$$S_1 = S_2 = M(1 - X)$$

This means that the equations of L_1 and L_2 are interchangeable, therefore, either of them may be used to determine the mean service times of the other line, making use of the symmetry property. In the same manner, it is possible to obtain the individual mean service times for the inverted bowl design (Λ). Note that the total area of imbalance for the bowl phenomenon (the large triangle) is composed of two equal parts (the two smaller triangles of L_1 and L_2). Consequently, the degree of imbalance (and hence the value of X) for each half should be $\frac{1}{2}\left(\frac{X}{1+X}\right)$. The same is true for the inverted bowl pattern.

The mean service times of the individual stations, along with their corresponding a and b values, for the bowl phenomenon and various N's and X's, are presented in Table 6.2. These values may be obtained for the inverted bowl pattern by reversing their order in the bowl pattern or, alternatively, by substituting X in place of -X in equation (1) and -X in place of X in equation (2).

Observe that the total work content for both patterns (V) and ($\wedge$) is equal to that of patterns (/), (\) and the random pattern. Since the (V) and ($\wedge$) patterns consist of two halves, i.e. (\) and (/), the condition was imposed that each half has an equal amount of work and a mean service time of nearly 10t.u.s, whatever the value of N and DI, and hence the overall mean service time for the whole line will be about 10t.u.s. Observe also that, for the same reason as that for patterns (/), (\) the equations of L_1 and L_2 are general and useful for both odd and even numbers of stations. In the case of an odd N the middle position will be taken by station $\frac{N+1}{2}$, and for an even N the middle position will lie in between the two middle stations.

MEAN SERVICE TIMES AND PARAMETERS a AND b FOR THE BOWL PHENOMENON PATTERN WITH MEDIUM COVARS

L = 5, X = 0.020, c = 1.60

STATION NUMBER	MEAN OPT	a	b
1,5	10.1	5.8378	4.7490
2,4	10.0	5.7800	4.7020
3	9.9	5.7222	4.6550

L = 5, X = 0.053, c = 1.60

1,5	10.26	5.9303	4.8242
2,4	10.00	5.7800	4.7020
3	9.74	5.6297	4.5798

L = 5, X = 0.136, c = 1.60

1,5	10.68	6.1730	5.0217
2,4	10.00	5.7800	4.7020
3	9.32	5.3870	4.3823

L = 5, X = 0.220, c = 1.60

1,5	11.1	6.4158	5.2192
2,4	10.0	5.7800	4.7020
3	8.9	5.7222	4.6550

L = 8, X = 0.020, c = 1.60

1,8	10.1000	5.8378	4.7490
2,7	10.0429	5.8048	4.7222
3,6	9.9857	5.7717	4.6953
4,5	9.9286	5.7387	4.6684

L = 8, X = 0.053, c = 1.60

1,8	10.2600	5.9303	4.8242
2,7	10.1114	5.8444	4.7544
3,6	9.9628	5.7585	4.6845
4,5	9.8142	5.6726	4.6146

TABLE 6. 2 (Cont)

L = 8, X = 0.136, c = 1.60

STATION NUMBER	MEAN OPT	a	b
1,8	10.6800	6.1730	5.0217
2,7	10.2914	5.9484	4.8390
3,6	9.9028	5.7238	4.6563
4,5	9.5142	5.4992	4.4736

L = 8, X = 0.220, c = 1.60

1,8	11.1000	6.4158	5.2192
2,7	10.4714	6.0525	4.9237
3,6	9.8428	5.6891	4.6281
4,5	9.2142	5.3258	4.3325

L = 10, X = 0.020, c = 1.60

1,10	10.1000	5.8378	4.7490
2,9	10.0556	5.8121	4.7281
3,8	10.0112	5.7865	4.7073
4,7	9.9668	5.7608	4.6854
5,6	9.9224	5.7352	4.6655

L = 10, X = 0.053, c = 1.60

1,10	10.2600	5.9303	4.8242
2,9	10.1444	5.8635	4.7699
3,8	10.0288	5.7967	4.7155
4,7	9.9132	5.7298	4.6612
5,6	9.7976	5.6630	4.6068

L = 10, X = 0.136, c = 1.60

1,10	10.6800	6.1730	5.0217
2,9	10.3778	5.9984	4.8796
3,8	10.0756	5.8237	4.7276
4,7	9.7734	5.6490	4.5955
5,6	9.4712	5.4744	4.4534

L = 10, X = 0.136, c = 1.60

1,10	11.1000	6.4158	5.2192
2,9	10.6111	6.1332	4.9893
3,8	10.1222	5.8506	4.7595
4,7	9.6333	5.5681	4.5296
5,6	9.1444	5.2855	4.2997

The individual mean service times for the random imbalance pattern have the same values as those for pattern (/), but these values, given DI, were distributed randomly among the stations, such that for N = 5, the mean service times' order: 1,2,3,4,5 in pattern (/) becomes 5,4,2,3,1 in the random pattern, whereas for N = 8, the order 1,2, 3,4,5,6,7,8 in pattern (/) becomes 6,8,5,4,2,3,1,7 in the random pattern. It is clear that the random patterns for N = 5,8 are not exactly identical and it seems that there is no feasible way to equate them. Therefore, when the statistical techniques are employed to analyse the simulation outputs, these unidentical patterns should be taken into consideration, i.e. a separate statistical analysis should be conducted on each line length's value.

INDIVIDUAL STATIONS' COVARS IMBALANCE

The objectives of this investigation are, firstly, to obtain a basis for conducting the investigations concerning the combined effect of the Covars and either of the means or the buffer capacities' imbalances, secondly, to compare the I's results of this research with those of De La Wyche and Wild (39), using the same general policies, and thirdly, to get results on stockholding, ST, and BL, which were not supplied by De La Wyche and Wild.

This investigation differs from that of El-Rayah (45) in that it uses the three Covars' values of 0.08, 0.27, 0.50 reflecting relatively steady, medium, and variable stations, rather than two values. In practice, the line designer is faced with a wide range of individual operators' Covars, since it is more likely that more than two levels of operators' speed variability will exist in the lines. However, increasing the range of variability to more than 3 levles, increases the number of feasible patterns in all the investigations containing Covars and this, in turn, raises the number of calls in each investigation.

In this investigation, each station has the same mean service time value of 10 time units and all the buffers are equal in terms of their capacity. The exogenous variables and their levels are as follows:

Line length	-	5, 8 (and 10 in the case of the best Covars pattern).
Buffer capacity	-	1, 2, 6 (and 3 for the best Covars pattern).
Policy of Covars imbalance	-	four policies were considered:

(1) Separating the variable stations from each other by steadier stations. This policy reflects various forms of Patterson's conjecture (refer to Chapter 3). In this case there are 3! or 6 possible permutations, giving rise to six patterns.

(2) Concentrating the steadiest stations towards the line's centre (i.e. the bowl phenomenon with regard to Covars imbalance). 2! or 2 possible patterns constitute this policy.

(3) The most varialbe stations are concentrated at the line's centre (the opposite of the Covars' bowl arrangement representing an inverted bowl shape). This policy comprises 2! or 2 patterns.

(4) The stations having medium variability are concentrated at the middle of the line. This policy represents both the increasing and the decreasing orders of Covars along the line and contains 2!, i.e. 2 feasible patterns.

It should be noted that there are several other alternative basic designs for each policy, of which only the aforementioned ones were considered, the reason being the impracticability of examining all the alternative possibilities of the Covars imbalance. Examples of such alternative patterns include MSSSSSSV, MMSSSSSV and MSSSSVVV for N = 8, which represent some feasible variations within policy 2. Note also that the number of alternative designs rises when N is increased and that it was not possible in this research to keep the same total amount of variability in the line for all the policies (a condition which was observed in El-Rayah's (45) study), e.g. the total magnitude of variability for pattern VSMVS (1.43) is $\neq$ that of pattern VSSSM (1.01). This is advantageous since constraining the total Covars' value is likely to reduce the number of possible policies and their corresponding patterns unnecessarily, resulting in more unpractical designs.

Figure 6.4 exhibits the experimental design for the Covars imbalance, where policies 1 - 4 are represented, respectively, by patterns 1 - 6, 7 - 8, 9 - 10, 11 - 12. Note that patterns 11 and 12 are some forms of the descending and

FIGURE 6.4

FULL FACTORIAL DESIGN - SERVICE TIMES' COVARS IMBALANCE

S = RELATIVELY STEADIER COVAR (COVAR = 0.08), M = MEDIUM COVAR (COVAR = 0.27), V = RELATIVELY MORE VARIABLE COVAR (COVAR = 0.50)

LINE LENGTH (N)			5			8		
BUFFER CAPACITY			1	2	6	1	2	6
PATTERN OF COVARS IMBALANCE (P_i)	P_1	= VMSVM (N=5) = VMSVMSVM (N=8)						
	P_2	= MSVMS (N=5) = MSVMSVMS (N=8)						
	P_3	= VSMVS (N=5) = VSMVSMVS (N=8)						
	P_4	= MVSMV (N=5) = MVSMVSMV (N=8)						
	P_5	= SMVSM (N=5) = SMVSMVSM (N=8)						
	P_6	= SVMSV (N=5) = SVMSVMSV (N=8)						
	P_7	= VSSSM (N=5) = VVSSSSMM (N=8)						
	P_8	= MSSSV (N=5) = MMSSSSVV (N=8)						
	P_9	= SVVVM (N=5) = SSVVVVMM (N=8)						
	P_{10}	= MVVVS (N=5) = MMVVVVSS (N=8)						
	P_{11}	= VMMMS (N=5) = VVMMMMSS (N=8)						
	P_{12}	= SMMMV (N=5) = SSMMMMVV (N=8)						

ascending Covars' sequences respectively. All in all, 2 x 3 x 12 = 72 basic + 6 extra = 78 different cells were considered.

The values of the Weibull parameters a and b in this investigation were derived from equations (3) and (4) (page 230), after accounting for the change in Q, and are given below:-

Covar (K)	a	b	Relative Variability
0.08	8.7500	1.3942	S (steady)
0.27	5.7800	4.7020	M (medium)
0.50	2.1874	8.7138	V (variable)

It should be noted that the Covars imbalance patterns for N = 5 are not exactly the same as those for N = 8, which in turn, are unidentical to those for N = 10 and hence, separate statistical analyses should be applied to each of the N's levels used.

UNEQUAL BUFFER SIZES' IMBALANCE

This third simulation investigation has three intended goals. Fristly, to provide information which can be used as a basis for studying the combined effect of the buffer sizes' imbalance, together with the means or the Covars' imbalances. Secondly, to make a general comparison with the results of De La Wyche and Wild (39) (where the patterns of imbalance agree), and to consider the effects of other patterns on the line's idle time and mean buffer level. Thirdly, to provide additional endogenous variables' data, i.e. ST, BL, L, SU, BU which were not hitherto reported.

It was decided not to use a total buffer capacity value which will result in zero buffer capacity assignments in one or more buffers. This was clearly shown in (39) to be a very bad design in terms of I. Therefore, a mean buffer capacity, MB, of unity (MB = the total buffer capacity for the whole line (TB) divided by the number of buffers), was not selected although it may be desirable to provide a basis whereby it can be compared with the corresponding B = 1 in the other investigations.
The above consideration was also observed when designing the means and buffers, and the Covars and buffers imbalanced line investigations.

All the stations in this investigation have values of mean service times and Covars of 10 time units and 0.274 respectively. In addition, the particular factors and their levels are as follows:

Line length	-	5 and 8
Total buffer capacity	-	8, 24 (for N = 5), and 14, 42 (for N = 8), giving rise to MB = 2,6 for N = 5,8.

An additional MB level, i.e. MB = 4, was obtained for the best pattern with respect to both I and ABL, in order to increase the number of the MB levels to three. The decision to utilize MB of 4 rather than 3 derives from the desirability to have MB values which are multiples of the initial MB value of 2, i.e. 4 and 6, the reason being that MB = 3 will result in several patterns which resemble, in their general direction and form (but unidentical to) that of MB = 2. This will add another source of dissimilarity between the shapes of the patterns for N = 5 and N = 8, to that which already exists, whereas MB = 4 produces patterns that are more similar to those of MB = 2.

Total buffer capacity allocation's policy: four policies were investigated:

(1) Concentrating the available buffer capacity at the end of the line. This portrays an increasing order of B of some form (not a straight line's arrangement).

(2) The available buffer capacity is concentrated towards the middle of the line. This policy depicts a bowl shaped B's sequence of some form.

(3) The total buffer capacity is amassed at the beginning of the line. This policy describes a descending B's sequence of some form.

(4) No concentration of TB. This policy is divided into three main sub-policies:

(a) General.

(b) Alternating B between high and low along the line (some form of Patterson's conjecture).

(c) The least B is positioned towards the centre (some form of an inverted bowl pattern).

A total of 2 x 2 x 15 = 60 basic + 4 extra = 64 cells were simulated.

Figure 6.5 displays the experimental design for the imbalance of buffer capacities. In this design, policies 1 through 4 are represented, respectively, by A_1-A_3, B_1-B_3, C_1-C3, D_1-D_6.

LINE LENGTH	5		8	
MEAN BUFFER CAPACITY	2	6	2	6
PATTERN OF BUFFER CAPACITIES IMBALANCE				
A_1	1,1,1,5	3,3,3,15	1,1,1,1,1,1,8	3,3,3,3,3,3,24
A_2	1,1,2,4	3,3,6,12	1,1,1,1,6,2,2	3,3,3,3,18,6,6
A_3	1,1,3,3	3,3,9,9	1,1,1,1,4,4,2	3,3,3,3,12,12,6
B_1	1,1,5,1	3,3,15,3	1,1,1,8,1,1,1	3,3,3,24,3,3,3
B_2	1,2,4,1	3,6,12,3	1,1,6,2,2,1,1	3,3,18,6,6,3,3
B_3	1,3,3,1	3,9,9,3	1,1,4,4,2,1,1	3,3,12,12,6,3,3
C_1	5,1,1,1	15,3,3,3	8,1,1,1,1,1,1	24,3,3,3,3,3,3
C_2	4,2,1,1	12,6,3,3	6,2,2,1,1,1,1	18,6,6,3,3,3,3
C_3	3,3,1,1	9,9,3,3	4,4,2,1,1,1,1	12,12,6,3,3,3,3
D_1	2,2,3,1	6,6,9,3	2,2,2,3,3,1,1	6,6,6,9,9,3,3
D_2	2,3,2,1	6,9,6,3	2,2,3,3,2,1,1	6,6,9,9,6,3,3
D_3	2,1,3,2	6,3,9,6	2,2,1,1,3,3,2	6,6,3,3,9,9,6
D_4	1,3,2,2	3,9,6,6	1,1,3,3,2,2,2	3,3,9,9,6,6,6
D_5	3,1,3,1	9,3,9,3	3,2,1,3,2,1,2	9,6,3,9,6,3,6
D_6	3,1,1,3	9,3,3,9	4,2,1,1,3,2,1	12,6,3,3,9,6,3
TOTAL BUFFER CAPACITY	8	24	14	42

FIGURE 6.5

FULL FACTORIAL DESIGN - SERVICE TIMES BUFFER CAPACITIES IMBALANCE

Sub-policies a - c of policy 4 are exhibited by D_1-D_4, D_5, D_6 respectively.

It must be noted that the imbalance degree of the buffer capacities' distribution for each pattern was not estimated, and that several alternative basic designs for each policy exist, of which only the subset used in this investigation was selected, in order that the number of alternatives is reduced to avoid an exhaustive exploration of all the existing possibilities, which is beyond the capacity of this work. Two such alternative patterns can be: 4,5,10,5 and 2,2,18,2 for policy 2 and N = 5. Note also that such alternative designs appreciably increase in number when there is an increase in N and/or MB. Observe also that for N = 5, only patterns B_3, D_1, D_4, D_6 are exactly the same as those of De La Wyche and Wild (39), and for N = 8 all the patterns of the two sets are nonidentical.

Again, since the patterns of the buffer capacities' imbalance for N = 5 are not exactly the same as their counterparts for N = 8, (in fact the differences between them are greater than those of the Covars imbalance patterns), the desired statistical procedures will be made separately on both N's levels. Moreover, to avoid the effect of having substantially dissimilar patterns of buffer capacities for the different N's values, it was decided not to obtain another level of N for the best unbalanced buffers' pattern.

INVESTIGATION OF TWO SOURCES OF IMBALANCE

This investigation is composed of three types of unbalanced lines' investigations, viz, the joint imbalance of the means and the Covars, the combined imbalance of the means and the buffer capacities, and the simultaneous imbalance of the Covars and the buffer capacities. A discussion on each of these investigations is presented below.

SERVICE TIMES' MEANS AND COVARS JOINT IMBALANCE

This investigation has the objective of studying the operating characteristics of lines that have two sources of imbalance at the same time, i.e. both the means and the Covars of the service times were allowed to differ among the stations, while all the buffers were balanced insomuch as each buffer has the same capacity. The motivation behind conducting this investigation lies in the lack of knowledge about the behaviour of such lines. El-Rayah (45) wrote: "a logical extension...is the consideration of the combined effect of various forms of imbalance, e.g. where some degree of imbalance exists both between mean operation times and their variability.. represents a fruitful area for future research". On the other hand, Carnall and Wild (26) said "If the results (of unbalancing the stations' mean service times and of unbalancing their Covars) are additive where the causes coexist, then the performance gain...may be significant". Such an investigation has not since been reported. It is hoped that this particular investigation will fill in some of the gaps in this area of the unbalanced manual flow lines.

Below is a summary of the design factors and the relevant levels:

Line length	-	5,8 (and 10 for the case of the best means imbalance pattern).
Buffer capacity	-	1,2,6 (and 3 for the best pattern of means imbalance).
Balancing loss %	-	2,5,12.
Pattern of means imbalance	-	the same as patterns 1-4 in the means imbalance investigation.
Policy of Covars imbalance	-	the same as policies 1-4 (and their patterns) in the Covars imbalance investigation.

The complete experimental design is given in Figures 6.6 and 6.7. In both these figures note that the Covars' patterns 1 through 8 were previously labelled as $P_2, P_1, P_5, P_7, P_8, P_{11}, P_{12}, P_{10}$ respecitvely, in the unbalanced Covars' investigation.

In this research, a decision had to be made concerning the experimental design's size. Recall that in the unbalanced means' investigation, five patterns of means imbalance and four values of DI were considered. Furthermore, in the Covars unbalance investigation, twelve Covars patterns were examined. Obviously, to include in this investigation all the patterns of the means and the Covars investigations, together with all the levels of DI, B, and N, would have required 1440 cells which, no doubt, is greatly excessive and impracticable and, had it been carried out, would have been at the expense of the two latter investigations.

FIGURE 6.6

FULL FACTORIAL DESIGN - SERVICE TIMES MEANS & COVARS IMBALANCE - (PATTERNS (/)&(\))

PATTERN OF MEANS IMBALANCE	PATTERN OF COVARS IMBALANCE (P_i)																			
	LINE LENGTH (N)	5									8									
	BUFFER CAPACITY	1			2			6			1			2			6			
	DEGREE OF MEANS IMBALANCE (%)	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	
(/)	P_1																			
(/)	P_2																			
(/)	P_3																			
(/)	P_4																			
(/)	P_5																			
(/)	P_6																			
(/)	P_7																			
(/)	P_8																			
(\)	P_1																			
(\)	P_2																			
(\)	P_3																			
(\)	P_4																			
(\)	P_5																			
(\)	P_6																			
(\)	P_7																			
(\)	P_8																			

(/) = MONOTONE INCREASING MEANS ORDER, (\) = MONOTONE DECREASING MEANS ORDER.

FOR N=5 : P_1 = MSVMS, P_2 = VSMVS, P_3 = SMVSM, P_4 = VSSSM, P_5 = MSSSV, P_6 = VMMMS, P_7 = SMMMV, P_8 = MVVVS

FOR N=8 : P_1 = MSVMSVMS, P_2 = VSMVSMVS, P_3 = SMVSMVSM, P_4 = VVSSSSMM, P_5 = MMSSSSVV, P_6 = VVMMMMSS, P_7 = SSMMMMVV, P_8 = MMVVVVSS.

LINE LENGTH		5									8									
BUFFER CAPACITY		1			2			6			1			2			6			
DEGREE OF MEANS IMBALANCE (%)		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	PATTERN OF MEANS IMBALANCE
PATTERN OF COVARS IMBALANCE (P_i)	P_1																			
	P_2																			
	P_3																			
	P_4																			(V)
	P_5																			
	P_6																			
	P_7																			
	P_8																			
	P_1																			
	P_2																			
	P_3																			
	P_4																			(Λ)
	P_5																			
	P_6																			
	P_7																			
	P_8																			

FIGURE 6.7

FULL FACTORIAL DESIGN - SERVICE TIMES MEANS & COVARS IMBALANCE - (PATTERNS (V)&(Λ))

Nevertheless, due to the importance attached to this particular investigation, the decision has been made to allocate it more computer time than any of the other investigations. Therefore, in order to reduce the number of experimental cells to an acceptable size, in view of the constraint imposed by the computer time limitation, the levels of the degree of imbalance were reduced to three, and the 18% balancing loss was dropped because it resulted in a high amount of I, as will be shown in the next chapter. In addition, the decision has been taken not to use the random pattern of the means imbalance investigation, since it did not differ sizably from patterns (/) and (\) with regard to I, and consequently, the inclusion of this pattern would unnecessarily increase the constrast among patterns, due to the fact that its shape for N = 5 does not exactly match that for N = 8.

As far as the patterns of the Covars imbalance are concerned, a decision had to be made on their reduction. Consequently, the number of patterns for policy 1 were reduced from six to three. Of these six patterns, two start with steady Covars, two with medium Covars, and two with variable ones. Since the resultant outcomes indicated that the differences between any two patterns of policy 1, with the same starting Covar, were nonsignificant, the pattern showing slightly better results of I and ABL in all or most points, was chosen from each pair. In addition, each of the patterns of policies 2 and 3 were chosen, because they were shown to be promising. Moreover, policy 4 showed the worst results and consequently, was reduced to a single pattern only; the one exhibiting slightly improved I, and much lower ABL, than the second.

Note that in this investigation the selection of the same means and the same Covars patterns, as those used in the investigations of the means and the Covars imbalances, is deliberate in order to make this investigation a natural extension of the two abovementioned separate investigations. This will provide a good basis for making valid comparisons among the three investigations. The same rule for the patterns' selection would be applied to the choice of the patterns of the means, Covars, and buffer capacities imbalances in the next two investigations.

The selection in this investigation of I rather than ABL as the decision criterion for getting extra points on the best pattern, is due to two reasons. Firstly, I is more important, in general, than ABL, and secondly, 694 more runs would be needed if additional data on the best pattern, in terms of ABL, were to be collected. For the same reasons above, I will be the deciding factor in reducing the number of patterns and in obtaining additional points for the best pattern of imbalance in the next two investigations.

The values of the individual stations' Weibull parameters a and b, along with their corresponding mean service times, when having steady and variable Covar values (0.08 and 0.50), are shown in Tables 6.3 and 6.4 for the monotone increasing

MEAN SERVICE TIMES AND PARAMETERS a AND b FOR
PATTERN (/) WITH STEADY (S) AND VARIABLE (V) COVARS

L = 5, X = 0.020, c = 1.60

STATION NUMBER	MEAN OPT	S		V	
		a	b	a	b
1	9.8000	8.5750	1.3661	2.1433	8.5397
2	9.9000	8.6625	1.3801	2.1651	8.6269
3	10.0000	8.7500	1.3942	2.1874	8.7138
4	10.1000	8.8375	1.4079	2.2089	8.8011
5	10.2000	8.9250	1.4219	2.2307	8.8883

L = 5, X = 0.053, c = 1.60

1	9.4700	8.2863	1.3201	2.0711	8.2522
2	9.7350	8.5181	1.3571	2.1291	8.4831
3	10.0000	8.7500	1.3942	2.1874	8.7158
4	10.2650	8.9819	1.4309	2.2450	8.9449
5	10.5300	9.2138	1.4679	2.3029	9.1758

L = 5, X = 0.136, c = 1.60

1	8.6400	7.5600	1.2044	1.8896	7.5289
2	9.3200	8.1550	1.2992	2.0383	8.1215
3	10.0000	8.7500	1.3942	2.1874	8.7138
4	10.6800	9.3450	1.4888	2.3357	9.3066
5	11.3600	9.9400	1.5836	2.4844	9.8991

L = 5, X = 0.220, c = 1.60

1	7.8000	6.8250	1.0873	1.7059	6.7969
2	8.9000	7.7875	1.2407	1.9464	7.7555
3	10.0000	8.7500	1.3942	2.1874	8.7138
4	11.1000	9.7125	1.5473	2.4276	9.6725
5	12.2000	10.6750	1.7007	2.6681	10.6311

TABLE 6. 3 (Cont)

L = 8, X = 0.020, c = 1.60

STATION NUMBER	MEAN OPT	S a	S b	V a	V b
1	9.8000	8.5750	1.3361	2.1433	8.5397
2	9.8571	8.6250	1.3741	2.1558	8.5895
3	9.9143	8.6750	1.3821	2.1683	8.6393
4	9.9714	8.7250	1.3904	2.1808	8.6891
5	10.0285	8.7749	1.3980	2.1932	8.7388
6	10.0856	8.8249	1.4059	2.2057	8.7886
7	10.1427	8.8749	1.4139	2.2182	8.8384
8	10.1998	8.9248	1.4219	2.2307	8.8881

L = 8, X = 0.053, c = 1.60

1	9.4700	8.2863	1.320	2.0711	8.2522
2	9.6214	8.4187	1.3412	2.1042	8.3841
3	9.7728	8.5512	1.3623	2.1373	8.5160
4	9.9242	8.6837	1.3834	2.1704	8.6480
5	10.0756	8.8162	1.4045	2.2035	8.7799
6	10.2270	8.9486	1.4256	2.2367	8.9118
7	10.3784	9.0811	1.4468	2.2698	9.0437
8	10.5298	9.2136	1.4679	2.3029	9.1757

L = 8, X = 0.136, c = 1.60

1	8.6400	7.5600	1.2044	1.8896	7.5289
2	9.0286	7.9000	1.2586	1.9746	7.8675
3	9.4172	8.2401	1.3128	2.0595	8.2062
4	9.8058	8.5801	1.3669	2.1445	8.5448
5	10.1944	8.9201	1.4211	2.2295	8.8834
6	10.5830	9.2601	1.4753	2.3145	9.2220
7	10.9716	9.6002	1.5294	2.3995	9.5607
8	11.3602	9.9402	1.5836	2.4845	9.8993

MEAN SERVICE TIMES AND PARAMETERS a AND b FOR THE BOWL PHENOMENON PATTERN WITH STEADY (S) AND VARIABLE (V) COVARS

L = 5, X = 0.020, c = 1.60

STATION NUMBER	MEAN OPT	S a	S b	V a	V b
1,5	10.1000	8.8375	1.4079	2.2089	8.8011
2,4	10.0000	8.7500	1.3942	2.1874	8.7138
3	9.9000	8.6625	1.3801	2.1651	8.6269

L = 5, X = 0.053, c = 1.60

STATION NUMBER	MEAN OPT	S a	S b	V a	V b
1,5	10.2600	8.9775	1.4302	2.2439	8.9406
2,4	10.0000	8.7500	1.3947	2.1874	8.7138
3	9.7400	8.5225	1.3578	2.1301	8.4874

L = 5, X = 0.136, c = 1.60

STATION NUMBER	MEAN OPT	S a	S b	V a	V b
1,5	10.6800	9.3450	1.4888	2.3357	9.3066
2,4	10.0000	8.7500	1.3942	2.1874	8.7138
3	9.3200	8.1550	1.2992	2.0383	8.1215

L = 5, X = 0.220, c = 1.60

STATION NUMBER	MEAN OPT	S a	S b	V a	V b
1,5	11.1000	9.7125	1.5473	2.4276	9.6725
2,4	10.0000	8.7500	1.3942	2.1874	8.7138
3	8.9000	7.7875	1.2407	1.9464	7.7969

L = 8, X = 0.020, c = 1.60

STATION NUMBER	MEAN OPT	S a	S b	V a	V b
1,8	10.1000	8.8375	1.4079	2.2089	8.8011
2,7	10.0429	8.7875	1.4000	2.1964	8.7514
3,6	9.9857	8.7375	1.3920	2.1839	8.7015
4,5	9.9286	8.6875	1.3841	2.1714	8.6518

L = 8, X = 0.53, c = 1.60

STATION NUMBER	MEAN OPT	S a	S b	V a	V b
1,8	10.2600	8.9775	1.4302	2.2439	8.9406
2,7	10.1114	8.8475	1.4095	2.2114	8.8111
3,6	9.9628	8.7175	1.3888	2.1789	8.6816
4,5	9.8142	8.5874	1.3681	2.1464	8.5521

TABLE 6. 4 (Cont)

L = 8, X = 0.130, c = 1.60

STATION NUMBER	MEAN OPT	S a	S b	V a	V b
1,8	10.6800	9.3450	1.4888	2.3357	9.3066
2,7	10.2914	9.0050	1.4346	2.2507	8.9679
3,6	9.9028	8.6650	1.3805	2.1657	8.6293
4,5	9.5142	8.3249	1.3263	2.0808	8.2907

L = 8, X = 0.220, c = 1.60

1,8	11.1000	9.7125	1.5473	2.4276	9.6725
2,7	10.4714	9.1625	1.4597	2.2901	9.1248
3,6	9.8428	8.6125	1.3721	2.1526	8.5770
4,5	9.2142	8.0624	1.2845	2.0152	8.0293

L = 10, X = 0.020, c = 1.60

1,10	10.1000	8.8375	1.4079	2.2089	8.8011
2,9	10.0556	8.7987	1.4018	2.1992	8.7625
3,8	10.0112	8.7598	1.3956	2.1895	8.7238
4,7	9.9668	8.7210	1.3894	2.1797	8.6851
5,6	9.9224	8.6821	1.3832	2.1700	8.6464

L = 10, X = 0.053, c = 1.60

1,10	10.2600	8.9775	1.4302	2.2439	8.9406
2,9	10.1444	8.8764	1.4141	2.2186	8.8398
3,8	10.0288	8.7752	1.3980	2.1933	8.7391
4,7	9.9132	8.6741	1.3819	2.1680	8.6384
5,6	9.7976	8.5729	1.3658	2.1427	8.5376

L = 10, X = 0.136, c = 1.60

1,10	10.6800	9.3450	1.4888	2.3357	9.3066
2,9	10.3778	9.0806	1.4467	2.2696	9.0432
3,8	10.0756	8.8162	1.4045	2.2035	8.7799
4,7	9.7734	8.5517	1.3624	2.1374	8.5165
5,6	9.4712	8.2873	1.3203	2.0714	8.2532

L = 10, X = 0.220, c = 1.60

1,10	11.1000	9.7115	1.5473	2.4276	9.6725
2,9	10.6111	9.2847	1.4792	2.3207	9.2465
3,1	10.1222	8.8569	1.4110	2.2137	8.8205
4,7	9.6333	8.4291	1.3429	2.1068	8.3945
[illegible]	[illegible]	8.001[illegible]	1.2747	1.9999	7.9684

order and the bowl phenomenon respectively. In the same way, these values for a descending mean service times' sequence and an inverted bowl, can be determined. To get these values for lines with medium Covars, refer to Tables 6.1 and 6.2. To carry out the whole investigation, a total of 2 x 3 x 3 x 8 x 4 = 576 basic + 18 additional = 594 (+ 100 extra for some patterns with interesting results (as will be explained in Chapter 7) = 694 simulation runs (cells) were needed.

UNEQUAL MEAN SERVICE TIMES AND BUFFER CAPACITIES'SIMULTANEOUS IMBALANCE

This series of simulation experiments aims at filling in another gap in the body of knowledge with respect to the unpaced manual flow lines' behaviour. All the stations in this investigation have equal Covars of 0.274 each. Apart from that, both the means and the buffer sizes were allowed to be unbalanced.

The following exogenous variables and their levels describe the way in which this investigation was designed:

Line length	-	5,8 (for the best I's pattern the value N = 10 was added).
Total buffer capacity	-	8,24 (for N = 5) and 14, 42 (for N = 8), resulting in MB of 2,6 for both N's levels (an MB of 4, i.e. TB of 16,28 was added in the case of the best pattern).
Balancing loss %	-	2, 5 and 12.

Imbalanced means' pattern	-	the same as patterns 1-4 in the means imbalance investigation.
Unbalanced buffer capacities' policy	-	the same as policies 1-4 (and their patterns) in the buffer capacities imbalance investigation.

In order to prevent the number of experimental cells from growing to an infeasible size, the degree of imbalance value of 18%, as well as the random means imbalance pattern, were not considered(dropping the same DI value and the same pattern of means unbalance as those of the means and Covars investigation, fascilitates the comparison of these two investigations). In addition, only one pattern of each of the buffer capacities' imbalance policies 1 to 3 was selected; the ones showing slightly lower, though not statistically significant, values of I than their counterparts, i.e. A_1,B_1,C_1 for N = 5 and A_2,B_2,C_2 for N = 8. On the other hand, since policy 4 resulted in better outcomes of I than policies 1 through 3, three patterns out of six were chosen from this policy, the first being the best among the general sub-policy, the second and the third being, respectively, better than their counterparts in the Patterson's conjecture, and the inverted bowl sub-policies. The total number of the simulation experiments (cells) required for this investigation was 2 x 2 x 3 x 6 x 4 = 288 basic + 15 additional = 303. The full factorial design is presented in Figures 6.8 and 6.9.

FIGURE 6.8

FULL FACTORIAL DESIGN - SERVICE TIMES MEANS & BUFFER CAPACITIES IMBALANCE -(PATTERNS (/)&(\))

LINE LENGTH (N)		5						8					
MEAN BUFFER CAPACITY (MB)		2			6			2			6		
% DEGREE OF IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE													
(/)	A												
	B												
	C												
	D_1												
	D_2												
	D_3												
(\)	A												
	B												
	C												
	D_1												
	D_2												
	D_3												

FIGURE 6.9

FULL FACTORIAL DESIGN - SERVICE TIMES MEANS & BUFFER CAPACITIES IMBALANCE - (PATTERNS (V)&(Λ))

LINE LENGTH	MEAN BUFFER CAPACITY	% DEGREE OF IMBALANCE	PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE (V) A	(V) B	(V) C	(V) D_1	(V) D_2	(V) D_3	(Λ) A	(Λ) B	(Λ) C	(Λ) D_1	(Λ) D_2	(Λ) D_3
5	2	2												
		5												
		12												
	6	2												
		5												
		12												
8	2	2												
		5												
		12												
	6	2												
		5												
		12												

STATIONS' COVARS AND BUFFER SIZES COMBINED IMBALANCE

The objectives of this investigation are to achieve viable results which can shed some light on this largely unexplored research area, and to relate them to those being generated from the foregoing investigations. Recall from Chapter 4 that De La Wyche and Wild (39) conducted a similar investigation, but under the transient state and for a four-station line with a total buffer capacity of 2 units, giving rise to zero buffer capacities' allocations. Furthermore, they considered unworthy Covars imbalance patterns (such as S,V,S,S,S) with only 2 levels of Covars, which were unrelated to those of their separate investigations of the Covars and the buffers imbalances. As a consequence of the aforementioned drawbacks, no useful results emerged from this study.

A similar criticism may be directed at the work of Smith and Brumbaugh (163), whose results were valid only for a three-station line operating under non-steady state conditions. The authors also considered 2 patterns of TB allocation only, namely, (/), (\), and a very small range of Covars, leading to unrepresentative Covars patterns. Therefore, the results of this study were of a limited value.

In this investigation, the mean service time for all the stations in the line was fixed at 10 time units, while the Covars were varied from steady (0.08) to medium (0.27) to variable (0.50) values, and the buffers' sizes were allowed to differ. The factors and their levels are shown below:

Line length	-	5,8.
Total buffer capacity	-	8,24 (for N = 5) and 14,42 (for N = 8), i.e. MB = 2,6. In the case of the best I's pattern, MB = 4 (TB = 16,28) for both N = 5,8, was also experimented with. The same MB value was also used for the best pattern, with respect to ABL, in order to increase the number of data points.
Unbalanced Covars policy	-	exactly the same policies (and their patterns) as those used in the means and Covars joint imbalance investigation.
Unbalanced buffer sizes policy	-	precisely the same policies (and their patterns) as those adopted in the investigation of the joint means and buffers imbalance.

For the same reasons as those of the buffers unbalance investigation, no extra levels of N were employed. Overall, a total of 2 x 2 x 8 x 6 = 192 basic + 4 additional = 196 experiments (cells) were required in this investigation. Figure 6.10 shows the experimental design. It must be noted that this investigation brings the grand total of the experimental cells simulated, for all the six investigations, to 1335.

FIGURE 6.10

FULL FACTORIAL DESIGN - SERVICE TIMES COVARS & BUFFER CAPACITIES IMBALANCE

MEAN BUFFER CAPACITY												PATTERN OF COVARS IMBALANCE (P_i)	LINE LENGTH
6						2							
PATTERN OF BUFFER CAPACITIES IMBALANCE													
D_3	D_2	D_1	C	B	A	D_3	D_2	D_1	C	B	A		
												P_1	5
												P_2	
												P_3	
												P_4	
												P_5	
												P_6	
												P_7	
												P_8	
												P_1	8
												P_2	
												P_3	
												P_4	
												P_5	
												P_6	
												P_7	
												P_8	

CHAPTER SEVEN

UNBALANCED STEADY STATE LINES - RESULTS

INTRODUCTION

Chapter 6 discussed some detailed design considerations pertaining to six major unbalanced lines' investigations. In this chapter the extensive results of these investigations are exhibited both in tabular and graphical forms, and conclusions are drawn from them. Furthermore, some additional conclusions, derived from subjecting the same data to various statistical techniques, will also be shown.

Two main performance criteria were adopted and data on them were gathered, namely, the % total line's idle time, I, and the mean buffer level for the whole line, ABL. In addition, several complimentary measures were obtained, including % line's starving and blocking idle times, ST and BL, together with the average total number of units in the line, L, space utilization, SU, and buffer utilization, BU.

Though the number of units produced at the end of the simulation run was calculated and printed by the simulator, it was decided not to show it, since it was observed in all the experimental cells and the investigations, that it was closely related to I, such that if I is relatively high, indicating a high proportion of line's inactivity, then fewer units will be outputed, and vice versa. This obvious relationship seemed not to warrant the introduction of more tables.

Moreover, the individual stations' I and the individual buffers' ABL were also computed and shown in the simulator's printout, but were not analysed due to the large quantity of I's and ABL's data being generated, which required a great deal of time and effort to handle them. Consequently, it was felt that the individual facilities' data will only contribute marginally to the objective of this research. Nevertheless, these data were preserved and may be used in any future research on uncovering the subtle behaviour of the individual stations and buffers in unbalanced lines.

The information on I and ABL, depicted in the tables and the graphs of the six unbalanced lines' investigations, were carefully examined in order to determine the following:

(1) the basic relationships between the exogenous and the main endogenous variables, I and ABL.

(2) The best, second best, and the worst patterns and policies.

(3) The same basic relationships in (1) above, but with regard to ST, BL, L, SU, and BU.

Note that because both I and ABL are important effectiveness criteria, the decision has been made in (2) to select two best patterns in each investigation; one in terms of I and another with respect to ABL, in order to see whether or not they are the same.

Additionally, the data on I and ABL in each investigation were subjected to ANOVA, multiple regression, canonical correlation, pairwise, and with control comparisons' procedures, and relevant conclusions extracted. The method of comparisons with control involved the repeated testing for each of the factors' levels, of the difference in both I and ABL between the control pattern (the balanced line) and each of the unbalanced patterns.

Inasmuch as the pairwise comparisons' procedure is concerned, similar sets of tests were conducted between the best unbalanced pattern and the remaining inferior unbalanced patterns. No attempt was made to perform pairwise comparisons among the inferior patterns, since this would lead to a substantial number of relatively important comparisons. It is clear that the main interest lies with the best pattern and, therefore, comparing the less important patterns is not worthy of the necessary effort and may even be confusing.

Furthermore, it was decided to use regression and canonical correlation analyses, as well as to determine the relationships in (3) for the best patterns only, because their extension to the rest of the unbalanced patterns is of little benefit. It should be noted that no effort was exerted on deriving optimal B formulae for any investigation, due to the unrealistic assumption embedded in such formulae that the inventory cost of partially completed units is universal along the line.

So as to avoid the unnecessary repetition of the investigations' titles, M, C, BC, M&C, M&B, C&B will hitherto refer, respectively, to the means, Covars, buffer capacities, means & Covars, means and buffers, and Covars & buffers unbalanced lines' investigations. These abbreviations will be used wherever it is judged necessary to do so in this and the two remaining chapters.

Before presenting the detailed conclusions, it should be noted that upon comparing the balanced line's results of this research (which are shown in Table A7.1) with those of Slack (160), it was found that they were very close for all the levels of N and B considered. On the other hand, no exact numerical comparisons were possible between the results of both investigations M and C of this thesis, and those of the preceding research, since the values of the exogenous variables used were different. However, it is quite feasible to make some general comparisons regarding the broad policies or patterns between the previous studies and this one.

As regards investigation M&B no comparisons were feasible, since no published results were available. With respect to investigations M&C and C&B, no numerical comparisons were practicable, because of the fact that the system simulated in this research is unique in terms of its patterns and factors' values, but the general policies and patterns of this and the previous works, can be compared and tested. Moreover, in investigation BC, the only fruitful numerical comparisons likely, are those between patterns B_3, D_1, D_4, D_5 of this research with N = 5, MB = 2, Covar = 0.274, and Weibull service times, and their counterparts in De La Wyche & Wild's study (39) with Covar = 0.3 and normal operation times. The comparison indicated that the two sets of results were fairly close, resulting in nonsignificant differences among them.

The results and conclusions of this thesis are classified as follows and will be presented and discussed in turn:

(1) Idle Time's Results:

(a) Indirect ranking of, and comparisons between, the policies and the patterns within them.

(b) Effects of the design variables N, B, DI, and the pattern of imbalance on I.

(c) A new phenomenon for the M&C imbalance.

(d) System's loss versus balancing loss.

(e) Rao's conjecture for the M&C imbalance.

(f) Testing some policies in **investigation C&B.**

(2) Blocking and Starving Idle Times' Results.

(3) Mean Buffer Level's Results:

(a) Indirect ranking of, and comparisons among the policies and the patterns within them.

(b) Effects of the design variables N, B, DI, and the pattern of imbalance on ABL.

(4) Stockholding Results:

(a) Effects of the exogenous variables on L.

(b) The design factors' influence on SU.

(c) Impacts of the exogenous variables on BU.

(5) ANOVA's Results.

(6) Multiple Regression's Results.

(7) Canonical Correlation's Results.

(8) Comparisons with Control's Results:

  (a) Idle time's conclusions.

  (b) Mean buffer level's conclusions.

(9) Pairwise Comparisons' Results:

  (a) Idle time's conclusions.

  (b) Mean buffer level's conclusions.

IDLE TIME'S RESULTS

Tables A7.2 - A7.19 show the I's results for the six unbalanced lines investigated. The corresponding information in graphical form for the best and the second best patterns (and for some good unbalanced patterns in the M&C and M&B investigations) are exhibited in Figures A7.1 - A7.41. The conclusions to be drawn from these tables and graphs are shown in Figures 7.1 and 7.2.

Presentation of Results - A Note

The investigations reported in this thesis produced a very large number of results. This is because of (a) the large number of individual investigations and (b) the use of two major measures of effectiveness - idle time, I, and average buffer level, ABL. Because of this it was decided to present the conclusions in the form of figures showing a series of conclusion statements and then comment on each statement as it applied to each investigation. This enables each conclusion statement to be evaluated for all six types of imbalance.

Each of these figures depicts a matrix which comprises eight columns. The last six columns represent the six imbalance investigations, M (means imbalance) through to B and C (Buffers and Covars imbalance combined). Each conclusion statement for each imbalance investigation is then declared to be either ...

√ - the result is true or valid for that investigation

√a - the result is true or valid in most or nearly all parts of the investigation

NA - not applicable in that investigation

X - unclear or non-existent for that investigation

≈ - the opposite of the statement obtains for that investigation.

Some statements include more detail. For example, in fig.7.1 conclusion statement number 2 (The best imbalance pattern is ...), the particular best patterns are given for each investigation. Where no single best pattern is evident the best pattern for each line length is given.

Notes on the Conclusions of Figure 7.1

Conclusion Number		Observation
1	*	For instance, in investigation C the bowl phenomenon's policy (policy (2)), as represented by patterns P_7 and P_8, is the best only for $N = 5$, but is not so for $N = 8$, since the six patterns of policy (1) are better than P_8 in most points. This particular example is in general agreement with the findings of De La Wyche and Wild (39) for unbalanced lines under transient conditions, with $N = 3,12$.

FIGURE 7.1

THE INDIRECT RANKING OF, AND COMPARISONS BETWEEN, THE POLICIES AND THE PATTERNS WITHIN THEM

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	B&C
1. There is no best or worst policy in all its variants (patterns), but there exist specific individual patterns which may be viewed as the best or the worst ones.	N,B/MB,(DI)	NA	✓	✓	✓	✓	✓
2. The best unbalanced pattern is:	N,B/MB,(DI)	(V)	P_7	D_2(N=5), D_1(N=8)	(∧)+P_5	(V)+D_1	C+P_4
3. The second best pattern, in most or almost all the points, may be regarded as:		(∧)	P_8(N=5), P_2(N=8)	D_1(N=5), X (N=8)	X	X	D_3+P_5(N= D_2+P_4(N=
4. The worst pattern(s), in all or most points, may be considered as:		(\\)	P_9	A_1	any means pattern + P_8	(\\)+A,(/) +C,(∧)+C	D_3+P_8

...Cont

FIGURE 7.1 (CONT)

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	B&C
5. The increase in I, as DI goes up, is lower in the best pattern than the rest, especially the poor ones.	MB/B,N	✓	NA	NA	✓	✓	NA
6. The difference in I between the best and the inferior patterns, especially the poor ones, is greater the higher DI is.	MB/B,N	✓	NA	NA	✓	✓	NA

NA = not applicable

X = unclear or nonexistent

* The only exception is in investigation C, where the third policy (the reversed bowl policy) is the worst for both patterns P_9 and P_{10}, which agrees with the results of El-Rayah (46).

2 * In investigation M: this means that the best pattern is that of the bowl phenomenon, which gives backing to Hillier and Boling's (75) and El-Rayah's (46) findings.

* In investigation C: the best pattern, in this case, is the bowl phenomenon in terms of the individual Covars' arrangement, which is in general agreement with De La Wyche and Wild's (39) results for the NSS conditions, and with those of El-Rayah (45).

* In investigation BC: the best pattern is whereby the available capacity is distributed as evenly as possible along the line. This is in line with El-Rayah's (45) general results.

* In investigation M&C: the best pattern, $(\Lambda)+P_5$, neither includes the best M pattern, (V), nor the best C pattern, P_4, (which was labelled as P_7 in investigation C). However, both P_5 and P_4 belong to the same policy and produce good results when combined with any of the patterns of investigation M.

* In investigation M&B: $(V)+D_1$, the best pattern, contains the best M pattern, (V), and the best BC pattern, D_1, (which was known as D_2 for $N=5$ and D_1 for $N=8$ in investigation BC).

* In investigation C&B: the best policy, $C+P_4$, contains the best C pattern, P_4, but does not include the best BC pattern, D_1. On the other hand, both P_4 and P_5 yield sound outcomes when joined with any of the BC patterns.

3 * In investigation M: this seems in contrast with El-Rayah's (46) findings where pattern (Λ) was amongst the worst. The discrepancy may be attributed to the fact that El-Rayah simulated different values of the exogenous variables, i.e. N=3, B=0, Exponential service times, which are all clearly inefficient.

4 * In investigation B&C: the Covars' imbalance pattern P_8, in conjunction with any of the buffer capacities' patterns of unbalance ($A-D_3$), is worse in general than the corresponding ($A-D_3 + P_1-P_7$) patterns.

5 * This implies that the best pattern is the least sensitive to DI's increase.

<u>Further Notes</u>

1 * In investigation M: pattern (/) is better than pattern (\) in most points. This confirms the finding of El-Rayah (46) and Payne et al (136). However, the difference in I between (/) and (\) is slight, for all N, B, and DI, whereas the difference between them in ABL is substantial and highly significant (see the conclusions of ABL).

2 * For investigation C, the following may be concluded:

(a) In policy (1), pattern P_2 is better than the other five patterns in most points.

(b) Pattern P_9 of policy (3) is better than P_{10} for all the values of N and B.

(c) Pattern P_{12} (policy(4)) is worse than P_{11} whatever the values of N and B are.

(d) Part of the reason why the bowl pattern, P_7, is the best is the fact that its total variability is less than that of any other pattern belonging to the other policies. However, the variability factor is not predominent in its effect on I, and there are other factors which also influence I, viz, B, N and the Covars unbalance pattern. These factors may neutralise the overall variability's impact. For this reason, pattern P_8 which also depicts the bowl configuration was inferior to the patterns representing policy (1), for N=8. In addition, pattern P_7 is better than P_8 in all the points, although they both have the same total Covars' value. This is in contrast with Anderson et al's (3) conclusions and it may be due to the fact that the conditions examined were not exact.

3 * For investigation BC the following observations may be made:

(a) When N=5, the best patterns within policies

(1),(2),(3) are A_3,B_3 and C_3 respectively, but when N=8, patterns A_2,B_2 and C_2 are the best within the aforementioned policies.

(b) Patterns A_1,B_1 and C_1 are the worst inside policies (1) through (3). This lends support to the strategy of avoiding extreme allocation of TB, i.e. most TB is assigned to one buffer, and the remainder to the other buffers. This appears in line with Soyster's (164) recommendations.

(c) If a balanced, or as close as possible to a balanced buffers' arrangement are infeasible, policy (2) should be selected as the best alternative.

(d) The effect of buffers imbalance pattern on I is much less than that of the means or the pattern of Covars imbalance, especially for low MB. This is in line with El-Rayah's (45) observation.

4 * Some good patterns in investigation M&C include: $(/)+P_4$; $(\backslash)+P_5$; $(V)+P_{1,3,4,5}$; $(\Lambda)+P_4$ (for N=5), and $(/)+P_4$; $(\backslash)+P_5$; $(V)+P_{1,3}$; $(\Lambda)+P_4$ (for N=8).

* In investigation M&B some good patterns are: $(\Lambda)+C,D_2,D_3$; $(V)+A\text{-}C,D_2,D_3$ (for N=5), and $(\Lambda)+B,D_1\text{-}D_3$; $(V)+A\text{-}C,D_2,D_3$ (for N=8).

* In investigation B&C some sound patterns are: D_3+P_4; $B+P_3$; $A+P_5$.

FIGURE 7.2

THE EFFECTS OF THE DESIGN VARIABLES N, B, DI, AND THE PATTERN OF IMBALANCE ON I

	RELATIONSHIP	GIVEN	M	C	BC	M&C	M&B	C&B
1.	Increasing N increases I.	MB/B,(DI), pattern *	✓	✓	✓a	✓a	✓a	✓a
2.	The increase in I, as N increases, is higher the lower B/MB is.	(DI), best pattern	✓a	✓a	✓	✓a	✓a	✓
3.	As B/MB is increased, I decreases.	N,(DI), pattern	✓	✓	✓	✓	✓	✓
4.	The marginal decrease in I decreases as B/MB increases.	N,(DI), best pattern **	✓	✓	✓	✓	✓	✓
5.	The effect of increasing B/MB on reducing I tends to be diminished as N is decreased.	(DI), best pattern	✓a	✓a	✓	✓a	✓a	✓
6.	I is increased as DI becomes higher.	N,B/MB, pattern	✓	NA	NA	✓a	✓	NA
7.	The marginal increase in I increases as DI continues to increase.	N,B/MB, best pattern	✓a	NA	NA	✓a	✓a	NA

...Cont

FIGURE 7.2 (CONT)

RELATIONSHIP	GIVEN	M	C	BC	M&C	M&B	C&B
8. The marginal increase in I (if exists), when DI increases, tends to increase as B/MB is increased.	N, best pattern	✓	NA	NA	X	✓	NA
9. The increase in I, as DI increases, is steeper for higher B/MB, and becomes more gradual as B/MB decreases, i.e. the slope of high B/MB is higher than that of low B/MB as DI rises.	N, pattern	✓	NA	NA	✓	✓	NA
10. The difference in I between the highest and lowest B/MB value decreases as DI increases.	N, pattern	✓	NA	NA	✓	✓	NA

* Refers to the pattern of means or Covars, ..., or Covars and buffers imbalance.

** The marginal change, in this and any other conclusions, is cited only for the best patterns, since it is of little interest for the other inferior patterns.

NA = not applicable

✓a = true in most or nearly all points

X = unclear or nonexistent

Notes on the Relationships of Figure 7.2

Relationship Number	Observation
1	* This relationship . for all the unbalanced lines investigations, is similar to that of Hunt (78) for the balanced lines, i.e. the effect of N on I is similar for both the balanced and the unbalanced lines.
	* 'Given DI, means pattern' is new in the means imbalance and has not been mentioned in El-Rayah's (46) study.
	* Though an N=5 pattern is unidentical to the corresponding pattern for N=8 which, in turn, is unidentical to that of N=10 (if exists), it is clear in all the investigations (except for patterns (/),(\),(V),(Λ) of M), that increasing N increases I, irrespective of the degree of similarity between the patterns.
	* In investigation C: this result confirms that of Anderson et al (3) for N=4.
	* In investigation BC: this is true, except for D_6 (all MB) and D_5 (MB=6).
	* In investigation M&C: this result is true for all the (V) patterns and most other means and Covars patterns (i.e. in 257 out of 288 cells).
	* In investigation M&B: this is valid in most cases (in 121 out of 144 cells), especially for patterns (V) and (\).
	* In investigation C&B: this is true in 185 out of 192 cells.

2 * In investigation C: true, with the exception of one point; B=6, $N=8\rightarrow 10$.

* In investigation M&C: the single exception occured at DI=0.12, $N=8\rightarrow 10$.

* In investigation M&B: the exception for this is that of DI=0.05, $N=5\rightarrow 8$.

* DI seems to have no clear influence on increasing I as N increases.

3 * This is in agreement with the result of Hunt (78) for the balanced line.

* 'Given DI, pattern' is new for the unbalanced lines.

* In investigation C: this gives support to Carnall & Wild's (26) imbalanced lines' results.

* The effect of TB (in investigations BC,M&B, and C&B) on I is the same as that of MB, i.e. it looks that TB and MB are two faces of the same general exogenous variable.

4 * This seems in line with the balanced line's results of Hunt (78), for all the unbalanced line's investigations.

5 * In all the investigations this appears to agree with the balanced line's conclusion of El-Rayah (44).

* The marginal decrease seems unclearly affected by DI.

* In investigation M: the exception for this is $N=8\rightarrow 10$.

* In investigation C: except for one instance; N=8 → 10, B=6.
* In investigation M&C: with the exception of N=8 → 10, DI=0.12, B=3,6.
* In investigation M&B: the single exception took place at N=5 → 8.

6
* In investigation M&C: except for some patterns, where the opposite is true. This will be explained later.
* This confirms El-Rayah's (46) results for the means imbalance.

7
* In investigation M: the exception is that of N=10, B=1,2, DI=0.02 → 0.05 → 0.12.
* In investigation M&C: the exceptions for this are N=5, B=2; N=10, B=1.
* In investigation M&B: except for one instance N=5, MB=2.
* The inverse is true in investigations M, M&C and M&B.

8
* The marginal increase appears to be uninfluenced by N.

9
* For example, for the best pattern of means unbalance increasing DI from 0.02 to 0.18 raised I by 15.7% (for N=8, B=1), while the same range of increasing DI for N=5, B=6, resulted in an increase in I of 330%. In addition, for the best pattern of investigation M&C in one extreme increasing DI from 0.02-0.12 led to an increase in I of 4%, for N=8, B=1,

TABLE 7.1

A NEW PHENOMENON FOR MEANS AND COVARS' COMBINED IMBALANCE

PATTERN	N	B	RANGE OF % DI WHERE I DECREASES	END VALUE OF I COMPARED TO ITS INITIAL VALUE
(/)+P_6	5	1	2-5	2
	5	2	2-5	2
	8	1	2-5-12†	1
	8	2	2-5	2
(/)+P_8	5	1	2-5	2
	5	2	2-5	2
	8	1	2-5	2
	8	2	2-5	2
	8	6	2-5	2
(V)+P_1	5	1	2-5-12†	1
	8	1	2-5*	1
	8	2	2-5-12	2
	10	1	2-5	2
(V)+P_3	5	1	2-5-12†	1
	5	2	2-5	2
	8	1	2-5-12†	1
	8	3	2-5	2
	10	1	5-12**	2
	10	2	2-5	2
	10	3	2-5	2
	10	6	2-5	2
(V)+P_6	5	1	2-5-12†	1
	8	1	no decrease in I	-
	10	1	2-5	2

...Cont

TABLE 7.1 (CONT)

PATTERN	N	B	RANGE OF % DI WHERE I DECREASES	END VALUE OF I COMPARED TO ITS INITIAL VALUE
	10	2	2-5	2
$(V)+P_8$	5	1	2-5-12-18†	1
	5	2	2-5-12	0
	5	3	2-5	2
	5	6	2-5	2
	8	1	2-5-12-18†	1
	8	2	2-5-12	1
	8	3	2-5-12	1
	8	6	5-12	2
	10	1	2-5-12-18†	1
	10	2	2-5-12	1
	10	3	2-5-12	2
	10	6	2-5	2

where

0 = end value of I is nearly the same as that of the initial value (for DI = 0.02)

1 = end value of I is lower than that of the initial value

2 = end value of I is higher than that of the initial value

† in these instances I continues to decline.

* in this case DI appears to have very little effect on I, i.e. the function of I is relatively insensitive to the rise in DI, where the successive increases in DI to 0.18, decrease I slightly first, then I goes up a little, until it finally converges to its initial level.

** raising DI from 0.02 to 0.05 in this case slightly increases I, but from DI=0.05 to DI=0.12, I tends to drop by a small amount. As DI becomes 0.18, I rises again and slightly overshoots its preliminary level.

whereas on the other extreme, the same increase in DI for N=5, B=6 resulted in a rise in I of 155%. Furthermore, for the best pattern in investigation M&B upon raising DI to 0.12 from an initial value of 0.02 for N=10, MB=2, I tended to go up by 13.1%, but for N=5, B=6 the same magnitude of DI's increase caused I to rise by 188.9%.

10 * In investigation M: with the exception of pattern (Λ).

* In investigation M&C: the single exception happened at N=10, DI=0.05.

A New Phenomenon for the M&C Imbalance

In some cases of investigation M&C it was observed that when DI is increased, I tended to go down. This phenomenon occurred for the patterns and under the conditions shown in Table 7.1.

It should be noted that in this table the additional values of N=10, B=3 and DI=0.18 were obtained for some of patterns (V)+P_i, but not for any of patterns (/)+P_i, because the former patterns' results were generally superior to those of the latter patterns.

Tables A7.16 and A7.17 show these additional data and Figures A7.15-A7.30 depict the graphical pictures of the basic and extra data which are indicative of this phenomenon.

From Table 7.1 the following observations can be made:

(1) The fall in I, when DI is increased, occurs always when B=1 and in most cases, for B=2 also.

(2) This phenomenon always takes place in patterns $(V)+P_{6,8}$; $(/)+P_{6,8}$.

(3) Except for one instance, if this phenomenon exists for N=5, it will also exist for N=8,10.

(4) With the exception of two instances, the phenomenon occurs at DI=0.02-0.05.

(5) All the cases where the drop in I continues as DI rises, take place at B=1 only.

(6) In most cases the marginal decrease in I diminishes as DI increases, i.e. the decline in I will ultimately stop at some degree of imbalance, where this phenomenon will disappear.

(7) As DI is increased beyond 0.18, more decrease in I may result, but at a decreasing rate, except for pattern $(V)+P_8$, N=10, where it may be expected that a further rise in DI will result in a somewhat high rate of decline in I, especially for B=1.

(8) In all the patterns whereby this phenomenon occurs I is initially higher than that of the balanced line, and despite the drop in I, as DI increases, it will still be above that of the balanced line. Therefore, this phenomenon serves only to reduce some of the inefficiencies incurred by its inferior patterns, as compared to that of the balanced arrangement.

This finding for investigation M&C appears to differ from that of El-Rayah (46) for the means imbalance, which indicates that when the initial value of I is higher than that achieved by the balanced counterpart, it continues to increase as DI tends to rise, and that only if I is initially lower than that obtainable by the balanced configuration, can it further decline.

System's Loss versus Balancing Loss

When DI = 0 (balanced line), the sole contributor to I is the system's loss, but when DI>0, the magnitude and nature of the contribution of system and balancing losses to the function of I, as well as their relationship, are completely unknown and, according to Slack (160), are worthy of exploration. In investigations M, M&C and M&B it was found that the balancing loss as expressed by DI is higher, in many instances, than I. This indicates that it may prove unreasonable to assume that I will always be $\geqslant$ the balancing and system's losses, taken individually. Upon examining the published graphs of El-Rayah's (46) work for N = 4, Covar = 0.3, normal operation times, and N = 3, Covar = 0.15, lognormal work times, support was found for this contention.

Therefore, when imbalance is present in the line, it may be sound to assume that both system's and balancing losses subscribe to I, but the determination of their relative share is not simple and, as yet, there is no known method to derive it.

Rao's Conjecture for the M&C Imbalance

According to Rao (142), if in a 3-station line having two exponential stations (each with a variable Covar, V) and an

FIGURE 7.3

THE EVALUATION OF SOME POLICIES - INVESTIGATION C&B

POLICY	AUTHORS	LIKELY PATTERNS (THIS RESEARCH)	TEST'S O
1. Allocating the highest Covar to the middle station is detrimental.	Smith & Brumbaugh (163)	P_8+A-D_3(N=5,8)	Tru
2. Assigning higher B around the stations with higher Covars is beneficial.	Smith & Brumbaugh and De La Wyche & Wild (39)	P_1+B,D_1;P_3+B;P_4+C,D_2,D_3; P_5+A,D_3;P_6+C,D_2,D_3;P_7+A,D_3 (N=5);$P_{4,6}$+C,D_2(N=8)	Tru
3. Allocating equal B around the high Covars' stations is beneficial.	De La Wyche & Wild	$P_{1,3}$+B,D_3(N=5);$P_{1,3}$+C(N=8)	True
4. Alloting higher amount of B around the variable stations than around the steadier stations is beneficial.	De La Wyche & Wild	$P_{1,3}$+D_1;$P_{4,6}$+C,D_2,D_3(N=5), $P_{4,6}$+C(N=8)	True

...Cont

FIGURE 7.3 (CONT)

POLICY	AUTHORS	LIKELY PATTERNS (THIS RESEARCH)	TEST'S O
5. More B should be placed between the variable station and the line's centre, than between it and the front or the rear of the line.	De La Wyche & Wild	$P_{1,3}$+B,D_1;P_2+B-D_2;P_4+C,D_1; $P_{5,7}$+A;P_8+B,D_1,(N=5);$P_{1,3}$+B, D_1;$P_{4,6}$+D_1;$P_{5,7}$+A;P_8+D_1 (N=8)	True
6. B should be assigned after the variable station, not before it.	De La Wyche & Wild	After: $P_{1,3}$+A,D_2(N=5);P_2+D_2; $P_{5,7}$+D_2(N=8). Before: $P_{1,3}$+C,D_1(N=5);$P_{4,6}$ +C,D_2;$P_{1,3}$+D_2(N=8)	True

Erlangian middle station, the difference between station's 2 Covar and the Covar of either of stations 1 or 3 is <0.5, i.e. pattern VVV or VMV, then the optimal pattern of the means imbalance is the bowl arrangement ((V)). On the other hand, if this difference is >0.5, i.e. pattern VSV, then the optimal pattern of the means is the reversed bowl (($\wedge$)).

Although the conditions of investigation M&C differ from those of Rao (i.e. using N>3, Weibull service times, and unequal (though close) Covars for the first and the last stations, in patterns VSSSM and VVSSSSMM of this research), the best pattern turned out to be ($\wedge$)+VSSSM/VVSSSSMM) which resembles, but is not identical to, Rao's ($\wedge$)+VSV pattern, more than his ($\wedge$)+VVV pattern. Consequently, it is possible to say that the conjecture of Rao is in general valid, and it is hoped that further research will throw more light on it.

Testing Some Policies in Investigation C&B

The previously reviewed unbalanced lines' works (Chapter 4) advanced some policies for unbalancing the Covars and buffer capacities.

Figure 7.3 presents these policies, together with their general evaluation, irrespective of the differences in the simulated conditions between this and the previous studies.

STARVING AND BLOCKING IDLE TIMES' RESULTS

Tables A7.20 and A7.21 exhibit the ST and BL data for the best patterns in terms if I, for all the imbalanced lines' investigations. These data are shown graphically in Figures

A7.66 - A7.74. The results of these data regarding the effects of the design factors N, B, and DI on ST and BL seemingly support the conclusions which are portrayed in Figure 7.4.

Notes on the Conclusions of Figure 7.4

Conclusion Number	Comment
1	* This appears in line with Slack's (160)balanced line results.
2	* In investigation C: true, with the exception of N = 5, ST.
	* In investigation C&B: the exception for this is N = 8, BL.
3	* In investigation M&C: true only for BL.
	* In investigation M&B: valid only for ST.
	* In investigation C&B: correct only for N = 8.
4	* In investigation M&C: except for N = 8, B = 1,2.
	* In investigation M&B: the only exception is that of N = 5, B = 2.
	* In investigation M&C: the marginal increase in ST rises with the increase in DI.
5	* In investigation C: the situation is reversed for all N and B.
	* In investigation BC: true, especially for low MB.
	* In investigation M&C: valid, especially for low B and DI.
6	* Note that in Slack's (160) results, N has little influence on ST.

FIGURE 7.4

THE EFFECTS OF THE DESIGN FACTORS N, B, AND DI ON ST AND BL

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	C&B
1. Both ST and BL are reduced as B/MB becomes higher	N,(DI)	X	✓	✓	✓	✓	✓
2. As B/MB continues to increase, both ST and BL decrease in diminishing proportions.	N,(DI)	X	✓a	✓	✓	X	✓a
3. The marginal decrease in both ST and BL increases as N tends to increase.	(DI)	X	X	✓	✓a	✓a	✓a
4. Both ST and BL rise with an increase in DI.	MB/B,N	X	NA	NA	✓a	✓a	NA
5. BL's amount is larger than that of ST.	MB/B,N,(DI)	X	X	✓	✓	✓	✓
6. Both BL and ST are likely to go up with the increasing of N.	MB/B,(DI)	X	✓	✓	X	X	✓
7. ST is lower than that of the balanced line.	MB/B,N,(DI)	X	X	✓	X	✓a	✓

where

✓a = true in most or nearly all points

X = unclear or nonexistent

NA = not applicable

7 * In investigation C: the opposite result occurred.

* In investigation BC: true, especially for low MB and high N.
* In investigation M&B: true, except for N=8, B=2, DI=0.02.
* In investigation C&B: valid, especially for lower MB.

MEAN BUFFER LEVEL'S RESULTS

Tables A7.22 - A7.31 summarise the ABL results for all the unbalanced lines' investigations. Figures A7.42 through A7.65 show the same data, but for the best and the second best patterns only (together with several good patterns in investigations M&C and M&B). The general conclusions that can be drawn out of these tables and figures are shown in Figures 7.5 and 7.6.

Notes on the Conclusions of Figure 7.5

Conclusion Number	Comment
2	* In investigation C: the bowl phenomenon pattern, P_7, is the best, which is the same result as that for I. This is the only incident where the best pattern is best in terms of both I and ABL.
	* In investigation M&C: the best pattern (pattern ($\diagdown$)+P_4) combines both the best M's pattern ($\diagdown$), and the best C's pattern, P_4.
	* In investigation M&B: both the best M's pattern, ($\diagdown$), and the best BC's pattern, A, are included in the M&B's best pattern (pattern ($\diagdown$)+A).

FIGURE 7.5

THE INDIRECT RANKING OF, AND COMPARISONS AMONG, THE POLICIES AND THE PATTERNS WITHIN THEM

CONCLUSION	GIVEN	M	C	B	M&C	M&B	B&C
1. There exists no best or worst policy in all its patterns. Specific individual patterns, however, exist which can be looked at as the best or the worst.	N,B/MB,(DI)	NA	✓	✓	✓	✓	✓
2. The best pattern in terms of ABL is:	N,B/MB,(DI)	(\)	P_7	A_1(N=5), A_2(N=8)	(\)+P_4	(\)+A	A+P_4
3. The second best pattern can be generally regarded as:		(∧)	P_{10}	A_2(N=5), A_3(N=8)	(\)+P_6	(\)+B	B+P_4(N A+P_6(N
4. Other good patterns are:		-	-	-	(\)+$P_{2,5}$	(\)+D_1,D_2	-

...Cont

FIGURE 7.5 (CONT)

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	B&C
5. The worst pattern(s) can be generally regarded as:		(/)	$P_{12,8}$	X(N=5), D_2(N=8)	(/)patterns†	(/)patterns	$C+P_5$,B (N=5),
6. The (\) pattern(s) is better than any other pattern(s).		✓	-	-	✓	✓	-
7. The difference in ABL between the best pattern and the inferior one is higher the larger DI is.	MB/B,N	✓	NA	NA	✓	✓	NA

† (/) patterns, in investigations M&C and M&B refer to the (/) means pattern combined with any of the Covars or the buffer capacities' patterns.

X = unclear or nonexistent

NA = not applicable

FIGURE 7.6

THE EFFECTS OF THE DESIGN VARIABLES N, B, DI, AND THE PATTERN OF IMBALANCE ON ABL

RELATIONSHIP	GIVEN	M	C	BC	M&C	M&B	C&E
1. Where N tends to increase, ABL rises, or may remain relatively constant.	B/MB,(DI), best pattern	✓	X	X	X	✓	X
2. ABL becomes larger as B/MB becomes higher.	N,(DI), pattern	✓	✓	✓	✓	✓	✓
3. The increase in ABL continues at a diminished rate when B/MB rises.	N,(DI), best pattern	✓	X	✓a	✓a	X	✓
4. The marginal increase in ABL, as B/MB increases, declines as N is reduced.	(DI), best pattern	✓	X	X	✓	X	X
5. ABL falls with an increase IN DI.	N,B/MB, (\)pattern(s)	✓	NA	NA	✓	✓	NA

...Cont

FIGURE 7.6 (CONT)

RELATIONSHIP	GIVEN	M	C	BC	M&C	M&B	C&B
6. The drop in ABL becomes less marked as DI continues to rise.	N,MB/B, best pattern	✓	NA	NA	✓	✓	NA
7. The marginal reduction in ABL, as DI goes up, decreases with the decrease in B/MB.	N,best pattern	X	NA	NA	✓	✓	NA
8. The decline in ABL, as DI is increased, is steeper the higher B/MB is.	N,(\) pattern(s)	✓	NA	NA	✓	✓	NA
9. The difference in ABL between the highest and the lowest B/MB lessens as DI rises.	N,(\) pattern(s)	✓	NA	NA	✓a	✓	NA

✓a = true in most or nearly all points

X = unclear or nonexistent

NA = not applicable.

* In investigation C&B: both the best patterns of C and BC, namely, patterns P_4 and A, are contained in the C&B's best pattern, P_4+A.
* The best patterns in investigations M, M&C and M&B are much better than any other inferior patterns, especially for high DI.

Other Notes

The following conclusions are related to the Covars imbalance:

(1) Although pattern P_7 of policy (2) is the best, pattern P_8 of the same policy is amongst the worst in terms of ABL.

(2) All the patterns of policy (1) have close values of ABL.

(3) Patterns P_{10} and P_{11} of policies (3) and (4) respectively are better than their counterparts P_9 and P_{12}, in all the levels of B and N.

(4) No relationship exists between the total Covars' value for the whole line and ABL, which is the same general finding as that of Anderson et al (3) for L. For example, both the patterns P_7 and P_8 have the same overall variability of 1.01, however, their ABL's values are, respectively, 0.397 and 0.668. This is also in line with I's results.

Notes on the Relationships of Figure 7.6

Relationship Number	Note
1	* The first part of this conclusion (where N tends to increase, ABL rises) is in agreement with Slack's (160) results for the balanced lines.

* This relationship is unclear in investigations C, BC, M&C, and C&B, partly as a result of the unidentical shapes of the corresponding patterns between N = 5 and N = 8.
* In investigation M: the increase in ABL (where exists), as N goes up, is larger for higher B.
* In investigation BC: this result is in line with that of El-Rayah (45) with regard to L.

2 * This agrees with the result of Slack (160) for the balanced lines.
* In investigation M: the ABL may stay relatively constant when B is up.
* In investigation C: this gives credence to Anderson et al's (3) results.
* 'Given DI, pattern' is new.

3 * This result looks to be in line with that of Slack (160) for the balanced line.
* In investigation BC: true only for N = 8. Note that the patterns for N = 5 are unidentical to those of N = 8 and, consequently, they may lead to different relationships.
* In investigation M&C: valid for N = 5 only.

8 * N appears to have no clear impact on the drop in ABL.
* In investigation M&C: this finding seems to be true for all the (\) patterns for N = 5 only.

Figure 7.6a shows some additional conclusions on the impacts of N, B, DI, and the imbalance pattern on ABL.

FIGURE 7.6a

ADDITIONAL CONCLUSIONS

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	C&
1. ABL rises as DI is increased.	N,B/MB, (/)pattern(s)	✓	NA	NA	✓	✓	NA
2. The upper bound of ABL in pattern(s) (/), as DI rises appears to be the maximum B/MB limit.	N,B/MB, (/)pattern(s)	✓	NA	NA	✓	✓	NA
3. The approach to the upper bound is closer the higher B/MB is.	N,(/) pattern(s)	✓	NA	NA	✓a	✓	NA
4. The pattern(s) whose rate of convergence to the maximum ABL's limit is higher than the rest is (are):	N,B/MB	-	NA	NA	(/)+$P_{5,4}$	(/)+B	NA

...Cont

FIGURE 7.6a (CONT)

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	C&
5. The lower bound of ABL in pattern(s) (\\), while DI increases, seems to be zero.	N,B/M, (\\)pattern(s)	✓	NA	NA	✓	✓	NA
6. The approach to this lower bound is closer wherever B/MB becomes higher.	N,(\\) pattern(s)	✓	NA	NA	✓	✓	NA
7. The pattern(s) whose convergence rate is higher than the rest is (are):	N,B/MB	-	NA	NA	(\\)+$P_{4,5}$	(\\)+A,B	NA
8. The ABL is approximately 0.5 of B/MB.	N,B/MB,(DI)	✓	✓	X	✓	✓	✓

✓a = true in most or nearly all the cases

X = unclear or nonexistent

NA = not applicable

Notes on the Conclusions of Figure 7.6a

Conclusion Number		Comment
1	*	This means that the behaviour of ABL varies substantially between pattern(s) (/) and pattern(s) (\). Furthermore, no clear ABL's behaviour could be found for pattern(s) (V) and (Λ).
3	*	In investigation M: this is true also for higher N.
	*	In investigation M&C: valid only for (/)+P_8.
4 and 6	*	The rate of convergence is greater in investigations M&C and M&B than in investigation M.
8	*	In investigation M: this is true for pattern (Λ) and pattern (V) (except for B=6).
	*	In investigation C: valid for all the patterns of policy (1) (the exception for this is N=8, B=6), pattern P_9 (the only exception took place at N=5, B=6), and pattern P_{11}.
	*	In investigation M&C: the conclusion is true for patterns (V)+$P_{1,2,3,8}$ (the exception is that of B=6), patterns (Λ)+P_8; (Λ)+$P_{1,2,3}$ (with the exception of B=6, DI=0.12).
	*	In investigation M&B: this is valid for patterns (Λ)+D_3 (all N); (Λ)+D_2 (N=8 only).
	*	In investigation C&B: true for patterns $P_{1,2,3}$+B; P_{1-3}+D_3(N=5); P_8+D_2(N=8).
	*	This result agrees with that of Wild (177) for the balanced lines.

Further Notes

(1) In investigation BC: patterns A_1-A_3 reduce ABL, whereas patterns C_1-C_3 increase ABL. This is in general agreement with El-Rayah's (45) result.

(2) A tentative explanation of why the ABL for the (/) pattern(s) is very high and that for the (\) pattern(s) is very low in investigations M, M&C and M&B, is that in pattern(s) (\), the slow worker is first in order, followed by a faster one, and so on. In this case, units enter the line at a relatively slow rate, and therefore, the ABL will be small in amount. However, once they leave operator 1, the units are processed in a comparatively shorter time. The opposite situation occurs for pattern(s) (/), where the fastest worker is first in order and hence, the units enter the system at a fast rate, leading to an increase in ABL, but the units wait for longer periods to be processed by the subsequent operators. In short, the first station controls the input to the system, given B/MB, which may result in high or low ABL, depending on the mean operation time of the controlling (first) operator.

STOCKHOLDING'S RESULTS

The results of the stockholding measures (other than that of ABL), for the best patterns with respect to ABL, in each of the six unbalanced lines' investigations, are summarised in Table A7.32 and presented graphically in Figures A7.75 - A7.84. On the basis of these tables and graphs, the apparent conclusions are exhibited in Figures 7.7 through 7.9.

FIGURE 7.7

THE EFFECTS OF THE EXOGENOUS VARIABLES ON L

RELATIONSHIP	GIVEN	M	C	BC	M&C	M&B	C&B
1. The higher B/MB the higher L is.	N,(DI)	✓	✓	✓	✓	✓	✓
2. The amount of the increase in L reduces as B/MB continues to rise.	N,(DI)	✓	X	✓	✓	X	X
3. The increase in L, as B/MB goes up, becomes higher with larger N.	(DI)	✓	X	✓	✓	✓	X
4. The rise in L tends to be greater wherever DI is lower.	N	✓a	NA	NA	✓	✓	NA
5. The higher N gives rise to increased L.	MB/B,(DI)	✓	✓	✓	✓	✓	✓
6. The increase in L, as a result of high N, tends to be greater the higher B/MB is.	(DI)	✓	✓a	✓	✓	✓	X

...Cont

FIGURE 7.7 (CONT)

RELATIONSHIP	GIVEN	M	C	BC	M&C	M&B	C&B
7. The rise in L, while N goes up, is larger as DI becomes lower.	B/MB	✓	NA	NA	✓	✓	NA
8. As DI increases, L is likely to decline.	N,B/MB	✓	NA	NA	✓	✓	NA
9. The continuing rise in DI leads to continuously diminishing fall in L.	N,B/MB	✓	NA	NA	✓	✓	NA
10. The decrease in L, as DI goes up, is larger the higher B/MB is.	N	✓a	NA	NA	✓a	✓	NA
11. The best pattern's L is lower than that of the balanced line.	N,B/MB,(DI)	✓	✓	✓	✓	✓	✓

✓a = true in most or nearly all the cases

X = unclear or nonexistent

NA = not applicable

FIGURE 7.8

THE DESIGN FACTORS' INFLUENCE ON SU

RELATIONSHIP	GIVEN	M	C	BC	M&C	M&B	C&B
1. Whenever B/MB increases, SU tends to diminish.	N,(DI)	✓	✓	✓	✓	✓	✓
2. The marginal decrease in SU lessens as B/MB is up.	N,(DI)	✓a	✓	X	✓	X	✓
3. Increasing DI reduces SU.	N,B/MB	✓	NA	NA	✓	✓	NA
4. The rate of reduction in SU becomes smaller as DI rises.	N,B/MB	✓a	NA	NA	✓	✓	NA
5. SU drops with the increase in N.	MB/B,(DI)	X	✓a	✓	✓	X	✓
6. The SU of the best pattern is lower than that of the balanced line counterpart.	N,B/MB,(DI)	✓	✓	✓	✓	✓	✓

✓a = true in most or nearly all the cases

X = unclear or nonexistent

NA = not applicable

Notes on the Relationships of Figure 7.7

Relationship Number	Note
1	* This is in general agreement with Slack's (160) results for the balanced line.
	* In investigation C: this seems to give backing to Anderson et al's (3) finding.
2	* This generally agrees with the balanced line's results of Slack (160).
4	* In investigation M: true, except for DI = $0.12 \rightarrow 0.18$, B = $2 \rightarrow 6$, N = 8.
5	* The result is in line with that of Slack (160) - balanced lines.
	* In investigation C: this confirms the finding of Anderson et al (3).
6	* In investigation C: valid, with the exception of B = 6.
10	* In investigation M: the exception is that of N = 8, B = $2 \rightarrow 6$, DI = $0.12 \rightarrow 0.18$.
	* In investigation M&C: the exception occurred at N = 5, B = $1 \rightarrow 2$, DI = $0.05 \rightarrow 0.12$.

Notes on the Relationships of Figure 7.8

Relationship Number	Observation
1	* This is generally in line with the balanced line's results of Slack (160).
	* In investigations BC and M&C: true, especially for low N.

2	*	In investigation M: valid, except for one instance; that of N = 5, DI = 0.02.
	*	In investigation M&B: the marginal decline diminishes, as DI rises.
3	*	In investigation M&B: the decrease is higher for larger MB.
4	*	In investigation M: with the exception of N = 5, B = 1.
5	*	This result generally agrees with that of Slack (160) for the balanced lines.
	*	In investigations M and M&B: the opposite is true.
	*	In investigation C: the exception is that of B = 1.
	*	In investigations BC and M&C: valid, especially for small B/MB.

ANOVA'S RESULTS

Tables A7.74 - A7.97 exhibit the analyses of variance for both I and ABL in the six unbalanced lines' investigations and for N = 5,8. The ANOVA was performed on the simulation data using the computer statistical package STATPAK. These ANOVA tables permit the drawing of some conclusions which are depicted in Figure 7.10.

Notes on the Conclusions of Figure 7.10

Conclusion Number		Observation
2	†*	In investigation C: except for the ABL's interaction, AB, for N = 5.
	†*	In investigation M&B: the exceptions for this

FIGURE 7.9

THE IMPACTS OF THE EXOGENOUS VARIABLES ON BU

RELATIONSHIP	GIVEN	M	C	BC	M&C	M&B	C&B
1. As B/MB becomes higher, BU declines.	N,(DI)	✓	✓	✓	✓	✓	✓
2. While DI rises, BU tends to fall.	N,B/MB	✓	NA	NA	✓	✓	NA
3. BU drops with increased N.	B/MB,(DI)	X	✓	✓	✓	X	✓
4. The balanced line's BU is higher than that of the best unbalanced pattern.	N,B/MB,(DI)	✓	✓	✓	✓	✓	✓

X = unclear or nonexistent

NA = not applicable

FIGURE 7.10

THE SIGNIFICANCE OF THE MAIN, INTERACTION, AND SUBRUN EFFECTS IN ANOVA

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	C&B
1. All the main effects are highly significant at the 0.99 level with very high F values, for both I and ABL.	N	√	√	√	√	√	√
2. All the interactions are significant at at least the 0.95 significance level. †This is true for both I and ABL.	N	√	√a	√	√	√a	√
3. The subruns' effect on I and ABL appears to be nonsignificant, giving backing to the contention that all the data represent the steady state conditions.	N	√	√	√	√	√	√

√a = true in most or nearly all the cases.

result are the I's interaction, AB, for N = 8, the ABL's interaction, ABC, for N = 5, and the ABL's interaction, BC, for N = 8.

In terms of the respective F values, it looks that the relative importance (order) of the factors affecting I and ABL in each of the unbalanced lines' investigations, and for both N = 5,8 is as follows:-

Endogenous Variable	Investigation	1st Factor	2nd	3rd	4th
I	M	DI	B	Means pattern	-
	C	B	Covars pattern	-	-
	BC	MB	Buffers pattern	-	-
	M&C	B	DI	Covars pattern	Means pattern
	M&B	MB	DI	Means pattern	Buffers pattern
	C&B	MB	Covars pattern	Buffers pattern	-
ABL	M	B	Means pattern	DI	-
	C	B	Covars pattern	-	-
	BC	MB	Buffers pattern	-	-
	M&C	B	DI	Means pattern	Covars pattern
	M&B	Means pattern	MB	DI	Buffers pattern
	C&B	MB	Covars pattern	Buffers pattern	-

From the above it seems justifiable to enlist the following conclusions:-

(1) For both I and ABL the buffers' pattern is always the last contributor to the main effect. This is compatible, in the case if I, with the result reported earlier in the chapter that the influence of the buffer capacities' imbalance is less important than that of the means or the Covars imbalance pattern.

(2) The most influential main effect in 5 out of 6 unbalanced lines' investigations, for both I and ABL, is B/MB.

(3) DI is a very important factor in terms of its impact on I (the first in investigation M, and the second in investigations M&C and M&B), but is much less significant with regard to its effect on ABL.

(4) The pattern of imbalance is always the least important regarding its influence on I, whereas it is more prominent (and the most prominent for investigation M&B) with respect to its effect on ABL.

(5) The F values of I for the investigations of two sources of imbalance are higher (more highly significant) than the corresponding values for the investigations of one cause of unbalance.

(6) The F values of ABL are higher for N = 8 than those for N = 5 in all the investigations, with the exception of investigation M&B.

MULTIPLE REGRESSION'S RESULTS

The simulated data were subjected to the multiple regression analysis and expressions were developed employing STATPAK. This computer package also conducts an ANOVA on the resulting regression equations, in order to determine whether or not the bulk of the variation in the endogenous variables is attributed to the regression line which was being fitted to the data.

It should be noted that the regression was only performed on the best patterns in terms of both I and ABL in each of the investigations because, firstly, these patterns are generally the ones of interest which warrant further examination and, secondly, a generalised regression equation which includes all the patterns is of little value compared to the great amount of time and effort needed to prepare the input data and to execute the regression analysis.

Note also that it was verified that linear equations were unrepresentative of the data on both I and ABL, resulting in low values of R^2. Consequently, it was decided to enlarge the regression model through the addition of more variables, and to use nonlinear (mainly quadratic) equations. These were found to be adequate and representative of the data. Furthermore, the particular variables to be included in the regression equation, were determined through the employment of the step-wise regression. Several formulations of the entering variables were tested and the one resulting in the highest R^2 was selected.

Since no study has, as yet, provided any expression relating I and ABL to the exogenous variables for unbalanced lines operating under SS conditions, it is hoped that the developed formulae will help in filling a gap in this area.

Tables 7.2 and 7.3 show the multiple regression equations for both I and ABL in each of the six unbalanced lines' investigations.

It should be observed that it is possible to derive regression formulae for L, SU and BUby substituting their expressions (see Chapter 1) into the preceding regression equations of ABL. From the foregoing tables it appears valid to conclude the following:

(1) The coefficient of multiple determination, R^2, for all the equations of I and ABL is very high, indicating a high degree of dependency of the endogenous on the exogenous variables. This means that all the regression equations provide very efficient fits to the data. This is true for all the investigations and the values of N.

(2) All the values of F are highly significant at the 0.99 level, implying that R^2 is statistically significant, i.e. the regression of I and ABL on the exogenous variables could not have occurred by chance. This is valid for all the unbalanced lines' investigations, given N.

It must be emphasised that although all the regression expressions have fitted the data remarkably well, these expressions, being obtained for simulated outcomes, may only be employed for situations whose factor levels are inside the range used in these simulation investigations.

CANONICAL CORRELATION'S RESULTS

The computerised STATPAK provides also for the execution of canonical correlation analysis on the data. This analysis

TABLE 7.2

% TOTAL IDLE TIME –
REGRESSION EQUATIONS FOR THE BEST PATTERNS

TYPE OF IMBALANCE	LINE LENGTH	REGRESSION EQUATION	R^2
Means	5,8,10	$I = 11.276 - 4.984N - 0.151B + 0.668DI + 0.519N^2 + 0.012B^2 - 0.019DI^2 + 0.056 N.B - 0.054N.DI + 0.002B.DI$ $(F = 122.695)^{**}$	0.9832
Covars	5	$I = 10.698 - 3.407B + 0.309B^2$ $(F = 50.720)^{**}$	0.9855
	8	$I = 16.612 - 4.461B + 0.372B^2$ $(F = 82.078)^{**}$	0.9910
Buffer Capacities	5	$I = 9.152 - 2.034MB + 0.135MB^2$ $(F = 351.915)^{**}$	0.9993
	8	$I = 11.634 - 2.581MB + 0.173MB^2$ $(F = 159.947)^{**}$	0.9984
Means & Covars	5	$I = 11.390 - 4.810B + 0.064DI + 0.523B^2 + 0.007DI^2 + 0.028B.DI$ $(F = 65.579)^{**}$	0.9955
	8	$I = 16.293 - 5.604B + 0.168DI + 0.540B^2 + 0.010DI^2 + 0.049B.DI$ $(F = 199.875)^{**}$	0.9994
Means & Buffer Capacities	5	$I = 7.364 - 1.072MB + 0.049DI + 0.051MB.DI$ $(F = 101.915)^{**}$	0.9968
	8	$I = 7.928 - 1.072MB + 0.075DI + 0.041MB.DI$ $(F = 9.152)^{**}$	0.9949
Covars & Buffer Capacities	5	$I = 5.473 - 1.238MB + 0.093MB^2$ $(F = 27.119)^{**}$	0.9909
	8	$I = 10.582 - 2.395MB + 0.167MB^2$ $(F = 105.786)^{**}$	0.9994

R^2 = co-efficient of multiple determination (correlation)
** = F is significant at the 0.99 level
B = buffer capacity
MB = mean buffer capacity
I = % mean total idle time
DI = % degree of imbalance

TABLE 7.5

MEAN BUFFER LEVEL -
REGRESSION EQUATIONS FOR THE BEST PATTERNS

TYPE OF IMBALANCE	LINE LENGTH	REGRESSION EQUATION	R^2
Means	5,8	$ABL = 0.218-0.325N+0.173B-0.091DI-0.033N^2-0.054B^2-0,027DI^2+0.039N.B-0.42N.DI-0.09B.DI$ $(F = 153.196)^{**}$	0.9718
Covars	5	$ABL = 0.482-0.086B+0.067B^2$ $(F = 64.675)^{**}$	0.9886
	8	$ABL = -0.025+0.301B-0.016B^2$ $(F = 224.457)^{**}$	0.9967
Buffer Capacities	5	$ABL = 0.353+0.051MB+0.024MB^2$ $(F = 133.642)^{**}$	0.9981
	8	$ABL = -0.067+0.315MB-0.011MB^2$ $(F = 194.426)^{**}$	0.9996
Means & Covars	5	$ABL = 0.246+0.199B-0.066DI-0.010B^2+0.004DI^2-0.008B.DI$ $(F = 22.578)^{**}$	0.9870
	8	$ABL = 0.166+0.166B-0.032DI-0.003B^2+0.002DI^2-0.008B.DI$ $(F = 128.219)^{**}$	0.9991
Means & Buffer Capacities	5	$ABL = 0.087+0.206MB+0.006DI-0.015MB.DI$ $(F = 13.901)^{**}$	0.9769
	8	$ABL =-0.104+0.304MB+0.025DI-0.023MB.DI$ $(F = 13.498)^{**}$	0.9762
Covars & Buffer Capacities	5	$ABL = 0.163+0.071MB+0.005MB^2$ $(F = 10.353)^{**}$	0.9767
	8	$ABL = 0.125+0.063MB+0.004MB^2$ $(F = 72.165)^{**}$	0.9966

R^2 = co-efficient of multiple determination (correlation)
** = F is significant at the 0.99 level
B = buffer capacity
MB = mean buffer capacity
ABL= mean buffer level
DI = % degree of imbalance

was performed on the data pertaining to investigations M, M&C and M&B for both I and ABL. The canonical correlation analysis for the remaining investigations, on the other hand, was not possible because in these investigations there is only one exogenous variable, viz, B/MB, whereas the analysis requires at least two such variables in order to determine their relationship with the endogenous variables I and ABL.

Tables 7.4 and 7.5 present the canonical correlations.

COMPARISONS WITH CONTROL'S RESULTS

After finding that all the overall F tests of the main effects in ANOVA are significant, it was decided that performing multiple comparisons (both pairwise and with control) would be an appropriate step. As far as the comparisons with control are concerned, Tables A7.33 - A7.42 depict their results, for both I and ABL, in all the unbalanced lines' investigations. From these tables, the conclusions exhibited in Figures 7.11 and 7.12 seem feasible.

Notes on the Conclusions of Figure 7.11

Conclusion Number		Comment
1a	*	In investigation M&C: except for the aforementioned cases, where the increase in DI diminishes I.
1b	*	In investigation M&C: the same exception as that for (1a). In addition, except for N=8, B=1, patterns ($\setminus$)+P_7; (V)+$P_{1,3,4}$; (Λ)+P_4. Furthermore, aside from the above exceptions,

TABLE 7.4

CANONICAL CORRELATION ANALYSIS - I

INVESTIGATION	LINE LENGTH	CANONICAL EQUATION	CANONICAL CORRELATION
M	5,8	0.472I-0.882ABL = 0.082DI-1.179B+0.100N	0.9684
M&C	5	0.015I-0.9998ABL = 0.121DI-1.005B	0.9934
	8	0.037I-0.999ABL = 0.160DI-1.023B	0.9829
M&B	5	0.441I-0.898MB = 0.103DI-1.231MB	0.9999
	8	0.468I-0.901MB = 0.128DI-1.406MB	0.9993

TABLE 7.5

CANONICAL CORRELATION ANALYSIS - ABL

INVESTIGATION	LINE LENGTH	CANONICAL EQUATION	CANONICAL CORRELATION
M	5,8	0.998I-0.060ABL = 0.319B-0.890DI-0.090N	0.9646
M&C	5	0.152I-0.988ABL = 0.655B-0.910DI	0.9745
	8	0.231I-0.973ABL = 0.913B-0.763DI	0.9763
M&B	5	0.306I-0.952ABL = 0.350MB-0.569DI	0.9874
	8	0.178I-0.984ABL = 0.528MB-0.630DI	0.9845

FIGURE 7.11

IDLE TIME'S CONCLUSIONS - COMPARISONS WITH CONTROL

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	C&B
1. (a) Increasing DI raises the control's advantage (if exists) over the unbalanced pattern, and	N,B/MB, pattern	✓	NA	NA	✓a	✓	NA
(b) moves it towards becoming significant, or more highly significant.	N,B/MB, pattern	✓	NA	NA	✓a	✓a	NA
2. When DI is relatively low, the control is not significantly better than the unbalanced pattern.	N,B/MB, pattern	✓a	NA	NA	X	✓a	NA
3. The increase in DI either reduces then negates, or immediately wipes out, the advantage of the unbalanced line's pattern (if exists) over the balanced line (control), especially for higher B.	N,B/MB pattern	✓	NA	NA	✓a	✓	NA
4. As B/MB becomes higher, the superiority of the control over the unbalanced pattern moves in the direction of being less highly significant, or nonsignificant, or it may remain the same, be it significant or insignificant.	N,(DI), pattern	✓✓	✓	✓	✓a	✓a	✓a

...Cont

FIGURE 7.11 (CONT)

CONCLUSION	GIVEN	M	C	BC	M&C	M&B
5. The rise in B/MB lessens, negates, or keeps constant the DI whereby an improvement in the unbalanced line's performance over that of the control takes place.	N,pattern	✓	NA	NA	✓	≈
6. Wherever B/MB goes up, the saving of the unbalanced pattern over the control diminishes then disappears, or remains the same, or disappears immediately.	N,(DI), pattern	✓	✓a	≈	✓a	≈
7. Point for point, the gain of the balanced line over the unbalanced counterpart becomes more significant (or stabilises) as N goes up from 5 to 8.	B/MB,(DI), pattern	≈	✓	✓	✓a	✓ a
8. The N = 5 (a) has more cells whereby an unbalanced design produces savings over the control and (b) achieves higher improvements than N = 8.	B/MB,(DI), pattern	≈	✓	✓	✓	✓

...Cont

FIGURE 7.11 (CONT)

	CONCLUSION	GIVEN	M	C	BC	M&C	M&B
9.	In total, the best unbalanced pattern has the least (or at least the same) number of significantly inferior cells, as compared to the control.	N,(DI)	✓	✓	X	✓	✓
10.	Overall, the worst pattern(s) results in the most (or at least the same) number of significantly unfavourable points, compared to those of the control.	N,(DI)	✓	✓	✓	✓	✓
11.	The total number of significantly or insignificantly superior cells of the unbalanced pattern over those of the control is higher in the best pattern than any other unbalanced pattern.		✓	✓	✓	✓	✓a
12.	The best and other unbalanced patterns result in significant advantages over the control especially for low values of N and B/MB.		X	X	X	✓	X

...Cont

FIGURE 7.11 (CONT)

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	C&
13. The gain of the best pattern over the control is greater than that obtainable by any other unbalanced pattern.	N,B/MB,(DI)	√	√	√	√	√	[illegible]
14. DI can be moderately or substantially increased, and still yields a slightly lower or higher amount of I than that achievable by a balanced line.	Best pattern	√	NA	NA	√	√	N

≉ = the opposite is true

NA = not applicable

√a = true in most or nearly all points

X = unclear or nonexistent

nearly all the points of DI=0.12 are significant at the 0.99 level.

* In investigation M&B: true, with the exception of some points in patterns (V)+A,B,D_1-D_3. In addition, except for patterns (V)+A-D_3, N=5, B=2,6, each of the DI=0.12 points is significant at the 0.99 level.

* In investigation M: all the points of DI=0.18 are significant at the 0.99 level, the single exception is that of B=1, N=8, pattern (V).

2 * In investigation M: when DI=0.05, this result is true for pattern (V) only.

3 * In investigation M&C: increasing DI in patterns $(\wedge)+P_{4,5}$ result in a reduction (but not the disappearance) in the unbalanced patterns' advantage, in all but one point.

4 * In investigation M&C: the reverse is true in a few cells, especially where (<) exists and DI=0.05.

* In investigation M&B: the inverse occurred at DI=0.02, N=5, patterns $(\backslash)+D_2$; $(/)+D_3$; $(\wedge)+D_3$.

* In investigation C&B: the opposite situation took place at N=5.

5 * In investigation M: this lends support to the conclusion of El-Rayah (46).

6 * In investigation M: this result in its general form does not contrast with that of El-Rayah (46).

* In investigation C: this is true for pattern P_8, while the inverse is valid for P_7. Support to the latter result can be found in De La Wyche

& Wild's (39) study, where increasing B from 0 to 1, raised the % superiority of patterns VSSV, VSVS, VSM, MSV over the datum (the balanced line), respectively, from 5.11 to 18.31, from 1.63 to 5.21, from 1.53 to 11.71, and from 1.46 to 3.28. On the other hand, in some other patterns, such as MVS and SMV, as B goes up, their improvement disappears.

* In investigation BC: the occurance of the reverse situation is supported by the work of De La Wyche & Wild (39), which indicated that for N=9, Covar = 0.3, TB=8 (i.e. MB=1) no gain over the balanced line was registered by any unbalanced pattern, but when TB was increased to 16 (MB of 2), a saving of 1.75% was achieved for pattern 1,2,2,3,3,2,1,1 which means that raising MB can create advantages for some unbalanced patterns over the control, or increase the existing ones.
* In investigation M&C: this supports the finding of Rao (142).
* In investigation M&C: the exception for that is pattern (V)+P_1, where the opposite took place.
* In investigation B&C: true, with the exception of patterns A+P_5 and B+P_3, where the inverse happened.

7 * In investigation M&C: valid only for patterns (V), (Λ)+P_i, while for patterns (/), (\)+P_i there is no clear relationship.

8a * In investigations C, BC, M&B, and C&B: the savings exist only for N=5.

8b * In investigation M: this agrees with Hillier & Boling's (75) and El-Rayah's (46) results.

* In investigation BC: this is not in line with El-Rayah's (45) finding. Part of the reason why this constrast occurs probably lies in the fact that the patterns of N=5 and 8 in this research are unidentical, as well as that the conditions simulated, especially the shapes of the patterns, are dissimilar between the two works.

9 * In investigations M, M&C and M&B: valid for all the levels of B/MB.

10 * In investigations M and BC: all the points are at least significant at the 0.95 level.

* In investigations C, M&C and C&B: all the cells are significant at the 0.99 level.

* In investigations M, M&C and M&B: true for all the values of B/MB.

11 * In investigations M&B and B&C: the exceptions are, respectively, patterns (V)+C and D_3+P_5, which have the same number of cells.

12 * In investigation M&C: these points are N=5, B=1, DI=0.02,0.05, patterns $(/)+P_4$ and $(\wedge)+P_5$.

* In investigation B&C: the relevant points are those of N=5, MB=2, patterns $C+P_4$, $D_3+P_{4,5}$.

14 * In investigation M: for low or moderate B in the best pattern DI can be 0.05 and I will still be slightly higher than that of the balanced line. In addition, DI can be as much as 0.12 for N=5,

B=1,2 and 0.18 for N=8, B=1, without rendering I significantly higher than that of the control. As B tends to become higher, however, I becomes significantly higher than that obtainable by the control even for a relatively small DI's value. This is, in general, the same result as El-Rayah's (46) though the values of N, B, DI differ between the two studies.

* In investigation M&C: for the best patterns it is quite feasible that DI will rise to 0.12 (when B=1 and N=5,8) and still some saving over the control will be produced. The general shape of this conclusion agrees with that of Rao (142).
* In investigation M&B: the DI for pattern $(V)+D_1$, N=5 can reach 0.12 and still I is not significantly higher than that of the balanced line, whereas for N=8, DI can be 0.05 without generating significantly higher I than that obtainable by a balanced line.
* In all the above investigations a DI in the range $0.05 < DI < 0.12$ was not considered, and therefore, it is not known if a DI within this range will still result in I which is very close to that of the balanced line.
* The DI in investigation M&C can go up much higher than in investigation M (for B=1), but this advantage is lost quickly, as B increases, even when DI is low. In investigation M, on the other

hand, the gain over the control for DI=0.02, all N continues for B as high as 6 units. Improvement for B > 6 may result, but the B=6 was the maximum value experimented with.

Further Conclusions

(1) In addition to the best pattern, at least one less favourable pattern also resulted in preferable performance over that of the control. In investigation M patterns (/), (∧), as well as the random one, showed some gains, but these were cancelled as DI went up to 0.05. On the other hand, pattern (\) did not show any improvement. In investigations C and M&B patterns P_8 and (V)+C,D_1 respectively achieved advantages. In investigations M&C patterns (/), (\), (V)+P_{1}, 4,5; (∧)+P_4, (\)+P_7 all generated gains. The above, in essence, implies that if the most promising unbalanced pattern is impracticable, one or more alternative designs may be utilized to ensure superior I's results over those achievable by a balanced line.

(2) In investigations M and M&C: the best pattern attains advantage over the balanced line within a wider range of DI than that of the other less efficient unbalanced patterns.

Notes on the Conclusions of Figure 7.12

Conclusion Number	Comment
1	* In investigation M: true also for the random pattern. * In investigation M&B: valid also for patterns (V)+C,D_2.

FIGURE 7.12

MEAN BUFFER LEVEL'S CONCLUSIONS - COMPARISONS WITH CONTROL

CONCLUSION	GIVEN	M	C	BC	M&C	M&B
1. The superiority of the control over the unbalanced (/) patterns increases and moves towards becoming significant (or more highly significant), as DI tends to rise.	N,B/MB,(/) pattern(s)	√	NA	NA	√	√
2. As DI goes up, the advantage of the unbalanced pattern over the control increases and becomes significant, or more highly significant.	N,B/MB,(\) pattern(s)	√	NA	NA	√	√
3. Increasing B/MB raised the unbalanced pattern's saving over the balanced line.	N,(DI)	√a	√a	√a	√a	√a
4. Whenever B/MB rises, the gain of the control over the unbalanced pattern becomes less significant, or maintains its level, be it significant or not.	N,(DI)	√a	√	√	√a	√a

...Cont

FIGURE 7.12 (CONT)

CONCLUSION	GIVEN	M	C	BC	M&C	M&B
5. The advantage of the control over the unbalanced pattern is more significant or remains relatively at the same level (significant or not) for N=8 than for N=5.	B/MB,(DI), (/)pattern(s)	√	X	√a	√a	√a
6. The potential improvement of the unbalanced pattern over the balanced one increases and becomes more significant (or stays relatively constant) for N=5 than for N=8.	B/MB,(DI)	√a	√a	√a	√a	√a
7. The best pattern has no inferior points as compared to those of the control.	N,B/MB,(DI)	√	√	√	√	√
8. There are more (or at least the same number of) significantly or insignificantly superior points over the control in the best pattern than any of the other unbalanced patterns.	N,B/MB,(DI)	√	√	√	√	√

...Cont

FIGURE 7.12 (CONT)

	CONCLUSION	GIVEN	M	C	BC	M&C	M&B
9.	Significantly advantageous cells, in comparison with those of the control, exist for the best and, in many cases, the other unbalanced patterns, especially for low N and B/MB.	N,B/MB	√	√	√	√	√
10.	The worst pattern(s) has the most significantly disadvantageous points, as compared to those of the balanced line.	N,B/MB,(DI)	√	√	√	√	√
11.	The unbalanced pattern's ABL is significantly lower than that of the control, in all the cells.	N,B/MB,(DI)	√a	√a	√a	√a	√a

NA = not applicable

√a = true in most or nearly all points

X = unclear or nonexistent

2 * In investigation M: the opposite is true for the random pattern.

* In investigation M&B: true also for patterns (V)+B, (Λ)+A (N=5). The opposite is true in M for the random pattern.

* In investigation M&C: the reverse is true for patterns (/)+$P_{4,6,8}$.

* In investigation M&B: the inverse situation took place for patterns (/)+A,D_2.

3 * In investigation M: this is true only for pattern ($\backslash$), with the exception of N=5, DI=0.02, B=1.

* In investigation C: the result is valid only for patterns $P_{1,2,4-6}$ (N=5); P_7 (N=8).

* In investigation BC: true for N=5, all the patterns (except for D_6); N=8, patterns C_1-C_3. The opposite is valid for N=8, patterns A_1-A_3, B_1, D_3-D_4.

* In investigation M&C: true for the ($\backslash$) patterns, except for ($\backslash$)+$P_{1,3,6,8}$ N=8, DI=0.02; P_7, N=8, DI=0.12; P_8, N=8, DI=0.15. True also for patterns (V)+P_6, N=8; (V)+P_8, N=5; (Λ)+P_3, N=5. The inverse occurred at patterns (Λ)+P_6, (/)+P_4, (/)+P_6 (N=8).

* In investigation M&B: true for the ($\backslash$) patterns, with the exceptions of patterns ($\backslash$)+A, N=5,8, DI=0.02; ($\backslash$)+B, N=8, DI=0.02. Valid also for pattern (Λ)+D_3. The reverse took place for patterns (V)+A, (Λ)+A.

* In investigation B&C: true for patterns A-D_3+P_3, 5,7,8 (N=5) and A-D_3+$P_{2,5,7}$ (N=8).

4 * In investigation M: true only for pattern (/).

* In investigation M&C: true for patterns $(/)+P_{1-3,5,7}$; $(V)+P_{5,7}$; $(\wedge)+P_{3,5,7}$.

* In investigation M&B: valid for the (/) patterns and pattern $(\wedge)+D_1$.

5 * In investigation BC: the exception for that is pattern C_2.

* In investigation M&B: the single exception occurred at pattern (/)+A.

* In investigation B&C: except for patterns $B+P_{2,3}$ MB=6.

6 * In investigation M: true only for pattern (\). The exceptions for this took place at B=1, DI=0.05; B=2, DI=0.02.

* In investigation C: valid only for patterns P_8 and P_{10}.

* In investigation BC: true, with the exception of patterns A_3, MB=2 and D_4, MB=2.

* In investigation M&C: valid for patterns $(\backslash)+P_{5,7}$; $(\backslash)+P_{1-3,8}$ (except for B=1, DI=0.02,0.05). The opposite happened for the (/) patterns and patterns $(V)+P_{4,6}$.

* In investigation M&B: true for the (\) patterns (except for MB=2, DI=0.02) and pattern (\)+D. The inverse occurred for pattern (/)+A.

11 This is true for the following investigations and patterns:-

Investigation	Pattern	Exceptions
M	$(\backslash)$	N=5, B=2, DI=0.02; N=8, DI=0.02, all B
C	P_7	-
C	P_{10}	B=6, N=5
BC	A_1-A_3	-
M&C	$(\backslash)+P_{4,6}$	-
M&B	$(\backslash)+A,B$	N=8, B=6, DI=0.02
B&C	Most patterns	-

Further Notes

(1) In investigations M, C and M&C: all the savings of the best patterns over the control are significant at the 0.99 level. The single exception in investigation M is at DI = 0.02, where the savings are significant at the 0.95 level only. The exception in investigation C, on the other hand, is at B = 6, N = 5, where no significant improvement was recorded.

(2) In all the unbalanced lines' investigations the best pattern in terms of ABL yielded gains over the control for all the levels of N, B/MB, (DI), i.e. the best pattern produced consistent improvements, whereas the best pattern with respect to I did not provide this consistency.

PAIRWISE COMPARISONS' RESULTS

Tables A7.53 - A7.73 provide summaries of the I's and ABL's pairwise comparisons for all the imbalanced lines' investigations. From these tables the conclusions portrayed in Figures 7.13 and 7.14 seem warranted.

FIGURE 7.13

IDLE TIME'S CONCLUSIONS - PAIRWISE COMPARISONS

CONCLUSION	GIVEN	M	C	BC	M&C	M&B
1. As DI goes up, it moves the best pattern's advantage over the other unbalanced patterns in the direction of being more significant, or at least keeps it at the same level (significant or nonsignificant).	N, B/MB, pattern	√a	NA	NA	√a	√
2. Whenever B/MB is up, the gain of the best pattern over the rest becomes less significant or may remain relatively constant (significant or not).	N, (DI), pattern	X	√	√	√a	√a
3. Decreasing N raises the saving of the best pattern over the other patterns, or stabilises it.	B/MB, (DI), pattern	√	√	√^	√a	√^

...Cont

FIGURE 7.13 (CONT)

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	C
4. The second best pattern(s) has the least (or sometimes the same) number of significantly detrimental points, as compared to those of the best pattern.	N,B/MB,(DI)	√	√	√	√	√	[illegible]
5. The worst pattern(s) contains most of the unfavourable points, in comparison with those of the best pattern.	N,B/MB,(DI)	√a	√	√	√	√	[illegible]
6. The best pattern is superior, in all its points, over the rest of the unbalanced patterns.	N,B/MB,(DI)	√	√	√	√	√	[illegible]

NA = not applicable

√a = true in most or nearly all points

X = unclear or nonexistent

≈ = the opposite is true

Notes on the Conclusions of Figure 7.13

Conclusion Number		Comment
1	*	In investigation M: except for N=5, B=1 in pattern (∧).
	*	In investigation M&C: the reverse is true for the cases where the curve declines.
2	*	In investigation M&C: true only for the (V) patterns.
	*	In investigation M&B: valid only for patterns (V), (∧)+A-D_3.
	*	In investigation B&C: the exceptions for this are patterns D_1+P_{5-8}.
3	*	In investigation M: the exceptions took place at B=1, DI=0.02,0.05.
5	*	In investigations M, BC, M&C and M&B: most of the differences between the best and the inferior patterns are at least significant at the 0.95 level.
	*	In investigations C and C&B: all the differences are significant at the 0.99 level.

Notes on the Conclusions of Figure 7.14

Conclusion Number		Note
1	*	In investigations M&C and M&B: true for the (/), (V), and (∧) patterns, but the opposite situation resulted in the (\) patterns.
2	*	In investigations M&C and M&B: true only for the (/) patterns.

FIGURE 7.14

MEAN BUFFER LEVEL'S CONCLUSIONS - PAIRWISE COMPARISONS

CONCLUSION	GIVEN	M	C	BC	M&C	M&B
1. As DI tends to increase, the advantage of the best pattern over the rest becomes more highly significant, or it remains at the same level (significant or not).	N,B/MB, pattern	✓	NA	NA	✓a	✓a
2. Whenever DI becomes higher, it accelerates the movement of the best pattern's saving over the other patterns towards being significant, faster than that of I.	N,B/MB, pattern	✓	NA	NA	✓a	✓a
3. Increasing B/MB decreases the improvement achieved by the best pattern over the other patterns and either makes it less significant, or maintains it at the same level of significance.	N,(DI), pattern	X	✓	✓	✓a	✓a

...Cont

FIGURE 7.14 (CONT)

CONCLUSION	GIVEN	M	C	BC	M&C	M&B
4. As N rises, the gain of the best pattern over the rest becomes larger and more significant (or at least stays at the same significance level).	B/MB,(DI), pattern	√a	√	√a	√a	≫
5. The second best pattern obtains the least inferior points to the best pattern, whereas the worst pattern has the most of such points.	N,B/MB,(DI)	√	√	√	√	√
6. The best pattern is favourable over the other unbalanced patterns for all the cells.	N,B/MB,(DI)	√	√	√	√	√

NA = not applicable

√a = true in most or nearly all points

X = unclear or nonexistent

≫ = the opposite is true

3 * In investigation M&C: except for patterns $(/)+P_4$, DI=0.02, N=8; $(V)+P_{4,6}$, N=5, DI=0.12; $(\wedge)+P_4$, DI=0.05 (N=5), DI=0.12 (N=8).

* In investigation M&B: with the exception of patterns (/)+A, N=5, DI=0.02; (V)+A, N=5, DI=0.05; ($\wedge$)+A, N=5, DI=0.12.

* In investigation B&C: the exception for this occurred at pattern D_1+P_4, N=8.

4 * In investigation M: except for the random pattern, B=6, DI=0.02.

* In investigation BC: the exception took place at patterns C_3 and D_5.

* In investigation M&C: true for all the points of the (\) patterns and for most of the other patterns' points.

* In investigation M&B: the opposite occurred in the (/), (\) and ($\wedge$) patterns, and in most of the points of the (V) patterns.

* In investigation C&B: the inverse is true in almost all the points.

Summary of Conclusions

Although the conclusions of these investigations have been presented in detail, it is worthwhile to summarise, in general terms, their findings. This is done under three headings -

(a) Which Imbalance Patterns are Best?

(b) What are the Attributes of the Best Patterms?

(c) How does the "Design" of the Line Affect the performance of an Unbalanced Line?

Best Patterns

One of the main conclusions of the S.S. investigations is that there is no overall best policy for all types of imbalance. So for example, although the bowl policy is generally a good one, there are some particular patterns of the bowl policy which are, in fact, inferior to other policies. Thus, the best policy will depend on other line design parameters, for example line length, as well as the degree of imbalance.

When imbalance is due to one source alone (M,C,BC) the best patterns (in terms of I) are as follows:

For means imbalance - a bowl pattern (V)

For Covars imbalance - a bowl arrangement (P_7)

For buffer capacity imbalance - as near as possible to balance (D_2 or D_1)

When two sources of imbalance are combined one would intuitively suspect that the best imbalance patterns, in terms of idle time, would combinations of the best imbalance patterns from the investigations when only one source of imbalance is used. This did not seem to be the case in all circumstances. However, combinations of the best patterns from single imbalance source investigations were always among the best for combined imbalance source investigations.

When we identify the best patterns in terms of average buffer level the following results obtain.

For means imbalance - a decreasing means order

For Covars imbalance - a bowl arrangement (P_7)

For buffer capacity imbalance - concentrating available total buffer capacity at the end of the line (A_1 and A_2)

Whereas in terms of idle time, combining the best patterns for single source imbalance provided only "among the best" patterns for double source imbalance, the results for average buffer level were far more definite. In all three cases double source imbalance patterns, which proved the best, could be precisely predicted by combining the optimal patterns from single source imbalance.

Attributes of Best Patterns

Perhaps the most surprising conclusion of the investigation is that under certain circumstances increasing the degree of deliberately induced imbalance can reduce the total idle time performance of the line. This phenomenon was noticed for M and C investigation. El-Rayah notes a similar phenomena but in his investigations he found it only for means imbalance and only when the initial idle time is lower than that of the balanced line. The implication of this result is that, whereas it may be assumed that both system and balancing losses subscribe to total idle time, it is unreasonable to assume that total idle time is always greater than, or equal to, the balancing and system losses taken individually. Furthermore, it would appear that certain types of imbalance can actively reduce the system loss component of total idle time. In fact, when El-Rayah's work is examined this conclusion may be drawn from his means imbalance results, although he does not discuss it.

Rao's conjecture is generally supported by the results. Thus if the imbalance of the lines Covars is variable, steady, medium (A form of bowl pattern) then the best pattern for any means imbalance combined with this is a reversed bowl.

Whereas the pattern of imbalance of means and Covars affects line behaviour significantly, the pattern of buffer imbalance has relatively little affect on line behaviour. This holds for performance both in terms of total idle time and average buffer level.

Effects of Design of Line

The primary line design parameters, line length and buffer capacity, affect the idle time performance of unbalanced lines in exactly the same way as they affect the performance of balanced lines. However, there is insufficient evidence to say whether the marginal rate of change of line performance with line length and buffer capacity is exactly the same for both balanced and unbalanced lines.

One of the most significant general results to come out of this investigation is that where means are imbalanced either alone or with anothersource of imbalance it is possible (using optimal or

CHAPTER EIGHT

UNBALANCED NON-STEADY STATE LINES - INVESTIGATIONS AND RESULTS

INTRODUCTION

The foregoing two chapters have focused on reporting the design factors, as well as the results and the conclusions, for six unbalanced production lines' investigations under steady state conditions. This chapter has the objective of complimenting these investigations, by attempting to uncover the operating characteristics of the same types of unbalanced lines when they operate under NSS conditions.

The motivation for researching into the transient behaviour of the unbalanced and unpaced manual lines, stems from the fact that very little is known about such lines. Since some real life manual flow lines spend a great deal of their working time under transient conditions (see Chapter 2), a natural extension of the work on the SS conduct is the investigation of the NSS counterpart, along with the comparison of their performance.

DESIGN OF EXPERIMENTS

The desire to experiment with a relatively extensive setting of factors' levels was hampered by the constraint of the limited availability of computer time. Therefore, it was decided to conduct a small-scale set of investigations by reducing the levels of most of the factors, while exploring the same six types of unbalanced lines, and adopting exactly the same basic factorial designs as those utilized in the SS near optimal imbalance patterns) to increase the degree of imbalance sometimes substantially, without signficantly increasing total idle time. In other words, idle time is relatively insensitive to degree of imbalance for good imbalance patterns.

investigations. This permits the achievement of the objective of this investigative part of the thesis without losing much information or affecting the generality of the results.

Thus, the particular exogenous variables (factors) included, together with their levels, are as follows:-

(1) Line length: 5, 8 for each investigation, i.e. the same values of N as those employed in the SS investigations.

(2) Buffer/mean buffer capacity: 1 (for investigations M, C, and M&C), 2 (for investigations B, M&B, and C&B), and 6 (for all the investigations), i.e. dropping the moderate values of B = 2 (for investigations M, C and M&C) and MB = 4 (for the remaining investigations), while maintaining the extreme values of B/MB to make the contrast between them clearly apparent.

(3) Degree of imbalance: 0.05, 0.12 for all the investigations, i.e. dropping DI = 0.02. The decision not to use this value was based on the finding of the SS investigations that, in almost all the cases, the effect of a very low DI on I and ABL, is minimal and consequently, only relatively moderate and high degrees of imbalance were chosen, in order to reduce the number of experimental cells. Furthermore, a DI of 0.18 was also not considered because it was shown, in the SS investigations, to greatly increase I.

(4) Pattern of Imbalance: since the number of patterns adopted in the SS investigations enlarged the sizes of the experiments, the decision was made that the brunt of the reduction in the exogenous variables' levels should be borne by the patterns of imbalance, and as follows:

(a) Investigation M: the random pattern was waived because its N = 5 patterns are not identical to those of N = 8.

(b) Investigations C and BC: only one pattern from each of the four basic policies is chosen; the one showing better I's performance than its counterparts. This effectively cut the number of patterns from 12 to 4 in the case of investigation C, and from 15 to 4 for investigation BC.

(c) Investigations M&C, M&B, and C&B: the same patterns used in investigations M, C, BC above were mixed, two at a time, to form the joint patterns. The exception took place in investigation M&C, where patterns MSSSV and MMSSSSVV (for N = 5,8), rather than patterns VSSSM and VVSSSSMM, were employed because the former patterns displayed lower I than the latter ones, in the SS investigations (i.e. patterns ($\wedge$)+MSSSV/MMSSSSVV were the best).

(5) <u>Number of cells</u>: 32 + 16 + 16 + 128 + 128 + 64 = 384 for the six investigations.

Tables A8.1 - A8.26 display the factorial design of each of these NSS investigations, together with its results of I and ABL. A key to the meanings of the symbols used in these tables is provided in Figure A8.1. Moreover, some additional factors' levels were simulated in order to obtain more points on the graphs. These are shown in Table A8.27.

As was explained in Chapter 5, each of the experimental cells were simulated 10 times with empty starting conditions, and a run length of 500p.c. divided into two subruns. The results on I and ABL for each investigation were then examined to determine the following:

(1) The manner in which the exogenous and the endogenous variables are related, and whether it conforms in general to that of the SS.

(2) The broad ranking of the policies and the patterns of imbalance.

In addition, when the best patterns regarding both I and ABL in each investigation were determined, two characteristics were computed, namely, the transient length and size (TS). The reason behind the concentration of the research on the best pattern is that in such scaled-down series of experiments it is mainly the superior pattern which figures prominently and, therefore, it is desirable to explore it further.

Insofar as the transient length is concerned, the Dunnett's t statistic for comparisons with control† was employed to test if significant differences exist between the NSS's I and ABL, and their SS counterparts (exhibited in Chapter 7), i.e. to determine if the transient run length of 500p.c. is sufficient. The procedure involves repeatedly testing the difference between the overall mean I/ABL of the two subruns and the corresponding SS mean.

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† This test is more appropriate than the z test utilized in Slack's (160) study on the transient behaviour of the balanced line, and required a far less number of observations.

With respect to TS, it may be indicated by several methods, e.g. the mean I/ABL after the elapse of an initial p.c. period, and the mean I/ABL after X% of the run length has elapsed. It was felt that the most convenient method would be the second and since the run consists of 2 subruns, TS is determined through dividing the mean I/ABL of the first subrun (the mean after 50% of the run has passed) by their SS counterparts.

Moreover, the decision was made not to obtain any endogenous variables other than I and ABL and not to apply any statisitical technique other than the Dunnett's test, due to the limited nature of this study. The results and conclusions on I and ABL for the six investigations will be presented in the following order:

(1) NSS Results - General.

(2) Transient Length's Results.

(3) Transient Size's Results:

(a) TS - Idle Time's Conclusions.

(b) TS - Mean Buffer Level's Conclusions.

NSS RESULTS - GENERAL

When consulting the aforementioned tables, several conclusions have energed, a summary of which is presented below:

(1) The NSS best patterns in terms of both I and ABL for all the six unbalanced lines' investigations are exactly the same as those of the SS.

(2) All the NSS unbalanced patterns exhibited similar

general relationships and operating characteristics to those identified with the stable conditions. This is true for all the values of N, B/MB, and DI. However, the absolute magnitude of these relationships and characteristics differed between the steady and non-steady states. This result is in parallel with that of Wild & Slack(179) for the balanced NSS lines.

(3) The difference in I (and ABL) between the first and the second transient subruns is much larger for B/MB=6 than for B/MB=1, given N,(DI). In addition, the difference for N = 8 is greater than that for N = 5, given B/MB,(DI). This indicates that the system will still be relatively unsettled, the higher B/MB and N are. This result is true for the following patterns in each investigation:

Endogenous Variable	Investigation	Pattern
I	M	(/), (V)
	C	P_1, P_2
	BC	A - D
	M&C	(/)+$P_{1,2,4}$; (\\)+$P_{2,3}$, (V)+P_1; (Λ)+P_2
	M&B	(/)+A-D; (V)+A-D
	C&B	P_1+B,D; P_2+B-D; P_3+A,B,D
ABL	M	(/),(\\),(V),(Λ)
	C	P_1 - P_4
	BC	A - D
	M&C	(/)+P_1-P_4; (V)+P_1-P_4; (Λ)+P_1-P_4
	M&B	(/)+A-D; (V)+A-D; (Λ)+A-D
	C&B	P_1+A-D; P_2+A,C; P_3+C,D; P_4+A-D

Note that the number of patterns as well as the amount of the difference between the two intervals are higher in ABL than in I.

to that of the SS, indicating that the system has started to settle down, or has already done so. The proximity of the values of the second period and those of the SS is higher in ABL than in I. The particular patterns whereby this result is valid, are listed below:

Endogenous Variable	Investigation	Pattern
I	M	-
	C	P_3
	BC	B
	M&C	$(/)+P_3$; $(\wedge)+P_3$
	M&B	-
	C&B	-
ABL	M	$(/)$, $(\wedge)$
	C	P_3
	BC	-
	M&C	$(/)+P_2-P_4$; $(\backslash)+P_{1,4}$; $(\wedge)+P_{1,2,4}$
	M&B	$(/)$+B,C; $(\backslash)$+A; $(\wedge)$+C
	C&B	-

(5) The buffer build up is very slow for B/MB=6, i.e. the difference between the second subrun's ABL and that of the SS is large. This result occurred for patterns (V), B, (V)+$P_{1,4}$ (V)+A-D in investigations M, B, M&C, and M&B respectively.

TRANSIENT LENGTH'S RESULTS

When the Dunnett's t value is insignificant, it implies that the mean I (or ABL) of the NSS period does not differ much from that of the steady state, i.e. 500 product cycles are sufficient for the SS to be approached, and vice versa. From Tables A8.1 - A8.26, it appears that the cases where the selected transient duration was insufficient are (for the best patterns with respect to I and ABL only) as follows:

Endogenous Variable	Investigation	N	B/MB	%DI	Level of Significance
I	M	8	6	5	*
	BC	8	6	-	***
	M&C	5	1	12	*
	M&C	8	6	5	**
	M&B	8	2	5	**
ABL	BC	8	6	-	*

Several conclusions seem to emerge from above. These are as follows:

(1) In the vast majority of the cells of I and ABL in each investigation, 500p.c. looks quite enough for the SS to be converged at. This largely justifies the selection of this period to account for the transient interval.

(2) The incidents at which significant differences were registered, are much less in ABL than in I. i.e. the chosen NSS duration is almost always sufficient in terms of ABL.

(3) In all but one instance the values of N and DI, whereby the interval of the transient was not enough, are N = 5, DI = 0.05.

(4) In most cases, the value of MB/B which resulted in insufficient warm-up period is B/MB = 6.

TRANSIENT SIZE'S RESULTS

Tables A8.1 - A8.13 and A8.14 - A8.26 show, respectively, the I's and ABL's transient sizes for all the unbalanced lines' investigations. The same information is portrayed graphically in Figures A8.2 - A8.7 and A8.8 - A8.13, respectively. From the examination of these tables and figures it looks reasonable to enlist the conclusions shown in Figures 8.1 and 8.2.

FIGURE 8.1

TS - IDLE TIME'S CONCLUSIONS

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	C
1. TS is > 1 since the mean I of the NSS period is greater than that of the SS counterpart.	N,B/MB,(DI)	✓	✓	✓	✓	✓	
2. TS is increased, and thus the transient period increases, as B/MB becomes higher.	N,(DI)	✓	✓	✓	✓	✓	
3. The increase in TS, as B/MB goes up, is higher the higher N is.	(DI)	X	≈	✓	≈	✓a	
4. With low DI the increase in TS, as a result of the rise in B/MB, is likely to become greater.	N	✓	NA	NA	✓a	✓	N
5. As DI rises, TS declines and, therefore, the NSS duration is shortened.	N,B/MB	✓	NA	NA	✓	✓	N
6. The fall in TS, while DI is increased, is more considerable for high N.	B/MB	✓a	NA	NA	X	✓	N

...Cont

FIGURE 8.1 (CONT)

CONCLUSION	GIVEN	M	C	BC	M&C	M&B	C
7. The drop in TS, as DI goes up, is likely to be higher whenever B/MB is high.	N	√	NA	NA	√	√	N
8. The TS for N=8 is higher than that for N=5, i.e. a higher N lengthens the transient interval.	B/MB,(DI)	√	√a	√	√a	√	
9. The increase in TS, when N rises, is larger as B/MB becomes higher.	(DI)	X	X	√	X	√	
10. The TS's increase, as N goes up, tends to be higher where DI is low.	B/MB	√a	NA	NA	X	√	N

√a = true in most or nearly all points

X = unclear or nonexistent

NA = not applicable

$\not\surd$ = the opposite is true

FIGURE 8.2

TS - MEAN BUFFER LEVEL'S CONCLUSIONS

	CONCLUSION	GIVEN	M	C	BC	M&C	M&B
1.	TS is < 1 since the mean ABL of the NSS interval is less than that of the SS counterpart.	N,B/MB,(DI)	√	√	√	√	√
2.	As B/MB is increased, TS is reduced, i.e. the buffer build up is decreased.	N,(DI)	√	√	√	√	√
3.	The reduction in TS, while B/MB goes up, is higher for low N.	(DI)	≈	√	√	≈	≈
4.	The drop in TS, when raising B/MB, is likely to become higher with the increasing of DI.	N	√a	NA	NA	√a	√
5.	As DI is increased, TS declines (buffer build up diminishes).	N,B/MB	√	NA	NA	≈	√
6.	The TS for N=8 is higher than that for N=5 (the lower N decreases the build up of the buffers).	B,(DI)	√a	√	≈	√	≈
7.	The rise in TS, when increasing N, becomes higher the larger B/MB is.	(DI)	X	√	≈	≈	√

NA = not applicable

√a = true in most or nearly all points

X = unclear or nonexistent

≈ = the opposite is true

Notes on the Conclusions of Figure 8.1

Conclusion Number	Comment
2	* This is in line with the NSS result of Slack (160) for the balanced lines.
	* In investigation C: the marginal increase in TS, when raising B, diminishes as B continues to go up.
3	* In investigation M&B: except for DI = 0.12.
4	* In investigation M&C: with the exception of DI = 0.05, N = 8.
6	* In investigation M: the exception for this is B = 6, DI = 0.05 → 0.12.
7	* In investigation M: the decrease in TS is very much higher.
8	* In investigations C and M&C: except for B = 6.
	* In investigation C&B: with the exception of MB = 2.
9	* In investigation M&B: the exception occurred at DI = 0.12.
10	* In investigation M: the exception for this took place at B = 6, DI = 0.05 → 0.12.

Notes on the Conclusions of Figure 8.2

Conclusion Number	
1	* True also for the vast majority of the other unbalanced patterns.
2	* This result agrees with that of Slack (160) for the NSS balanced lines.

4 * In investigation M and M&C: except for N = 5.

6 * This is not in agreement with Slack's (160) finding with regard to balanced lines operating under transient conditions.

* In investigation M: with the exception of B = 6.

* In investigation M&C: the exception for that occurred at DI = 0.12, B = 1,6.

<u>Summary of Conclusions</u>

One of the main objectives of this N.S.S. investigation was to determine the robustness of the best imbalance patterns when operating conditions were non-steady state. The most important conclusion is that the relative merit of the imbalance patterns, both in terms of I and ABL, is exactly the same under non-steady state and steady state conditions. This finding held for all types of imbalance. Furthermore the general relationships between line design and line performance were exactly the same under non-steady state and steady state conditions for all types of imbalance, although their absolute values were different.

The magnitude of the transient was shown to be influenced by line length and buffer capacity in generally the same form as has been previously demonstrated for balanced lines (160). In addition, the higher the degree of imbalance the shorter the transient magnitude was found to be when the means are imbalanced. If this result is combined with the results from chapter 7 it can be seen that the best imbalance patterns with certain degrees of imbalance have performance characteristics which are superior under all operating conditions to those of balanced lines.

P A R T   F O U R

DISCUSSIONS AND CONCLUSIONS

Part Four is comprised of one chapter:

CHAPTER NINE

DISCUSSIONS AND IMPLICATIONS

INTRODUCTION

Chapters 6 to 8 presented some detailed aspects of simulation experiments executed for six types of unbalanced production lines under SS and NSS conditions with several levels of B, N, and DI and different patterns. The outcomes of this series of simulations were then analysed, employing various statistical procedures and pertinent conclusions were derived. In this chapter an attempt will be made to compare the relative efficiencies of the six unbalanced lines' designs with respect to both I and ABL for their best unbalanced patterns. Furthermore, an attempt will be made to tackle the multi-response problem, by resorting to a simple utility function in order to evaluate the relative advantage of each investigation. Finally, some broad implications of this research to the theory and practice of the design of manual unpaced lines will be offered, together with several recommendations for future research in this area.

COMPARING THE PERFORMANCE OF THE BEST PATTERNS

Tables 9.1 and 9.2 (extracted from Tables A7.33 - A7.42) summarise the % reduction (-), or % increase in I and ABL for the best pattern in each investigation, in comparison with those of the balanced line. From these two tables the following observations seem to be appropriate:

(1) The lowest % gain in I over that of the control, among the six investigations, is achieved by investigation M for

TABLE 9.1

% REDUCTION OR INCREASE IN THE BEST UNBALANCED PATTERN'S I OVER THAT OF THE BALANCED LINE

N	B/MB	%DI	M	C	BC	M&C	M&B	C&B
5	1	2	-1.27	-10.17	-	-20.73	-	-
	1	5	0.31	-	-	-20.58	-	-
	1	12	6.41	-	-	- 4.48	-	-
	2	2	-0.88	-11.57	12.30	-16.12	9.03	-31.46
	2	5	1.79	-	-	- 0.18	22.27	-
	2	12	30.95	-	-	26.48	40.16	-
	6	2	-0.31	-43.08	-16.14	- 3.58	-11.57	-25.27
	6	5	22.80	-	-	20.43	17.33	-

...Cont

TABLE 9.1 (CONT)

N	B/MB	%DI	M	C	BC	M&C	M&B	C&B
	6	12	141.77	-	-	147.68	153.63	-
8	1	2	- 3.46	12.28	-	- 5.31	-	-
	1	5	- 2.46	-	-	- 4.56	-	-
	1	12	2.18	-	-	- 2.25	-	-
	2	2	- 2.74	41.63	19.83	20.37	5.73	9.30
	2	5	14.50	-	-	21.92	6.42	-
	2	12	23.90	-	-	30.68	30.43	-
	6	2	- 0.82	33.21	1.75	16.05	7.68	5.75
	6	5	18.86	-	-	27.65	29.16	-
	6	12	160.26	-	-	135.19	149.63	-

FIGURE 9.2

% REDUCTION OR INCREASE IN THE BEST PATTERN'S ABL, AS COMPARED TO THAT OF THE BALANCED LINE

N	B/MB	%DI	M	C	BC	M&C	M&B	C&B
5	1	2	-11.3	-24.53	-	-45.06	-	-
	1	5	-25.29	-	-	-61.22	-	-
	1	12	-52.66	-	-	-74.72	-	-
	2	2	-10.94	-38.24	-48.60	-59.25	-54.79	-69.31
	2	5	-48.11	-	-	-67.76	-65.83	-
	2	12	-71.83	-	-	-78.36	-79.14	-
	6	2	-49.65	-26.74	-56.13	-71.91	-51.22	-77.93
	6	5	-70.16	-	-	-83.35	-77.34	-

...Cont

FIGURE 9.2 (CONT)

N	B/MB	%DI	M	C	BC	M&C	M&B	C&B
	6	12	-87.56	-	-	-90.06	-89.58	-
8	1	2	-10.02	-49.02	-	-54.12	-	-
	1	5	-27.37	-	-	-64.94	-	-
	1	12	-49.20	-	-	-75.85	-	-
	2	2	-14.74	-52.17	-47.11	-59.95	-51.96	-73.35
	2	5	-33.92	-	-	-68.97	-60.31	-
	2	12	-64.23	-	-	-77.78	-74.64	-
	6	2	-15.96	-53.75	-45.52	-64.36	-37.37	-75.76
	6	5	-59.67	-	-	-73.16	-63.80	-
	6	12	-82.66	-	-	-84.95	-84.58	-

both N = 5,8, whereas the highest percentage saving depends on N, i.e. for N = 5 it is obtained by investigation C, while for N = 8 it is attained by investigation M&C.

(2) Investigation C produced the least % improvement in ABL over that of the balanced line for N = 5, while investigation BC yielded the least % advantage for N = 8. On the other hand, investigation M&C resulted in the greatest % saving for both N = 5 and 8.

(3) The highest % improvement in I over that of the control for investigation M&C, is greater than that of M, but less than that of C. In addition, in investigation M&B this greatest % saving is higher than that of M, but lower than that of BC, whereas for investigation C&B it is respectively, $>$ and $<$ those of BC and C.

(4) The largest % improvement in ABL over that of the control, for any of the two sources of imbalance investigations, is higher than that achieved by each of the investigations of one source of imbalance. This is true for N = 5 and 8.

(5) The greatest % I's gain for investigation M&C is more than 16 times higher than that of investigation M (for N = 5). This is in line with the results of Rao (142). However, for N = 8 this % gain decreases substantially, although it remains appreciably higher than that of investigation M. On the other hand, all the other investigations achieved much higher savings than that of investigation M, but for N = 5 only. The fact that the patterns of N = 5 and N = 8 are unidentical may have contributed to this result.

(6) It seems that there is no clear best or worst unbalanced line's investigation in terms of its expected % savings in I

over that of the control, but the superiority or inferiority of any particular investigation depends on the values of B/MB and N. The same is true regarding the lowest % gain in ABL, but investigation M&C may be viewed as being generally the best and investigation M&B as the second best.

(7) The highest % gain in ABL over that of the balanced, is larger than that of I, for all the N,B/MB and DI levels experimented with.

PAIRWISE COMPARISONS AMONG THE BEST PATTERNS

Pairwise comparisons were conducted between the best patterns of the six unbalanced lines' investigations, with respect to both I and ABL, in order to determine if significant differences exist among their outcomes.

Table 9.3 shows the values of the critical differences for these pairwise comparisons.

From Table 9.3, the cases where a particular type of imbalance yielded significantly lower I or ABL than another one, are exhibited in Table 9.4.

THE MULTI-RESPONSE PROBLEM

So far, the best patterns in terms of both I and ABL, taken individually, for all the unbalanced lines' investigations were identified. However, the question of whether the best pattern with regard to I is preferrable to that with respect to ABL, remains unanswered. This is a facet of the still underdeveloped multiple response problem (discussed in Chapter 5) which is frequently encountered when conducting scientific

TABLE 9.3

PAIRWISE COMPARISONS AMONG THE BEST PATTERNS - CRITICAL DIFFERENCES

ENDOGENOUS VARIABLE	N	B/MB	d_1	d_2
I	5	1	1.879	2.561
	5	2	3.291	3.926
	5	6	2.069	2.587
	8	1	2.622	3.573
	8	2	2.554	3.181
	8	6	2.308	2.875
ABL	5	1	0.068	0.093
	5	2	0.346	0.431
	5	6	1.972	2.457
	8	1	0.084	0.114
	8	2	0.173	0.216
	8	6	0.574	0.715

TABLE 9.4

PAIRWISE COMPARISONS BETWEEN THE BEST PATTERNS

ENDOGENOUS VARIABLE	N	B/MB	%DI	A		B	SIGNIFICANCE LEVEL
I	5	1	5	M&C	<	M	0.95
	8	2	2	M	<	C	0.95
ABL	5	1	2-12	M&C	<	M,C	0.99
	5	2	2	M&B	<	M	0.99
	5	2	12	M&B	<	C	0.99
	5	6	5,12	M&C	<	C	0.95
	8	1	2-12	M&C	<	M	0.99
	8	1	5	M&C	<	C	0.95
	8	1	12	M&C	<	C	0.99
	8	2	2,5	M&C	<	M	0.99
	8	2	12	M&C	<	C	0.99
	8	2	5	M&C	<	BC	0.95
	8	2	12	M&C	<	BC	0.99
	8	6	2	M&C	<	M	0.99
	8	6	5	M&C	<	C	0.95
	8	6	12	M&C	<	C	0.99
	8	6	5,12	M&C	<	BC	0.95
	8	6	2	M&C	<	M&B	0.95

where

A = the investigation whereby the endogenous variable is significantly lower than that of the corresponding investigation.

B = the investigation whereby the endogenous variable is significantly higher than that of the corresponding investigation.

research. Wild and Slack (179) alluded to this difficulty by stating that "whether the benefits of increased efficiency or reduced inventory space are chosen, will depend on the relative cost of inventory and lost production".

The economical effect of decreased I and/or reduced ABL depends to a large extent on the unique cost function of each system. These functions are largely unknown and, at present, there is no mathematically feasible way to estimate them with reasonable accuracy. To tackle this problem an attempt was made to compare the best I's pattern with that of the best ABL's pattern, for each of the imbalanced lines' investigations, in order to determine which one is the better. Such an attempt will no doubt require the use of a utility weighing function where weights (representing the relative importance in cost terms) are assigned to I and ABL to form total cost functions for the best pattern with regard to I + its corresponding ABL, and the best pattern with reference to ABL + its corresponding I.

The relative weights of I and ABL may differ from one industry to another. In some industries, where the demand is high and the working schedules are tight, i.e. the operators are nearly fully utilized, it may be reasonable to expect the cost of I to be much higher than the inventory cost. In other industries, on the other hand, there is redundant capacity and therefore, it may not be unreasonable to anticipate that the cost of inventory is probably higher than that of I.

Since the simulation model utilized in this research portrays a generalised work situat on and not a specific one, it was decided to assign arbitrary weights to both I and ABL using the simplest form of utility functions. To do this, only the best patterns' points of I and ABL, whereby improvements over the control have occurred, were examined. Furthermore, only two relationships between I and ABL were considered, i.e. I = 2ABL and I = ABL.

Table 9.5 shows the % total savings over the balanced line for the best pattern with respect to I plus its ABL's counterpart, and the best pattern in terms of ABL plus its corresponding I. Note that when the I and ABL of the same best pattern are added together, three cases may arise:

(1) Both the I and ABL of the best pattern achieve improvements over the control, in which case the total amount of improvement will increase.

(2) I yields advantages over the control, whereas its corresponding ABL does not.

(3) ABL obtains gains over the balanced line, while its I's counterpart does not.

The total outcome may be positive, i.e. an overall saving exists (which is the case always in (1)) or negative, resulting in an overall loss.

From the aforementioned table the following may be concluded:

(1) The highest total I's saving is larger than that of ABL in the investigations of one source of imbalance, but is smaller in the investigations of the two causes of imbalance. This is true for both I = ABL and 2ABL.

TABLE 9.5

% TOTAL SAVINGS OF THE I'S AND ABL'S BEST PATTERNS

ENDOGENOUS VARIABLE	INVESTIGATION	N	B/MB	%DI	I=ABL	I=2ABL
I	M	5	1	2	-	1.21
	M	5	2	2	0.98	1.86
	M	5	6	2	24.91	25.22
	M	8	1	2	1.67	5.13
	M	8	1	5	3.36	5.82
	M	8	2	2	-	0.94
	C	5	1	-	34.70	44.87
	C	5	2	-	49.81	61.38
	C	5	6	-	$\boxed{69.82}$	$\boxed{122.90}$
	C	8	1	-	36.74*	24.46*
	C	8	2	-	10.54*	-
	C	8	6	-	20.54*	-
	BC	5	6	-	32.07	48.21
	M&C	5	1	2	-	11.61
	M&C	5	1	5	-	12.07
	M&C	5	2	2	-	2.13
	M&B	5	6	2	4.61	16.18
	C&B	5	2	-	44.43	75.90
	C&B	5	6	-	20.72	46.00
	C&B	8	6	-	16.99*	11.24*
ABL	M	5	1	2	6.54	1.78
	M	5	1	5	7.87	-
	M	5	2	2	9.33	7.72
	M	5	2	5	1.63	-
	M	5	6	2	24.48	-

...Cont

TABLE 9.5 (CONT)

ENDOGENOUS VARIABLE	INVESTIGATION	N	B/MB	%DI	I=ABL	I=2ABL
	M	8	1	2	7.78	5.54
	M	8	1	5	17.22	7.08
	M	8	1	12	5.55	-
	C	The best pattern is the same as that for I				
	BC	-	-	-	-	-
	M&C	5	1	2	45.76	46.46
	M&C	5	1	5	38.92	16.62
	M&C	5	1	12	7.20	-
	M&C	5	2	2	33.67	8.10
	M&C	5	2	5	4.99	-
	M&C	8	1	2	24.96	-
	M&C	8	1	5	12.67	-
	M&B	5	6	2	25.32	-
	C&B	5	2	-	19.66	-
	C&B	5	6	-	(72.07)	(66.22)

where

* The control was better than the unbalanced pattern in terms of I, but the addition of the corresponding ABL resulted in a total improvement over the balanced line.

\- The control is better than the unbalanced pattern.

▭ The highest total saving regarding I.

( ) The highest total saving with respect to ABL.

(2) The total ABL's gain is lower than the gain of ABL alone. This result is true for all the investigations and for I = 1 and 2 ABL. On the other hand, the relationship between the total I's improvement and I, taken individually, is unclear.

(3) The relatively high % advantage of I in investigation M&C (compared to that of investigation M) disappears when calculating the % total saving when I equals ABL, but when I equals 2ABL, the % total improvement is reduced by nearly two thirds. Therefore, when the total function of both I and ABL for a particular pattern or imbalance type is taken into account, a different judgement regarding its preferability may emerge.

(4) The biggest total savings of both I and ABL in each investigation occur at N = 5, B = 6 (in most cases), and DI = 0.02 (in almost all cases). This is to be expected since those levels of N, B, and DI result in small amounts of I.

(5) The highest total advantage of I is obtained by investigation C, followed by C&B. This is the same sequence as that of the saving of the best I's pattern alone. The lowest total improvement, on the other hand, is achieved by investigations M&B (for I = 2ABL) and M&C (for I = ABL).

(6) The ranking of the investigations in terms of their ABL's total advantage changes as the weight of I, in relation to that of ABL, differs, i.e. C&B, C, ..., BC for I = ABL, and C, C&B, ..., BC for I = 2ABL. Both these sequences differ from that of the ABL's best pattern alone.

(7) When the weight of I relative to that of ABL rises, the total saving in I (if exists) tends to become higher than that in ABL, rendering the best pattern in terms of I

favourable and vice versa.

(8) Increasing the weight of I decreases or negates the total saving in ABL, but has no clear effect on the total saving in I.

(9) The advantage of the best pattern, in terms of I/ABL over the control, may disappear (i.e. the control becomes better) when computing the total function of I + ABL. This occurred in several occassions in most investigations and for I = 1 and 2ABL. In general, the disappearance is more common in ABL's than in I's best patterns. The importance of this finding may be realised when considering the fact that in all the investigations the best pattern with regard to ABL yielded savings over the control, for all the values of N, B/MB and DI.

(10) The advantage of the control over the total I + ABL function (where exists) goes up as DI and B/MB are increased in most investigations. This is valid for the best patterns with reference to both I and ABL.

IMPLICATIONS OF THE RESEARCH

This research offers new and additional findings and extends theoretical knowledge in an important area. It is hoped that the results provided in this thesis will help in supplying general guidelines to the practitioner on how to design unpaced manual production lines in a more efficient manner, and how to utilize operators more economically. The most important implications of this thesis are as follows:

(1) This research demonstrated that superior performance over that of the balanced line can be achieved by each of the

six methods of unbalancing the system. In each instance, at least one unbalanced pattern produced some gains over the balanced line in terms of either I or ABL or both. In fact, some appreciably unbalanced lines can attain approximately equal performance to that achievable by a balanced line. The highest percentage savings in I over that of the balanced design, obtained by the best patterns in investigations M through C&B are, respectively, 3.46, 43.08, 16.14, 20.73, 11.57, and 31.46. Therefore, it is apparent that even the lowest of these figures is not trivial, while the highest is sizeable, especially when both are considered over the entire life span of a production line, and therefore, savings of such magnitudes justify unbalancing the line. Likewise, the advantage of the best pattern with respect to ABL, over that of a balanced line arrangement for each investigation, is a worthwhile one and much higher than that of the best I's pattern, when making the assumption that I and ABL are equal in weight. The importance of this implication is heightened when taking into account the fact that in practice it is seldom possible to achieve even a nominal balance from a technical viewpoint, and consequently, there is in most lines a certain degree of imbalance.

(2) The likely improvements in I and ABL of the best patterns should be considered in the light of the other inferior unbalanced patterns. The results of this research have indicated that if a line's imbalance is carried out in the wrong direction, the resulting pattern will yield substantially high amounts of I and/or ABL. The results also imply, on the other hand, that by deliberately unbalancing the line in the correct manner

an appreciable magnitude of gain in I/ABL will result over that of any less fortunate unbalanced patterns. It remains up to the production line's designer to utilize the general guidelines of these investigations which point out the proper way of line's unbalance, after finding out the type(s) of imbalance being present in the work situation.

(3) Another implication of this thesis is on the line balancing practice. The primary goal of the line balancing techniques, viz, to equally apportion the work tasks to each station on the line has to be questioned. Consequently, line's designers should not strive to obtain a nominal balance of mean operation times (or even a slight imbalance, whereby the direction of the mean service times is very close to that of the balanced configuration). Therefore, line 'unbalancing' procedures need to be developed (or alternatively the currently available line balancing techniques need to be modified) to distribute the work elements times among the stations in accordance with a prescribed manner (e.g. the bowl phenomenon pattern for the means imbalance), such that some specific limits of mean work times are met, given the desired degree of unbalance, rather than assigning these work elements in equal proportions to all the stations. This is done while heeding the precedence and zoning restrictions on the direction of the line unbalancing.

(4) There is a lack of knowledge on how workers with different capabilities, be it with regard to their work times' means or Covars, ought to be placed along the line. As a consequence of that, pragmatic principles were applied concerning the placement of operators. The findings of this research on the

favourable patterns (orders) of arranging workers with different potentials, provide general guidelines to the designer on how to obtain an adequate labour placement. This may also imply that better selection procedures may be developed to choose workers who are compatible with the adopted placement policy.

(5) Since this research clearly shows that the expected proportion of savings over the balanced line's arrangement depend on the type of imbalance present in the line, one obvious implication for the designer is the fact that it is necessary to make a thorough study, which aims at determining the existing kind(s) of imbalance in his production unit, so that the expected improvements in I and ABL from selecting a particular pattern of imbalance may be computed. Following that, he should attempt to derive the practical cost functions of I and ABL, and conduct a cost benefit evaluation to determine the net total feasible saving of the favourable unbalanced pattern over the balanced counterpart.

RECOMMENDATIONS FOR FURTHER RESEARCH

It is recommended that research be continued in the area of unbalanced and unpaced manual flow lines, utilizing an appropriate experimental design methodology. The real-life behaviour of these production lines is very complex and should be fully understood if it is desired to operate them efficiently. The results obtained from this research point to several future research extensions, the most important of which are:

(1) Double-Operator Unbalanced Lines

All the investigations reported in this thesis are confined to the single-operator unbalanced lines. It is suggested to examine the same types of unbalanced lines, with the addition that double or multiple manning exists at some or all the stations. Such an investigation will contribute towards the understanding and the design of these lines.

(2) Optimal and Breakeven Degrees of Imbalance

As was indicated in Chapter 5, no attempt was made to determine the optimal and maximal degrees of imbalance for all the investigations. A possible future study includes the use of a search procedure to determine such imbalance degrees. The contribution of this study will be the discovery of the highest possible improvement in both I and ABL over those of the balanced lines, and the exploration of the range and sensitivity of these improvements.

(3) Effect or Learning on the Unbalanced Line's Behaviour

No learning parameters were inserted in any of the simulation investigations. A further research can enter the learning factor into account, by assuming that different learning curves and timings of replacements, various absenteeism and turnover rates, and different composition of trainees and experienced operators exist in the line. Such a comprehensive study will provide deep insights into the effects of the learning conditions on the effectiveness of the line.

(4) Transient Behaviour of the Unbalanced Lines

It was stated in Chapter 8 that, due to the use of a relatively lengthy subrun's duration, it was not possible to determine the impact of the major design parameters on the rate of the decrease in I, and the rise in ABL, during the early part of the transient, or where this process would stop, etc. It may be necessary, in a future research, to employ spectral analysis if the autocorrelation among the subruns cannot be avoided. Such a research will shed more light on some of the delicate behavioural aspects of the unbalanced lines under unstable conditions.

(5) Rao's Conjecture

The results of Chapter 7 suggested that the conclusion of Rao (142) that the amount of the difference between the stations' Covars may dictate the optimal pattern of the means imbalance is, in general, valid. It is suggested that a study be made to test Rao's conjecture for which two levels of Covars, i.e. steady and variable (S and V), are considered, each with a range varying between, say, 0.10 and 0.90 in steps of 0.05. Following that, various ranges of the difference in the values of S and V are constructed and the system is simulated for several factors' levels, using the (V) and (Λ) patterns of means imbalance, along with several patterns of unbalanced Covars. The contribution of such a study lies in its exploration of the possible optimal designs for lines unbalanced in terms of both their means and Covars.

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UNBALANCED MANUAL FLOW LINE OPERATING CHARACTERISTICS

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Vol II

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VOLUME TWO

APPENDICES

APPENDIX 5

ASPECTS OF THE WEIBULL DISTRIBUTION AND THE COMPUTER SIMULATION PROGRAMME

This general appendix is divided into four appendices:

5.1 - Tables of Some Characteristics of the Weibull Distribution

5.2 - The Simulation Programme

5.3 - Data Arrays 'A' and 'Q'

5.4 - Description of Variables and Parameters

APPENDIX 5.1

TABLES OF SOME CHARACTERISTICS OF THE WEIBULL DISTRIBUTION

TABLE A5.1

Weibull Probability Density Function

$\eta = 1$

x \ β	.10000000	.20000000	.30000000	.40000000	.50000000	.60000000
.10000000	.35894478	.67144238	.91087429	1.0694593	1.1524818	1.1723613
.20000000	.18169408	.35110596	.49937392	.62130483	.71487912	.78053075
.30000000	.12177530	.23875804	.34713653	.44409894	.52788158	.59760227
.40000000	9.1596065E- 2	.18105425	.26654103	.34657324	.42001816	.48607649
.50000000	7.3403352E- 2	.14580711	.21631444	.28414781	.34865222	.40929435
.60000000	6.1234639E- 2	.12200932	.18189748	.24051124	.29750297	.35256604
.70000000	5.2521179E- 2	.10484755	.15679379	.20818548	.25885953	.30866461
.80000000	4.5973566E- 2	9.1879659E- 2	.13765279	.18322971	.22854946	.27355356
.90000000	4.0873230E- 2	8.1732966E- 2	.12256587	.16335885	.20409899	.24477354
1.0000000	3.6787945E- 2	7.3575890E- 2	.11036384	.14715178	.18393973	.22072767
1.1000000	3.3442062E- 2	6.6874943E- 2	.10028936	.13367592	.16702511	.20032734
1.2000000	3.0651494E- 2	6.1271993E- 2	9.1829856E- 2	.12229276	.15262779	.18280142
1.3000000	2.8288595E- 2	5.6517598E- 2	8.4625677E- 2	.11254979	.14022532	.16758632
1.4000000	2.6262064E- 2	5.2432636E- 2	7.8416927E- 2	.10411695	.12943205	.15425939
1.5000000	2.4504869E- 2	4.8885144E- 2	7.3010990E- 2	9.6747691E- 2	.11995668	.14249675
1.6000000	2.2966681E- 2	4.5775726E- 2	6.8262056E- 2	9.0253928E- 2	.11157480	.13204551
1.7000000	2.1608978E- 2	4.3028092E- 2	6.4057671E- 2	8.4469650E- 2	.10411057	.12270510
1.8000000	2.0401771E- 2	4.0582703E- 2	6.0309706E- 2	7.9339708E- 2	9.7424137E- 2	.11431418
1.9000000	1.9321368E- 2	3.8392409E- 2	5.6948094E- 2	7.4712038E- 2	9.1402818E- 2	.10674136
2.0000000	1.8348802E- 2	3.6419389E- 2	5.3916421E- 2	7.0532131E- 2	8.5954749E- 2	9.9878480E- 2
2.1000000	1.7468707E- 2	3.4632940E- 2	5.1468756E- 2	6.6739015E- 2	8.1004262E- 2	9.3635639E- 2
2.2000000	1.6668508E- 2	3.3007890E- 2	4.8667315E- 2	6.3282320E- 2	7.6488448E- 2	8.7937459E- 2
2.3000000	1.5937807E- 2	3.1523394E- 2	4.6380730E- 2	6.0120054E- 2	7.2354561E- 2	8.2720246E- 2
2.4000000	1.5267935E- 2	3.0162050E- 2	4.4282744E- 2	5.7216921E- 2	6.8558019E- 2	7.7929754E- 2
2.5000000	1.4651611E- 2	2.8909215E- 2	4.2354200E- 2	5.4543046E- 2	6.5060911E- 2	7.3519543E- 2

x \ β	.70000000	.80000000	.90000000	1.0000000	1.1000000	1.2000000
.10000000	1.1440498	1.0820790	.99900613	.90483752	.80704086	.71085196
.20000000	.82039050	.83761077	.83582132	.81873079	.78985840	.75237383
.30000000	.65311978	.69487255	.72372294	.74081827	.74747269	.74507237
.40000000	.54425327	.59432135	.63628117	.67032008	.69677255	.71608400
.50000000	.46565837	.51743918	.56443364	.60653066	.64370095	.67598585
.60000000	.40543472	.45588339	.50372545	.54881163	.59102766	.63029220
.70000000	.35746122	.40512192	.45153117	.49658528	.54019236	.58227156
.80000000	.31818587	.36239284	.40612338	.44932894	.49196369	.53398415
.90000000	.28537005	.32587627	.36628004	.40656966	.44673345	.48676002
1.0000000	.25751562	.29430356	.33109151	.36787945	.40466740	.44145534
1.1000000	.23357291	.26675199	.29985472	.33287110	.36579104	.39860443
1.2000000	.21277961	.24252782	.27201112	.30119422	.33004148	.35851709
1.3000000	.19456570	.22109552	.24710718	.27253180	.29730044	.32134444
1.4000000	.17849464	.20203269	.22476828	.24659697	.26741590	.28712473
1.5000000	.16422536	.18500019	.20468076	.22313016	.24021692	.25581695
1.6000000	.15148671	.16972163	.18657907	.20189653	.21552351	.22732500
1.7000000	.14006006	.15596884	.17023625	.18268353	.19315350	.20151581
1.8000000	.12976681	.14355141	.15545686	.16529889	.17292729	.17823291
1.9000000	.12045943	.13230889	.14207148	.14956862	.15467092	.15730650
2.0000000	.11201485	.12210498	.12993237	.13533528	.13821814	.13856086
2.1000000	.10432950	.11282303	.11891016	.12245643	.12341179	.12181989
2.2000000	9.7315552E- 2	.10436257	.10889103	.11080316	.11010439	.10691109
2.3000000	9.0897986E- 2	9.6635523E- 2	9.9774428E- 2	.10025884	9.8158464E- 2	9.3668345E- 2
2.4000000	8.5012308E- 2	8.9569063E- 2	9.1471342E- 2	9.0717959E- 2	8.7446586E- 2	8.1933806E- 2
2.5000000	7.9602750E- 2	8.3093841E- 2	8.3902617E- 2	8.2085008E- 2	7.7851177E- 2	7.1559123E- 2

x \ β	1.3000000	1.4000000	1.5000000	1.6000000	1.7000000	1.8000000
.10000000	.61969378	.53559752	.45957645	.39193234	.33249392	.28079510
.20000000	.70901835	.66208324	.61342547	.56450551	.51643574	.47003281
.30000000	.73500271	.71859517	.69709193	.67162303	.64319573	.61269077
.40000000	.72877812	.73542852	.73663523	.73300500	.72513582	.71360491
.50000000	.70348687	.72635547	.74478346	.75899387	.76923344	.77576507
.60800000	.66655430	.69979083	.73000376	.75721739	.78147623	.80284176
.70000000	.62275348	.66157904	.69869966	.73407626	.76767920	.79948758
.80000000	.57534954	.61602160	.65596460	.69514544	.73353324	.77109981
.90000000	.52663823	.56635719	.60590619	.64527481	.68445276	.72343019
1.0000000	.47824329	.51503123	.55181918	.58860712	.62539507	.66218301
1.1000000	.43130101	.46387049	.49630256	.52858674	.56071252	.59266937
1.2000000	.38658513	.41420936	.44135387	.46798261	.49405963	.51954945
1.3000000	.34459578	.36698752	.38845388	.40893097	.42835698	.44667264
1.4000000	.30562664	.32282924	.33864563	.35299534	.36580559	.37701190
1.5000000	.26981556	.28210944	.29260853	.30123818	.30794070	.31267716
1.6000000	.23718468	.24500840	.25072696	.25429893	.25571304	.25498959
1.7000000	.20767156	.21155800	.21315203	.21247309	.20958481	.20459534
1.8000000	.18115369	.18167971	.17985670	.17578737	.16963088	.16159954
1.9000000	.15746782	.15521609	.15068298	.14406873	.13563672	.12570425
2.0000000	.13642500	.13195628	.12538222	.11700471	.10718723	9.6337488E- 2
2.1000000	.11782218	.11165641	.10364795	9.4194316E- 2	8.3743219E- 2	7.2765655E- 2
2.2000000	.10145105	9.4055532E- 2	8.5142122E- 2	7.5189196E- 2	6.4703689E- 2	5.4184768E- 2
2.3000000	8.7104593E- 2	7.8867843E- 2	6.9515669E- 2	5.9525844E- 2	4.9453831E- 2	3.9789419E- 2
2.4000000	7.4581913E- 2	6.5891826E- 2	5.6423782E- 2	4.6749226E- 2	3.7399860E- 2	2.8820817E- 2
2.5000000	6.3691505E- 2	5.4816659E- 2	4.5536705E- 2	3.6429470E- 2	2.7992198E- 2	2.0596419E- 2

TABLE A5.1 (CONT)

Weibull Probability Density Function

x \ β	1.9000000	2.0000000	2.1000000	2.2000000	2.3000000	2.4000000	2.5000000
.10000000	.23620345	.19801002	.16548921	.13793757	.11469681	9.5166136E- 2	7.8807364E- 2
.20000000	.42586839	.38431582	.34559149	.30979044	.27691588	.24690378	.21954238
.30000000	.58086516	.54835880	.51570324	.48333258	.45159414	.42075893	.39103295
.40000000	.69896048	.68171502	.66234190	.64127301	.61889845	.59556693	.57158722
.50000000	.77886158	.77880079	.77586035	.77031419	.76242949	.75246424	.74066515
.60000000	.82139043	.83721167	.85040476	.86107792	.86934580	.87532803	.87914765
.70000000	.82948847	.85767702	.88405501	.90863122	.93142026	.95244227	.97172246
.80000000	.80781921	.84366791	.87862461	.91267042	.94578835	.97796376	1.0091840
.90000000	.75219724	.80074455	.83906274	.87714287	.91497609	.95255403	.98986818
1.0000000	.69897096	.73575890	.77254685	.80933479	.84612274	.88291068	.91969863
1.1000000	.62444674	.65603402	.68742045	.71859541	.74954821	.78026801	.81074420
1.2000000	.54441670	.55862662	.59214461	.61493759	.63697198	.65821579	.67863768
1.3000000	.46382156	.47975077	.49441089	.50775688	.51974810	.53034885	.53952869
1.4000000	.38655943	.39440358	.40051112	.40486067	.40744362	.40826442	.40734120
1.5000000	.31542868	.31619767	.31500862	.31190833	.30696648	.30027462	.29194567
1.6000000	.25218164	.24737520	.24068846	.23227038	.22229804	.21097314	.19851786
1.7000000	.19765609	.18895914	.17873257	.16723459	.15474613	.14156208	.12798199
1.8000000	.15195298	.14099004	.12903799	.11644037	.10354327	9.0681286E- 2	7.8164048E- 2
1.9000000	.11462975	.10279702	9.0597354E- 2	7.8410634E- 2	6.6587084E- 2	5.5430663E- 2	4.5185914E- 2
2.0000000	8.4886052E- 2	7.3262556E- 2	6.1871559E- 2	5.1069928E- 2	4.1148120E- 2	3.2317168E- 2	2.4702694E- 2
2.1000000	6.1725935E- 2	5.1051748E- 2	4.1107304E- 2	3.2172246E- 2	2.4429272E- 2	1.7961430E- 2	1.2759228E- 2
2.2000000	4.4087555E- 2	3.4791045E- 2	2.6574038E- 2	1.9602669E- 2	1.3930525E- 2	9.5111904E- 3	6.2205108E- 3
2.3000000	3.0938029E- 2	2.3192103E- 2	1.6716798E- 2	1.1551752E- 2	7.6279269E- 3	4.7958926E- 3	2.8597314E- 3
2.4000000	2.1335189E- 2	1.5125336E- 2	1.0233908E- 2	6.5834163E- 3	4.0096512E- 3	2.3014016E- 3	1.2384937E- 3
2.5000000	1.4461627E- 2	9.6522695E- 3	6.0974656E- 3	3.6281905E- 3	2.0227412E- 3	1.0503846E- 3	

x \ β	2.6000000	2.7000000	2.8000000	2.9000000	3.0000000	4.0000000
.10000000	6.5145221E- 2	5.3764717E- 2	4.4306746E- 2	3.6462912E- 2	2.9970027E- 2	3.9996020E- 3
.20000000	.19498780	.17277626	.15283345	.13498142	.11904385	3.1948845E- 2
.30000000	.36256613	.33546152	.30978320	.28556318	.26280753	.10712875
.40000000	.54722977	.52272984	.49828867	.47407827	.45024247	.24952961
.50000000	.72726642	.71248852	.69653810	.67960705	.66187271	.46970656
.60000000	.88092971	.88080035	.87888522	.87530887	.87019410	.75897801
.70000000	.98929054	1.0051805	1.0194291	1.0320774	1.0431683	1.0791455
.80000000	1.0394383	1.0687179	1.0970158	1.1243270	1.1506480	1.3596996
.90000000	1.0269107	1.0636736	1.1001494	1.1363308	1.1722105	1.5130279
1.0000000	.95648657	.99327452	1.0300625	1.0668504	1.1036384	1.4715178
1.1000000	.84096583	.87092228	.90060278	.92999651	.95902289	1.2313670
1.2000000	.69820750	.71889504	.73467543	.75151919	.76740194	.86906188
1.3000000	.54726285	.55353240	.55832477	.56163376	.56345984	.50524457
1.4000000	.40470568	.40040359	.39449429	.38705056	.37815801	.23553663
1.5000000	.28211247	.27092582	.25855224	.24517131	.23097233	8.5451170E- 2
1.6000000	.18516952	.17117474	.15678351	.14224238	.12778828	2.3346817E- 2
1.7000000	.11430004	.10079550	8.7723296E- 2	7.5306135E- 2	6.3727999E- 2	4.6359193E- 3
1.8000000	6.6263883E- 2	5.5205767E- 2	4.5160414E- 2	3.6240528E- 2	2.8501052E- 2	
1.9000000	3.6029186E- 2	2.8065088E- 2	2.1328293E- 2	1.5790440E- 2	1.1371109E- 2	
2.0000000	1.8346445E- 2	1.3214810E- 2	9.2131828E- 3	6.2039246E- 3	4.0255537E- 3	
2.1000000	8.7363026E- 3	5.7507678E- 3	3.6290875E- 3	2.1888680E- 3	1.2576473E- 3	
2.2000000	3.8844529E- 3	2.3078498E- 3	1.2995266E- 3			
2.3000000	1.6102761E- 3					

x \ β	5.0000000	6.0000000	7.0000000	8.0000000	9.0000000	10.000000
.10000000						
.20000000	7.9974413E- 3	1.9198775E- 3				
.30000000	4.0401708E- 2	1.4569377E- 2	5.1018842E- 3	1.7494853E- 3		
.40000000	.12669597	6.1188862E- 2	2.8625063E- 2	1.3098615E- 2	5.8966944E- 3	2.6211653E- 3
.50000000	.30288539	.18459309	.10852386	6.2256346E- 2	3.5087658E- 2	1.9512190E- 2
.60000000	.59952087	.44529220	.31757636	.22021876	.14964970	.10016944
.70000000	1.0147763	.89649350	.75843834	.62192800	.49831219	.39229670
.80000000	1.4757758	1.5127026	1.4878522	1.4185916	1.3202993	1.2055296
.90000000	1.8175799	2.0823894	2.3058520	2.4879293	2.6298322	2.7337166
1.0000000	1.8393973	2.2072767	2.5751562	2.9430356	3.3109151	3.6787945
1.1000000	1.4625313	1.6433706	1.7665952	1.8276373	1.8253257	1.7623258
1.2000000	.86105603	.75380878	.58080441	.38902009	.22224318	.10558311
1.3000000	.34852863	.17848903	6.3624569E- 2	1.4388650E- 2	1.8210183E- 3	
1.4000000	8.8662952E- 2	1.7328286E- 2	1.3925683E- 3			
1.5000000	1.2747103E- 2					
1.6000000						
1.7000000						

TABLE A5.2

Weibull Cumulative Distribution Function

x \ β	.1000000	.2000000	.3000000	.4000000	.5000000	.6000000	.7000000	.8000000	.9000000	1.0000000
.1000000	.5481153	.4679178	.3941590	.3284099	.2711065	.2221243	.1808812	.1465679	.1182904	.0951625
.2000000	.5731573	.5155687	.4604575	.4086253	.3605926	.3166381	.2768447	.2411460	.2093683	.1812692
.3000000	.5879325	.5443375	.5018456	.4608712	.4217347	.3846680	.3498236	.3172850	.2870781	.2591317
.4000000	.5984581	.5550626	.5321735	.4999988	.4587143	.4384643	.4093625	.3814948	.3549215	.3296799
.5000000	.6066411	.5812790	.5551428	.5313308	.5069313	.4830214	.4596683	.4369287	.4148498	.3934693
.6000000	.6133364	.5945997	.5759556	.5574461	.5391103	.5209843	.5031008	.4854897	.4681775	.4511883
.7000000	.6190019	.6058997	.5928295	.5798065	.5668451	.5539589	.5411610	.5284634	.5158777	.5034146
.8000000	.6239122	.6157079	.6075116	.5993271	.5911582	.5830085	.5748816	.5667808	.5587095	.5506710
.9000000	.6282446	.6243691	.6204944	.6166211	.6127494	.6088798	.6050127	.6011485	.5972875	.5934303
1.0000000	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205
1.1000000	.6356267	.6391326	.6426379	.6461421	.6496451	.6531465	.6566459	.6601430	.6636374	.6671289
1.2000000	.6388274	.6455320	.6522320	.6589252	.6656092	.6722816	.6789399	.6855816	.6922044	.6988057
1.3000000	.6417712	.6514152	.6610456	.6706552	.6802370	.6897836	.6992876	.7087415	.7181375	.7274682
1.4000000	.6444963	.6568578	.6691903	.6814788	.6937078	.7058617	.7179243	.7298796	.7417113	.7534030
1.5000000	.6470326	.6619195	.6767557	.6915140	.7061673	.7206876	.7350465	.7492156	.7631662	.7768698
1.6000000	.6494045	.6666494	.6838144	.7008575	.7177356	.7344047	.7508204	.7669381	.7827136	.7981034
1.7000000	.6516320	.6710868	.6904262	.7095891	.7285128	.7471337	.7653876	.7832107	.8005406	.8173164
1.8000000	.6537313	.6752550	.6966411	.7177758	.7385836	.7589773	.7788701	.7981763	.8168130	.8347011
1.9000000	.6557165	.6792118	.7025010	.7254744	.7480198	.7700237	.7913739	.8119602	.8316734	.8504313
2.0000000	.6575992	.6829508	.7080417	.7327332	.7568832	.7803492	.8029907	.8246727	.8452688	.8646647
2.1000000	.6593894	.6865019	.7132937	.7395937	.7652273	.7900201	.8138020	.8364119	.8577021	.8775435
2.2000000	.6610956	.6898827	.7182836	.7460922	.7730985	.7990945	.8238790	.8472647	.8690842	.8891968
2.3000000	.6627254	.6931082	.7230344	.7522600	.7805377	.8076236	.8332850	.8573089	.8795104	.8997411
2.4000000	.6642852	.6961915	.7275561	.7581248	.7875807	.8156528	.8420763	.8666139	.8890662	.9092920
2.5000000	.6657808	.6991442	.7318965	.7637110	.7942593	.8232222	.8503033	.8752424	.8978291	.9179150

TABLE A5.2 (CONT)

x \ β	1.1000000	1.2000000	1.3000000	1.4000000	1.5000000	1.6000000	1.7000000	1.8000000	1.9000000	2.0000000
.1000000	.0763599	.0511464	.0422635	.0390286	.0311280	.0248060	.0197548	.0157239	.0125103	.0099501
.2000000	.1565612	.1349395	.1160959	.0997304	.0855593	.0733192	.0627697	.0536936	.0458980	.0392105
.3000000	.2335382	.2100620	.1886482	.1691784	.1515267	.1355640	.1211609	.1081910	.0965324	.0860688
.4000000	.3057876	.2832450	.2620381	.2421410	.2235183	.2061271	.1899190	.1748415	.1608395	.1478562
.5000000	.3728157	.3529134	.3337739	.3154058	.2978144	.2809878	.2649274	.2496192	.2350488	.2211992
.6000000	.4345424	.4182580	.4023502	.3868317	.3717128	.3570016	.3427040	.3288240	.3153636	.3023236
.7000000	.4910942	.4786955	.4668574	.4549774	.4432628	.4317200	.4203548	.4091724	.3981773	.3873736
.8000000	.5426582	.5347042	.5267818	.5189033	.5110728	.5032913	.4955619	.4878866	.4802673	.4727075
.9000000	.5895771	.5257294	.5218844	.5780457	.5742125	.5703852	.5665641	.5627497	.5589421	.5551419
1.0000000	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205
1.1000000	.6706170	.6741015	.6775819	.6810580	.6845293	.6879956	.6914564	.6949114	.6983603	.7018027
1.2000000	.7053530	.7119337	.7184552	.7249450	.7314005	.7378191	.7441982	.7505351	.7568273	.7630722
1.3000000	.7367256	.7459022	.7549901	.7639815	.7728689	.7816445	.7903008	.7988304	.8072260	.8154804
1.4000000	.7649363	.7763011	.7874752	.7984429	.8091948	.8197097	.8299753	.8399778	.8497009	.8591415
1.5000000	.7902933	.8034242	.8162208	.8286622	.8407240	.8523833	.8636168	.8744112	.8847434	.8946007
1.6000000	.8130852	.8275584	.8415444	.8549876	.8678552	.8801163	.8917519	.9027355	.9130535	.9226952
1.7000000	.8334905	.8489787	.8637517	.8777857	.8910132	.9034139	.9149652	.9256529	.9354713	.9444237
1.8000000	.8517674	.8679458	.8831786	.8974184	.9106284	.9227845	.9338750	.9439017	.9528794	.9608361
1.9000000	.8681316	.8847038	.9000869	.9142355	.9271220	.9387371	.9490902	.9582097	.9661416	.9729481
2.0000000	.8827517	.8994798	.9147603	.9285684	.9408942	.9517535	.9611873	.9692603	.9760582	.9816843
2.1000000	.8958201	.9124828	.9274532	.9407255	.9523174	.9622798	.9706946	.9776705	.9833384	.9878448
2.2000000	.9074940	.9239048	.9383991	.9509897	.9617314	.9707192	.9780828	.9839798	.9885874	.9920929
2.3000000	.9178963	.9339206	.9478109	.9596177	.9694418	.9774291	.9837616	.9886469	.9923054	.9949582
2.4000000	.9271558	.9426887	.9558208	.9668396	.9757190	.9827206	.9880600	.9920518	.9948331	.9968488
2.5000000	.9354229	.9503526	.9627817	.9728600	.9808000	.9868608	.9913297	.9945024	.9966632	.9980695

TABLE A5.2 (CONT)

x \ β	2.1000000	2.2000000	2.3000000	2.4000000	2.5000000	2.6000000	2.7000000	2.8000000	2.9000000	3.0000000
.1000000	.0079118	.0062397	.0049993	.0039731	.0031572	.0025087	.0019932	.0015836	.0012581	.0009995
.2000000	.0334302	.0285749	.0243792	.0207930	.0177294	.0151138	.0128815	.0109771	.0093529	.0079680
.3000000	.0765908	.0682961	.0607399	.0540845	.0480997	.0427621	.0380050	.0337677	.0299954	.0266387
.4000000	.1358344	.1247174	.1144490	.1049745	.0962413	.0881983	.0807971	.0739915	.0677379	.0619950
.5000000	.2080609	.1955831	.1837732	.1725979	.1620331	.1520541	.1425362	.1337548	.1253851	.1175030
.6000000	.2897029	.2774993	.2657092	.2543260	.2433502	.2327694	.2225784	.2127696	.2033345	.1942646
.7000000	.3767645	.3663531	.3561416	.3461319	.3363254	.3267232	.3173256	.3081331	.2991453	.2903617
.8000000	.4652077	.4577703	.4503969	.4430893	.4358490	.4286774	.4215759	.4145458	.4075882	.4007042
.9000000	.5513492	.5475645	.5437380	.5400201	.5362610	.5325111	.5287706	.5250399	.5213192	.5176088
1.0000000	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205	.6321205
1.1000000	.7052381	.7086664	.7120870	.7154996	.7189039	.7222994	.7256859	.7290629	.7324301	.7357870
1.2000000	.7692672	.7754097	.7814972	.7875272	.7934971	.7994045	.8052469	.8110219	.8167273	.8223606
1.3000000	.8235858	.8315383	.8393285	.8469512	.8544006	.8616710	.8687571	.8756542	.8823573	.8888639
1.4000000	.8682791	.8771063	.8856139	.8937935	.9016382	.9091424	.9163017	.9231131	.9295750	.9356874
1.5000000	.9039709	.9128445	.9212148	.9290782	.9364339	.9432843	.9496346	.9554932	.9608710	.9657818
1.6000000	.9316554	.9399342	.9475371	.9544752	.9607644	.9664259	.9714850	.9759715	.9799181	.9833609
1.7000000	.9525222	.9597871	.9662473	.9719387	.9769040	.9811914	.9848534	.9879455	.9905249	.9926495
1.8000000	.9678115	.9738570	.9790328	.9834070	.9870533	.9900489	.9924723	.9944010	.9959094	.9970677
1.9000000	.9787055	.9835013	.9874315	.9905966	.9930986	.9950377	.9965092	.9976009	.9983917	.9989500
2.0000000	.9862551	.9898956	.9927342	.9948975	.9965065	.9976722	.9984935	.9990550	.9994267	.9996645
2.1000000	.9913451	.9939966	.9959514	.9973513	.9983229	.9989748	.9993966	.9996590	.9998156	.9999049
2.2000000	.9946841	.9965407	.9978268	.9986858	.9992374	.9995768	.9997762	.9998877	.9999457	.9999762
2.3000000	.9968155	.9980673	.9988768	.9993773	.9996720	.9998366	.9999233	.9999653	.9999862	.9999948
2.4000000	.9981396	.9989534	.9994413	.9997184	.9998667	.9999411	.9999758	.9999908	.9999969	.9999990
2.5000000	.9989402	.9994507	.9997327	.9998736	.9999489	.9999802	.9999930	.9999977	.9999993	.9999998

APPENDIX 5.2

THE SIMULATION PROGRAMME

This appendix contains a listing of the computer programme utilized in the simulation investigations.

APPENDIX 5.2

THE SIMULATION PROGRAMME

```
/ID SHABAN                    003 050
/ETC          461 MUSIC JOB                                              8
/LOAD FORTG1

 FORTRAN IV G1  RELEASE 2.0                MAIN

                    DIMENSION A(21,21),B(2,21,11),Q(21,13),BQ(2,21,11),
                   1WQ(21,13),FINI(15,20),FINQ(20),FSTA(20,3,15)
                    DIMENSION ANTVAL(90),ENTIME(90)
                    DATA START/5000.0/
                    REAL MEAN
                    READ(5,1110)NASH
               1110 FORMAT(I2)
                    DO 6000 IP=1,NASH
                    READ(5,6010)ANTVAL(IP),ENTIME(IP)
               6010 FORMAT(F7.1,1X,F7.1)
               6000 CONTINUE
                    DO 888 II=1,NASH
                    IX=15849
                    CALL RANDU(IX,IY,YFL)
                    READ(5,1009)ITYP
                    DO 8 I=1,21
                    DO 8 J=1,21
                  8 A(I,J)=0.0
                    DO 12 I=1,2
                    DO 12 J=1,21
                    DO 12 K=1,11
                    B(I,J,K)=0.0
                 12 BQ(I,J,K)=0.0
                    DO 14 I=1,21
                    DO 14 J=1,13
                    Q(I,J)=0.0
                 14 WQ(I,J)=0.0
                    DO 1 I=1,15
                    DO 1 J=1,20
                  1 FINI(I,J)=0.0
                    DO 2 I=1,20
                  2 FINQ(I)=0.0
                    DO 3 I=1,20
                    DO 3 J=1,3
                    DO 3 K=1,15
                  3 FSTA(I,J,K)=0.0
                    READ(5,1006)IST
                    IF(ITYP)31,31,19
                 31 READ(5,1007)BUC,AAA,BBB,CCC,BUFIN
                    DO 119 N=1,IST
                    A(N,18)=BUC
                    A(N,17)=AAA
                    A(N,19)=BBB
                    A(N,20)=CCC
                119 A(N,4)=BUFIN
                    GO TO 18
                 19 CONTINUE
                    DO 13 N=1,IST
                 13 READ(5,1007)A(N,18),A(N,17),A(N,19),A(N,20),A(N,4)
               1000 FORMAT(5F5.2)
                 18 CONTINUE
               1005 FORMAT(2F10.2)
               1006 FORMAT(2I3)
               1007 FORMAT(5F10.3)
```

```
 1008 FORMAT(F10.3)
 1009 FORMAT(I2)
      COUNT=1.0
      FITIME=0.0
      A(1,4)=10000.0
      A(1,18)=10000.0
      A(IST+1,18)=10000.0
C SET STARTING CONDITIONS
      DO 11 N=1,IST
      IX=IY
      CALL RANDU(IX,IY,YFL)
      BERT=ALOG(1.0/(1.0-YFL))
      OPT=(((BERT)**(1.0/A(N,20)))*A(N,19))+A(N,17)
      Q(N,1)=OPT
   11 A(N,1)=OPT
C FIND NEXT EVENT
  110 CONTINUE
      J=1
      DO 111 M=2,IST
      IF(A(J,1)-A(M,1))111,111,32
   32 J=M
  111 CONTINUE
      N=J
      IF(A(N,4))40,40,41
   40 IF(A(N+1,4)-A(N+1,18))42,43,43
   41 IF(A(N+1,4)-A(N+1,18))44,45,45
C EXAMINES STARVED CONDITION
   42 IF(A(N,8))33,33,52
   33 A(N+1,4)=A(N+1,4)+1.0
      IF(N.GE.IST)GO TO 52
      QTIM=A(N,1)-Q(N+1,12)
      JQ=Q(N+1,13)
      Q(N+1,JQ+1)=Q(N+1,JQ+1)+QTIM
      Q(N+1,12)=A(N,1)
      Q(N+1,13)=A(N+1,4)
      IF(A(N+1,4)-A(N+1,9))52,52,34
   34 A(N+1,9)=A(N+1,4)
   52 DS=A(N-1,1)-A(N,1)
      IF(DS.LT.0.0)GO TO 99
      A(N,2)=A(N,2)+DS
      A(N,1)=A(N,1)+DS+0.0001
      A(N,8)=1.0
      GO TO 400
C EXAMINES BLOCKED AND STARVED CONDITION
   43 IF(A(N,8))35,35,53
   35 A(N,5)=A(N,5)+1.0
   53 DS=A(N-1,1)-A(N,1)
      IF(DS.LT.0.0)GO TO 99
      A(N,8)=1.0
      K=2
  147 IF(A(N+K,4)-A(N+K,18))36,247,247
   36 DB=A(N+K-1,1)-A(N,1)
      IF(DB.LT.0.0)GO TO 99
      GO TO 48
```

FORTRAN IV G1 RELEASE 2.0   MAIN

```
  247 K=K+1
      GO TO 147
   48 IF(DS-DB)37,37,148
   37 A(N,1)=A(N,1)+DB+0.0001
      A(N,3)=A(N,3)+DB
```

```
      GO TO 400
  148 A(N,1)=A(N,1)+DS+0.0001
      A(N,2)=A(N,2)+DS
      GO TO 400
C EXAMINES BLOCKED CONDITION
   45 IF(A(N,8))38,38,55
   38 A(N,5)=A(N,5)+1.0
   55 K=2
  145 IF(A(N+K,4)-A(N+K,18))39,254,254
   39 DB=A(N+K-1,1)-A(N,1)
      A(N,1)=A(N,1)+DB+0.01
      A(N,3)=A(N,3)+DB
      A(N,8)=1.0
      IF(DB.LT.0.0)GO TO 99
      GO TO 400
  254 K=K+1
      GO TO 145
C  EXAMINES OK CONDITION
   44 A(N,4)=A(N,4)-1.0
      IF(N.LE.1)GO TO 499
      QTIM=A(N,1)-Q(N,12)
      JQ=Q(N,13)
      Q(N,JQ+1)=Q(N,JQ+1)+QTIM
      Q(N,12)=A(N,1)
      Q(N,13)=A(N,4)
  499 IF(A(N,8))321,321,54
  321 A(N+1,4)=A(N+1,4)+1.0
      IF(N.GE.IST)GO TO 244
      QTIM=A(N,1)-Q(N+1,12)
      JQ=Q(N+1,13)
      Q(N+1,JQ+1)=Q(N+1,JQ+1)+QTIM
      Q(N+1,12)=A(N,1)
      Q(N+1,13)=A(N+1,4)
      IF(A(N+1,4)-A(N+1,9))244,244,322
  322 A(N+1,9)=A(N+1,4)
  244 IX=IY
      CALL RANDU(IX,IY,YFL)
  500 BERT=ALOG(1.0/(1.0-YFL))
      OPT=(((BERT)**(1.0/A(N,20)))*A(N,19))+A(N,17)
  502 A(N,1)=A(N,1)+OPT
      GO TO 400
   54 IF(A(N,5))144,144,323
  323 A(N,5)=A(N,5)-1.0
      A(N+1,4)=A(N+1,4)+1.0
      IF(N.GE.IST)GO TO 144
      QTIM=A(N,1)-Q(N+1,12)
      JQ=Q(N+1,13)
      Q(N+1,JQ+1)=Q(N+1,JQ+1)+QTIM
      Q(N+1,12)=A(N,1)

FORTRAN IV G1  RELEASE 2.0              MAIN

      Q(N+1,13)=A(N+1,4)
      IF(A(N+1,4)-A(N+1,9))144,144,324
  324 A(N+1,9)=A(N+1,4)
  144 IX=IY
      CALL RANDU(IX,IY,YFL)
  503 BERT=ALOG(1.0/(1.0-YFL))
      OPT=(((BERT)**(1.0/A(N,20)))*A(N,19))+A(N,17)
  505 A(N,1)=A(N,1)+OPT
      A(N,8)=0.0
  400 CONTINUE
```

```
C END OF PERIOD CHECK
      IF(A(N,1).LT.START)GO TO 110
      TIME=START+((COUNT-1.0)*ANTVAL(II))
      IF(A(N,1).GE.ENTIME(II))FITIME=1.0
      IF(A(N,1).GE.TIME)GO TO 81
      GO TO 110
   81 COUNT=COUNT+1.0
      KOUNT=COUNT-1.0
C RECORD RELEVANT PERIOD DATA
      DO 65 N=2,IST
      DO 65 I=1,11
   65 BQ(2,N,I)=BQ(1,N,I)
      DO 68 N=1,IST
      B(2,N,1)=B(1,N,1)
      B(2,N,2)=B(1,N,2)
      B(2,N,3)=B(1,N,3)
   68 CONTINUE
      DO 82 N=1,IST
      B(1,N,1)=A(N,1)
      B(1,N,2)=A(N,2)
   82 B(1,N,3)=A(N,3)
      DO 83 N=2,IST
      DO 83 I=1,11
   83 BQ(1,N,I)=Q(N,I)
      B(1,IST+1,4)=A(IST+1,4)
      DO 60 N=1,IST
      DO 60 I=1,11
   60 BQ(1,N,I)=Q(N,I)
C CALCULATE AND PRINT PERIODIC SUMMARY DATA
      SSTY=0.0
      SSTYS=0.0
      SSTYB=0.0
      DO 1041 N=1,IST
      TDIF=B(1,N,1)-B(2,N,1)
      STYS=(B(1,N,2)-B(2,N,2))*100.0/TDIF
      STYB=(B(1,N,3)-B(2,N,3))*100.0/TDIF
      STY=(B(1,N,2)+B(1,N,3)-B(2,N,2)-B(2,N,3))*100.0/TDIF
      SSTYS=SSTYS+STYS
      SSTYB=SSTYB+STYB
 1041 SSTY=SSTY+STY
      FINI(1,KOUNT)=SSTY/IST
      FINI(2,KOUNT)=SSTYS/IST
      FINI(3,KOUNT)=SSTYB/IST
      CUMEAN=0.0
```



```
      DO 1042 N=2,IST
      SUMQ=0.0
      DO 1043 I=1,11
      WQ(N,I)=(BQ(1,N,I)-BQ(2,N,I))*100.0/TDIF
 1043 SUMQ=SUMQ+(I*WQ(N,I))
      MEAN=(SUMQ/100.0)-1.0
 1042 CUMEAN=CUMEAN+MEAN
      FINQ(KOUNT)=CUMEAN/(IST-1)
  800 IF(FITIME-1.0)325,801,325
  325 NEXT=0
      IF(NEXT)110,110,326
  326 CONTINUE
  801 CONTINUE
C WRITE PERIODIC I%
      DO 1044 J=1,3
```

```
0221           1044 WRITE(6,6020)(FINI(J,I),I=1,3)
0222                WRITE(6,6020)(FINQ(I),I=1,3)
0223                WRITE(6,5000)A(IST+1,4)
              C FORMAT STATEMENT LISTING
0224           6020 FORMAT(1H ,20F6.3)
0225           6030 FORMAT(1H ,5F6.3)
0226           5000 FORMAT(1H ,F8.3,15F7.3)
0227           5001 FORMAT(1H1,7X)
0228           5009 FORMAT(1H ,F10.3)
              C  BLOW-UP CHECK
0229                GO TO 98
0230             99 WRITE(6,9999)N,TIME
0231                WRITE(6,5000)(A(N,1),N=1,IST)
0232           9999 FORMAT(1H1,2I5,F8.2)
0233             98 CONTINUE
0234            888 CONTINUE
0235                STOP
0236                END

 *OPTIONS IN EFFECT*  NOTERM,ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
 *OPTIONS IN EFFECT*  NAME = MAIN    , LINECNT =       56
 *STATISTICS*    SOURCE STATEMENTS =       236,PROGRAM SIZE = 005286
 *STATISTICS*  NO DIAGNOSTICS GENERATED
/END                                                              18.4S
008C98 BYTES USED
```

APPENDIX 5.3

DATA ARRAYS 'A' AND 'Q'

This appendix contains details of the two major data arrays A(I,J) and Q(I,J) utilized in the simulation programme.

APPENDIX 5.3

DATA ARRAY 'A' - ADAPTED FROM SLACK (160)

CUMULATIVE 'CLOCK' TIME	1
STARVING IDLE TIME	2
BLOCKING IDLE TIME	3
PRECEDING BUFFER LEVEL	4
MACHINE QUEUE (ZERO OR ONE)	5
SPARE	6
SPARE	7
'WAS-DELAYED' SIGNAL (ZERO OR ONE)	8
MAXIMUM QUEUE	9
STARVING IDLE TIME %	10
BLOCKING IDLE TIME %	11
TOTAL IDLE TIME %	12
SPARE	13
SPARE	14
SPARE	15
SPARE	16
WEIBULL PARAMETER 'A'	17
BUFFER CAPACITY	18
WEIBULL PARAMETER 'B'	19
WEIBULL PARAMETER 'C'	20

← STATION NUMBER →

APPENDIX 5.3
DATA ARRAY 'Q'

I \ J		PRECEDING BUFFER LEVEL						
		0	1	2	3	4	5	6
STATION NUMBER	1	1,1	1,2	1,3	1,4	1,5	1,6	1,7
	2	2,1	2,2	2,3	2,4	2,5	2,6	2,7
	3	3,1	3,2	3,3	3,4	3,5	3,6	2,7
	¦							
	¦							
	¦							
	1st	1st,1	1st,2	1st,3	1st,4	1st,5	1st,6	1st,7

APPENDIX 5.4

DESCRIPTION OF VARIABLES AND PARAMETERS

This appendix contains the definitions and notations of the exogenous, status, and endogenous variables, along with the parameters of the simulation programme.

APPENDIX 5.4

DESCRIPTION OF VARIABLES AND PARAMETERS

EXOGENOUS VARIABLES

START	=	The period throughout which no data are collected by the experimenter, and = 5000 time units for the SS investigations (representing the NSS period), = 0 time units for the NSS investigations.
NASH	=	Number of simulation runs to be executed within a single continuous run (batch).
ENTIME (IP)	=	Length of the simulation run for the run IP, IP = 1,2,....,NASH (= 35000 time units for the SS experiments and 5000 time units for the NSS experiments).
X	=	Random numbers generator starting (seed) value (= 0.5 for the SS runs, 0.1,0.2,..., 0.90,0.95 for the NSS runs).
ITYP	=	a(0,1) variable to determine if the service times' means are the same for all the stations, i.e. the line is balanced (ITYP=0), or the line is unbalanced (ITYP=1).
IST	=	Number of stations in the line.
BUC	=	Buffer capacity for ITYP = 0.
A(N,18)	=	Buffer capacity for station N and ITYP = 1.
A(N,17),A(N,19) A(N,20)	=	Weibull service times' parameters a,b,c, respectively for station N and ITYP = 1.
OPT	=	Station's operation time.
FITIME	=	a(0,1) variable which is given (0) if a station's cumulative clock time is $<$ the simulation run's end time, and assigned (1) when both times become equal.

Status Variables

COUNT	=	Period number + 1.
KOUNT	=	Period (subrun) number and 1,2,3,...,12.
A(N,1)	=	Cumulative clock time for station N.
TDIF	=	Difference between the cumulative clock times of the current and previous periods for station N, i.e. TDIF = a KOUNT period.
SSTYS	=	Line's cumulative mean starving idle time % for period KOUNT.

SSTYB	=	Line's cumulative mean blocking idle time % for period KOUNT.
SSTY	=	Line's cumulative mean total idle time % for period KOUNT.
WQ(N,I)	=	% mean number of times the buffer level is at level I for station N and period KOUNT, I = 0,1,2,..,6.
SUMQ	=	% mean total buffer levels for all the I levels, station N, and period KOUNT.
CUMEAN	=	Cumulative mean buffer levels for all the (N-1) buffers and period KOUNT.
TIME	=	Current cumulative time of the master clock.
B(J,N,I)	=	Cumulative results (I) for station N and period J, I = 1 (cumulative clock time), 2 (starving idle time), 3 (blocking idle time), 4 (preceding buffer level), J=1 (current period), 2 (previous period).
A(N,8)	=	a(0,1) variable to distinguish between whether station N was delayed (=0) or was not delayed (=1).
BQ(J,N,I)	=	Cumulative number of times the buffer level was at level I for station N and period J, I,0,1,2,...,6, J = 1 (current period), 2 (previous period).
BUFIN	=	Initial buffer level (except for the buffer preceding the first station), for ITYP = 0.
A(N,4)	=	Preceding buffer level for station N and ITYP = 1.
A(N,5)	=	a(0,1) switch to signify if machine queue for station N is zero or unity.

Endogenous Variables

FSTA(KOUNT,1,N) = STYS	=	% mean starving idle time for station N and period KOUNT.
FSTA(KOUNT,2,N) = STYB	=	% mean blocking idle time for station N and period KOUNT.
FSTA(KOUNT,3,N) = STY	=	% mean total idle time for station N and period KOUNT.

FINI(1,KOUNT)	=	Mean line's total % idle time for period KOUNT.
FINI(2,KOUNT)	=	Mean line's starving % idle time for period KOUNT.
FINI(3,KOUNT)	=	Mean Line's blocking % idle time for period KOUNT.
MEAN	=	Mean buffer level for the buffer succeeding station N for period KOUNT.
FINQ(KOUNT)	=	Mean buffer level for the whole line and period KOUNT.
A(IST+1,4)	=	Level of the buffer succeeding the last station (the number of units outputed).

Parameters

CCC	=	Weibull service times' parameters c for ITYP = 0 (= 1.60).
A(N,20)	=	Weibull service times' parameters c for station N and ITYP = 1 (= 1.60 for all the unbalanced lines' investigations).
AAA,BBB	=	Weibull service times' parameters a,b for ITYP = 0 (a = 5.780, b = 4.702).
A(N,17),A(N,18)	=	Weibull service times' parameters a,b for station N and ITYP = 1 (buffer capacities' imbalance only), where a = 5.780 and b = 4.702

where

N = 1,2,3...,IST

A = data array 'A'

APPENDIX 7

STEADY STATE RESULTS

This general appendix contains the results of the steady state simulation investigations and is divided into four appendices:

7.1 - Index of Tables

7.2 - Tables

7.3 - Index of Figures

7.4 - Figures

APPENDIX 7.1

INDEX OF TABLES

This appendix contains a list of the tables showing the steady state's results.

INDEX OF TABLES - STEADY STATE INVESTIGATIONS

where (in tables A7.33 - A7.72)

*	=	significantly different from the control at the 0.95 significant level
**	=	significantly different from the control at the 0.99 significant level
(figure)	=	% saving of the unbalanced pattern over the control (tables A7.33 - A7.52 only)
✓	=	the best unbalanced pattern (tables A7.53 - A7.72 only)

APPENDIX 7.2

TABLES

This appendix contains the results of the steady state simulation investigations, shown in tabular forms.

TABLE A7.1

THE BALANCED LINE - % TOTAL IDLE TIME AND MEAN BUFFER LEVEL RESULTS

LINE LENGTH	BUFFER CAPACITY	% IDLE TIME	MEAN BUFFER LEVEL
5	1	9.522	0.526
5	2	4.985	1.033
5	3	3.727	1.416
5	6	2.066	3.321
8	1	11.522	0.559
8	2	5.935	0.970
8	3	4.250	1.438
8	6	2.174	2.601
10	1	12.189	0.543
10	2	6.196	1.064
10	3	4.408	1.396
10	6	2.229	2.691

TABLE A7.2
MEANS IMBALANCE - % TOTAL IDLE TIME RESULTS

LINE LENGTH		5											
BUFFER CAPACITY		1				2				6			
% DEGREE OF IMBALANCE		2	5	12	18	2	5	12	18	2	5	12	18
PATTERN OF MEANS IMBALANCE	/	9.740	10.221	14.705	19.512	5.396	6.611	12.585	18.398	2.378	5.349	11.815	18.385
	\	9.975	11.181	14.978	19.507	5.065	7.302	13.247	18.636	2.586	5.443	12.010	17.535
	RAND	10.162	11.071	14.367	18.878	5.092	6.381	12.199	18.357	2.652	4.583	10.967	18.135
	^	9.996	10.141	12.335	14.790	5.442	6.116	9.335	10.829	2.381	2.928	7.902	8.806
	v	9.401	9.551	10.132	11.809	4.941	5.074	6.528	8.747	2.060	2.537	4.995	8.559

LINE LENGTH		8										
BUFFER CAPACITY		1				2				6		
% DEGREE OF IMBALANCE		2	5	12	18	2	5	12	18	2	5	12
PATTERN OF MEANS IMBALANCE	/	11.469	12.429	16.173	20.580	6.413	7.943	13.068	18.970	3.293	5.366	11.902
	\	11.780	12.691	16.551	20.965	6.368	7.486	13.479	19.352	2.969	6.003	12.802
	RAND	11.973	12.894	16.187	19.317	5.900	6.925	13.082	18.741	2.782	5.449	11.785
	^	11.251	12.269	13.205	15.699	5.837	7.154	9.347	10.832	2.589	3.421	7.909
	v	11.123	11.239	11.774	12.856	5.772	6.080	7.357	9.686	2.156	2.584	5.658

TABLE A7.3

COVARS IMBALANCE - % TOTAL IDLE TIME RESULTS

LINE LENGTH	5			8		
BUFFER CAPACITY / PATTERN OF COVARS IMBALANCE (P_i)	1	2	6	1	2	6
P_1	13.719	8.383	2.821	15.228	9.197	3.538
P_2	11.079	6.656	2.634	14.737	8.409	2.901
P_3	13.255	7.942	2.195	15.171	8.829	3.480
P_4	12.718	7.859	2.925	15.176	8.857	3.624
P_5	11.224	6.694	2.987	13.862	8.431	3.003
P_6	12.903	7.433	2.291	15.003	8.454	3.295
P_7	8.554	4.409	1.176	12.937	8.406	2.896
P_8	8.704	4.679	2.036	12.977	9.302	3.799
P_9	18.295	11.486	5.283	20.578	12.728	6.624
P_{10}	17.926	11.464	4.130	19.961	12.176	5.630
P_{11}	11.576	6.590	2.845	15.582	9.517	4.150
P_{12}	12.190	6.930	3.054	15.913	9.589	4.251

TABLE A7.4

BUFFER CAPACITIES IMBALANCE -
% TOTAL IDLE TIME RESULTS

LINE LENGTH		5		8	
MEAN BUFFER CAPACITY		2	6	2	6
PATTERN OF BUFFER CAPACITIES IMBALANCE	A_1	9.104	3.499	10.982	4.123
	A_2	7.978	3.094	9.315	3.571
	A_3	7.440	2.799	9.606	3.910
	B_1	7.532	2.696	9.143	3.599
	B_2	6.240	2.463	7.542	2.949
	B_3	6.007	2.342	7.846	3.260
	C_1	9.009	3.149	10.872	4.088
	C_2	7.661	2.906	9.976	3.511
	C_3	7.380	2.682	10.110	3.918
	D_1	5.707	2.082	7.112	2.212
	D_2	5.598	1.733	7.497	3.492
	D_3	5.980	2.229	7.938	3.497
	D_4	6.065	2.166	7.524	2.259
	D_5	6.175	3.030	7.501	2.352
	D_6	7.721	3.099	7.663	2.230

TABLE A7.5

MEANS & COVARS IMBALANCE PATTERN (/)) - % TOTAL IDLE TIME RESULTS

	LINE LENGTH	5									8								
	BUFFER CAPACITY	1			2			6			1			2			6		
	% DEGREE OF MEANS IMBALANCE	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	11.252	11.942	14.929	6.510	8.150	12.248	3.352	5.554	12.029	14.189	15.292	18.166	8.371	9.159	13.694	3.868	5.567	12.424
	P_2	13.026	13.130	16.644	8.365	10.056	14.511	3.476	5.736	11.997	15.625	16.354	18.821	8.639	10.086	15.697	3.420	6.543	12.998
	P_3	11.567	12.243	15.160	6.758	7.359	12.222	2.767	6.896	11.582	14.272	15.004	17.685	8.958	9.184	14.075	4.281	5.635	13.754
	P_4	7.554	7.568	12.255	4.185	5.826	11.972	2.439	4.024	11.709	10.920	11.002	15.106	8.010	8.245	12.782	2.532	5.769	11.538
	P_5	9.442	11.954	16.542	6.221	9.630	14.117	4.519	6.716	12.001	14.656	16.528	21.499	10.986	12.215	17.411	5.441	6.921	15.469
	P_6	11.353	10.988	12.989	6.018	6.138	11.869	2.147	5.241	11.546	15.247	13.731	13.151	9.001	7.334	12.042	3.947	4.633	11.378
	P_7	12.497	13.846	18.905	7.941	10.387	14.933	3.880	7.445	12.750	16.878	18.403	22.071	11.433	13.063	17.831	5.949	8.163	12.921
	P_8	18.219	17.667	19.373	12.347	11.532	15.057	6.032	6.199	12.092	20.200	19.836	21.008	12.872	12.569	14.536	5.598	5.323	13.883

TABLE A7.6

MEANS & COVARS IMBALANCE PATTERN (\)) - % TOTAL IDLE TIME RESULTS

	LINE LENGTH	5									8								
	BUFFER CAPACITY	1			2			6			1			2			6		
	% DEGREE OF MEANS IMBALANCE	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	12.047	12.264	15.644	7.207	7.437	12.829	2.838	5.234	12.429	14.837	15.015	18.640	8.265	10.630	13.317	4.900	5.767	11.654
	P_2	13.512	14.737	18.605	7.999	10.222	15.158	4.143	5.823	12.788	16.072	16.244	20.184	8.923	10.375	16.192	4.162	6.579	12.897
	P_3	11.488	12.412	15.536	6.639	7.560	12.608	2.907	5.253	11.430	14.664	14.713	17.214	8.623	9.803	13.932	4.301	6.054	11.522
	P_4	9.456	11.646	15.951	6.260	8.114	14.021	3.812	6.033	11.342	14.882	17.545	22.320	11.044	13.608	17.990	5.152	8.482	14.247
	P_5	7.773	8.028	12.733	4.352	5.783	10.698	2.352	5.139	12.027	11.328	11.347	15.150	7.575	8.003	13.219	2.661	5.888	12.137
	P_6	12.820	14.393	17.666	7.591	9.729	14.654	4.077	6.510	12.489	17.221	18.686	23.726	11.533	13.908	18.503	6.002	8.246	15.301
	P_7	10.831	10.852	13.782	5.845	6.168	12.262	2.055	5.162	12.091	13.038	13.303	13.341	8.421	8.620	11.956	2.784	4.651	11.935
	P_8	18.464	18.734	20.722	11.691	12.486	16.304	5.090	7.313	12.825	20.145	20.316	22.420	13.487	13.516	15.920	6.197	6.531	11.844

TABLE A7.7

MEANS & COVARS IMBALANCE PATTERN (V)) - % TOTAL IDLE TIME RESULTS

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	11.193	10.612	10.284	6.325	6.249	6.542	2.004	2.954	5.119	14.326	13.982	14.153	8.230	8.136	7.915	3.107	3.304	5.913
	P_2	13.232	13.697	14.824	8.187	8.297	10.160	2.920	4.323	6.438	14.803	14.857	15.580	8.993	9.613	10.345	3.759	4.239	6.684
	P_3	11.392	10.662	9.803	6.113	5.795	6.322	2.455	2.491	5.296	14.243	13.869	13.366	7.824	7.893	8.399	3.144	3.077	5.121
	P_4	9.325	9.533	11.184	4.991	5.682	7.669	2.101	3.031	5.714	14.252	14.697	16.363	9.018	9.519	12.427	5.988	6.249	7.937
	P_5	9.076	10.619	10.911	5.747	5.990	8.107	2.304	3.170	5.985	13.723	14.151	15.853	10.105	10.313	11.938	6.198	5.235	6.277
	P_6	12.377	12.440	12.193	7.425	7.651	9.257	3.066	3.226	5.697	16.307	16.781	16.899	9.523	9.960	12.470	5.001	5.133	8.733
	P_7	11.698	11.832	12.757	7.093	7.177	8.255	2.976	3.349	6.879	15.770	16.350	16.911	10.571	11.298	12.345	4.669	4.804	7.241
	P_8	18.042	17.606	16.491	11.193	10.633	10.331	5.326	4.465	6.237	20.100	19.204	17.843	12.221	12.021	10.133	5.357	5.562	5.285

TABLE A7.8

MEANS & COVARS IMBALANCE PATTERN(Λ)) - % TOTAL IDLE TIME RESULTS

LINE LENGTH	5									8								
BUFFER CAPACITY	1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i): P_1	11.710	12.544	14.762	7.066	7.814	10.981	4.014	4.369	8.525	14.846	15.034	16.299	8.508	9.658	10.618	3.233	4.464	7.937
P_2	13.679	13.879	14.467	7.891	7.976	9.850	3.167	3.456	7.584	15.196	15.978	16.845	9.638	9.766	11.649	3.327	4.908	6.277
P_3	11.481	13.788	15.126	7.133	7.532	10.543	3.210	4.584	9.303	14.944	15.285	16.667	8.768	9.496	11.560	3.907	4.163	8.733
P_4	8.255	8.457	9.100	4.505	7.555	7.732	2.146	2.891	7.765	13.012	12.430	12.621	7.690	8.052	8.133	3.290	3.638	5.529
P_5	7.549	7.563	9.096	4.181	4.976	6.305	1.992	2.488	5.117	10.911	10.996	11.263	7.144	7.236	7.756	2.523	2.775	5.113
P_6	12.098	12.405	13.216	6.894	7.171	8.466	3.209	3.672	7.986	15.499	15.585	15.731	9.490	9.571	10.014	3.851	3.963	6.587
P_7	11.571	11.971	12.868	7.053	7.251	9.035	3.299	3.489	7.084	15.744	15.696	16.032	9.715	9.998	9.981	3.159	4.129	6.316
P_8	18.451	19.201	20.352	11.833	13.286	15.436	4.790	7.075	9.718	20.690	21.238	22.868	13.146	14.708	16.730	5.818	7.277	9.590

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (/) & (\)) - % TOTAL IDLE TIME RESULTS

LINE LENGTH			5						8					
MEAN BUFFER CAPACITY			2			6			2			6		
% DEGREE OF MEANS IMBALANCE			2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE	(/)	A	6.246	7.322	11.985	2.725	4.937	11.894	8.235	8.559	13.286	3.433	5.107	11.602
		B	6.416	8.181	13.871	2.910	5.713	12.189	8.401	11.045	15.803	5.150	6.526	13.857
		C	8.239	10.017	14.915	3.129	5.264	11.939	10.674	11.808	15.703	3.551	5.541	12.405
		D_1	6.620	8.610	14.392	2.919	5.351	11.915	6.515	8.883	13.453	2.532	5.141	13.218
		D_2	7.242	8.999	14.747	2.390	4.848	12.320	8.710	9.747	14.691	3.213	5.888	11.398
		D_3	7.627	8.026	12.422	2.775	4.652	12.143	7.379	9.301	13.802	4.492	5.180	11.765
	SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE												
	SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE												
	(\)	A	8.835	10.206	14.390	2.601	5.476	12.014	10.973	11.927	16.951	3.217	5.543	13.159
		B	6.454	8.785	14.068	2.991	6.077	11.969	8.435	10.418	15.556	3.047	6.380	12.296
		C	6.794	6.879	11.973	2.433	5.304	11.159	9.024	9.108	12.063	4.601	5.399	11.869
		D_1	5.436	6.984	13.004	2.303	5.510	11.563	7.067	8.634	14.108	3.340	5.496	11.701
		D_2	6.251	7.765	12.714	3.494	5.672	12.513	8.098	8.901	12.955	3.324	5.808	12.671
		D_3	7.019	7.995	11.789	3.099	4.736	11.717	7.030	8.016	14.703	3.555	5.822	12.484
	SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE												
	SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE												

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (V)&(Λ)) - % TOTAL IDLE TIME RESULTS.

LINE LENGTH		5						8					
MEAN BUFFER CAPACITY		2			6			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE (V)	A	7.774	7.897	8.422	2.431	3.013	5.431	9.752	9.812	10.131	3.234	3.304	5.437
(V)	B	6.202	6.666	7.742	2.551	3.046	5.422	7.999	8.692	9.648	3.261	4.020	6.717
(V)	C	6.599	7.081	7.614	1.973	3.211	5.314	9.483	9.583	9.967	2.780	2.851	6.280
(V)	D_1	5.435	6.095	6.987	1.827	2.424	5.260	6.275	6.316	7.714	2.341	2.808	5.427
(V)	D_2	6.371	6.438	6.995	2.819	2.952	5.627	6.843	7.116	9.179	2.349	3.324	5.569
(V)	D_3	6.289	6.478	7.385	2.242	3.097	5.697	6.642	6.749	7.753	3.852	3.929	6.049
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE												
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE												
(Λ)	A	7.627	7.776	10.740	2.147	4.016	8.401	9.760	10.282	12.088	3.469	4.921	7.020
(Λ)	B	6.010	6.234	8.749	2.635	3.386	8.336	7.826	7.958	9.641	2.755	2.815	5.95[illegible]
(Λ)	C	7.420	7.802	10.248	2.706	3.459	8.018	10.125	10.671	12.164	3.791	4.967	7.431
(Λ)	D_1	5.697	6.101	8.693	2.301	3.368	7.141	6.283	6.330	8.706	2.928	3.441	6.24[illegible]
(Λ)	D_2	6.121	7.896	9.381	2.615	2.800	8.001	8.081	8.727	11.274	2.961	3.630	6.60[illegible]
(Λ)	D_3	7.601	8.549	11.019	3.599	4.468	7.801	6.997	7.551	9.492	4.832	5.118	8.124
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE												
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE												

LINE LENGTH			5								8							
PATTERN OF COVARS IMBALANCE (P_i)			P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
MEAN BUFFER CAPACITY	2	PATTERN OF BUFFER CAPACITIES IMBALANCE																
	2	A	8.615	10.785	9.668	7.460	3.857	11.353	6.647	14.709	9.866	14.225	12.155	13.642	9.155	15.897	9.625	18.168
	2	B	5.916	9.917	5.433	7.389	7.660	8.341	8.019	10.879	10.368	11.291	9.722	13.539	12.779	14.379	14.747	12.839
	2	C	9.186	10.574	8.241	3.417	7.373	6.576	10.705	14.779	12.455	13.735	12.069	6.487	12.964	9.140	15.855	18.124
	2	D_1	6.135.	10.120	6.996	5.219	7.159	6.895	9.539	11.416	8.871	9.859	9.415	10.198	11.371	12.014	12.524	11.859
	2	D_2	7.484	9.119	8.893	3.932	7.317	6.638	9.399	13.472	10.363	11.528	9.310	7.250	12.267	9.897	13.009	14.648
	2	D_3	10.639	10.028	10.051	3.781	3.422	7.914	8.156	15.916	11.292	10.127	9.923	7.866	10.805	9.371	12.293	18.552
	6	A	3.926	4.245	3.995	2.187	1.583	4.985	2.139	7.028	4.675	6.093	5.155	6.023	3.574	7.486	4.438	9.147
	6	B	3.265	4.716	1.948	3.723	3.888	4.543	4.392	4.659	4.050	5.252	4.036	6.725	6.635	7.253	7.525	6.048
	6	C	4.148	5.690	3.805	1.544	3.491	2.873	5.769	6.065	4.719	5.588	4.682	2.299	5.438	3.663	7.374	8.958
	6	D_1	2.607	4.733	2.908	2.717	3.507	2.736	4.428	5.814	4.225	4.985	4.289	4.384	6.897	7.004	7.422	6.849
	6	D_2	3.695	4.328	3.921	1.632	3.316	2.846	4.643	6.370	4.038	4.319	4.851	2.537	4.458	3.953	5.569	6.609
	6	D_3	5.138	4.232	4.604	1.737	1.706	3.468	3.879	8.673	4.657	4.642	4.092	2.472	5.481	3.617	5.639	9.259

TABLE A7.11

COVARS & BUFFER CAPACITIES IMBALANCE -
% TOTAL IDLE TIME RESULTS

MEANS IMBALANCE - ADDITIONAL % TOTAL IDLE TIME RESULTS FOR (V) (THE BOWL PHENOMENON)

LINE LENGTH	BUFFER CAPACITY	% DEGREE OF IMBALANCE	% IDLE TIME
5	3	2	3.704
5	3	5	3.981
5	3	12	6.007
5	3	18	8.702
8	3	2	4.161
8	3	5	4.418
8	3	12	6.549
8	3	18	9.133
10	1	2	11.537
10	1	5	11.800
10	1	12	12.089
10	1	18	13.249
10	2	2	5.935
10	2	5	6.807
10	2	12	7.778
10	2	18	10.262
10	3	2	4.260
10	3	5	4.722
10	3	12	7.025
10	3	18	9.526
10	6	2	2.201
10	6	5	2.684
10	6	12	5.751
10	6	18	9.256

TABLE A7.13

COVARS IMBALANCE - ADDITIONAL DATA FOR THE BEST PATTERN (P_7)

LINE LENGTH	BUFFER CAPACITY		IDLE TIME	MEAN BUFFER LEVEL
5	3	V,S,S,S,M	3.225	0.921
8	3	V,V,S,S,S,S,M,M	6.740	0.736
10	1	V,V,V,S,S,S,S,M,M,M	17.378	0.281
10	2	V,V,V,S,S,S,S,M,M,M	11.035	0.494
10	3	V,V,V,S,S,S,S,M,M,M	8.037	0.792
10	6	V,V,V,S,S,S,S,M,M,M	5.363	1.265

TABLE A7.14

BUFFER CAPACITIES IMBALANCE - ADDITIONAL POINTS FOR THE BEST PATTERNS(D_2 & D_1)

LINE LENGTH	MEAN BUFFER CAPACITY	PATTERN	TOTAL BUFFER CAPACITY DISTRIBUTION	IDLE TIME	MEAN BUFFER LEVEL
5	4	A_1	2,2,2,10	5.008	0.988
	4	D_2	4,6,4,2	3.261	2.525
8	4	A_2	2,2,2,2,12,4,4	5.592	1.038
	4	D_1	4,4,4,6,6,2,2	4.225	2.660

MEANS & COVARS IMBALANCE - ADDITIONAL % TOTAL IDLE TIME

RESULTS FOR THE BEST PATTERN $((\Lambda)+P_5)$

LINE LENGTH	BUFFER CAPACITY	% DEGREE OF IMBALANCE	% IDLE TIME
5	3	2	3.298
5	3	5	4.226
5	3	12	5.862
8	3	2	5.303
8	3	5	5.607
8	3	12	6.825
10	1	2	13.793
10	1	5	14.468
10	1	12	14.871
10	2	2	8.316
10	2	5	8.435
10	2	12	9.594
10	3	2	6.183
10	3	5	6.326
10	3	12	7.216
10	6	2	3.019
10	6	5	3.134
10	6	12	5.682

TABLE A7.16

MEANS & COVARS IMBALANCE - ADDITIONAL % TOTAL IDLE TIME RESULTS FOR THE (V) PATTERNS WITH DOWNTREND CURVES

LINE LENGTH	BUFFER CAPACITY	% DEGREE OF IMBALANCE	PATTERN (V) + P_1	PATTERN (V) + P_3	PATTERN (V) + P_6	PATTERN (V) + P_8
5	1	18	10.807	10.536	NA	16.157
5	2	18	NA	NA	NA	11.159
5	3	2	4.702	4.041	5.052	8.271
5	3	5	4.757	4.262	5.294	7.472
5	3	12	6.006	5.752	7.239	7.896
5	3	18	NA	NA	NA	8.702
5	6	18	NA	NA	NA	7.964
8	1	18	14.248	13.887	NA	17.288
8	2	18	9.433	NA	NA	10.827
8	3	2	5.283	6.221	6.485	9.845
8	3	5	5.776	5.738	7.681	8.919
8	3	12	6.591	6.347	9.731	7.389
8	3	18	7.769	NA	NA	8.572
8	6	18	7.382	NA	NA	8.346
10	1	2	15.643	15.752	18.921	19.402
10	1	5	14.834	15.802	18.840	18.632
10	1	12	15.409	15.499	20.894	17.857
10	1	18	15.805	16.172	NA	16.468
10	2	2	8.940	9.207	12.704	12.485
10	2	5	8.926	8.606	12.528	11.366
10	2	12	10.121	9.886	15.029	10.556
10	2	18	12.336	11.888	NA	10.825
10	3	2	6.916	6.547	9.837	8.834

TABLE A7.17

MEANS & COVARS IMBALANCE - ADDITIONAL % TOTAL IDLE TIME RESULTS FOR THE (V) PATTERNS WITH DOWNTREND CURVES - CONTINUED

LINE LENGTH	BUFFER CAPACITY	% DEGREE OF IMBALANCE	PATTERN (V) + P_1	PATTERN (V) + P_3	PATTERN (V) + P_6	PATTERN (V) + P_8
10	3	5	7.328	6.369	9.890	8.434
10	3	12	8.757	8.063	11.706	7.440
10	3	18	10.567	10.236	NA	9.852
10	6	2	3.247	3.631	5.110	6.062
10	6	5	4.348	3.151	5.237	4.622
10	6	12	6.649	5.908	7.761	6.059
10	6	18	9.239	9.012	NA	9.421

TABLE A7.18

MEANS & BUFFER CAPACITIES IMBALANCE - ADDITIONAL % TOTAL IDLE TIME RESULTS FOR THE BEST PATTERN ((V)+D_1)

LINE LENGTH	MEAN BUFFER CAPACITY	% DEGREE OF IMBALANCE	TOTAL BUFFER CAPACITY DISTRIBUTION	% IDLE TIME
5	4	2		4.329
5	4	5	4,6,4,2	4.372
5	4	12		6.306
8	4	2		3.946
8	4	5	4,4,4,9,9,2,2	4.413
8	4	12		6.644
10	2	2		9.648
10	2	5	2,3,1,2,3,1,2,3,1	9.699
10	2	12		11.211
10	4	2		5.589
10	4	5	4,6,2,4,6,2,4,6,2	6.073
10	4	12		7.539
10	6	2		2.564
10	6	5	6,9,3,6,9,3,6,9,3	2.815
10	6	12		5.798

TABLE A7.19

COVARS AND BUFFER CAPACITIES IMBALANCE - ADDITIONAL DATA FOR THE BEST PATTERN (C+P_4)

LINE LENGTH	MEAN BUFFER CAPACITY	PATTERN	TOTAL BUFFER CAPACITY DISTRIBUTION	IDLE TIME	MEAN BUFFER LEVEL
5	4	A + 4	2,2,6,6	4.298	0.536
	4	C + 4	6,6,2,2	1.875	1.918
8	4	A + 4	2,2,2,2,12,4,4	8.603	0.472
	4	C + 4	12,4,4,2,2,2,2	3.587	1.821

TABLE A7.20

% STARVING & BLOCKING IDLE TIMES RESULTS

TYPE AND PATTERN OF IMBALANCE	LINE LENGTH	BUFFER CAPACITY	% DEGREE OF MEANS IMBALANCE	% STARVING IDLE TIME	% BLOCKING IDLE TIME
MEANS IMBALANCE - PATTERN (V)	5	1	2	3.775	5.626
	5	1	5	3.344	6.207
	5	1	12	2.890	7.242
	5	1	18	5.093	6.716
	5	2	2	2.014	2.927
	5	2	5	2.479	2.595
	5	2	12	2.624	3.904
	5	2	18	3.358	5.389
	5	6	2	0.883	1.177
	5	6	5	1.141	1.396
	5	6	12	1.414	3.581
	5	6	18	0.171	8.388
	8	1	2	3.740	7.384
	8	1	5	3.987	7.252
	8	1	12	3.817	7.957
	8	1	18	4.624	8.232
	8	2	2	2.451	3.321
	8	2	5	2.784	3.296
	8	2	12	3.412	3.945
	8	2	18	3.563	6.123
	8	6	2	1.320	0.826
	8	6	5	1.520	1.064
	8	6	12	2.144	3.514
	8	6	18	7.743	1.155

TABLE A7.20 - CONTINUED

% STARVING AND BLOCKING IDLE TIMES RESULTS - CONTINUED

TYPE AND PATTERN OF IMBALANCE		LINE LENGTH	BUFFER/MEAN BUFFER CAPACITY	% DEGREE OF MEANS IMBALANCE	% STARVING IDLE TIME	% BLOCKING IDLE TIME
COVARS IMBALANCE - PATTERN P_7		5	1	0	4.628	3.926
		5	2	0	3.175	1.234
		5	6	0	0.793	0.383
		8	1	0	8.642	4.296
		8	2	0	6.819	1.586
		8	6	0	2.369	0.527
BUFFER CAPACITIES IMBALANCE - PATTERN	D_2	5	2	0	1.744	3.854
		5	6	0	0.647	1.086
	D_1	8	2	0	1.854	5.258
		8	6	0	0.805	1.408
MEANS AND COVARS IMBALANCE - PATTERN $(\Lambda) + P_5$		5	1	2	2.200	5.349
		5	1	5	2.024	5.539
		5	1	12	3.395	5.701
		5	2	2	1.196	2.985
		5	2	5	1.711	3.265
		5	2	12	2.766	3.539
		5	6	2	0.643	1.349
		5	6	5	0.903	1.585
		5	6	12	3.741	4.024
		8	1	2	1.816	9.095
		8	1	5	2.002	8.994
		8	1	12	2.679	8.584
		8	2	2	1.350	5.794
		8	2	5	1.467	5.769

TABLE A7.20 - CONTINUED

% STARVING AND BLOCKING IDLE TIMES RESULTS - CONTINUED

TYPE AND PATTERN OF IMBALANCE	LINE LENGTH	BUFFER/MEAN BUFFER CAPACITY	% DEGREE OF MEANS IMBALANCE	% STARVING IDLE TIME	% BLOCKING IDLE TIME
MEANS AND COVARS IMBALANCE - PATTERN $(\Lambda)+P_5$	8	2	12	2.203	5.553
	8	6	2	0.425	2.098
	8	6	5	0.622	2.153
	8	6	12	2.106	3.008
MEANS AND BUFFER CAPACITIES IMBALANCE - PATTERN $(V)+D_1$	5	2	2	0.744	4.691
	5	2	5	0.801	5.294
	5	2	12	1.125	5.862
	5	6	2	0.518	1.309
	5	6	5	0.542	1.882
	5	6	12	0.675	4.565
	8	2	2	2.987	3.287
	8	2	5	2.577	3.739
	8	2	12	1.801	5.940
	8	6	2	0.280	2.061
	8	6	5	0.441	2.367
	8	6	12	0.482	4.945
COVARS AND BUFFER CAPACITIES IMBALANCE - PATTERN $C+P_4$	5	2	0	1.441	1.976
	5	6	0	0.279	0.454
	8	2	0	1.655	4.832
	8	6	0	0.962	1.337

TABLE A7.21

ADDITIONAL % STARVING AND BLOCKING IDLE TIMES DATA FOR THE BEST PATTERNS + ADDITIONAL STOCKHOLDING DATA FOR THE BEST PATTERNS

LINE LENGTH	BUFFER / MEAN BUFFER CAPACITY	COVARS IMBALANCE		BUFFER CAPACITIES IMBALANCE		COVARS AND BUFFER CAPACITIES IMBALANCE	
		% STARVING	% BLOCKING	% STARVING	% BLOCKING	% STARVING	% BLOCKING
5	3	2.613	0.612	—	—	—	—
5	4	—	—	0.995	2.266	1.386	0.489
8	3	5.509	1.231	—	—	—	—
8	4	—	—	1.072	3.153	2.359	1.228

LINE LENGTH	BUFFER / MEAN BUFFER CAPACITY	COVARS IMBALANCE		BUFFER CAPACITIES IMBALANCE		COVARS AND BUFFER CAPACITIES IMBALANCE	
		TOTAL NUMBER OF UNITS	SPACE UTILIZATION	TOTAL NUMBER OF UNITS	SPACE UTILIZATION	TOTAN NUMBER OF UNITS	SPACE UTILIZATION
5	3	8.523	0.521	—	—	—	—
5	4	—	—	8.702	0.414	6.929	0.330
8	3	12.613	0.435	—	—	—	—
8	4	—	—	14.819	0.256	10.616	0.295

TABLE A7.22

MEANS IMBALANCE - MEAN BUFFER LEVEL RESULTS

LINE LENGTH		5												8										
BUFFER CAPACITY		1				2				6				1				2				6		
% DEGREE OF IMBALANCE		2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18	2	5	1
PATTERN OF MEANS IMBALANCE	/	0.591	0.673	0.814	0.880	1.143	1.420	1.724	1.841	4.197	4.953	5.047	5.843	0.614	0.698	0.812	0.864	1.280	1.433	1.683	1.809	4.324	4.804	5.5
	\	0.468	0.398	0.249	0.153	0.920	0.536	0.291	0.160	1.672	0.991	0.413	0.156	0.503	0.406	0.284	0.191	0.827	0.641	0.347	0.225	2.186	1.049	0.4
	RAND	0.528	0.549	0.561	0.813	1.129	1.253	1.699	1.804	3.247	3.655	4.138	5.792	0.530	0.527	0.525	0.753	1.094	1.498	1.129	1.598	2.865	2.847	3.1
	∧	0.542	0.526	0.542	0.529	1.272	1.018	1.005	1.011	2.976	2.946	2.961	2.985	0.552	0.548	0.544	0.541	1.089	1.084	1.046	1.007	3.021	3.020	2.4
	∨	0.533	0.560	0.588	0.550	1.032	1.024	1.046	1.109	2.504	2.690	2.666	3.956	0.569	0.554	0.575	0.553	1.014	1.003	0.971	1.136	2.760	2.752	3.2

TABLE A7.23

COVARS IMBALANCE - MEAN BUFFER LEVEL RESULTS

LINE LENGTH		5			8		
BUFFER CAPACITY		1	2	6	1	2	6
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.525	0.966	2.914	0.500	1.019	2.661
	P_2	0.510	0.982	3.005	0.531	1.046	3.234
	P_3	0.513	1.043	2.905	0.496	1.092	2.849
	P_4	0.567	1.048	3.026	0.567	0.975	2.810
	P_5	0.533	1.012	3.066	0.555	1.034	2.669
	P_6	0.540	1.047	3.297	0.557	1.066	3.402
	P_7	0.397	0.638	2.433	0.285	0.464	1.203
	P_8	0.668	1.373	3.618	0.722	1.575	4.756
	P_9	0.564	1.083	3.356	0.574	1.072	3.138
	P_{10}	0.503	0.992	2.883	0.506	0.941	2.816
	P_{11}	0.402	0.727	2.437	0.326	0.576	1.537
	P_{12}	0.674	1.271	3.719	0.755	1.452	4.548

TABLE A7.24

BUFFER CAPACITIES IMBALANCE -
MEAN BUFFER LEVEL RESULTS

LINE LENGTH		5		8	
MEAN BUFFER CAPACITY		2	6	2	6
PATTERN OF BUFFER CAPACITIES IMBALANCE	A_1	0.531	1.457	0.531	1.444
	A_2	0.534	1.679	0.513	1.417
	A_3	0.615	1.733	0.514	1.429
	B_1	0.677	1.843	1.012	2.648
	B_2	0.798	2.170	0.942	2.664
	B_3	1.090	3.341	1.076	2.689
	C_1	1.537	1.684	1.574	2.543
	C_2	1.507	2.506	1.136	1.965
	C_3	1.468	4.417	1.577	2.480
	D_1	1.235	2.758	1.314	3.458
	D_2	1.218	2.792	1.411	4.248
	D_3	0.886	2.678	0.814	2.281
	D_4	0.810	2.510	0.829	2.760
	D_5	1.284	4.135	1.198	3.140
	D_6	1.025	2.741	1.144	3.258

TABLE A7.25

MEANS & COVARS IMBALANCE (PATTERN (/)) -
MEAN BUFFER LEVEL RESULTS

	LINE LENGTH	5									8								
	BUFFER CAPACITY	1			2			6			1			2			6		
	% DEGREE OF MEANS IMBALANCE	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.550	0.625	0.759	1.120	1.257	1.660	3.826	5.027	5.669	0.577	0.632	0.805	1.130	1.308	1.552	3.467	4.590	5.379
	P_2	0.576	0.651	0.787	1.114	1.383	1.375	3.595	4.716	5.329	0.597	0.668	0.766	1.193	1.379	1.647	3.792	4.784	5.358
	P_3	0.568	0.618	0.784	1.087	1.262	1.381	3.684	5.037	5.520	0.588	0.644	0.744	1.187	1.280	1.590	3.598	4.691	4.917
	P_4	0.469	0.636	0.857	1.087	1.467	1.828	4.079	5.229	5.828	0.362	0.507	0.844	0.718	1.299	1.749	3.592	5.153	5.613
	P_5	0.754	0.828	0.879	1.551	1.694	1.795	5.252	5.432	5.607	0.813	0.846	0.887	1.626	1.711	1.775	5.202	5.199	5.609
	P_6	0.456	0.535	0.795	0.867	1.210	1.482	3.613	5.059	5.758	0.325	0.408	0.680	0.628	0.918	1.734	1.762	4.934	4.989
	P_7	0.690	0.748	0.835	1.412	1.549	1.638	4.420	5.167	5.490	0.783	0.801	0.843	1.558	1.578	1.708	4.768	4.926	5.301
	P_8	0.542	0.574	0.687	1.064	1.156	1.512	3.530	4.066	5.190	0.511	0.545	0.613	1.001	1.055	1.309	3.085	3.923	4.712

TABLE A7.26

MEANS & COVARS IMBALANCE (PATTERN (\)) -

MEAN BUFFER LEVEL RESULTS

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.491	0.430	0.294	0.894	0.708	0.366	2.289	1.090	0.393	0.500	0.444	0.338	0.892	0.699	0.478	2.021	1.492	0.682
	P_2	0.496	0.403	0.260	0.849	0.643	0.391	1.833	1.336	0.549	0.482	0.432	0.292	0.827	0.644	0.383	1.964	1.226	0.694
	P_3	0.504	0.439	0.299	0.917	0.736	0.395	2.346	1.235	0.441	0.517	0.468	0.338	0.954	0.728	0.468	2.169	1.721	0.750
	P_4	0.289	0.204	0.133	0.421	0.333	0.224	0.933	0.553	0.332	0.257	0.196	0.135	0.389	0.301	0.216	0.927	0.698	0.392
	P_5	0.626	0.441	0.181	0.985	0.476	0.240	1.971	0.559	0.349	0.724	0.574	0.239	1.348	0.599	0.242	2.115	0.858	0.400
	P_6	0.346	0.290	0.225	0.645	0.499	0.340	1.536	1.137	0.590	0.278	0.245	0.193	0.476	0.371	0.301	1.337	0.956	0.615
	P_7	0.625	0.543	0.300	1.146	0.885	0.264	2.485	1.081	0.334	0.740	0.670	0.430	1.344	1.060	0.533	2.810	1.054	1.033
	P_8	0.475	0.429	0.351	0.867	0.799	0.510	2.350	1.630	0.875	0.483	0.447	0.390	0.831	0.819	0.612	2.719	2.172	1.157

TABLE A7.27

MEANS & COVARS IMBALANCE (PATTERN (V)) -

MEAN BUFFER LEVEL RESULTS

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.519	0.519	0.512	1.002	0.963	0.989	2.641	3.180	4.179	0.566	0.525	0.563	1.013	0.978	1.089	3.134	3.518	4.436
	P_2	0.537	0.546	0.512	0.926	1.021	0.917	3.320	3.663	2.935	0.555	0.570	0.560	0.892	1.135	1.130	2.762	3.018	2.240
	P_3	0.531	0.534	0.559	1.018	0.995	1.192	3.752	2.062	4.183	0.535	0.547	0.559	1.019	1.066	1.036	2.717	3.273	2.891
	P_4	0.351	0.328	0.415	0.719	0.665	0.769	3.282	1.666	3.789	0.294	0.287	0.278	0.481	0.492	0.440	1.529	1.454	1.279
	P_5	0.668	0.695	0.657	1.396	1.356	1.335	3.503	2.967	2.177	0.757	0.782	0.760	1.611	1.567	1.598	4.944	4.749	4.317
	P_6	0.376	0.375	0.398	0.653	0.624	0.694	2.236	2.213	2.037	0.338	0.286	0.259	0.570	0.490	0.414	1.305	1.238	1.030
	P_7	0.669	0.678	0.696	1.334	1.258	1.311	3.741	3.663	3.401	0.761	0.785	0.793	1.532	1.572	1.536	4.596	4.278	4.868
	P_8	0.504	0.510	0.508	0.948	0.962	0.982	2.919	2.805	2.049	0.496	0.500	0.472	0.931	0.968	0.950	2.676	2.614	3.295

TABLE A7.28

MEANS & COVARS IMBALANCE (PATTERN (Λ)) -
MEAN BUFFER LEVEL RESULTS

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.511	0.520	0.522	1.011	0.984	1.013	2.916	3.048	3.107	0.535	0.541	0.527	0.991	1.006	1.023	2.990	3.001	1.279
	P_2	0.548	0.532	0.536	1.072	1.066	1.010	2.849	3.047	3.359	0.553	0.533	0.542	1.024	1.016	1.002	2.659	2.943	4.317
	P_3	0.541	0.581	0.515	1.031	1.019	0.999	3.060	2.954	2.926	0.557	0.548	0.556	1.025	0.941	1.050	2.977	3.041	1.230
	P_4	0.382	0.344	0.393	0.768	0.872	0.877	1.836	2.504	2.894	0.309	0.314	0.328	0.552	0.478	0.679	1.375	1.642	3.034
	P_5	0.683	0.679	0.628	1.344	1.139	1.140	3.931	3.739	3.119	0.780	0.770	0.738	1.496	1.499	1.394	4.670	4.165	3.237
	P_6	0.386	0.401	0.421	0.757	0.807	0.874	2.376	2.393	2.879	0.327	0.347	0.387	0.585	0.626	0.736	1.720	2.097	2.903
	P_7	0.648	0.651	0.612	1.262	1.208	1.163	3.994	3.855	3.195	0.740	0.727	0.684	1.476	1.437	1.258	4.254	3.758	3.381
	P_8	0.607	0.507	0.516	0.998	0.986	0.982	2.907	3.013	2.994	0.490	0.500	0.501	0.964	0.982	1.011	2.837	2.915	3.041

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (/)&(\)) - MEAN BUFFER LEVEL RESULTS

		LINE LENGTH	5						8					
		MEAN BUFFER CAPACITY	2			6			2			6		
		% DEGREE OF MEANS IMBALANCE	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE	(/)	A	0.777	1.264	1.642	3.886	4.699	5.667	0.592	0.844	1.673	2.862	4.702	5.534
	(/)	B	1.380	1.540	1.804	4.460	5.097	5.681	1.419	1.684	1.803	5.112	5.298	5.707
	(/)	C	1.550	1.688	1.824	4.549	5.105	5.610	1.622	1.702	1.796	4.163	5.002	5.560
	(/)	D_1	1.432	1.599	1.819	4.298	5.127	5.595	1.251	1.560	1.782	3.984	4.778	5.649
	(/)	D_2	1.482	1.627	1.812	3.197	4.930	5.613	1.344	1.508	1.776	4.194	5.027	5.447
	(/)	D_3	1.175	1.303	1.688	4.439	4.774	5.706	1.313	1.535	1.680	3.188	4.387	5.500
	SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE												
	SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE												
	(\)	A	0.467	0.353	0.216	1.251	0.753	0.346	0.466	0.385	0.246	1.629	0.942	0.401
	(\)	B	0.690	0.438	0.222	1.458	0.777	0.376	0.689	0.438	0.248	2.044	1.154	0.520
	(\)	C	1.361	0.835	0.369	2.014	0.977	0.491	1.474	1.377	0.518	2.869	1.626	0.621
	(\)	D_1	1.109	0.611	0.304	1.991	0.756	0.439	0.808	0.583	0.350	1.637	1.154	0.541
	(\)	D_2	0.988	0.741	0.382	1.257	0.820	0.435	1.063	0.904	0.555	1.926	0.951	0.640
	(\)	D_3	0.909	0.705	0.446	2.016	1.259	0.549	1.029	0.745	0.473	2.523	1.426	0.459
	SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE												
	SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE												

TABLE A7.30

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (V)&(Λ)) - MEAN BUFFER LEVEL RESULTS

LINE LENGTH			5						8					
MEAN BUFFER CAPACITY			2			6			2			6		
% DEGREE OF MEANS IMBALANCE			2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE	(V)	A	0.646	0.671	0.711	2.365	3.575	3.940	0.503	0.528	0.509	2.510	3.423	2.061
		B	1.293	1.256	1.201	3.278	3.116	2.565	1.222	1.166	0.927	4.429	4.290	4.014
		C	1.412	1.401	1.381	3.121	2.626	3.064	1.564	1.597	1.614	3.715	3.873	3.129
		D_1	1.315	1.455	1.479	3.552	3.683	4.329	0.928	1.085	1.344	3.994	4.109	4.281
		D_2	1.179	1.245	1.262	2.220	2.237	2.298	1.201	1.199	1.123	2.781	2.873	2.394
		D_3	0.789	0.981	0.977	2.473	2.717	3.619	1.245	1.239	1.168	3.226	3.138	3.024
	SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE												
	SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE												
	(Λ)	A	0.582	0.568	0.539	2.816	2.590	2.428	0.526	0.522	0.506	2.414	2.483	2.468
		B	0.986	0.943	1.025	3.646	3.104	3.023	0.990	1.288	1.326	2.776	2.849	2.859
		C	1.470	1.468	1.494	3.901	3.943	3.977	1.545	1.568	1.558	3.587	3.406	3.377
		D_1	1.217	1.309	1.299	2.959	3.259	3.288	0.971	0.999	0.989	2.834	3.023	3.001
		D_2	1.123	1.109	1.110	2.088	2.567	2.548	1.184	1.177	1.109	3.167	3.149	3.025
		D_3	1.022	1.025	1.012	2.935	2.980	3.065	1.214	1.136	1.185	3.038	3.110	3.082
	SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE												
	SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE												

LINE LENGTH		5								8							
PATTERN OF COVARS IMBALANCE (P_i)		P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
MEAN BUFFER CAPACITY 2 — PATTERN OF BUFFER CAPACITIES IMBALANCE	A	0.539	0.592	0.544	0.317	0.759	0.356	0.850	0.617	0.733	0.545	0.564	0.259	1.485	0.291	1.321	0.515
	B	1.010	1.111	1.010	0.417	1.649	0.547	1.434	0.936	1.227	1.093	0.981	0.303	1.753	0.345	1.747	0.910
	C	1.487	1.418	1.488	0.899	1.627	1.107	1.646	1.418	1.512	1.551	1.530	1.333	1.772	1.146	1.759	1.479
	D_1	1.184	1.357	1.287	0.793	1.661	0.763	1.542	1.163	0.908	1.282	0.788	0.459	1.708	0.495	1.663	0.802
	D_2	1.031	1.244	1.074	1.005	1.651	0.880	1.478	1.182	1.214	1.199	1.162	0.960	1.743	0.818	1.644	1.050
	D_3	1.002	0.971	1.052	0.818	1.100	0.837	1.196	1.025	1.169	1.022	1.175	0.671	1.608	0.672	1.545	1.098
MEAN BUFFER CAPACITY 6 — PATTERN OF BUFFER CAPACITIES IMBALANCE	A	1.596	2.107	1.668	0.733	2.126	1.531	3.595	1.534	1.675	1.386	1.463	0.631	1.847	0.822	2.134	1.463
	B	2.815	2.928	2.873	0.741	5.184	1.328	4.872	2.644	2.528	2.278	2.316	0.888	2.627	0.896	2.614	1.779
	C	4.423	4.333	4.417	3.472	5.228	3.539	4.878	4.008	0.809	2.154	1.743	2.010	2.724	1.634	2.491	1.782
	D_1	3.852	4.028	3.871	2.536	5.129	2.626	4.669	3.849	4.054	4.124	3.914	1.511	5.287	1.486	5.168	3.094
	D_2	2.557	3.908	3.226	2.179	4.746	2.219	4.677	3.387	3.083	3.332	3.535	2.112	4.838	2.077	4.342	3.119
	D_3	2.966	2.998	2.982	2.170	3.880	2.117	3.941	3.144	2.080	2.037	2.111	2.001	3.881	2.026	2.980	2.026

TABLE A7.31

COVARS & BUFFER CAPACITIES IMBALANCE - MEAN BUFFER LEVEL RESULTS

TABLE A7.32

STOCKHOLDING RESULTS

TYPE AND PATTERN OF IMBALANCE	LINE LENGTH	BUFFER CAPACITY	% DEGREE OF MEANS IMBALANCE	NUMBER OF UNITS IN THE BUFFERS	NUMBER OF UNITS AT THE STATIONS	TOTAL NUMBER OF UNITS IN THE LINE	BUFFER UTILIZATION	SPACE UTILIZATION
MEANS - MONOTONE DECREASING ORDER (\)	5	1	2	1.872	4.501	6.373	0.468	0.708
	5	1	5	1.592	4.441	6.033	0.398	0.670
	5	1	12	0.996	4.251	5.247	0.249	0.583
	5	1	18	0.612	4.025	4.637	0.153	0.515
	5	2	2	3.680	4.747	8.427	0.460	0.648
	5	2	5	2.144	4.635	6.779	0.268	0.522
	5	2	12	1.164	4.338	5.502	0.146	0.423
	5	2	18	0.640	4.068	4.708	0.080	0.362
	5	6	2	6.688	4.871	11.559	0.279	0.398
	5	6	5	3.964	4.728	8.692	0.165	0.300
	5	6	12	1.652	4.400	6.052	0.069	0.209
	5	6	18	0.624	4.123	4.747	0.026	0.167
	8	1	2	3.521	7.058	10.579	0.503	0.705
	8	1	5	2.842	6.985	9.827	0.406	0.655
	8	1	12	1.988	6.676	8.664	0.284	0.578
	8	1	18	1.337	6.323	7.660	0.191	0.511
	8	2	2	5.789	7.491	13.280	0.414	0.604
	8	2	5	4.487	7.401	11.888	0.321	0.540
	8	2	12	2.429	6.922	9.351	0.174	0.425
	8	2	18	1.575	6.452	8.027	0.113	0.365
	8	6	2	15.302	7.763	23.065	0.364	0.461
	8	6	5	7.343	7.520	14.863	0.175	0.297
	8	6	12	3.157	6.976	10.133	0.075	0.203
	8	6	18	2.583	6.590	9.173	0.062	0.184

TABLE A7.32 - CONTINUED
STOCKHOLDING RESULTS

TYPE AND PATTERN OF IMBALANCE		LINE LENGTH	BUFFER/MEAN BUFFER CAPACITY	% DEGREE OF MEANS IMBALANCE	NUMBER OF UNITS IN THE BUFFERS	NUMBER OF UNITS AT THE STATIONS	TOTAL NUMBER OF UNITS IN THE LINE	BUFFER UTILIZATION	SPACE UTILIZATION
COVARS - PATTERN P_7		5	1	0	1.588	4.572	6.160	0.397	0.684
		5	2	0	2.552	4.780	7.331	0.319	0.564
		5	3	0	3.684	4.839	8.523	0.307	0.510
		5	6	0	9.732	4.941	14.673	0.406	0.506
		8	1	0	1.995	6.965	8.960	0.285	0.597
		8	2	0	3.248	7.328	10.576	0.232	0.481
		8	3	0	5.152	7.301	12.480	0.215	0.430
		8	6	0	8.421	7.768	16.189	0.201	0.324
BUFFERS	PATTERN A_1	5	2	0	2.124	4.545	6.669	0.266	0.513
		5	6	0	5.828	4.823	10.653	0.243	0.367
	PATTERN A_2	8	2	0	3.591	7.255	10.846	0.257	0.493
		8	6	0	9.919	7.714	17.633	0.236	0.353
MEANS AND COVARS - PATTERN ($\diagdown$) + P_4		5	1	2	1.156	4.527	5.683	0.289	0.631
		5	1	5	0.816	4.418	5.234	0.204	0.582
		5	1	12	0.532	4.203	4.735	0.133	0.526
		5	2	2	1.684	4.687	6.371	0.211	0.490
		5	2	5	1.332	4.594	5.926	0.167	0.456
		5	2	12	0.896	4.299	5.195	0.112	0.400
		5	6	2	3.732	4.809	8.541	0.156	0.284
		5	6	5	2.212	4.698	6.910	0.092	0.238
		5	6	12	1.328	4.433	5.761	0.055	0.199
		5	1	2	1.799	6.810	8.609	0.257	0.574
		5	1	5	1.372	6.596	7.968	0.196	0.532
		5	1	12	0.945	6.214	7.159	0.135	0.477

TABLE A7.32 - CONTINUED

STOCKHOLDING RESULTS

TYPE AND PATTERN OF IMBALANCE	LINE LENGTH	BUFFER CAPACITY	% DEGREE OF MEANS IMBALANCE	NUMBER OF UNITS IN THE BUFFERS	NUMBER OF UNITS AT THE STATIONS	TOTAL NUMBER OF UNITS IN THE LINE	BUFFER UTILIZATION	SPACE UTILIZATION
MEANS AND COVARS - PATTERN (↖) + P_4	8	2	2	2.723	7.117	9.840	0.195	0.447
	8	2	5	2.107	6.911	9.018	0.151	0.410
	8	2	12	1.512	6.561	8.073	0.108	0.367
	8	6	2	6.489	7.588	14.077	0.155	0.282
	8	6	5	4.886	7.321	12.207	0.116	0.244
	8	6	12	2.744	6.860	9.604	0.065	0.192
MEANS AND BUFFER CAPACITIES - PATTERN (↖) + A	5	2	2	1.868	4.558	6.426	0.234	0.494
	5	2	5	1.412	4.490	5.902	0.177	0.454
	5	2	12	0.864	4.281	5.145	0.108	0.396
	5	6	2	5.004	4.870	9.874	0.209	0.341
	5	6	5	3.012	4.726	7.738	0.126	0.267
	5	6	12	1.384	4.399	5.783	0.058	0.199
	8	2	2	3.262	7.122	10.384	0.233	0.472
	8	2	5	2.695	7.046	9.741	0.193	0.443
	8	2	12	1.772	6.644	8.366	0.123	0.380
	8	6	2	11.403	7.743	19.146	0.272	0.383
	8	6	5	6.594	7.557	14.151	0.157	0.283
	8	6	12	2.807	6.947	9.754	0.067	0.195
COVARS AND BUFFER CAPACITIES - PATTERN A+P_4	5	2	0	1.268	4.627	5.895	0.159	0.454
	5	6	0	2.932	4.891	7.823	0.122	0.270
	8	2	0	1.813	6.909	8.722	0.130	0.397
	8	6	0	4.417	7.518	11.935	0.105	0.239

TABLE A7.33

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - MEANS IMBALANCE

LINE LENGTH		5												8											
BUFFER CAPACITY		1				2				6				1				2				6			
% DEGREE OF IMBALANCE		2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18
PATTERN OF MEANS IMBALANCE	/			**	**		*	**	**		**	**	**	< 0.46		**	**		*	**	**		**	**	**
	\		*	**	**		**	**	**		**	**	**		*	**	**		*	**	**		**	**	**
	RAND		*	**	**		*	**	**		**	**	**		*	**	**			**	**		**	**	**
	∧			**	**			**	**			**	**	< 2.35		**	**	< 1.65		**	**			**	**
	∨	< 1.27			**	< 0.88			**	< 0.31		**	**	< 3.46	< 2.46			< 2.74		*	**	< 0.82		**	**
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	1.417	1.436	1.665	1.463	0.999	1.388	1.838	2.160	1.506	1.371	1.717	1.691	0.569	0.986	1.146	1.888	0.911	1.459	1.390	1.380	1.298	1.601	1.429	1.162
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	2.012	2.038	2.364	2.076	1.418	1.966	2.611	3.066	2.138	1.946	2.435	2.402	0.808	1.399	1.624	2.680	1.293	2.069	1.973	1.959	1.842	2.274	2.031	1.648

TABLE A7.34

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - COVARS IMBALANCE

LINE LENGTH			5			8		
BUFFER CAPACITY			1	2	6	1	2	6
PATTERN OF COVARS IMBALANCE (P_i)		P_1	**	**		**	**	
		P_2	*			**	*	
		P_3	**	**		**	**	
		P_4	**	**		**	**	
		P_5	*			**	**	
		P_6	**	*		**	**	
		P_7	$<$ 10.17	$<$ 11.57	$<$ 43.08	**	*	
		P_8	$<$ 8.59	$<$ 6.13	$<$ 1.48	**	**	
		P_9	**	**	**	**	**	**
		P_{10}	**	**	**	**	**	*
		P_{11}	**			**	**	**
		P_{12}	**			**	**	*
LEVEL OF SIGNIFICANCE	0.05	CRITICAL DIFFERENCE	1.521	2.132	1.613	1.117	1.971	1.796
	0.01		1.914	2.683	2.030	1.407	2.681	2.261

TABLE A7.35

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - BUFFER CAPACITIES IMBALANCE

LINE LENGTH			5		8	
MEAN BUFFER CAPACITY			2	6	2	6
PATTERN OF BUFFER CAPACITIES IMBALANCE		A_1	**	*	**	**
		A_2	**		**	
		A_3	**		**	*
		B_1	**		**	
		B_2	*		*	
		B_3			**	
		C_1	**		**	**
		C_2	**		**	
		C_3	**		**	*
		D_1				
		D_2		< 16.14	*	
		D_3			**	
		D_4			*	
		D_5	*		*	
		D_6	**		**	
LEVEL OF SIGNIFICANCE	0.05	CRITICAL DIFFERENCE	1.157	1.415	1.308	1.436
	0.01		1.447	1.771	1.637	1.798

TABLE A7.36

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & COVARS IMBALANCE (PATTERN (/))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	**	**	**	**	**	**		**	**	**	**	**	*	**	**	*	**	**
	P_2	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**		**	**
	P_3	*	**	**		**	**		**	**	**	**	**	**	**	**		**	**
	P_4	< 0.70	*	**		**	**	**	**	**	**	**	**	**	**	**	*	**	**
	P_5	< 18.37	< 15.69	**	< 12.70		**		**	**	< 1.68	< 1.52	**			**		**	**
	P_6	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
	P_7	< 0.53		**			**		**	**				*	*	**		**	**
	P_8	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
SIGNIFICANCE LEVEL / CRITICAL DIFFERENCE	0.05	1.788	1.928	1.757	1.680	1.915	2.682	0.887	2.151	2.557	2.332	1.973	1.895	1.925	2.515	2.072	2.569	1.830	2.434
	0.01	2.348	2.532	2.308	2.208	2.449	3.522	1.165	2.825	3.358	3.062	2.573	2.491	2.528	3.490	2.719	3.374	2.405	3.196

TABLE A7.37

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & COVARS IMBALANCE (PATTERN (\))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	*	**	**		**	**		**	**	**	**	**		**	**		*	**
	P_2	**	**	**	**	**	**		**	**	**	**	**	*	**	**		**	**
	P_3	**	**	**		**	**		**	**	**	**	**	*	**	**	*	*	**
	P_4	** < 20.67	** < 20.52	**	< 16.04		**		**	**	< 5.23	< 4.52	**		**	**		*	**
	P_5	< 0.85	**	**		**	**	**	**	**	**	**	**	**	**	**	**	**	**
	P_6	*	*	**			**		**	**	**	**		*	*	**			**
	P_7	**	**	**	*	**	**	*	**	**	**	**	**	**	**	**	**	**	**
	P_8	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	*	**
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	1.369	1.117	1.460	2.466	1.598	1.698	1.676	0.973	2.079	1.665	1.591	1.644	2.664	1.388	2.074	1.984	2.920	2.187
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	1.892	1.467	1.917	3.238	2.099	2.230	2.201	1.276	2.719	2.187	2.089	2.158	3.498	1.822	2.723	2.605	3.835	2.872

TABLE A7.38

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & COVARS IMBALANCE (PATTERN (V))

LINE LENGTH	BUFFER CAPACITY	% DEGREE OF MEANS IMBALANCE	PATTERN OF COVARS IMBALANCE (ρ_i): ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	CRITICAL DIFFERENCE, SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE, SIGNIFICANCE LEVEL 0.01
5	1	2		**		< 2.08	< 4.69	**	*	**	1.580	2.147
		5		**				**	*	**	1.779	2.335
		12		**				**	**	**	1.737	2.280
	2	2		**				**	*	**	1.750	2.299
		5		**				**	*	**	1.680	2.206
		12		**		*	**	**	**	**	2.137	2.806
	6	2	< 3.00					*		**	0.959	1.260
		5		**				*	*	**	1.157	1.636
		12	**	**	**	**	**	**	**	**	2.023	2.785
8	1	2	**	**	**	**	**	**	**	**	1.283	1.717
		5	**	**	*	**	**	**	**	**	1.712	2.505
		12	**	**	*	**	**	**	**	**	1.534	2.013
	2	2	**	**	*	**	**	**	**	**	1.696	2.225
		5	**	**	**	**	**	**	**	**	1.341	1.764
		12	**	**	**	**	**	**	**	**	1.321	1.924
	6	2				**	**	*	*	**	2.269	2.979
		5				**	**	*	*	**	2.099	2.976
		12	**	**	**	**	**	**	**	**	1.439	1.890

TABLE A7.39

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & COVARS IMBALANCE (PATTERN (Λ))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	**	**	**	**	**	**			**	**	**	**	**	**	**		**	**
	P_2	**	**	**	**	**	**			**	**	**	**	**	**	**		**	**
	P_3	*	**	**	**	**	**		*	**	**	**	**	**	**	**	*	**	**
	P_4	< 13.31	< 11.18	< 4.43	< 9.64	**	**			**				*	**	**		*	**
	P_5	< ** 20.73	< ** 20.58	< 4.48	< 16.12	< 0.18		< 3.58		**	< 5.31	< 4.56	< 2.25			*			**
	P_6	**	**	**	**	**	**			**	**	**	**	**	**	**	*	**	**
	P_7	**	**	**	**	**	**			**	**	**	**	**	**	**		**	**
	P_0	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	1.497	1.121	1.489	0.997	1.652	1.418	2.026	2.312	1.941	1.434	1.573	1.563	1.501	1.557	1.811	1.610	1.276	1.709
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	1.970	1.474	1.955	1.309	2.168	1.862	2.661	3.035	2.549	1.884	2.064	2.028	1.971	2.046	2.379	2.114	1.673	2.264

TABLE A7.40

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (/)&(\))

LINE LENGTH			5						8					
MEAN BUFFER CAPACITY			2			6			2			6		
% DEGREE OF MEANS IMBALANCE			2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE	(/)	A	*	**	**		**	**	**	**	**		**	**
		B	*	**	**		**	**	**	**	**	**	**	**
		C	**	**	**		**	**	**	**	**		**	**
		D_1	*	**	**		**	**		**	**		**	**
		D_2	**	**	**		**	**	**	**	**		**	**
		D_3	**	**	**		**	**		**	**	*	**	**
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	1.190	0.784	2.345	2.168	0.957	1.944	1.551	1.480	1.304	1.819	1.253	1.370
	0.01		1.637	1.078	3.227	2.982	1.316	2.672	2.134	2.037	1.796	2.502	1.722	1.886
	(\)	A	**	**	**		**	**	**	**	**		**	**
		B		**	**		**	**	**	**	**		**	**
		C	*	**	**		**	**	**	**	**	*	**	**
		D_1		**	**		**	**	*	**	**		**	**
		D_2		**	**	*	**	**	**	**	**		**	**
		D_3	*	**	**		**	**	*	**	**		**	**
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	1.788	1.075	1.377	1.334	1.179	1.604	1.038	1.284	1.332	1.952	1.567	1.154
	0.01		2.459	1.477	1.892	1.835	1.623	2.205	1.428	1.769	1.835	2.685	2.156	1.58[illegible]

TABLE A7.41

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (V)&(∧))

LINE LENGTH		5						8					
MEAN BUFFER CAPACITY		2			6			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE (V)	A	**	**	**				**	**	**			**
	B			*				**	**	**		*	**
	C		*	*	< 4.48			**	**	**			**
	D_1				< 11.57					**			**
	D_2							*	*	**			**
	D_3			*						*		*	**
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	1.897	1.691	2.163	1.727	2.308	1.247	0.711	1.149	1.451	1.900	1.535	2.225
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	2.609	2.325	2.974	2.376	3.175	1.715	0.967	1.578	1.996	2.614	2.112	3.061
(∧)	A	**	**	**		**	**	**	**	**		**	**
	B	**	**	**	*	*	**	**	**	**			**
	C	**	**	**	*	*	**	**	**	**	*	**	**
	D_1	*	*	**		*	**			**			**
	D_2	**	**	**			**	**	**	**			**
	D_3	**	**	**	**	**	**		**	**	**	**	**
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	0.708	0.621	2.379	0.543	1.287	2.251	1.339	1.127	1.279	1.514	1.497	0.807
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	0.974	1.154	3.271	0.750	1.769	3.095	1.842	1.548	1.756	2.083	2.060	1.111

LINE LENGTH			5								8							
PATTERN OF COVARS IMBALANCE (P_i)			P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
MEAN BUFFER CAPACITY	2	PATTERN OF BUFFER CAPACITIES IMBALANCE A	**	**	**	**	< 22.64	**		**	**	**	**	**	**	**	**	**
		B		**		**	**	**	**	**	**	**	*	**	**	**	**	**
		C	**	**	**	<** 31.46	*		**	**	**	**	**		**	**	**	**
		D_1		**	**				**	**	*	**	**	**	**	**	**	**
		D_2	**	**	**	< 21.13	**		**	**	**	**	**		**	**	**	**
		D_3	**	**	**	<* 24.15	<** 31.34	**	**	**	**	**	**		**	**	**	**
	6	A	*	**	**		< 23.38	**		**	**	**	**	**		**	**	**
		B		**	< 5.71	*	**	**	**	**	*	**	*	**	**	**	**	**
		C	*	**	*	< 25.27			**	**	*	**	*		**		**	**
		D_1		**					**	**	*	**	*	*	**	**	**	**
		D_2				< 21.01			*	**	**	**	**		**	**	**	**
		D_3	**	**	**	< 15.90	< 17.45	**	**	**	**	**	**		**	*	**	**

TABLE A7.42

% TOTAL IDLE TIME'S MULTIPLE COMPARISONS WITH CONTROL - COVARS & BUFFER CAPACITIES IMBALANCE

TABLE A7.43

MEAN BUFFER LEVEL'S MULTIPLE COMPARISONS WITH CONTROL - MEANS IMBALANCE

LINE LENGTH		5											
BUFFER CAPACITY		1				2				6			
% DEGREE OF IMBALANCE		2	5	12	18	2	5	12	18	2	5	12	18
PATTERN OF MEANS IMBALANCE	/	*	**	**	**		*	**	**		*	**	**
	\	*< 11.3	<** 25.29	**< 52.66	**< 70.91	< 10.94	**< 48.11	**< 71.83	**< 84.51	*< 49.65	**< 70.16	**< 87.56	**< 95.30
	RAND				**			**	**	< 2.23		*	*
	∧						< 1.45	< 2.71	< 2.13	< 10.39	< 11.29	< 10.84	< 10.12
	∨			*		< 0.10	< 0.87			< 24.60	< 19.00	< 19.72	
SIGNIFICANCE LEVEL / CRITICAL DIFFERENCE	0.05	0.051	0.081	0.051	0.051	0.255	0.305	0.197	0.325	1.178	1.315	0.654	1.672
	0.01	0.072	0.114	0.072	0.072	0.361	0.428	0.280	0.457	1.672	1.864	0.928	2.499

LINE LENGTH		8											
BUFFER CAPACITY		1				2				6			
% DEGREE OF IMBALANCE		2	5	12	18	2	5	12	18	2	5	12	18
PATTERN OF MEANS IMBALANCE	/	**	**	**	**	*	**	**	**	*	**	**	**
	\	**< 10.02	**< 27.37	**< 49.20	**< 65.83	< 14.74	*< 33.92	**< 64.23	**< 76.80	< 15.96	**< 59.67	**< 82.66	**< 85.81
	RAND	< 5.19	< 5.73	< 6.08	**		**	**	**				**
	∧	< 1.25	< 1.97	< 2.68	< 3.22								
	∨		< 0.90		< 1.07								
SIGNIFICANCE LEVEL / CRITICAL DIFFERENCE	0.05	0.031	0.066	0.036	0.081	0.228	0.250	0.325	0.405	1.308	1.068	0.917	1.203
	0.01	0.041	0.089	0.043	0.114	0.323	0.361	0.457	0.583	1.857	1.516	1.303	1.710

TABLE A7.44

MEAN BUFFER LEVEL'S MULTIPLE COMPARISONS WITH CONTROL - COVARS IMBALANCE

LINE LENGTH		5			8		
BUFFER CAPACITY		1	2	6	1	2	6
PATTERN OF COVARS IMBALANCE (P_i)	P_1	< 0.19	< 6.49	< 12.26	< 10.56		
	P_2	< 3.23	< 4.94	< 9.52	< 1.43		
	P_3	< 2.47		< 12.53	< 11.27		
	P_4	**		< 8.88			
	P_5		< 2.03	< 7.68	< 0.72		
	P_6			< 0.72	< 0.36		
	P_7	** < 26.53	** < 38.24	< 26.74	** < 49.02	** < 52.17	** < 53.75
	P_8	**	*		**	**	**
	P_9	**					
	P_{10}	< 4.37	< 3.97	< 14.69	< 9.48	< 2.99	
	P_{11}	** < 23.57	* < 29.62	< 26.62	** < 41.68	** < 40.62	** < 40.91
	P_{12}	**			**	**	**
LEVEL OF SIGNIFICANCE 0.05	CRITICAL DIFFERENCE	0.030	0.285	1.691	0.083	0.186	0.962
LEVEL OF SIGNIFICANCE 0.01	CRITICAL DIFFERENCE	0.038	0.358	2.101	0.105	0.234	1.211

TABLE A7.45

MEAN BUFFER LEVEL'S MULTIPLE COMPARISONS WITH CONTROL - BUFFER CAPACITIES IMBALANCE

LINE LENGTH		5		8	
MEAN BUFFER CAPACITY		2	6	2	6
PATTERN OF BUFFER CAPACITIES IMBALANCE	A_1	** < 48.60	** < 56.13	** < 45.26	* < 44.83
	A_2	** < 48.31	* < 49.44	** < 47.11	* < 45.52
	A_3	** < 40.47	* < 47.82	** < 47.01	* < 45.06
	B_1	** < 34.46	* < 44.51		
	B_2	* < 72.75	< 34.66	< 2.89	
	B_3				
	C_1	**	* < 49.29	**	< 2.35
	C_2	**	< 24.54		< 26.45
	C_3	**		**	< 4.65
	D_1		< 16.95	**	
	D_2		< 15.93	**	**
	D_3	< 14.23	< 19.36	< 13.30	< 12.30
	D_4	< 21.59	< 24.62	< 29.12	
	D_5	*		*	
	D_6	< 0.77	< 17.47		
LEVEL OF SIGNIFICANCE 0.05	CRITICAL DIFFERENCE	0.204	1.226	0.228	0.998
LEVEL OF SIGNIFICANCE 0.01	CRITICAL DIFFERENCE	0.255	1.535	0.285	1.249

TABLE A7.46

MEAN BUFFER LEVEL'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & COVARS IMBALANCE (PATTERN (/))

LINE LENGTH	5									8								
BUFFER CAPACITY	1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i): P_1		**	**		**	**		**	**		*	**		**	**		**	**
P_2		**	**		**	**		**	**	*	**	**	*	**	**		**	**
P_3		**	**		**	**		**	**		**	**	*	**	**		**	**
P_4	< 10.84	**	**		**	**		**	**	**< 35.24	< 9.30	**	**< 25.98	**	**		**	**
P_5	**	**	**	**	**	**		**	**	**	**	**	**	**	**	**	**	**
P_6	*< 13.31		**	< 16.07	*	**		**	**	**< 41.86	**< 27.01		**< 35.36	< 5.36	**	< 32.26	**	**
P_7	**	**	**	**	**	**		**	**	**	**	**	**	**	**	**	**	**
P_8		**	**			**		**	**	**< 8.59	< 2.51				**		**	**
SIGNIFICANCE LEVEL 0.05 — CRITICAL DIFFERENCE	0.060	0.035	0.016	0.091	0.156	0.127	1.959	0.523	0.270	0.035	0.060	0.127	0.191	0.156	0.104	1.533	0.647	0.220
SIGNIFICANCE LEVEL 0.01 — CRITICAL DIFFERENCE	0.079	0.046	0.020	0.250	0.204	0.145	2.570	0.694	0.354	0.046	0.079	0.145	0.250	0.204	0.137	2.013	0.855	0.289

TABLE A7.47

MEAN BUFFER LEVEL'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & COVARS IMBALANCE (PATTERN (\))

LINE LENGTH			5									8								
BUFFER CAPACITY			1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE			2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (ρ_i)	ρ_1		< 6.65	**< 18.25	**< 44.11	< 13.46	**< 31.46	**< 64.57	< 31.08	**< 67.18	**< 88.17	< 10.56	**< 20.51	**< 39.54	< 8.04	**< 27.94	**< 50.72	< 22.30	**< 42.64	**< 73.78
	ρ_2		< 5.70	**< 23.38	**< 50.57	< 17.81	**< 37.75	**< 62.15	*< 44.81	**< 59.77	**< 83.47	*< 13.78	**< 22.72	**< 47.76	< 14.74	**< 33.61	**< 60.52	< 24.50	**< 52.87	**< 73.32
	ρ_3		< 4.18	**< 16.56	**< 43.16	< 11.23	*< 28.75	**< 61.76	< 29.36	**< 62.81	**< 86.72	< 5.90	*< 16.28	**< 39.54	< 1.65	*< 24.95	**< 51.75	< 16.61	*< 33.83	**< 71.17
	ρ_4		**< 45.06	**< 61.22	**< 74.72	**< 59.25	**< 67.76	**< 78.36	**< 71.91	**< 83.35	**< 90.06	**< 54.12	**< 64.94	**< 75.85	**< 59.95	**< 68.97	**< 77.78	**< 64.36	**< 73.16	**< 84.95
	ρ_5		**	**< 16.16	**< 65.59	< 4.65	**< 58.76	**< 76.82	< 40.65	**< 83.17	**< 89.49	**		**< 57.25	*	**< 38.30	**< 75.10	< 18.69	**< 67.01	**< 84.62
	ρ_6		**< 34.22	**< 44.87	**< 57.22	**< 37.56	**< 51.69	**< 67.09	*< 53.75	**< 65.76	**< 82.23	**< 50.27	**< 56.17	**< 65.47	**< 50.93	**< 61.75	**< 68.97	*< 48.60	**< 63.25	**< 81.48
	ρ_7		**		**< 42.97		< 14.13	**< 74.44	< 25.17	**< 67.45	**< 89.94	**	*	**< 23.08	*		**< 68.25		**< 59.48	**< 60.29
	ρ_8		*< 9.70	**< 18.44	**< 33.27	< 16.07	< 22.65	**< 50.63	< 29.24	**< 50.92	**< 73.65	*< 13.60	**< 20.04	**< 30.23	< 14.33	< 15.57	*< 36.91		< 16.49	**< 55.52
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	0.049	0.060	0.060	0.220	0.256	0.092	1.472	0.561	0.311	0.070	0.082	0.070	0.291	0.183	0.104	1.117	0.685	0.348
	0.01		0.065	0.079	0.079	0.289	0.323	0.121	1.934	0.737	0.409	0.091	0.112	0.091	0.382	0.250	0.137	1.467	0.903	0.457

TABLE A7.48

MEAN BUFFER LEVEL'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & COVARS IMBALANCE (PATTERN (V))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	<1.33	<1.33	<2.66	<3.00	<6.78	<4.26	<20.48	<4.40			<6.08						*	**
	P_2			<2.66	<10.36	<1.16	<11.23	<0.04		<11.62	<0.72			<8.04					<13.88
	P_3				<1.45	<3.68			<37.91		<4.29	<2.15							
	P_4	**<33.27	**<37.64	*<21.10	<30.40	*<35.62	<25.56	<1.17	<49.83		**<44.61	**<48.66	**<50.27	**<50.41	**<49.28	**<54.64	<41.22	*<44.10	**<50.83
	P_5	**	**	*	*				<10.66	*<34.45	**	**	**	**	**	**	**	**	**
	P_6	**<28.52	**<28.71	*<24.34	*<36.79	*<39.59	<37.82	<32.67	<33.36	**<38.66	**<39.54	**<48.84	**<53.67	**<41.24	**<49.49	**<57.32	*<4983	**<52.40	**<60.40
	P_7	**	**	**							**	**	**	**	**	**	**	**	**
	P_8	<4.18	<3.04	<3.42	<8.23	<6.87	<4.94	<12.11	<15.54	**<38.30	*<11.18	*<10.56	*<15.56	<4.02	<0.21	<2.06			
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	0.070	0.052	0.104	0.330	0.330	0.412	1.144	1.988	0.911	0.062	0.049	0.070	0.191	0.220	0.330	1.230	0.694	0.978
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	0.091	0.075	0.137	0.434	0.434	0.541	1.502	2.609	1.200	0.083	0.065	0.091	0.250	0.289	0.434	1.616	1.317	1.285

TABLE A7.49

MEAN BUFFER LEVEL'S COMPARISONS WITH CONTROL - MEANS & COVARS IMBALANCE (PATTERN (Λ))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (Pi)	P_1	✓ 2.85	✓ 1.14	✓ 0.76	✓ 2.13	✓ 4.74	✓ 1.94	✓ 12.20	✓ 8.22	✓ 6.44	✓ 4.29	✓ 3.22	✓ 5.73						**✓ 50.83
	P_2						✓ 2.23	✓ 14.21	✓ 8.25		✓ 1.07	✓ 4.65	✓ 3.04						**
	P_3		*	✓ 2.09	✓ 0.19	✓ 1.36	✓ 3.29	✓ 7.86	✓ 11.05	*✓ 11.89	✓ 0.36	✓ 1.97	✓ 0.54		✓ 2.99				**✓ 60.40
	P_4	**✓ 27.38	**✓ 34.60	*✓ 25.29	**✓ 27.59	✓ 15.59	✓ 15.15	**✓ 44.72	*✓ 24.60	*✓ 12.86	**✓ 44.72	**✓ 43.83	**✓ 41.32	**✓ 43.14	**✓ 50.72	**✓ 30.00	**✓ 47.14	✓ 36.87	
	P_5	**	**		**					✓ 6.08	**	**	**	**	**	**	**		
	P_6	**✓ 26.62	**✓ 23.76	*✓ 19.96	**✓ 26.72	**✓ 21.88	✓ 15.39	*✓ 28.46	*✓ 27.94	*✓ 13.31	**✓ 41.50	**✓ 37.93	**✓ 30.86	**✓ 39.69	**✓ 35.46	*✓ 24.12	*✓ 33.87	✓ 19.38	
	P_7	**	**		**					✓ 3.79	**	**	**	**	**	**	**		
	P_8	✓ 3.61	✓ 3.61	✓ 1.90	✓ 3.39	✓ 4.55	✓ 4.94	✓ 12.47	✓ 9.27	✓ 9.85	**✓ 12.34	**✓ 10.56	**✓ 10.38	✓ 0.62					
SIGNIFICANCE LEVEL / CRITICAL DIFFERENCE	0.05	0.060	0.049	0.211	0.156	0.156	0.191	0.925	0.724	0.368	0.035	0.070	0.035	0.191	0.164	0.191	0.755	1.118	0.867
	0.01	0.079	0.065	0.289	0.206	0.204	0.250	1.309	1.009	0.479	0.046	0.091	0.046	0.250	0.204	0.250	0.911	1.467	1.138

TABLE A7.50

MEAN BUFFER LEVEL'S MULTIPLE COMPARISONS WITH CONTROL - MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (/)&(\))

LINE LENGTH			5						8				
MEAN BUFFER CAPACITY			2			6			2			6	
% DEGREE OF MEANS IMBALANCE			2	5	12	2	5	12	2	5	12	2	5
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE	(/)	A	< 24.78	*	**		**	**	**< 39.07	< 12.99	**		**
		B	*	**	**		**	**	**	**	**	**	**
		C	**	**	**		**	**	**	**	**	**	**
		D1	*	**	**		**	**	**	**	**	**	**
		D2	**	**	**	< 3.73	**	**	**	**	**	**	**
		D3		*	**		**	**	**	**	**		**
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	0.296	0.224	0.085	1.301	0.847	0.194	0.194	0.247	0.073	0.823	0.83
	0.01		0.408	0.308	0.119	1.789	1.163	0.267	0.267	0.344	0.097	1.132	1.142
	(\)	A	**< 54.79	**< 65.83	**< 79.14	**< 51.22	**< 77.34	**< 89.58	**< 51.96	**< 60.31	**< 74.64	< 37.37	** 63.8
		B	**< 33.20	**< 57.60	**< 78.51	**< 56.10	**< 76.60	**< 88.68	*< 28.97	**< 54.85	**< 74.43	< 21.42	** 55.6
		C	**	*< 19.17	**< 64.28	**< 39.36	**< 70.58	**< 85.22	**	**	**< 46.60		< 37.4
		D1		**< 40.85	**< 70.57	**< 40.05	**< 77.24	**< 86.78	< 16.70	**< 39.90	**< 63.92	< 37.06	** 55.6
		D2	< 4.36	**< 28.27	**< 63.02	**< 62.15	**< 75.31	**< 86.90		< 6.86	**< 42.78	< 25.95	** 63.4
		D3	< 12.00	**< 31.75	**< 56.83	**< 39.30	**< 62.09	**< 83.47		*< 23.20	**< 51.24	< 3.00	* 45.1
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	0.224	0.196	0.104	0.776	0.604	0.284	0.224	0.194	0.180	1.615	1.03
	0.01		0.308	0.267	0.146	1.067	0.829	0.377	0.308	0.267	0.267	1.221	1.4

TABLE A7.51

MEAN BUFFER LEVEL'S MULTIPLE COMPARISONS WITH CONTROL -

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (V)&(Λ))

LINE LENGTH		5						8			
MEAN BUFFER CAPACITY		2			6			2			6
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE (V)	A	**< 37.46	**< 35.04	< 31.17	< 28.79			**< 48.14	**< 45.57	*< 47.53	< 3.50	
(V)	B	*			< 1.30	< 6.17	< 22.76	*		< 4.43	**	
(V)	C	**	**		< 6.02	< 20.93	< 7.74	**	**	**		
(V)	D_1	*	**					< 4.33		*	*	
(V)	D_2				< 33.15	< 32.64	< 30.80	*				
(V)	D_3	*< 37.46	< 5.03	< 5.42	< 22.54	< 18.19		*				
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	0.224	0.239	0.510	1.829	2.601	1.903	0.224	0.274	0.364	1.190	2.98
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	0.308	0.344	0.706	2.516	3.579	2.618	0.308	0.377	0.511	1.637	4.11
(Λ)	A	**< 43.66	**< 45.02	**< 47.82	< 15.21	*< 22.01	**< 26.89	**< 45.77	**< 46.19	**< 47.84	< 7.19	< 4.5
(Λ)	B	< 4.55	< 8.71	< 0.77		< 6.53	< 8.97		*	**		
(Λ)	C	**	**	**			**	**	**	**	*	*
(Λ)	D_1		*	**	< 10.90	< 1.87	< 0.99					
(Λ)	D_2				*< 37.13	*< 22.70	**< 23.28					
(Λ)	D_3	< 1.07	< 0.77	< 2.03	< 11.62	< 10.27	< 7.71					
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	0.317	0.194	0.158	0.909	0.629	0.394	0.250	0.296	0.158	0.823	0.71
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	0.436	0.267	0.218	1.251	0.871	0.534	0.344	0.408	0.218	1.132	0.98

TABLE A7.52

MEAN BUFFER LEVEL'S MULTIPLE COMPARISONS WITH CONTROL - COVARS & BUFFER CAPACITIES IMBALANCE

LINE LENGTH		5								8							
PATTERN OF COVARS IMBALANCE (P_i)		P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
MEAN BUFFER CAPACITY	PATTERN OF BUFFER CAPACITIES IMBALANCE																
2	A	**< 47.82	**< 42.69	**< 47.34	**< 69.31	< 26.53	**< 65.54	< 17.72	*< 40.32	< 24.49	**< 43.81	**< 41.91	**< 73.35	**	**< 71.55	**	**< 46.96
	B	< 2.13		< 2.23	**< 59.63	**	**< 67.05	**	< 9.44				**< 68.76	**	**< 64.43	**	< 6.24
	C	**	**	**	< 12.97	**		**	**	**	**	**	**	**		**	**
	D_1		*		< 23.23	**	< 26.16	**		< 6.39	**	< 18.76	**< 52.68	**	**< 68.97	**	< 17.32
	D_2	< 0.19	**		< 2.71	**	< 14.81	**		**	**	*	< 1.03	**	< 15.67	**	
	D_3	< 3.00	< 6.00		< 20.81		< 18.97		< 0.82				**< 30.83	**	**< 30.72	**	
6	A	*< 51.96	< 36.56	*< 50.39	**< 77.93	< 35.98	*< 53.90		*< 53.81	**< 35.62	**< 46.71	**< 43.77	**< 75.76	*< 28.99	**< 68.42	< 17.97	**< 43.75
	B	< 15.25	< 11.83	< 13.49	**< 77.70	**	**< 74.65	**	*< 20.40	< 2.83	< 12.44	< 10.98	**< 65.86		**< 65.57		*< 31.62
	C	**	**	**		**		**	*	**< 68.90	< 17.21	**< 32.99	< 22.74		**< 37.18	< 6.25	**< 31.51
	D_1				< 23.66		< 20.96			**	**	*	*< 41.91	**	*< 42.89	**	
	D_2	< 23.01		< 2.86	< 34.39	*	< 33.18						< 18.80	**	< 20.17	**	

TABLE A7.53

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS - MEANS IMBALANCE

LINE LENGTH		5												8											
BUFFER CAPACITY		1				2				6				1				2				6			
% DEGREE OF IMBALANCE		2	5	12	13	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18
PATTERN OF MEANS IMBALANCE	/			**	**			**	**		*	**	**			**	**			**	**		*	**	**
	\		*	**	**		*	**	**		**	**	**		*	**	**			**	**		**	**	**
	RAND			**	**			**	**		*	**	**	*	*	**	**			**	**		*	**	**
	∧			*	**			*				*					*			*				*	
	∨	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	1.545	1.578	1.965	1.624	1.214	1.805	2.458	2.918	2.088	1.901	2.381	2.346	0.676	1.305	1.538	2.586	1.116	1.938	1.840	1.823	1.552	2.026	1.759	1.328
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	2.289	2.338	2.911	2.405	1.799	2.675	3.641	4.322	3.093	2.816	3.527	3.475	1.001	1.933	2.279	3.830	1.653	2.872	2.725	2.700	2.299	3.003	2.606	1.968

TABLE A7.54

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS - COVARS IMBALANCE

LINE LENGTH	BUFFER CAPACITY	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	CRITICAL DIFFERENCE (LEVEL OF SIGNIFICANCE 0.05)	CRITICAL DIFFERENCE (LEVEL OF SIGNIFICANCE 0.01)
5	1	**	*	**	**	*	**	✓		**	**	**	**	2.365	2.946
	2	*		*				✓		**	**			3.528	4.394
	6							✓		**	*			2.696	3.359
8	1	*		*	*		*	✓		**	**	**	**	1.847	2.300
	2							✓		**	*			3.268	4.071
	6							✓		**				2.925	3.644

PATTERN OF COVARS IMBALANCE (P_i)

TABLE A7.55

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS - BUFFER CAPACITIES IMBALANCE

LINE LENGTH		5		8	
MEAN BUFFER CAPACITY		2	6	2	6
PATTERN OF BUFFER CAPACITIES IMBALANCE	A_1	**		**	
	A_2	*			
	A_3			*	
	B_1				
	B_2				
	B_3				
	C_1	**		**	
	C_2			*	
	C_3			**	
	D_1			✓	✓
	D_2	✓	✓		
	D_3				
	D_4				
	D_5				
	D_6				
LEVEL OF SIGNIFICANCE 0.05	CRITICAL DIFFERENCE	2.133	2.674	2.437	2.629
LEVEL OF SIGNIFICANCE 0.01	CRITICAL DIFFERENCE	2.594	3.253	2.964	3.198

TABLE A7.56

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS - MEANS & COVARS IMBALANCE (PATTERN (/))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	*	*																
	P_2	**	**	**	*						*		**						
	P_3	*	*																
	P_4												**						
	P_5		*										**						
	P_6	**	**								**	**	**						
	P_7	**	**	*							**	**	**						
	P_8	**	**	**	**	**		*			**	**	**	**	*				
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	3.473	3.019	3.317	3.940	5.036	4.485	3.289	4.017	4.752	3.877	4.062	3.634	4.412	4.331	6.594	4.668	4.968	4.337
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	3.880	3.374	3.706	4.402	5.627	5.012	3.675	4.489	5.310	4.333	4.539	4.060	4.930	4.839	7.368	5.216	5.551	4.846

TABLE A7.57

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS - MEANS & COVARS IMBALANCE (PATTERN (\))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	**	**	**			*				*		**						
	P_2	**	**	**							*	**	**						
	P_3	**	**	**							*	*	**						
	P_4																		
	P_5	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	P_6	**	**	**							**	**	**						
	P_7	**	**	**							**	**	**						
	P_8	**	**	**	**	**	**		**		**	**	**	**	**	**			*
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	3.473	3.019	3.317	3.940	5.036	4.485	3.289	4.017	4.752	3.877	4.062	3.634	4.412	4.331	6.594	4.668	4.968	4.337
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	3.880	3.374	3.706	4.402	5.627	5.012	3.675	4.489	5.310	4.333	4.539	4.060	4.930	4.839	7.368	5.216	5.551	4.846

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS - MEANS & COVARS IMBALANCE (PATTERN (V))

LINE LENGTH		5									8							
BUFFER CAPACITY		1			2			6			1			2			6	
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	*	**	**			**			**		*	**						**
	P_2	**	**	**	*	*	**			**	**	**	**			**			**
	P_3	**	**	**			**			**			**						**
	P_4						**			**			*						**
	P_5		**	**			**		*	**		**	**		**	**			**
	P_6	*	**	**			**			**	**								**
	P_7	**	**	**		*	**		**	**	**	**	**		**	**		*	**
	P_8	**	**	**	**	**	**	**		**	**	**	**	**	**	*			**
SIGNIFICANCE LEVEL / CRITICAL DIFFERENCE	0.05	3.473	3.019	3.317	3.940	5.036	4.485	3.289	4.017	4.752	3.877	4.062	3.634	4.412	4.331	6.594	4.668	4.968	4.337
	0.01	3.880	3.374	3.706	4.402	5.627	5.012	3.675	4.489	5.310	4.333	4.539	4.060	4.930	4.839	7.368	5.216	5.551	4.846

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS -
MEANS & COVARS IMBALANCE (PATTERN (Λ))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (ρ_i)	ρ_1	**	**	**			**			**	*		**						**
	ρ_2	**	**	**		*	**			**	**	**	**			**			**
	ρ_3	**	**	**			**			**			**						**
	ρ_4		**	**			**			**	*	**	**		**	**		**	**
	ρ_5			*						**			*						**
	ρ_6	**	**	**			**		*	**	**	**	**		**	**			**
	ρ_7		*	**			**			**									**
	ρ_8	**	**	**	**	**	**		**	**	**	**	**	**	**	**			**
SIGNIFICANCE LEVEL / CRITICAL DIFFERENCE	0.05	3.473	3.019	3.317	3.940	5.036	4.485	3.289	4.017	4.752	3.877	4.062	3.634	4.412	4.331	6.594	4.668	4.968	4.337
	0.01	3.880	3.374	3.706	4.402	5.627	5.012	3.675	4.489	5.310	4.333	4.539	4.060	4.930	4.839	7.368	5.216	5.551	4.846

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS -

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (/)&(\))

LINE LENGTH			5						8					
MEAN BUFFER CAPACITY			2			6			2			6		
% DEGREE OF MEANS IMBALANCE			2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE	(/)	A			**			**			**			**
		B			**		*	**		**	**		**	**
		C		**	**			**	**	**	**			**
		D_1			**			**		*	**			**
		D_2		*	**			**	*	**	**		*	**
		D_3			**			**		**	**			**
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	2.903	2.534	4.311	3.285	3.144	3.709	2.338	2.498	2.741	3.513	2.766	2.995
	0.01		3.364	2.937	4.996	3.808	3.644	4.298	2.710	2.895	3.177	4.072	3.206	3.472
	(\)	A	**	**	**			**	**	**	**			**
		B		*	**		**	**		**	**			**
		C			*			**	**	*	**			**
		D_1			**			**			**			**
		D_2			**		*	**		*	**		*	**
		D_3			*			**			**		*	**
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	2.903	2.534	4.311	3.285	3.144	3.709	2.338	2.498	2.741	3.513	2.766	2.9[illegible]
	0.01		3.364	2.937	4.996	3.808	3.644	4.298	2.710	2.895	3.177	4.072	3.206	3.472

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS -
MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (V)&(Λ))

LINE LENGTH			5						8					
MEAN BUFFER CAPACITY			2			6			2			6		
% DEGREE OF MEANS IMBALANCE			2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE	(V)	A							**	**				
		B												
		C							**	**				
		D_1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		D_2												
		D_3												
	SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	2.903	2.534	4.311	3.285	3.144	3.709	2.338	2.498	2.741	3.513	2.766	2.995
	SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	3.364	2.937	4.996	3.808	3.644	4.298	2.710	2.895	3.177	4.072	3.206	3.472
	(Λ)	A							**	**	**			
		B												
		C							**	**	**			
		D_1												
		D_2									**			
		D_3												
	SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	2.903	2.534	4.311	3.285	3.144	3.709	2.338	2.498	2.741	3.513	2.766	2.995
	SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	3.364	2.937	4.996	3.808	3.644	4.298	2.710	2.895	3.177	4.072	3.206	3.472

LINE LENGTH			5								8							
PATTERN OF COVARS IMBALANCE (P_i)			P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
MEAN BUFFER CAPACITY	2	PATTERN OF BUFFER CAPACITIES IMBALANCE: A	**	**	**			**		**		**		**		**		**
		B		**				*		**				**	*	**	**	*
		C	**	**	*	✓			**	**		**		✓	*		**	**
		D_1		**					**	**								
		D_2		**	**				**	**							*	**
		D_3	**	**	**				*	**								**
	6	A					●			**						**		**
		B												*	*	**	**	
		C				✓				*				✓			**	**
		D_1								*					*	**	**	*
		D_2								**								*
		D_3								**								**

TABLE A7.62

% TOTAL IDLE TIME'S PAIRWISE COMPARISONS -
COVARS & BUFFER CAPACITIES IMBALANCE

TABLE A7.63

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS - MEANS IMBALANCE

LINE LENGTH		5												8										
BUFFER CAPACITY		1				2				6				1				2				6		
% DEGREE OF IMBALANCE		2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12	18	2	5	12
PATTERN OF MEANS IMBALANCE	/	**	**	**	**		**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	*	**	**
	\	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	RAND		*	**	**		**	**	**	*	**	**	**		*	**	**	*	**	**	**		**	**
	∧	*	*	**	**	*	*	**	**		*	**	**	*	**	**	**	*	**	**	**		**	**
	∨	*	*	**	**		*	**	**			**	**	**	**	**	**		*	**	**		**	**
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	0.064	0.112	0.064	0.064	0.250	0.353	0.129	0.381	1.538	1.741	0.720	2.680	0.035	0.091	0.020	0.112	0.250	0.288	0.408	0.520	1.538	1.125	0.82
0.01		0.096	0.165	0.096	0.096	0.370	0.523	0.191	0.565	2.279	2.579	1.067	3.970	0.052	0.135	0.030	0.165	0.370	0.427	0.603	0.770	2.279	1.667	1.22

TABLE A7.64

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS - COVARS IMBALANCE

LINE LENGTH		5			8		
BUFFER CAPACITY		1	2	6	1	2	6
PATTERN OF COVARS IMBALANCE (P_i)	P_1	**	*		**	**	*
	P_2	**	*		**	**	**
	P_3	**	**		**	**	*
	P_4	**	**		**	**	*
	P_5	**	**		**	**	*
	P_6	**	**		**	**	**
	P_7	✓	✓	✓	✓	✓	✓
	P_8	**	**		**	**	**
	P_9	**	**		**	**	**
	P_{10}	**	*		**	**	*
	P_{11}						
	P_{12}	**	**		**	**	**
LEVEL OF SIGNIFICANCE 0.05	CRITICAL DIFFERENCE	0.049	3.528	2.762	0.145	0.245	1.426
LEVEL OF SIGNIFICANCE 0.01	CRITICAL DIFFERENCE	0.061	4.394	3.441	0.180	0.305	1.777

TABLE A7.65

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS - BUFFER CAPACITIES IMBALANCE

LINE LENGTH		5		8	
MEAN BUFFER CAPACITY		2	6	2	6
PATTERN OF BUFFER CAPACITIES IMBALANCE	A_1	✓	✓		
	A_2			✓	✓
	A_3				
	B_1			**	
	B_2			*	
	B_3	**		**	
	C_1	**		**	
	C_2	**		**	
	C_3	**	*	**	
	D_1	**		**	*
	D_2	**		**	**
	D_3				
	D_4				
	D_5	**	*	**	
	D_6	**		**	*
LEVEL OF SIGNIFICANCE 0.05	CRITICAL DIFFERENCE	0.373	2.282	0.417	1.738
LEVEL OF SIGNIFICANCE 0.01	CRITICAL DIFFERENCE	0.453	2.776	0.507	2.114

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS -

MEANS & COVARS IMBALANCE (PATTERN (/))

LINE LENGTH			5									8								
BUFFER CAPACITY			1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE			2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (ρ_i)	ρ_1		**	**	**	**	**	**		**	**	**	**	**	**	**	**	*	**	**
	ρ_2		**	**	**	**	**	**		**	**	**	**	**	**	**	**	**	**	**
	ρ_3		**	**	**	**	**	**		**	**	**	**	**	**	**	**	**	**	**
	ρ_4		**	**	**	**	**	**		**	**		**	**		**	**	**	**	**
	ρ_5		**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
	ρ_6		*	**	**		**	**		**	**		**	**		**	**		**	**
	ρ_7		**	**	**	**	**	**	*	**	**	**	**	**	**	**	**	**	**	**
	ρ_8		**	**	**	**	**	**		**	**	**	**	**	**	**	**		**	**
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	0.147	0.102	0.229	0.465	0.468	0.512	3.256	2.476	1.074	0.147	0.145	0.192	0.393	0.324	0.395	2.372	2.444	1.254
	0.01		0.164	0.114	0.256	0.520	0.522	0.672	3.638	2.766	1.200	0.164	0.162	0.214	0.440	0.362	0.441	2.650	2.731	1.401

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS -

MEANS & COVARS IMBALANCE (PATTERN (\))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	**	**		*						**	**	*	**	**				
	P_2	**	**								**	**		*	*				
	P_3	**	**		*						**	**	*	**	**				
	P_4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	P_5	**	**		**						**	**		**					
	P_6																		
	P_7	**	**		**	**					**	**	**	**	**				
	P_8	**	**								**	**	**	**	**	*			
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	0.147	0.102	0.229	0.465	0.468	0.512	3.256	2.476	1.074	0.147	0.145	0.192	0.393	0.324	0.395	2.372	2.444	1.254
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	0.164	0.114	0.256	0.520	0.522	0.572	3.638	2.766	1.200	0.164	0.162	0.214	0.440	0.362	0.441	2.650	2.731	1.401

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS - MEANS & COVARS IMBALANCE (PATTERN (V))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (P_i)	P_1	**	**	**	**	**	**		*	**	**	**	**	**	**	**		**	**
	P_2	**	**	**	*	**	**		**	**	**	**	**	**	**	**			**
	P_3	**	**	**	**	**	**			**	**	**	**	**	**	**		*	**
	P_4		**	**			*			**									
	P_5	**	**	**	**	**	**			**	**	**	**	**	**	**	**	**	**
	P_6		**	**						**									
	P_7	**	**	**	**	**	**		**	**	**	**	**	**	**	**	**	**	**
	P_8	**	**	**	**	**	**			**	**	**	**	**	**	**			**
SIGNIFICANCE LEVEL / CRITICAL DIFFERENCE	0.05	0.147	0.102	0.229	0.465	0.468	0.512	3.256	2.476	1.074	0.147	0.145	0.192	0.393	0.324	0.395	2.372	2.444	1.254
	0.01	0.164	0.114	0.256	0.520	0.522	0.572	3.638	2.766	1.200	0.164	0.162	0.214	0.440	0.362	0.441	2.650	2.731	1.401

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS -
MEANS & COVARS IMBALANCE (PATTERN (Λ))

LINE LENGTH		5									8								
BUFFER CAPACITY		1			2			6			1			2			6		
% DEGREE OF MEANS IMBALANCE		2	5	12	2	5	12	2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF COVARS IMBALANCE (ρ_i)	ρ_1	**	**	**	**	**	**		*	**	**	**	**	**	**	**			
	ρ_2	**	**	**	**	**	**		*	**	**	**	**	**	**	**			**
	ρ_3	**	**	**	**	**	**			**	**	**	**	**	**	**			
	ρ_4			**		**	**			**			*			**			**
	ρ_5	**	**	**	**	**	**		**	**	**	**	**	**	**	**	**	**	**
	ρ_6		**	**		*	**			**		*	**		*	**			**
	ρ_7	**	**	**	**	**	**		**	**	**	**	**	**	**	**	**	**	**
	ρ_8	**	**	**	**	**	**			**	**	**	**	**	**	**			**
SIGNIFICANCE LEVEL 0.05	CRITICAL DIFFERENCE	0.147	0.102	0.229	0.465	0.468	0.512	3.256	2.476	1.074	0.147	0.145	0.192	0.393	0.324	0.395	2.372	2.444	1.254
SIGNIFICANCE LEVEL 0.01	CRITICAL DIFFERENCE	0.164	0.114	0.256	0.520	0.522	0.572	3.638	2.766	1.200	0.164	0.162	0.214	0.440	0.362	0.441	2.650	2.731	1.401

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS -

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (/)&(\\))

PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE		LINE LENGTH	5						8					
		MEAN BUFFER CAPACITY	2			6			2			6		
		% DEGREE OF MEANS IMBALANCE	2	5	12	2	5	12	2	5	12	2	5	12
(/)		A		**	**	*	**	**		**	**		**	**
		B	**	**	**	**	**	**	**	**	**	**	**	**
		C	**	**	**	**	**	**	**	**	**	**	**	**
		D_1	**	**	**	**	**	**	**	**	**	**	**	**
		D_2	**	**	**	**	**	**	**	**	**	**	**	**
		D_3	**	**	**		**	**	**	**	**		*	**
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	0.367	0.302	0.493	2.570	2.887	1.968	0.370	0.427	0.370	2.002	2.594	2.980
	0.01		0.435	0.350	0.576	2.978	3.346	2.280	0.428	0.495	0.428	2.320	3.102	3.248
(\\)		A	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		B												
		C	**	**					**	**				
		D_1	**											
		D_2	**	**					**	**				
		D_3	**	**					**					
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	0.367	0.302	0.493	2.570	2.887	1.968	0.376	0.427	0.370	2.002	2.594	2.980
	0.01		0.435	0.350	0.576	2.978	3.346	2.280	0.428	0.495	0.428	2.320	3.102	3.248

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS -

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (V)&(Λ))

LINE LENGTH			5						8					
MEAN BUFFER CAPACITY			2			6			2			6		
% DEGREE OF MEANS IMBALANCE			2	5	12	2	5	12	2	5	12	2	5	12
PATTERN OF MEANS AND BUFFER CAPACITIES IMBALANCE	(V)	A		*	*			**						
		B	**	**	**			*	**	**	**	**	**	**
		C	**	**	**			**	**	**	**	*	*	
		D_1	**	**	**		*	**	**	**	**	**	**	**
		D_2	**	**	**				**	**	**			
		D_3		**	**			**	**	**	**			
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	0.367	0.302	0.493	2.570	2.887	1.968	0.370	0.427	0.370	2.002	2.594	2.980
	0.01		0.435	0.350	0.576	2.978	3.346	2.280	0.428	0.495	0.428	2.320	3.102	3.248
	(Λ)	A						*						
		B	**	**	**			**	**	**	**			
		C	**	**	**	*	*	**	**	**	**			
		D_1	**	**	**			**	**	**	**			
		D_2	**	**	**			*	**	**	**			
		D_3	**	**	**			**	**	**	**			
SIGNIFICANCE LEVEL	0.05	CRITICAL DIFFERENCE	0.367	0.302	0.493	2.570	2.887	1.968	0.370	0.427	0.370	2.002	2.594	2.980
	0.01		0.435	0.350	0.576	2.978	3.346	2.280	0.428	0.495	0.428	2.320	3.102	3.248

LINE LENGTH			5								8							
PATTERN OF COVARS IMBALANCE (P_i)			P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
MEAN BUFFER CAPACITY	PATTERN OF BUFFER CAPACITIES IMBALANCE																	
2		A				✓								✓	**		**	
		B	**	**	**		**		**	**	**	**	**		**		**	**
		C	**	**	**	*	**	**	**	**	**	**	**	**	**	**	**	**
		D_1	**	**	**		**		**	**	**	**			**		**	*
		D_2	**	**	**	**	**	*	**	**	**	**	**	**	**	*	**	**
		D_3	**	**	**		**		**	**	**	**	**		**		**	**
6		A				✓								✓			*	
		B					**		**		**	*	**		**		**	
		C	*	*	*		**		**			*			**		**	
		D_1					**		**		**	**	**		**		**	**
		D_2					**		**		**	**	**		**		**	**
		D_3													**		**	

TABLE A7.72

MEAN BUFFER LEVEL'S PAIRWISE COMPARISONS - COVARS & BUFFER CAPACITIES IMBALANCE

TABLE A7.73

CRITICAL DIFFERENCES FOR "WITH CONTROL & PAIRWISE" COMPARISONS - COVARS & BUFFER CAPACITIES IMBALANCE

LINE LENGTH	MEAN BUFFER CAPACITY	BUFFER CAPACITIES IMBALANCE PATTERN	LEVEL OF SIGNIFICANCE	CRITICAL DIFFERENCE – COMPARISONS WITH CONTROL		CRITICAL DIFFERENCE – PAIRWISE COMPARISONS	
				IDLE TIME	MEAN BUFFER LEVEL	IDLE TIME	MEAN BUFFER LEVEL
5	2	A	0.05	1.737	0.311		
5	2	A	0.01	2.280	0.409		
5	2	B	0.05	1.032	0.220		
5	2	B	0.01	1.356	0.289		
5	2	C	0.05	1.845	0.246	4.674	0.542
5	2	C	0.01	2.423	0.323	5.131	0.595
5	2	D_1	0.05	2.901	0.311		
5	2	D_1	0.01	3.810	0.409		
5	2	D_2	0.05	1.676	0.156		
5	2	D_2	0.01	2.201	0.204		
5	2	D_3	0.05	1.195	0.191		
5	2	D_3	0.01	1.570	0.250		
5	6	A	0.05	1.431	1.621		
5	6	A	0.01	1.879	2.129		
5	6	B	0.05	1.388	0.572		
5	6	B	0.01	1.822	0.751		
5	6	C	0.05	1.719	0.678	4.238	3.582
5	6	C	0.01	2.257	0.891	4.653	3.932
5	6	D_1	0.05	1.701	2.478		
5	6	D_1	0.01	2.234	3.254		
5	6	D_2	0.05	2.378	1.357		
5	6	D_2	0.01	3.123	1.782		
5	6	D_3	0.05	0.593	0.705		
5	6	D_3	0.01	0.778	0.925		

TABLE A7.73 - CONTINUED

LINE LENGTH	MEAN BUFFER CAPACITY	BUFFER CAPACITIES IMBALANCE PATTERN	LEVEL OF SIGNIFICANCE	CRITICAL DIFFERENCE - COMPARISONS WITH CONTROL		CRITICAL DIFFERENCE - PAIRWISE COMPARISONS	
				IDLE TIME	MEAN BUFFER LEVEL	IDLE TIME	MEAN BUFFER LEVEL
8	2	A	0.05	2.242	0.246		
8	2	A	0.01	2.944	0.323		
8	2	B	0.05	3.167	0.348		
8	2	B	0.01	4.159	0.457		
8	2	C	0.05	1.591	0.191		
8	2	C	0.01	2.089	0.250	6.049	0.542
8	2	D_1	0.05	2.637	0.191	6.640	0.595
8	2	D_1	0.01	3.462	0.250		
8	2	D_2	0.05	2.053	0.156		
8	2	D_2	0.01	2.696	0.204		
8	2	D_3	0.05	2.020	0.220		
8	2	D_3	0.01	2.653	0.289		
8	6	A	0.05	1.595	0.632		
8	6	A	0.01	2.094	0.830		
8	6	B	0.05	1.754	0.696		
8	6	B	0.01	2.303	0.914	4.035	1.483
8	6	C	0.05	1.938	0.623	4.429	1.628
8	6	C	0.01	2.545	0.818		
8	6	D_1	0.05	1.877	1.032		
8	6	D_1	0.01	2.465	1.356		
8	6	D_2	0.05	1.044	0.959		
8	6	D_2	0.01	1.371	1.260		
8	6	D_3	0.05	1.338	0.660		
8	6	D_3	0.01	1.822	0.867		

TABLE A7.74

ANALYSIS OF VARIANCE TABLES

Means Imbalance - Line Length = 5 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Buffer Capacity)	522.960	2	261.479	1634.250**
B (Degree of Imbalance)	1704.418	3	568.139	3550.869**
AB	75.398	6	12.566	78.538**
C (Means Imbalance Pattern)	389.315	4	97.329	608.306**
AC	14.952	8	1.869	11.681**
BC	276.689	12	23.057	144.106**
ABC	15.784	24	0.658	4.113**
R (Subrun)	0.576	1	0.576	3.600
Within Cell	9.430	59	0.160	
Total	3009.519	119		

TABLE A7.75

Means Imbalance - Line Length = 8 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Buffer Capacity)	783.582	2	391.791	2859.788**
B (Degree of Imbalance)	1573.410	3	524.470	3828.248**
C (Pattern of Means Imbalance)	400.578	4	100.145	730.985**
AB	95.934	6	15.989	116.708**
AC	11.981	8	1.498	10.934**
BC	245.244	12	20.437	149.175**
ABC	16.698	24	0.696	5.080
R (Subrun)	0.202	1	0.202	1.475
Within Cell (error term)	8.099	59	0.137	
Total	3135.728	119		

** = significant at the 0.99 level

TABLE A7.76

Means Imbalance - Line Length = 5 (Mean Buffer Level Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Buffer Capacity)	158.612	2	79.306	1468.630**
B (Degree of Imbalance)	1.904	3	0.635	11.759**
AB	2.202	6	0.367	6.796**
C (Means Imbalance Pattern)	51.691	4	12.923	239.315**
AC	39.908	8	4.989	92.389**
BC	9.264	12	0.772	14.296**
ABC	5.068	24	0.211	3.907**
R (Subrun)	0.059	1	0.059	1.093
Within Cell	3.183	59	0.054	
Total	271.890	119		

TABLE A7.77

Means Imbalance - Line Length = 8 (Mean Buffer Level Results)

A (Buffer Capacity)	146.998	2	73.499	2624.964**
B (Degree of Imbalance)	1.006	3	0.335	11.964**
AB	1.288	6	0.215	7.679**
C (Means Imbalance Pattern)	42.251	4	10.563	377.250**
AC	31.924	8	3.991	142.536**
BC	8.087	12	0.674	24.071**
ABC	5.400	24	0.225	8.036**
R (Subrun)	0.001	1	0.001	0.036
Within Cell	1.631	59	0.028	
Total	238.585	119		

** = significant at the 0.99 level

TABLE A7.78

Covars Imbalance - Line Length = 5 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Buffer Capacity)	1115.597	2	557.798	2446.482**
B (Covars Imbalance Pattern)	290.213	11	26.383	115.715**
AB	59.284	22	2.695	11.82**
R (Subrun)	0.694	1	0.694	3.044
Within Cell	7.995	35	0.228	
Total	1473.783	71		

TABLE A7.79

Covars Imbalance - Line Length = 8 (Idle Time Results)

A (Buffer Capacity)	1613.204	2	806.602	3432.349**
B (Covars Imbalance Pattern)	167.182	11	15.198	64.672**
AB	29.084	22	1.322	5.626**
R (Subrun)	0.655	1	0.655	2.787
Within Cell	8.239	35	0.235	
Total	1818.363	71		

TABLE A7.80

Covars Imbalance - Line Length = 5 (Mean Buffer Level Results)

A (Buffer Capacity)	85.910	2	42.955	482.64**
B (Covars Imbalance Pattern)	3.281	11	0.298	3.348**
AB	1.201	22	0.055	0.618
R (Subrun)	0.013	1	0.013	0.146
Within Cell	3.124	35	0.089	
Total	93.529	71		

** = significant at the 0.99 level

TABLE A7.81

Covars Imbalance - Line Length = 8 (Mean Buffer Level Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Buffer Capacity)	79.599	2	39.800	1658.333**
B (Covars Imbalance Pattern)	15.362	11	1.397	58.208**
AB	9.852	22	0.448	18.667**
R (Subrun)	0.001	1	0.001	0.042
Within Cell	0.844	35	0.024	
Total	105.657	71		

TABLE A7.82

Buffer Capacities Imbalance - Line Length = 5 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	280.111	1	280.111	1795.583**
B (Buffer Capacities Imbalance Pattern)	33.463	14	2.390	15.321**
AB	8.182	14	0.584	3.744**
R (Subrun)	0.390	1	0.390	2.500
Within Cell	4.515	29	0.156	
Total	326.659	59		

TABLE A7.83

Buffer Capacities Imbalance - Line Length = 8 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	447.305	1	447.305	2471.298**
B (Buffer Capacities Imbalance Pattern)	52.341	14	3.739	20.658**
AB	11.301	14	0.807	4.459**
R (Subrun)	0.258	1	0.258	1.425
Within Cell	5.253	29	0.181	
Total	516.457	59		

** = significant at the 0.99 level

TABLE A7.84

Buffer Capacities Imbalance - Line Length = 5 (Mean Buffer Level Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	40.531	1	40.531	513.051**
B (Buffer Capacities Imbalance Pattern)	19.004	14	1.357	17.177*
AB	8.467	14	0.605	7.658**
R (Subrun)	0.001	1	0.001	0.013
Within Cell	2.290	29	0.079	
Total	70.292	59		

TABLE A7.85

Buffer Capacities Imbalance - Line Length = 8 (Mean Buffer Level Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	25.329	1	25.329	550.630**
B (Buffer Capacities Imbalance Pattern)	17.058	14	1.218	26.478**
AB	10.007	14	0.715	15.544**
R (Subrun)	0.045	1	0.045	0.978
Within Cell	1.334	29	0.046	
Total	53.772	59		

** = significant at the 0.99 level

TABLE A7.86

Means and Covars Imbalance - Line Length = 5 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Buffer Capacity)	850.563	1	850.563	4622.625**
B (Degree of Imbalance)	612.297	2	306.148	1663.848**
AB	25.852	2	12.926	70.250**
C (Covars Imbalance Pattern)	396.285	4	99.071	538.429**
AC	39.830	4	9.958	54.120**
BC	14.613	8	1.827	9.929**
ABC	3.262	8	0.408	2.217*
D (Means Imbalance Pattern)	168.456	3	56.152	305.174**
AD	10.104	3	3.368	18.304**
BD	180.664	6	30.111	163.647**
ABD	2.556	6	0.426	2.315*
CD	187.502	12	15.625	84.919**
ACD	9.740	12	0.812	4.413**
BCD	38.118	24	1.588	8.630**
ABCD	10.352	24	0.431	2.432**
R (Subrun)	0.432	1	0.432	2.348
Within Cell	21.928	119	0.184	
Total	2572.549	239		

* = significant at the 0.95 level
** = significant at the 0.99 level

TABLE A7.87

Means and Covars Imbalance - Line Length = 8 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Buffer Capacity)	8982.309	1	8982.309	20744.362**
B (Degree of Imbalance)	1779.566	2	889.783	2050.191**
AB	84.949	2	42.475	98.095**
C (Covars Imbalance Pattern)	871.284	4	217.821	501.892**
AC	84.202	4	21.050	48.614**
BC	67.733	8	8.467	19.554**
ABC	55.976	8	6.997	16.159**
D (Means Imbalance Pattern)	589.668	3	196.556	453.940**
AD	626.693	3	208.898	482.443**
BD	1009.438	6	168.240	388.545**
ABD	936.135	6	156.023	360.330**
CD	147.316	12	12.276	28.351**
ACD	142.343	12	11.862	27.395**
BCD	354.900	24	14.788	34.152**
ABCD	365.507	24	15.230	35.173**
R (Subrun)	0.848	1	0.848	1.954
Within Cell	51.594	119	0.434	
Total	13650.391	239		

** = significant at the 0.99 level

TABLE A7.88

Means & Covars Imbalance - Line Length = 5 (Mean Buffer Level Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Buffer Capacity)	95.478	1	95.478	875.945**
B (Degree of Imbalance)	28.300	2	14.150	129.817**
AB	6.473	2	3.237	29.618**
C (Covars Imbalance Pattern)	2.468	4	0.617	5.661**
AC	1.250	4	0.312	2.859*
BC	6.557	8	0.822	7.451**
ABC	3.198	8	0.399	3.658**
D (Means Imbalance Pattern)	13.126	3	4.375	40.138**
AD	4.682	3	1.561	14.283**
BD	64.461	6	10.744	98.569**
ABD	10.217	6	1.703	15.583**
CD	22.076	12	1.840	16.881*
ACD	5.104	12	0.425	3.892**
BCD	32.811	24	1.367	12.541**
ABCD	9.237	24	0.385	3.522**
R (Subrun)	0.121	1	0.121	1.110
Within Cell	13.004	119	0.109	
Total	318.583	239		

* = significant at the 0.95 level
** = significant at the 0.99 level

TABLE A7.89

Means & Covars Imbalance - Line Length = 8 (Mean Buffer Level Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Buffer Capacity)	93.975	1	93.975	989.211**
B (Degree of Imbalance)	25.322	2	12.661	133.274**
AB	7.911	2	3.956	41.840**
C (Covars Imbalance Pattern)	3.213	4	0.803	8.453**
AC	1.874	4	0.469	4.955**
BC	7.431	8	0.929	9.779**
ABC	3.196	8	0.399	4.225**
D (Means Imbalance Pattern)	11.224	3	3.741	39.379**
AD	4.615	3	1.538	16.270**
BD	61.130	6	10.188	107.242**
ABD	18.523	6	3.087	32.651**
CD	19.750	12	1.646	17.326**
ACD	6.074	12	0.506	5.353**
BCD	31.247	24	1.302	13.705**
ABCD	8.773	24	0.731	7.732**
R (Subrun)	0.161	1	0.161	1.695
Within Cell	11.252	119	0.095	
Total	315.671	239		

** = significant at the 0.99 level

TABLE A7.90

Means & Buffer Capacities Imbalance - Line Length = 5 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	1179.910	1	1179.910	4111.185**
B (Degree of Imbalance)	836.365	2	418.182	1457.080**
AB	3.432	2	1.716	5.979**
C (Buffer Capacities Imbalance Pattern)	59.219	4	14.805	51.585**
AC	8.904	4	2.226	7.756**
BC	8.066	8	1.008	3.512**
ABC	16.281	8	2.035	7.091**
D (Means Imbalance Pattern)	778.137	3	259.379	903.76**
AD	155.502	3	51.834	180.606**
BD	569.157	6	94.860	330.523**
ABD	90.105	6	15.018	52.328**
CD	141.035	12	11.753	40.951**
ACD	102.961	12	8.580	29.896**
BCD	42.468	24	1.770	6.167**
ABCD	28.499	24	1.188	4.139**
R (Subrun)	1.038	1	1.038	3.617
Within Cell	34.125	119	0.287	
Total	4055.199	239		

** = significant at the 0.99 level

TABLE A7.91

Means & Buffer Capacities Imbalance - Line Length = 8 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	1729.019	1	1729.019	5681.081**
B (Degree of Imbalance)	698.231	2	349.116	1183.444**
AB	1.681	2	0.841	2.851
C (Buffer Capacities Imbalance Pattern)	83.514	4	20.879	70.776**
AC	47.079	4	11.770	39.898**
BC	15.455	8	1.932	6.549
ABC	17.961	8	2.245	7.610**
D (Means Imbalance Pattern)	819.402	3	273.134	925.878**
AD	196.440	3	65.480	222.966**
BD	511.408	6	85.235	288.932**
ABD	134.796	6	22.466	76.156**
CD	233.618	12	19.468	65.993**
ACD	110.500	12	9.208	31.214**
BCD	49.252	24	0.052	6.956**
ABCD	32.319	24	1.347	4.566**
R (Subrun)	0.009	1	0.009	0.031
Within Cell	35.092	119	0.295	
Total	4715.738	239		

** = significant at the 0.99 level

TABLE A7.92

Means and Buffer Capacities Imbalance - Line Length = 5 (Mean Buffer Level Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	48.426	1	48.426	768.667**
B (Degree of Imbalance)	7.294	2	3.647	57.889**
AB	6.237	2	3.119	49.508**
C (Buffer Capacities Imbalance Pattern)	2.786	4	0.697	11.064**
AC	5.289	4	1.322	20.984**
BC	1.134	8	0.142	2.254*
ABC	0.920	8	0.115	1.825
D (Means Imbalance Pattern)	240.029	3	80.010	1270.001**
AD	139.090	3	46.030	730.635**
BD	42.086	6	7.014	111.333**
ABD	28.664	6	4.777	75.825**
CD	5.081	12	0.423	6.714**
ACD	6.190	12	0.516	8.191**
BCD	3.276	24	0.137	2.175**
ABCD	2.530	24	0.105	1.683*
R (Subrun)	0.164	1	0.164	2.603
Within Cell	7.432	119	0.063	
Total	545.626	239		

* = Significant at the 0.95 level
** = Significant at the 0.99 level

TABLE A7.93

Means & Buffer Capacities Imbalance - Line Length = 8 (Mean Buffer Level Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	69.290	1	69.290	1004.203**
B (Degree of Imbalance)	13.176	2	6.588	95.478**
AB	11.887	2	5.943	86.130**
C (Buffer Capacities Imbalance Pattern)	4.596	4	1.149	16.652**
AC	3.570	4	0.892	12.928**
BC	0.864	8	0.108	1.565
ABC	0.564	8	0.071	1.029**
D (Means Imbalance Pattern)	225.544	3	75.181	1089.580**
AD	133.016	3	44.339	642.594**
BD	49.254	6	8.209	118.971**
ABD	37.500	6	6.250	90.580**
CD	8.035	12	0.670	9.710**
ACD	12.236	12	1.020	14.783**
BCD	4.849	24	0.202	2.928**
ABCD	3.948	24	0.165	2.391**
R (Subrun)	0.002	1	0.002	0.029
Within Cell	8.210	119	0.069	
Total	586.535	239		

** = significant at the 0.99 level

TABLE A7.94

Covars and Buffer Capacities Imbalance - Line Length = 5 (Idle Time Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	982.172	1	982.172	3692.376**
B (Buffer Capacities Imbalance Pattern)	17.896	5	3.579	13.455**
AB	11.465	5	2.293	8.620**
C (Covars Imbalance Pattern)	620.064	7	88.581	333.011**
AC	80.755	7	11.536	43.368**
BC	242.855	35	6.939	26.087**
ABC	30.280	35	0.865	3.252**
R (Subrun)	0.909	1	0.909	3.417
Within Cell	25.245	95	0.266	
Total	2011.639	191		

TABLE A7.95

Covars and Buffer Capacities Imbalance - Line Length = 8 (Idle Time Results)

A (Mean Buffer Capacity)	1965.178	1	1965.178	5746.135**
B (Buffer Capacities Imbalance Pattern)	67.125	5	13.425	39.254**
AB	21.333	5	4.267	12.477**
C (Covars Imbalance Pattern)	395.451	7	56.493	165.184**
AC	22.682	7	3.240	9.474**
BC	398.611	35	11.389	33.301**
ABC	39.497	35	1.129	3.301**
R (Subrun)	1.115	1	1.115	3.260
Within Cell	32.487	95	0.342	
Total	2943.478	191		

** = significant at the 0.99 level

TABLE A7.96

Covars & Buffer Capacities Imbalance - Line Length - 5 (Mean Buffer Level Results)

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	OBSERVED F VALUE
A (Mean Buffer Capacity)	211.319	1	211.319	2374.371**
B (Buffer Capacities Imbalance Pattern)	31.783	5	6.357	71.422**
AB	15.471	5	3.094	34.764**
C (Covars Imbalance Pattern)	46.981	7	6.712	75.416**
AC	13.248	7	1.893	21.270**
BC	13.660	35	0.390	4.382**
ABC	7.616	35	0.218	2.449**
R (Subrun)	0.130	1	0.130	1.461
Within Cell	8.474	95	0.089	
Total	371.679	191		

TABLE A7.97

Covars & Buffer Capacities Imbalance - Line Length = 8 (Mean Buffer Level Results)

A (Mean Buffer Capacity)	82.711	1	82.711	4865.353**
B (Buffer Capacities Imbalance Pattern)	32.117	5	6.423	377.824**
AB	24.450	5	4.890	287.647**
C (Covars Imbalance Pattern)	48.376	7	6.911	406.529**
AC	4.388	7	0.627	36.882**
BC	11.662	35	0.333	19.588**
ABC	8.741	35	0.250	14.706**
R (Subrun)	0.019	1	0.019	1.118
Within Cell	1.570	95	0.017	
Total	214.033	191		

** = significant at the 0.99 level

APPENDIX 7.3

INDEX OF FIGURES

This appendix contains a list of the figures exhibiting the steady state's results.

APPENDIX 7.3

INDEX OF FIGURES - STEADY STATE INVESTIGATIONS

FIGURE NUMBER	TITLE
A7.31	% Total Idle Time Curves for Means & Buffer Capacities Imbalance - Pattern (V) + B, (N = 5)
A7.32	% Total Idle Time Curves for Means & Buffer Capacities Imbalance - Pattern (V) + B, (N = 8)
A7.33	% Total Idle Time Curves for Means & Buffer Capacities Imbalance - Pattern (V) + D_1, (N = 5)**
A7.34	% Total Idle Time Curves for Means & Buffer Capacities Imbalance - Pattern (V) + D_1, (N = 8)**
A7.35	% Total Idle Time Curves for Means & Buffer Capacities Imbalance - Pattern (V) + D_1, (N = 10)**
A7.36	% Total Idle Time Curves for Means & Buffer Capacities Imbalance - Pattern (V) + D_2, (N = 5)
A7.37	% Total Idle Time Curves for Means & Buffer Capacities Imbalance - Pattern (V) + D_2, (N = 8)
A7.38	% Total Idle Time Curves for Means & Buffer Capacities Imbalance - Pattern (V) + D_3, (N = 5)
A7.39	% Total Idle Time Curves for Means & Buffer Capacities Imbalance - Pattern (V) + D_3, (N = 8)
A7.40	% Total Idle Time Curves for Covars & Buffer Capacities Imbalance - Pattern P_4 + C** and Other Patterns,(N = 5)
A7.41	% Total Idle Time Curves for Covars and Buffer Capacities Imbalance - Pattern P_4 + C** and Other Patterns,(N = 8)
A7.42	Mean Buffer Level Curves for Means Imbalance - Pattern (\\), (N = 5,8)**
A7.43	Mean Buffer Level Curves for Means Imbalance - Pattern (∧),(N = 5,8)
A7.44	Mean Buffer Level Curves for Covars Imbalance - Pattern P_7** and Other Patterns, (N = 5)
A7.45	Mean Buffer Level Curves for Covars Imbalance - Pattern P_7** and Other Patterns, (N = 8)

** = the best pattern

APPENDIX 7.4

MEANINGS OF GRAPHS' SYMBOLS

This appendix contains the meanings of the symbols used to identify the different curves in each steady state figure.

APPENDIX 7.4

MEANINGS OF GRAPHS' SYMBOLS - SS INVESTIGATIONS

FIGURE NUMBER	CURVES - SYMBOLS AND MEANINGS
A7.1	(□)N=10,B=1; (■)N=8,B=1; (△)N=5,B=1; (▲)N=10,B=2; (●)N=8,B=2; (◑)N=5,B=2; (○)N=10,B=3; (▼)N=8,B=3; (▽)N=5,B=3; (◇)N=10,B=6; (◆)N=8,B=6; (◐)N=5,B=6.
A7.2	(●)N=8,B=1; (○)N=5,B=1; (■)N=8,B=2; (□)N=5,B=2; (△)N=8,B=6; (▲)N=5,B=6.
A7.3	(□)P_2; (■)P_5; (△)P_8; (▲)P_7.
A7.4	(□)P_5; (■)P_2; (●)P_7.
A7.5	(△)P_8; (▲)P_7.
A7.6	(■)B_3; (□)D_3; (△)D_1; (▲)D_2.
A7.7	(△)D_6; (▲)D_5; (■)D_4; (□)D_1.
A7.8	(△)B=1; (▲)B=2; (■)B=6.
A7.9	(▲)B=1; (△)B=2; (●)B=6.
A7.10	(△)B=1; (▲)B=2; (■)B=6.
A7.11	(▲)B=1; (△)B=2; (■)B=6.
A7.12	(△)B=1; (▲)B=2; (□)B=3; (■)B=6.
A7.13	(▲)B=1; (△)B=2; (○)B=3; (●)B=6.
A7.14	(▲)B=1; (△)B=2; (■)B=3; (□)B=6.
A7.15	(▲)B=1; (△)B=2; (■)B=6.
A7.16	(▲)B=1; (△)B=2; (■)B=6.
A7.17	(■)B=1; (▲)B=2; (△)B=6.
A7.18	(▲)B=1; (△)B=2; (■)B=6.
A7.19	(●)B=1; (□)B=2; (■)B=3; (○)B=6.
A7.20	(○)B=1; (●)B=2; (■)B=3; (□)B=6.
A7.21	(○)B=1; (●)B=2; (□)B=3; (■)B=6.
A7.22	(●)B=1; (○)B=2; (□)B=3; (■)B=6.
A7.23	(▲)B=1; (■)B=2; (□)B=3; (△)B=6.

FIGURE NUMBER	CURVES - SYMBOLS AND MEANINGS
A7.24	(□)B=1; (●)B=2; (○)B=3; (▲)B=6.
A7.25	(●)B=1; (■)B=2; (○)B=3; (□)B=6.
A7.26	(▲)B=1; (△)B=2; (○)B=3; (●)B=6.
A7.27	(■)B=1; (□)B=2; (●)B=3; (○)B=6.
A7.28	(▲)B=1; (△)B=2; (●)B=3; (○)B=6.
A7.29	(△)B=1; (□)B=2; (○)B=3; (●)B=6.
A7.30	(○)B=1; (□)B=2; (■)B=3; (●)B=6.
A7.31	(□)MB=2; (●)MB=3; (■)MB=6.
A7.32	(■)MB=2; (□)MB=3; (●)MB=6.
A7.33	(□)MB=2; (■)MB=3; (●)MB=6.
A7.34	(■)MB=2; (●)MB=3; (○)MB=6.
A7.35	(△)MB=2; (▲)MB=3; (△)MB=6.
A7.36	(○)MB=2; (●)MB=3; (□)MB=6.
A7.37	(□)MB=2; (■)MB=3; (●)MB=6.
A7.38	(▲)MB=2; (△)MB=3; (■)MB=6.
A7.39	(△)MB=2; (▲)MB=3; (■)MB=6.
A7.40	(△)P_3+B; (■)P_5+A; (□)P_4+D_2; (○)P_4+D_3; (●)P_5+D_5; (▲)P_4+C.
A7.41	(■)P_5+A; (▲)P_4+D_3; (△)P_4+D_2; (□)P_4+C.
A7.42	(△)N=5,B=6; (○)N=8,B=6; (□)N=5,B=2; (●)N=8,B=2; (■)N=8,B=1; (▲)N=5,B=1.
A7.43	(○)N=5,B=6; (●)N=8,B=6; (□)N=5,B=2; (■)N=8,B=2; (▲)N=8,B=1; (△)N=5,B=1.
A7.44	(□)P_3; (■)P_{11}; (●)P_{10}; (○)P_7.
A7.45	(●)P_5; (○)P_1; (□)P_1 ; (■)P_7.
A7.46	(△)D_3; (▲)A_1; (□)A_3; (■)A_2.
A7.47	(□)B_1; (■)A_3; (▲)A_2; (△)A_1.
A7.48	(●)B=6; (○)B=2; (□)B=1.
A7.49	(●)B=6; (○)B=2; (□)B=1.

FIGURE NUMBER	CURVES - SYMBOLS AND MEANINGS
A7.50	(●)B=6; (○)B=2; (□)B=1.
A7.51	(●)B=6; (○)B=2; (□)B=1.
A7.52	(●)B=6; (○)B=2; (□)B=1.
A7.53	(●)B=6; (○)B=2; (□)B=1.
A7.54	(●)B=6; (○)B=2; (□)B=1.
A7.55	(●)B=6; (○)B=2; (□)B=1.
A7.56	(●)MB=6; (○)MB=2.
A7.57	(●)MB=6; (○)MB=2.
A7.58	(●)MB=6; (○)MB=2.
A7.59	(●)MB=6; (○)MB=2.
A7.60	(●)MB=6; (○)MB=2.
A7.61	(●)MB=6; (○)MB=2.
A7.62	(●)MB=6; (○)MB=2.
A7.63	(●)MB=6; (○)MB=2.
A7.64	(▲)P_6+B; (□)P_4+B; (■)P_6+A; (△)P_4+A.
A7.65	(▲)P_6+B; (△)P_4+B; (■)P_6+A; (□)P_4+A.
A7.66	(●)B=1,BL; (○)B=1,ST; (■)B=2,BL; (▽)B=2,ST; (▼)B=3,ST; (▲)B=3,BL; (□)B=6,BL; (△)B=6,ST.
A7.67	(○)B=1,BL; (●)B=1,ST; (△)B=2,ST; (◇)B=3,ST; (□)B=3,BL; (▲)B=6,ST; (■)B=2,BL; (◆)B=6,BL.
A7.68	(□)N=10,ST; (■)N=8,ST; (△)N=10,BL; (▲)N=5,ST; (●)N=8,BL; (○)N=5,BL.
A7.69	(●)N=8,BL; (○)N=5,BL; (■)N=8,ST; (□)N=5,ST
A7.70	(●)B=1,BL; (○)B=2,BL; (△)B=6,BL; (▲)B=1,ST; (□)B=2,ST; (■)B=6,ST.
A7.71	(▲)B=1,BL; (△)B=2,BL; (■)B=1,ST; (□)B=6,BL; (●)B=2,ST; (○)B=6,ST.
A7.72	(●)MB=2,BL; (△)MB=2,ST; (○)MB=6,BL; (□)MB=6,ST.
A7.73	(○)MB=2,BL; (●)MB=6,BL; (▲)MB=2,ST; (△)MB=6,ST.

FIGURE NUMBER	CURVES - SYMBOLS AND MEANINGS
A7.74	(○)N=8,BL; (●)N=5,BL; (■)N=8,ST; (▭)N=5,ST.
A7.75	(●)B=1,SU; (○)B=2,SU; (◇)B=6,L; (◆)B=6,SU; (△)B=2,L; (▲)B=1,L.
A7.76	(○)B=6,L; (●)B=1,SU; (▭)B=2,SU; (▲)B=2,L; (△)B=6,SU; (■)B=1,L.
A7.77	(●)SU; (▭)L.
A7.78	(●)SU; (○)L.
A7.79	(▲)N=5,SU; (■)N=8,L; (▭)N=5,L; (△)N=8,SU.
A7.80	(■)B=1,SU; (▭)B=6,L; (△)B=2,SU; (●)B=2,L; (○)B=1,L; (▲)B=6,SU.
A7.81	(○)B=1,SU; (●)B=2,SU; (▭)B=6,L; (■)B=6,SU; (▲)B=2,L; (△)B=1,L.
A7.82	(■)MB=6,L; (▭)MB=2,SU: (○)MB=2,L; (●)MB=6,SU.
A7.83	(■)MB=6,L; (○)MB=2,SU; (▭)MB=2,L; (●)MB=6,SU.
A7.84	(△)N=5,SU; (▲)N=8,SU; (■)N=8,L; (▭)N=5,L.

where

N	=	line length
B	=	buffer capacity
MB	=	mean buffer capacity
ST	=	starving idle time
BL	=	blocking idle time
L	=	mean total number of units in the line
SU	=	space utilization
P_i	=	pattern of the Covars imbalance
B_i, D_i	=	pattern of the buffer capacities imbalance
$P_i+A-P_i+D_3$	=	pattern of Covars and buffer capacities joint imbalance

APPENDIX 7.5

FIGURES

This appendix contains the results of the steady state simulation investigations, exhibited in graphical forms.

FIGURE A7.1

% TOTAL IDLE TIME CURVES FOR MEANS IMBALANCE - PATTERN (V), (N = 5,8,10,)

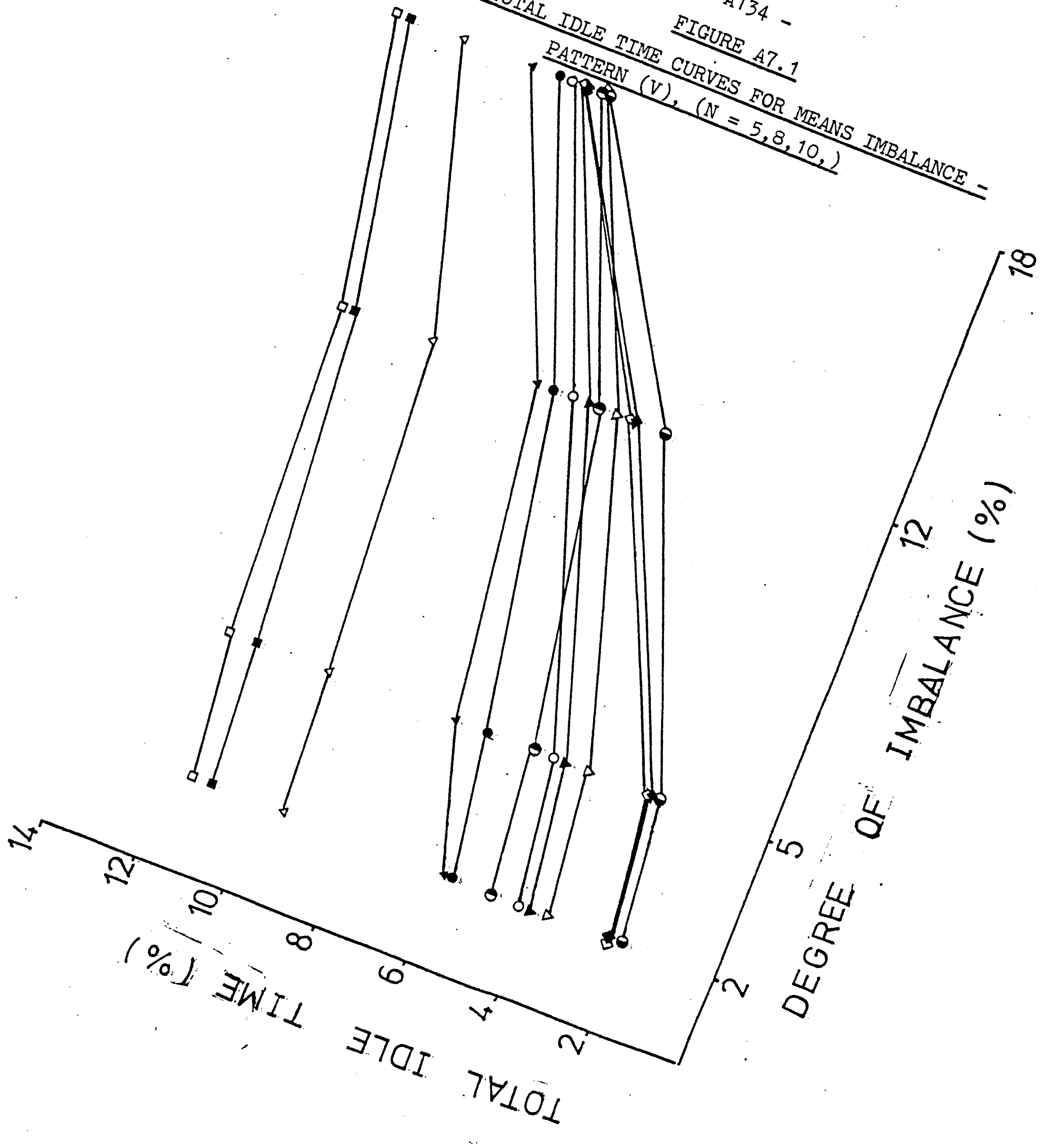
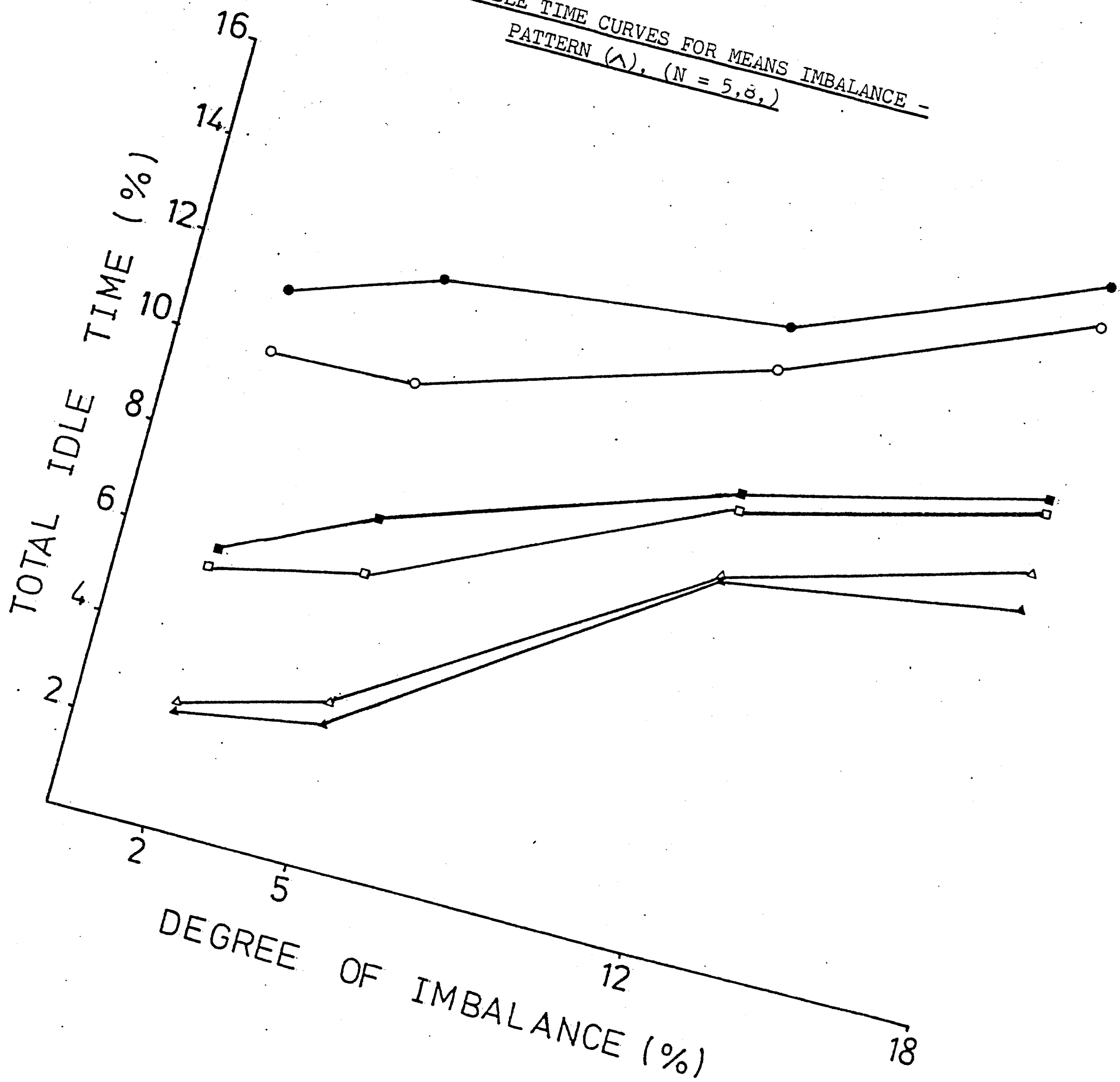


FIGURE A7.2

% TOTAL IDLE TIME CURVES FOR MEANS IMBALANCE - PATTERN (∧), (N = 5,8,)



% TOTAL IDLE TIME CURVES FOR COVARS IMBALANCE - PATTERN P_7 AND OTHER PATTERNS, (N = 5)

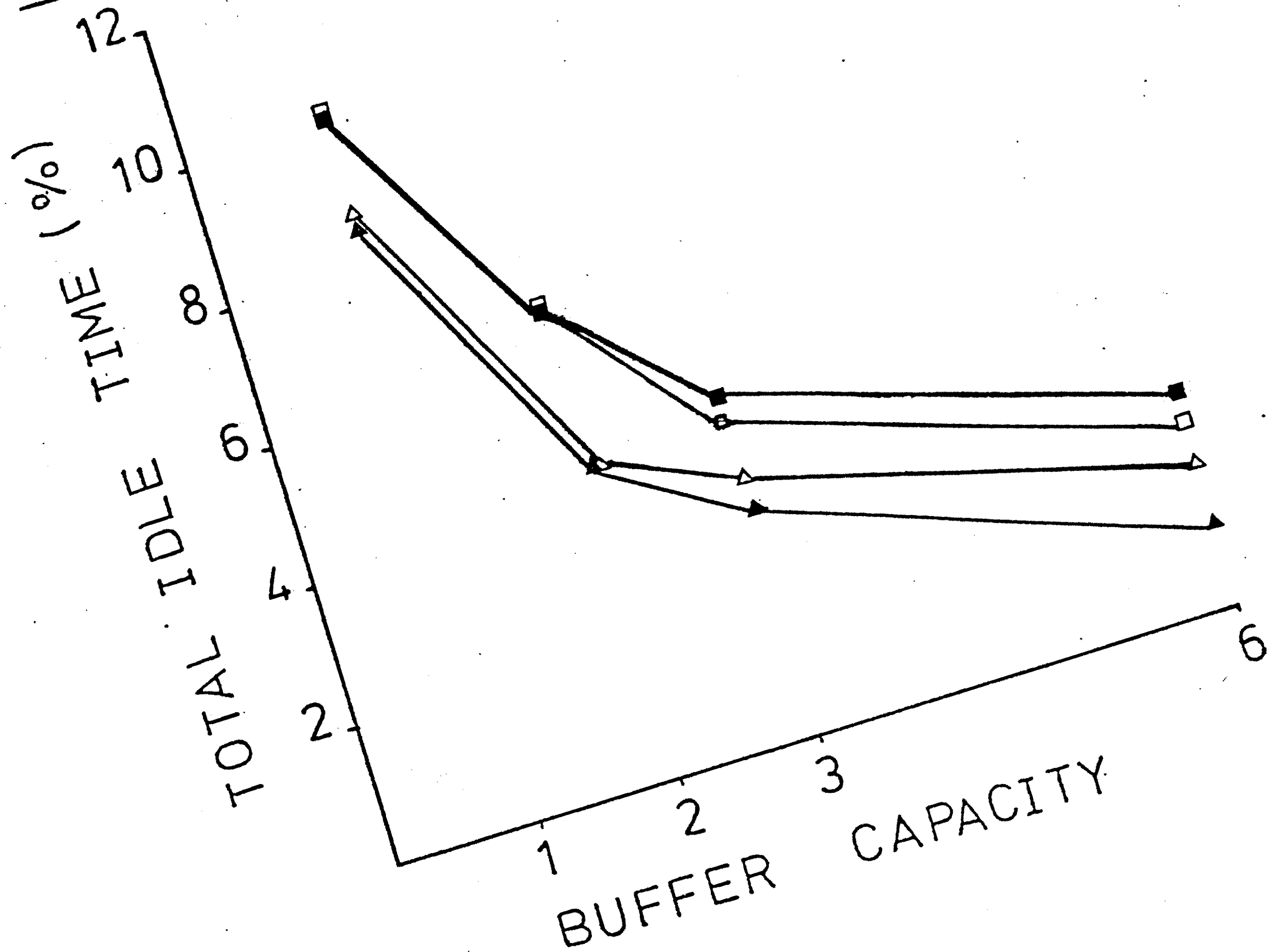
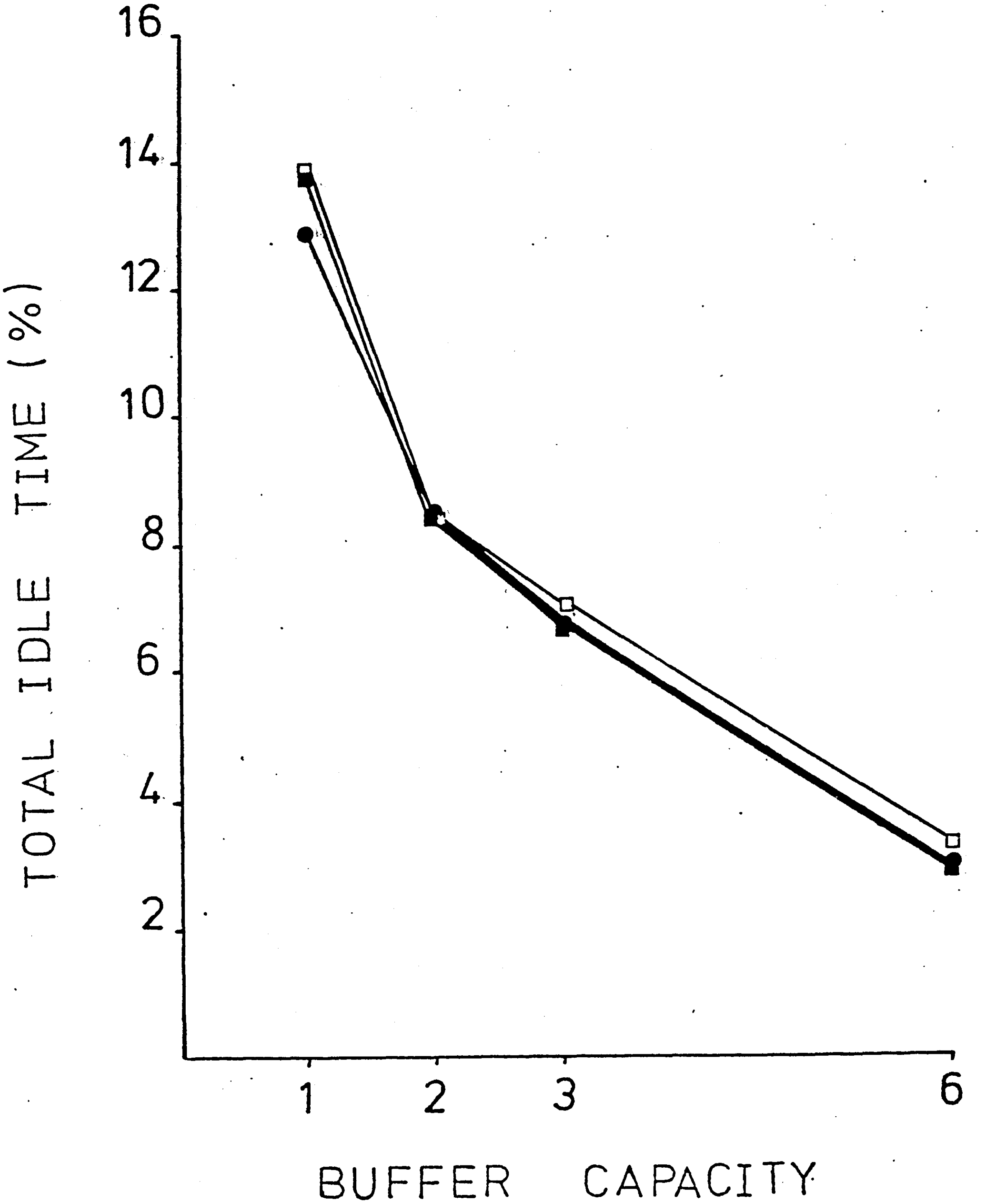


FIGURE A7.4

% TOTAL IDLE TIME CURVES FOR COVARS IMBALANCE - PATTERN P_7 AND OTHER PATTERNS, (N = 8)



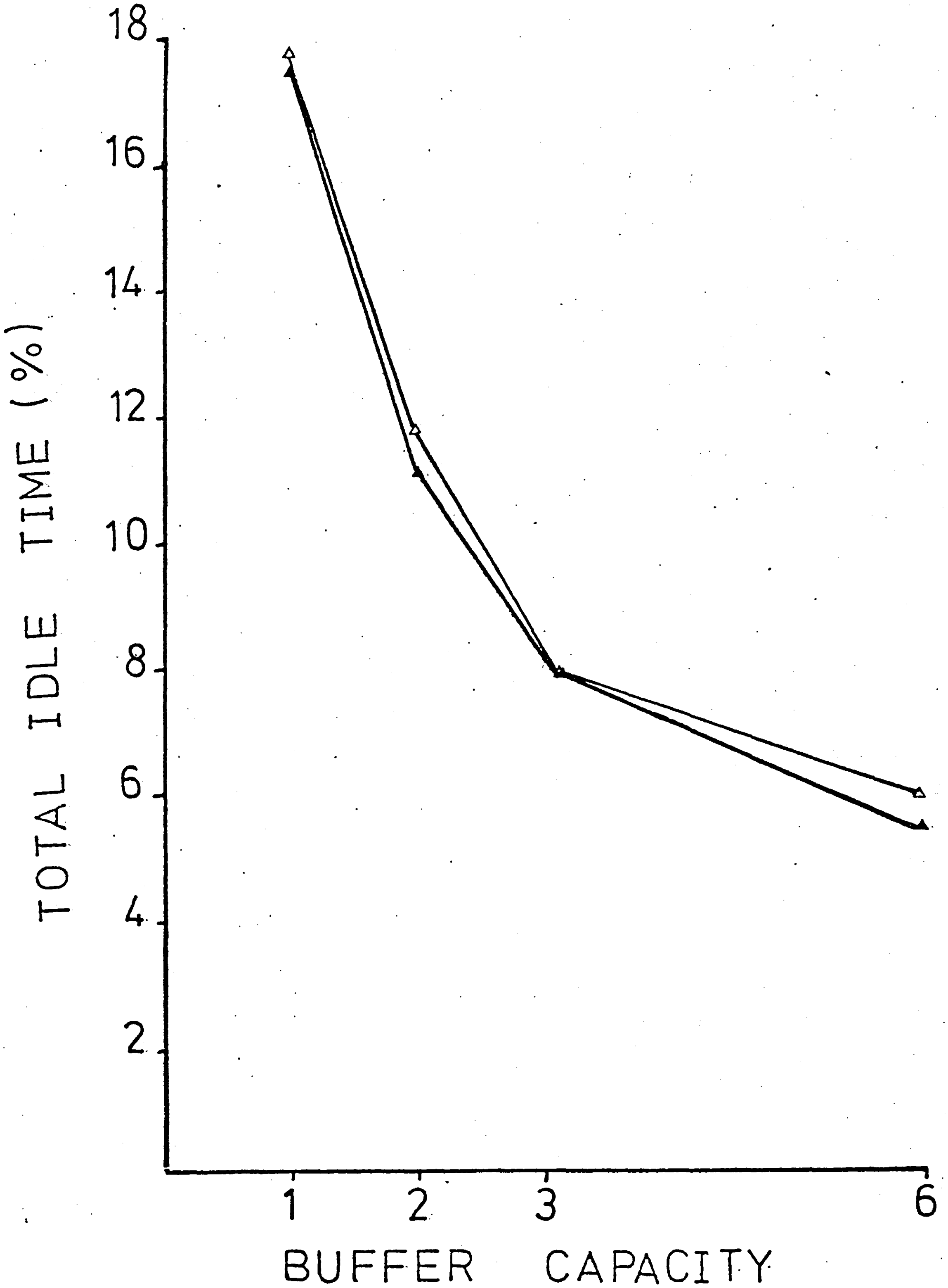
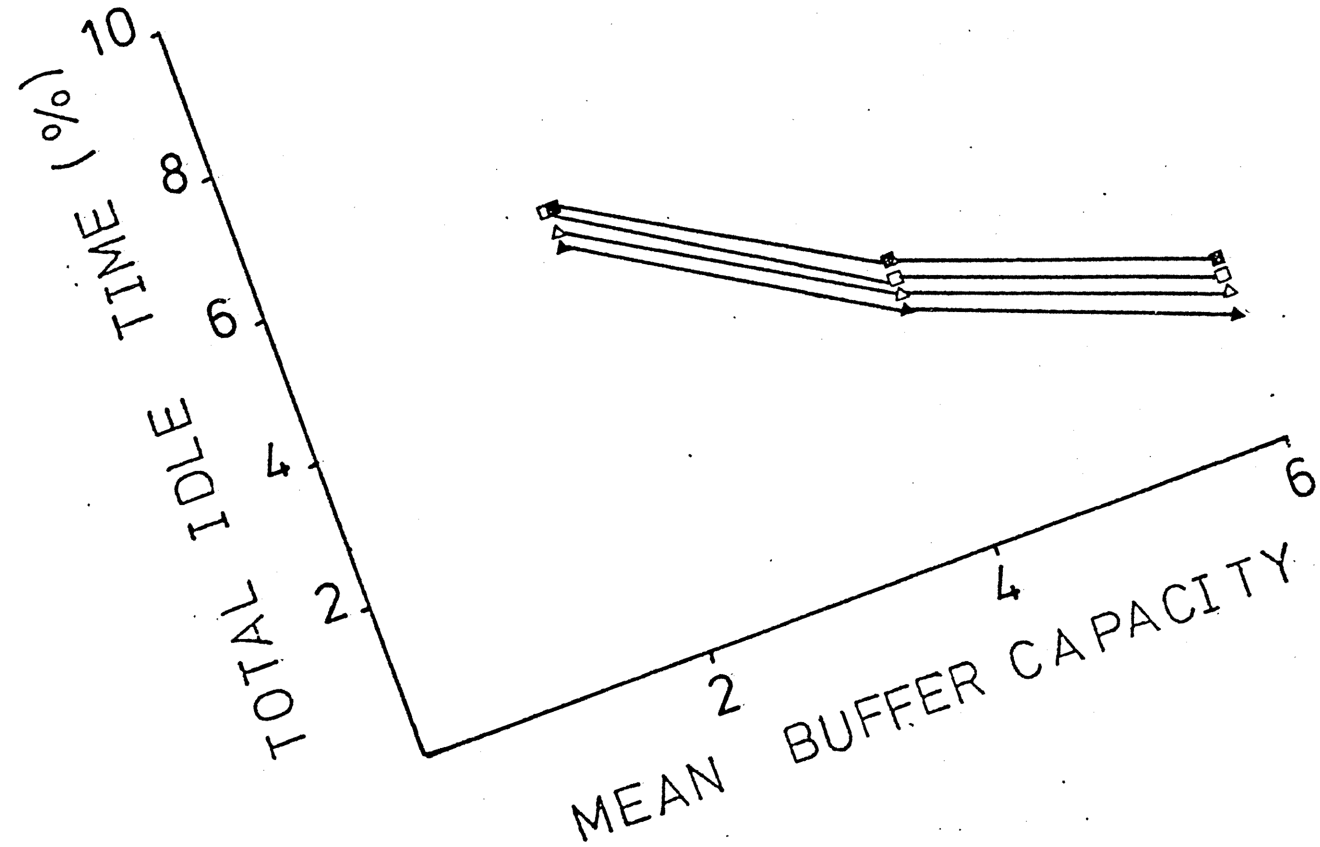


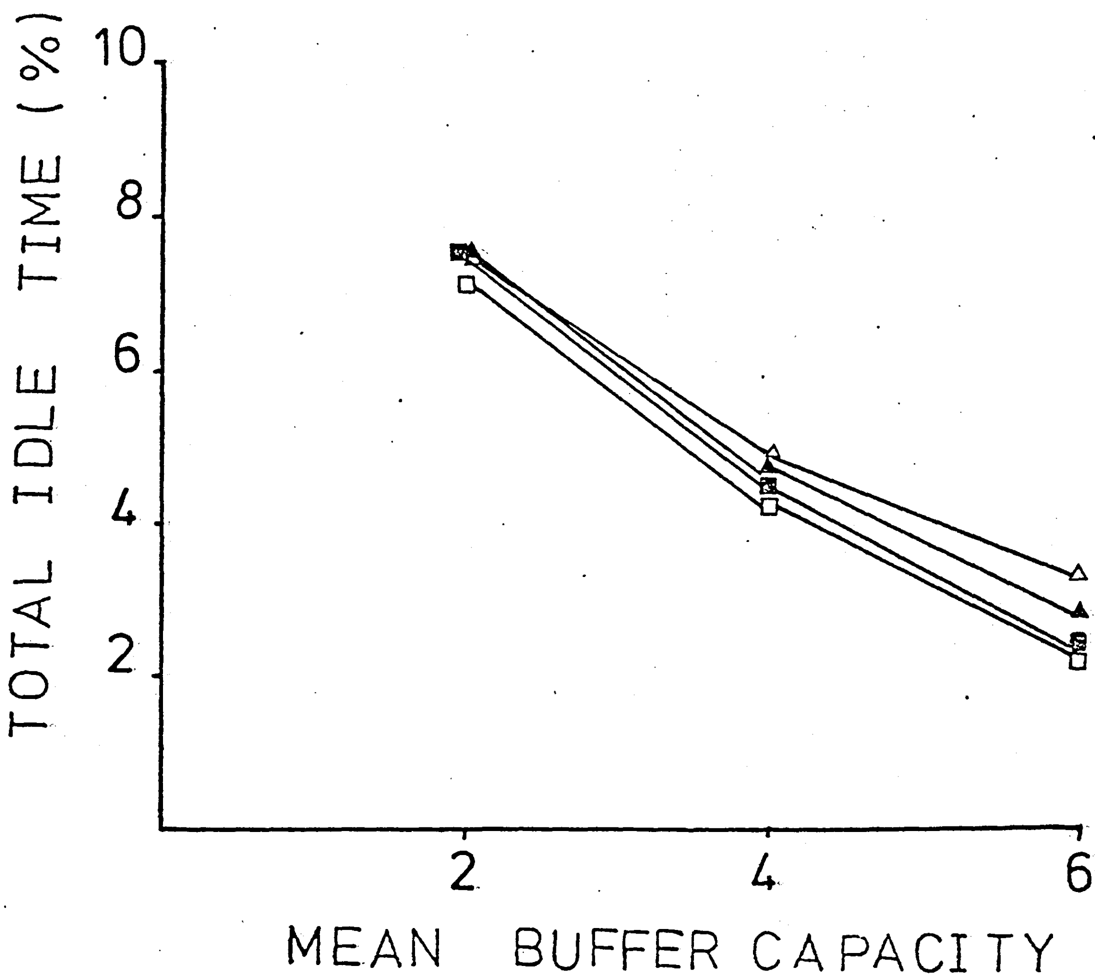
FIGURE A7.5

% TOTAL IDLE TIME CURVES FOR COVARS IMBALANCE - PATTERN P_7 AND P_8(N = 10)

% TOTAL IDLE TIME CURVES FOR BUFFER CAPACITIES IMBALANCE - PATTERN D_2 AND OTHER PATTERNS; (N = 5)



% TOTAL IDLE TIME CURVES FOR BUFFER CAPACITIES IMBALANCE - PATTERN D_1 AND OTHER PATTERNS, (N = 8)



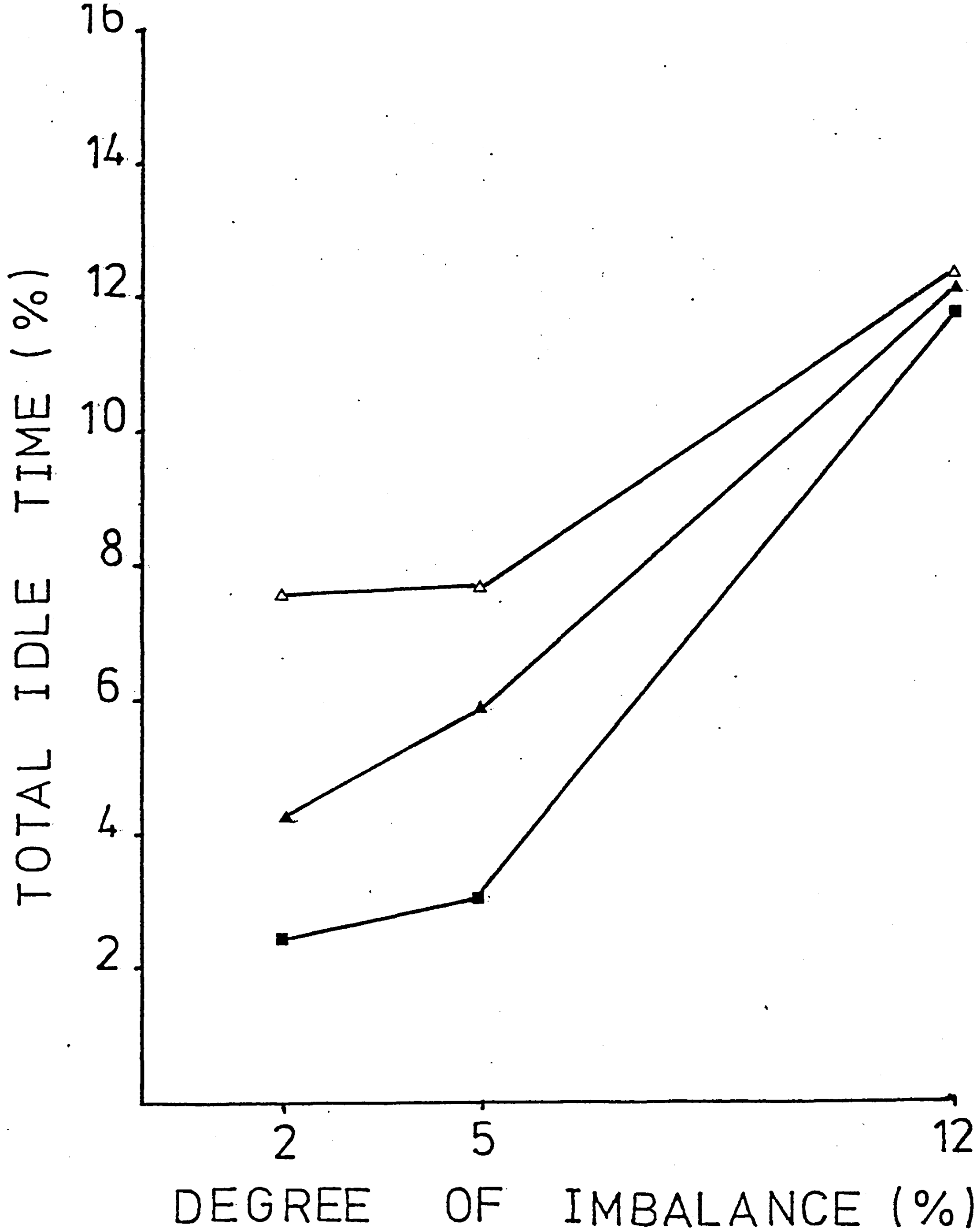


FIGURE A7.8

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (/) + P_4, (N = 5)

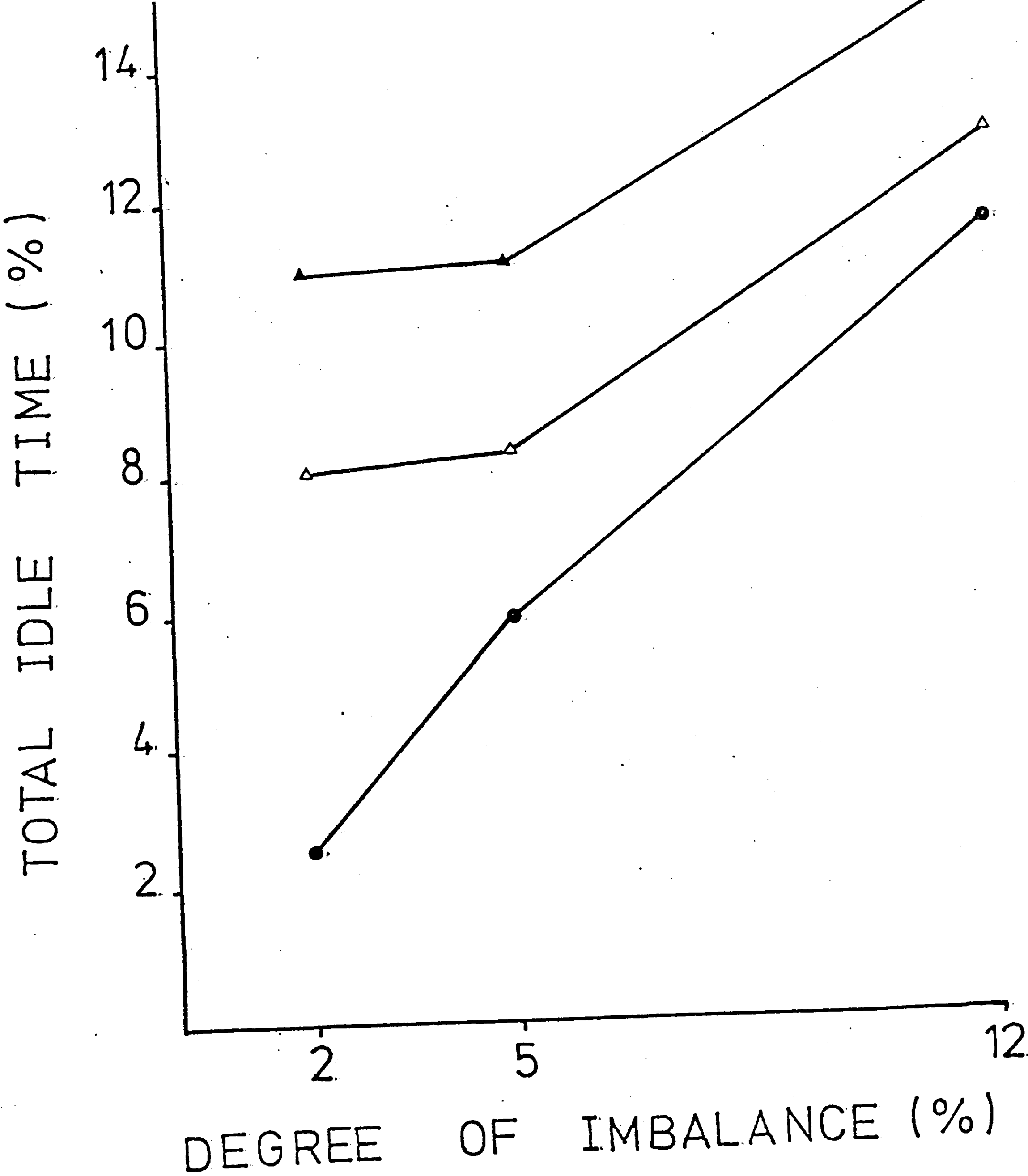


FIGURE A7.9

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (/) + P_4, (N = 8)

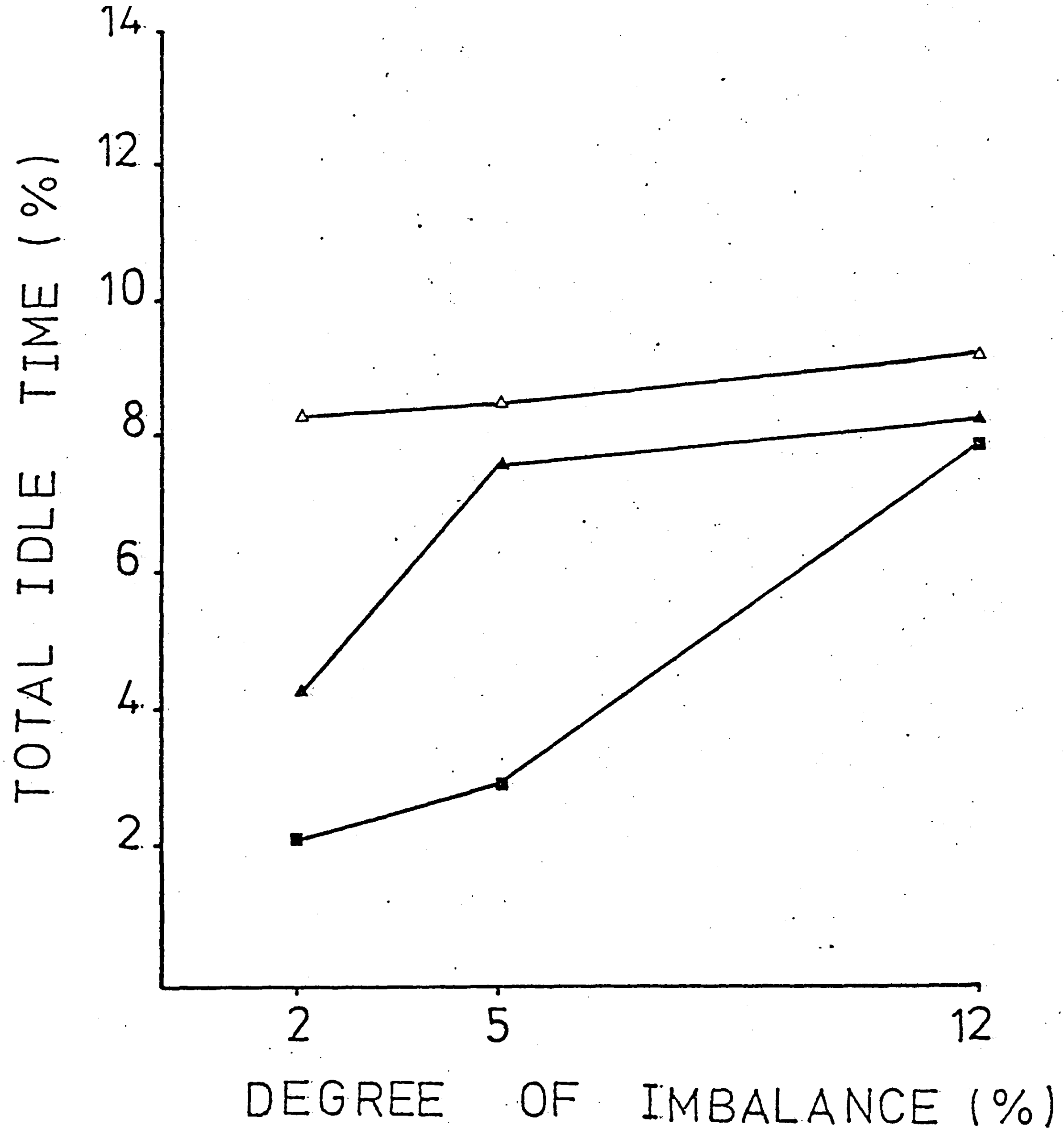


FIGURE A7.10

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (Λ) + P_4, (N = 5)

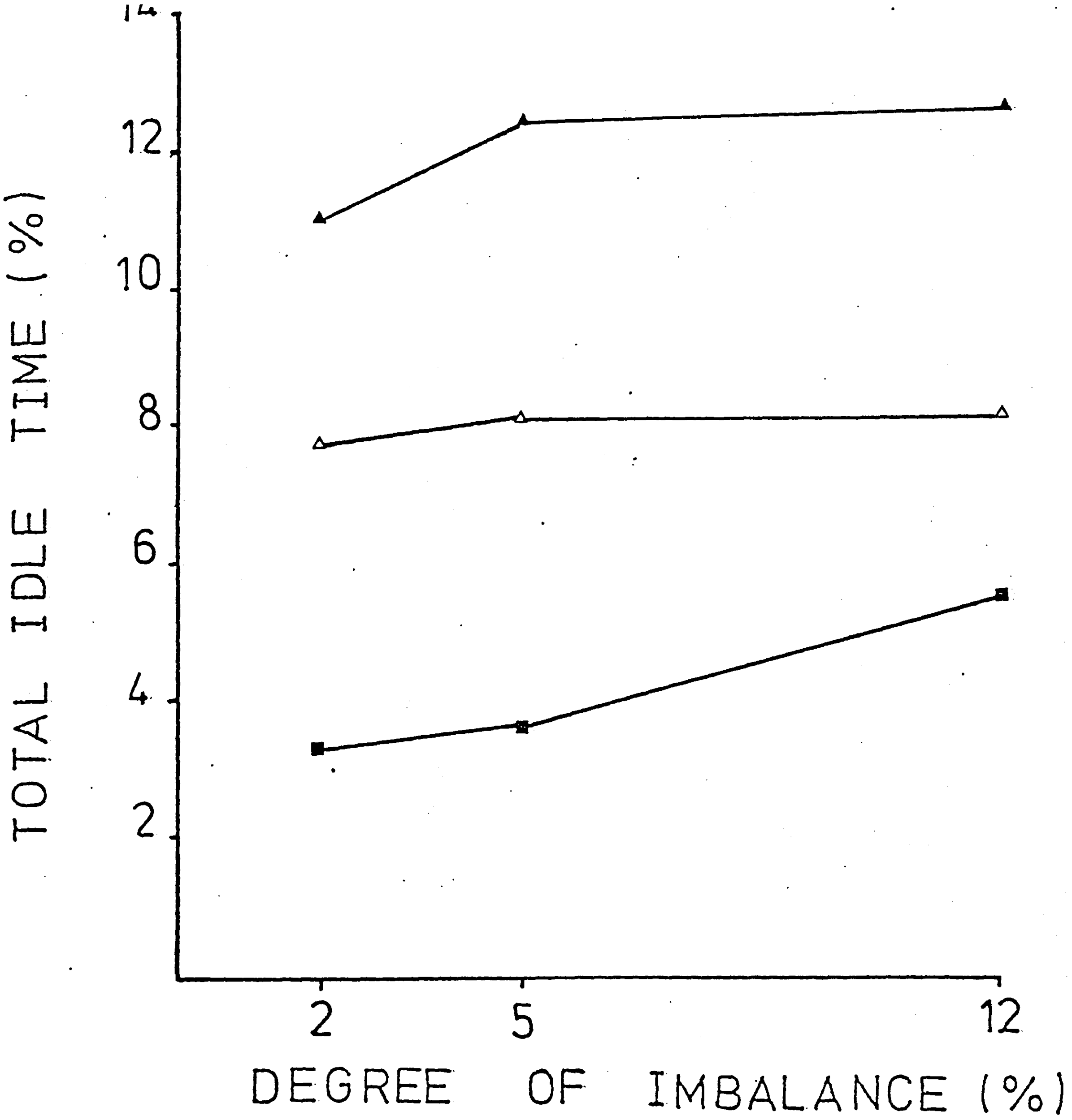
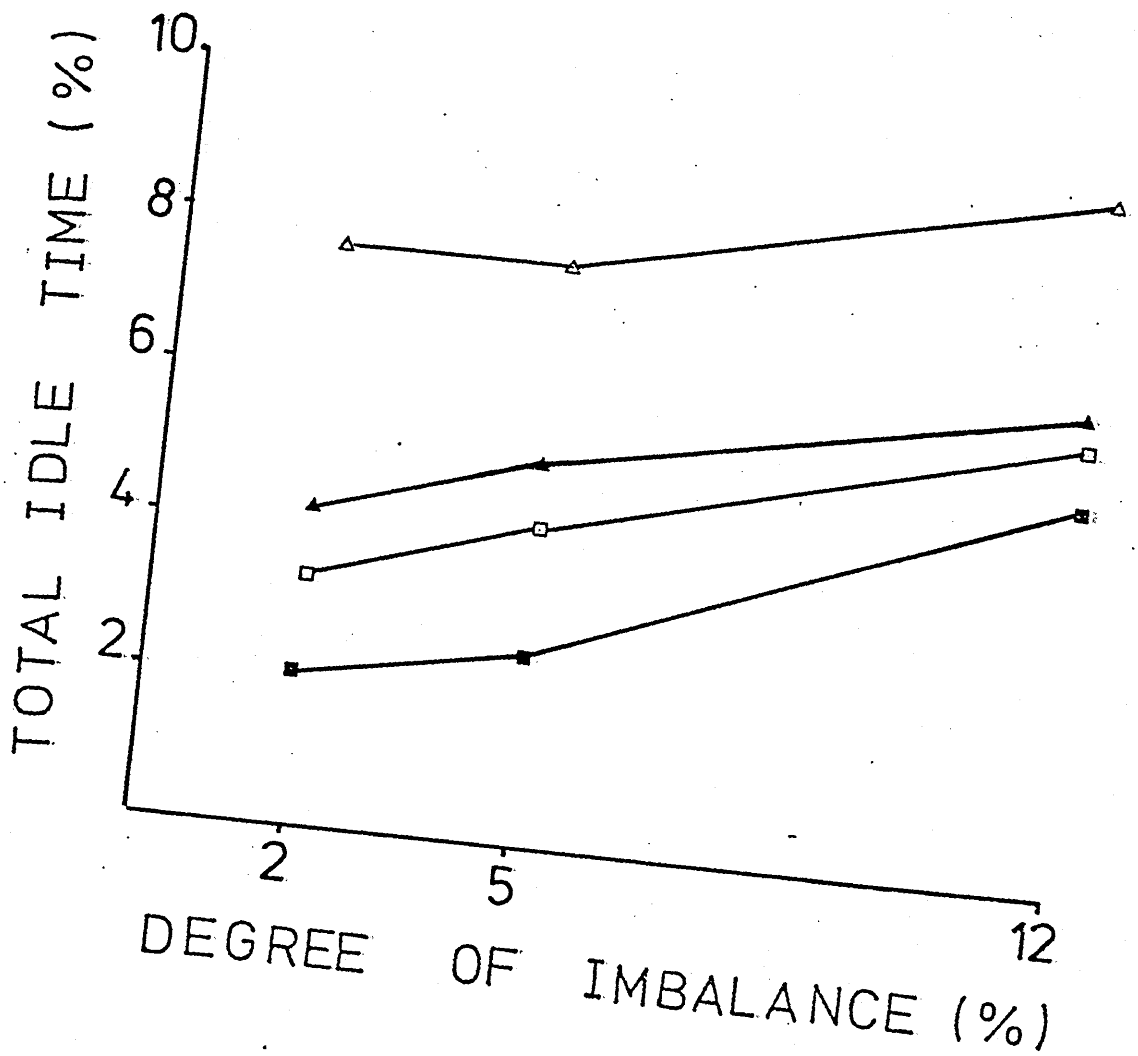


FIGURE A7.11

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (Λ) + P_4, (N = 8)

FIGURE A7.12

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE – PATTERN (Λ) + P_5, (N = 5)



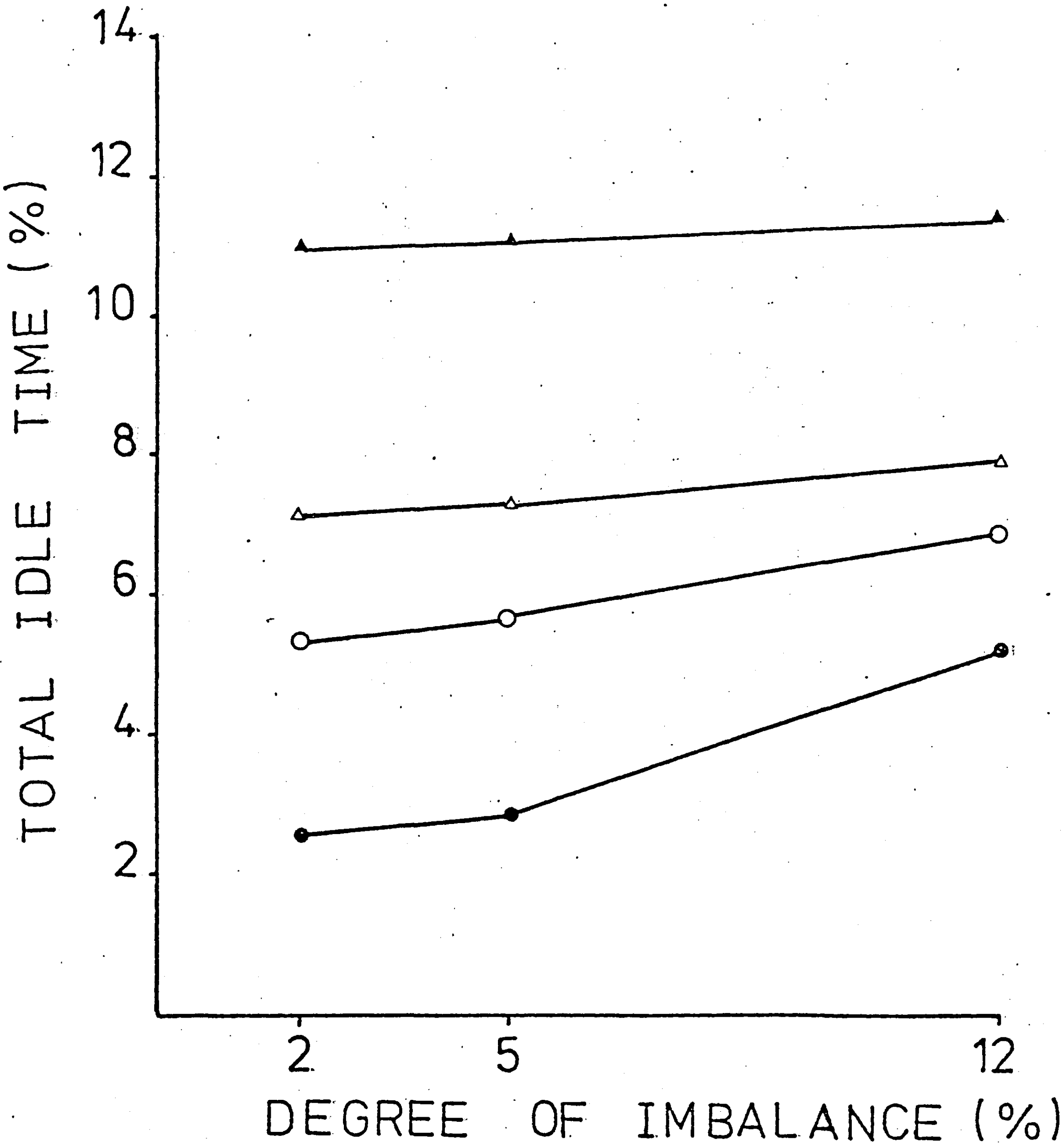


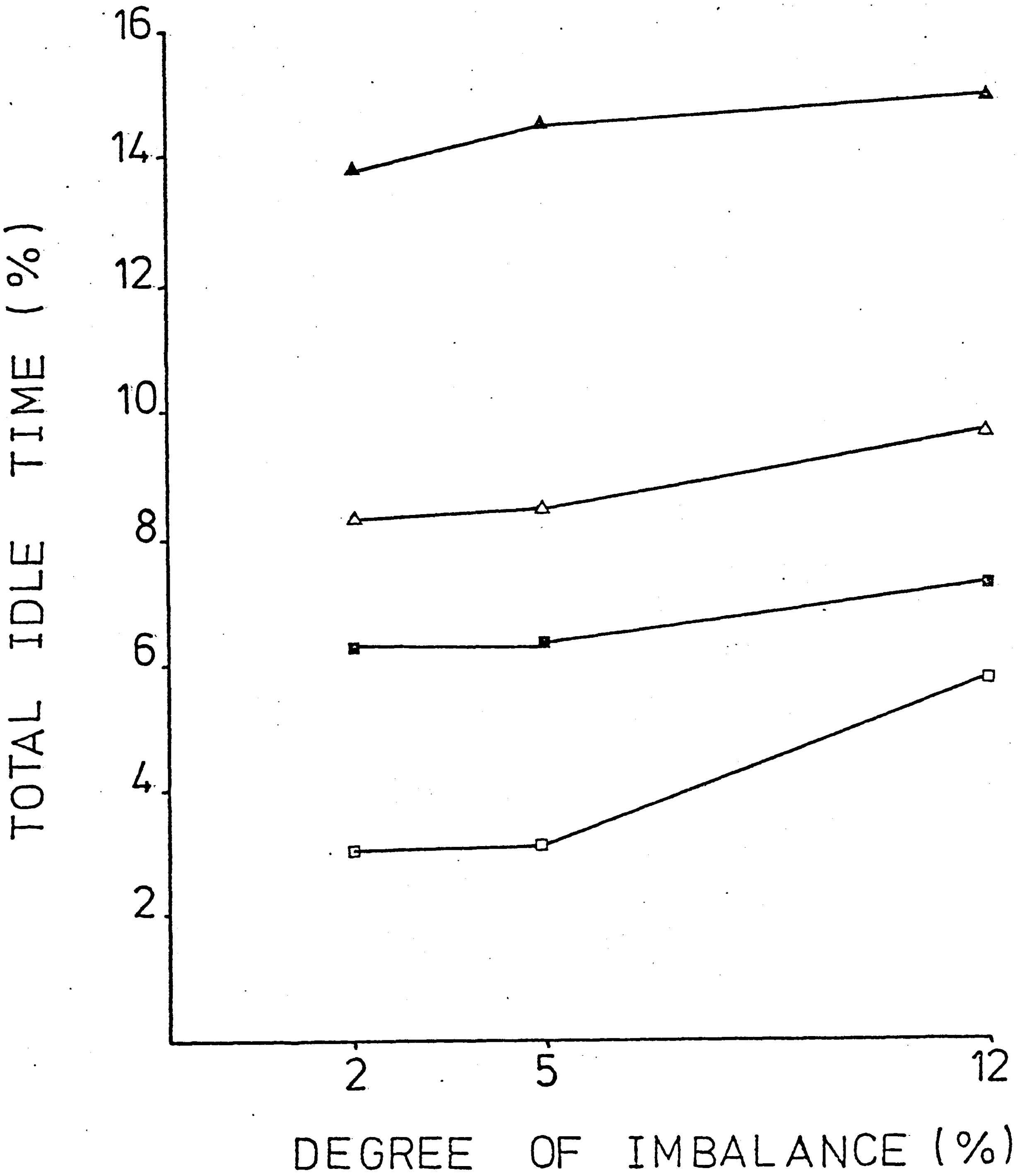
FIGURE A7.13

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (Λ) + P_5(N = 8)

FIGURE A7.14

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE -

PATTERN (∧) + P_5, (N = 10)



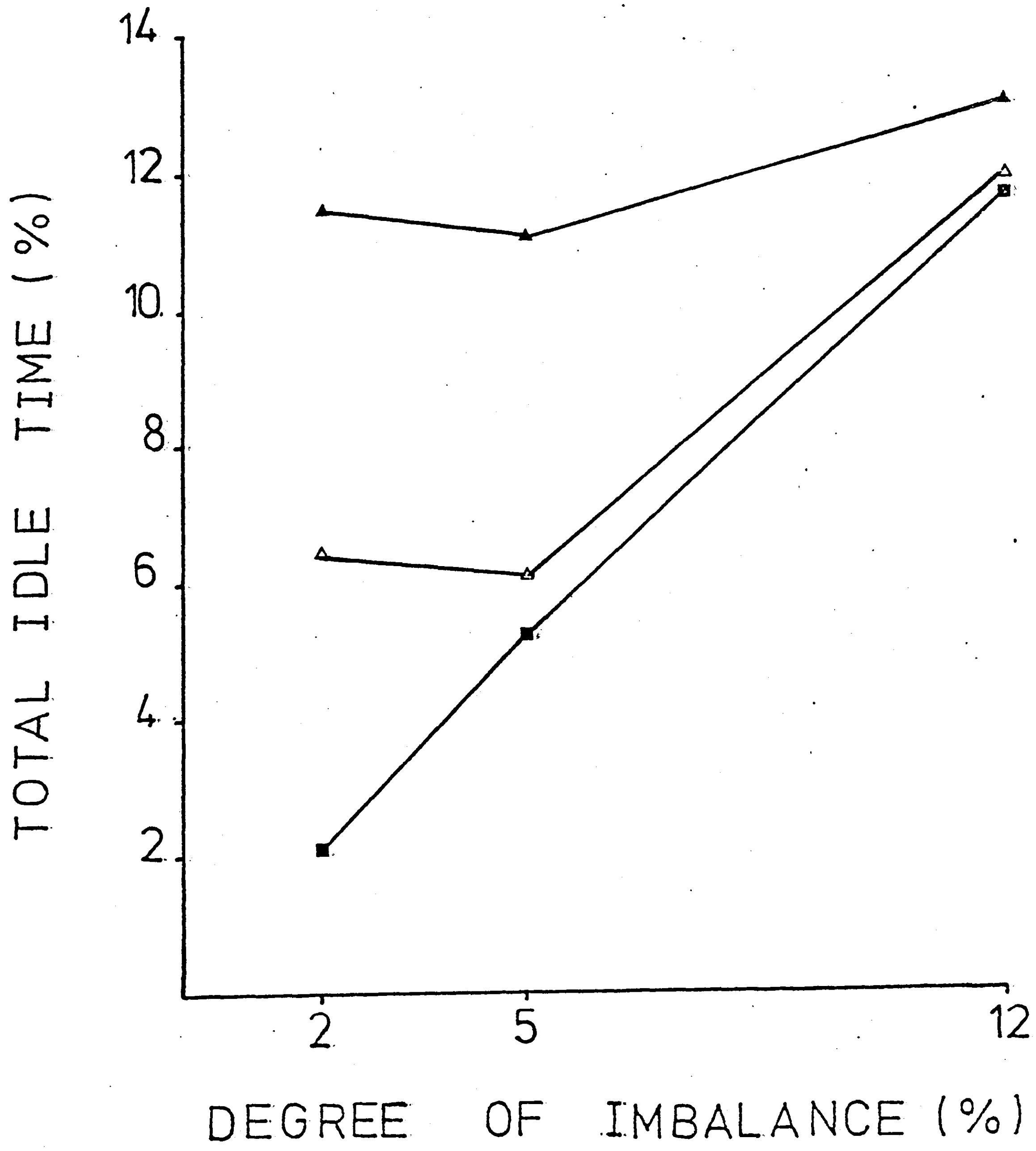


FIGURE A7.15

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (/) + P_6, (N = 5)

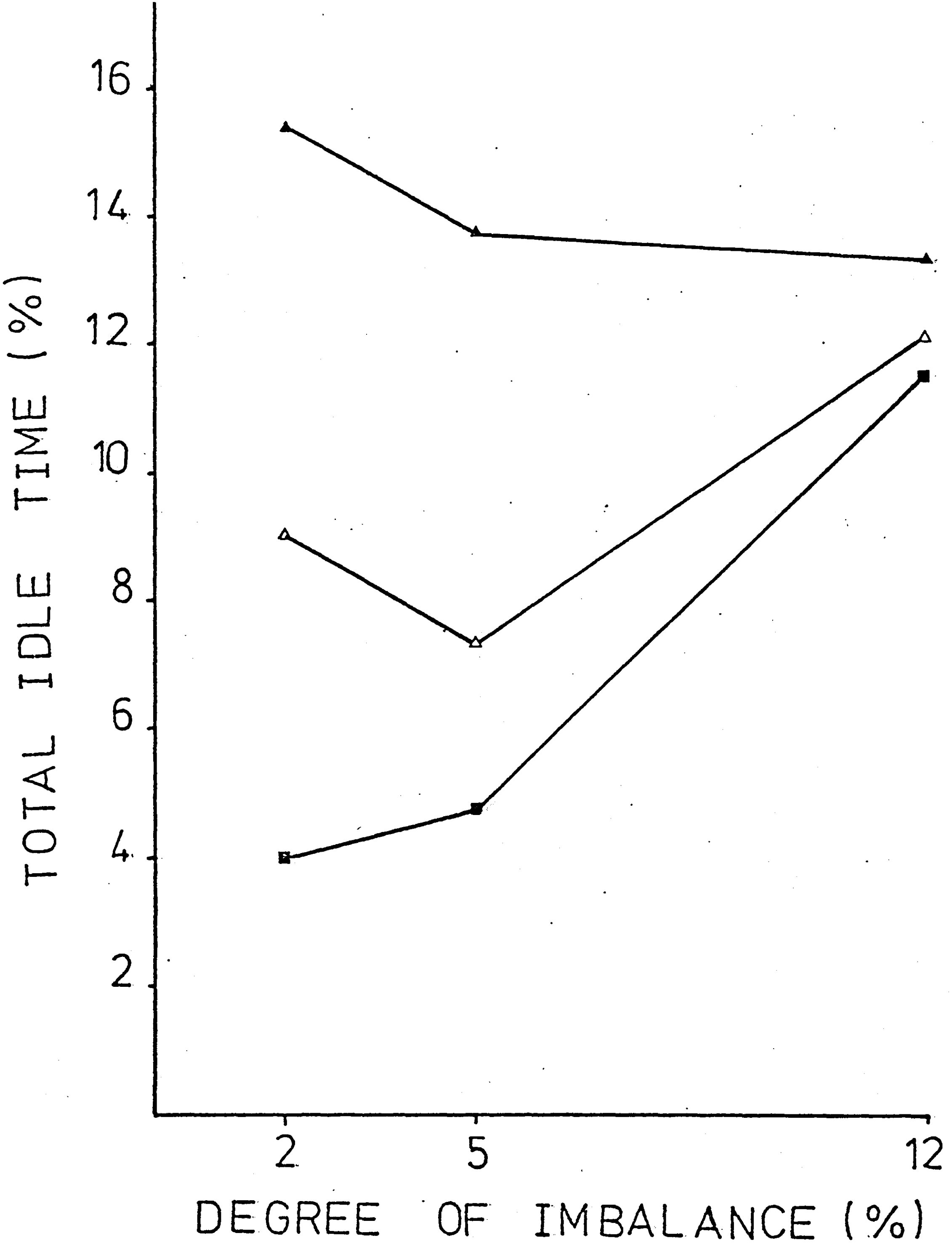


FIGURE A7.16

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (/) + P_6, (N = 8)

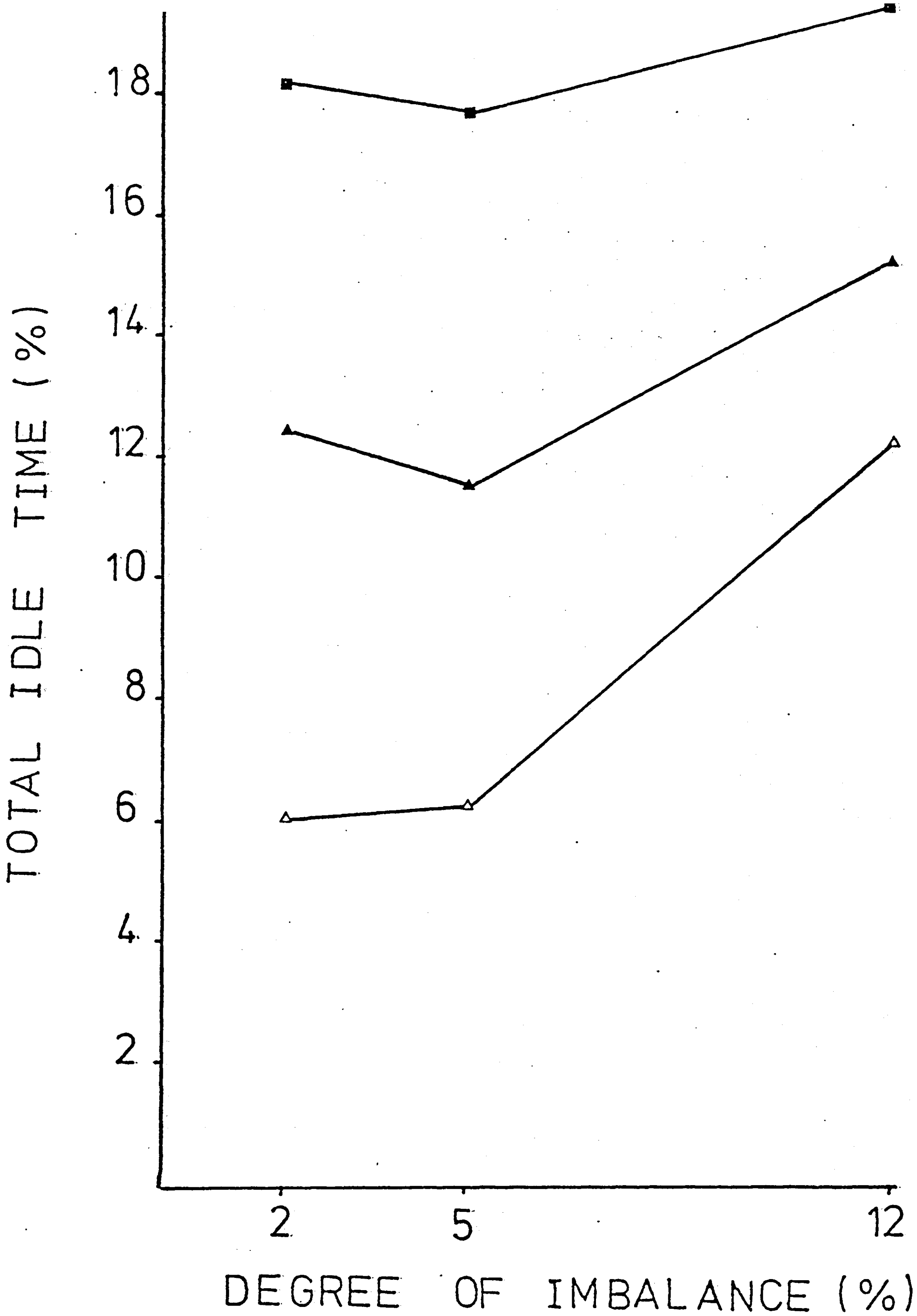


FIGURE A7.17

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (/) + P_8, (N = 5)

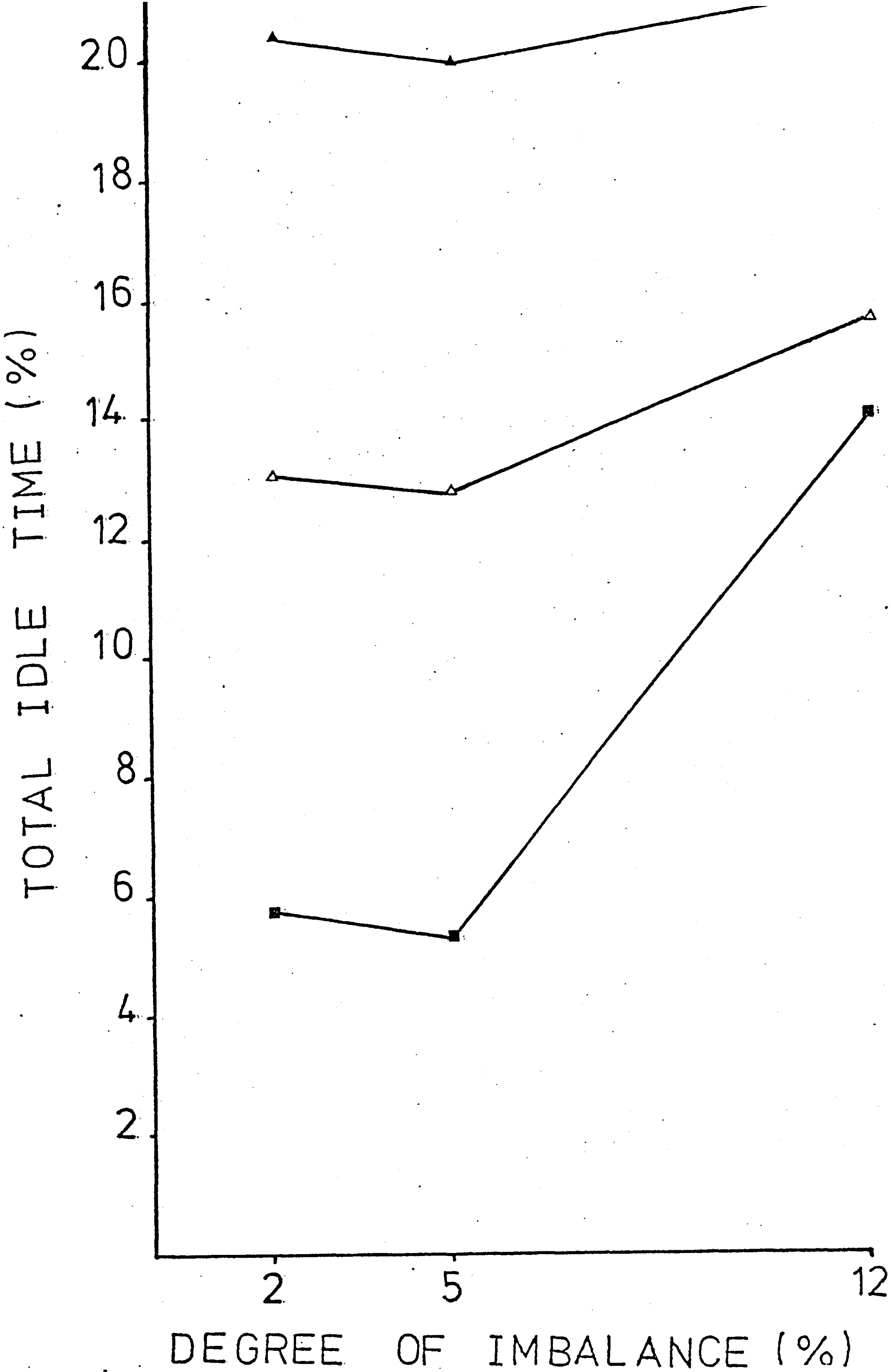


FIGURE A7.18

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (/) + P_8, (N = 8)

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (V) + P_1,(N = 5)

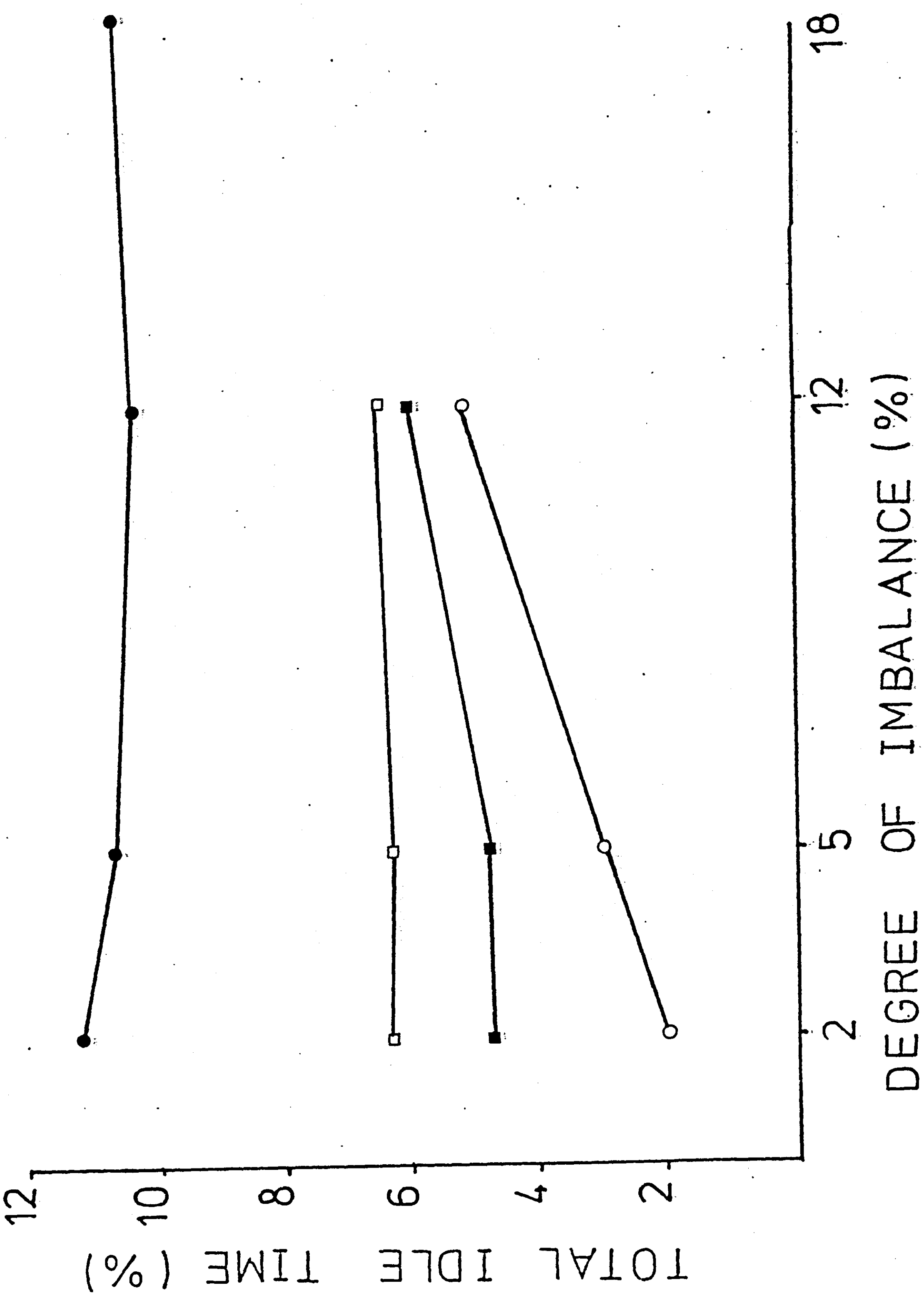


FIGURE A7.20

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (V) + P_1, (N = 8)

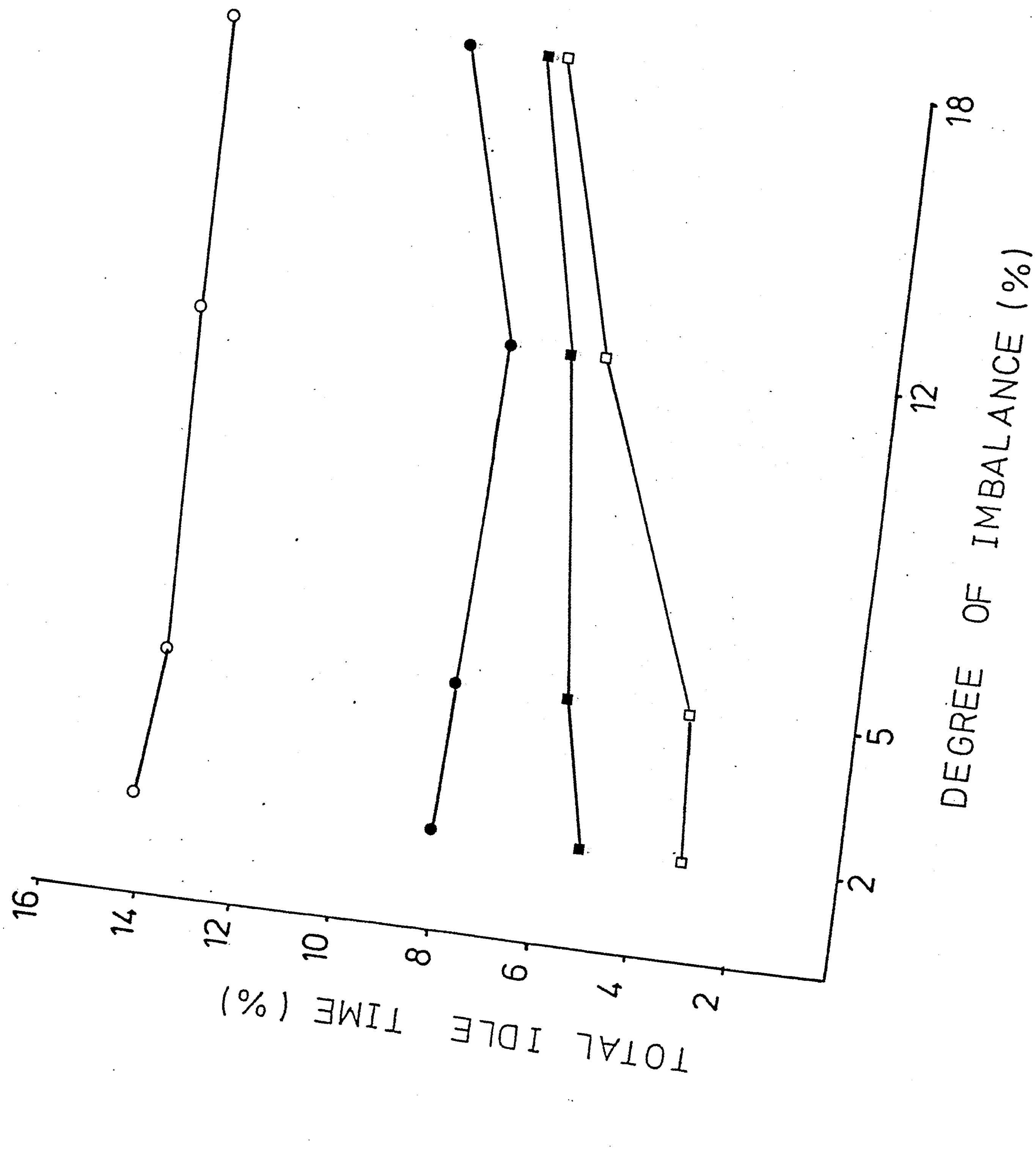
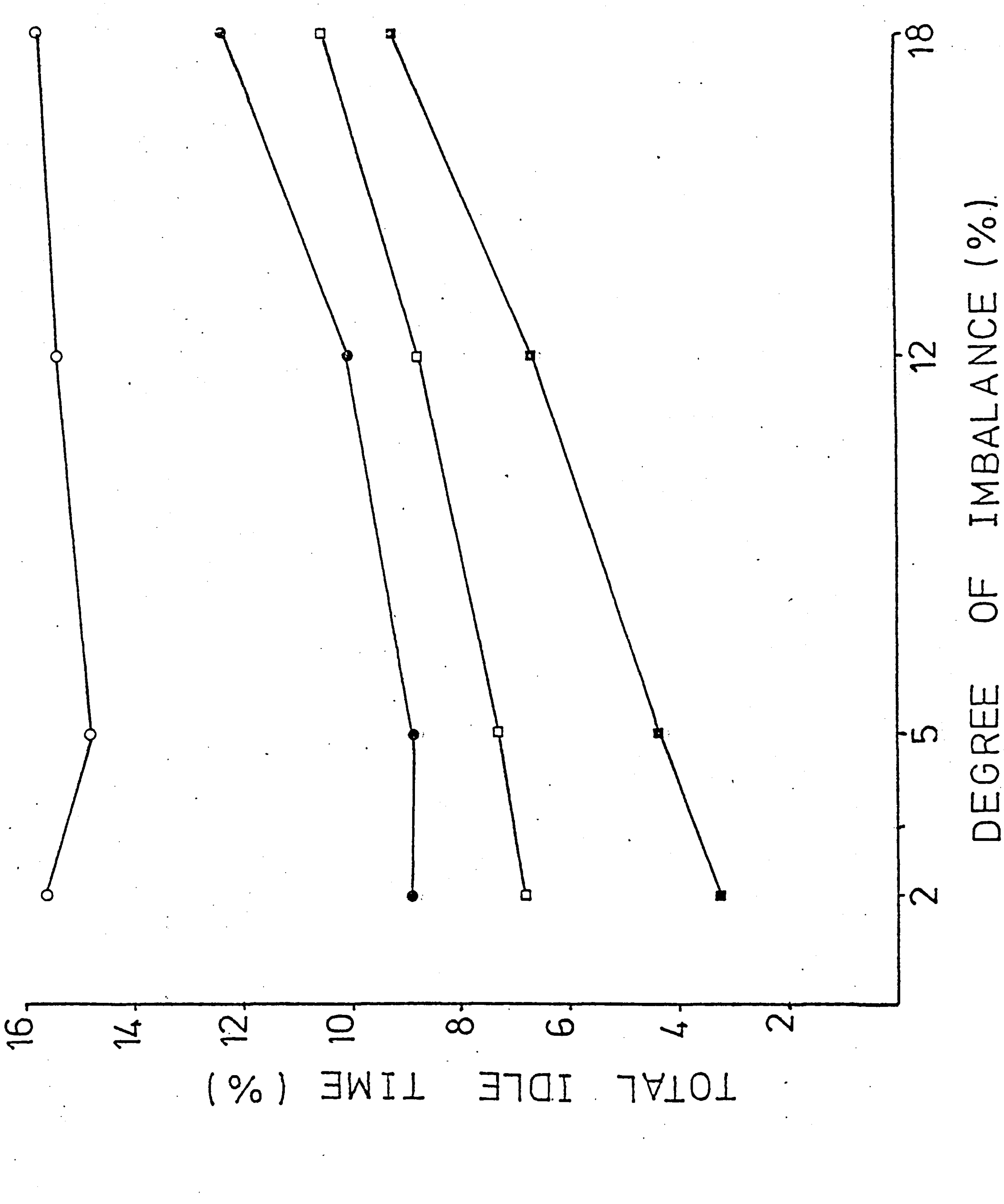


FIGURE A7.21

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (V) + P_1, (N = 10)



% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE

PATTERN (V) + P_3, (N = 5)

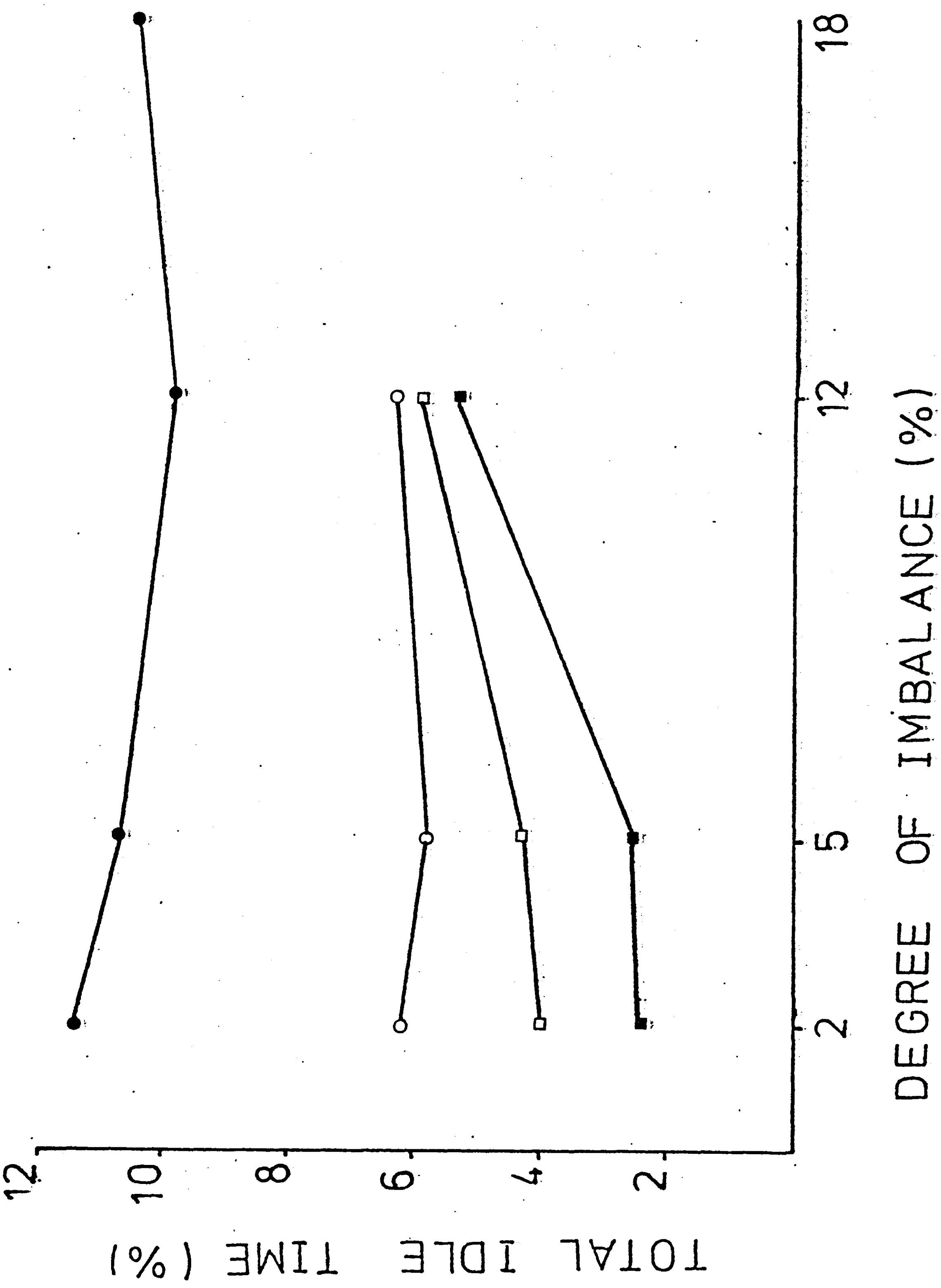


FIGURE A7.23

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (V) + P_3,(N = 8)

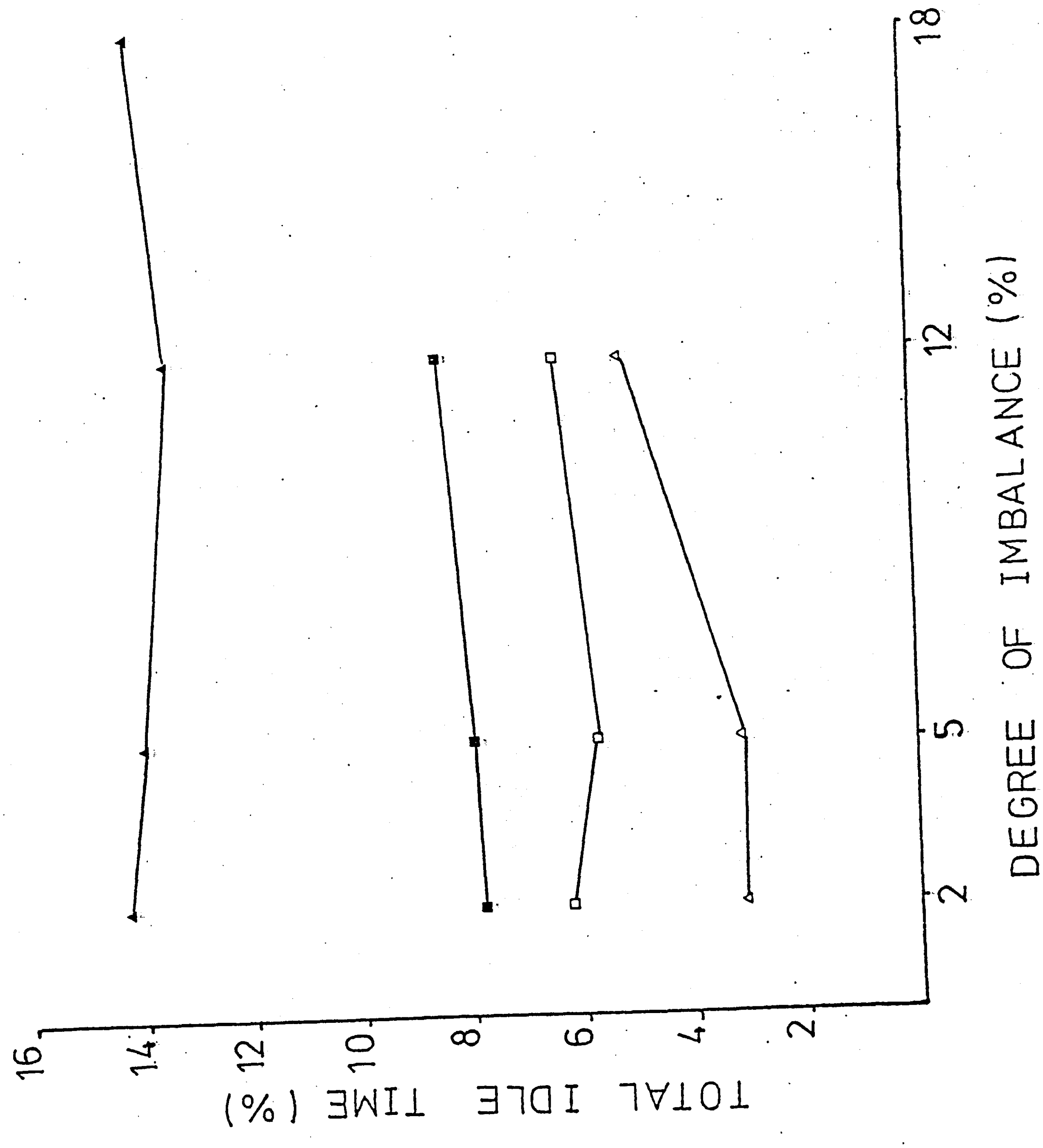
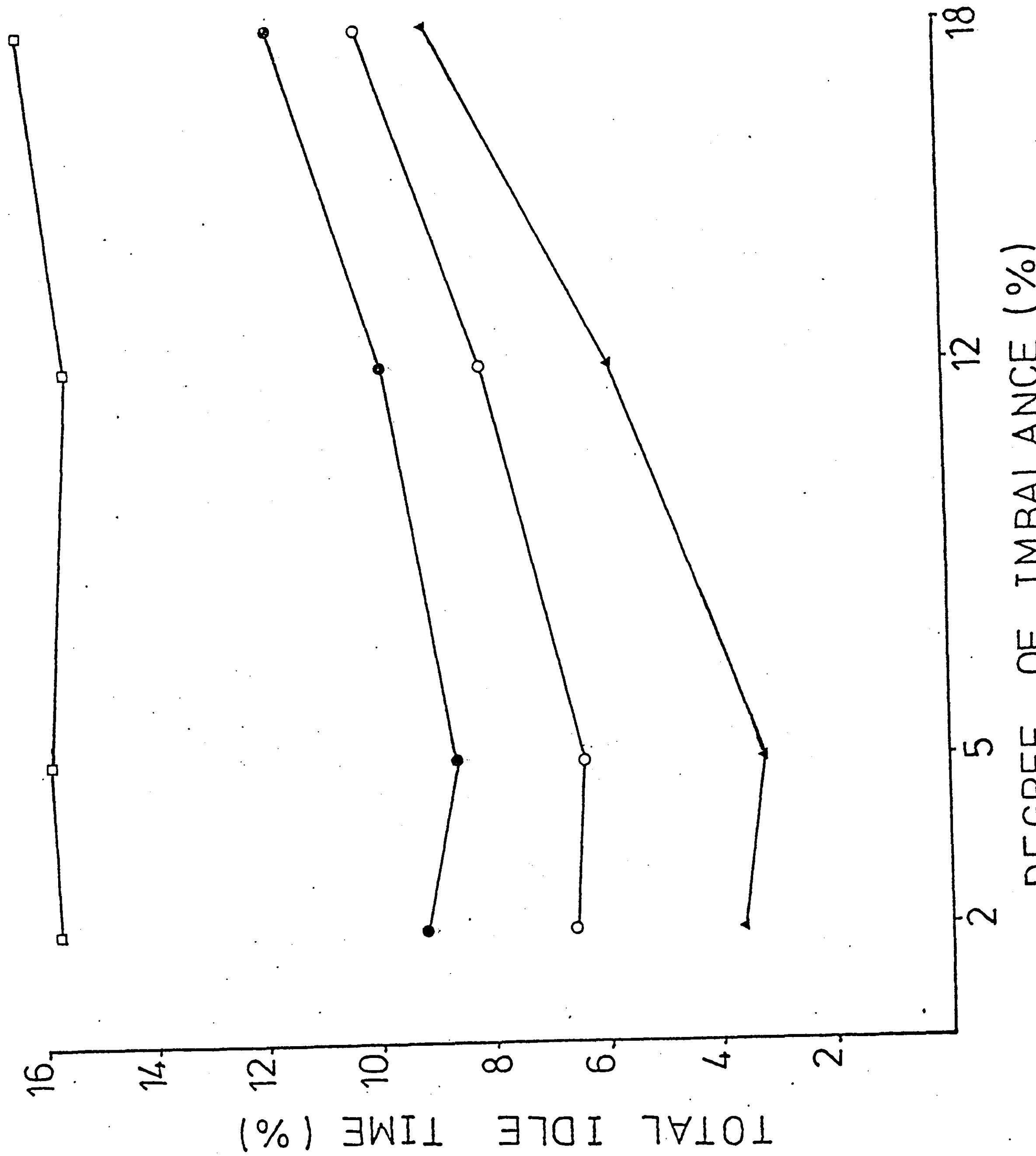


FIGURE A7.24

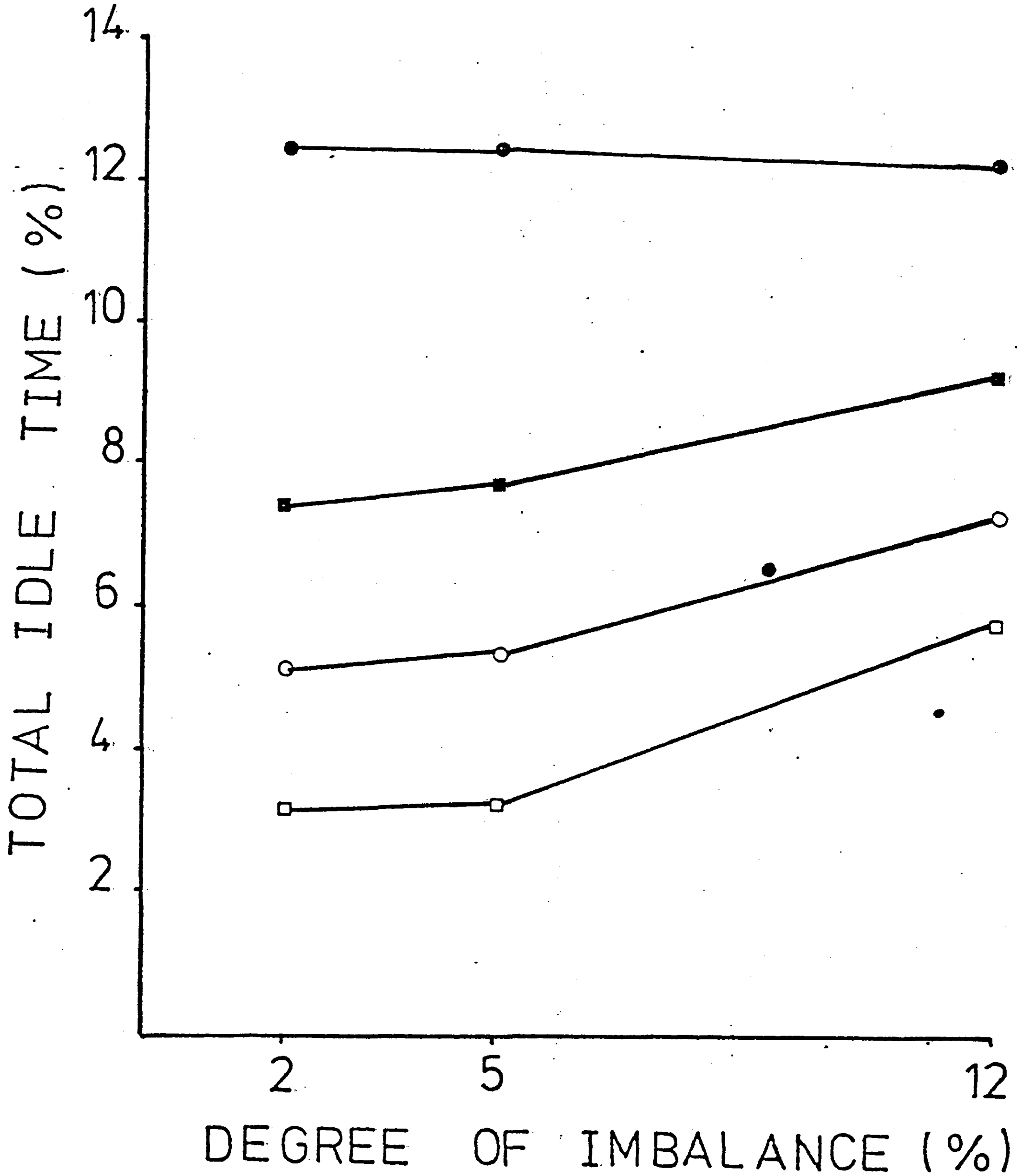
% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE -

PATTERN (V) + P_3, (N = 10)



% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE -

PATTERN (V) + P_6, (N = 5)



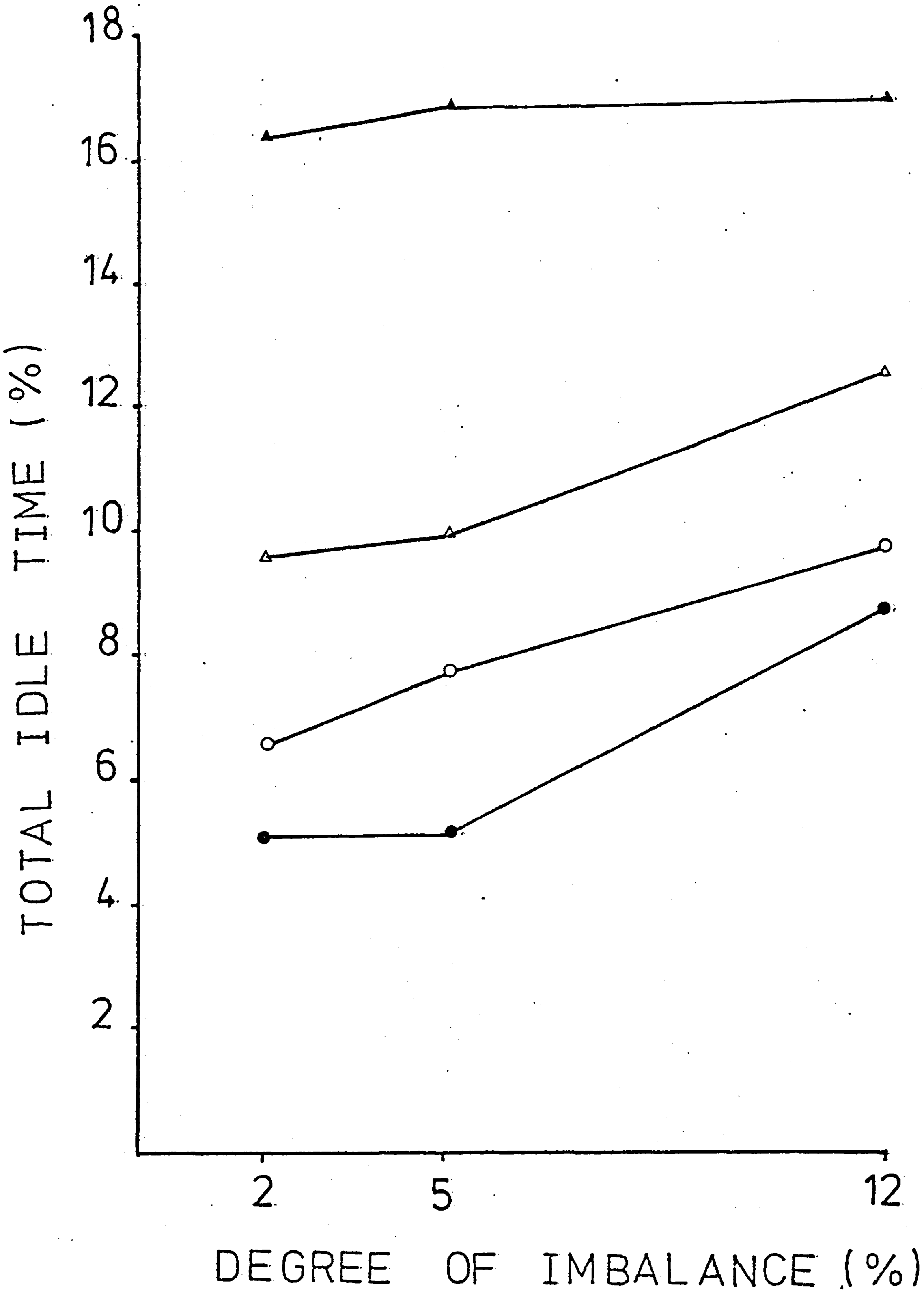


FIGURE A7.26

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE -
PATTERN (V) + P_6, (N = 8)

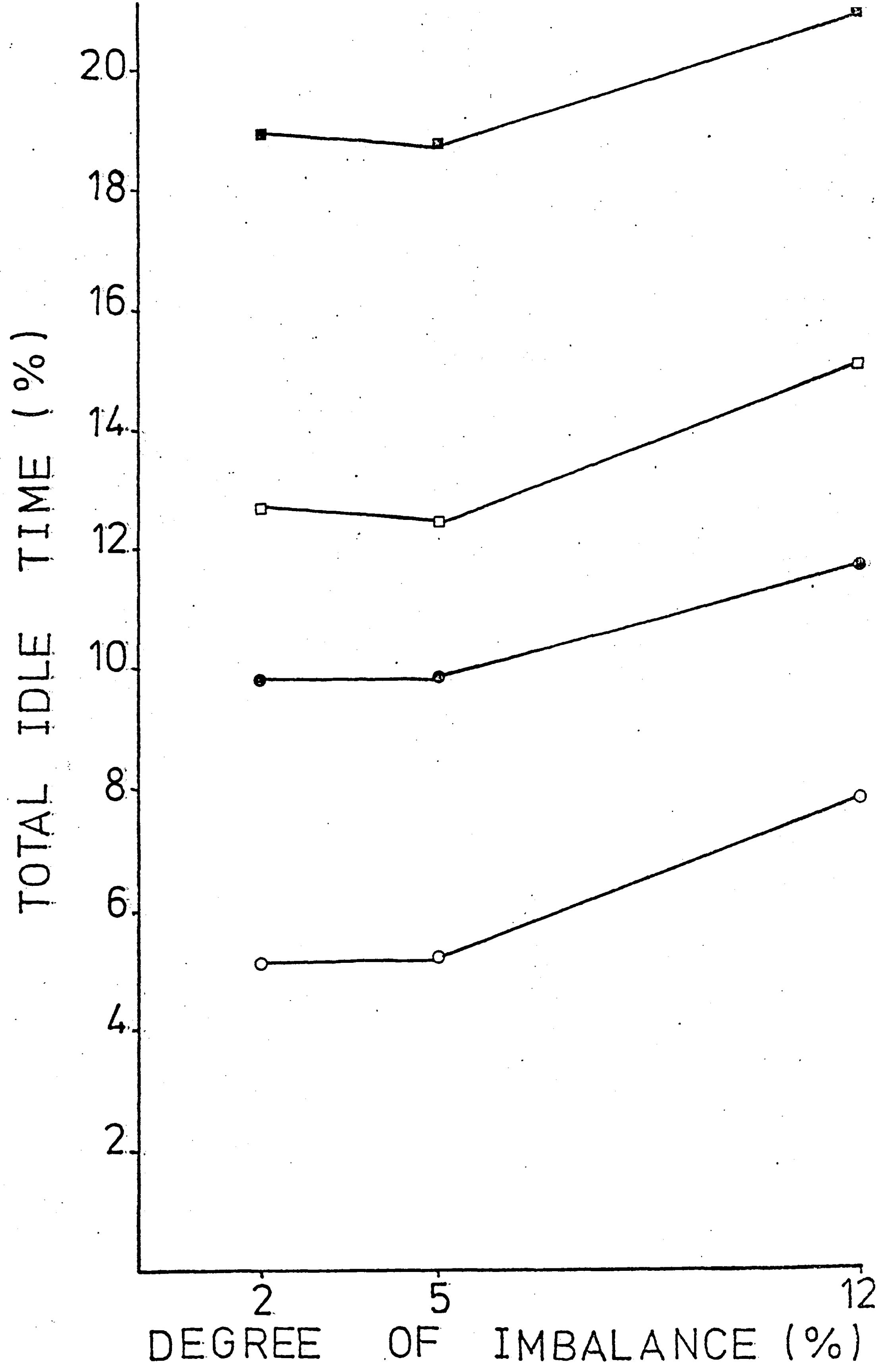
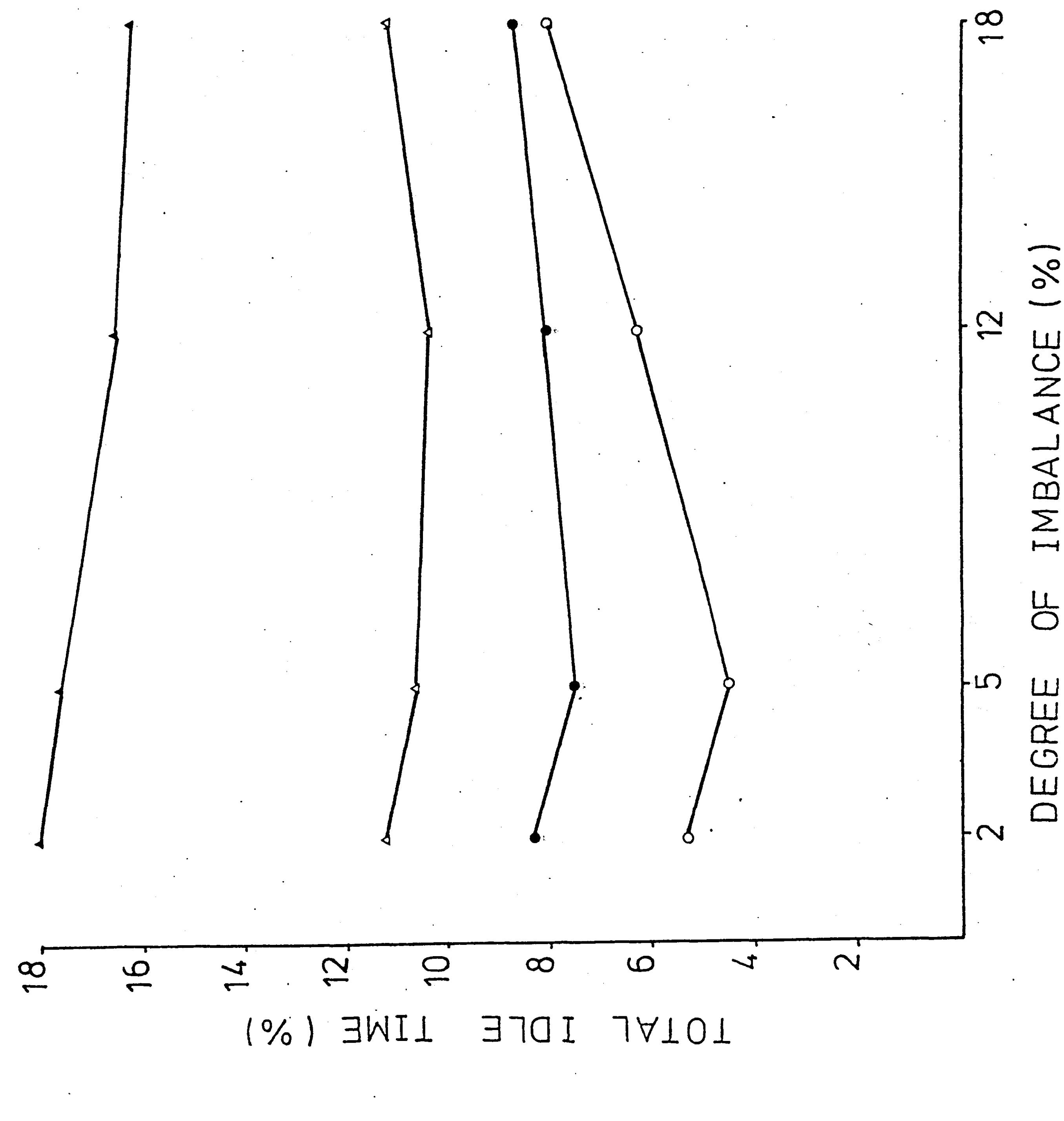


FIGURE A7.27

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (V) + P_6, (N = 10)

FIGURE A7.28

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (V) + P_8,(N = 5)



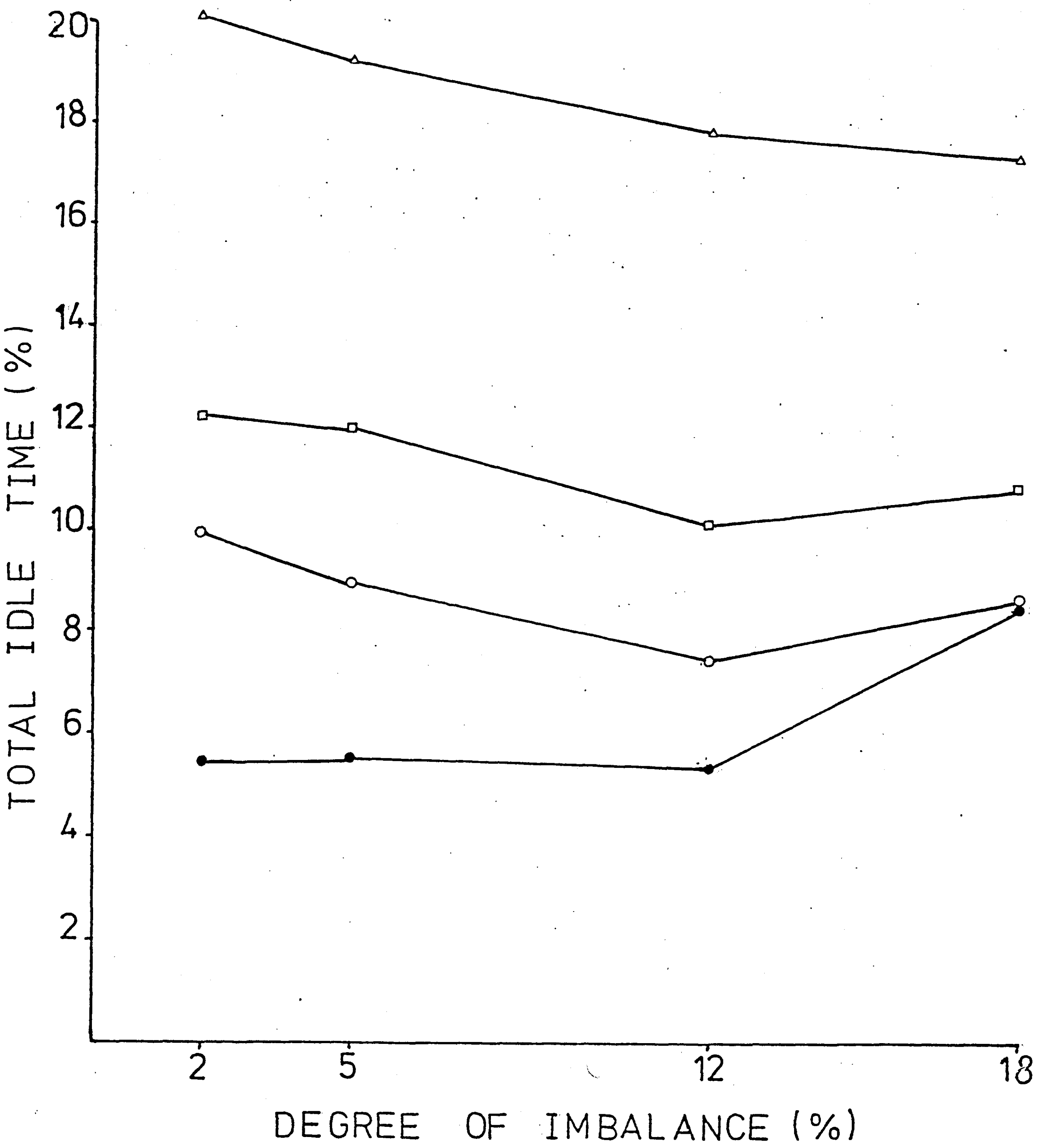


FIGURE A7.29

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (V) + P_8, (N = 8)

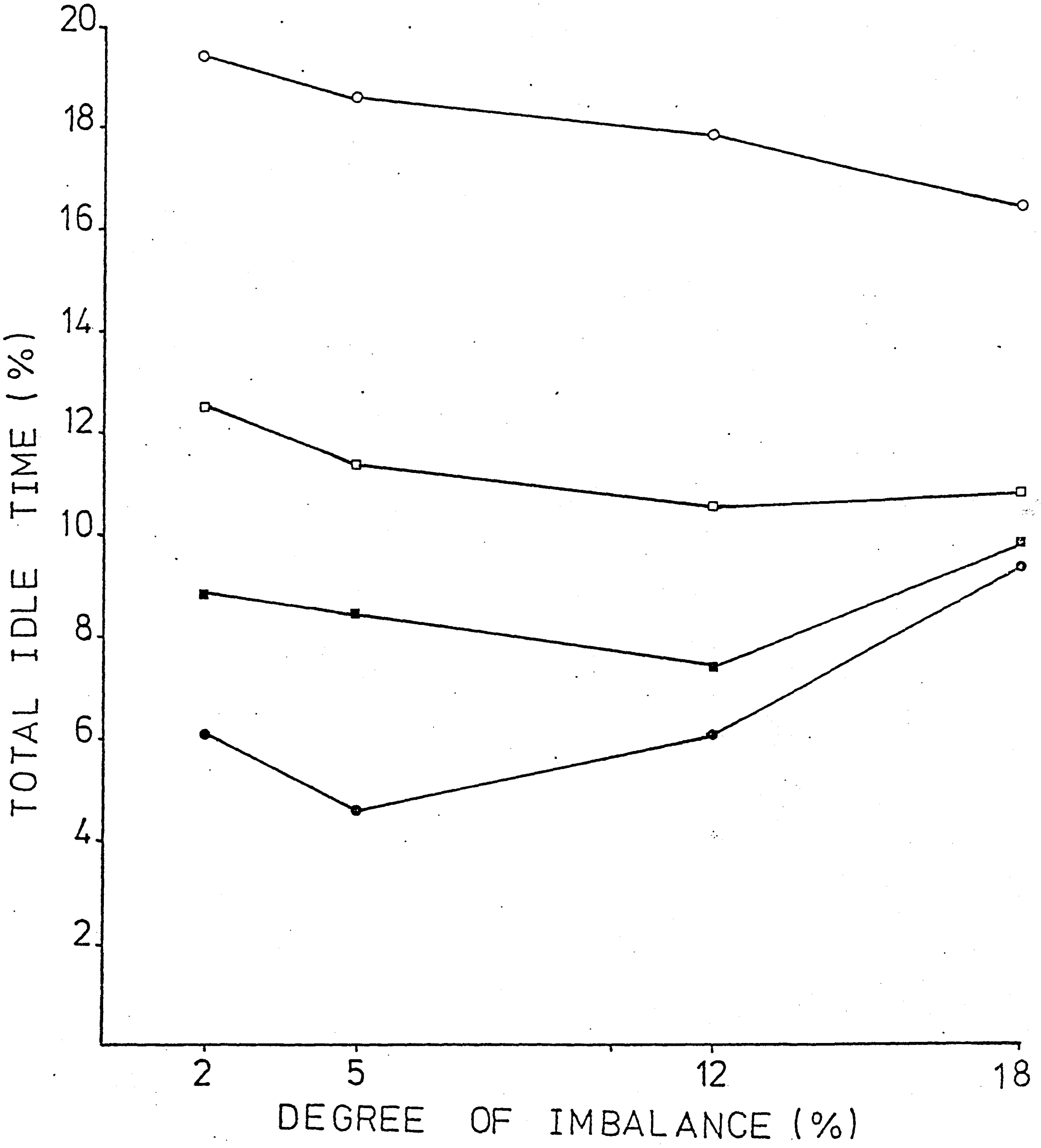


FIGURE A7.30

% TOTAL IDLE TIME CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (V) + P_8,(N = 10)

% TOTAL IDLE TIME CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE

PATTERN (V) + B ,(N = 5)

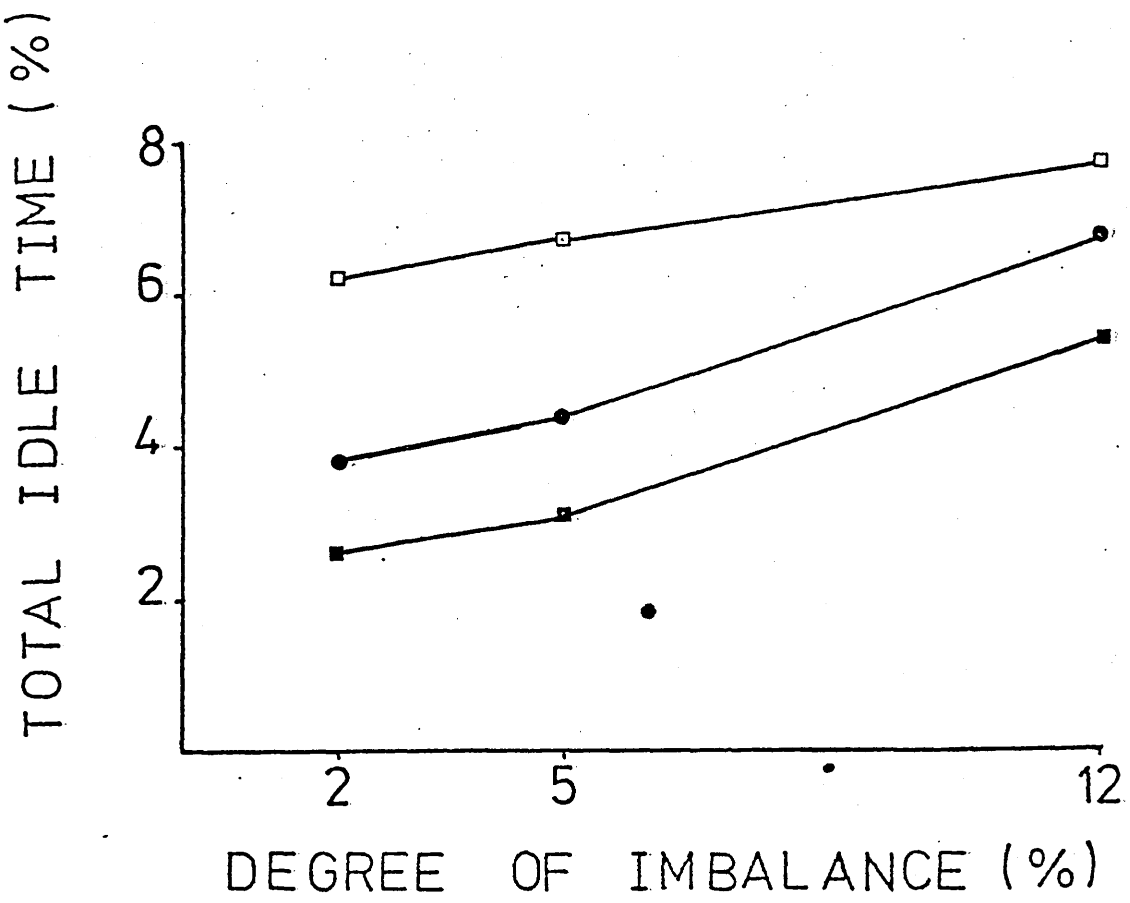


FIGURE A7.32

% TOTAL IDLE TIME CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (V) +B ,(N = 8)

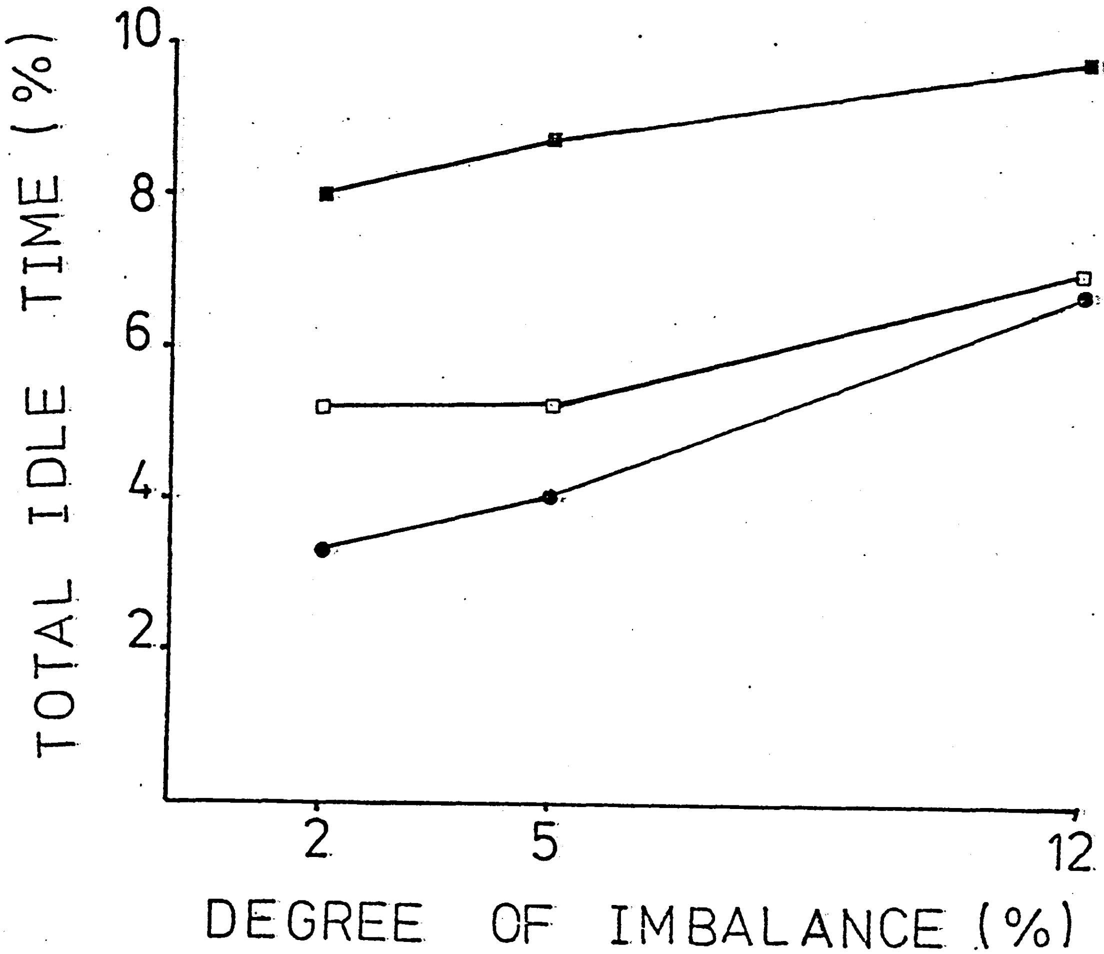


FIGURE A7.33

% TOTAL IDLE TIME CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE

PATTERN (V) + D_1, (N = 5)

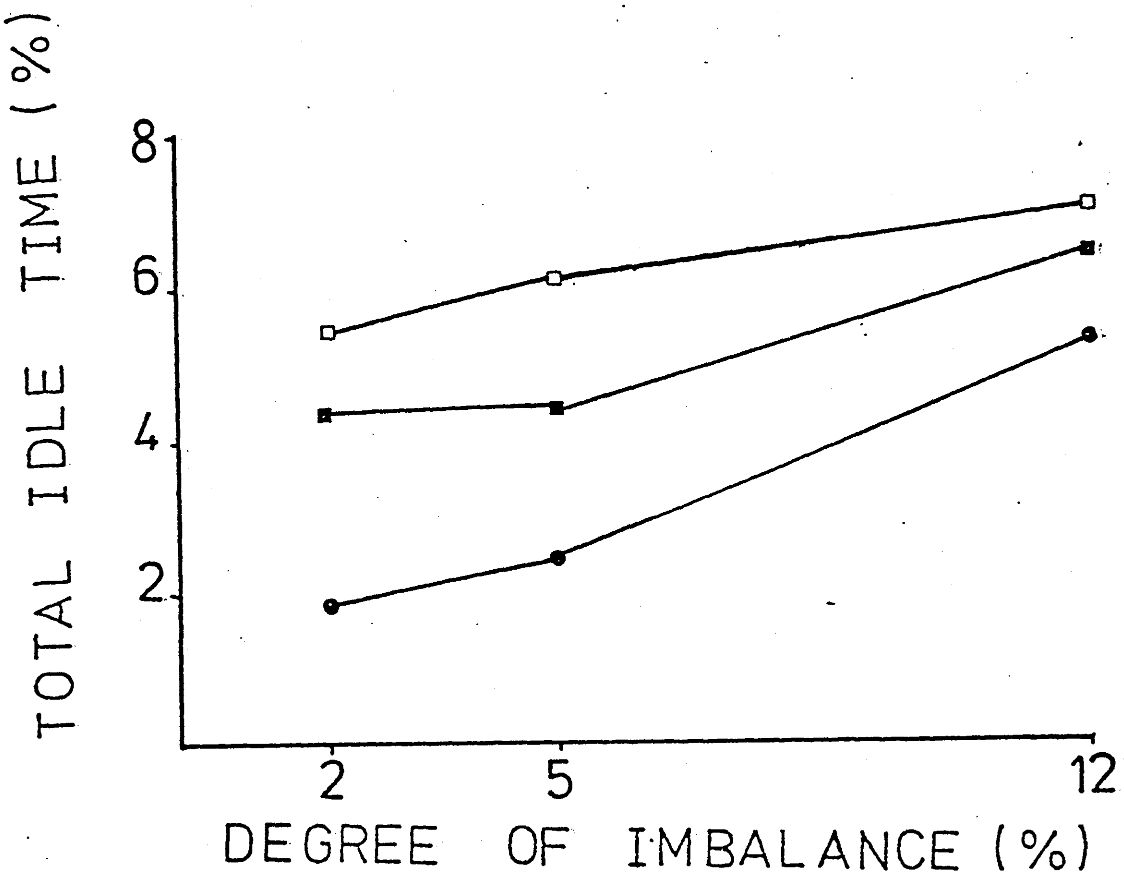
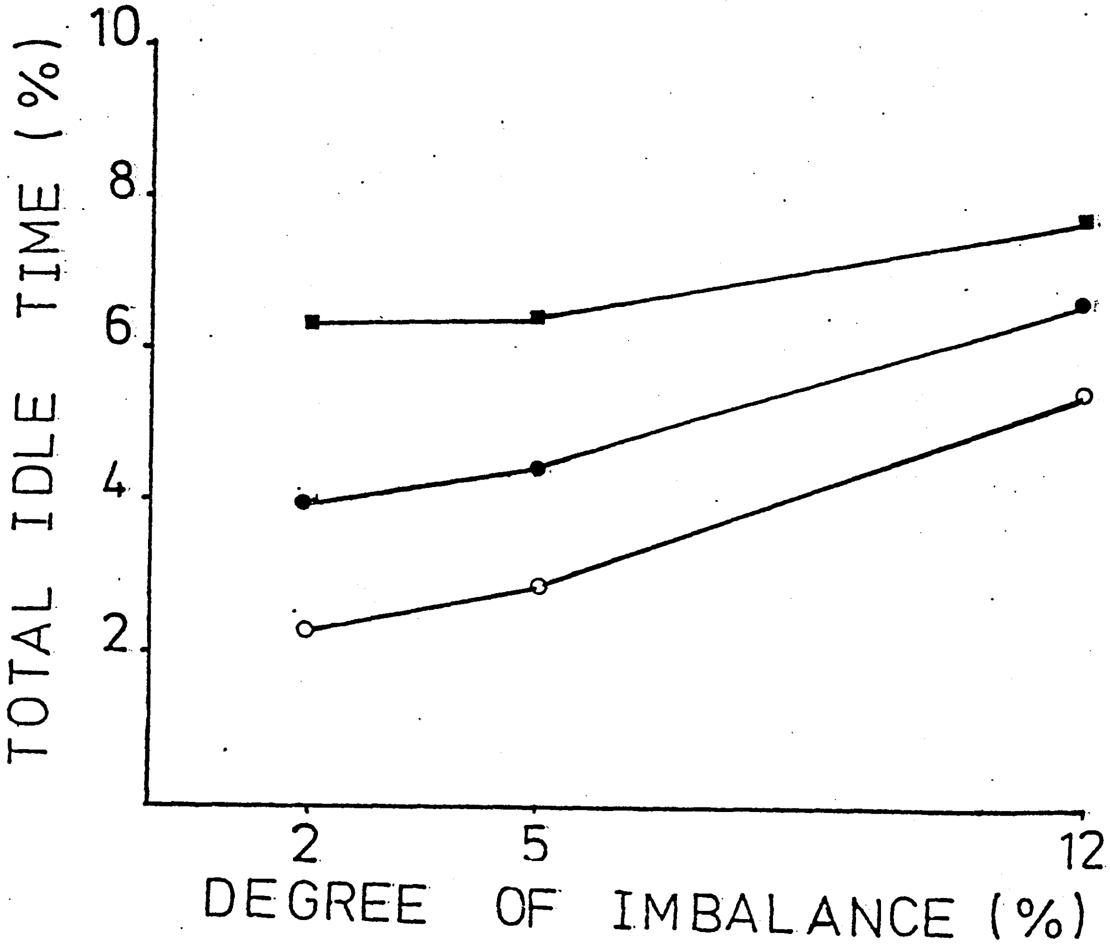


FIGURE A7.34

% TOTAL IDLE TIME CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE -

PATTERN (V) + D_1, (N = 8)



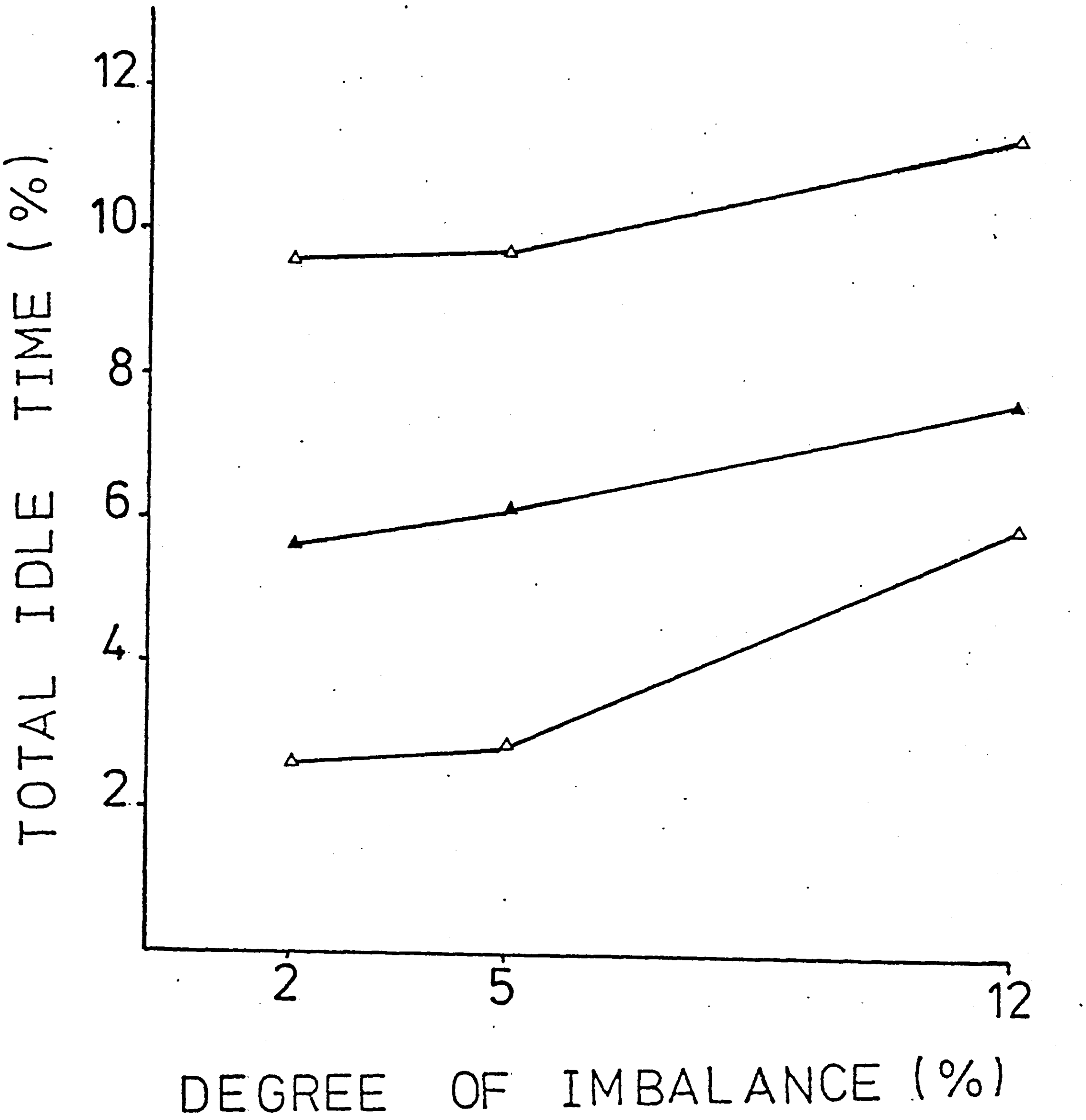


FIGURE A7.35

% TOTAL IDLE TIME CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (V) + D_1,(N = 10)

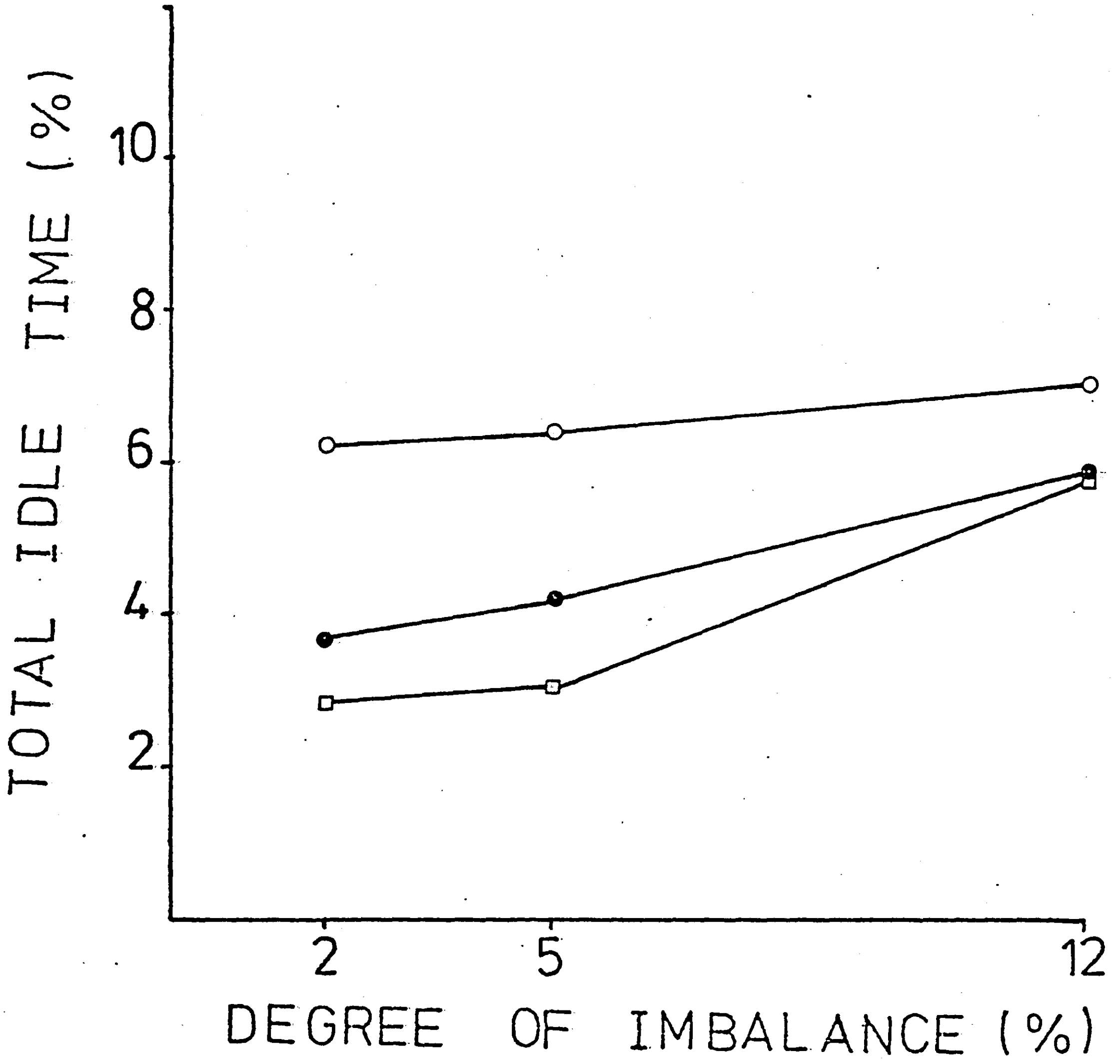


FIGURE A7.36

% TOTAL IDLE TIME CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE -

PATTERN (V) + D_2,(N = 5)

FIGURE A7.37

% TOTAL IDLE TIME CURVES FOR MEANS & BUFFER CAPACITIES

IMBALANCE - PATTERN (V) + D_2, (N = 8)

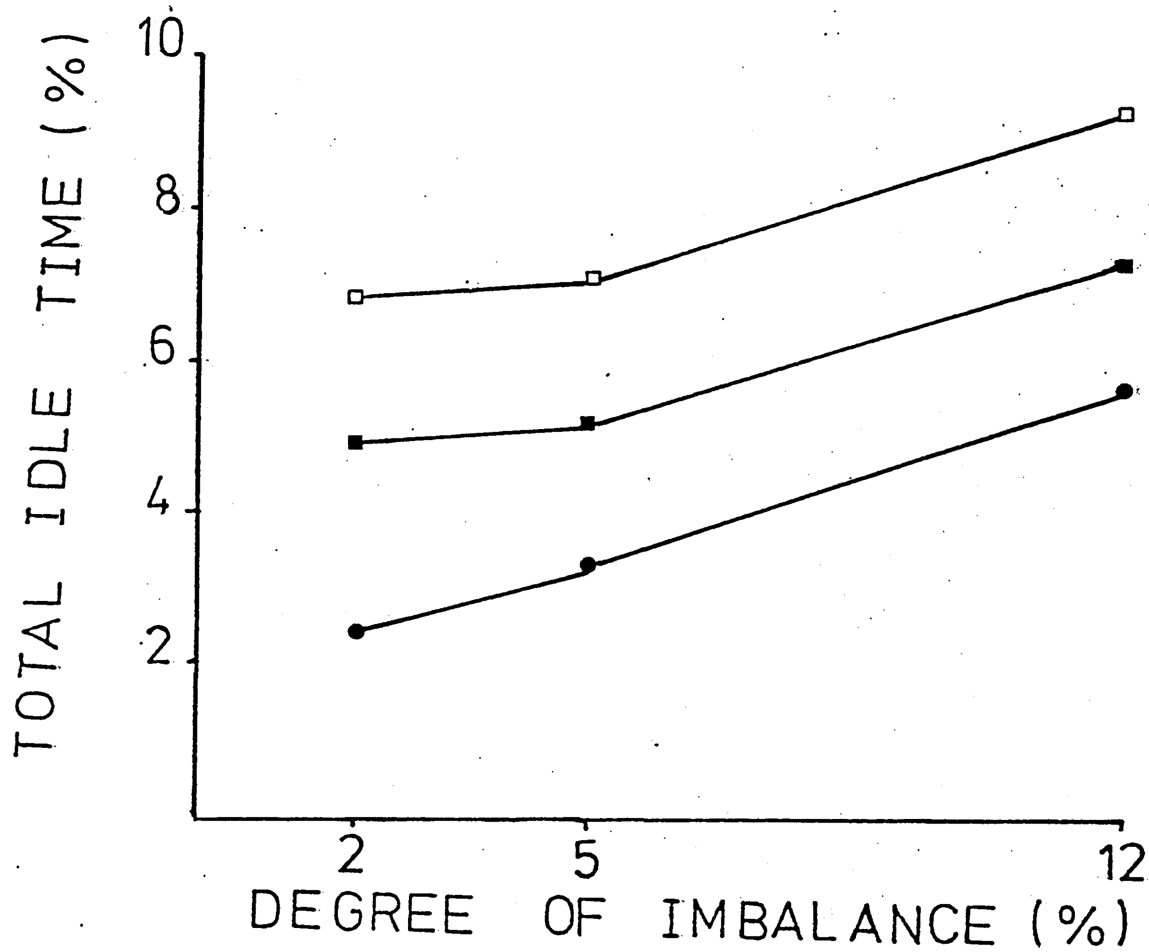


FIGURE A7.38

% TOTAL IDLE TIME CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (V) + D_3, (N = 5)

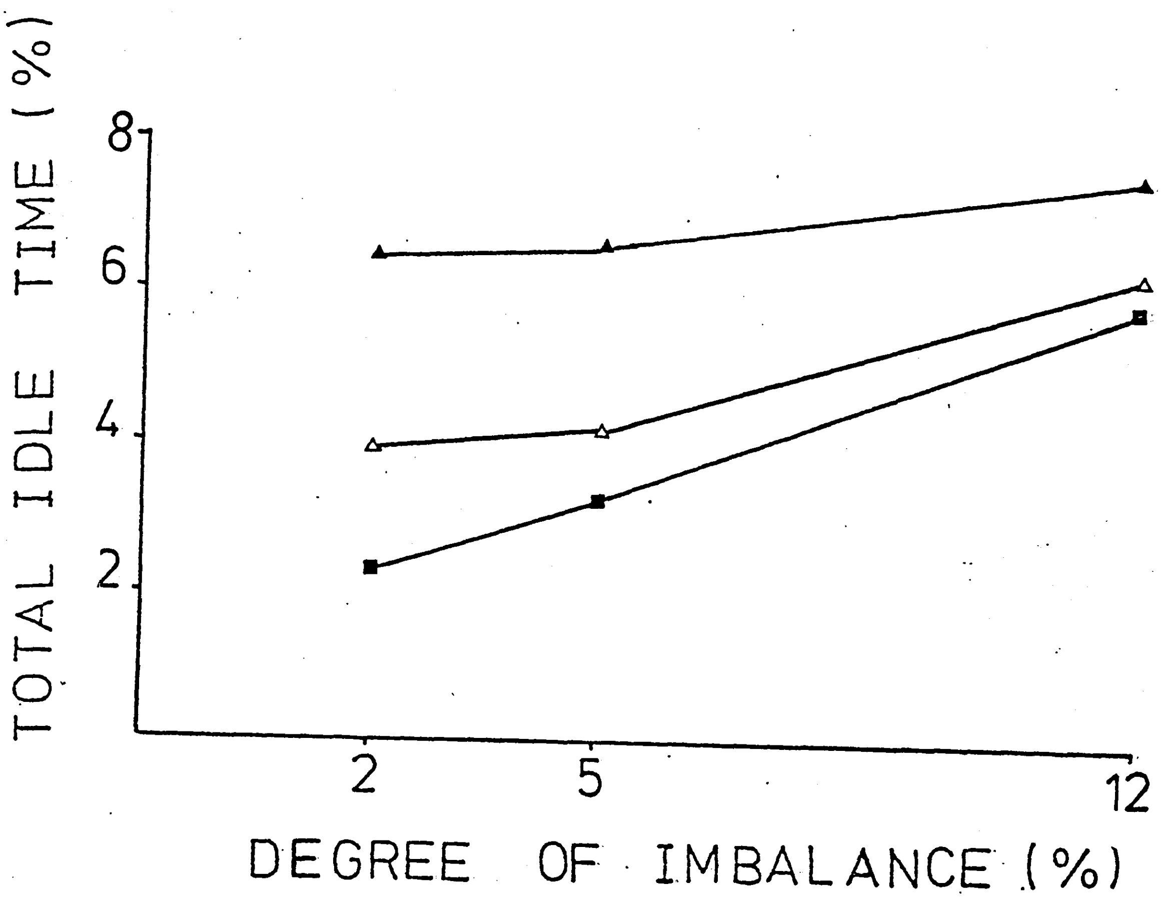
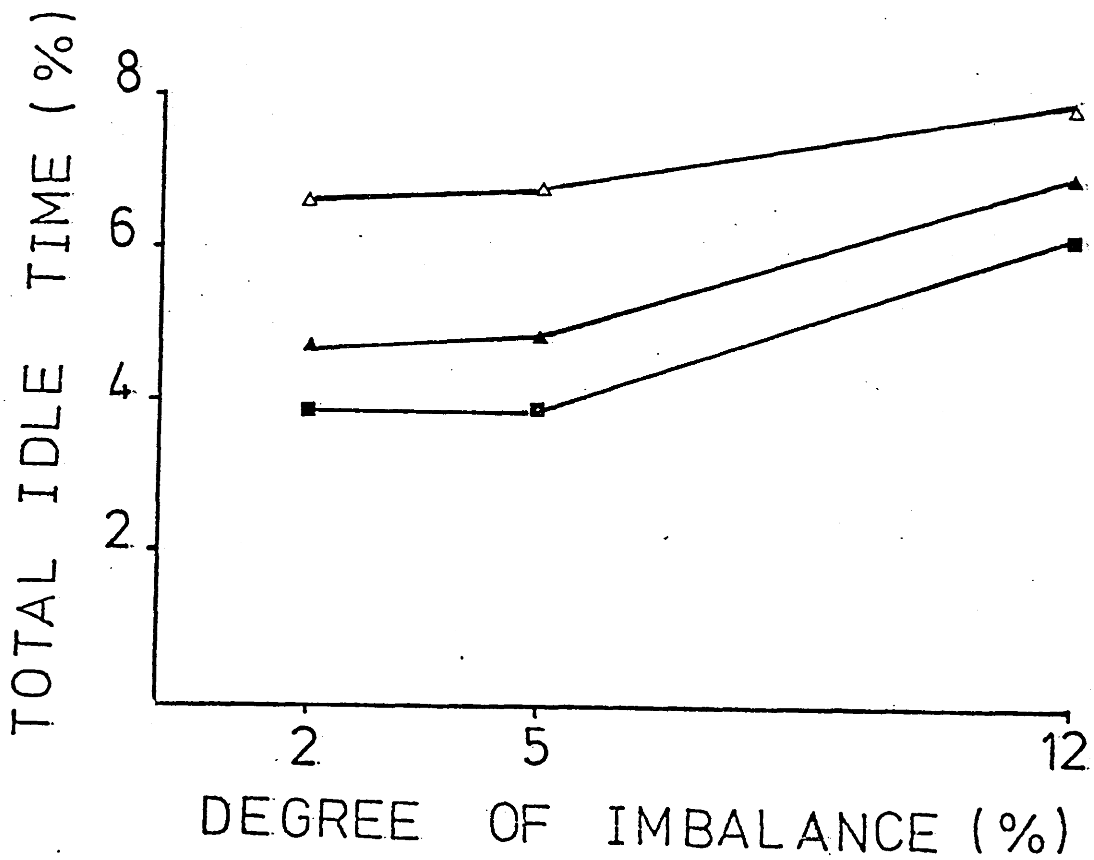


FIGURE A7.39

% TOTAL IDLE TIME CURVES FOR MEANS AND BUFFER CAPACITIES IMBALANCE - PATTERN (V) + D_3, (N = 8)



% TOTAL IDLE TIME CURVES FOR COVARS AND BUFFER CAPACITIES

IMBALANCE - PATTERN P_4 + C AND OTHER PATTERNS, (N = 5)

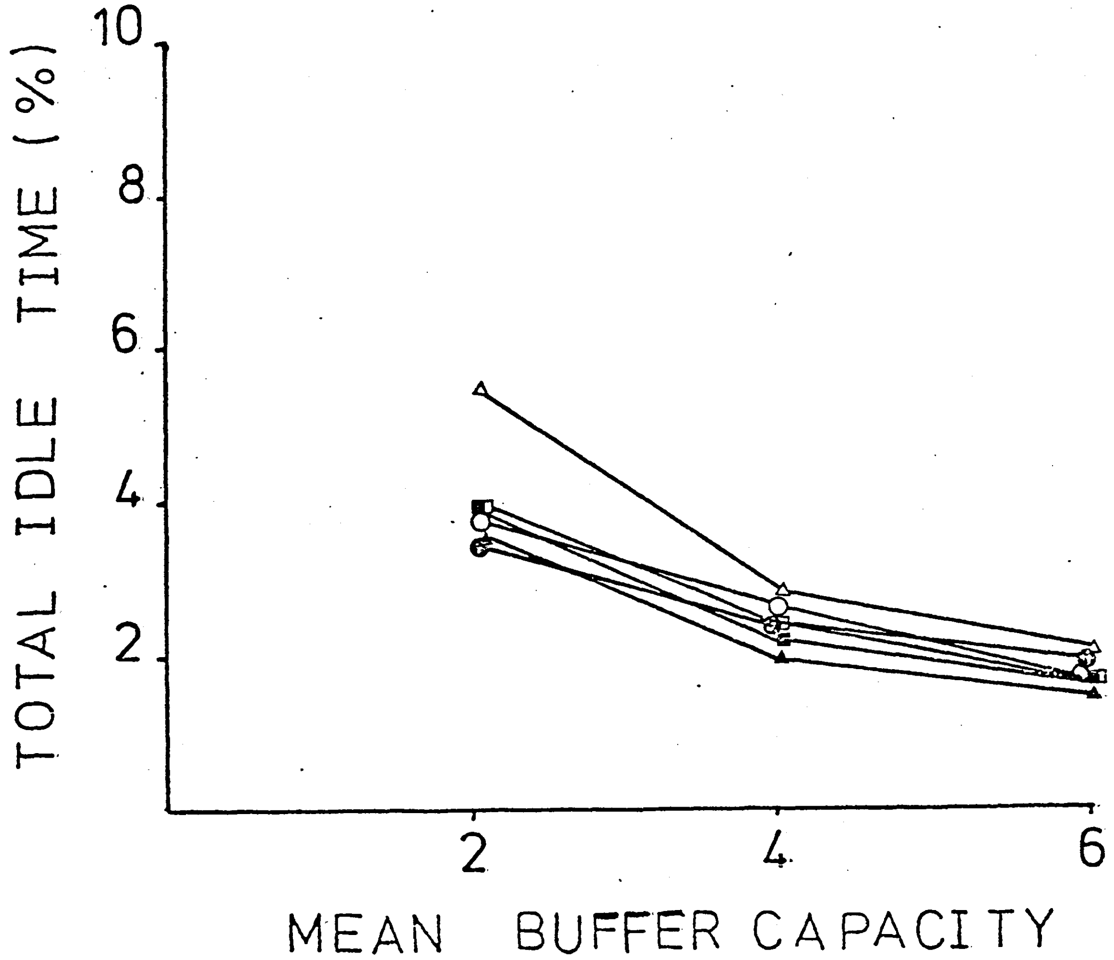


FIGURE A7.41

% TOTAL IDLE TIME CURVES FOR COVARS & BUFFER CAPACITIES

IMBALANCE - PATTERN P_4 + C AND OTHER PATTERNS, (N = 8)

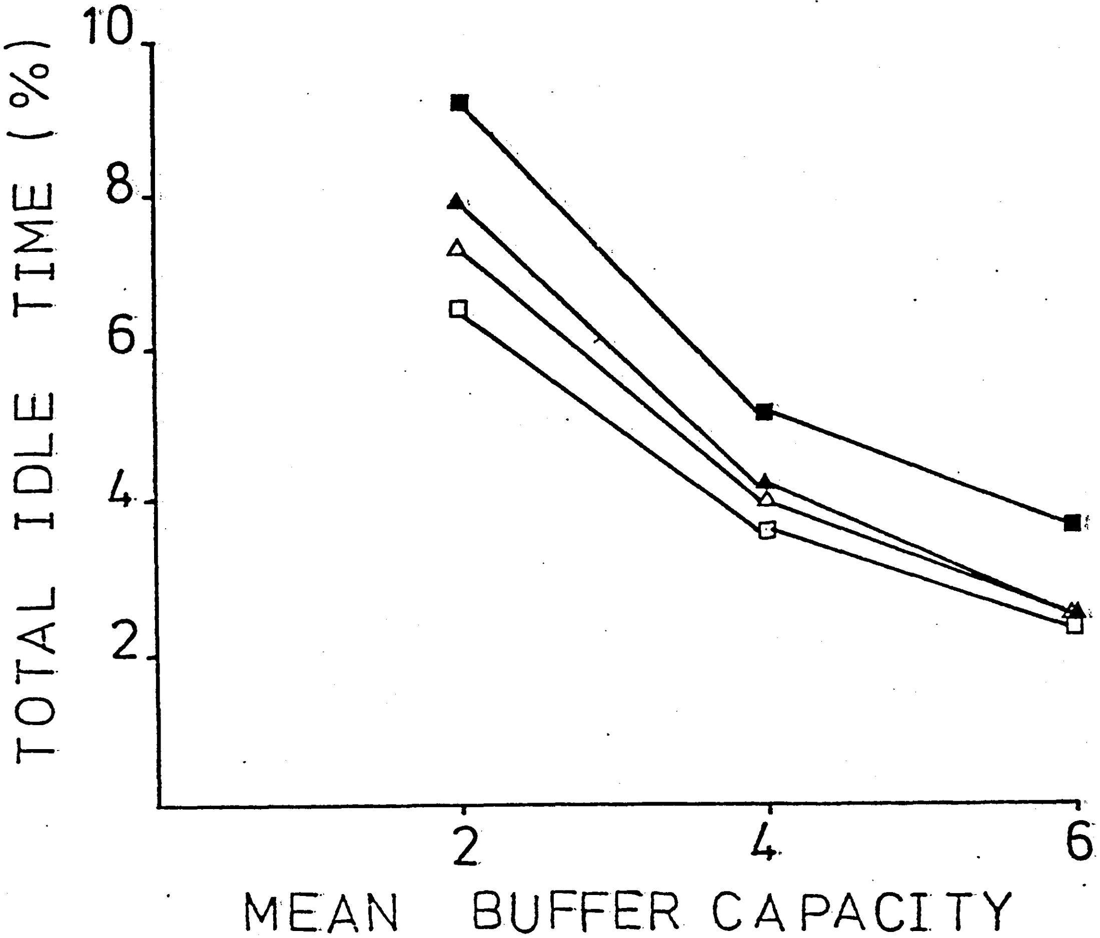
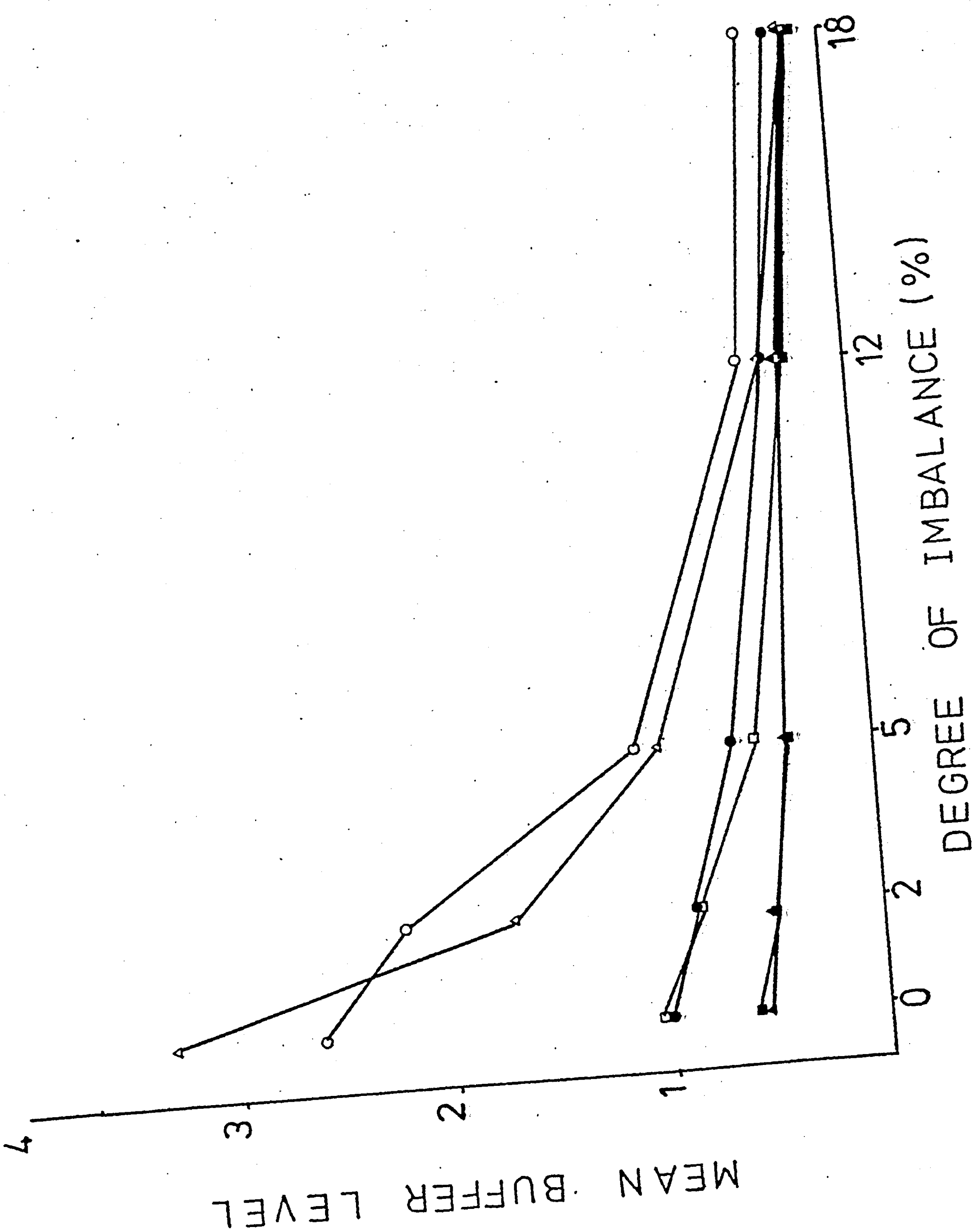


FIGURE A7.42

MEAN BUFFER LEVEL CURVES FOR MEANS IMBALANCE -

PATTERN (\),(N = 5,8)



MEAN BUFFER LEVEL CURVES FOR MEANS IMBALANCE

PATTERN (Λ), (N = 5,8)

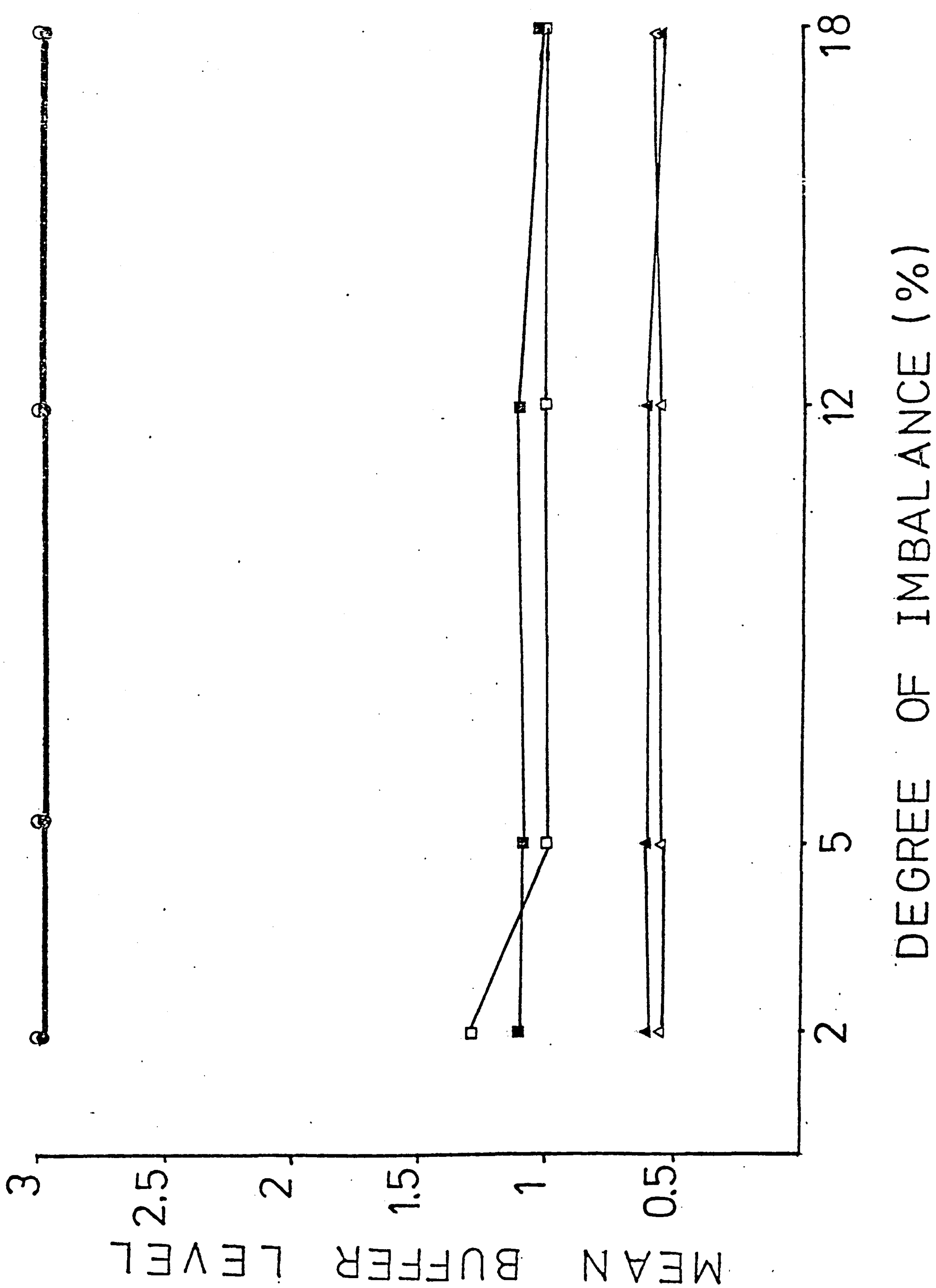
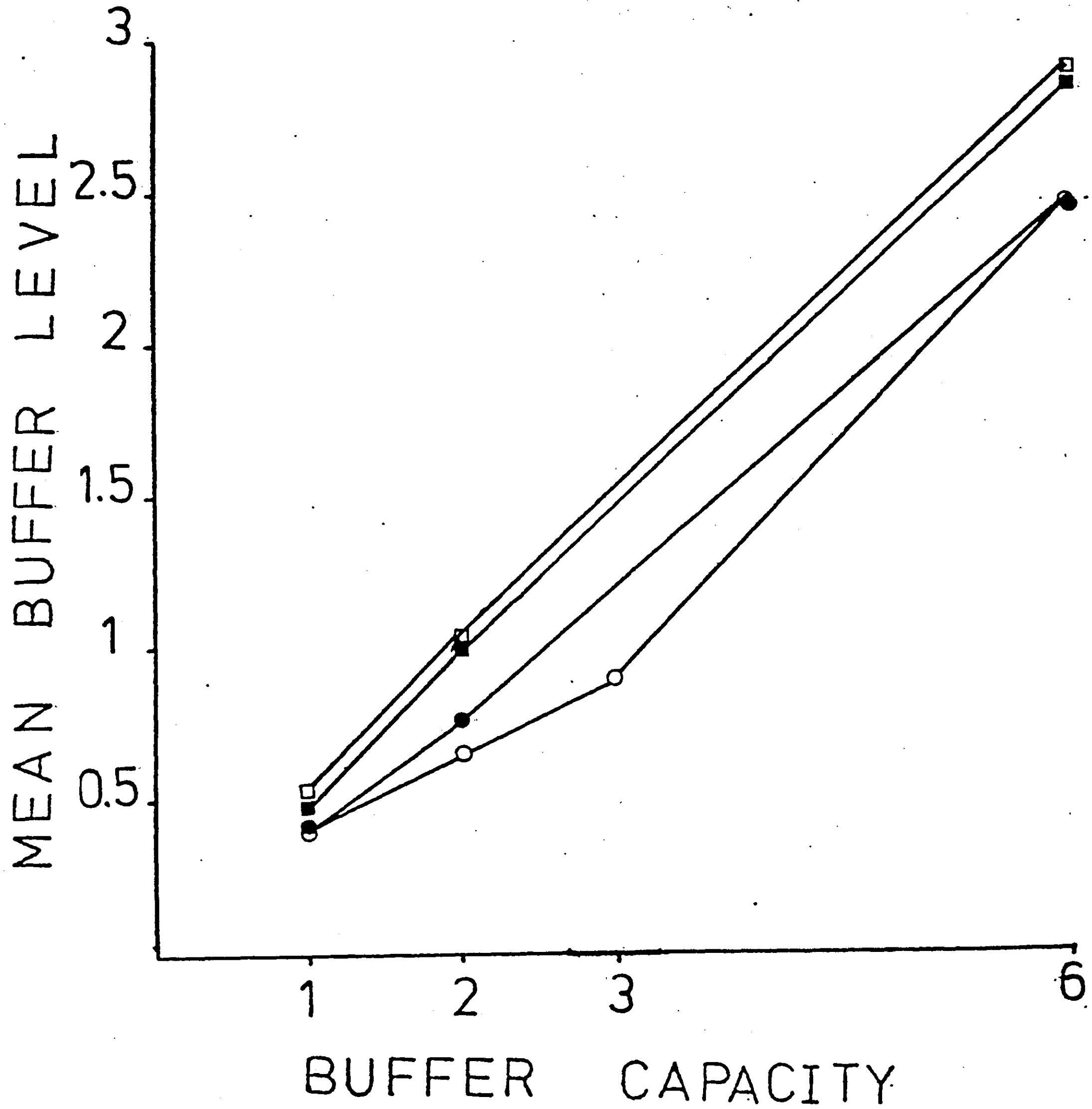
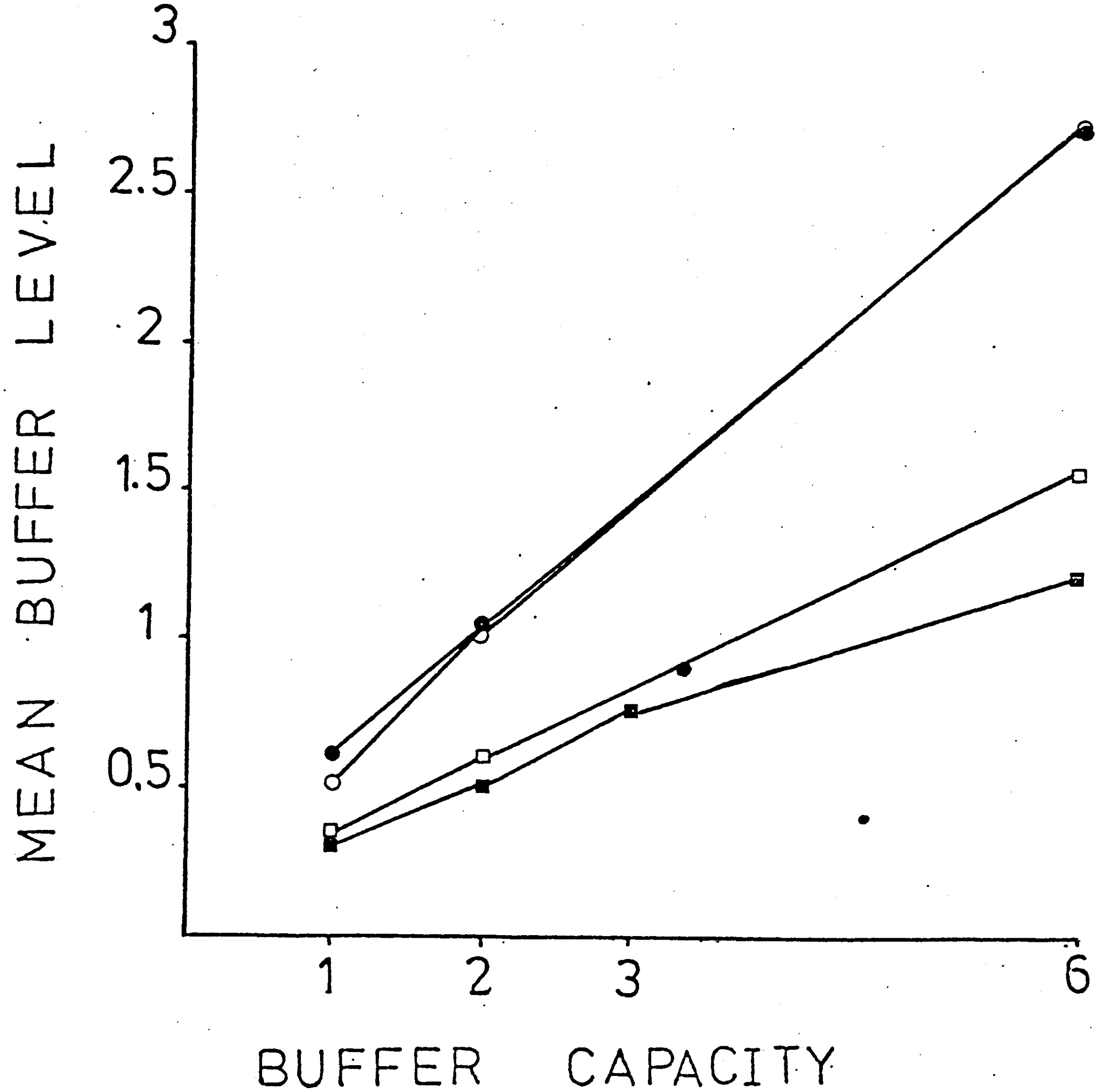


FIGURE A7.44

MEAN BUFFER LEVEL CURVES FOR COVARS IMBALANCE - PATTERN P_7 AND OTHER PATTERNS, (N = 5)



MEAN BUFFER LEVEL CURVES FOR COVARS IMBALANCE - PATTERN P_7 AND OTHER PATTERNS, (N = 8)



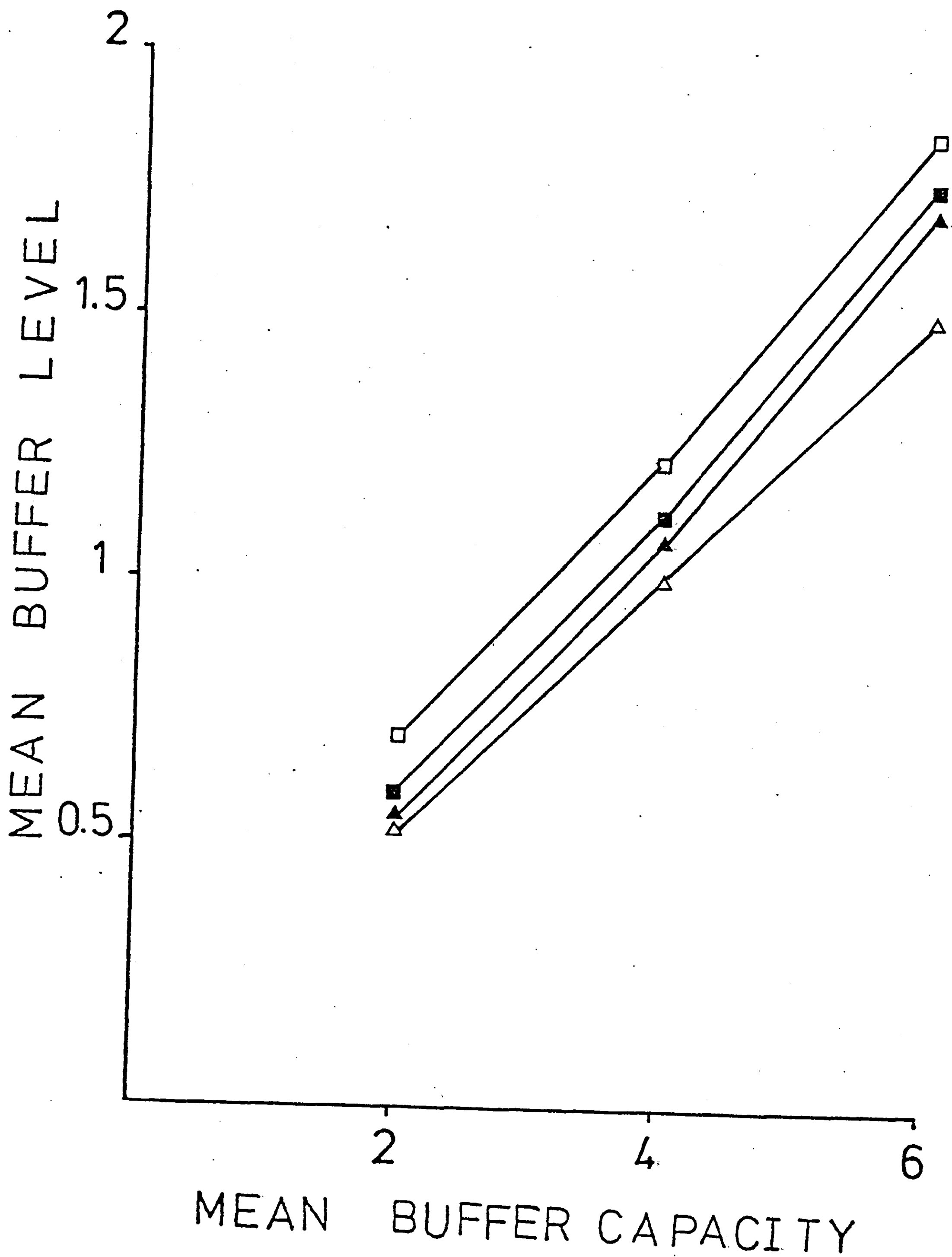


FIGURE A7.46

MEAN BUFFER LEVEL CURVES FOR BUFFER CAPACITIES IMBALANCE - PATTERN A_1 AND OTHER PATTERNS, (N = 5)

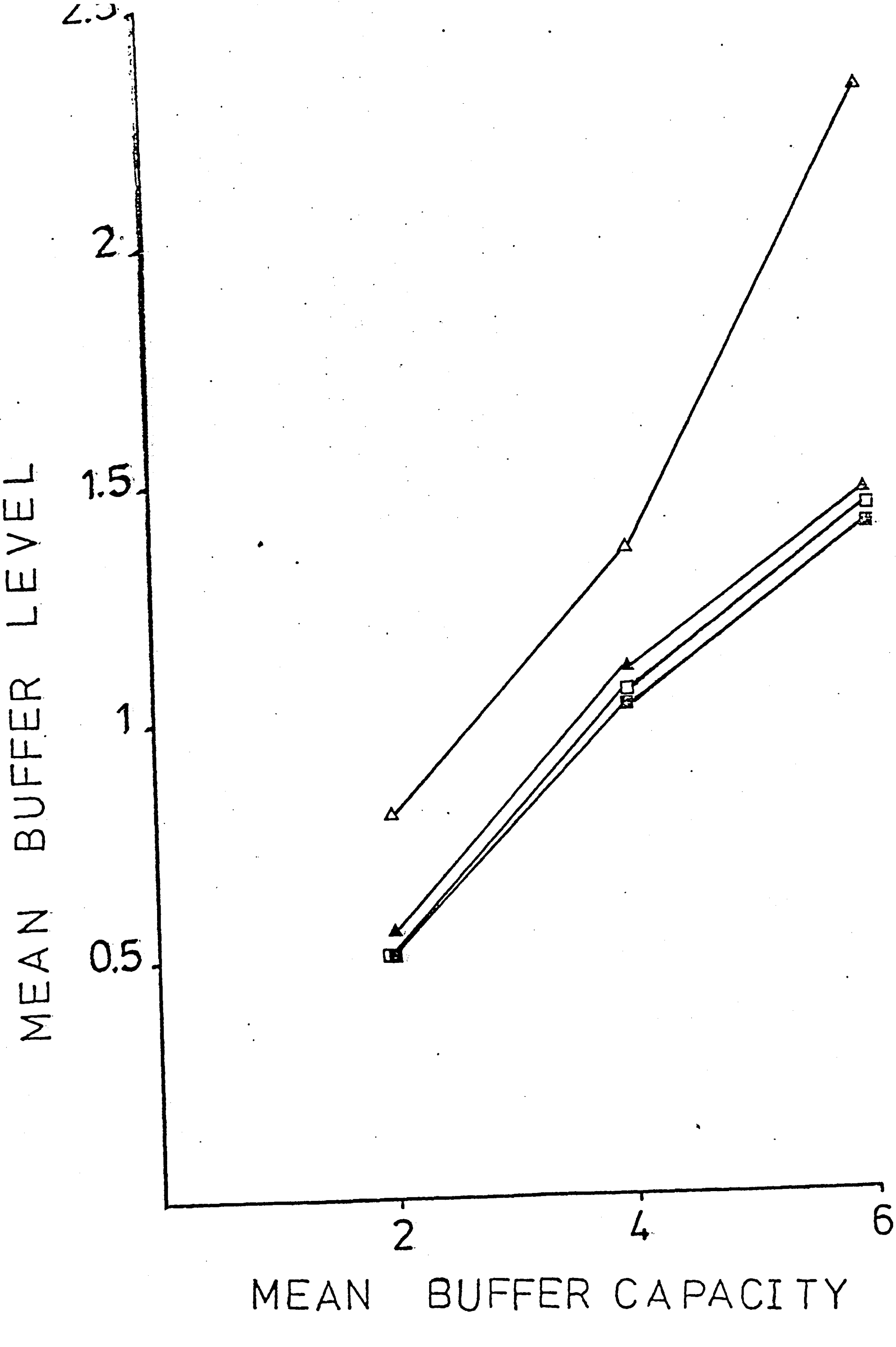


FIGURE A7.47

MEAN BUFFER LEVEL CURVES FOR BUFFER CAPACITIES IMBALANCE PATTERN A_2 AND OTHER PATTERNS, (N = 8)

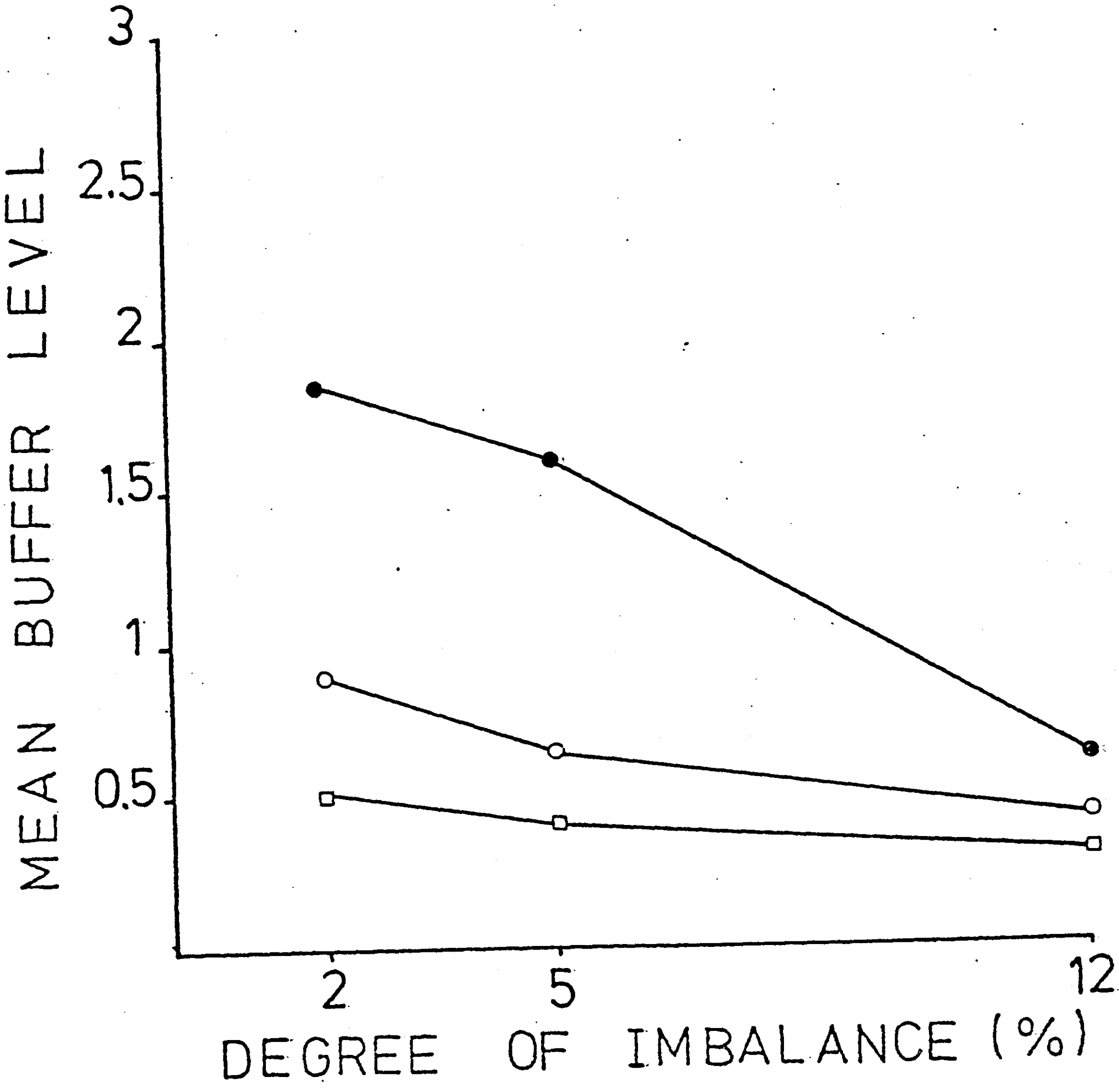


FIGURE A7.48

MEAN BUFFER LEVEL CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (\) + P_2, (N = 5)

FIGURE A7.49

MEAN BUFFER LEVEL CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (\) + P_2,(N = 8)

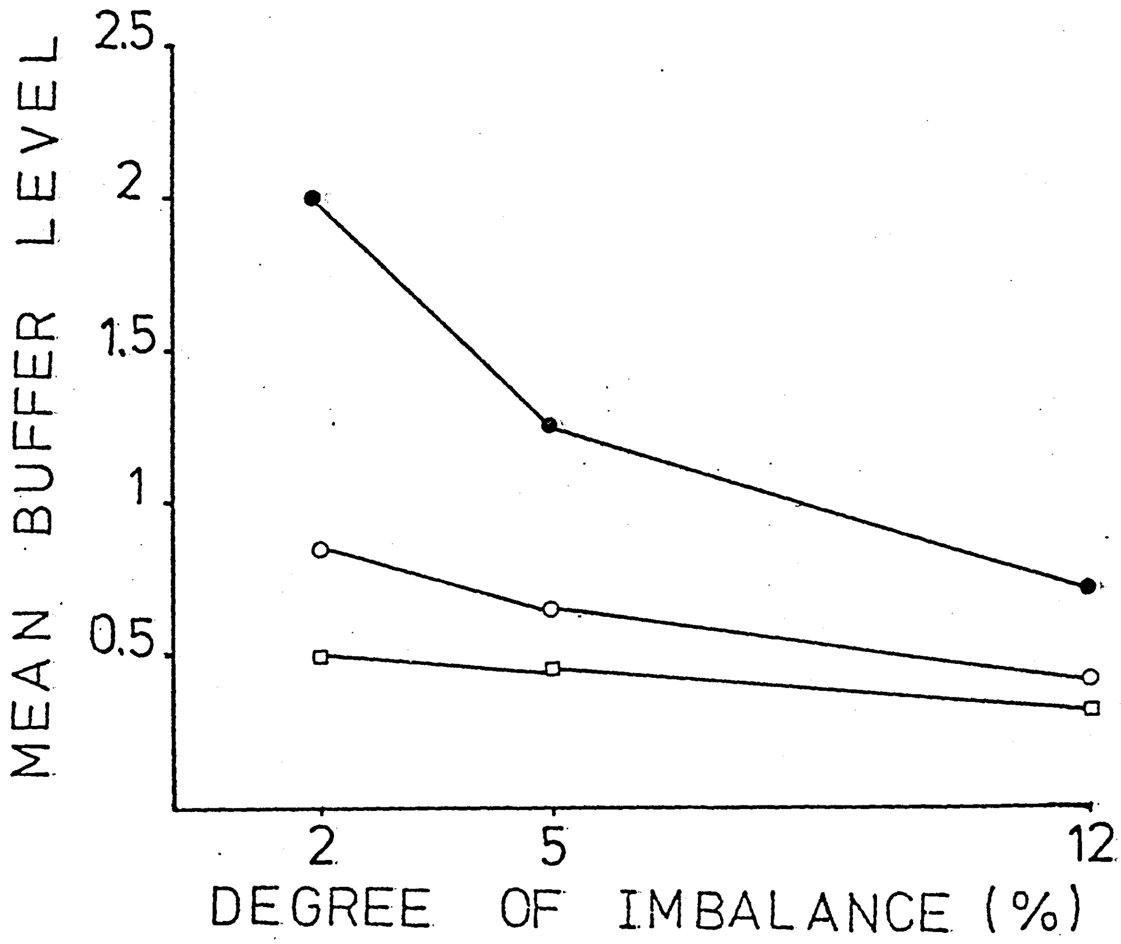
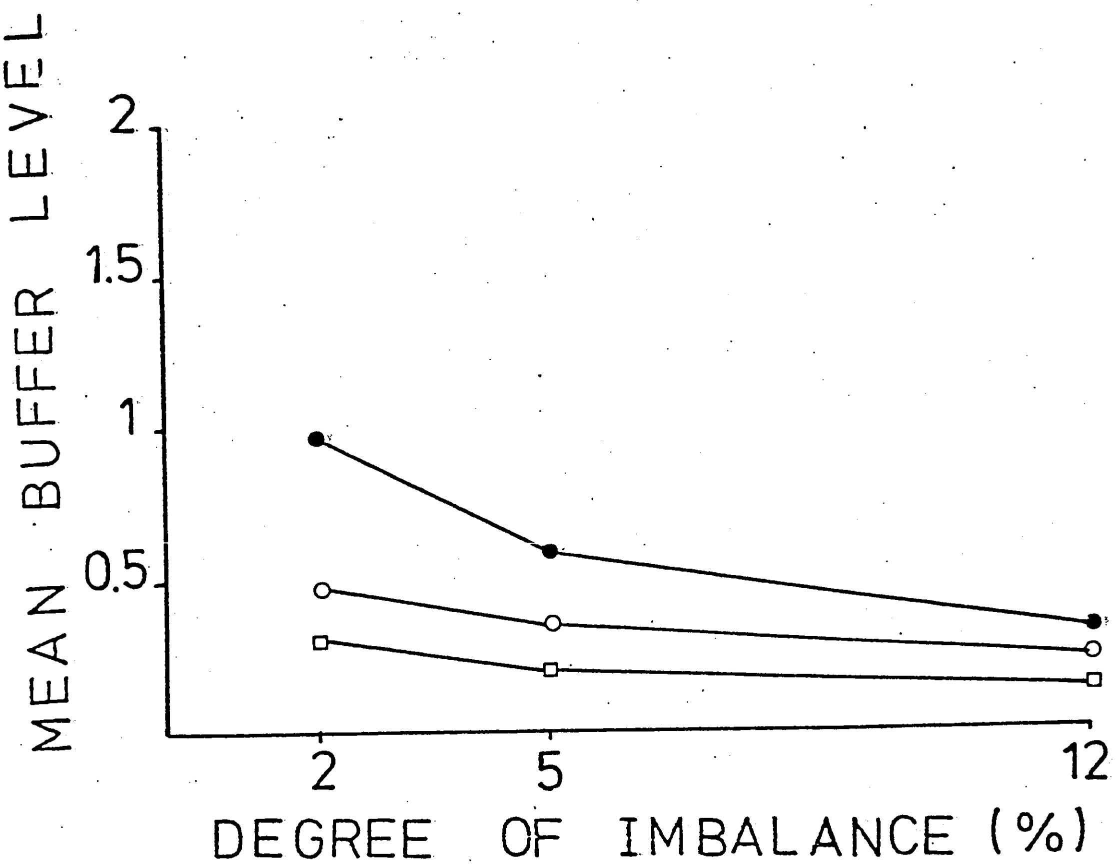


FIGURE A7.50

MEAN BUFFER LEVEL CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (\) + P_4, (N = 5)



MEAN BUFFER LEVEL CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (\) + P_4, (N = 8)

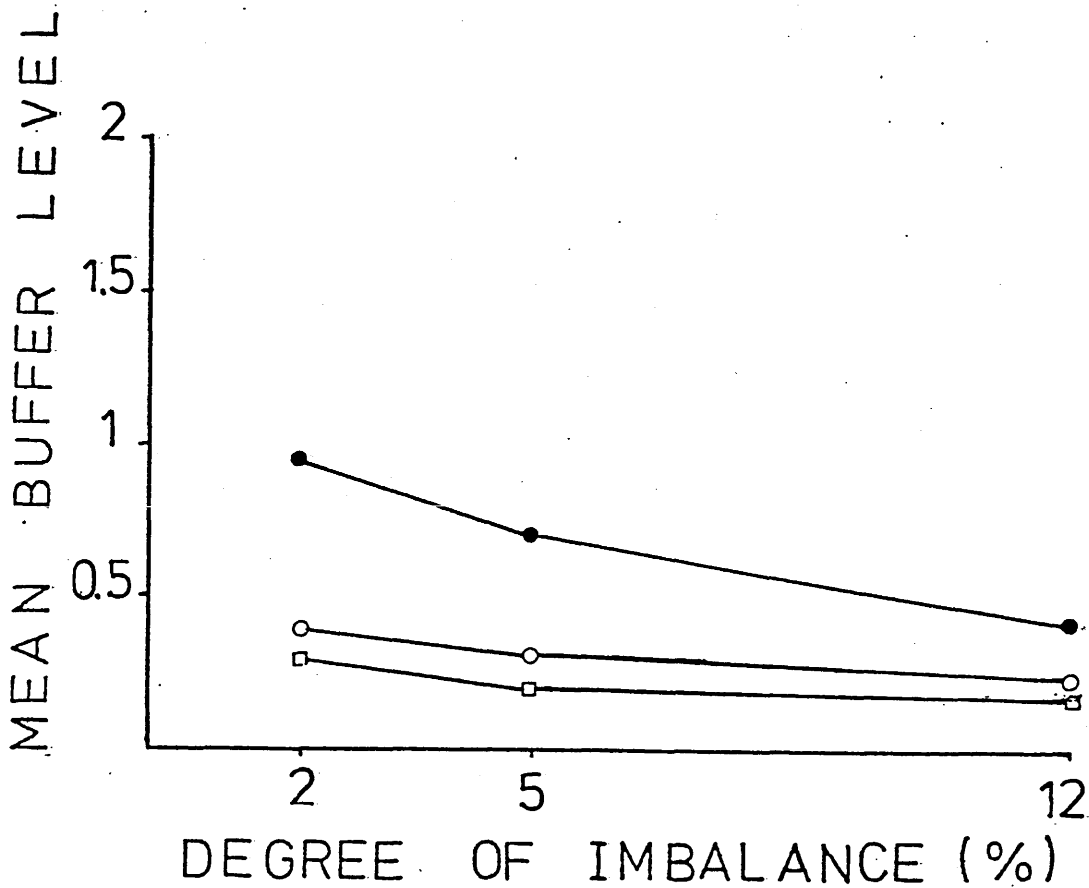


FIGURE A7.52

MEAN BUFFER LEVEL CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (↘) + P_5, (N = 5)

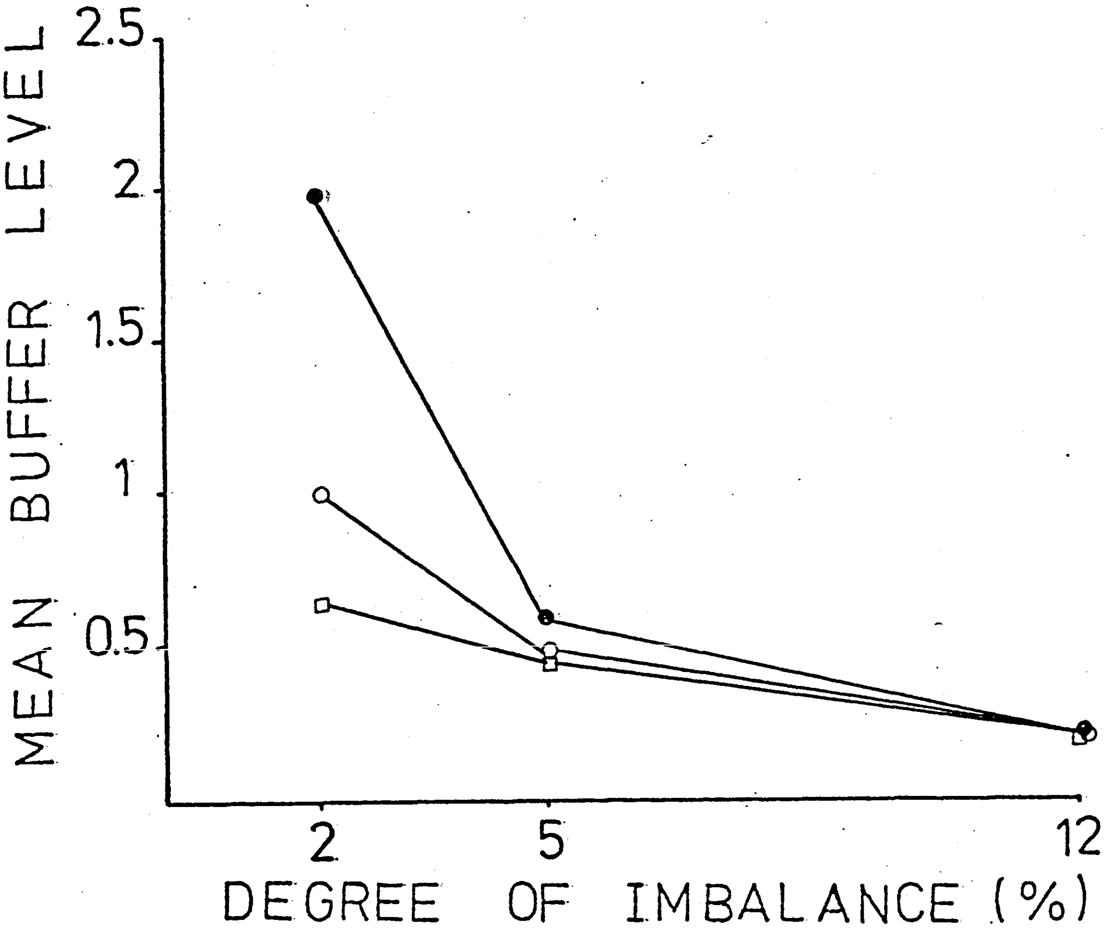
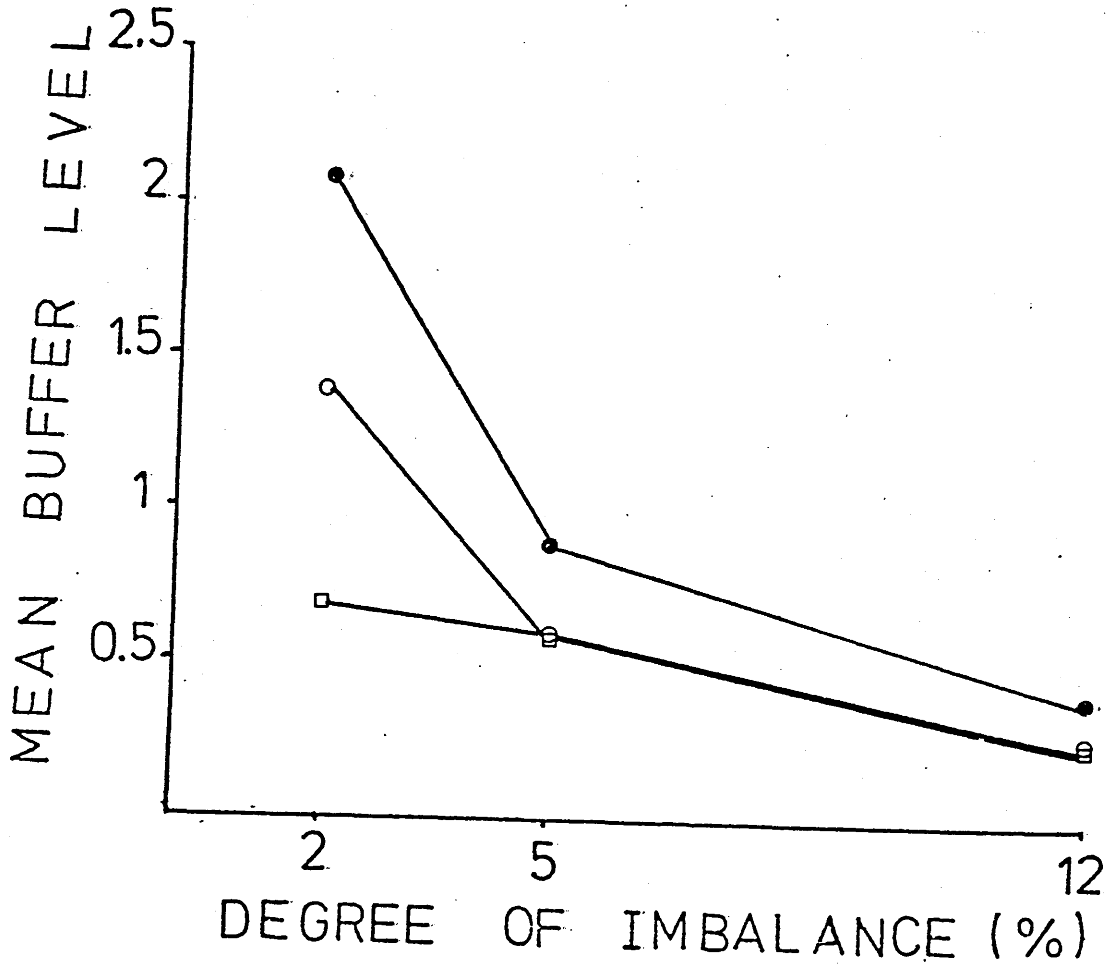
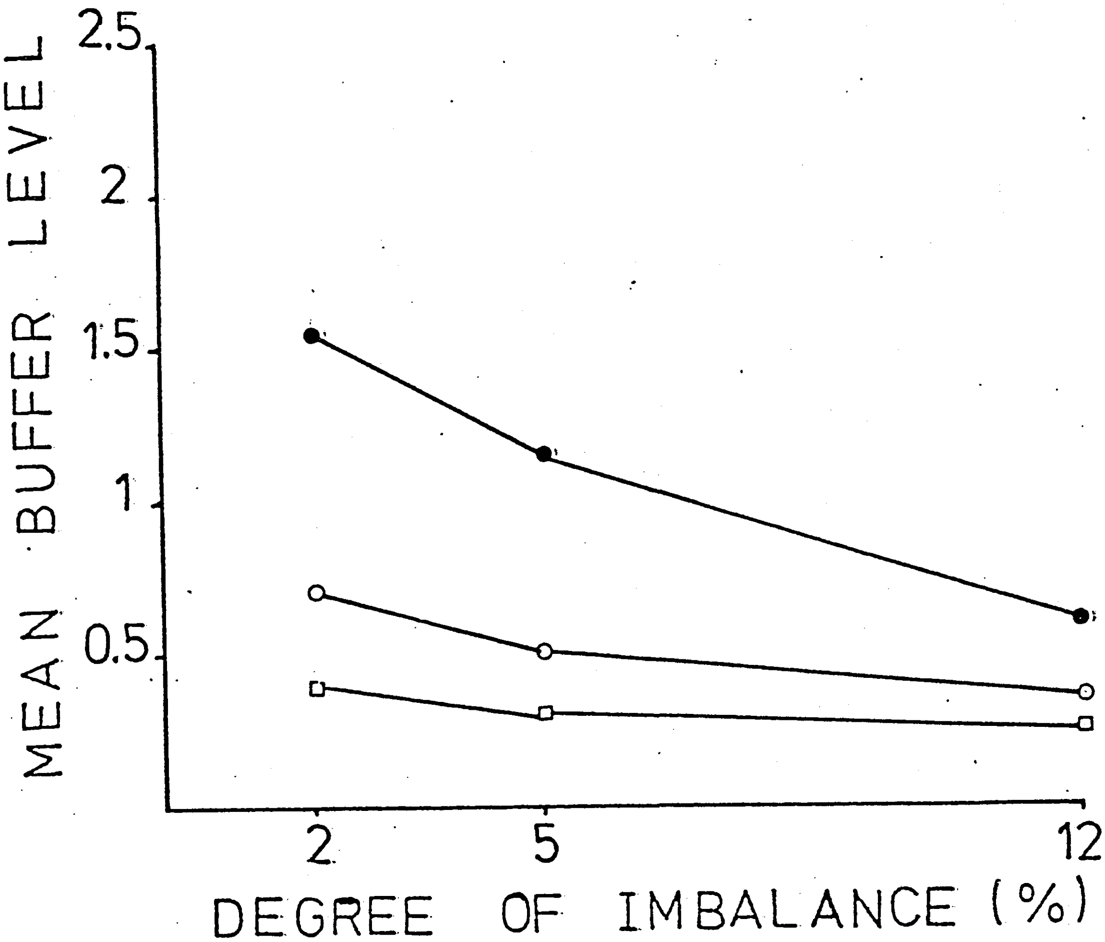


FIGURE A7.53

MEAN BUFFER LEVEL CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (↘) + P_5, (N = 8)



MEAN BUFFER LEVEL CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (↘) + P_6, (N = 5)



MEAN BUFFER LEVEL CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (↘) + P_6,(N = 8)

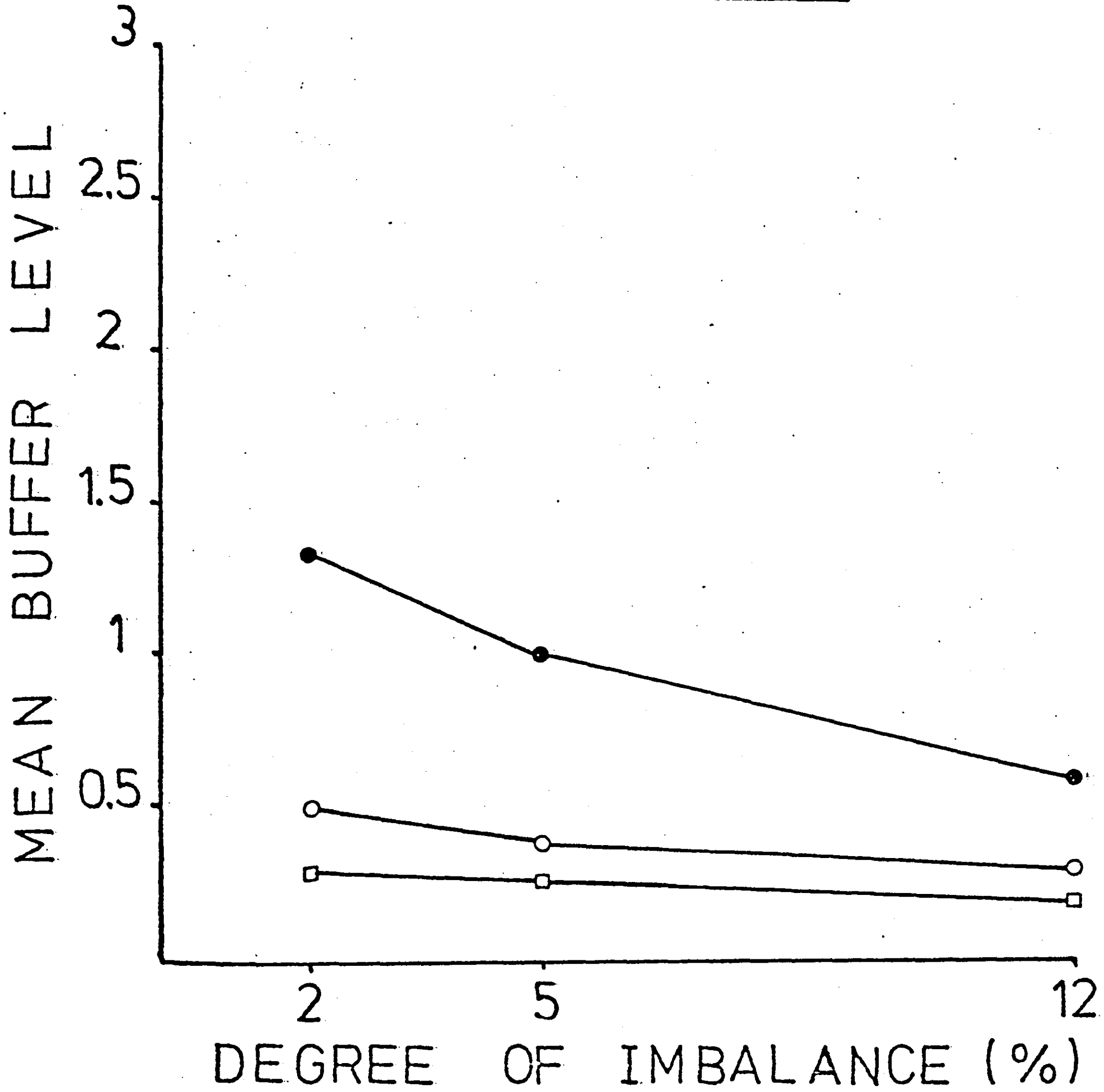


FIGURE A7.56

MEAN BUFFER LEVEL CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (\) + A, (N = 5)

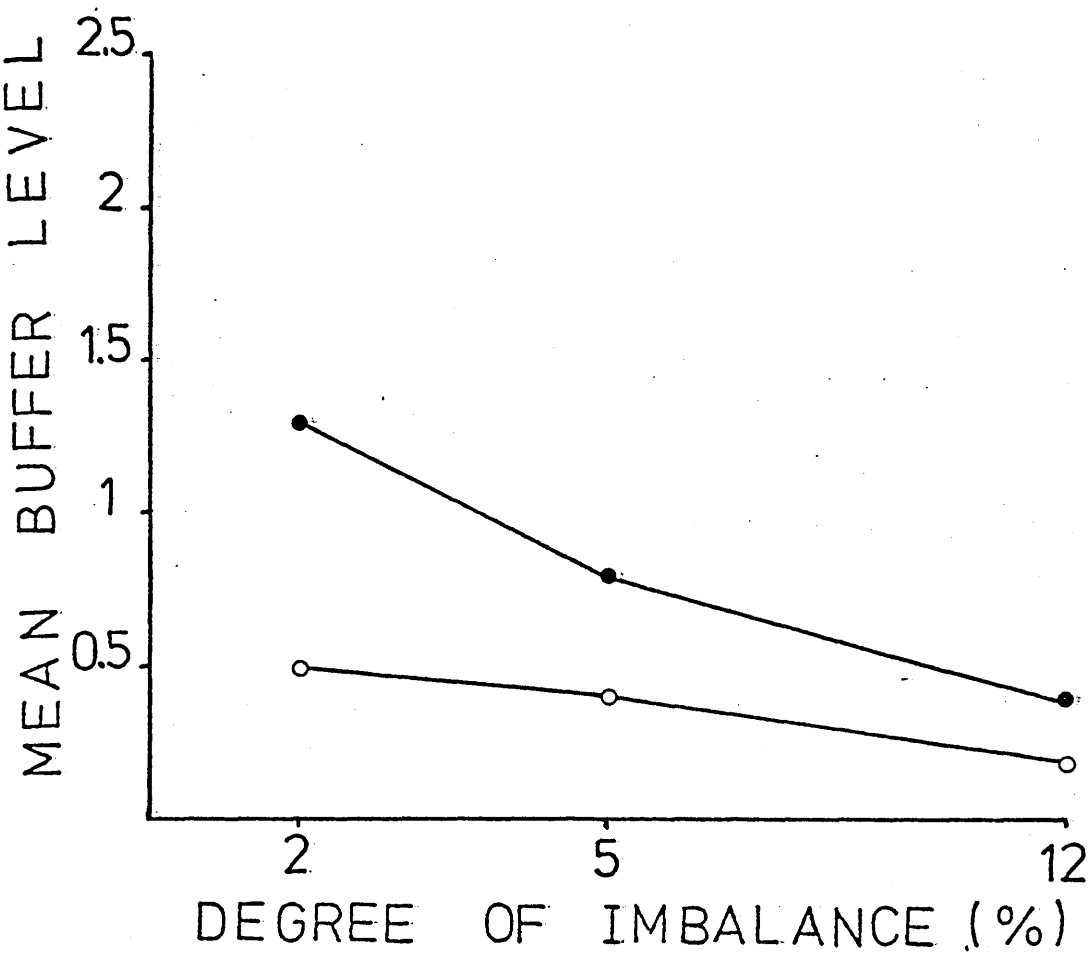


FIGURE A7.57

MEAN BUFFER LEVEL CURVES FOR MEANS & BUFFER CAPACITIES
IMBALANCE - PATTERN (\) +A, (N = 8)

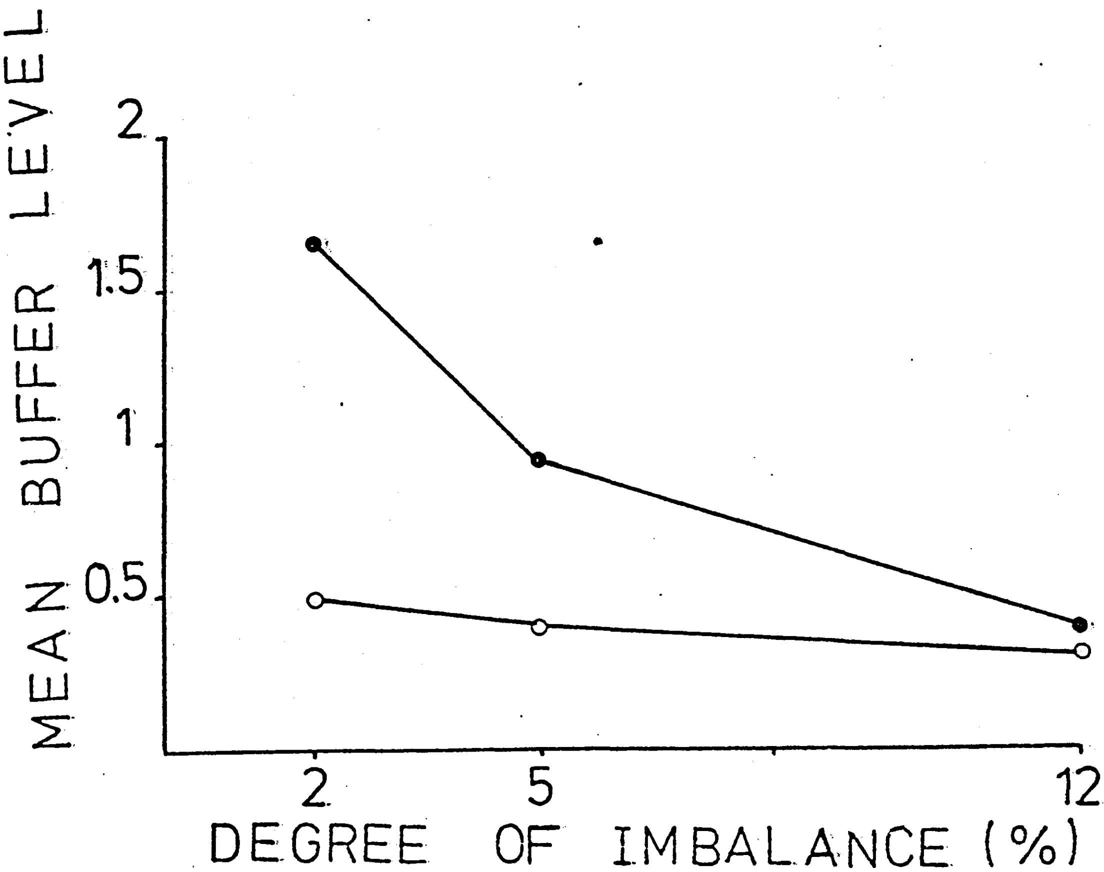
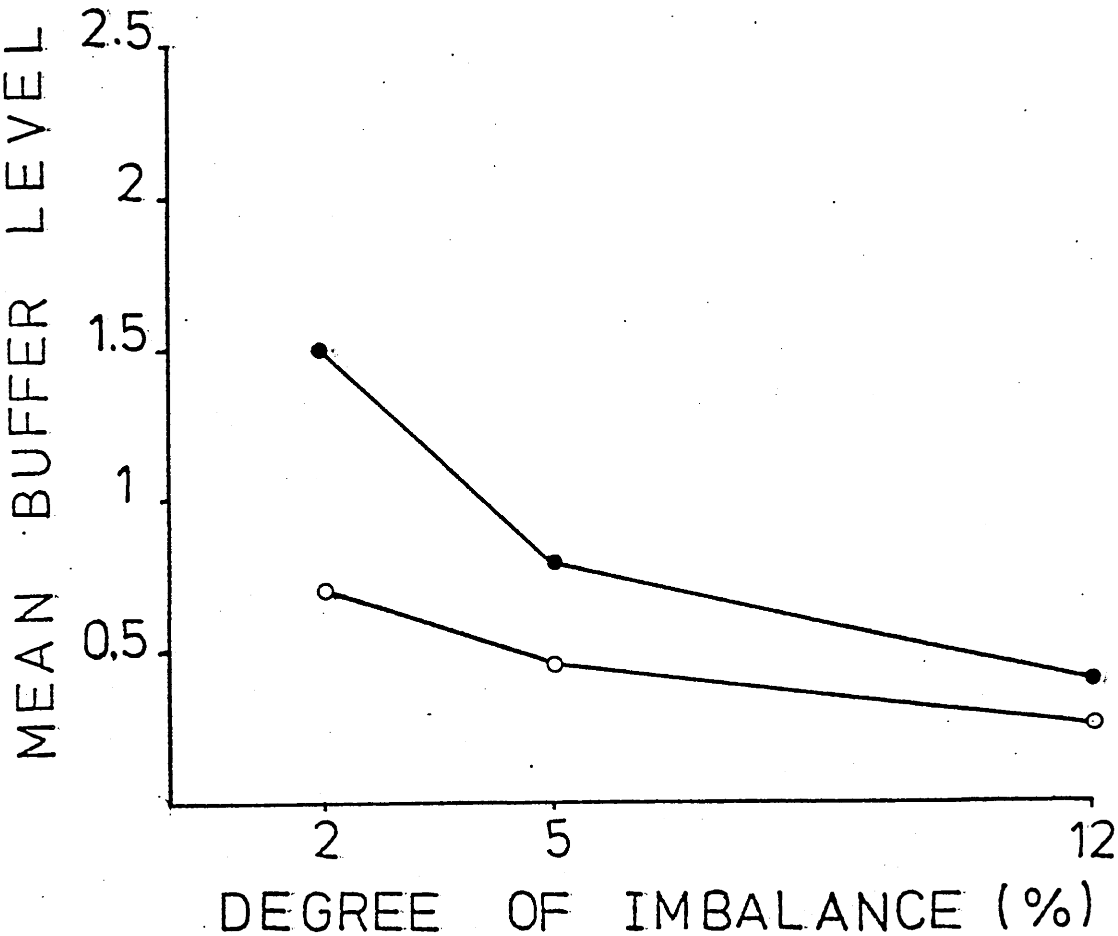


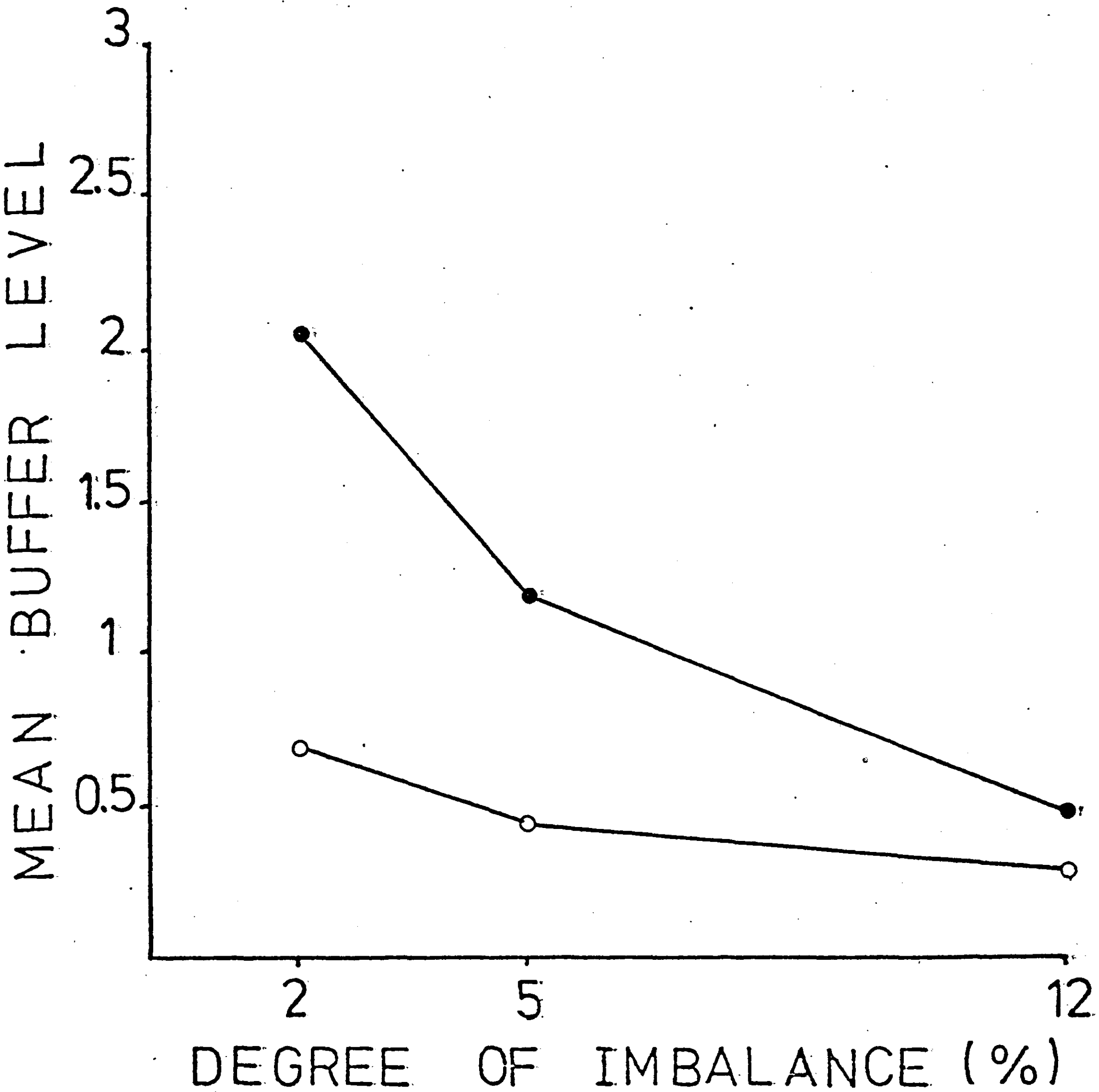
FIGURE A7.58

MEAN BUFFER LEVEL CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (\) + B, (N = 5)



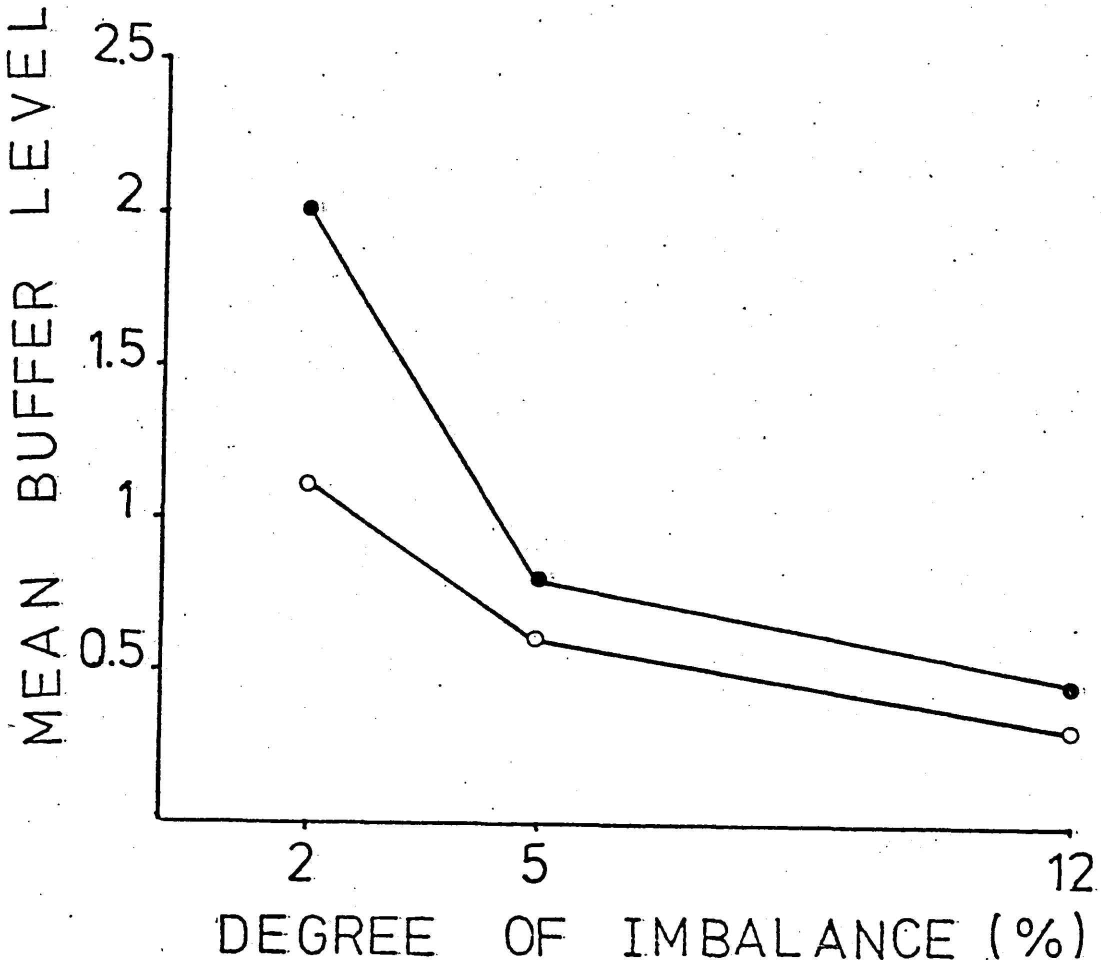
MEAN BUFFER LEVEL CURVES FOR MEANS & BUFFER CAPACITIES

IMBALANCE - PATTERN (\) + B, (N = 8)



MEAN BUFFER LEVEL CURVES FOR MEANS & BUFFER CAPACITIES

IMBALANCE - PATTERN (\) + D_1, (N = 5)



MEAN BUFFER LEVEL CURVES FOR MEANS & BUFFER CAPACITIES

IMBALANCE - PATTERN (\) + D_1,(N = 8)

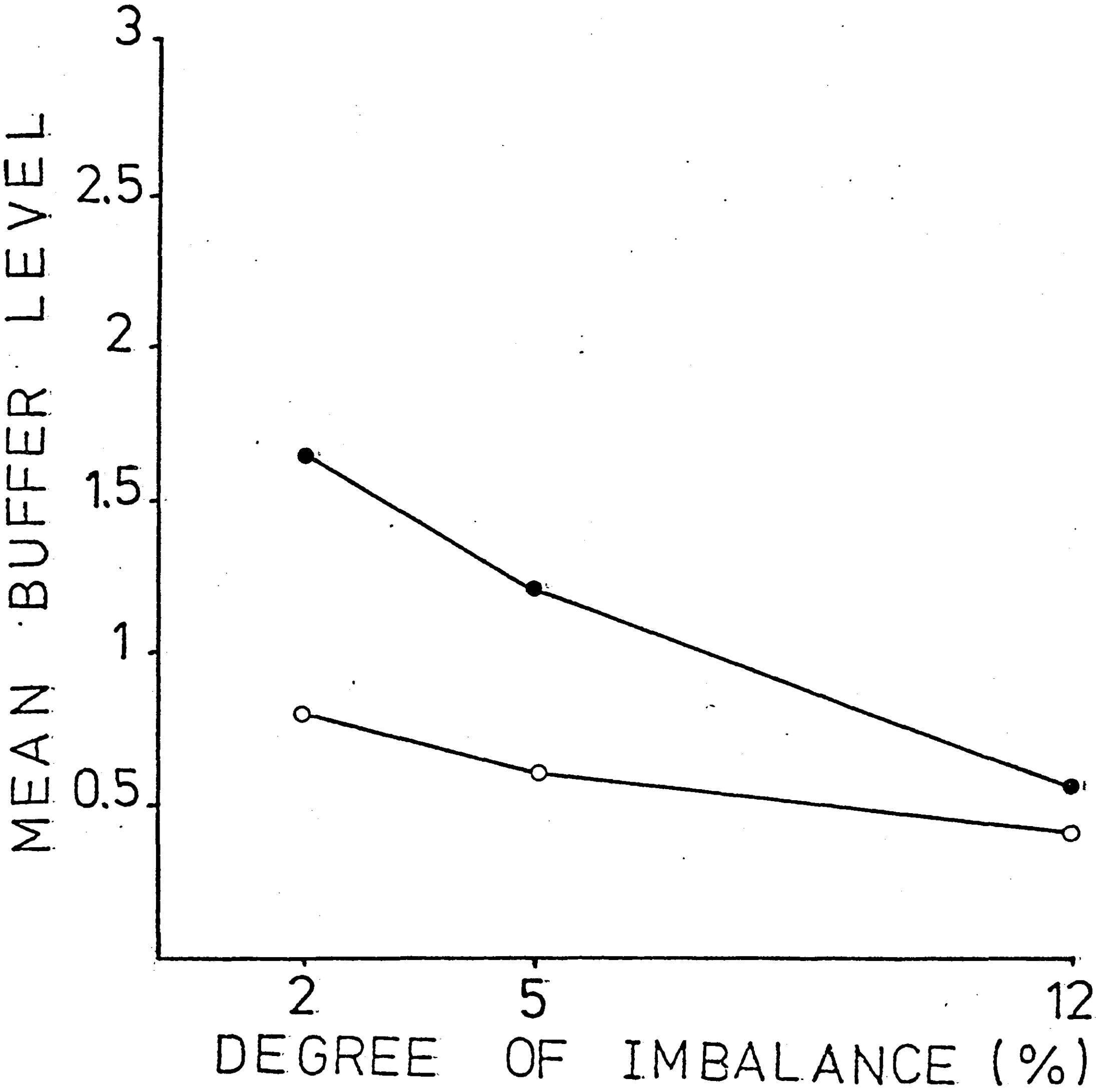


FIGURE A7.62

MEAN BUFFER LEVEL CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (↘) + D_2.(N = 5)

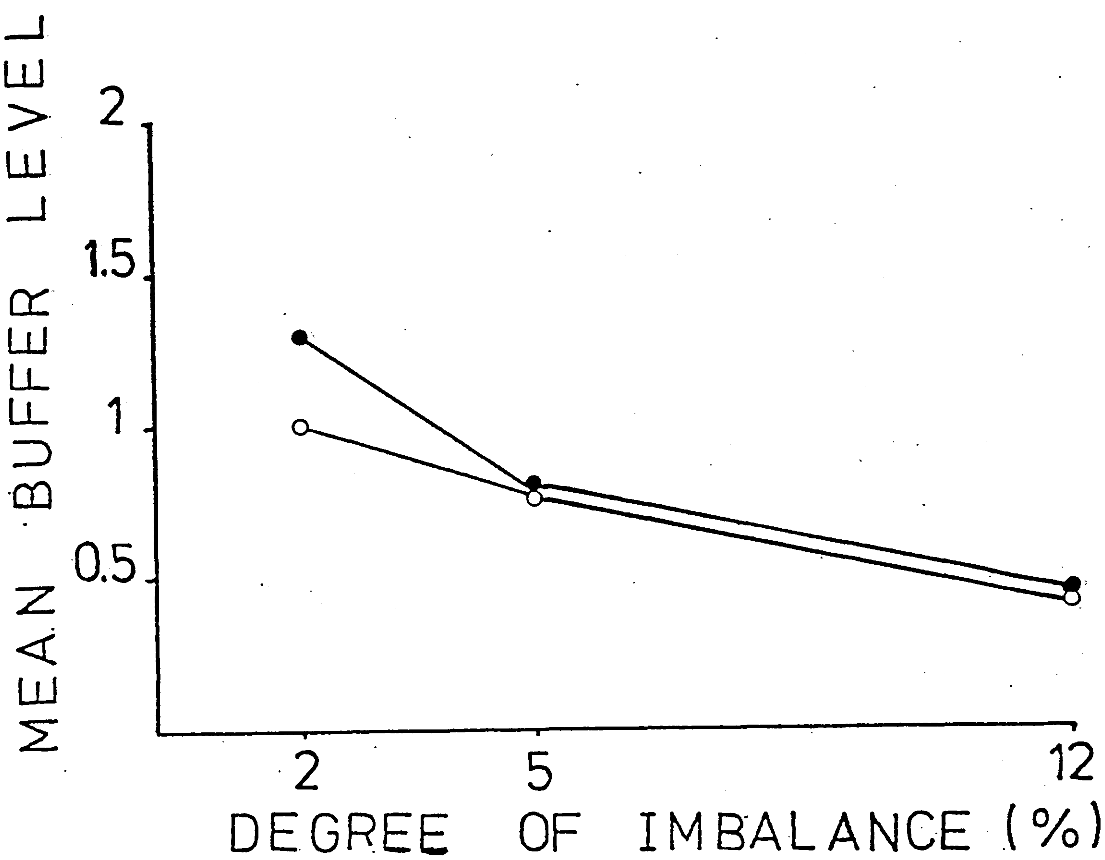


FIGURE A7.63

MEAN BUFFER LEVEL CURVES FOR MEANS & BUFFER CAPACITIES

IMBALANCE - PATTERN (\) + D_2,(N = 8)

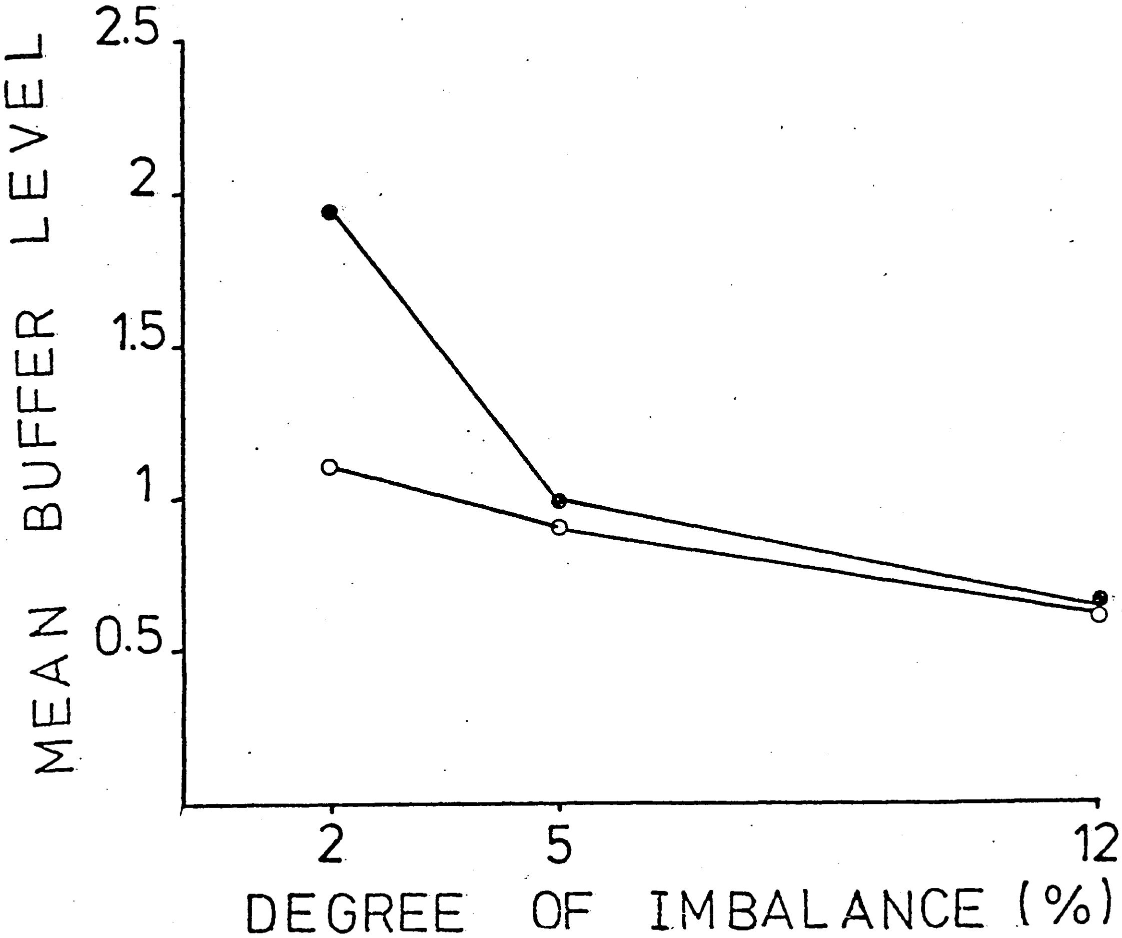
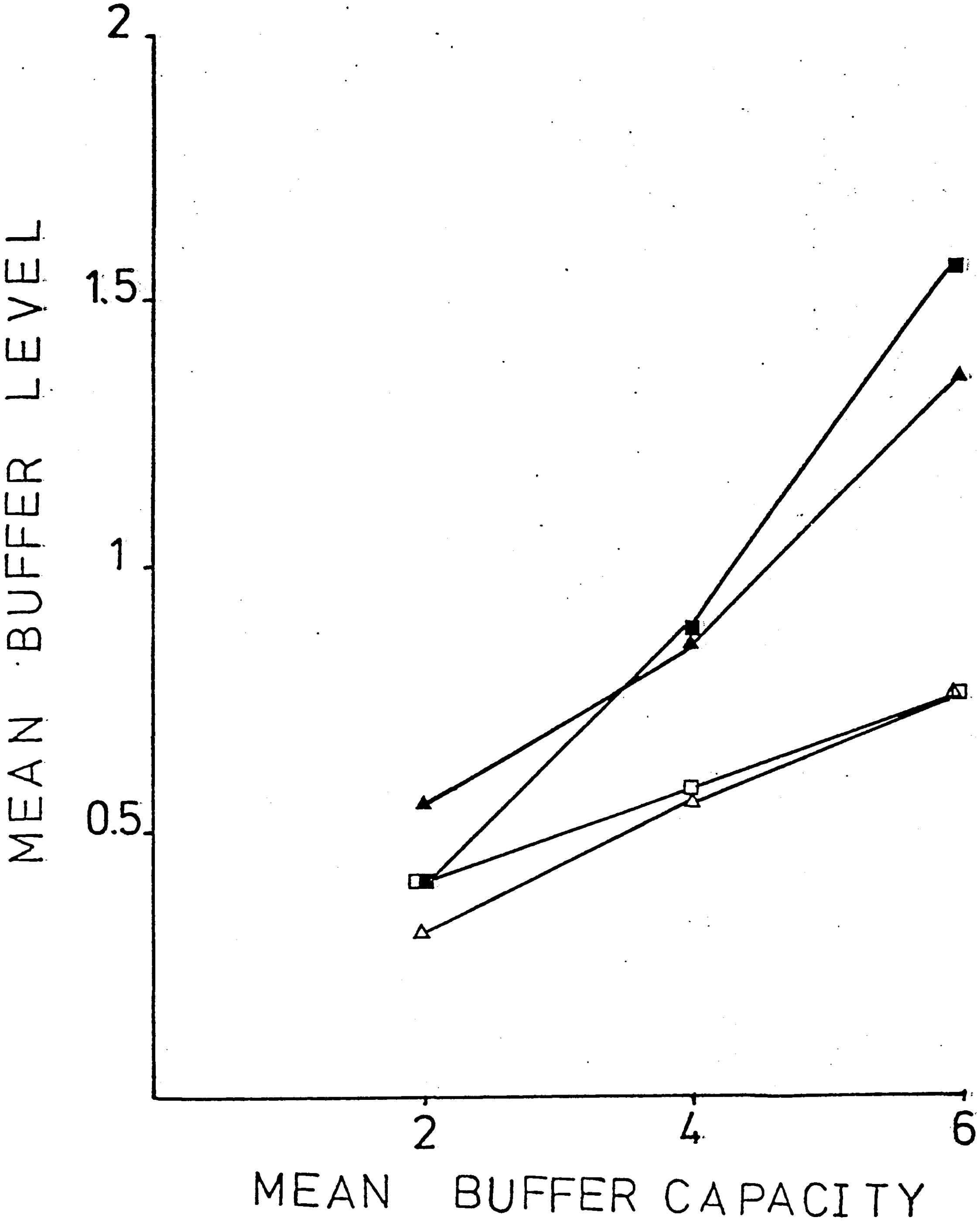


FIGURE A7.64

MEAN BUFFER LEVEL CURVES FOR COVARS & BUFFER CAPACITIES

IMBALANCE - PATTERN P_4 + A AND OTHER PATTERNS. (N = 5)



MEAN BUFFER LEVEL CURVES FOR COVARS & BUFFER CAPACITIES IMBALANCE - PATTERN P_4 + A AND OTHER PATTERNS, (N = 8)

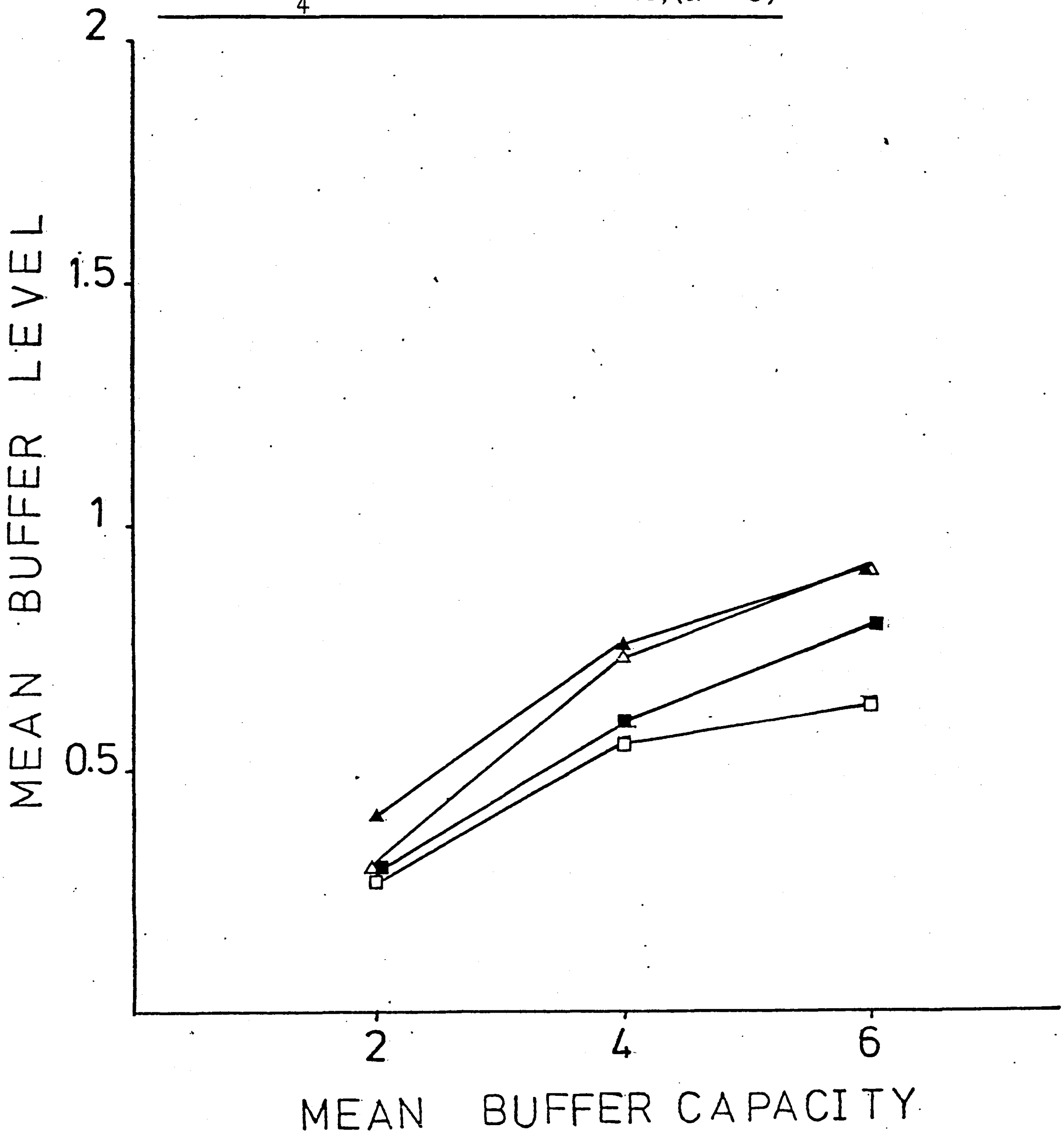
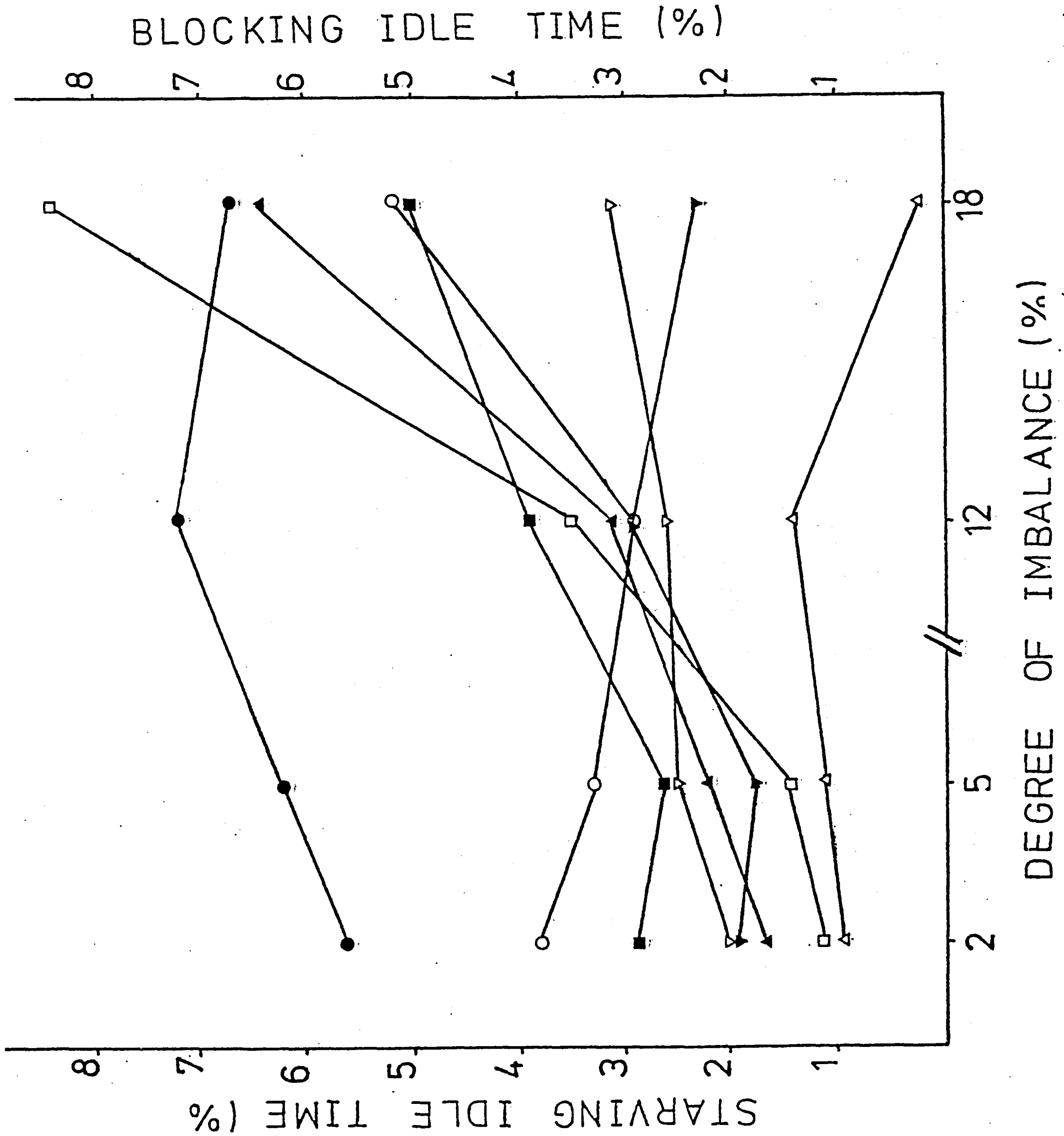


FIGURE A7.66

% STARVING AND BLOCKING IDLE TIMES CURVES FOR MEANS IMBALANCE - PATTERN (V), (N = 5)



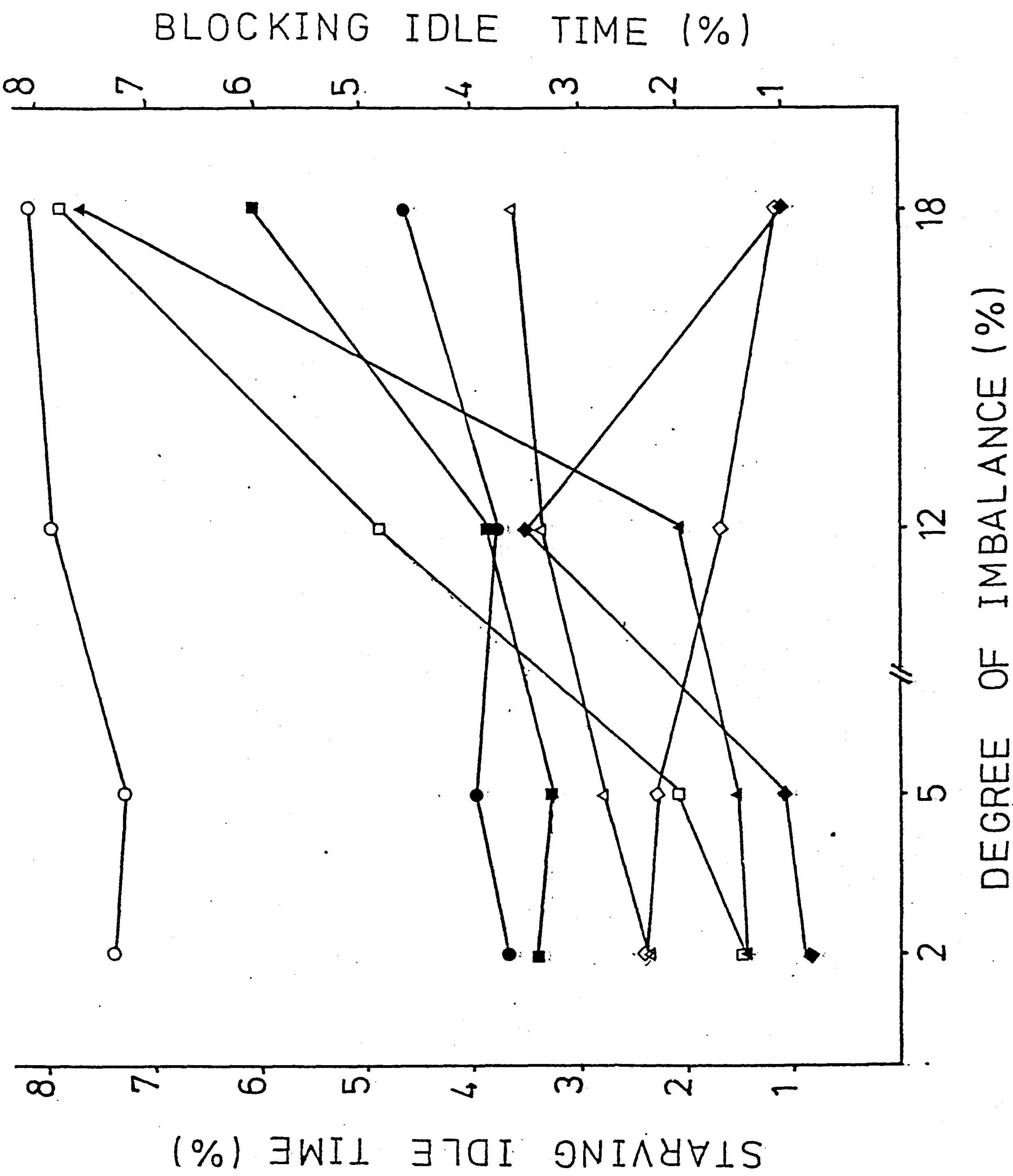


FIGURE A7.67

% STARVING AND BLOCKING IDLE TIMES CURVES FOR MEANS IMBALANCE - PATTERN (V), (N = 8)

FIGURE A7.68

% STARVING AND BLOCKING IDLE TIMES CURVES FOR COVARS IMBALANCE - PATTERN P_7, (N = 5,8)

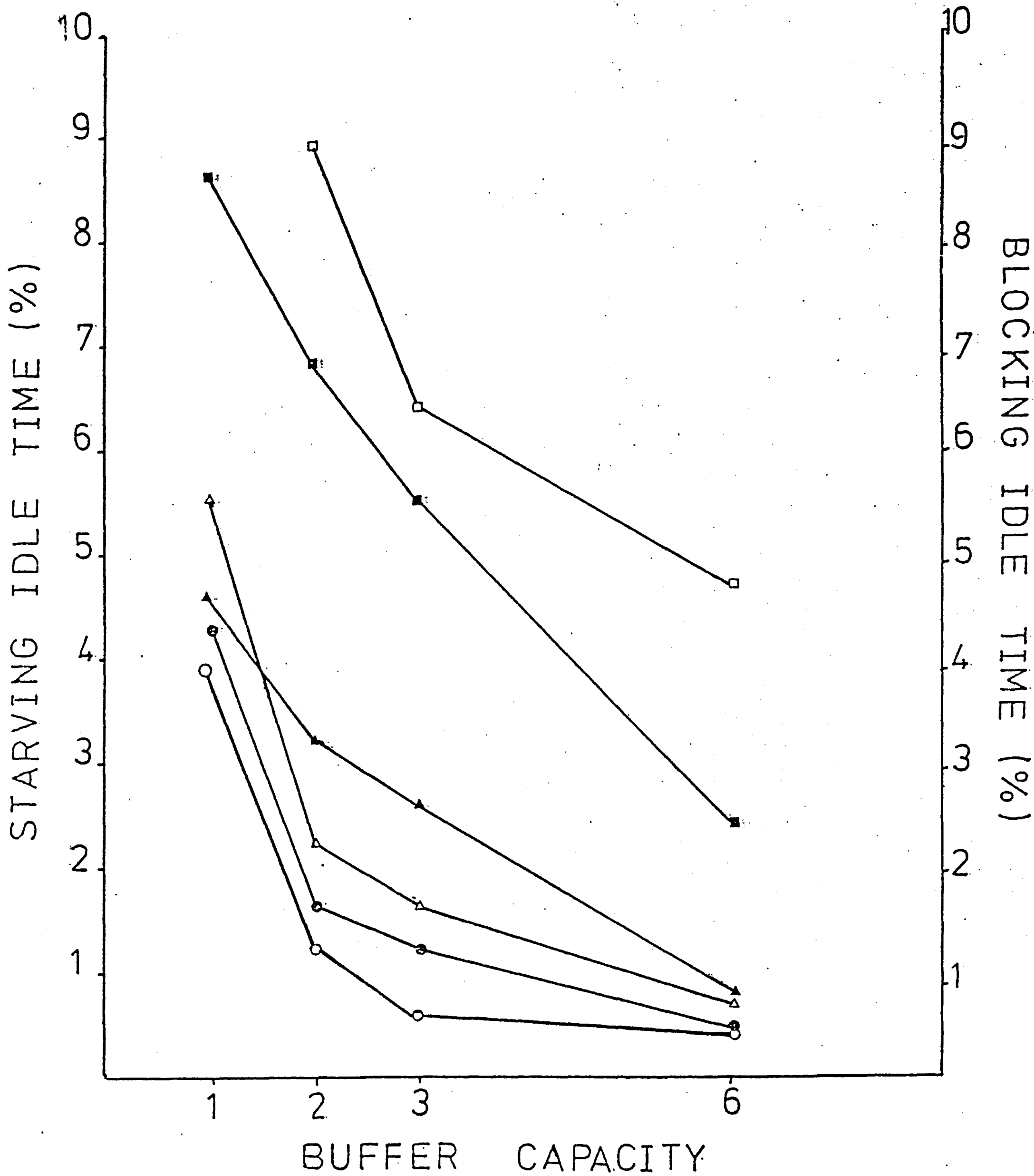


FIGURE A7.69

% STARVING AND BLOCKING IDLE TIMES CURVES FOR BUFFER CAPACITIES IMBALANCE - PATTERNS $D_2(N = 5)$&$D_1(N = 8)$

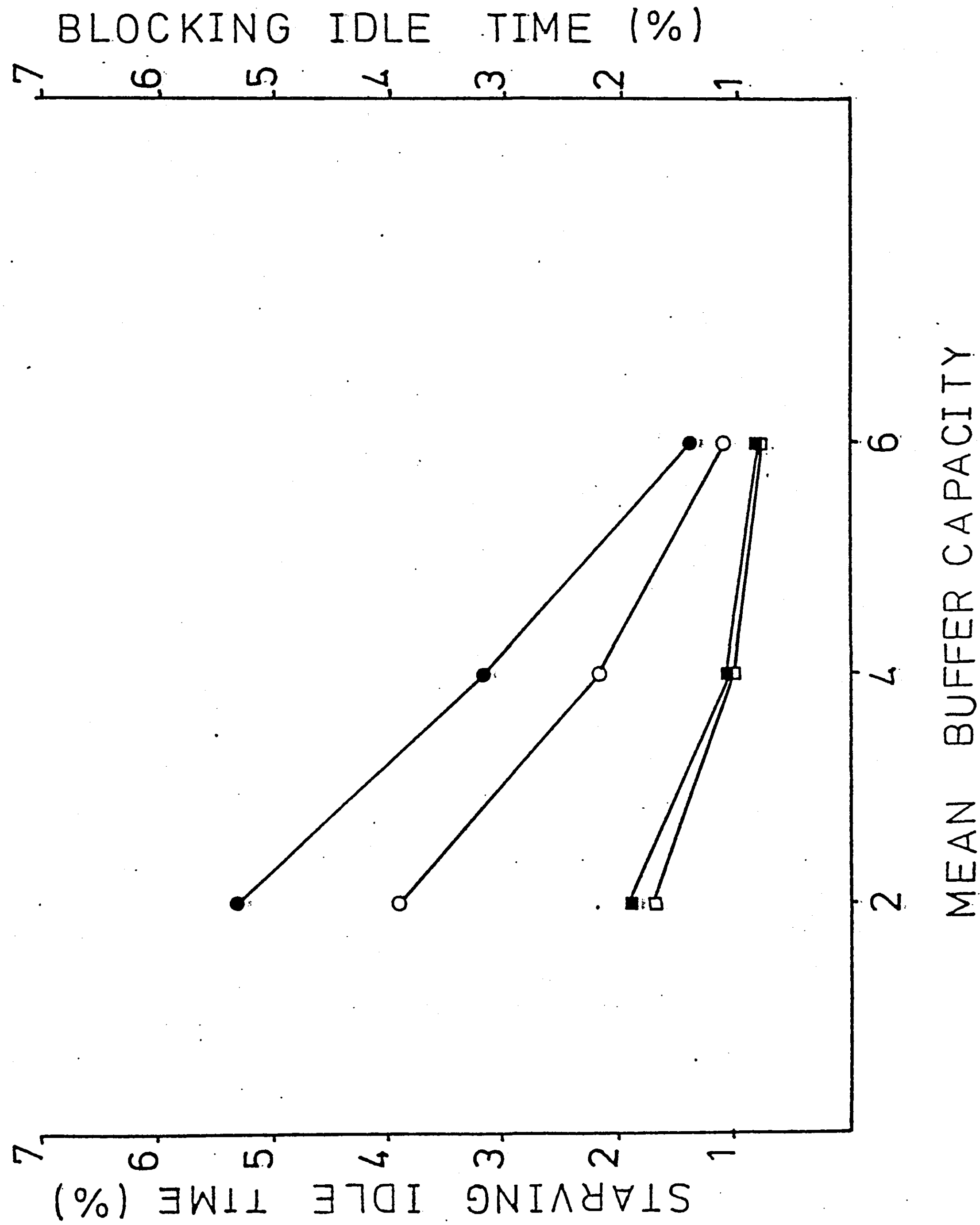
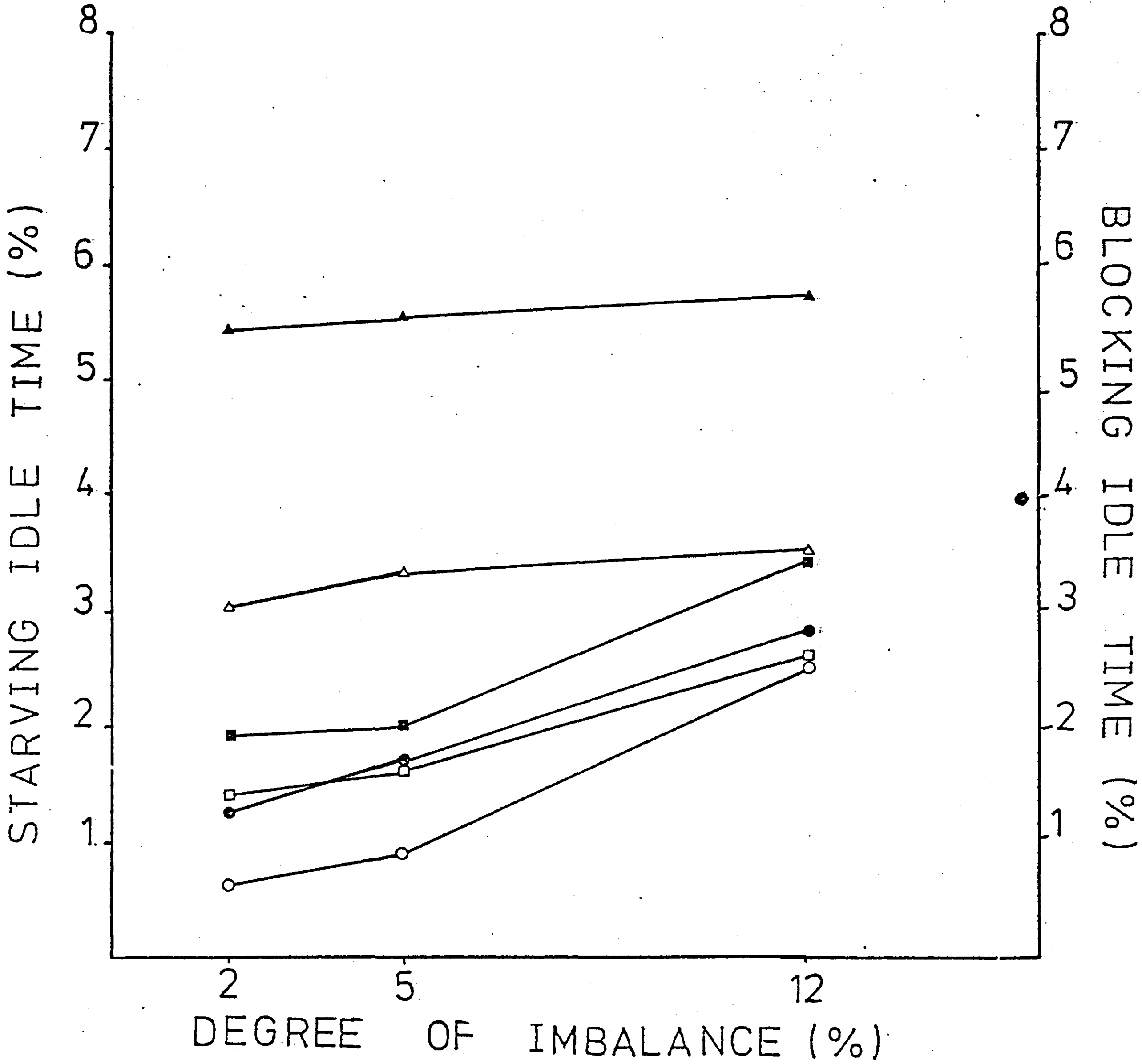


FIGURE A7.70

% STARVING AND BLOCKING IDLE TIMES CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (Λ) + P_5, (N = 5)



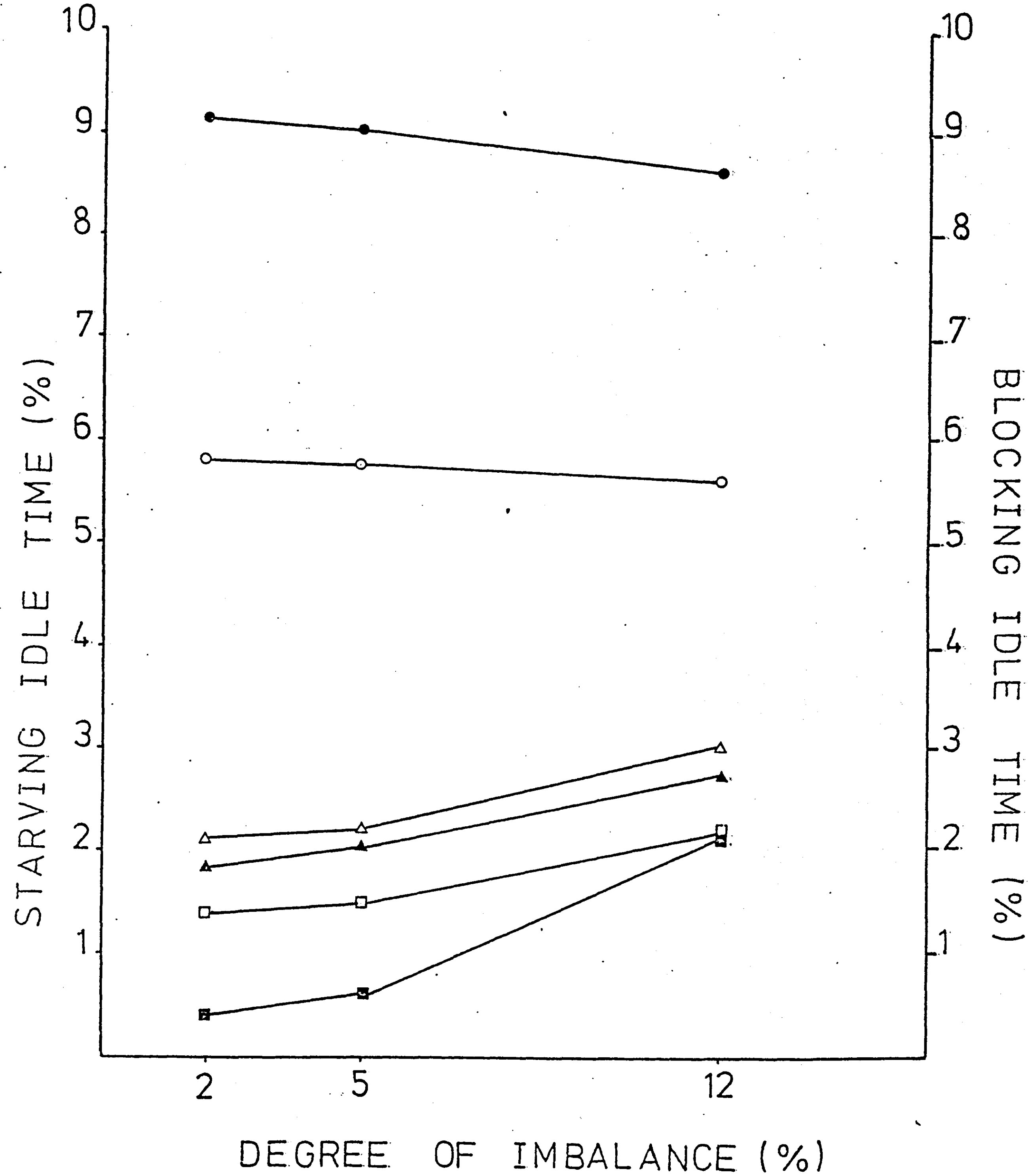
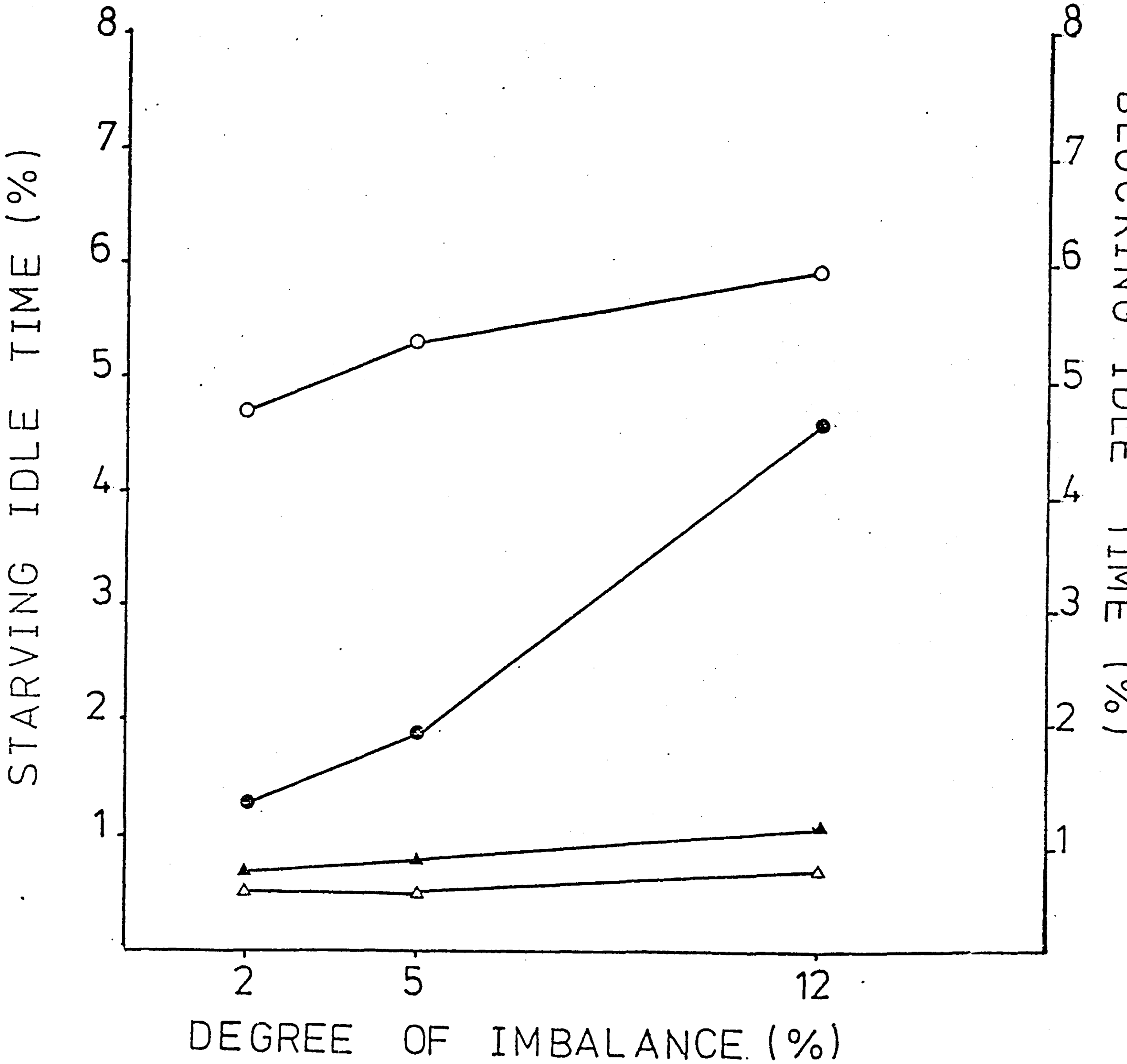


FIGURE A7.71

% STARVING AND BLOCKING IDLE TIMES CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (Λ) + P_5, (N = 8)

FIGURE A7.72

% STARVING AND BLOCKING IDLE TIMES CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (V) + D_1,(N = 5)



% STARVING AND BLOCKING IDLE TIMES CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (V) + D_1, (N = 8)

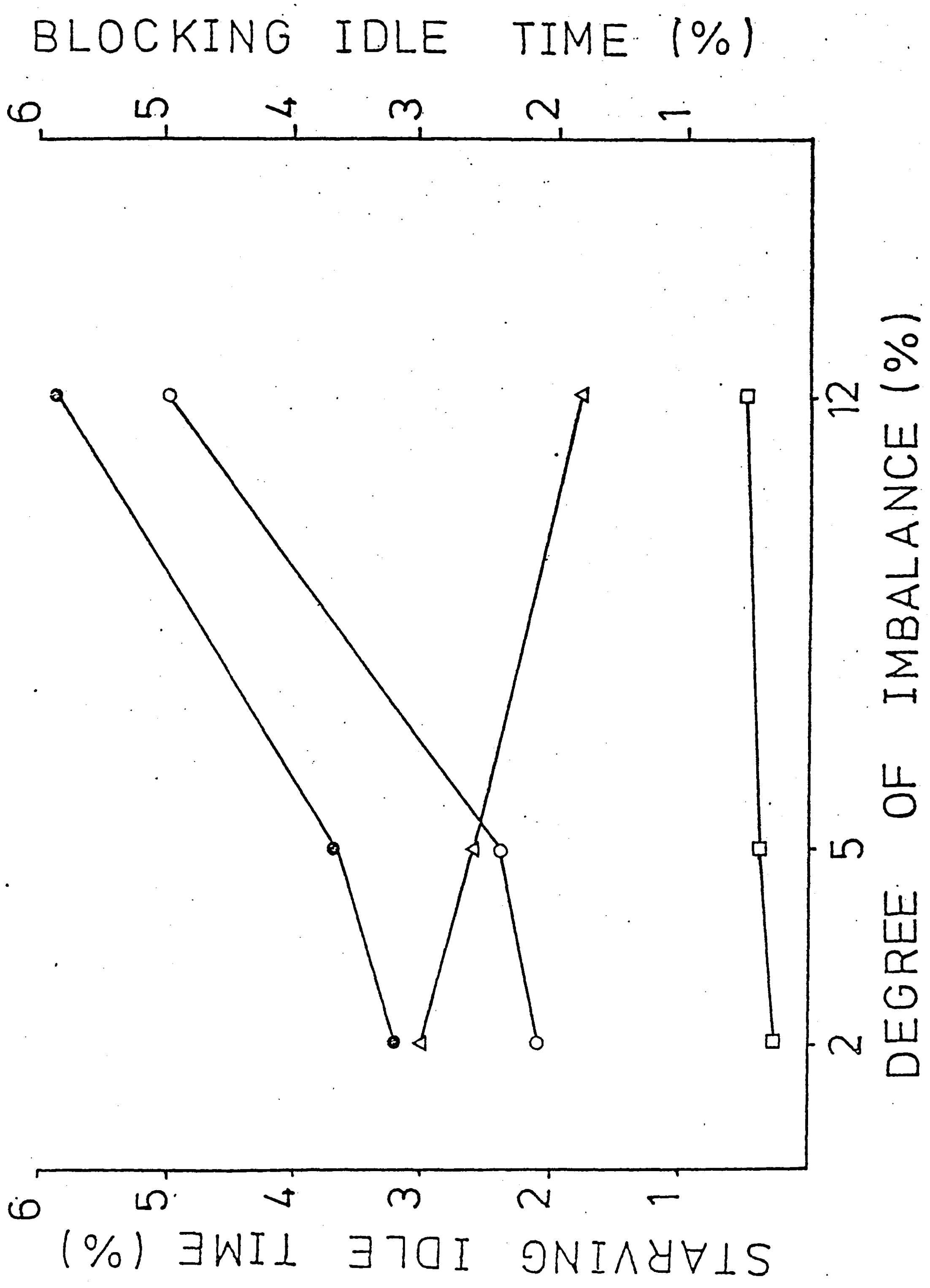


FIGURE A7.74

% STARVING AND BLOCKING IDLE TIMES CURVES FOR COVARS & BUFFER CAPACITIES

IMBALANCE - PATTERN P_4 + C.(N = 5.8)

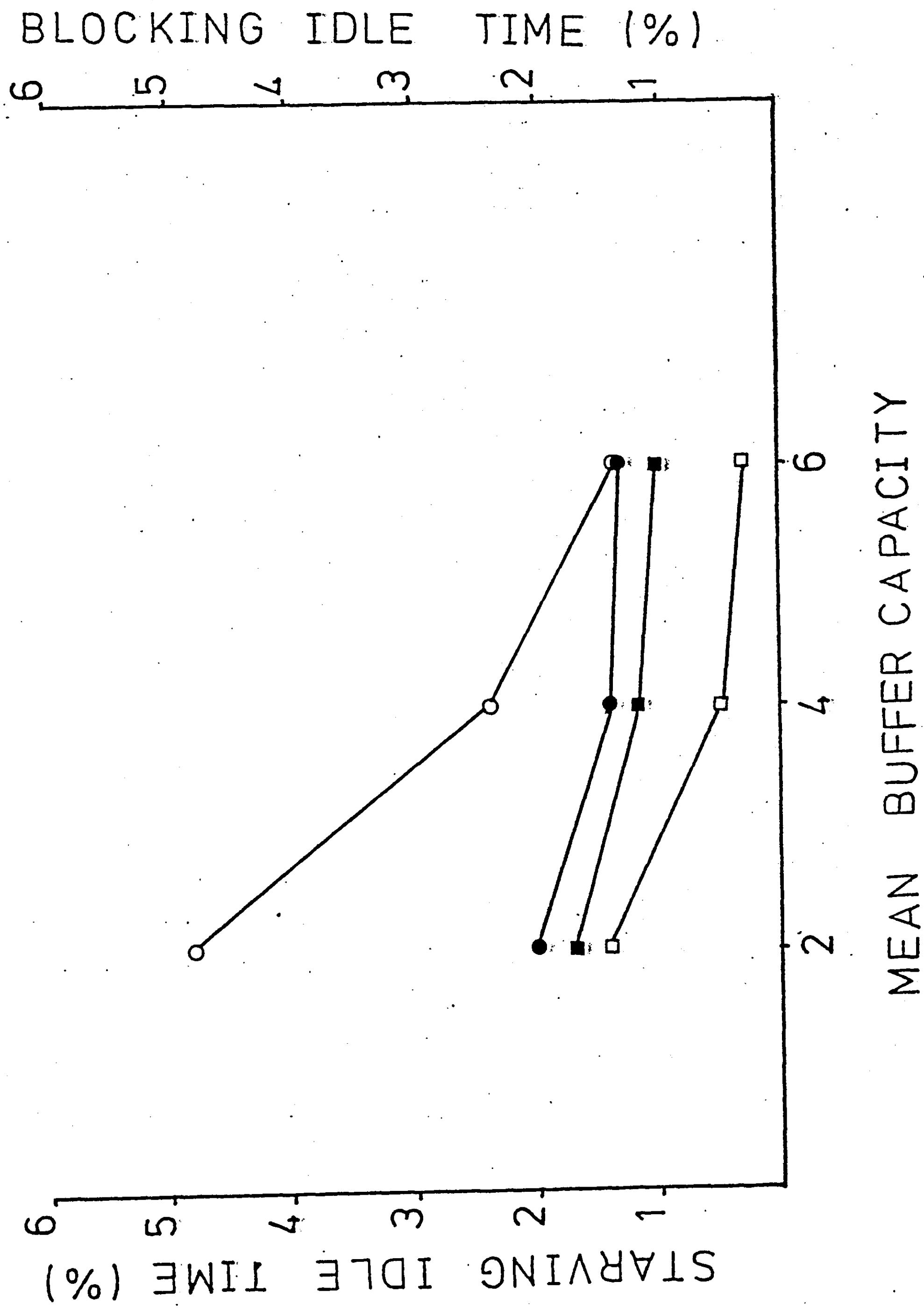


FIGURE A7.75

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR MEANS IMBALANCE - PATTERN (↘),(N = 5)

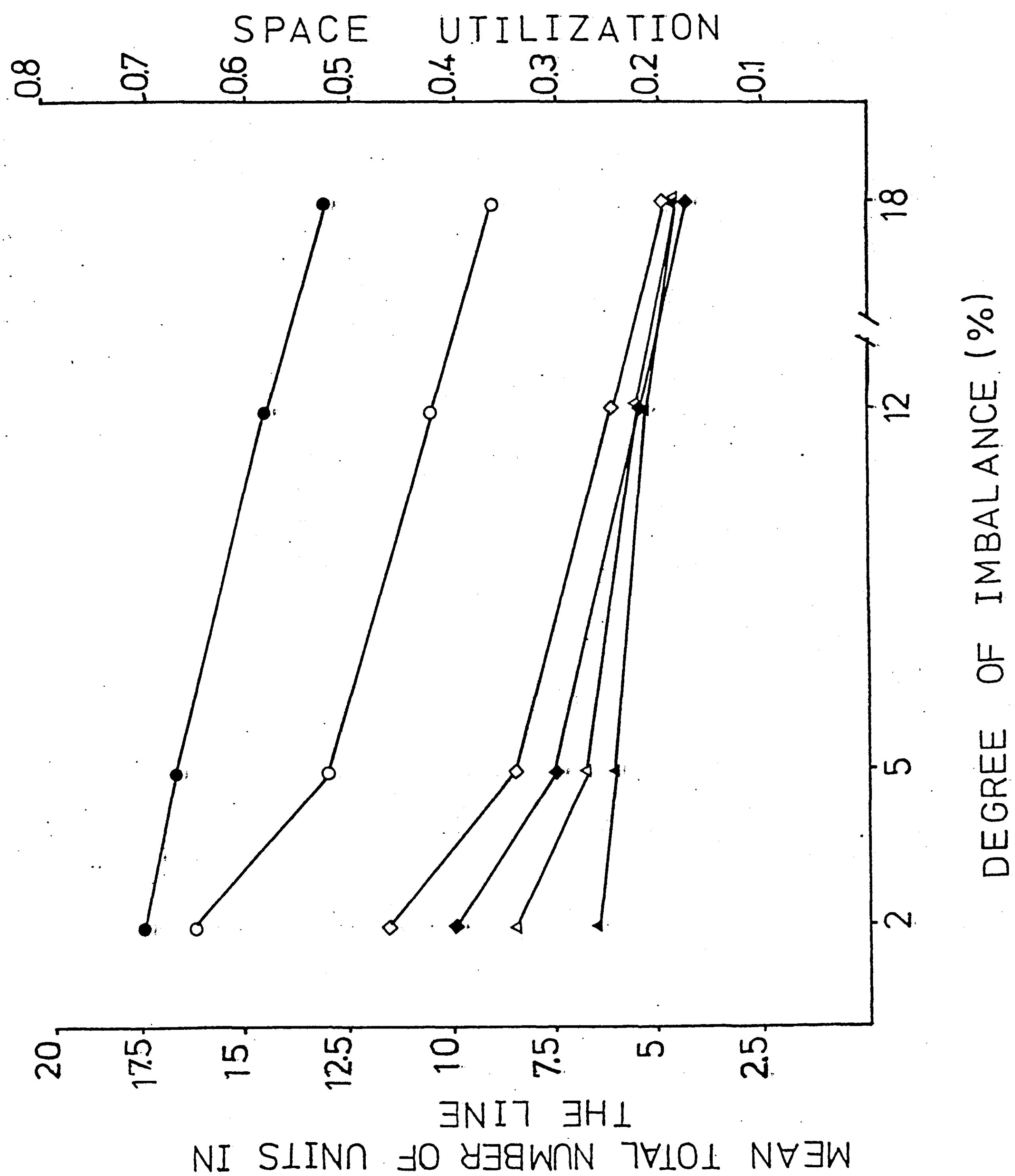


FIGURE A7.76

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR MEANS IMBALANCE - PATTERN (↘),(N = 8)

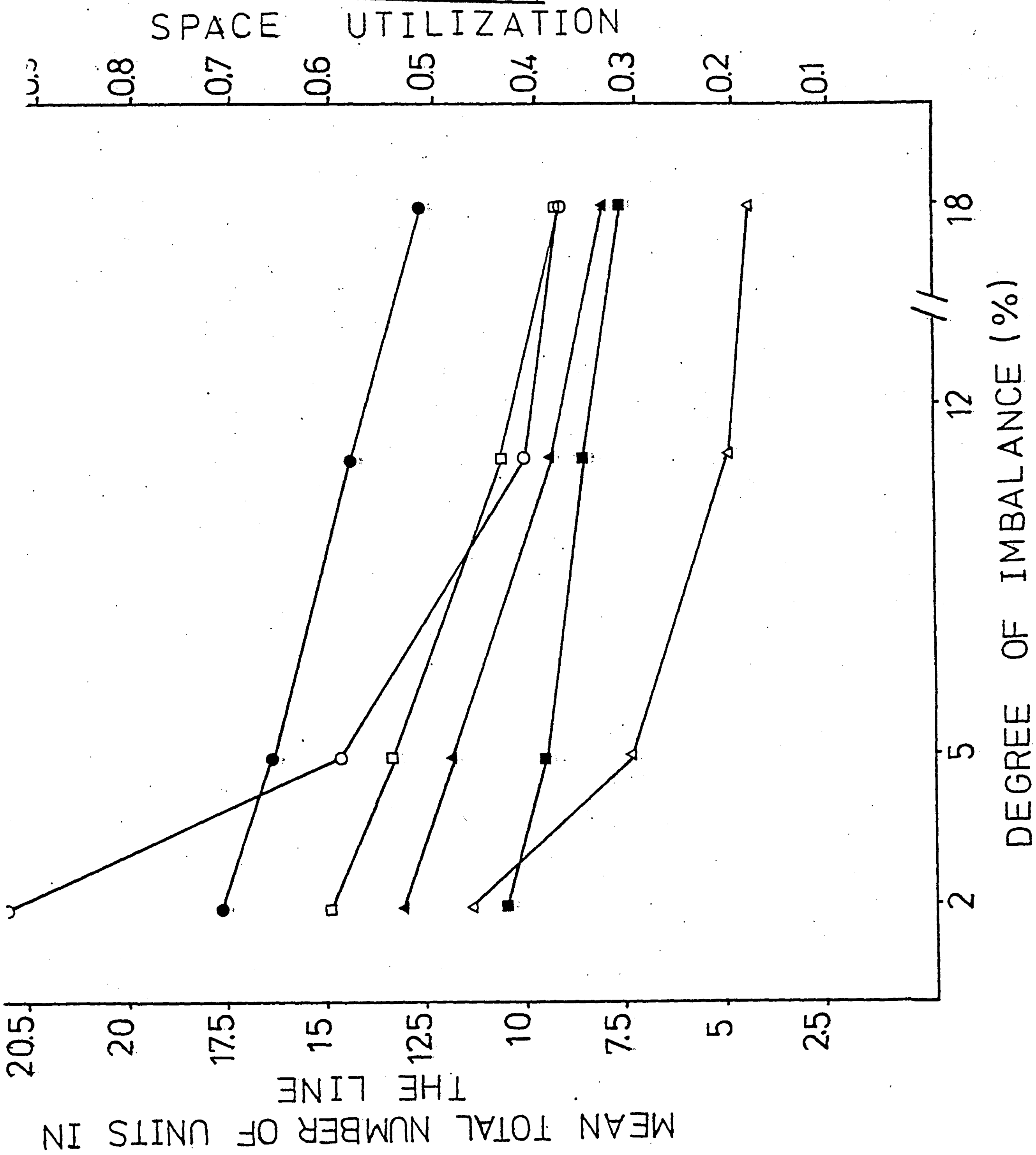


FIGURE A7.77

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR COVARS IMBALANCE - PATTERN P_7, (N = 5)

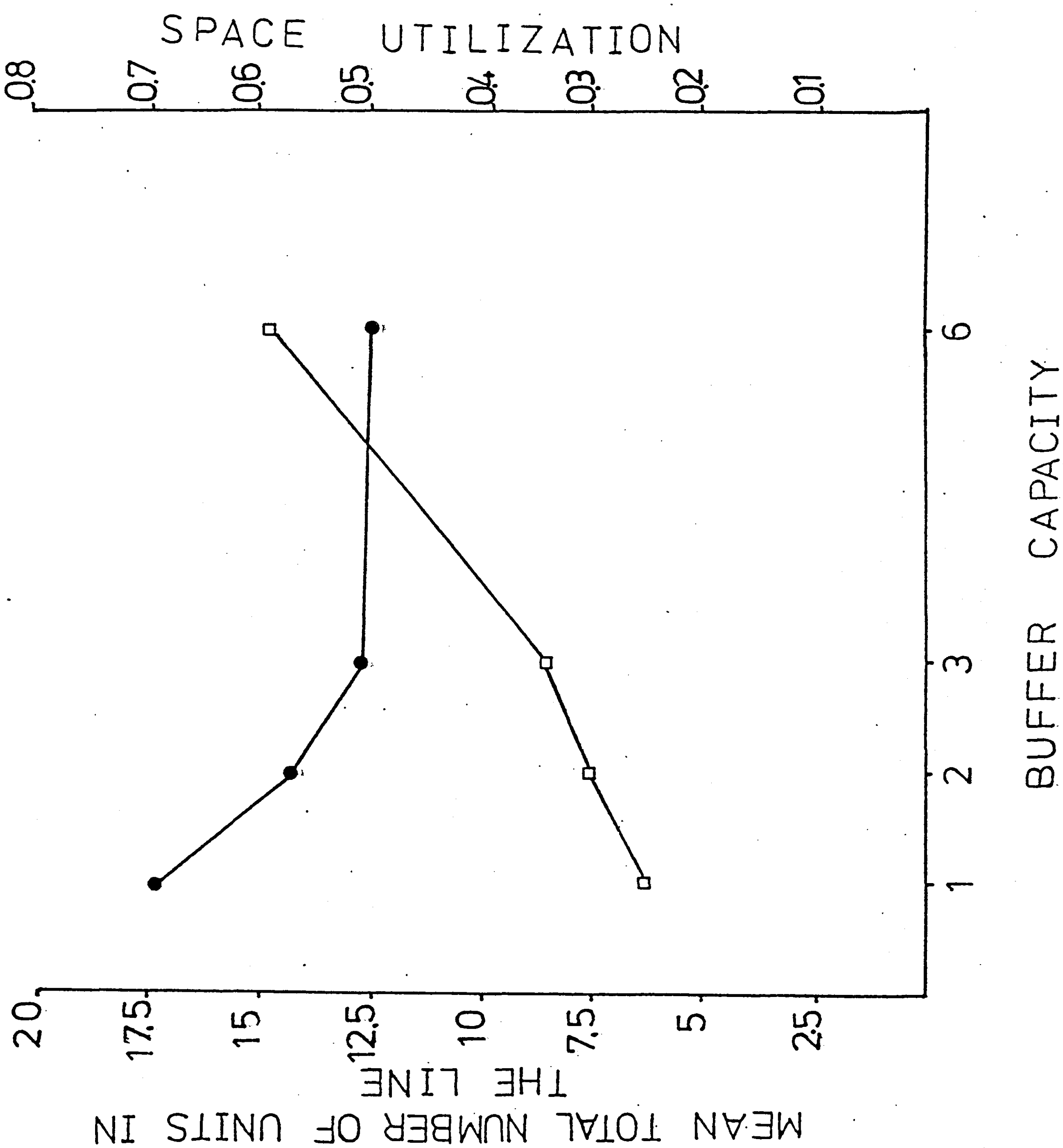
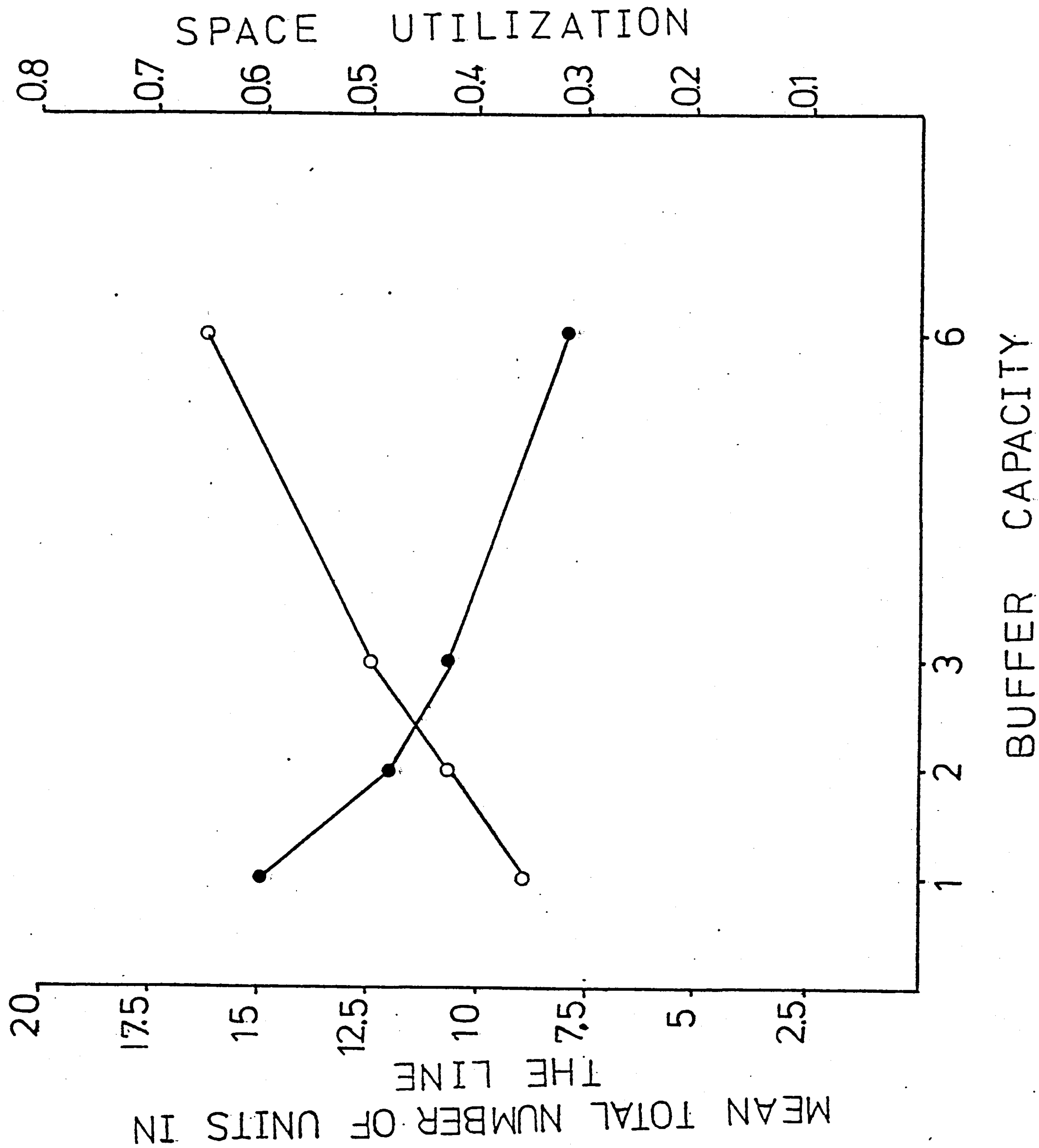


FIGURE A7.78

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR COVARS IMBALANCE - PATTERN P_7, (N = 8)



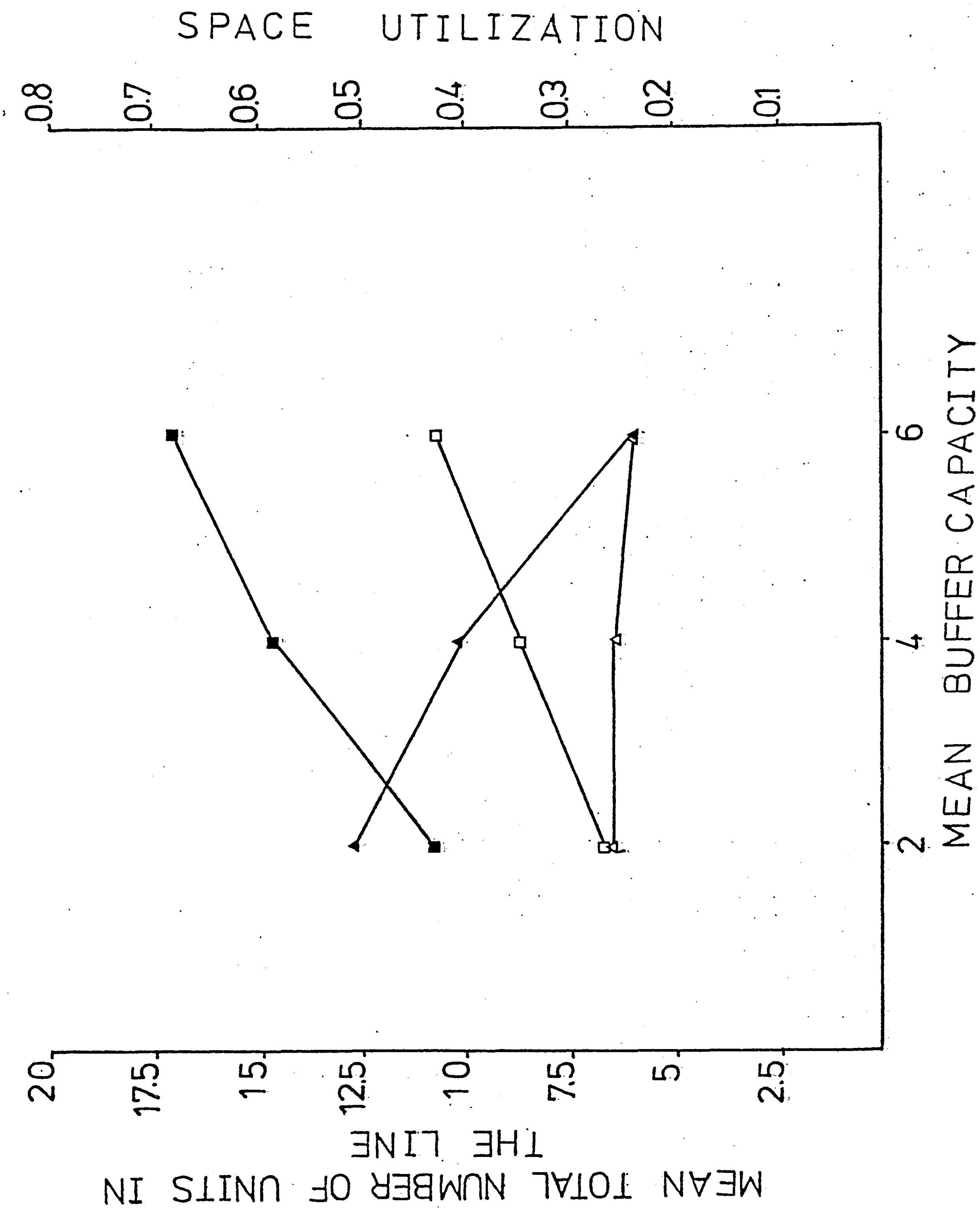


FIGURE A7.79

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR BUFFER CAPACITIES

IMBALANCE - PATTERNS A_1 (N = 5)&A_2 (N = 8)

FIGURE A7.80

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR MEANS & COVARS IMBALANCE - PATTERN (\) + P_4,(N = 5)

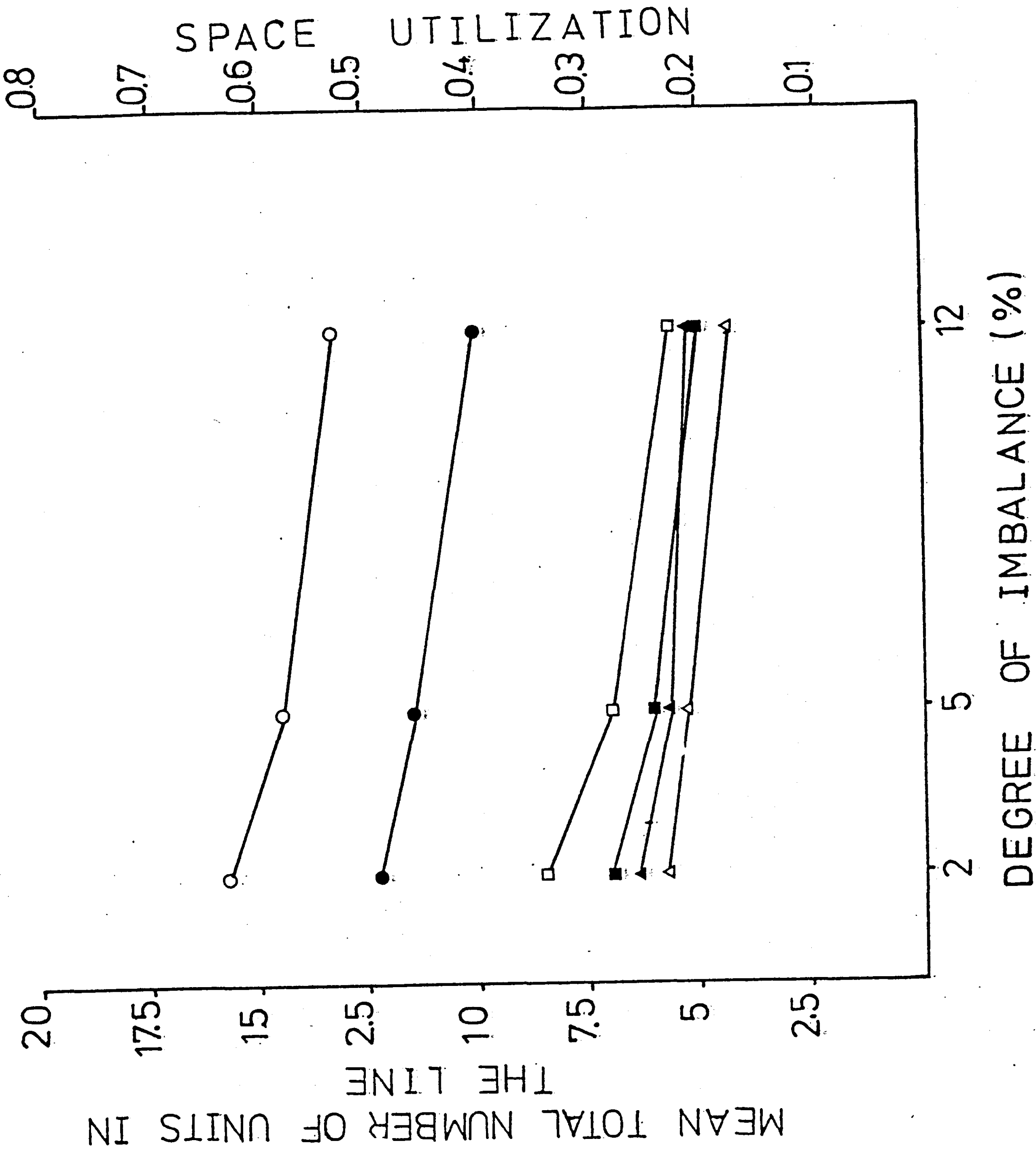


FIGURE A7.81

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR MEANS & COVARS

IMBALANCE - PATTERN (\) + P_4.(N = 8)

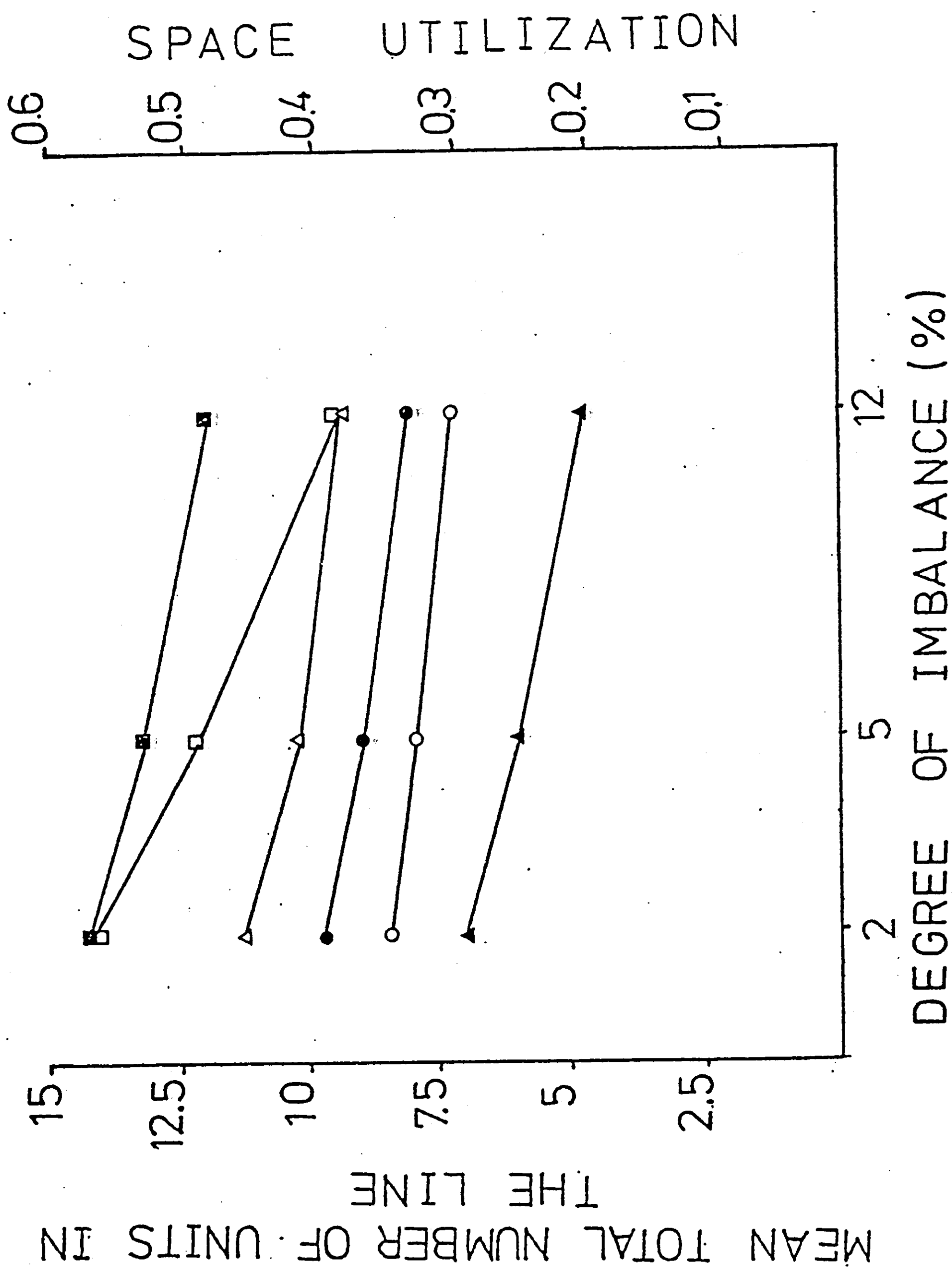
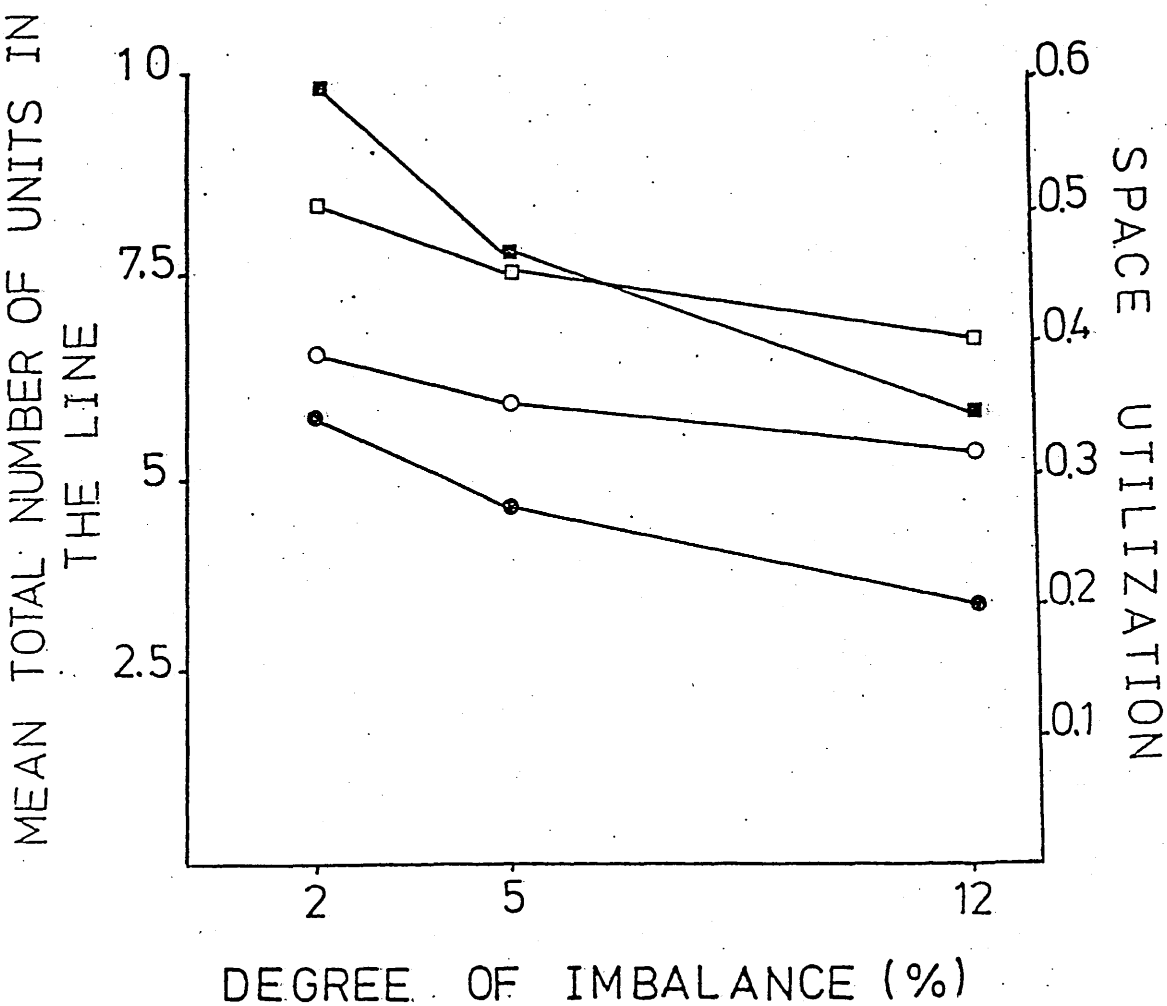


FIGURE A7.82

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (\) + A, (N = 5)



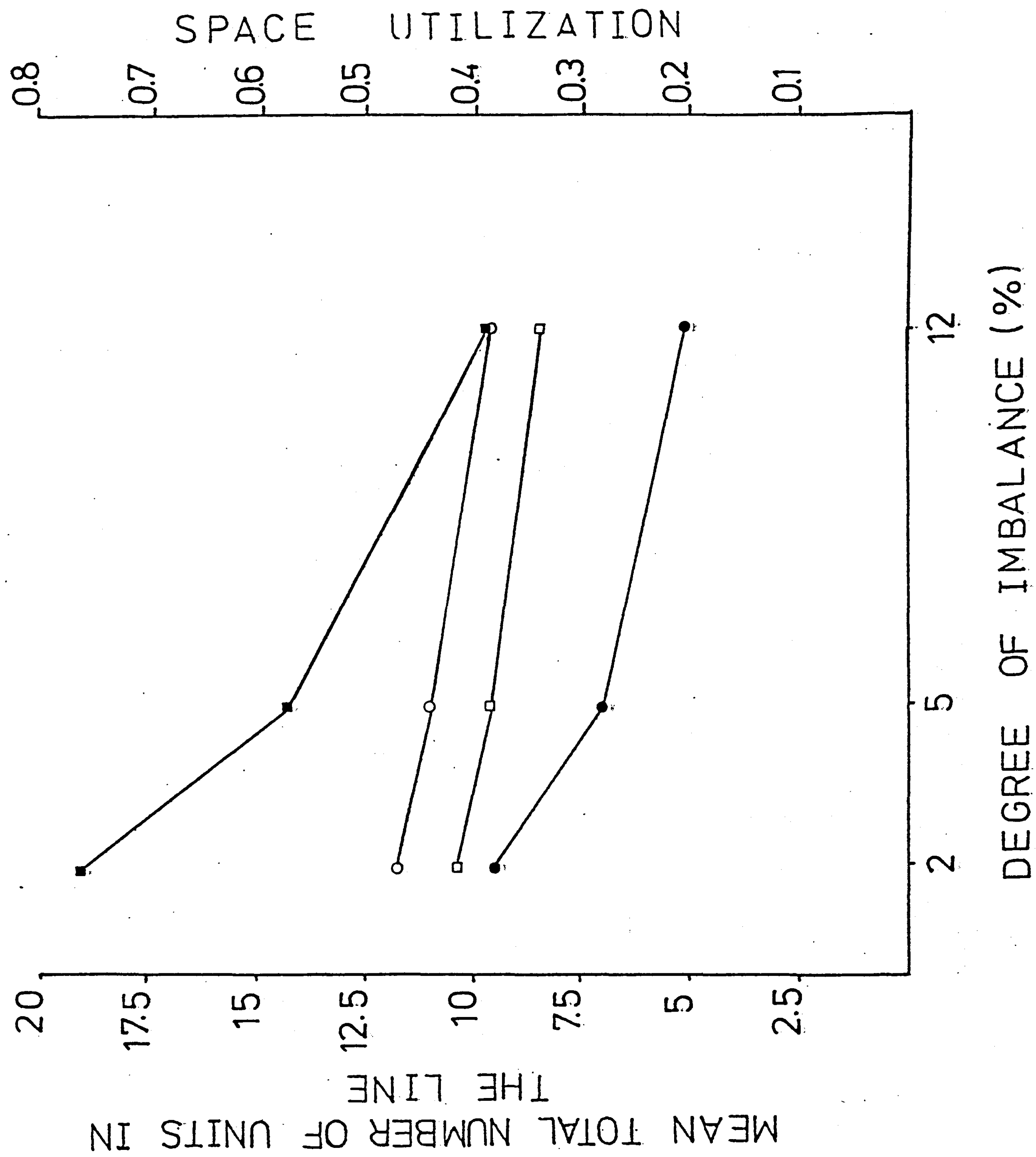
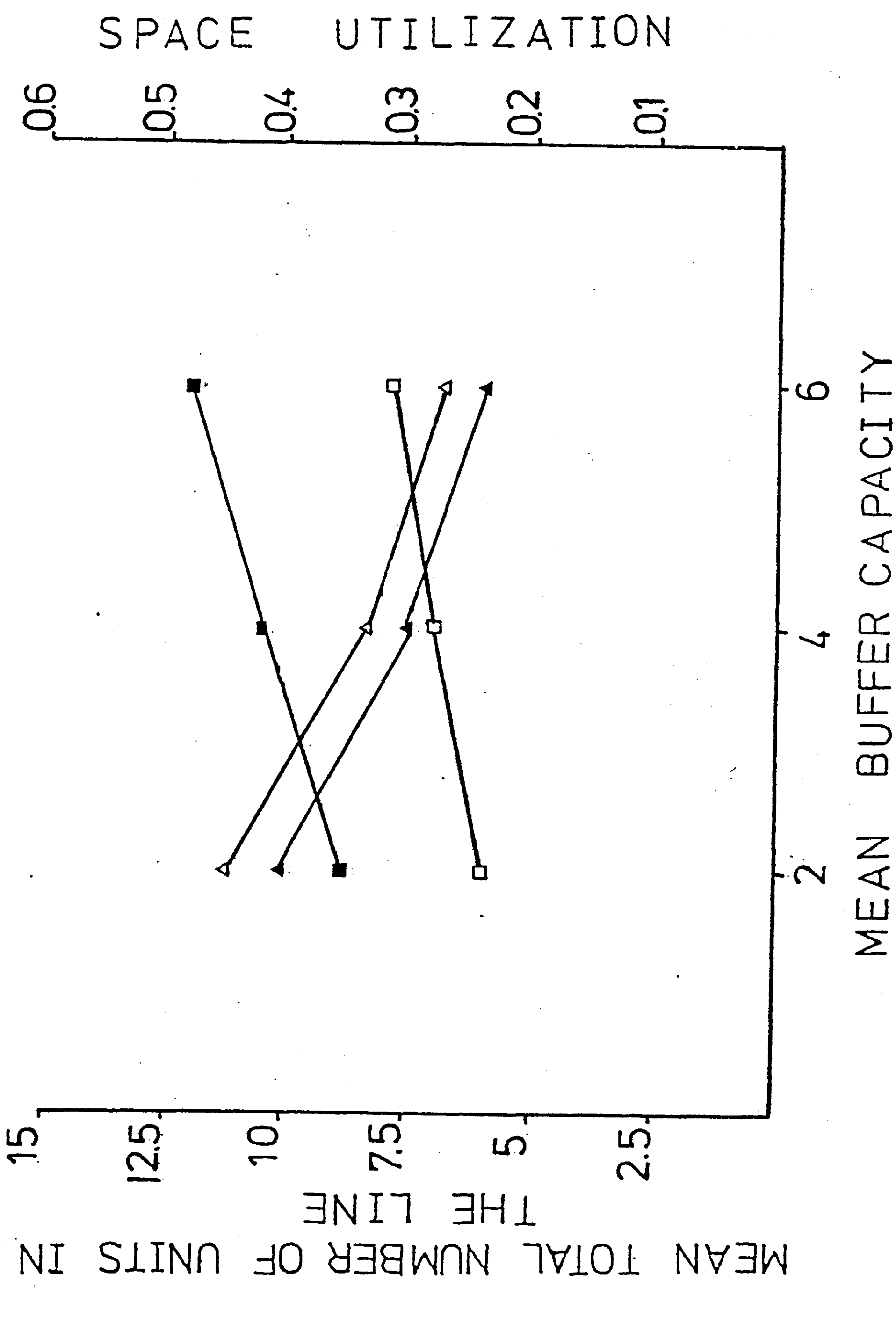


FIGURE A7.83

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (\) + A, (N = 8)

FIGURE A7.84.

TOTAL LINE'S UNITS AND SPACE UTILIZATION CURVES FOR COVARS & BUFFER CAPACITIES IMBALANCE - PATTERN P_4 + A, (N = 5,8)



APPENDIX 8

NON-STEADY STATE RESULTS

This general appendix contains the results of the non-steady state simulation investigations and is divided into four appendices:

8.1 - Index of Tables

8.2 - Tables

8.3 - Index of Figures

8.4 - Figures

APPENDIX 8.1

INDEX OF TABLES

This appendix contains a list of the tables showing the non-steady state's results and their patterns' symbols and meanings.

APPENDIX 8.1

INDEX OF TABLES - NON-STEADY STATE INVESTIGATIONS

TABLE NUMBER	TITLE

where (in the tables above)

* = significantly different from the control (the steady-state counterpart) at the 0.90 significance level.

** = significantly different from the control at the 0.95 level of significance.

*** = significantly different from the control at the 0.99 level of significance.

APPENDIX 8.2

TABLES

This appendix contains the results of the non-steady state simulation investigations, presented in tabular forms.

TABLE A8.1

NON-STEADY STATE % TOTAL IDLE TIME RESULTS - MEANS IMBALANCE

LINE LENGTH		5												8										
BUFFER CAPACITY		1						6						1						6				
DEGREE OF IMBALANCE(%)		5			12			5			12			5			12			5			12	
RESULTS OF : ① : 1ST PERIOD ② : 2ND PERIOD ③ : MEAN OF THE TWO PERIODS		①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	③
PATTERN OF MEANS IMBALANCE	/	10.731	10.538	10.635	13.477	14.258	13.868	3.177	5.216	4.197	8.268	4.238	9.743	12.342	12.187	12.265	16.004	15.762	15.883	3.170	5.361	4.266	7.189	12.708
	\	10.832	11.448	11.140	14.805	14.719	14.762	6.237	5.313	5.775	12.201	11.623	11.912	13.337	12.882	13.110	17.028	16.701	16.865	7.051	5.253	6.152	13.249	12.416
	V	9.658	9.519	9.589	10.192	10.032	10.112	4.199	2.886	3.543	6.207	5.289	5.748	11.643	11.395	11.519	12.021	11.647	11.834	4.263	2.935	3.609*	7.067	5.540
	Λ	10.755	10.451	10.603	12.453	12.361	12.407	3.784	3.448	3.616	7.042	6.915	6.979	12.488	12.012	12.255	13.926	13.613	13.770	4.319	2.983	3.650	7.339	6.730
TRANSIENT SIZE (PATTERN V ONLY)		1.011			1.006			1.655			1.242			1.036			1.021			1.658			1.249	
DUNNETT'S t VALUE (PATTERN V ONLY)		0.079			0.039			1.504			1.116			0.473			-0.110			1.919*			1.281	

TABLE A8.2

NON-STEADY STATE % TOTAL IDLE TIME RESULTS - COVARS IMBALANCE

LINE LENGTH	BUFFER CAPACITY	RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: TWO PERIODS' MEAN	PATTERN OF COVARS IMBALANCE (P_i): P_1	P_2	P_3	P_4	TRANSIENT SIZE (PATTERN P_2 ONLY)	DUNNETT'S t VALUE (PATTERN P_2 ONLY)
5	1	①	11.896	8.571	19.164	11.937	1.002	0.021
		②	11.505	8.564	18.405	11.489		
		③	11.701	8.568	18.785	11.713		
	6	①	3.413	2.467	5.721	2.940	2.098	1.458
		②	2.057	1.844	5.503	2.225		
		③	2.735	2.156	5.612	2.583		
8	1	①	14.838	14.235	21.542	15.896	1.100	1.490
		②	14.163	14.199	20.562	15.249		
		③	14.501	14.217	21.052	15.573		
	6	①	4.938	4.306	6.993	3.534	1.487	0.598
		②	3.474	2.895	6.362	3.688		
		③	4.206	3.601	6.678	3.611		

NON-STEADY STATE % TOTAL IDLE TIME RESULTS - BUFFER CAPACITIES IMBALANCE

LINE LENGTH	5						8					
MEAN BUFFER CAPACITY	2			6			2			6		
RESULTS OF: ①:1ST PERIOD ②:2ND PERIOD ③:2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE A	7.917	7.872	7.895	3.164	2.132	2.648	10.524	10.072	10.298	5.635	4.211	4.923
B	6.441	6.211	6.326	3.121	2.638	2.880	8.856	8.110	8.483	4.755	3.302	4.029
C	7.420	7.335	7.378	3.371	1.781	2.576	9.756	9.350	9.553	4.973	2.988	3.981
D	5.661	5.529	5.595	2.972	2.051	2.512	7.380	6.884	7.132	4.781	3.168	3.975**
TRANSIENT SIZE (PATTERN D ONLY)	1.011			1.715			1.038			2.161		
DUNNETT'S t VALUE (PATTERN D ONLY)	0.006			1.035			0.063			4.119***		

LINE LENGTH	5											
BUFFER CAPACITY	1						6					
DEGREE OF IMBALANCE (%)	5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEANS	①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF COVARS IMBALANCE (P_i) P_1	12.272	12.146	12.209	15.353	15.143	15.248	4.711	6.607	5.659	8.500	12.193	10.347
P_2	8.270	8.248	8.259	12.237	12.158	12.198	2.079	5.333	3.254	6.959	11.760	9.360
P_3	18.682	17.843	18.263	19.731	19.522	19.627	6.583	6.222	6.403	9.593	12.279	10.936
P_4	14.001	13.638	13.820	17.843	17.581	17.712	3.947	5.715	4.831	9.420	12.765	11.093

LINE LENGTH	8											
BUFFER CAPACITY	1						6					
DEGREE OF IMBALANCE (%)	5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEANS	①	②	③	①	②	③	①	②	③	①	②	(
PATTERN OF COVARS IMBALANCE (P_i) P_1	15.115	15.017	15.066	18.020	17.042	17.531	3.302	5.218	4.260	6.759	11.890	9
P_2	12.392	12.077	11.761	14.551	14.428	14.490	3.949	4.413	4.181	6.114	12.186	9.
P_3	20.612	19.873	20.243	21.136	21.119	21.128	6.638	6.276	6.457	10.093	13.789	11.
P_4	18.069	17.802	17.936	22.978	21.764	22.371	4.770	8.310	6.540	9.454	14.333	11.

TABLE A8.4

NON-STEADY STATE % TOTAL IDLE TIME RESULTS - MEANS & COVARS IMBALANCE (PATTERNS(/))

LINE LENGTH		5											
BUFFER CAPACITY		1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①:1ST PERIOD ②:2ND PERIOD ③:2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF COVARS IMBALANCE (P_i)	P_1	13.203	12.855	13.029	15.727	15.025	15.376	6.380	5.225	5.803	12.591	12.333	12.462
	P_2	13.347	11.619	12.483	15.933	15.231	15.582	7.265	6.102	6.684	12.023	10.269	11.146
	P_3	19.256	19.125	19.191	21.356	21.034	21.195	8.006	6.937	7.472	13.854	12.186	13.020
	P_4	11.059	11.035	11.047	13.947	13.388	13.668	5.440	4.849	5.145	12.093	11.653	11.873

LINE LENGTH		8											
BUFFER CAPACITY		1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①:1ST PERIOD ②:2ND PERIOD ③:2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	(
PATTERN OF COVARS IMBALANCE (P_i)	P_1	15.425	15.394	15.410	19.230	17.618	18.424	7.883	5.592	6.738	14.596	14.336	14
	P_2	17.234	16.629	16.932	22.327	22.069	22.198	9.044	7.703	8.374	16.199	14.192	15
	P_3	21.006	20.926	20.966	22.937	22.875	22.906	8.723	6.735	7.729	13.624	11.701	12
	P_4	14.147	13.628	13.888	13.909	13.164	13.537	6.104	4.871	5.488	13.124	13.400	13

TABLE A8.5

NON-STEADY STATE % TOTAL IDLE TIME RESULTS - MEANS & COVARS IMBALANCE (PATTERNS(\))

LINE LENGTH		5												8											
BUFFER CAPACITY		1						6						1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12			5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	(
PATTERN OF COVARS IMBALANCE (P_i)	P_1	11.431	11.109	11.270	10.479	10.230	10.355	4.204	2.416	3.309	5.850	4.699	5.275	14.622	13.928	14.275	13.873	13.516	13.695	5.285	3.991	4.638	6.726	6.427	6.
	P_2	9.535	8.596	9.066	11.015	9.886	10.451	3.335	2.753	3.044	6.871	5.726	6.299	14.746	14.648	14.697	17.109	15.729	16.419	6.456	6.214	6.335	8.780	7.167	7.
	P_3	18.386	17.999	18.192	17.177	16.822	17.000	5.982	5.532	5.757	7.259	5.652	6.456	19.794	19.542	19.668	18.846	18.027	18.437	6.702	4.565	5.634	6.339	5.473	5.
	P_4	12.195	12.033	12.114	12.496	12.334	12.415	3.764	3.404	3.584	5.601	5.526	5.564	15.605	15.132	15.369	17.415	17.024	17.220	4.329	4.137	4.233	6.646	6.250	6.

TABLE A8.6

NON-STEADY STATE % TOTAL IDLE TIME RESULTS - MEANS & COVARS IMBALANCE (PATTERNS(V))

LINE LENGTH		5											
BUFFER CAPACITY		1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF : ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF COVARS IMBALANCE (P_i)	P_1	13.410	12.831	13.121	15.315	15.015	15.165	5.380	5.003	5.192	8.100	8.705	8.403
	P_2	8.646	7.654	8.150	9.240	9.014	9.127*	3.308	2.590	2.949	5.972	4.545	5.251
	P_3	19.759	19.311	19.535	21.042	20.506	20.774	7.608	6.456	7.032	8.388	9.501	8.945
	P_4	12.201	11.935	12.068	13.549	13.322	13.436	4.076	3.732	3.904	7.821	7.305	7.563
TRANSIENT SIZE (PATTERN $\Lambda + P_2$ ONLY)		1.143			1.016			1.329			1.167		
DUNNETT'S t VALUE (PATTERN $\Lambda + P_2$ ONLY)		0.764			1.811*			0.539			0.186		

LINE LENGTH		8											
BUFFER CAPACITY		1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF : ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	(
PATTERN OF COVARS IMBALANCE (P_i)	P_1	15.388	14.704	15.046	16.650	15.953	16.302	5.099	3.851	4.475	7.171	6.279	6.
	P_2	12.535	10.922	11.729	12.580	12.330	12.455	3.318	4.986	4.152**	5.900	4.955	5
	P_3	21.476	21.399	21.438	23.076	22.886	22.981	8.681	7.092	7.887	9.944	9.839	9
	P_4	15.842	15.455	15.649	15.884	15.473	15.679	3.934	3.932	3.933	7.209	6.273	6
TRANSIENT SIZE (PATTERN $\Lambda + P_2$ ONLY)		1.140			1.117			1.196			1.154		
DUNNETT'S t VALUE (PATTERN $\Lambda + P_2$ ONLY)		0.750			1.256			2.875**			0.714		

TABLE A8.7

NON-STEADY STATE % TOTAL IDLE TIME RESULTS - MEANS & COVARS IMBALANCE (PATTERNS(Λ))

LINE LENGTH		5											
BUFFER CAPACITY		2						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF : ①:1ST PERIOD ②:2ND PERIOD ③:2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	6.680	6.725	6.703	11.210	12.893	12.052	3.267	4.029	3.648	7.967	11.774	9.871
	B	6.899	8.232	7.566	13.362	13.850	13.606	2.959	5.535	4.247	9.301	11.792	10.547
	C	9.440	9.791	9.616	13.642	13.717	13.680	3.970	5.746	2.858	8.799	12.581	10.690
	D	8.554	8.762	8.658	13.649	14.863	14.256	3.545	5.838	4.692	8.451	12.422	10.437

LINE LENGTH		8											
BUFFER CAPACITY		2						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF : ①:1ST PERIOD ②:2ND PERIOD ③:2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	[illegible]
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	8.633	8.661	8.647	12.033	13.256	12.645	4.468	5.052	4.760	6.403	10.625	[illegible]
	B	9.657	9.859	9.758	14.229	15.384	14.807	3.711	5.283	4.497	6.194	13.059	[illegible]
	C	10.225	11.927	11.076	15.634	15.879	15.757	3.809	5.374	4.592	6.969	11.835	[illegible]
	D	8.290	8.568	8.429	12.942	14.068	13.505	3.117	5.130	4.124	6.690	11.733	[illegible]

TABLE A8.8

NON-STEADY STATE % TOTAL IDLE TIME RESULTS -

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS(/))

LINE LENGTH	5											
BUFFER CAPACITY	2						6					
DEGREE OF IMBALANCE (%)	5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE: A	10.256	10.245	10.251	14.945	14.826	14.886	5.815	4.982	5.399	11.819	11.687	11.753
B	8.618	8.566	8.592	14.380	13.396	13.888	6.677	5.347	6.012	11.812	11.746	11.779
C	7.391	6.504	6.948	12.727	12.216	12.471	5.857	4.933	5.395	11.918	11.743	11.831
D	7.971	6.791	7.381	12.682	12.566	12.624	5.889	4.985	5.437	11.535	11.417	11.476

LINE LENGTH	8										
BUFFER CAPACITY	2						6				
DEGREE OF IMBALANCE (%)	5			12			5			12	
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②
PATTERN OF BUFFER CAPACITIES IMBALANCE: A	12.351	12.801	12.576	13.334	16.368	16.351	7.549	5.690	6.620	12.985	11.673
B	11.878	10.777	11.328	16.563	16.126	16.345	7.516	5.927	6.722	13.223	12.563
C	9.786	9.041	9.414	12.852	11.913	12.383	8.106	6.295	7.151	12.752	11.763
D	9.287	8.855	9.071	14.972	14.190	14.581	6.930	5.403	6.167	13.345	12.687

TABLE A8.9

NON-STEADY STATE % TOTAL IDLE TIME RESULTS -

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS(\))

LINE LENGTH		5											
BUFFER CAPACITY		2						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2 ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	8.114	7.695	7.905	8.210	8.165	8.188	4.217	2.161	3.489	6.110	5.178	5.644
	B	6.494	6.156	6.325	8.096	7.948	8.022	4.333	2.960	3.647	6.586	5.463	6.025
	C	7.482	6.503	6.993	8.156	7.940	8.048	4.238	3.099	3.669	6.875	5.916	6.396
	D	6.389	5.884	6.137	7.267	7.761	7.514	3.723	2.811	3.267	6.055	5.215	5.635
TRANSIENT SIZE (PATTERN V+D ONLY)		1.048			1.040			1.536			1.156		
DUNNETT'S t VALUE (PATTERN V+D ONLY)		0.045			0.727			1.519			0.493		

LINE LENGTH		8											
BUFFER CAPACITY		2						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2 ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	(
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	10.122	9.506	9.814	11.218	10.345	10.782	5.007	3.361	4.184	7.454	6.049	6
	B	8.969	8.452	8.711	10.716	9.921	10.319	5.133	3.488	4.311	6.518	5.766	6
	C	9.756	9.754	9.755	10.313	9.853	10.08[illegible]	5.023	2.947	3.985	6.613	5.422	6
	D	7.550	7.042	7.296**	9.201	8.453	8.827	4.872	2.793	3.833	6.577	5.437	6
TRANSIENT SIZE (PATTERN V+D ONLY)		1.195			1.189			1.735			1.212		
DUNNETT'S t VALUE (PATTERN V+D ONLY)		2.590**			1.642			1.604			0.936		

TABLE A8.10

NON-STEADY STATE % TOTAL IDLE TIME RESULTS -
MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS(V))

LINE LENGTH		5											
BUFFER CAPACITY		2						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	8.441	8.565	8.506	10.920	10.020	10.470	4.673	4.487	4.580	7.137	8.218	7.678
	B	6.626	5.671	6.149	8.013	8.001	8.007	3.327	3.225	3.276	7.445	7.443	7.444
	C	8.346	8.212	8.279	10.134	10.098	10.116	3.762	3.630	3.696	6.719	8.058	7.389
	D	6.713	6.603	6.658	9.150	7.871	8.511	3.552	3.502	3.527	6.672	7.974	7.323

LINE LENGTH		8											
BUFFER CAPACITY		2						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	[illegible]
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	11.521	11.321	11.421	11.867	12.349	12.108	4.818	4.907	4.863	6.616	7.346	[illegible]
	B	8.521	8.066	8.294	9.406	9.403	9.405	4.589	3.114	3.852	6.316	6.144	[illegible]
	C	10.663	10.080	10.372	12.124	11.996	12.060	4.816	4.531	4.674	7.718	7.022	[illegible]
	D	7.423	7.212	7.318	9.218	9.031	9.125	4.310	3.627	3.969	6.360	6.211	[illegible]

TABLE A8.11

NON-STEADY STATE % TOTAL IDLE TIME RESULTS -
MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS(Λ))

TABLE A8.12

NON-STEADY STATE % TOTAL IDLE TIME RESULTS - COVARS & BUFFER CAPACITIES IMBALANCE (N = 5)

LINE LENGTH (N)	5											
MEAN BUFFER CAPACITY	2											
PATTERN OF COVARS IMBALANCE	P_1			P_2			P_3			P_4		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE A	9.342	8.688	9.015	8.154	8.081	8.118	15.021	14.298	14.660	6.692	6.458	6.575
B	6.351	5.712	6.032	7.958	6.765	7.362	11.878	11.674	11.776	8.783	8.214	8.499
C	9.296	8.618	8.957	4.350	4.296	4.323	14.123	13.533	13.828	11.062	10.665	10.864
D	6.931	6.571	6.751	5.541	4.362	4.952	11.718	10.920	11.319	9.769	9.129	9.449
TRANSIENT SIZE (PATTERN C + P_2 ONLY)				1.273								
DUNNETT'S t VALUE (PATTERN C + P_2 ONLY				1.181								

LINE LENGTH (N)	5											
MEAN BUFFER CAPACITY	6											
PATTERN OF COVARS IMBALANCE	P_1			P_2			P_3			P_4		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE A	3.847	3.380	3.614	2.875	2.588	2.732	6.586	5.646	6.116	2.693	2.203	2.
B	3.812	2.132	2.972	3.070	2.285	2.678	6.010	4.553	5.282	2.923	3.461	3.
C	3.803	3.680	3.742	2.551	1.382	1.967	6.433	6.052	6.243	3.710	3.363	3.
D	3.785	2.437	3.111	2.705	2.080	2.393	5.530	4.603	5.067	3.399	2.996	3.
TRANSIENT SIZE (PATTERN C + P_2 ONLY)				1.652								
DUNNETT'S t VALUE (PATTERN C + P_2 ONLY				1.732								

<u>TABLE A8.13</u>

<u>NON-STEADY STATE % TOTAL IDLE TIME RESULTS - COVARS & BUFFER CAPACITIES IMBALANCE (N = 8)</u>

LINE LENGTH (N)	8											
MEAN BUFFER CAPACITY	2											
PATTERN OF COVARS IMBALANCE	P_1			P_2			P_3			P_4		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE A	11.295	13.709	11.002	13.940	13.454	13.697	18.784	18.550	18.667	9.917	9.378	9.648
B	11.094	9.297	10.196	13.008	13.984	13.496	13.650	12.695	13.173	13.892	14.631	14.262
C	14.301	14.297	14.299	7.115	6.236	6.676	18.353	17.838	18.096	14.797	16.007	15.402
D	9.197	8.487	8.842	10.804	8.771	9.788	13.301	12.799	13.050	10.274	11.001	10.638
TRANSIENT SIZE (PATTERN C+P_2 ONLY)				1.097								
DUNNETT'S t VALUE (PATTERN C+P_2 ONLY)				0.429								

LINE LENGTH (N)	8											
MEAN BUFFER CAPACITY	6											
PATTERN OF COVARS IMBALANCE	P_1			P_2			P_3			P_4		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	(
PATTERN OF BUFFER CAPACITIES IMBALANCE A	5.723	5.369	5.546	6.520	3.862	5.191	9.464	9.006	9.235	4.231	3.871	4.
B	4.884	4.156	4.520	6.606	4.406	5.506	7.047	4.377	5.712	5.313	6.749	6.
C	5.453	5.385	5.419	5.397	2.521	3.959	8.761	9.958	9.360	4.445	4.210	4.
D	5.018	3.504	4.261	5.460	5.850	4.655	7.261	5.318	6.293	4.270	3.749	4.
TRANSIENT SIZE (PATTERN C+P_2 ONLY)				2.348								
DUNNETT'S t VALUE (PATTERN C+P_2 ONLY)				1.302								

TABLE A8.14

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS - MEANS IMBALANCE

RESULTS OF:
① : 1st PERIOD
② : 2ND PERIOD
③ : MEAN OF THE TWO PERIODS

LINE LENGTH		5											
BUFFER CAPACITY		1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF MEANS IMBALANCE	/	0.663	0.672	0.668	0.786	0.812	0.799	3.286	4.902	4.094	4.492	5.632	5.062
	\	0.380	0.397	0.389	0.212	0.244	0.228	0.887	0.675	0.776	0.333	0.444	0.389
	∨	0.507	0.528	0.518	0.503	0.548	0.526	1.127	1.805	1.466	1.004	2.258	1.631
	∧	0.556	0.566	0.561	0.521	0.531	0.526	2.080	2.842	2.461	2.416	2.941	2.679
TRANSIENT SIZE (PATTERN \ ONLY)		0.955			0.851			0.885			0.806		
DUNNETT'S t VALUE (PATTERN \ ONLY)		0.360			1.235			1.014			0.147		

LINE LENGTH		8											
BUFFER CAPACITY		1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF		①	②	③	①	②	③	①	②	③	①	②	[illegible]
PATTERN OF MEANS IMBALANCE	/	0.663	0.691	0.677	0.757	0.798	0.778	2.476	4.669	3.573	3.923	5.504	[illegible]
	\	0.346	0.403	0.400	0.267	0.271	0.269	0.916	1.110	1.013	0.383	0.502	[illegible]
	∨	0.539	0.549	0.544	0.518	0.583	0.551	0.988	1.901	1.445	0.984	1.676	[illegible]
	∧	0.518	0.564	0.541	0.532	0.536	0.534	1.752	2.757	2.255	2.176	3.022	[illegible]
TRANSIENT SIZE (PATTERN \ ONLY)		0.975			0.940			0.873			0.849		
DUNNETT'S t VALUE (PATTERN \ ONLY)		0.273			0.652			0.160			0.119		

TABLE A8.15

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS - COVARS IMBALANCE

LINE LENGTH	5						8					
BUFFER CAPACITY	1			6			1			6		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: TWO PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF COVARS IMBALANCE P_1	0.520	0.523	0.522	2.138	2.817	2.478	0.506	0.536	0.518	1.853	2.941	2.397
P_2	0.384	0.387	0.386	1.266	1.776	1.521	0.275	0.281	0.278	0.984	1.378	1.181
P_3	0.501	0.523	0.512	2.341	2.693	2.517	0.466	0.489	0.478	1.889	2.721	2.305
P_4	0.635	0.654	0.645	2.221	3.417	2.819	0.709	0.760	0.735	1.728	3.508	2.618
TRANSIENT SIZE (PATTERN P_1 ONLY)	0.967			0.520			0.965			0.818		
DUNNETT'S t VALUE (PATTERN P_1 ONLY)	0.204			1.550			0.159			0.107		

TABLE A8.16

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS -
BUFFER CAPACITIES IMBALANCE

LINE LENGTH		5						8					
MEAN BUFFER CAPACITY		2			6			2			6		
RESULTS OF : ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	0.524	0.531	0.528	1.041	1.850	1.446	0.453	0.472	0.463	0.945	1.256	1.101*
	B	0.816	1.188	1.032	1.582	2.377	1.980	0.800	1.003	0.902	1.046	1.567	1.307
	C	1.347	1.454	1.401	1.857	3.184	2.521	1.322	1.514	1.418	1.270	2.456	1.863
	D	1.083	1.236	1.160	1.875	3.206	2.541	0.819	0.930	0.875	1.153	1.865	1.509
TRANSIENT SIZE (PATTERN A ONLY)		0.987			0.715			0.883			0.667		
DUNNETT'S t VALUE (PATTERN A ONLY)		0.041			0.046			1.042			2.194*		

LINE LENGTH		5												8										
BUFFER CAPACITY		1						6						1						6				
DEGREE OF IMBALANCE (%)		5			12			5			12			5			12			5			12	
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.616	0.625	0.621	0.723	0.767	0.745	3.147	4.479	3.813	4.605	5.617	5.111	0.615	0.639	0.627	0.712	0.741	0.727	2.181	3.816	2.999	3.779	5.427
	P_2	0.614	0.644	0.629	0.826	0.868	0.847	2.537	5.114	3.826	4.543	5.834	5.189	0.480	0.590	0.535	0.655	0.835	0.745	1.828	3.624	2.726	3.813	5.691
	P_3	0.570	0.586	0.578	0.663	0.666	0.665	3.090	4.061	3.576	4.272	5.187	4.730	0.517	0.535	0.526	0.589	0.597	0.593	2.655	3.586	3.121	3.641	4.706
	P_4	0.717	0.733	0.725	0.798	0.824	0.811	3.469	4.789	4.129	4.591	5.503	5.047	0.760	0.795	0.778	0.805	0.856	0.831	3.148	5.045	4.097	4.139	5.381

TABLE A8.17

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS - MEANS & COVARS IMBALANCE (PATTERNS(/))

LINE LENGTH		5											
BUFFER CAPACITY		1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①:1ST PERIOD ②:2ND PERIOD ③:2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.435	0.437	0.436	0.310	0.319	0.315	0.814	1.114	0.964	0.386	0.427	0.407
	P_2	0.179	0.218	0.199	0.129	0.134	0.132	0.442	0.660	0.551	0.292	0.358	0.325
	P_3	0.437	0.450	0.444	0.344	0.356	0.350	1.602	1.736	1.669	0.826	1.015	0.921
	P_4	0.527	0.554	0.541	0.281	0.318	0.300	0.923	0.946	0.935	0.290	0.333	0.312
TRANSIENT SIZE (PATTERN \ + P_2 ONLY)		0.878			0.970			0.799			0.885		
DUNNETT'S t VALUE (PATTERN \ + P_2 ONLY)		0.192			0.063			0.010			0.057		

LINE LENGTH		8												
BUFFER CAPACITY								6						
DEGREE OF IMBALANCE (%)		5			12			5			12			
RESULTS OF: ①:1ST PERIOD ②:2ND PERIOD ③:2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.427	0.451	0.439	0.299	0.331	0.315	1.079	1.511	1.295	0.683	1.059	0
	P_2	0.178	0.203	0.191	0.130	0.131	0.131	0.513	0.738	0.626	0.324	0.454	0
	P_3	0.427	0.445	0.436	0.371	0.381	0.376	1.519	1.896	1.708	0.913	0.997	0.
	P_4	0.635	0.670	0.653	0.433	0.440	0.437	0.839	1.089	0.964	0.670	1.023	0
TRANSIENT SIZE (PATTERN \ + P_2 ONLY)		0.908			0.963			0.812			0.827			
DUNNETT'S t VALUE (PATTERN \ + P_2 ONLY)		0.385			0.364			0.901			0.032			

TABLE A8.18

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS - MEANS & COVARS IMBALANCE (PATTERNS (x))

LINE LENGTH		5											
BUFFER CAPACITY		1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.525	0.531	0.528	0.495	0.547	0.521	1.481	2.742	2.112	0.933	1.664	1.299
	P_2	0.376	0.419	0.398	0.407	0.430	0.419	1.276	2.351	1.814	0.935	1.736	1.336
	P_3	0.517	0.575	0.546	0.502	0.504	0.503	2.010	2.863	2.437	1.659	2.692	2.176
	P_4	0.666	0.678	0.672	0.658	0.664	0.661	1.478	2.843	2.161	1.423	2.172	1.798

LINE LENGTH		8											
BUFFER CAPACITY		1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	
PATTERN OF COVARS IMBALANCE (P_i)	P_1	0.489	0.524	0.507	0.509	0.535	0.522	1.405	1.989	1.697	1.008	1.601	1.
	P_2	0.287	0.300	0.294	0.263	0.304	0.284	0.899	1.047	0.973	0.786	1.227	1.
	P_3	0.482	0.487	0.485	0.457	0.482	0.470	1.689	2.561	2.125	1.402	2.261	1.
	P_4	0.715	0.746	0.731	0.758	0.810	0.784	1.480	2.636	2.058	1.273	1.947	1.

TABLE A8.19

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS - MEANS & COVARS IMBALANCE (PATTERNS(V))

LINE LENGTH		5												8											
BUFFER CAPACITY		1						6						1						6					
DEGREE OF IMBALANCE (%)		5			12			5			12			5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF COVARS IMBALANCE (P)	P_1	0.513	0.520	0.517	0.514	0.520	0.517	2.379	3.129	2.754	2.734	3.115	2.925	0.508	0.536	0.522	0.513	0.526	0.520	1.925	2.878	2.402	2.167	3.162	2.[illegible]
	P_2	0.341	0.407	0.374	0.377	0.440	0.409	1.288	2.098	1.693	2.203	2.836	2.520	0.321	0.332	0.327	0.326	0.344	0.335	1.090	1.464	1.277	1.517	2.513	2.[illegible]
	P_3	0.489	0.506	0.495	0.504	0.511	0.508	2.529	2.886	2.708	2.612	3.081	2.847	0.468	0.505	0.487	0.501	0.515	0.508	2.084	2.857	2.471	2.480	2.915	2.[illegible]
	P_4	0.617	0.643	0.630	0.612	0.613	0.613	2.541	3.704	3.123	2.576	3.161	2.869	0.681	0.730	0.706	0.678	0.679	0.679	2.126	3.556	2.841	2.160	3.341	2.[illegible]

TABLE A8.20

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS - MEANS & COVARS IMBALANCE (PATTERNS(Λ))

LINE LENGTH		5											
BUFFER CAPACITY		2						6					
DEGREE OF IMBALANCE (%)		5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	0.946	1.057	1.002	1.494	1.686	1.590	2.787	4.589	3.688	4.481	5.639	5.060
	B	1.224	1.538	1.381	1.680	1.809	1.745	2.453	5.021	3.987	4.701	5.631	5.166
	C	1.542	1.664	1.603	1.729	1.799	1.764	3.511	5.140	4.326	4.731	5.629	5.180
	D	1.470	1.635	1.553	1.719	1.814	1.767	3.523	5.081	4.302	4.644	5.666	5.155

LINE LENGTH		8										
BUFFER CAPACITY		2						6				
DEGREE OF IMBALANCE (%)		5			12			5			12	
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	0.931	1.079	1.005	1.433	1.665	1.549	2.369	3.662	3.016	3.560	5.269
	B	1.273	1.655	1.464	1.591	1.808	1.700	2.465	4.908	3.687	3.782	5.656
	C	1.530	1.684	1.607	1.667	1.795	1.731	2.797	4.931	3.864	4.073	5.548
	D	1.317	1.477	1.397	1.591	1.753	1.672	2.526	4.788	3.657	3.985	5.475

TABLE A8.21

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS -
MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS(/))

LINE LENGTH	5												8											
BUFFER CAPACITY	2						6						2						6					
DEGREE OF IMBALANCE (%)	5			12			5			12			5			12			5			12		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	
PATTERN OF BUFFER CAPACITIES IMBALANCE A	0.347	0.353	0.350	0.211	0.217	0.214	0.738	0.764	0.751	0.317	0.360	0.339	0.370	0.363	0.367	0.233	0.235	0.234	0.858	0.925	0.892	0.347	0.448	0
B	0.463	0.478	0.471	0.227	0.251	0.239	0.791	0.879	0.835	0.371	0.411	0.391	0.366	0.437	0.402	0.228	0.240	0.234	0.830	1.018	0.924	0.371	0.458	0
C	0.892	1.006	0.949	0.357	0.389	0.373	0.996	0.958	0.977	0.402	0.439	0.421	0.616	0.949	0.783	0.434	0.457	0.446	0.858	0.925	0.892	0.418	0.519	0
D	0.629	0.700	0.665	0.309	0.336	0.323	0.898	0.955	0.927	0.417	0.426	0.422	0.503	0.567	0.535	0.315	0.341	0.328	0.895	1.028	0.962	0.352	0.505	0
TRANSIENT SIZE (PATTERN \ + A ONLY)	0.982			0.977			0.980			0.916			0.961			0.947			0.911			0.865		
DUNNETT'S t VALUE (PATTERN \ + A ONLY)	0.104			0.125			0.015			0.233			0.818			0.571			0.260			0.038		

TABLE A8.22

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS - MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS(\))

LINE LENGTH	5												8												
BUFFER CAPACITY	2						6						2						6						
DEGREE OF IMBALANCE (%)	5			12			5			12			5			12			5			12			
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	1	2	3
PATTERN OF BUFFER CAPACITIES IMBALANCE: A	0.517	0.601	0.559	0.631	0.701	0.666	1.265	2.545	1.905	1.061	1.886	1.474	0.457	0.624	0.544	0.420	0.530	0.475	0.986	1.635	1.311	0.802	1.504	1.1
B	0.777	0.824	0.801	0.885	0.832	0.859	1.199	2.156	1.678	0.932	1.847	1.390	0.754	1.102	0.928	0.554	0.938	0.746	1.173	1.933	1.553	0.950	1.687	1.3
C	1.124	1.521	1.323	1.226	1.268	1.247	1.273	1.874	1.574	0.983	1.897	1.440	1.221	1.676	1.449	1.076	1.539	1.308	1.162	2.239	1.701	0.964	1.513	1.2
D	1.198	1.406	1.302	0.937	1.322	1.130	1.429	2.032	1.731	1.002	1.657	1.330	0.770	1.080	0.925	0.715	1.125	0.920	0.914	1.785	1.350	0.897	1.497	1.1

TABLE A8.23

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS -
MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS (V))

LINE LENGTH	5												8											
BUFFER CAPACITY	2						6						2						6					
DEGREE OF IMBALANCE (%)	5			12			5			12			5			12			5			12	
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②	③	①	②
PATTERN OF BUFFER CAPACITIES IMBALANCE A	0.521	0.640	0.581	0.522	0.548	0.535	1.966	2.536	2.251	2.115	2.480	2.298	0.489	0.509	0.499	0.500	0.508	0.504	1.687	2.548	2.48	2.014	2.456
B	0.774	1.026	0.900	0.983	1.038	1.011	1.965	3.238	2.602	2.374	2.916	2.645	0.867	1.226	1.047	1.010	1.314	1.162	1.671	2.655	2.463	1.842	2.603
C	1.353	1.479	1.416	1.423	1.492	1.458	2.562	3.920	3.241	3.104	3.997	3.551	1.348	1.563	1.456	1.390	1.524	1.457	1.866	3.433	2.650	2.590	3.417
D	1.149	1.265	1.207	1.198	1.268	1.233	2.031	3.292	2.662	2.529	3.175	2.852	0.835	0.983	0.909	0.889	0.985	0.937	1.741	2.771	2.256	2.162	3.013

TABLE A8.24

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS -

MEANS & BUFFER CAPACITIES IMBALANCE (PATTERNS(∧))

TABLE A8.25

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS - COVARS & BUFFER CAPACITIES IMBALANCE (N = 5)

LINE LENGTH (N)		5											
MEAN BUFFER CAPACITY		2											
PATTERN OF COVARS IMBALANCE		P_1			P_2			P_3			P_4		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	0.548	0.561	0.555	0.280	0.294	0.287	0.583	0.604	0.594	0.663	0.814	0.739
	B	0.994	1.006	1.000	0.343	0.480	0.412	0.835	0.851	0.843	1.409	1.410	1.410
	C	1.320	1.415	1.368	0.964	1.037	1.001	1.362	1.394	1.378	1.517	1.653	1.585
	D	1.163	1.256	1.205	0.687	0.902	0.795	1.093	1.179	1.136	1.216	1.564	1.390
TRANSIENT SIZE (PATTERN A+P_2 ONLY)					0.883								
DUNNETT'S t VALUE (PATTERN A+P_2 ONLY)					0.638								

LINE LENGTH (N)		5											
MEAN BUFFER CAPACITY		6											
PATTERN OF COVARS IMBALANCE		P_1			P_2			P_3			P_4		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIODS' MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	1.724	2.459	2.092	0.474	0.796	0.635	1.992	2.458	2.225	2.342	3.564	[illegible]
	B	2.063	2.921	2.492	0.916	1.289	1.103	2.166	2.719	2.443	2.291	4.163	[illegible]
	C	2.532	3.838	3.185	1.473	2.083	1.781	2.774	3.810	3.292	2.473	4.375	[illegible]
	D	2.295	3.064	2.680	1.203	1.657	1.430	2.351	3.147	2.749	2.516	4.108	[illegible]
TRANSIENT SIZE (PATTERN A+P_2 ONLY)					0.646								
DUNNETT'S t VALUE (PATTERN A+P_2 ONLY)					0.205								

TABLE A8.26

NON-STEADY STATE MEAN BUFFER LEVEL RESULTS - COVARS & BUFFER CAPACITIES IMBALANCE (N = 8)

LINE LENGTH (N)		8											
MEAN BUFFER CAPACITY		2											
PATTERN OF COVARS IMBALANCE		P_1			P_2			P_3			P_4		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIOD'S MEAN		①	②	③	①	②	③	①	②	③	①	②	③
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	0.576	0.589	0.583	0.236	0.253	0.245	0.506	0.517	0.512	0.984	1.287	1.136
	B	0.894	1.302	1.098	0.273	0.312	0.293	0.925	1.119	1.022	1.496	1.734	1.615
	C	1.379	1.581	1.480	1.173	1.302	1.238	1.370	1.480	1.425	1.536	1.744	1.640
	D	0.823	0.834	0.829	0.417	0.496	0.457	0.823	0.860	0.842	1.372	1.553	1.463
TRANSIENT SIZE (PATTERN A + P_2 ONLY)						0.911							
DUNNETT'S t VALUE (PATTERN A + P_2 ONLY)						0.636							

LINE LENGTH (N)		8											
MEAN BUFFER CAPACITY		6											
PATTERN OF COVARS IMBALANCE		P_1			P_2			P_3			P_4		
RESULTS OF: ①: 1ST PERIOD ②: 2ND PERIOD ③: 2 PERIOD'S MEAN		①	②	③	①	②	③	①	②	③	①	②	
PATTERN OF BUFFER CAPACITIES IMBALANCE	A	1.576	2.447	2.012	0.423	0.807	0.615	1.943	2.350	2.147	1.703	3.003	2
	B	1.824	2.714	2.269	0.844	1.102	0.973	1.715	2.364	2.040	2.223	4.716	3
	C	1.906	3.301	2.604	1.105	1.897	1.501	2.310	3.413	2.862	1.761	3.519	2
	D	1.834	2.775	2.305	0.987	1.352	1.170	1.912	2.964	2.438	1.863	3.745	2
TRANSIENT SIZE (PATTERN A + P_2 ONLY)						0.670							
DUNNETT'S t VALUE (PATTERN A + P_2 ONLY)						0.082							

TABLE A8.27

NON-STEADY STATE - ADDITIONAL % TOTAL IDLE TIME & MEAN BUFFER LEVEL DATE FOR THE BEST PATTERNS

TYPE OF IMBALANCE	LINE LENGTH	BUFFER/ MEAN BUFFER CAPACITY	% DEGREE OF IMBALANCE	% IDLE TIME			MEAN BUFFER LEVEL		
				1ST PERIOD	2ND PERIOD	TRANSIENT SIZE	1ST PERIOD	2ND PERIOD	TRANSIENT SIZE
MEANS IMBALANCE	5	1	2	9.617	9.466	1.023	0.461	0.483	0.984
	5	6	2	3.566	2.432	1.731	1.609	1.586	0.962
	8	1	2	11.768	11.528	1.058	0.498	0.506	0.989
	8	6	2	4.776	3.659	2.215	2.035	2.267	0.931
COVARS IMBALANCE	5	2	-	7.046	6.792	1.598	0.533	0.548	0.835
	8	2	-	11.113	10.935	1.322	0.410	0.415	0.883
BUFFER CAPACITIES IMBALANCE	5	4	-	4.605	4.383	1.412	0.857	0.936	0.867
	8	4	-	6.659	6.210	1.576	0.843	0.993	0.812
MEANS AND COVARS IMBALANCE	5	1	2	8.734	8.194	1.157	0.237	0.250	0.819
	5	6	2	2.717	2.032	1.364	0.673	0.819	0.721
	8	1	2	12.908	12.656	1.183	0.217	0.252	0.845
	8	6	2	3.076	2.015	1.219	0.681	0.755	0.734
MEANS AND BUFFER CAPACITIES IMBALANCE	5	2	2	5.712	5.251	1.051	0.464	0.471	0.994
	5	6	2	2.817	2.076	1.542	1.229	1.253	0.982
	8	2	2	7.536	6.949	1.201	0.454	0.459	0.975
	8	6	2	4.237	3.163	1.810	1.556	1.611	0.955
COVARS AND BUFFER CAPACITIES IMBALANCE	5	4	-	2.584	2.459	1.378	0.423	0.436	0.789
	8	4	-	5.793	3.262	1.615	0.384	0.401	0.814

APPENDIX 8.3

INDEX OF FIGURES

This appendix contains a list of the figures portraying the non-steady state's results.

INDEX OF FIGURES - NON-STEADY STATE INVESTIGATIONS

APPENDIX 8.4

MEANINGS OF GRAPHS' SYMBOLS

This appendix contains the meanings of the symbols used to identify the different curves in each steady state figure.

APPENDIX 8.4

MEANINGS OF GRAPHS' SYMBOLS - NSS INVESTIGATIONS

FIGURE NUMBER	CURVES - SYMBOLS AND MEANINGS
A8.2	(■)N=8,B=6; (□)N=5,B=6; (▲)N=8,B=1; (△)N=5,B=1.
A8.3	(●)N=8; (○)N=5.
A8.4	(■)N=8; (□)N=5.
A8.5	(□)N=5,B=6; (■)N=8,B=6; (▲)N=8,B=1; (△)N=5,B=1.
A8.6	(●)N=5; (○)N=8.
A8.7	(▲)N=8,MB=6; (●)N=5,MB=6; (△)N=8,MB=2; (○)N=5,MB=2.
A8.8	(□)N=8,B=1; (■)N=5,B=1; (●)N=5,B=6; (△)N=8,B=6.
A8.9	(△)N=8; (▲)N=5.
A8.10	(□)N=5; (■)N=8.
A8.11	(▲)N=8,B=1; (△)N=5,B=1; (○)N=8,B=6; (●)N=5,B=6.
A8.12	(●)N=5,MB=2; (○)N=5,MB=6; (■)N=8,MB=2; (□)N=8,MB=6.
A8.13	(■)N=8; (□)N=5.

where

N = line length

B = buffer capacity

MB = mean buffer capacity

APPENDIX 8.5

FIGURES

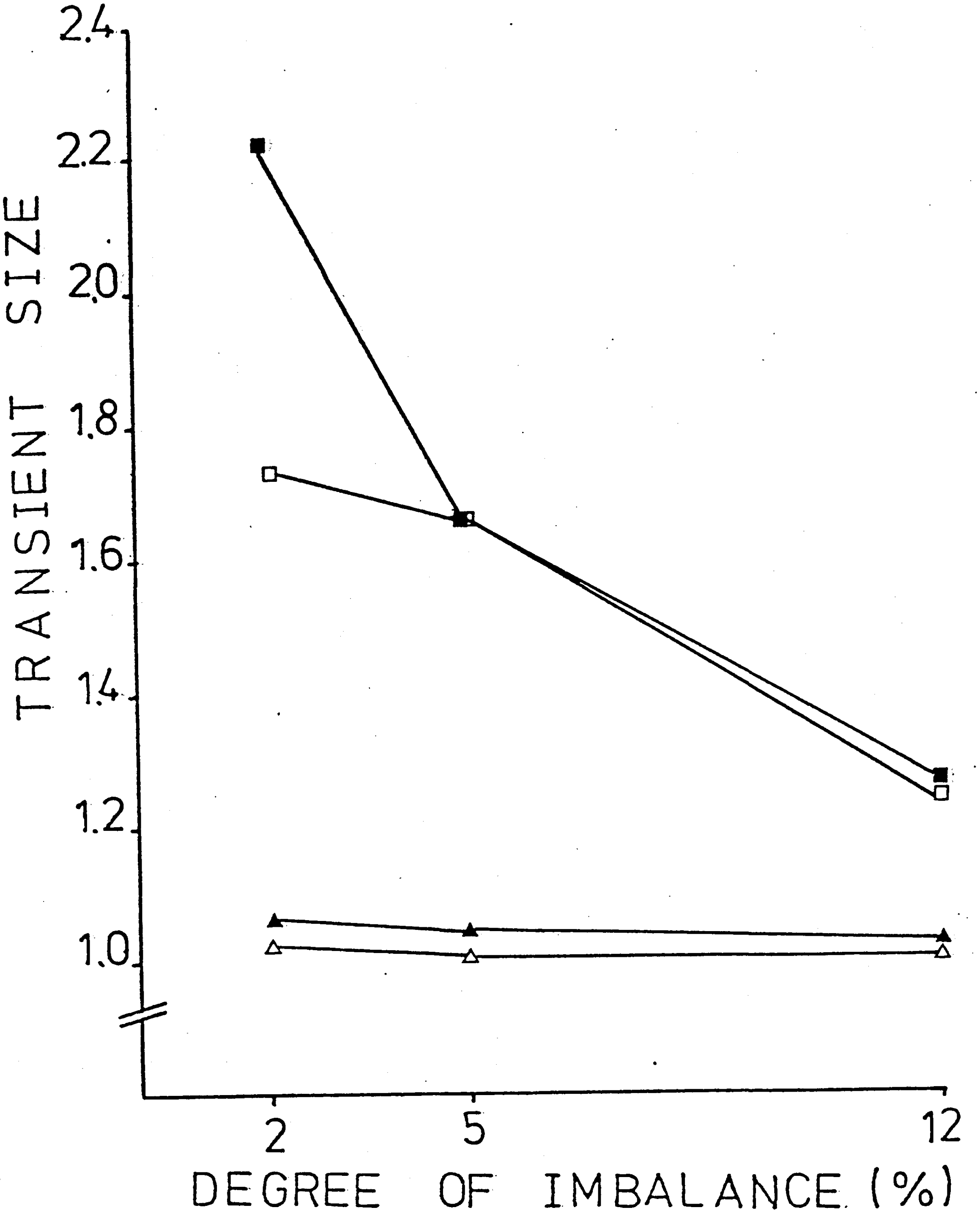
This appendix contains the results of the non-steady state simulation investigations, exhibited graphically.

(SEE TABLES A8.1 - A8.27)

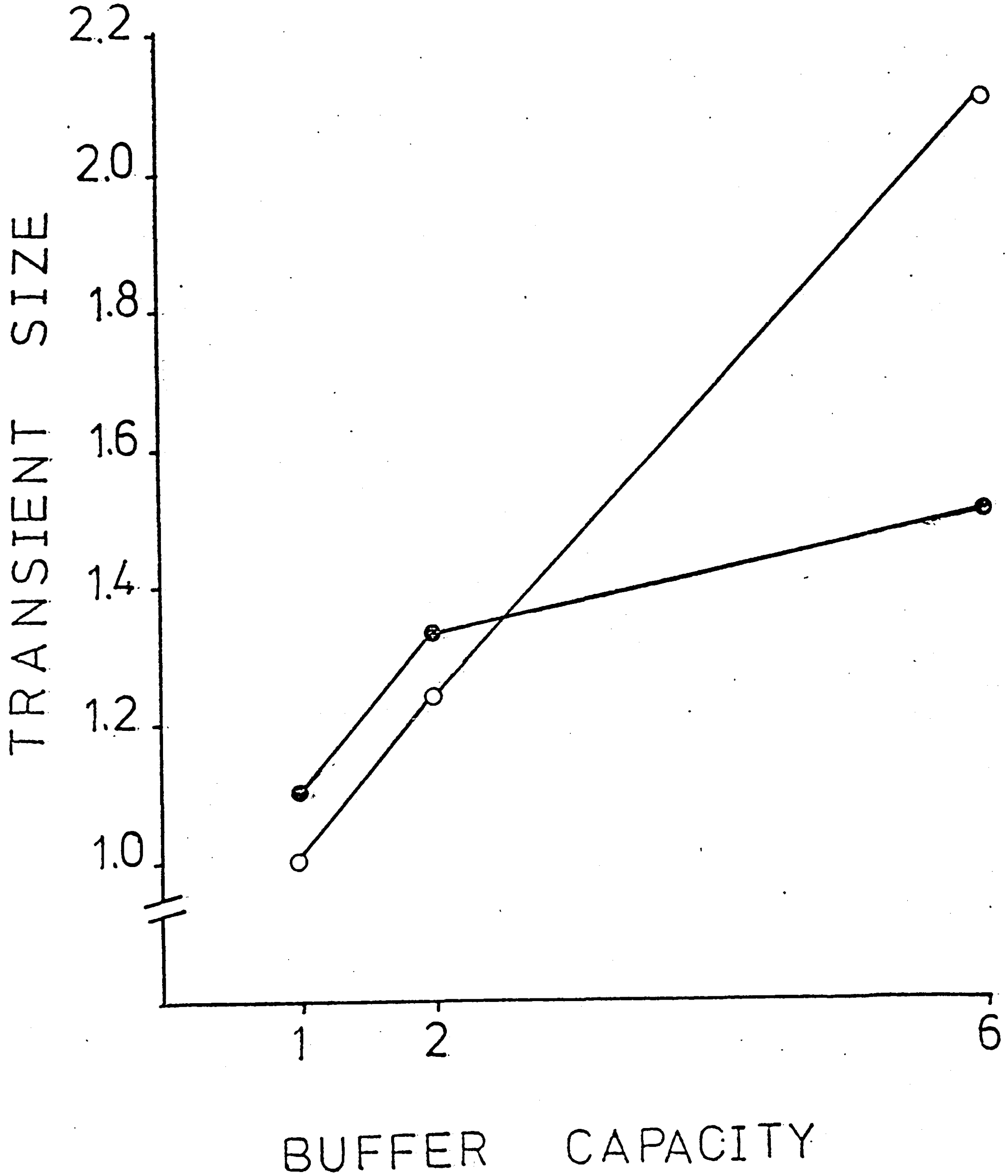
N = LINE LENGTH
MB = MEAN BUFFER CAPACITY

SYMBOL	MEANING	
/	MONOTONE INCREASING MEANS' ORDER ALL N, B	
\	MONOTONE DECREASING MEANS' ORDER ALL N, B	
V	MEANS' BOWL PHENOMENON ALL N, B	
Λ	MEANS' INVERTED BOWL ARRANGEMENT ALL N, B	
P_1	MSVMS (N = 5) MSVMSVMS (N=8)	ALL B
P_2	VSSSM (N=5) VVSSSMM (N=8) MSSSV (N=5) MMSSSSVV (N=8) — FOR (N) MEANS AND COVARS IMBALANCE	ALL B
P_3	MVVVS (N=5) MMVVVVSS (N=8)	ALL B
P_4	SMMMV (N=5) SSMMMMVV (N=8)	ALL B
A	1,1,3,3 (N=5, MB = 2) 3,3,9,9 (N= 5, MB= 6) 1,1,1,1, 6, 2, 2 (N = 8, MB = 2) 3,3,3,3, 18, 6, 6 (N = 8, MB = 6)	
B	1,3,3,1 (N = 5, MB = 2) 3,9,9,3 (N = 5, MB = 6) 1,1,6,2,2,1,1 (N = 8, MB = 2) 3,3,18,6,6,3,3 (N = 8, MB = 6)	
C	3,3,1,1 (N = 5, MB = 2) 9,9,3,3 (N = 5, MB = 6) 6,2,2,1,1,1,1 (N = 8, MB = 2) 18,6,6,3,3,3,3 (N = 8, MB = 6)	
D	2,3,2,1 (N = 5, MB = 2) 6,9,6,3 (N = 5, MB = 6) 2,2,2,3,3,1,1 (N = 8, MB = 2) 6,6,6,9,9,3,3 (N = 8, MB = 6)	

% TOTAL IDLE TIME TRANSIENT SIZE CURVES FOR MEANS IMBALANCE - PATTERN (V)

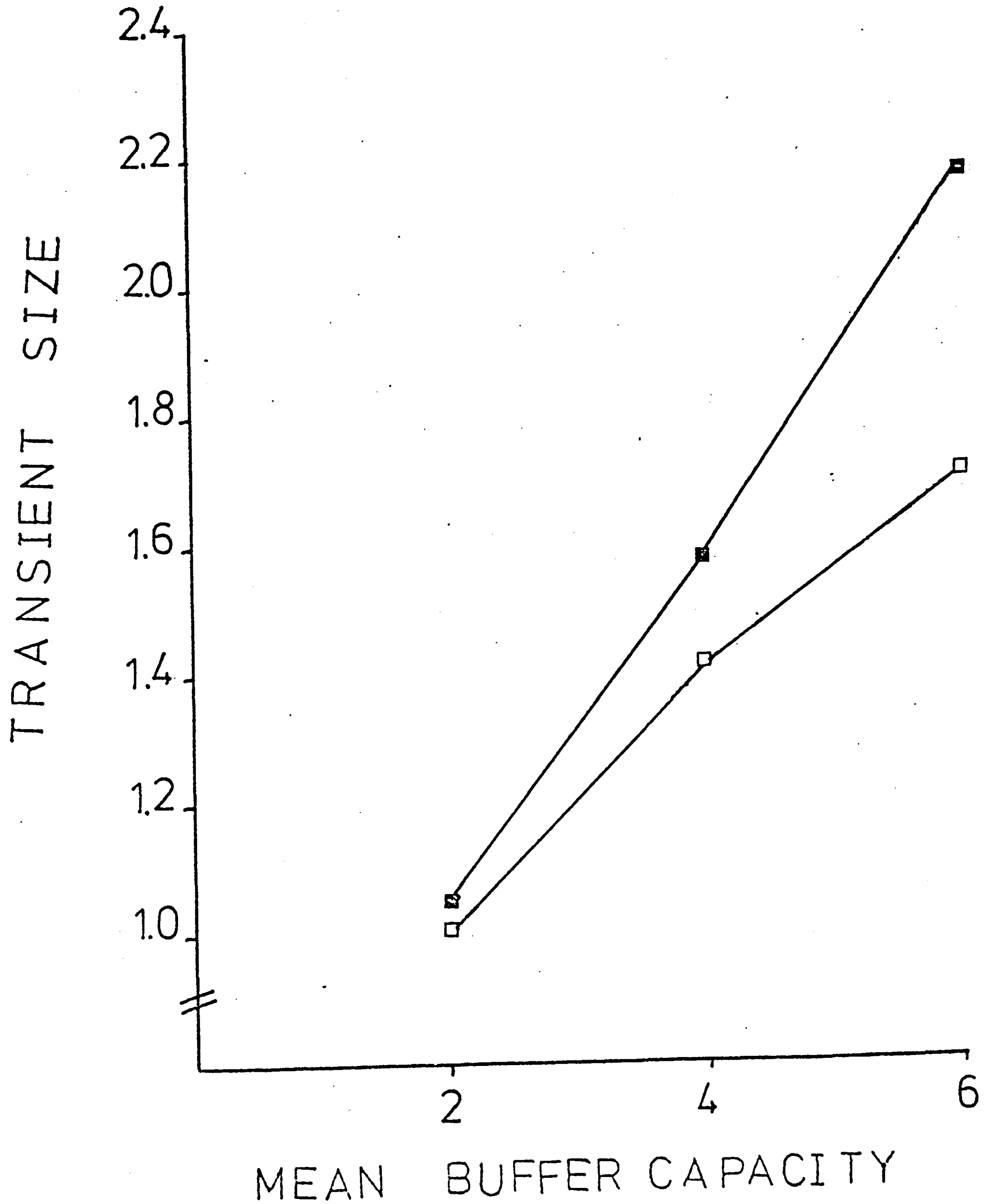


% TOTAL IDLE TIME TRANSIENT SIZE CURVES FOR COVARS IMBALANCE - PATTERN P_7, (N = 5,8)



% TOTAL IDLE TIME TRANSIENT SIZE CURVES FOR BUFFER CAPACITIES

IMBALANCE - PATTERNS $D_2(N = 5)$ AND $D_1(N = 8)$



% TOTAL IDLE TIME TRANSIENT SIZE CURVES FOR MEANS & COVARS

IMBALANCE - PATTERN (∧) $+P_5$, (N = 5,8)

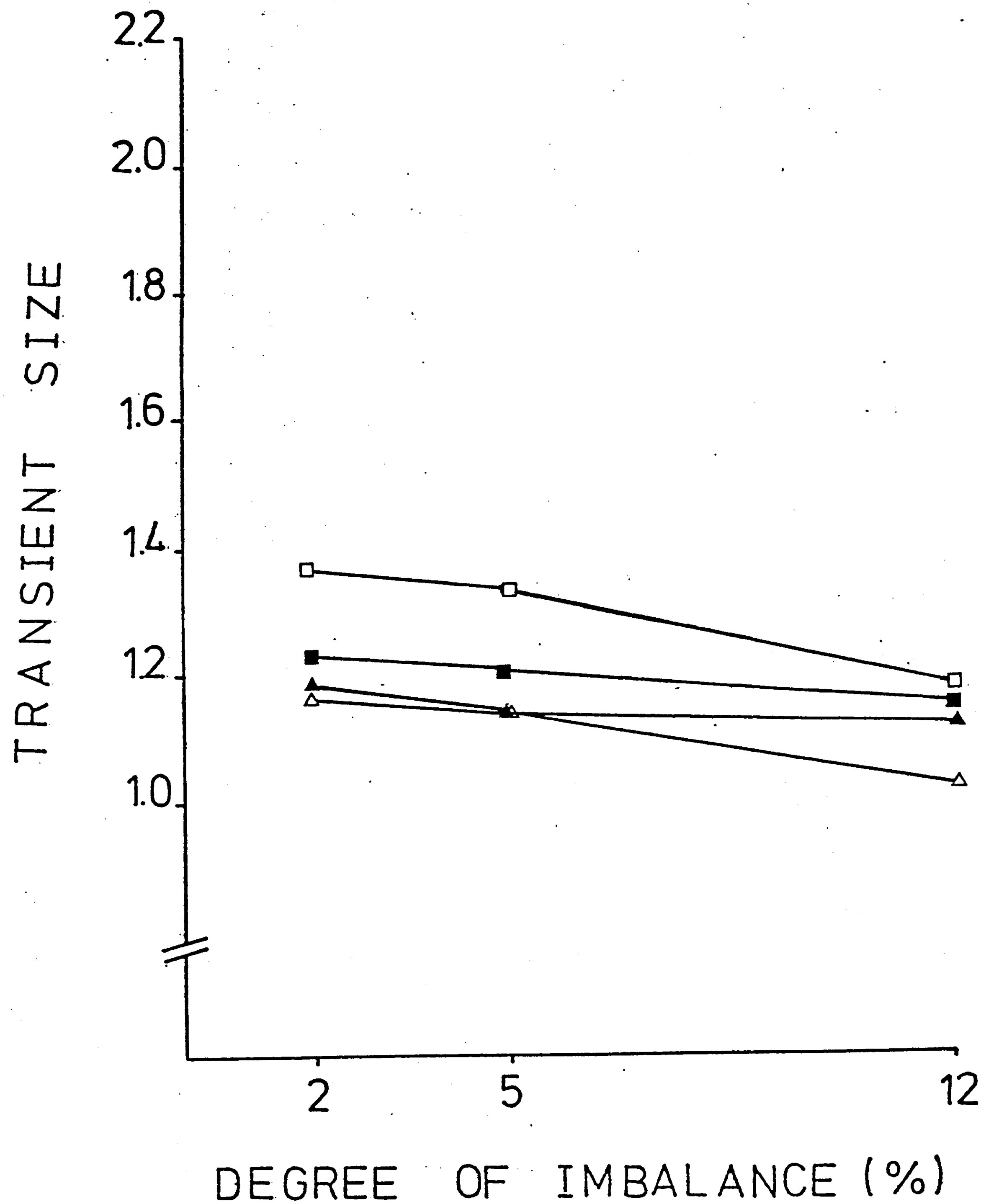


FIGURE A8.6

% TOTAL IDLE TIME TRANSIENT SIZE CURVES FOR COVARS & BUFFER CAPACITIES
IMBALANCE - PATTERN P_4 + C,(N = 5,8)

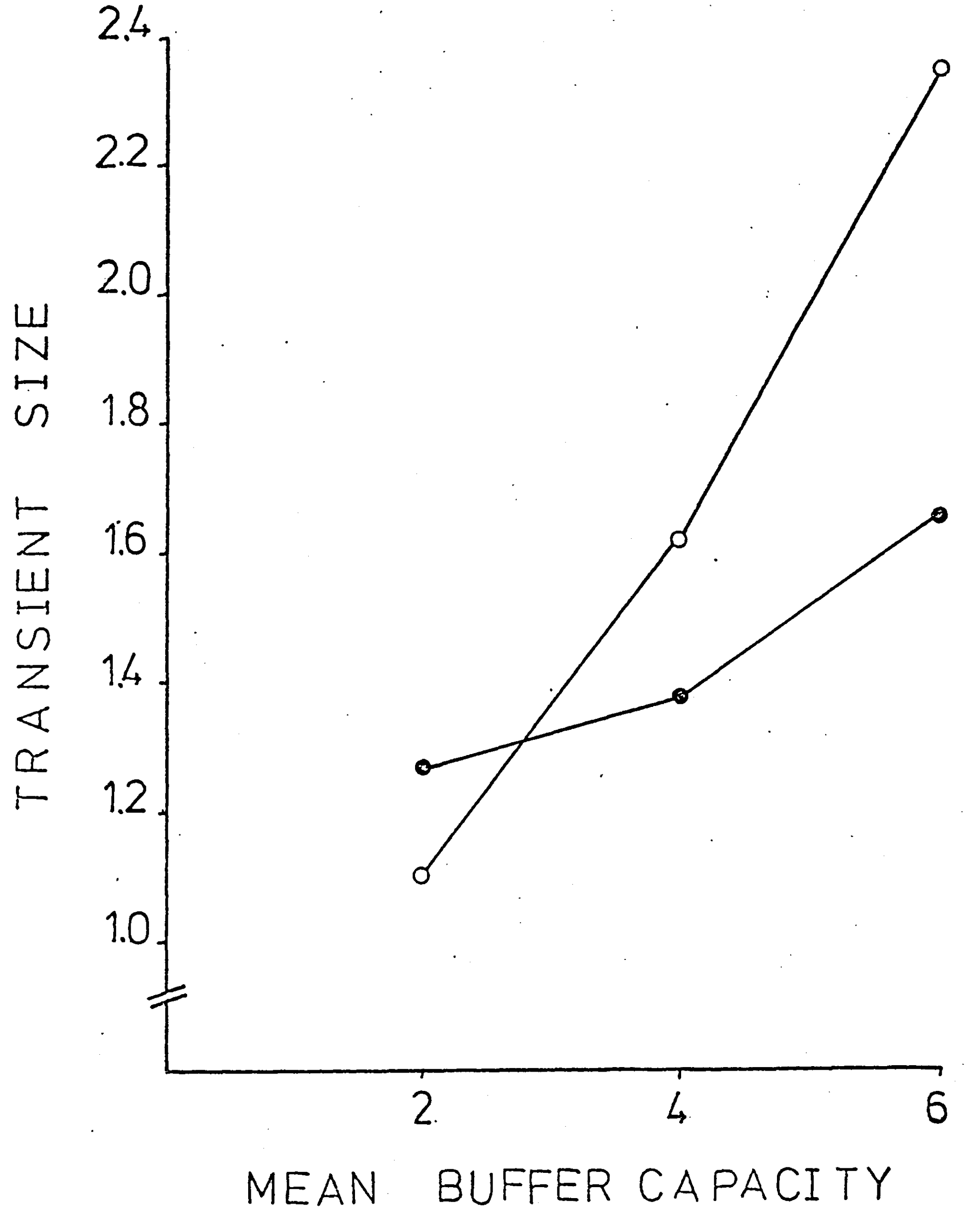


FIGURE A8.7

% TOTAL IDLE TIME TRANSIENT SIZE CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERN (V) + D_1,(N = 5,8)

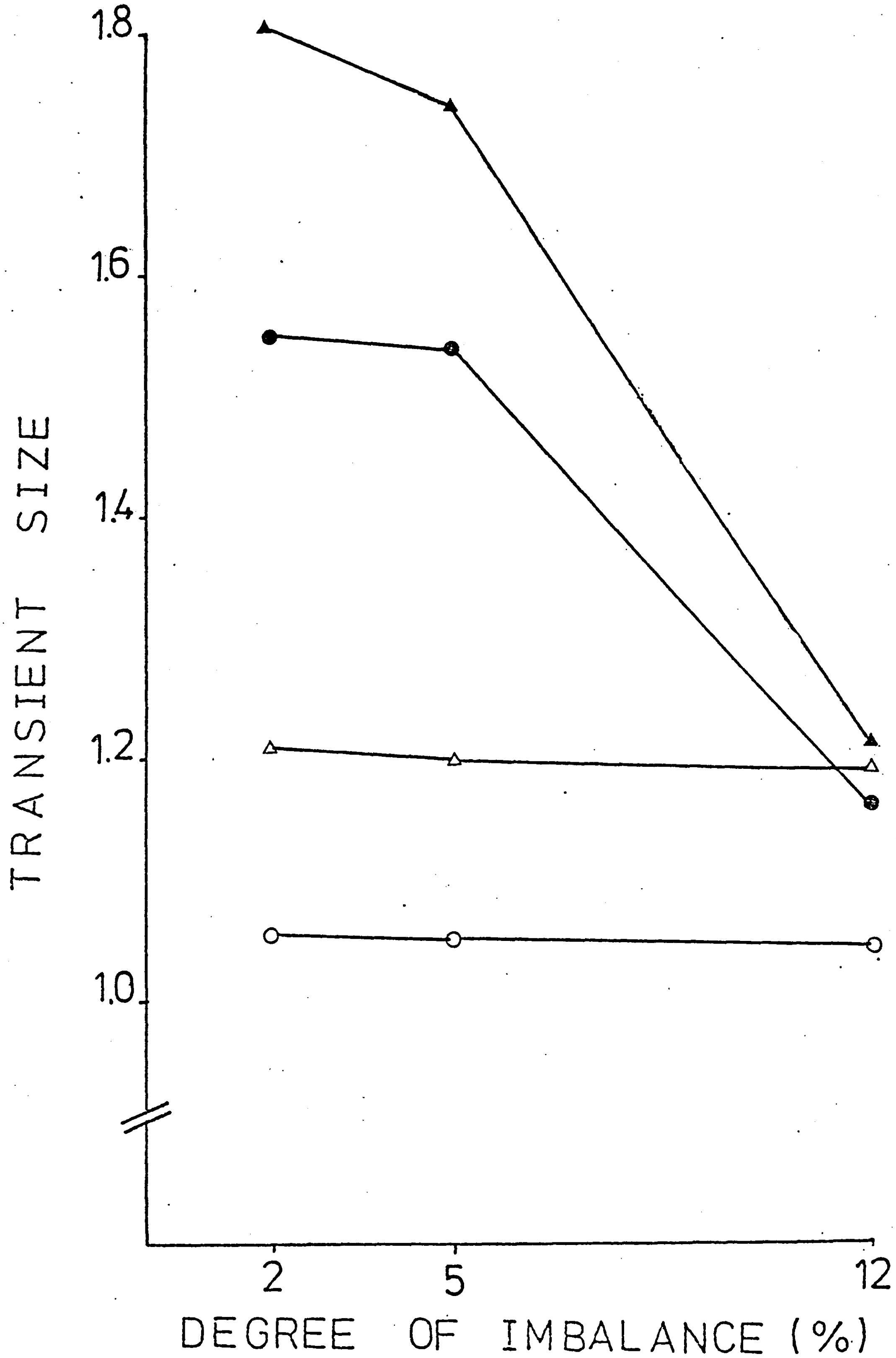


FIGURE A8.8

MEAN BUFFER LEVEL TRANSIENT SIZE CURVES FOR MEANS IMBALANCE - PATTERN (\\), (N = 5,8)

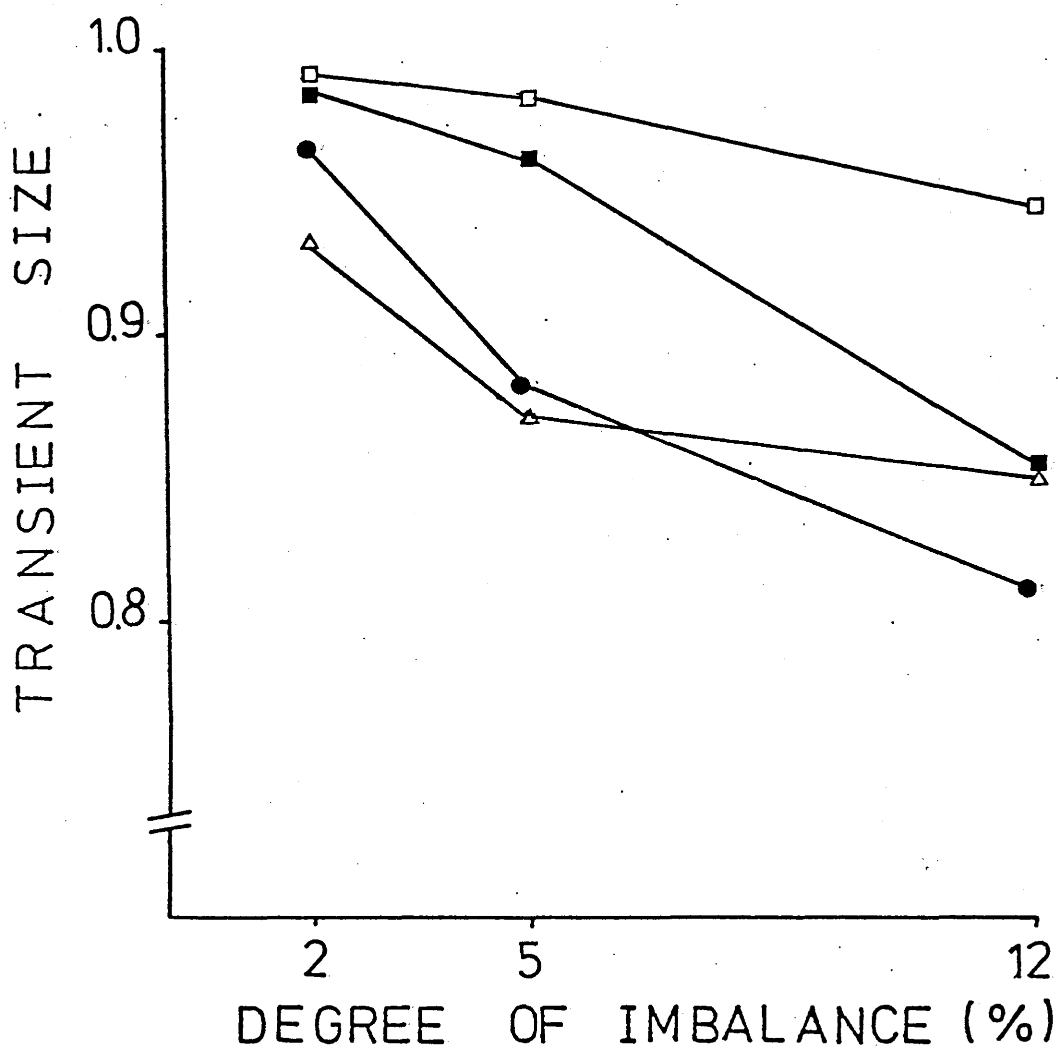


FIGURE A8.9

MEAN BUFFER LEVEL TRANSIENT SIZE CURVES FOR COVARS IMBALANCE - PATTERN P_7 (N = 5,8)

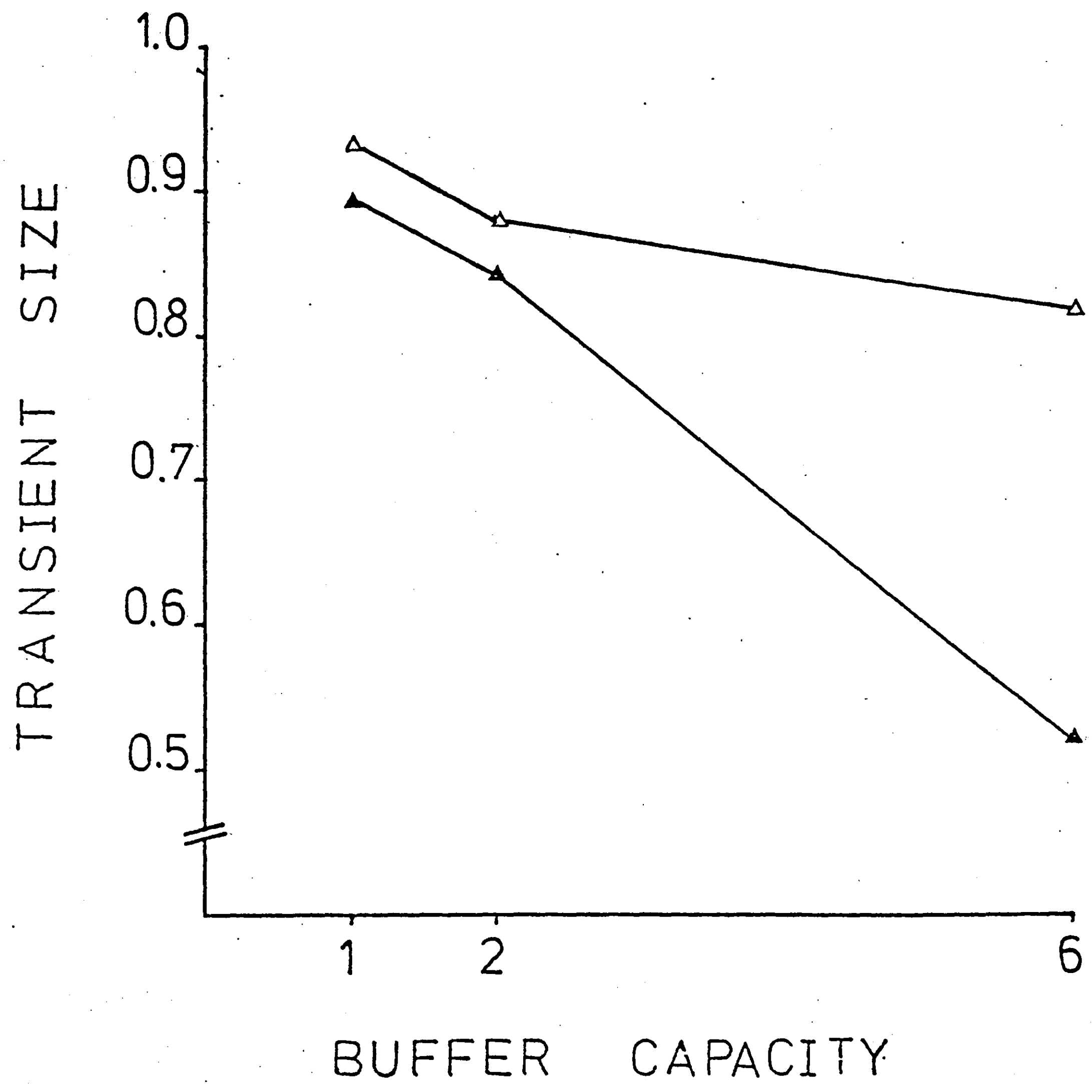


FIGURE A8.10

MEAN BUFFER LEVEL TRANSIENT SIZE CURVES FOR BUFFER CAPACITIES

IMBALANCE - PATTERNS $A_1(N = 5)$ & $A_2(N = 8)$

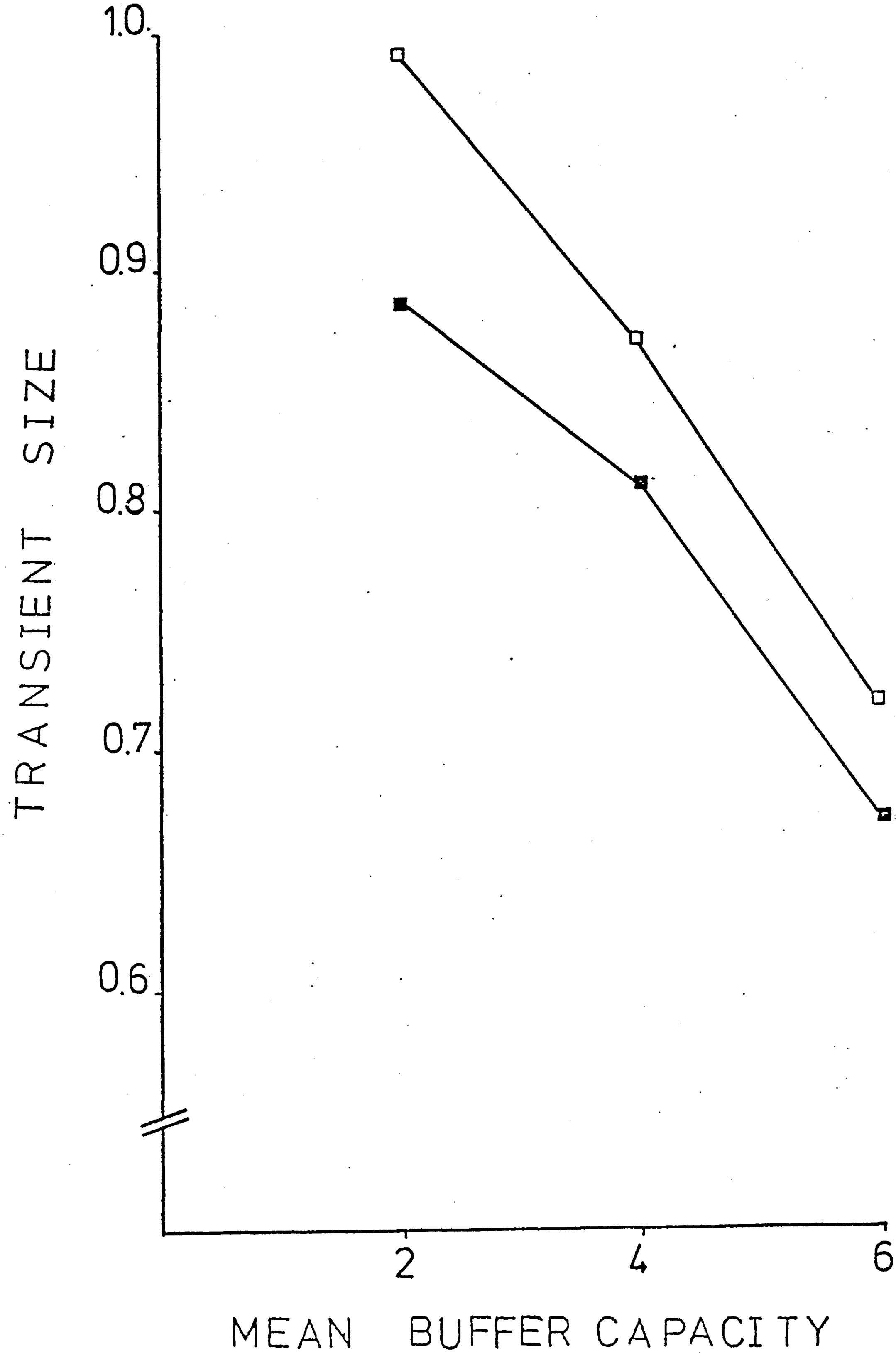


FIGURE A8.11

MEAN BUFFER LEVEL TRANSIENT SIZE CURVES FOR MEANS & COVARS

IMBALANCE - PATTERNS (↘) + P_4,(N = 5,8)

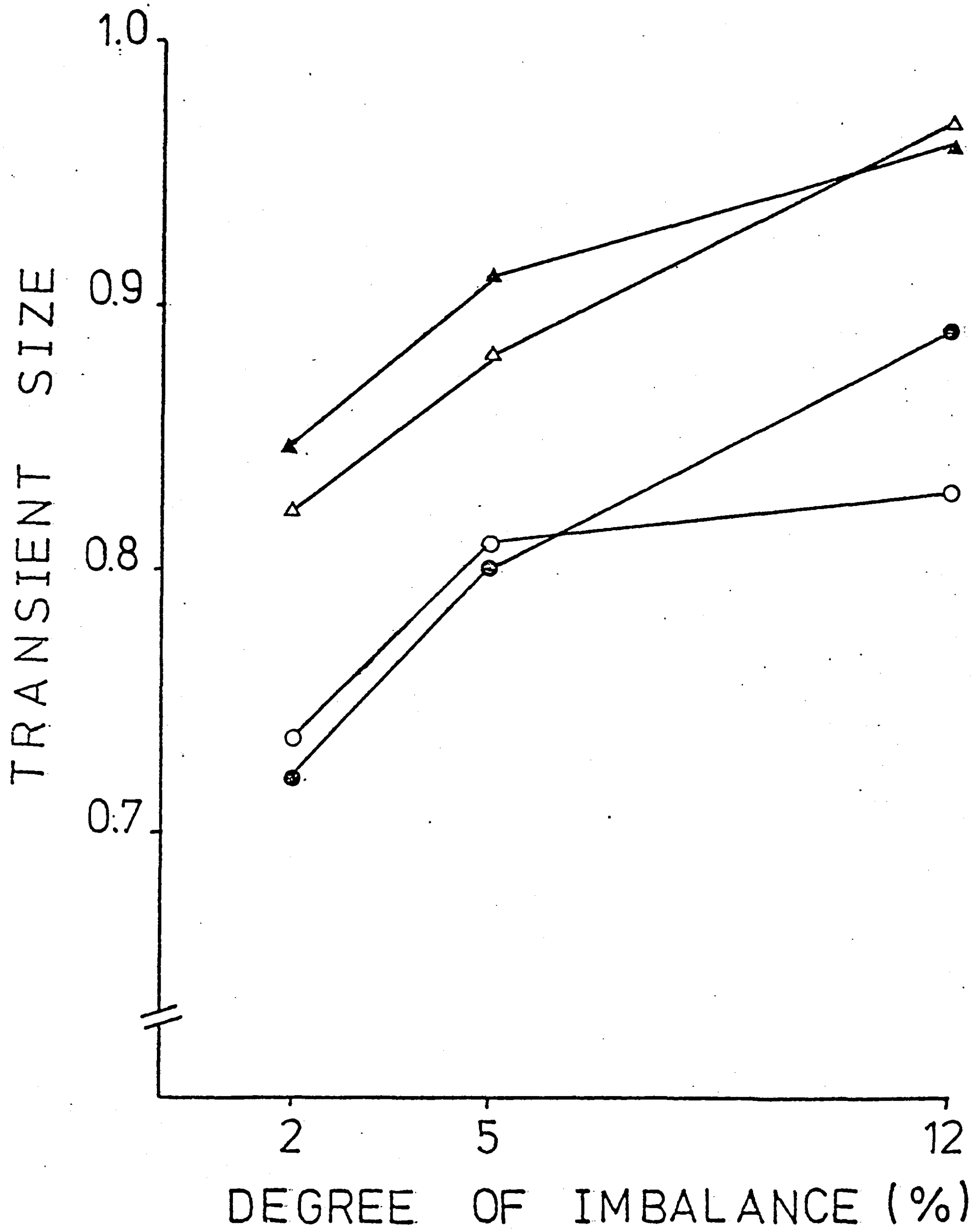


FIGURE A8.12

MEAN BUFFER LEVEL TRANSIENT SIZE CURVES FOR MEANS & BUFFER CAPACITIES IMBALANCE - PATTERNS (↘) + A, (N = 5,8)

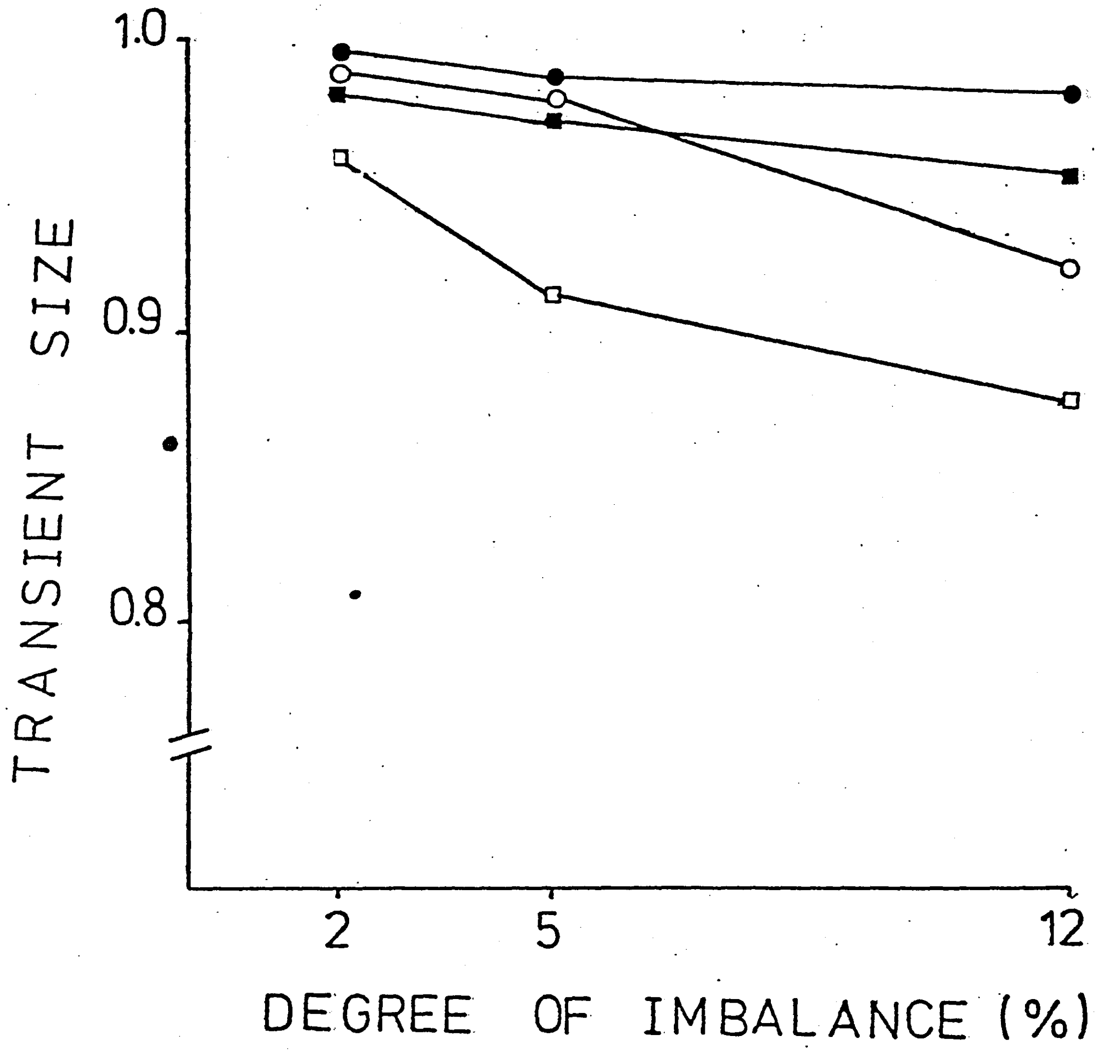


FIGURE A8.15

MEAN BUFFER LEVEL TRANSIENT SIZE CURVES FOR COVARS &
BUFFER CAPACITIES IMBALANCE - PATTERNS P_4 + A,(N = 5,8)