Causes and Macroeconomic Consequences of Time Variations in Wage Indexation

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Causes and Macroeconomic Consequences of Time Variations in Wage Indexation

Oorzaken en macroeconomische gevolgen van tijdsvariatie in loonindexatie

Thesis

to obtain the degree of Doctor from the
Erasmus University Rotterdam
by command of the rector magnificus

Prof.dr. H.A.P. Pols

and in accordance with the decision of the Doctorate Board.

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by

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Dedication

TO

MY FATHER ENOS DAVID KOMLA ATTEY

For his unconditional love and support, and whose exemplary sacrifice almost single-handedly contributed to what I am today

PROF.DR. JAN WILLEM GUNNING

For strongly making the case for me to be accepted into the MPhil/PhD programme at the Tinbergen Institute

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For her kindheartedness, without which I would not have been able to complete my PhD training

AND

MY BROTHER DONUDENU ADJORLOLO

For his strong belief in me, for his kindness and for having been there for me whenever I needed him throughout my whole life

Acknowledgements

The bus from Flåm just stopped at Fagernes. I have an hour to wait for the connecting bus to Oslo. The old me would have fretted at the delay, but I should be the most ungrateful person to complain. The view outside the bus station is really beautiful. Houses are dotted on the surrounding hills as if a chef had placed them there meticulously to serve as garnishment. Besides, this is a great time and an inspiring environment to begin writing out this acknowledgement. In short, I have learnt to enjoy the little things in life and the time spent as a PhD candidate perhaps is largely responsible for this valuable life lesson.

There are other precious life lessons I learnt during my years as a PhD candidate: to make time for people and things I enjoy and to enjoy the process of research no matter how gruelling it might be. Of course, my personal growth would not have been possible had it not been for the wonderful and inspiring people I was fortunate enough to meet during my PhD journey. It is some of these people I would like to thank in the subsequent paragraphs. For those that I forgot to mention, please know that I am still appreciative of the impact you had in my life.

My doctoral journey began on the 2nd of February 2008, when I received an email from Arianne indicating the Tinbergen Institute's decision to admit me into the MPhil programme. Needless to say, I was very excited at the prospects that this opportunity offered me. One can therefore understand the frustrations I felt at the Dutch embassy in Ghana, when the security would not understand that I was there to pick up my visa and not to apply for one. That incidence initiated my contact with Judith, who was patient enough to put up with all my tantrums. She has been of valuable assistance ever since, the assistance for which I am very grateful. I would also like to thank the following (ex) TI staff members, who were always ready to offer me help whenever I needed one: Babs, Esther, Carolien, Christina and Carine. Also, I learnt a lot from my interactions with the following TI MPhil/PhD colleagues: Rui, Chin-Kuan, Mark, Pedro, Max, Patrick, Philipe, Ona, Pinar, Ivan, Wei, Uyanga, Yun, Sait, Shen and Yang. Esmee, thanks for going through the Dutch translation of the summary.

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Travelling has been a favourite pastime I picked up during my doctoral training. I soon discovered that there are people with whom one shares connections that withstand the test of long distances. Ismanou, Sarah, Sebastien and Amin, I count myself really lucky to have you in my life. Thanks for being there. To my Ehsani family (Amin, Amir, Ehsan, Akdas and Ahmad) in Andimeshk Iran, I love you and will definitely see you again soon. Irakli, I will try to keep in touch a bit more often. Jael, thanks for your timely words of encouragement when I felt like giving up. Marica was someone whose experience I could count on when dealing with problems. Mtchris and Sefakor, you guys have often been my source of encouragement. I could not have finished this thesis without you. Kate, your legendary courage is always an inspiration. Cristelle and Chimene, let us meet up again soon.

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Finally to my paranymphs Rei and Irene, I say a big thank you. Rei, your warmth is nothing short of being legendary. I could go on and on about how you have always been supportive of me, be it concerning hiking, academics or in times of material need. I often had to rely on you for emotional support and you took all in without ever complaining. No words can explain how grateful I am. Irene, I am thankful not only for your company but also for drawing on your experience when dealing with life issues. Of course, there is no way I would write such a pretty long acknowledgement without acknowledging the person mainly responsible for the completion of my thesis: my muse. She often visited me in the weirdest of places: washroom, cycling in the rain and in the shower. Please be sure to increase the frequency of your visitations because I will need them in my intended career in policy oriented research.

Jonathan Agbeko Attey

Rotterdam, November 2016

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Chapter 1

Introduction and Outline

'What can Labor do for itself? The answer is not difficult. Labor can organize, it can unify; it can consolidate its forces. This done, it can demand and command'.

–Eugene V. Debs

1.1 Introduction

Somewhere in the middle of October 2015, a lively discussion ensued between my colleague, Rutger Kaput, and me. The discussion was about my claim that powerful unions can potentially play a destabilizing role in the economy. Mr. Kaput, who was a former policy advisor to the municipal sector of the Federatie Nederlandse Vakbeweging (FNV)[1], was of the view that people often ignore the constructive roles that unions play in the economy and society as a whole. He cited the case of the Tunisian National Dialogue Quartet as an example.[2] He drew my attention to the fact that about half of the quartet were representatives of trade (labour) and employers unions. Mr. Kaput's position is similar to the views advanced by proponents of neo-

[1]The FNV is the largest labour union in the Netherlands

[2]The quartet was awarded the 2015 Nobel Peace Prize in recognition for its role in Tunisia's transition to democracy in the wake of the Jasmine Revolution on the Friday prior to the time of the discussion.

corporatism.[3] This theory implies that a highly centralized bargaining system better internalizes the macroeconomic consequences of the actions of negotiating parties than a decentralized bargaining system. Thus, settlements to negotiations reached are better aligned with the interests of economic policy makers.

After several minutes of arguments, we converged on a common position regarding two very interrelated motivations for union behaviour: first, union behaviour is mainly motivated by the quest for survival and for justification of its existence. It is indeed this basic motivation that forces unions into national politics. Thus, one may posit that unions find their *raison d'être* in a political atmosphere characterized by protracted uncertainty. This conjecture, if true, might explain the involvement of union representatives in the National Dialogue Quartet of Tunisia. The second motivation entails safeguarding the economic interests of the employers or the employees that the unions represent. The range of activities that can be regarded as emanating from the second motivation include bargaining for wages and employment conditions. Western (1995) identifies increased global competition and worldwide recession as some factors contributing to the decline of union coverage. Perhaps, the increasing interconnectedness leading to increasing risks of crisis contagion limits the ability of unions to insulate their members from the adverse effects of economic crises. Therefore, unions are seen as not being able to secure the interests of their members when the economy is often subjected to external shocks. This might explain why the decline of union coverages in the majority of OECD member countries coincides with increased globalization.

The interest of this dissertation lies in one particular union behaviour stemming from the latter of the aforementioned motivations: wage indexation. Wage indexation can be seen as a form of wage setting that allows automatic adjustment to the evolution or future realizations of some particular factors. Aizenman (2008) defines wage indexation as :

> '... *mechanism designed to adjust wages to information that cannot be foreseen*

[3] Schmitter (1974) defines (neo)corporatism as '...*a system of interest representation in which the constituent units are organized into a limited number of singular, compulsory, noncompetitive, hierarchically ordered and functionally differentiated categories, recognized or licensed (if not created) by the state and granted a deliberate representational monopoly within their respective categories in exchange for observing certain controls on their selection of leaders and articulation of demands and supports*'.

> *when the wage contract is negotiated. A wage contract with indexation clauses will specify the wage base (that is, the money wage applicable in the absence of new information), the indexation formula that will be used to update wages, and how often updating will occur...'.*

The definition above does not preclude wages from being indexed to factors other than inflation, such as productivity. However, it is observed in Caju et al. (2008) that inflation remains the most important variable influencing wage (indexation) negotiations in practice. Furthermore, it is plausible to assume that informational lags tend to place significant constraints on the ability of unions to include the inflation rate as a basis for wage negotiations. These constraints are not as binding when it comes to including productivity as a basis for wage negotiations. This might explain why most indexation clauses are based on inflation or price level, rather than output or productivity. Following these observations, I therefore define wage indexation as the elasticity of wages to prices or lags of prices for the purposes of this dissertation. In the subsequent chapters, I use the terms *wage indexation* and *the degree of wage indexation* interchangeably.

1.2 Time-varying degree of wage indexation

The institutional settings within which wage (indexation) negotiations take place vary across countries. It is therefore plausible that the wage indexation outcomes also vary across countries. Furthermore, there is substantial evidence in support of time variations in these institutional settings. This coupled with the observation that the frequency of wage bargaining is influenced by business cycles strongly suggests time variation in the degree of wage indexation. In contrast to these observations, studies on wage indexation generally assume a time-invariant degree of wage indexation. The practice of assuming a constant degree of wage indexation is mostly done out of convenience rather than out of the attempt to mimic reality. This might be due to the difficulty faced with measuring wage indexation. The few works that include time variation of wage indexation in their analysis normally rely on US data. These works include Holland (1986), Ascari et al. (2011), and Hofmann et al. (2010). A readily available proxy variable for measuring wage indexation makes it easier to work with US data.[4] Comparable data for other countries are difficult to come by. Other works on time-varying wage indexation are mostly theoretical (see Carrillo et al. (2014) for example). One study that deserves special mention is that of Schryder et al. (2014). The authors use a panel estimation methodology to show that wage indexation varies according to the presence or absence of an inflation targeting regime.

Having gone through the scant literature on time-varying wage indexation, I have identified the following gaps. First, only a few studies explain the processes that lead to wage indexation as a random (time-varying) bargaining outcome between unions. Establishing wage indexation as a random bargaining outcome permits one to draw additional inferences on the effects of union behaviour on macroeconomic stability. Second, no study has attempted so far to produce country specific estimates of the wage indexation variable. Not knowing the cross-country variations in wage indexation limits our understanding of the factors that explain these variations. Also, it may be difficult to empirically test existing theoretical implications regarding wage indexation. In an attempt to close these existing gaps, this dissertation investigates the causes and the macroeconomic implications of time and cross country variations in wage indexation. Both theoretical and empirical methodologies are employed in this dissertation. The next section describes the outline and the research questions addressed in each of the subsequent chapters.

1.3 Outline and research questions

The remainder of this dissertation consists of four chapters which are based on studies mostly written with co-authors. The chapters are organized around specific research questions related to the causes and consequences of time variations in wage indexation and an be read independently. The outline below details the research questions addressed, the methodologies employed, and the summary of the chapters' results.

Chapter 2 is based on my Job Market Paper titled 'Time-Varying Degree of Wage Indexation and the New Keynesian Wage Phillips Curve'. The main contribution of this chapter to general

[4] Proportion of cost-of-living-adjusted contracts (COLA) is usually used as a proxy variable for wage indexation.

literature lies in the new estimation methodology proposed to measure time-varying wage indexation. The chapter addresses these three main research questions. First, can one find evidence for time variation in the existing estimates of the New Keynesian Wage Phillips Curve? Second, is there a way of estimating time-varying wage indexation using available data? Finally, what macroeconomic or other variables explain variations in the degree of wage indexation? In order to answer these questions, this chapter draws on the structural New Keynesian Wage Phillips Curve (NKWPC) developed by Gali (2011). This model assumes a constant degree of wage indexation. A rolling window regression based on the NKWPC is run to provide preliminary evidence in support of the time-varying nature of wage indexation. Also, diagnostics tests performed on the estimated NKWPC reveal an instability in the parameters that are linked to wage indexation. Subsequently, the chapter derives another version of the NKWPC with time-varying wage indexation. The resulting model is labeled the 'Time-Varying New Keynesian Wage Phillips Curve' (TV-NKWPC). The TV-NKWPC is then estimated with quarterly data of US and 10 other OECD countries using a state-space estimation methodology. The resulting estimates of wage indexation for the US are closer to the generally accepted proxy of this variable than the estimates from any other methodology. Finally, the chapter investigates the variables that explain the time variations in wage indexation. The results suggest a strong evidence for the positive effects of trend inflation. There is some evidence in support of the negative effects of the variance of productivity shocks (as predicted by Gray (1976)).

Chapter 3 is partly based on Attey and de Vries (2011) and Attey and de Vries (2013) In a way, it can be seen as complementary to the previous chapter in that it also seeks to explain the origin of the randomness of wage indexation. While the previous chapter links variation in wage indexation to macroeconomic variables, this chapter theoretically models it as a random bargaining outcome between unions. To this end, the chapter considers two settings under which wage indexation bargaining takes place. The first setting is a bargaining process in the presence of arbitration while the other setting is a 'war of attrition' type of bargaining between unions. It is shown that the bargaining outcome regarding wage indexation is random. In the case of the former setting, the distribution of wage indexation is bounded while the distribution in the latter case is unbounded from below. The chapter subsequently shows that the distribution of equilibrium inflation under optimal monetary policy exhibits fat-tailed characteristics when wages are indexed to current inflation. The random wage indexation exacerbates the effects of extreme realizations of other shocks in the model, thereby producing the fat tails in the distribution of equilibrium inflation. This property implies that one sees more extreme values of inflation more frequently than what would be predicted by current models based on a constant wage indexation. However, this fat-tailed property does not extend to the output gap. This is due to the fact that the equilibrium output gap under optimal monetary policy is simply normally distributed under the assumptions made in the model.

Chapter 4 is based on Attey and de Vries (2013). This chapter continues the analysis contained in the second part of the previous chapter by theoretically examining the implications of random wage indexation for the conduct of monetary policy. The model employed is similar in many ways to the one used in the previous chapter. The only difference is the assumption made with regards to how wages are indexed: wages are indexed to the lag of inflation rather than current inflation. This assumption appears plausible since informational lags regarding inflation make it difficult for wage setters to index their wages to the prevailing inflation rate. Under this model, inflation and the output gap tend to have a similar behaviours in their distributions. The equilibrium inflation under optimal monetary policy is shown to be an autoregressive (AR) process with a random coefficient due to the random wage indexation. It is subsequently shown that under some conditions, the unconditional distribution of the interest rate, equilibrium inflation, and output gap do exhibit heavy-tailed properties. These properties do not apply to the conditional distributions of the aforementioned three macroeconomic variables. Finally, it is shown that a Taylor rule which allows the interest rate to react to current inflation performs better than the one under which the interest rate reacts to the lag of inflation.

Chapter 5 is based on Attey and Kouame (2015). The main research question addressed in this chapter is as follows: what are the effects of the behaviour of unions on inflation volatility? Addressing this research question can be seen as a partial attempt to investigating the effects of organized union behaviour on macroeconomic stability. This chapter focuses on inflation volatility due to the often conflicting predictions regarding the effect of union behaviour on this variable. Proponents of neocorporatism usually point to the cases of Scandinavian countries to showcase the positive effects of a highly centralized bargaining system on inflation stability. Other studies meanwhile claim that decentralizing the wage bargaining process leads to more restrained wage changes. Theoretical studies backing the latter claim are difficult to come by. Herein lies the novelty of this chapter: it begins with the simple model presented in **Chapter 3** and derives unambiguous testable implications regarding the effect of the number of independent wage indexation negotiations on inflation volatility. In particular, the model predicts a negative relationship between number of independent negotiations and inflation volatility. The intuition behind this prediction follows from a simple law of large numbers logic: a higher number of independent negotiations results in a smaller variance of aggregate wage indexation which then leads to a lower volatility of inflation. This prediction can be made only if one considers a time-varying wage indexation as a bargaining outcome. Thus, one may view testing this prediction as an indirect test of the assumption made in **Chapter 3** when deriving the random wage indexation outcome.

The hypothesis is tested using a panel data estimation methodology. The panel consists of 15 OECD countries. Estimation results do indeed indicate a negative relationship between independence of negotiations and the volatility of inflation. They also indicate a positive relationship between bargaining power of negotiating unions and inflation volatility. Thus, concerning the question as to whether bargaining should be centralized or not, the message is simple: 'Decentralization is better, in the absence of which powerful and centralized unions should not be left to their own devices!'

Chapter 2

Estimating Time-Varying Wage Indexation

'In our lust for measurement, we frequently measure that which we can rather than that which we wish to measure... and forget that there is a difference'.

–George Udny Yule

2.1 Introduction

The proportion of wage contracts with cost-of-living-adjustment (COLA) clauses in the US has been observed to vary between 20% and 62% since the 1950s. This can be construed as evidence for time variation in the degree of wage indexation since this percentage of COLA coverage is a widely accepted proxy for the degree of wage indexation in the US.[1] In spite of this evidence, a substantial proportion of theoretical research on the topic typically assumes a constant degree of wage indexation. Furthermore, some recent empirical studies devoted to estimating the degree of wage indexation give a time invariant estimate to this parameter.[2]

The COLA coverage figures suggest that models regarding wage indexation should incorpo-

[1]Figure 2.5a contains a time plot of COLA coverage in the US for the period spanning 1955 to 1995 after which it was discontinued

[2]Smets and Wouters (2003) estimate a dynamic stochastic general equilibrium (DSGE) model for the Euro area. The estimate for the degree of wage indexation has a posterior mean of 0.728. More recently, Gali (2011) and Muto and Shintani (2014) estimate a New Keynesian Wage Phillips Curve for the US and Japan respectively. The estimates for the degree of wage indexation in both are statistically significant estimates.

rate the time-varying nature of wage indexation. In addition to this, there are other motivations as to why wage indexation models should consider incorporating the time-varying nature of wage indexation. We outline three of such motivations in the subsequent paragraphs.

First, the time-varying nature of wage indexation has implications for the unconditional distributions of macroeconomic variables. Current models work under the assumption of a constant degree of wage indexation. A consequence of this assumption is that macroeconomic variables are normally distributed. However, empirical evidence as documented in Chang (2012), for instance, supports the existence of fat tails in the distribution of inflation. Attey and de Vries (2011) provide a possible theoretical explanation for this empirical observation. This explanation is linked to the time-varying nature of wage indexation. The aforementioned study derives a new Classical Phillips curve under the assumption of random wage indexation and solves for equilibrium inflation in a version of the Barro-Gordon model. It is subsequently shown that under this model, an unconditional distribution of inflation exists and is fat tailed. The intuition behind this result is that shocks to the degree of wage indexation may act as multiplicative shocks rather than additive shocks. Therefore, these multiplicative shocks exacerbate the effects of any extreme realizations of other (additive) shocks to inflation, thus producing the fat tail.

Second, incorporating time variation in the degree of wage indexation into models enables one to gain additional insights into factors explaining the volatility of macroeconomic variables. For instance, it is conceivable that the distribution of the degree wage of indexation is determined ,at least in part, by labour market institutional variables such as bargaining power of unions. One can exploit this link to investigate the relationship between labour market institutional factors and the volatility of inflation since the latter variable depends on the distribution of the time-varying degree of wage indexation. Attey and Kouame (2015) confirm that the correlations between inflation volatility and labour market institutional variables are often significant. This correlation would not have been obvious if wage indexation models abstract from the time-varying nature of wage indexation.

Finally, the preceding two motivations imply that the presence of a time variation in the degree of wage indexation does have some implications for the conduct of monetary policy.

A monetary policy conducted by a Taylor rule, for instance, necessitates the response of the interest rate to shocks stemming from the degree of wage indexation. Also, the determinacy of a system associated with a policy rule depends on the degree of wage indexation. Ascari et al. (2011) show how the probability of determinacy of a system characterized by a Taylor rule depends on the level of wage indexation. In particular, the study finds that a higher degree of wage indexation increases the probability of a system being determinate. Thus, the response of macroeconomic indicators to monetary policy in the US during the period spanning the mid-1970s to early 1980s when wage indexation was relatively higher might differ from the response in other periods when wage indexation was lower, given the same Taylor rule parameters.

It stands to reason that the descriptive and prescriptive performances of wage indexation models would be greatly improved by incorporating a time-varying degree of wage indexation. However the unobservable nature of this variable limits the accuracy of this class of models. The main purpose of this paper is to develop an estimation methodology for the time-varying degree of indexation. The model employed in this study augments that of Gali (2011) with a time-varying degree of indexation and productivity growth. The resulting reduced-form expression which is labeled the time-varying New Keynesian Wage Phillips Curve (TV-NKWPC) is estimated using a state-space methodology. This methodology permits one to capture the time variations in the degree of wage indexation. The estimation is done using data of US and 10 other OECD countries, namely: Austria, Belgium, Canada, Finland, Germany, Japan, Netherlands, Norway, Sweden and the UK.

This study further investigates the factors that explain the time variation in wage indexation. In order to do this, the study performs country specific OLS estimations of wage indexation equations with trend inflation, (time-varying) variance of productivity shocks and other labour market institutional variables as explanatory variables. Given the findings in numerous studies indicating a positive relationship between inflation uncertainty (which is positively correlated with trend inflation) and wage indexation, it is expected that trend inflation will have a significant effect.

The variance of productivity shocks is included in the list of regressors in order to have an ad hoc test of the empirical validity of a hypothesis derived from Gray (1976), which predicts a negative relationship between the degree of wage indexation and productivity shocks. The test might best be described as ad hoc since the original hypothesis relies on the assumption that wage indexation is a policy instrument used by a policy maker. This study makes no such assumption. The labour market institutional variables are included to control for the bargaining power of unions and other variables that might explain the time variation in wage indexation.

Our study is not the only attempt at estimating time variations in the degree of wage indexation. Ascari et al. (2011) and Hofmann et al. (2010) have also attempted to estimate the time variations in the degree of wage indexation. The former study employs a methodology based on a rolling-window OLS regression of wage inflation on its lags and lags of price inflation. The latter study adopts a methodology based on a Bayesian VAR approach with time-varying coefficients. The estimates of the time-varying degree of wage indexation in the two studies are consistent with the general belief that wage indexation continuously fell during and after the great moderation. However, the specific values of the estimates do sometimes deviate from wage indexation figures suggested by the proportion of COLA covered contracts. For instance, estimates provided by Ascari et al. (2011) peaked around 0.9 during the 'Great Inflation' period while COLA figures suggest 0.61 as the highest value for wage indexation during this period. Also, the estimates by Hofmann et al. (2010) are less than 0.5 throughout the whole sample period. Furthermore, the methodology adopted in this paper is simpler than those of Ascari et al. (2011) and Hofmann et al. (2010).

The remainder of this paper is organized as follows. The time-varying New Keynesian Phillips Curve is derived in Section 2.2, where the NKWPC is shown to be a special case of the more general TV-NKWPC. The first part of Section 2.3 includes some diagnostic tests on an estimated NKWPC using US data in order to provide evidence for the presence of time variation in the degree of wage indexation. The second part develops and estimates the state-space regression model of the TV-NKWPC. Section 2.4 provides country-specific estimates of the TV-NKWPC for 11 OECD countries and also estimates for the OLS regression of the wage indexation equation. Finally, Section 2.5 concludes.

2.2 The New Keynesian Wage Phillips Curve

Gali (2011) derives a New Keynesian Wage Phillips Curve (NKWPC) based on the assumption of staggered wage setting by the representative household. We extend this model by incorporating a time-varying degree of wage indexation. The resulting expression is designated as the Time-Varying New Keynesian Wage Phillips Curve (TV-NKWPC). This section briefly explains the theoretical derivation of the TV-NKWPC and shows how the NKWPC is a special case of the more general TV-NKWPC.

Consider a representative household with members who can be represented by the unit square and indexed by a pair $(i, j) \in [0, 1] \times [0, 1]$, with the first dimension i representing labour type and the second dimension determining their disutility from work. Let the disutility from supplying labour type j be $\chi_t j^{\varphi}$ where the variable χ_t denotes the exogenous labour supply shock. Assume that consumption (C_t) enters utility function in a loglinear manner. This implies the following expression for the utility function:

$$\begin{aligned} U(C_t, N_t(i), \chi_t) &= \log C_t - \chi_t \int_0^1 \int_0^{N_t(i)} j^{\varphi} dj \;\; di \\ &= \log C_t - \chi_t \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} di. \end{aligned}$$

Further assume each household member supplies specialized labour which is an imperfect substitute to other members' labour supply. The aggregate labour index by the household has the following Dixit-Stiglitz form:

$$N_t \equiv \left[\int_0^1 N_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}},$$

whereby ϵ denotes the elasticity of substitution between the different labour types. An intratemporal problem of cost minimization given a wage rate $W_t(i)$ by the members of the household yields the following expression for labour supply of type i:

$$N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon} N_t.$$

The variable W_t denotes the aggregate wage index with its expression implicitly given as follows:

$$W_t \equiv \left[\int_0^1 W_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$

The representative household seeks to maximize its lifetime utility subject to its budget constraint. The objective function of the household and the budget constraint are respectively given below:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t(i)) \tag{2.1}$$

$$P_t C_t + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(i) N_t(i) di + \Upsilon_t. \tag{2.2}$$

The variable P_t represents the price level while B_t represents one-period riskless bond purchased at price Q_t. The variable Υ_t denotes the lump-sum component of income. The constraint in (2.2) is supplemented by the usual transversality conditions to prevent bubble solutions.

2.2.1 Time-varying wage indexation

In each period, a worker resets their nominal wage with probability $1-\theta$. Workers who do not get the opportunity to reset have their wages automatically indexed according to the following indexation rule:

$$W_{t+k|t} = W_t^* X_{t+k|t}, \tag{2.3}$$

where W_t^* is the optimal nominal wage level prevailing at time t for a worker who resets their wages in that period. The X_{t+k} is generally a function of inflation and other variables to which wages are indexed. Similar to the indexation rule found in the studies of Fischer (1983) and Jadresic (2002), it is assumed that workers index to both productivity and inflation.[3] Let $X_{t+k|t} = \exp(x_{t+k|t})$. The following expression for log indexation ($x_{t+k|t}$) is proposed:

$$x_{t+k|t} = \begin{cases} 0 & k = 0 \\ \sum_{s=0}^{k-1} \left(\gamma_{t+s+1}\bar{\pi}_{t+s}^{p} + (1-\gamma_{t+s+1})\pi^{p} + \phi\pi_{t+s+1}^{z} + (1-\phi)\pi^{z}\right) & k \geq 1. \end{cases} \tag{2.4}$$

where $\bar{\pi}_t^p$ and π^p denote the inflation rate (or its moving average) implied by the indexation agreement and the steady-state inflation rate respectively. The variables π_t^z and π^z denote the growth in productivity and its steady-state value respectively.

While the general features of the wage indexation rules found in the literature allow for log wages (w_t) to react in a deterministic manner to an inflation measure ($\bar{\pi}_t^p$) and productivity growth (π_t^z), our indexation rule additionally allows for the possibility of time variation in the degree of wage indexation to inflation, γ_t. Empirical estimates such as those found in Holland (1986), Ascari et al. (2011) can be interpreted as evidence for the time-varying nature of wage indexation to inflation. For this reason, we time index γ while assuming that the influence of productivity growth on the indexed part of wages is time invariant.[4] The variation in wage indexation might reflect, for instance, the varying bargaining power of unions. Also, a time-varying γ_t is more compatible with the observation that wage indexation is higher in the presence of a higher level of (trend) inflation.

As will be shown later in this section, the New Keynesian Wage Phillips Curve (NKWPC) derived under the assumption of time-varying degree of wage indexation exhibits time-varying parameters. The expression for aggregate wages (W_t) implied by the indexation expression (2.4)

[3] While the indexation rules used in the literature cited imply that wages are indexed to output and inflation, it is assumed here that wages are indexed to productivity instead of output.

[4] While this assumption may seem arbitrary, estimations provided in Table 2.12 do not reject this assumption.

is given as follows:

$$W_t = \left[\theta\left(W_{t-1}X_{t|t-1}\right)^{1-\epsilon} + (1-\theta)(W_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{2.5}$$

2.2.2 Staggered wage setting and the NKWPC

Similar to the wage setting mechanism in Erceg et al. (2000), it is assumed that a worker resets their nominal wages with probability $1-\theta$. A worker that resets their wages in period t chooses nominal wages to maximize their lifetime utility given by the equation (2.1) subject to the constraint implied by the demand for their labour. The first order condition for the household is given as follows:

$$\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[N_{t+k|t}U_c(\cdot)\left(\frac{W_t^* X_{t+k|t}}{P_{t+k}} - \mathcal{M}MRS_{t+k|t}\right)\right] = 0, \tag{2.6}$$

where MRS_t denotes the marginal rate of substitution between consumption and labour, $\mathcal{M} \equiv \epsilon/(\epsilon-1)$ denotes the wage mark-up under flexible prices and β denotes the discount factor. The specification of the utility function implies that the expression for marginal rate of substitution can be derived as follows: $MRS_{t+k|t} = C_{t+k}N^{\varphi}_{t+k|t}$. We loglinearize the expression (2.5) around a deterministic steady state. Substituting the resulting expression as well as (2.4) into a loglinearized version of (2.6) results in the following expression:

$$\pi_t^w - \nu_t = \beta E_t(\pi_{t+1}^w - \nu_{t+1}) - \lambda(\mu_t - \mu), \tag{2.7}$$

where $\nu_t = x_{t|t-1}$ and $\lambda = [(1-\theta)(1-\beta\theta)]/[(1+\epsilon\varphi)\theta]$. The variable π_t^w indicates the growth rate (defined as log-difference) of wages. The variable μ_t denotes the average markup defined as the difference between the log of real wages and the marginal rate of substitution. The expression of μ_t is given below:

$$\mu_t = w_t - p_t - [c_t + \varphi n_t + \log(\chi_t)]. \tag{2.8}$$

In the flexible price steady state, log markup only consists of the distortion caused by the presence of monopolistic competition. It can be shown from household's optimizing conditions that the steady-state markup is:[5]

$$\mu \equiv \log(\mathcal{M}) = w - p - mrs. \tag{2.9}$$

In giving an intuition behind a version of (2.7) without indexation ν_t, Gali (2008) notes the following: 'When the average wage in the economy is below the level consistent with maintaining (on average) the desired markup, households readjusting nominal wage will tend to increase the latter, thus generating positive wage inflation'. A similar intuition lies behind (2.7). We first note that average wage inflation exclusive of indexation $(\pi_t^w - \nu_t)$ is identical to wage inflation as defined by Gali (2008). Thus, the intuition behind the expression (2.7) is as follows: when the perceived markup gap is bigger, wage setting household members see less incentive to increase nominal wages, thus resulting in less wage inflation.

Unemployment is introduced into the model in a way identical to that by Gali (2011). Let $l_t(i)$ be the log labour supply of individual i in the absence of real and nominal distortions. The expression for the log of individual labour supply in this case is given by the following first order condition:

$$w_t - p_t = c_t + \varphi l_t + \log(\chi_t), \tag{2.10}$$

where $l_t = \int_0^1 l_t(i)di$. It should be noted once again that the presence of risk sharing among individuals in a household implies that the marginal utility of consumption is equal across all individuals, further implying that $c_t = c_t(i)$. We note that the unemployment associated with labour supply $l_t(i)$ is voluntary unemployment. Also, $n_t \equiv \log(N_t)$ is the effective log labour demand under monopolistic wage setting. Using these two observations, we can define the

[5]See Appendix 2.B.1 for a detailed derivation.

unemployment rate as follows:

$$u_t = l_t - n_t. \tag{2.11}$$

Substitute (2.10), and (2.11) into the definition of average wage markup in (2.8) to obtain the following:

$$\mu_t = \varphi u_t. \tag{2.12}$$

It follows from (2.12) that the natural rate of unemployment is defined as follows: $u^n = (1/\varphi)\mu$. In other words, the natural rate of unemployment in a flexible price equilibrium is solely a function of wage markup.

Finally a substitution of the expressions for unemployment and its natural rate into (2.7) permits us to derive the NKWPC as follows:

$$\pi_t^w - \nu_t = \beta E_t(\pi_{t+1}^w - \nu_{t+1}) - \lambda\varphi(u_t - u^n). \tag{2.13}$$

In order to derive a reduced-form version of the expression in (2.13) it is assumed that the unemployment gap follows the following autoregressive process of order 2 (AR(2)):[6]

$$\hat{u}_t = u_t - u^n = \phi_1 \hat{u}_{t-1} + \phi_2 \hat{u}_{t-2} + \eta_t.$$

Following Gali (2011), we suggest this process for unemployment because it seems to describe the US data quite well. Substituting this AR representation for unemployment into (2.13) and solving the resulting difference equation after assuming rational expectations yields the follow-

[6]It may be argued that this AR(2) process for unemployment seems rather ad hoc. Nevertheless, this study adopts this process in order to facilitate comparison with Gali (2011). Table 2.7 gives the results of the estimated process.

ing time-varying New Keynesian Wage Phillips Curve expression:

$$\pi_t^w = \alpha_t' + \gamma_t \bar{\pi}_{t-1}^p + \phi \pi_t^z + \psi_0 u_t + \psi_1 u_{t-1} + \xi_t, \tag{2.14a}$$

where

$$\begin{aligned}
\alpha_t' &\equiv (1-\gamma_t)\pi^p + (1-\phi)\pi^z - (\psi_0 + \psi_1)u^n \\
\psi_0 &\equiv -\frac{\lambda\varphi}{1-\beta(\phi_1 + \beta\phi_2)} \\
\psi_1 &\equiv -\frac{\lambda\varphi\beta\phi_2}{1-\beta(\phi_1 + \beta\phi_2)}.
\end{aligned}$$

The random variable ξ_t is assumed to be measurement error [7] which is uncorrelated to all the other independent variables and could possibly be an autocorrelated process. It is worth noting that the sum $\gamma_t \bar{\pi}_{t-1}^p + \phi \pi_t^z$ and the function of the time-varying parameter, α_t', are included in TV-NKWPC due to the presence of the indexed part of wages $\nu_t = x_{t|t-1}$.

We note that α_t and γ_t are negatively correlated. This property will later prove important in supporting our claim for the time-varying nature of the degree of wage indexation. The expression (2.14a) is a more general version of the NKWPC in that it also takes into account the time variation in the degree of wage indexation. Estimating the dynamics of wage inflation has the advantage of combining the microfounded nature of the model by Gali (2011) with the additional benefit of estimating the time variation in wage indexation. The TV-NKWPC nests the specification employed in Gali (2011) and Muto and Shintani (2014) as a special case in which the degree of wage indexation γ is assumed constant and there is no indexation to productivity (i.e. $\gamma_t = \gamma$ and $\phi = 0$). In this case, the specific form that (2.14a) assumes is the following expression:

$$\pi_t^w = \alpha' + \gamma \bar{\pi}_{t-1}^p + \psi_0 u_t + \psi_1 u_{t-1} + \xi_t, \tag{2.14b}$$

where $\alpha' = (1-\gamma)\pi^p - (\psi_0 + \psi_1)u^n$.

[7] It has been suggested by Gali (2011) that the error term could also capture the time variation in the desired wage mark-up.

2.3 Estimating the TV-NKWPC

The empirical fit of the TV-NKWPC is investigated in this section. First, baseline estimations of the standard NKWPC are performed using US data. Diagnostic tests are then conducted on these estimations in order to look for possible evidences supporting the instability of the estimated constant term and the coefficient of inflation(α' and γ). The basic idea behind the tests is as follows: the estimates of α' and γ will not exhibit any instability if indeed the degree of wage indexation is constant. The diagnostic tests conducted can therefore be seen as indirect tests as to whether there is time variation in the degree of wage indexation. The final part of this section demonstrates the empirical fit of TV-NKWPC when estimates are conducted using US data. Not only does the use of US data facilitate comparison of the two methodologies (Gali (2011) and our study), but also it permits one to easily compare the time-varying degree of wage indexation obtained from the TV-NKWPC estimation to corresponding figures suggested by the extent of COLA coverage.

2.3.1 Data and preliminary evidence

This study uses quarterly data spanning the period from 1948Q1 to 2012Q4 obtained from the Bureau of Labour Statistics (BLS).[8] For the measure of inflation, Consumer Price Index (CPI) inflation is used. Wages are measured using compensation data.[9] We make use of the compensation based measure of wages in order to take advantage of its relatively longer time span. Also, according to Gali (2011), both measures yield remarkably similar results. The index of output per hour is used as a proxy for labour productivity.

Table 2.1 presents the results of the estimation. The first two columns of the baseline estimation represent a model in which wages are indexed to lagged inflation ($\bar{\pi}^p_{t-1} = \pi_{t-1}$) and a model in which wages are indexed to a moving average of lags of inflation ($\bar{\pi}^p_{t-1} = \pi^{(4)}_{t-1} = \left(\sum_{k=1}^{4} \pi_{t-k}\right)/4$). A preliminary diagnostic test run on the residuals suggests that including pro-

[8]Gali (2011) uses unemployment data obtained from the Haver Database.

[9]Gali (2011) makes use of earnings data in the main part of the study due to the possibility of the presence of measurement errors in compensation data.

ductivity growth adds some explanatory power to the baseline equation.[10] The regressions in the last two columns therefore include productivity growth in the list of regressors. It is theoretically possible that productivity enters the model by means of wage indexation, i.e. wages are indexed to lags of inflation and current productivity. It can be seen from the values of the R^2 that the fit of the model is improved when productivity growth is introduced into the model. Also, the residuals from estimations in the cases of all models shown in Table 2.1 display a significant level (1%) of autocorrelation.[11]

Ascari et al. (2011) document how wage indexation rises when trend inflation increases and falls when trend inflation decreases. This observation suggests the existence of instability in the NKWPC when wages are indexed to inflation. Guided by this observation, we conduct further diagnostic tests on the residuals from the regressions in Table 2.1 by including a nonlinear term, namely: the product of trend inflation and the measure of inflation indexed to, i.e. $\pi^{\tau}_{t-1}\pi_{t-1}$ or $\pi^{\tau}_{t-1}\pi^{4}_{t-1}$. Trend inflation is obtained by means of applying the HP filter to the quarterly inflation series. Results from Table (2.2) indicate a strong effect of a nonlinear term in both cases of wage indexation considered. We interpret this finding as evidence in support of our claim concerning the improved fit of the TV-NKWPC.

Finally a rolling window regression on (2.14a) is performed in order to obtain an idea of the time-varying parameters α_t and γ_t. This is done using the following procedure. First, the constant parameters in the expression contained in (2.14b) are estimated. A rolling regression is subsequently performed in order to obtain rough estimates on the parameters in the following expression:

$$x_t = \alpha_t + \gamma_t \bar{\pi}^p_{t-1} + \epsilon_t,$$

where $x_t \equiv \pi^w_t - \hat{\psi}_0 \hat{u}_t - \hat{\psi}_1 \hat{u}_{t-1}$ and ϵ_t is an independent and identically distributed (iid) zero mean normally random distributed error term. The results under the two assumptions regarding wage indexation considered are respectively presented in Figure 2.1 and Figure 2.2. In both

[10]Formal causality tests indicate that productivity growth Granger causes unemployment

[11]This is reported in Table 2.9 in the appendix

figures, the estimated time-varying degree of wage indexation first rises to a point, after which it falls. Again, it is interesting to note that the later periods' values of wage indexation do not significantly differ from zero. This is generally in line with empirical evidence that the degree of wage indexation initially rose to high levels during the 1970s and diminished thereafter. Also, the time-varying wage indexation parameter varies between 0 and 1 in both cases. It is worth noting that the correlation between $\hat{\alpha}_t$ and $\hat{\gamma}_t$ estimated under this rolling regression technique is negative (see Table 2.8 in the appendix). This is expected if one holds the assumption that the reduced form specification in (2.14a) describes the dynamics between output and unemployment.

Figure 2.1: Rolling regression estimates for $\bar{\pi}_t^p = \pi_{t-1}$

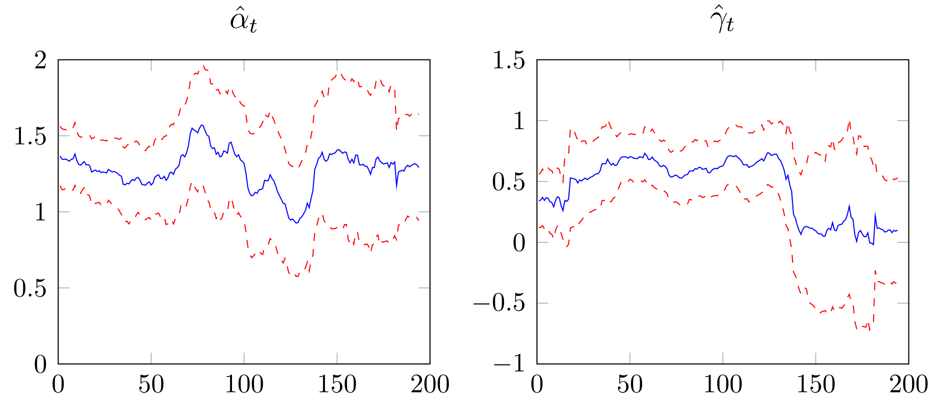


Figure 2.2: Rolling regressions estimates for $\bar{\pi}_t^p = \pi_{t-1}^{(4)}$

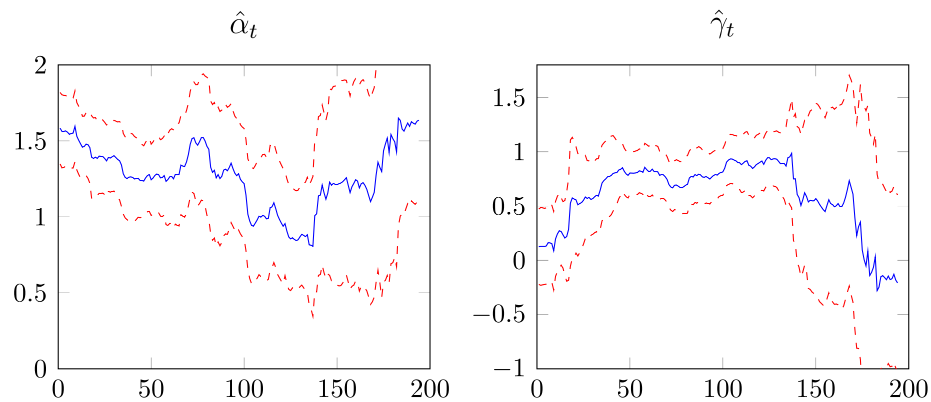


Table 2.1: Estimated NKWPC (π_t^w)

	(1)	(2)	(3)	(4)
u_t	-0.17	-0.33**	-0.114	-0.28**
	(0.091)	(0.094)	(0.091)	(0.093)
u_{t-1}	0.1	0.23*	0.032	0.175
	(0.092)	(0.093)	(0.091)	(0.093)
π_{t-1}	0.544**		0.582**	
	(0.050)		(0.050)	
π_{t-1}^4		0.673**		0.707**
		(0.062)		(0.061)
π_t^z			0.167**	0.15**
			(0.045)	(0.045)
const	1.217**	1.262**	1.139**	1.197**
	(0.143)	(0.143)	(0.141)	(0.142)
Adj-R^2	0.319	0.326	0.352	0.352
AIC	1.865	1.86	1.820	1.825

Estimation of $\pi_t^w = \alpha_0 + \gamma\bar{\pi}_{t-1}^p + \psi_0 u_t + \psi_1 u_{t-1} + +\psi_2 \pi_t^z + \epsilon_t$.
Standard errors of estimates are indicated in parenthesis.
* $p < 0.05$, ** $p < 0.01$.

Table 2.2: Estimating NKWPC residuals (ϵ_t)

	(1)	(2)	(3)	(4)
$\pi_{t-1}^{\tau}\pi_{t-1}$	0.323**		0.34**	
	(0.056)		(0.054)	
$\pi_{t-1}^{\tau}\pi_{t-1}^{(4)}$		0.420**		0.432**
		(0.074)		(0.071)
π_{t-1}	-0.518**		-0.541**	
	(0.098)		(0.095)	
$\pi_{t-1}^{(4)}$		-0.793**		-0.814**
		(0.147)		(0.143)
ϵ_{t-1}	0.111	0.117	0.074	0.099
	(0.061)	(0.061)	(0.060)	(0.061)
const	0.0120*	0.257**	0.123*	0.264**
	(0.056)	(0.72)	(0.055)	(0.070)
Adj-R^2	0.136	0.147	0.143	0.152

Estimation of $\hat{\epsilon}_t = \beta_0 + \beta_1\hat{\epsilon}_{t-1} + \beta_2\pi_{t-1}^{(p)p} + \beta_3\pi_{t-1}^{\tau}\pi_{t-1}^{(p)} + \nu_t$.
Standard errors of estimates are indicated in parenthesis.
* $p < 0.05$, ** $p < 0.01$.

2.3.2 Estimation results

In order to estimate the expression contained in (2.14a), we propose a state-space methodology with the time-varying degree of wage indexation (γ_t) and the measurement error (ξ_t) as the unobserved state variables. This estimation method requires one to give the law of motion for the time-varying wage indexation. As noted earlier, empirical findings suggest that wage indexation is positively correlated to trend inflation. If one assumes a simple linear relationship between wage indexation and trend inflation, it is possible to propose a highly persistent process for the wage indexation parameter.[12] It is therefore assumed that wage indexation behaves as if it were a random walk process over the sample period in consideration. Given that no restrictions are placed *a priori* with regards to the autocorrelation structure of the random process ξ_t, a stationary AR(1) process is assumed for this variable. We estimate the following empirical model:

$$\begin{cases} \pi_t^w & = \varphi_1 u_t + \varphi_2 u_{t-1} + \varphi_3 \pi_t^z + \mu_t + \varphi_4 \gamma_t + \gamma_t \bar{\pi}_{t-1}^p \\ \mu_t & = (1-\rho_\xi)\varphi_5 + \rho_\xi \mu_{t-1} + \varepsilon_t \\ \gamma_t & = \gamma_{t-1} + \eta_t \end{cases} . \tag{2.15}$$

where $\mu_t = \varphi_5 + \xi_t$, $\varepsilon_t \sim iid\, N(0, \sigma_\varepsilon^2)$ and $\eta_t \sim iid\, N(0, \sigma_\eta^2)$. A definition of all the coefficients contained in expression (2.15) above in terms of the structural parameters in the previous section is given in Table 2.3 below:

Table 2.3: Definition of coefficients

coef	definition	coef	definition
φ_1	$-\frac{\lambda\varphi}{1-\beta(\phi_1+\beta\phi_2)}$	φ_4	$-\pi^p$
φ_2	$\beta\phi_2\varphi_1$	φ_5	$(1-\phi)\pi^z + \pi^p - (\varphi_1+\varphi_2)u^n$
φ_3	ϕ		

[12] Formal unit-root tests run on trend inflation do not statistically reject the existence of a unit-root. We acknowledge that this specification may come off as economically implausible. An alternative specification might suggest a highly persistent but stationary process (e.g. with AR(1) coefficient 0.99). We nevertheless stick to the random walk assumptions due to the following reasons: First, there is very little difference in results between a random walk specification and the persistent AR(1) specification. Secondly, it is common practice in recent literature to assume a random walk process for trend inflation.

Following Gali (2011), the TV-NKWPC is first estimated under two assumptions with regards to price inflation: that wages are indexed to a quarter lag of price inflation (π_{t-1}) and that wages are indexed to an average inflation over the last four quarters ($\pi_{t-1}^{(4)}$). Additional estimates of the TV-NKWPC are then made under the assumption that μ_t (or ξ_t) is autocorrelated, and subsequently under the assumption that μ_t (or ξ_t) is iid normal distributed. Finally, the TV-NKWPC is estimated under the assumption that there is no autocorrelation in ξ_t ($\rho_\xi = 0$) and the coefficient of lag of unemployment is zero ($\varphi_2 = 0$).

The estimations of all versions of Equation 2.15 were performed using 7th edition of the EVIEWS statistical package. The same package was used for all other estimations in this study except for the rolling regressions which were done in MATLAB. The results from the six estimations are presented in Table 2.4. There are some observations worth noting concerning the estimates of the various versions of TV-NKWPC. First, results obtained from the estimations of the various versions of the TV-NKWPC show rather striking similarities to those obtained from estimations of the NKWPC in Table 2.1. In most cases, the values of the constant terms (φ_5 in (2.15)) imply that the coefficients of unemployment and the coefficients of productivity are roughly similar under the various specifications.[13] An implication of these similarities could be that the error term ξ_t in the NKWPC posited to be measurement error in wage inflation by Gali (2011) is most likely explained by variations in the trend inflation (as can be seen from Table 2.2). Estimates for ξ_t under the various TV-NKWPC models are independent of the time-varying wage indexation and are not autocorrelated (see Figure 2.4). Also, estimates for the linear effect of the time-varying degree of wage indexation (φ_4 or the coefficient of γ in the table) are either not statistically significant or significantly negative as predicted by the expression (2.14a). [14] Finally, as indicated by the AIC values, all the versions of TV-NKWPC estimated in the table above outperform the estimation of all the versions of the NKWPC contained in Table 2.1. One can interpret these observations as evidence in support of the relatively better empirical fit of the TV-NKWPC to US data.

[13] This similarity only holds to the extent that wages are indexed similarly under the various specifications.

[14] The identity of the coefficient φ_4 as contained in Table 2.3 implies that $\varphi_4 \leq 0$ for $\pi^p \geq 0$

Table 2.4: Estimated TV-NKWPC (π_t^w)

	$\rho_\xi \neq 0$		$\rho_\xi = 0$		$\rho_\xi = 0, \varphi_2 = 0$	
	π_{t-1}	$\pi_{t-1}^{(4)}$	π_{t-1}	$\pi_{t-1}^{(4)}$	π_{t-1}	$\pi_{t-1}^{(4)}$
	(1)	(2)	(3)	(4)	(5)	(6)
u_t	-0.194	-0.302**	-0.194	-0.309**	-0.114**	-0.158**
	(0.107)	(0.097)	(0.106)	(0.090)	(0.037)	(0.0249)
u_{t-1}	0.085	0.151	0.085	0.157		
	(0.106)	(0.100)	(0.105)	(0.094)		
π_t^z	0.157**	0.139**	0.158**	0.142**	0.166**	0.157**
	(0.038)	(0.038)	(0.038)	(0.387)	(0.037)	(0.039)
γ_t	0.484	-0.365*	0.434	-0.365*	0.539	-0.304**
	(0.625)	(0.145)	(0.585)	(0.128)	(0.596)	(0.108)
φ_5	1.265**	1.739**	1.282**	1.744**	1.269**	1.763**
	(0.302)	(0.167)	(0.290)	(1.156)	(0.299)	(0.155)
ρ_ξ	0	0.062				
	(0.054)	(0.051)				
$\ln(\sigma_\varepsilon^2)$	-1.264**	-1.235**	-1.264**	-1.236**	-1.261**	-1.226**
	(0.072)	(0.072)	(0.072)	(0.072)	(0.072)	(0.071)
$\ln(\sigma_\eta^2)$	-7.016**	-5.862**	-6.972	-5.840**	-7.079**	-5.884**
	(1.06)	(0.857)	(1.031)	(0.821)	(0.962)	(0.773)
AIC	1.782	1.787	1.774	1.782	1.770	1.786

[1] Estimation of the various versions of the TV-NKWPC in Equation (2.15).
[2] Standard errors of estimates are indicated in parenthesis
[3] * $p > 0.05$, **$p > 0.01$

Furthermore, after taking into account the time variation in the degree of wage indexation when estimating the TV-NKWPC, the lag of unemployment plays no significant role in explaining wage inflation under all the versions of the TV-NKWPC estimated. This is possibly due to the fact that the persistence in wage inflation is mostly accounted for by changes in the degree of wage indexation(which is in itself a persistent process).[15] This result and the fact that ξ_t is not an autocorrelated process ($\rho_\xi = 0$ is not rejected at 10% significance level) imply that the TV-NKWPC models (5) and (6) should be preferred to the others.

The estimates for φ_4 under the aforementioned two versions of the TV-NKWPC imply two

[15] This is similar to the findings of Cogley and Sbordone (2008) who argue that taking into account the variation of trend inflation makes the NKPC purely forward looking, with no need for an ad hoc backwards price indexation

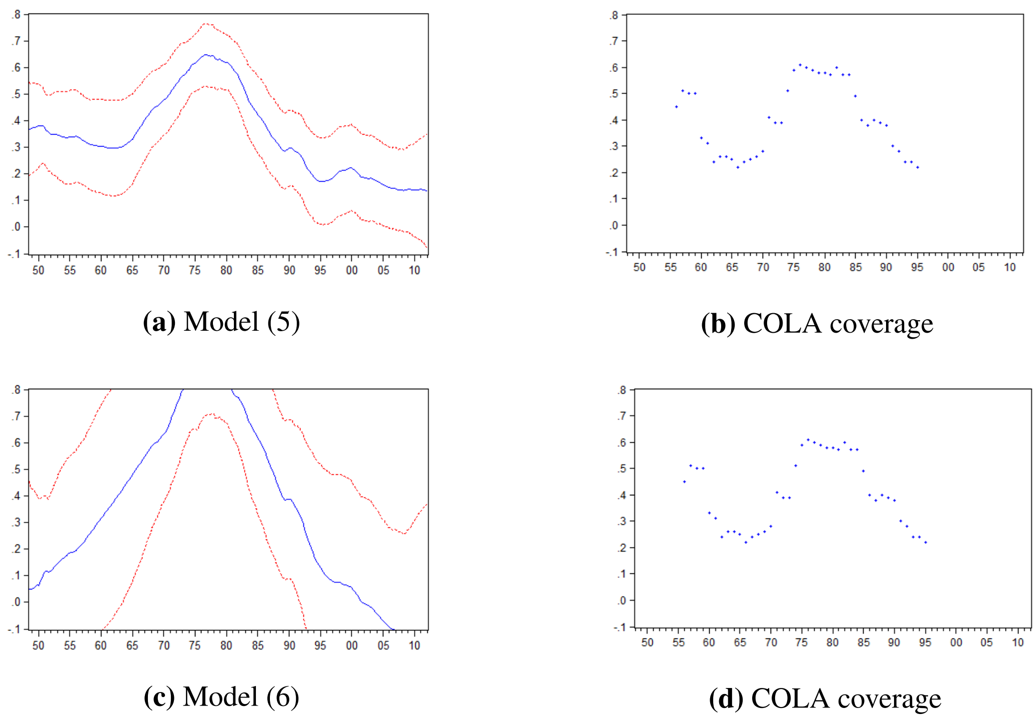
different values for non-varying steady-state inflation. In model (5), φ_4 (i.e. the coefficient of γ_t) is not statistically significant. This implies that after taking into account the effect of time-varying degree of wage indexation, the constant steady-state value of inflation is not statistically different from 0. In contrast, the estimate of φ_4 in model (5) implies that the constant steady-state value of inflation is 0.304. When one considers (as will be shown later) that the degree of wage indexation is a function of trend inflation, it is easy to see why $\varphi_4 = 0$ is more plausible. In other words, it makes sense that the constant steady-state value of detrended inflation should be 0. Also, comparing the AIC values of models (5) and (6) suggests that one should opt for the former. Finally, the estimated time-varying degrees of wage indexation obtained under the former version of the TV-NKWPC (Figure 2.3) are more comparable to those suggested by COLA coverage figures. Given the result just mentioned, the next section of this study only estimates the model (5) version of the TV-NKWPC for various countries.

2.3.3 Time-varying degree of wage indexation

If the dynamics of wage inflation are indeed described by the reduced form equation (2.15), one would expect the state variable γ_t to effectively capture the time-varying degree of wage indexation. The estimated log variance of the γ_t's disturbance term ($\ln(\sigma_\eta^2)$) is significant at 1% under all estimated versions of TV-NKWPC. This can be interpreted as evidence in support of the time-varying nature of the degree of wage indexation. Figure 2.3 gives the values of the time-varying degrees of wage indexation as indicated by the smoothed estimates for γ_t under the models (5) and (6) in Table 2.4. The two sets of estimates for γ_t reveal a general story: the degree of wage indexation rose during the period of the Great Inflation and fell during the period of the Great Moderation, a story consistent with other empirical investigations. One main difference however exists between the two models. The magnitudes of the estimates for γ_t under model (6) slightly exceed those suggested by the proportion of workers under COLA[16] contracts. This suggests that model (5) better describes the dynamics of wage inflation.

[16]The COLA coverage figures are obtained from the Bureau of Labour Statistics (BLS) and Weiner (1996).

Figure 2.3: Smoothed estimates for γ_t



(a) Model (5)

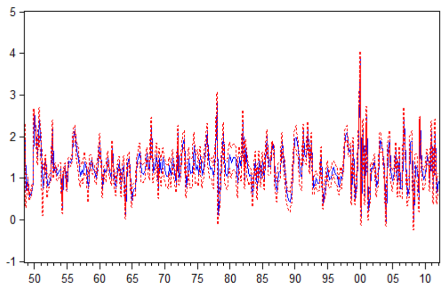
(b) COLA coverage

(c) Model (6)

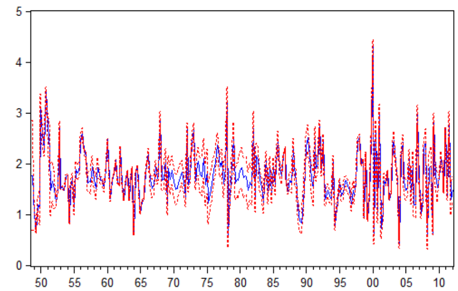
(d) COLA coverage

Our study is not the first attempt at estimating the time-varying degree of wage indexation. A comparison with other approaches found in existing literature reveals that our estimates for the time-varying degree of wage indexation are the closest to the figures suggested by the percentage of COLA coverage. Hofmann et al. (2010) and Ascari et al. (2011) provide estimates for the time-varying degree of wage indexation. While the estimates from their approaches produce reasonable measures for the time-varying degree of wage indexation, our approach is relatively simple, but nonetheless effectively measures this variable. Estimates for time-varying degree of wage indexation obtained in the two works just cited are compared to estimates obtained under TV-NKWPC and COLA coverage figures in Figure 2.5. It can be seen from this figure that the approach that best reproduces the estimates for the degree of wage indexation (γ_t) as suggested by COLA contracts coverage is the TV-NKWPC estimation. Similar to the figures suggested by COLA coverage, estimates for γ_t under the TV-NKWPC peaked at over 60% during the late $1970s$ and decreased to around 20% afterward. Thus, the subsequent part of this work will focus

Figure 2.4: Smoothed estimates for μ_t



(a) Model (5)



(b) Model (6)

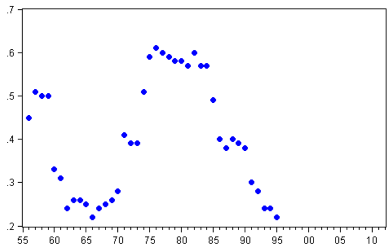
on estimating the model (5) of TV-NKWPC for selected OECD countries.

To recap, the analysis performed in this section indicates that there is indeed an empirical support for instability of the NKWPC. This instability stems from the time-varying nature of the degree of wage indexation. In particular, estimates for the time-varying degree of wage indexation (obtained from estimating the TV-NKWPC derived in the previous section) yield results strikingly similar to the percentage of COLA coverage. The latter variable is generally accepted as the proxy for the degree of wage indexation regarding US data. The estimates for the coefficients of productivity growth and unemployment under the NKWPC in Table (2.1) and under the TV-NKWPC in Table (2.4) are similar.

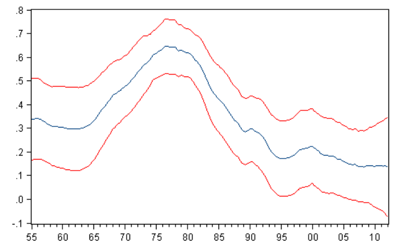
2.4 The TV-NKWPC in selected OECD countries

This section estimates the TV-NKWPC for 10 OECD countries and subsequently investigates the possible reasons for the time variations in the degrees of wage indexation. The countries are Austria, Belgium, Canada, Finland, Germany, Japan, Netherlands, Norway, Sweden and the United Kingdom. These countries are selected based solely on the availability of relevant data spanning a relatively long time period. For the sake of comparability, the analysis period is restricted to the period between 1970 and 2011. This is done because the data pertaining to some countries only begins from 1970.

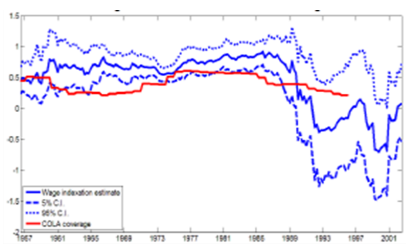
Figure 2.5: Various estimates for γ_t



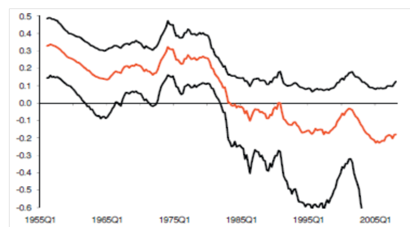
(a) COLA Coverage(BLS and Weiner (1996))



(b) TV-NKWPC



(c) Ascari et al 2011



(d) Hofmann et al, 2010

The main variables used for the estimations performed are in most ways identical to those used in the previous section. For instance, inflation is measured by the quarter-on-quarter change in the log of CPI, while union density and union strike variables (when available) are included in the analysis as proxies for bargaining power.

However, there are some minor differences. First, hourly earnings in the manufacturing sector rather than compensation based data is used as proxy variable to measure wages. There is a possibility that this variable might not actually reflect wages in an economy dominated by the service sector. However this is the best option available as data on other potential proxy variables is scant, or in some cases, non-existent for most of the countries. Secondly, one of the following three types of unemployment data was used for the case of each country: the unemployment rate of the labour force over 15 years old, the registered unemployment rate, and the harmonized unemployment rate. Our choice of the particular type of unemployment variable for each country is motivated by the duration of the data available. It is not expected that this will qualitatively affect our result as all types of unemployment are highly correlated. Furthermore, the use of country specific OLS estimation does not require a consistent measurement of unemployment rate across countries, as a panel regression estimation would for instance. Data on unemployment rate and wages are obtained from the OECD Main Economic Indicators database. Finally, quarterly data for GDP per hour is used as the proxy for labour productivity. Data for this variable is obtainable in annual frequency from the economic data published on the website of St Louis Federal Reserve Bank. A spline interpolation is used to obtain quarterly data from available annual data.

2.4.1 Impressions from data

The original Phillips curve relation posits a negative relationship between wage inflation and unemployment rate. The unstable nature of the Phillips curve has often been noted by authors. Gali (2011) for instance documents this instability, especially during the period from 1970 to 1985. As a result, the correlation between wage inflation and unemployment becomes weaker when the sample period is extended to cover the period from the 1960s to the 2010s.

In order to get a crude test of the stability of the negative correlation between unemployment and wage inflation, we plot scatter diagrams depicting the relationships for each of the 10 countries. The plots in Figure 2.9 reveal that in most of the countries, there is at least a reasonable amount of correlation between wage inflation and unemployment. From the figure, the magnitude of the correlations between the two aforementioned variables are generally higher than 0.5. The exceptions are in the cases of Netherlands, Canada and the UK in which relatively low correlations are reported. The lowest two correlations occur in Canada and the UK. This observation could potentially hint at the poor empirical fit of the TV-NKWPC to the data of these two countries.

2.4.2 Results

This section presents the results obtained from the country specific estimations of the TV-NKWPC. The result for the US is included to facilitate comparison. The specific version of the TV-NKWPC estimated is repeated below:

$$\begin{cases} \pi_t^w &= \varphi_1 u_t + \varphi_3 \pi_t^z + \mu_t + \varphi_4 \gamma_t + \gamma_t \bar{\pi}_{t-1}^p \\ \mu_t &= \varphi_5 + \varepsilon_t \\ \gamma_t &= \gamma_{t-1} + \eta_t. \end{cases} \tag{2.16}$$

The version of the TV-NKWPC in (2.16) above implies that after taking into account the persistence in wage inflation accounted for by time-varying wage indexation, the possible effects of lagged unemployment are negligible. The TV-NKWPC estimated in order to investigate the robustness of our estimation excludes lagged unemployment as an explanatory variable given its low explanatory power. Table 2.5 gives the estimated coefficients of the TV-NKWPC for the OECD countries considered in this study.

With the exception of the UK, the estimates for the coefficients of unemployment (u_t) are significant at 1% or 5% in all countries. The magnitudes of these estimates are lowest for Sweden, Finland, the US and the UK. This may suggest the presence of a relatively higher degree of nominal wage rigidity in these countries than the others in this study. This finding is partially corroborated by Dickens et al. (2007) who find that the degree of nominal wage rigidity is indeed higher in Sweden, Finland, and the US. Also, estimates for Austria, Japan and Norway suggest that unemployment in these countries are relatively less responsive to changes in wage inflation than in the others.

There is generally no conclusive evidence in support of the explanatory role of productivity growth in the TV-NKWPC from the estimation results. For Finland, including this variable resulted in estimates for the TV-NKWPC which are difficult to explain, hence the removal of productivity growth from the list of regressors. With the exception of Belgium, Germany, Norway and the US, country specific estimates for the coefficients of productivity growth (π_t^z) imply that wages are generally more indexed to inflation than productivity growth. This still holds even when available proxies for productivity other than real GDP per hour are used. Remarkably, all the country specific estimates for the variance of the shock to the wage indexation process

Table 2.5: Estimated TV-NKWPC (π_t^w)

	Estimated Coefficients						
	u_t	π_t^z	γ_t	φ_5	$\ln(\sigma_\varepsilon^2)$	$\ln(\sigma_\eta^2)$	AIC
Austria	-0.416**	0.226	-0.663**	3.888**	0.541**	-4.352**	3.619
	(0.088)	(0.21)	(0.136)	(0.477)	(0.091)	(1.135)	
Belgium	-0.274**	-0.067**	0.582	3.543**	-0.258*	-4.595**	2.938
	(0.074)	(0.159)	(0.414)	(0.717)	(0.115)	(0.642)	
Canada	-0.267*	0.2325	1.648*	2.814**	0.274 **	-5.707**	3.438
	(0.106)	(0.393)	(0.723)	(0.910)	(0.096)	(0.768)	
Finland	-0.151*		1.47	2.68*	0.803**	-5.703**	3.99
	(0.076)		(1.184)	(1.272)	(0.064)	(0.975)	
Germany	-0.214**	0.355**	-0.427**	2.912**	-0.664**	-3.292**	2.50
	(0.031)	(0.1)	(0.158)	(0.268)	(0.094)	(0.492)	
Japan	-0.659**	0.193	0.158	2.954**	0.872**	-2.433**	4.092
	(0.164)	(0.292)	(0.196)	(0.687)	(0.098)	(0.552)	
Netherlands	-0.390**	0.091	6.268	5.563**	-0.255**	-5.97**	3.19
	(0.147)	(0.233)	(4.024)	(1.429)	(0.132)	(1.075)	
Norway	-0.709*	0.768*	1.053	5.054**	0.988**	-3.344**	4.268
	(0.314)	(0.379)	(0.581)	(1.008)	(0.1)	(0.454)	
Sweden	-0.194*	0.027	2.702	3.305**	0.013	-0.168**	3.272
	(0.094)	(0.326)	(2,108)	(0.785)	(0.134)	(1.326)	
UK	-0.177	0.399	2.744*	5.368**	0.342*	-4.323**	3.77
	(0.147)	(0.363)	(1.091)	(0.944)	(0.158)	(0.578)	
the US	-0.1**	0.135*	0.765	1.213**	-1.134**	-7.187**	2.046
	(0.043)	(0.06)	(0.92)	(0.398)	(0.096)	(1.227)	

[1] The EVIEWS package used for the state-space estimation converged to two sets of estimates for the UK.The selected output shown in the table has a lower AIC value and has estimates similar to those of the model used for the robustness checks.
[2] Productivity growth was omitted from the list of regressors for Finland since including them yields unintuitive estimates for the coefficient of unemployment.
[3] Standard errors of estimates are indicated in parenthesis.
[4] * $p < 0.05$, ** $p < 0.01$

($\ln(\sigma_\eta^2)$) are statistically significant at 1% . This result gives credence to the assertion that the degree of wage indexation is indeed time-varying. Furthermore, the time-varying wage indexation expression given in (2.4) requires the following condition to hold for the coefficient φ_4 in the presence of positive steady-state inflation: $\varphi_4 \leq 0$. This condition is due to the following identity: $\varphi_4 = -\bar{\pi}^p$. It can be seen from Table 2.5 that with the exception of the UK, all country specific estimates for φ_4 (the coefficient of γ_t) are either significantly negative or not statistically significant.

The constant term (φ_5) is remarkably significant and positive for all countries. In order to explain this result we recall the following definition, $\varphi_5 = (1-\phi)\pi^z + \pi^p - (\varphi_1 + \varphi_2)u^n$. In other words, φ_5 is the sum of linear functions of steady-state productivity, steady-state inflation and steady-state unemployment rate.[17] Thus, a significantly positive estimate for φ_5 in each of the countries results from the presence of positive steady-state figures for the unemployment rates and the productivity growth rates in these countries.[18]

The results indicate a good empirical fit of the TV-NKWPC to the data of the OECD countries , with the exception being the case of the UK. This is not entirely surprising as it has already been demonstrated that the correlation between wage inflation and unemployment is lowest for the UK. In spite of the poor empirical fit of the TV-NKWPC to UK data, the time-varying wage indexation estimates obtained do follow the reasonable pattern of peaking in the late 1970s and falling thereafter due to fall in trend inflation since the 1970s.

2.4.3 Explaining the time variation in wage indexation in OECD countries

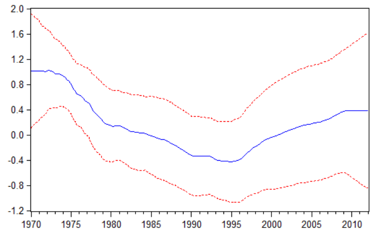
The results obtained from estimating the TV-NKWPC as presented in Table 2.5 support the case for time-varying wage indexation in each of the OECD countries: the estimated log variance of the shocks to time-varying wage indexation ($\ln(\sigma_\eta^2)$) is significant at 1% level for each country. The estimated time-varying indexation ($\hat{\gamma}_t$) for each the OECD countries is presented in Figure 2.6. While one can be reasonably certain that the estimated degrees of wage indexation for the US do come close to the figures suggested by the generally accepted proxy for wage indexation, a similar conclusion is hard to reach for the other countries.[19] However, it can be seen from the estimations that the degree of wage indexation has been falling in the majority of these countries

[17]Note that by definition $-(\varphi_1 + \varphi_2) > 0$

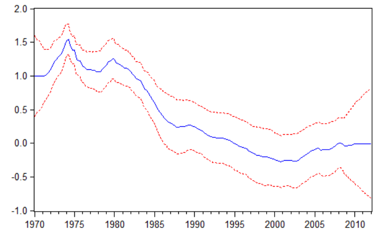
[18]steady-state inflation ($\bar{\pi}^p$) estimates are not statistically significant in most countries as seen from the estimates for $\varphi_4 = -\bar{\pi}^p$.

[19]Most countries in our panel do not keep data on wage indexation. Even though data for percentage COLA coverage is available for Canada, no study has established its usefulness as a proxy for wage indexation to the best of our knowledge.

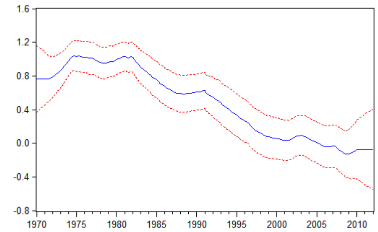
Figure 2.6: Smoothed estimates for γ_t



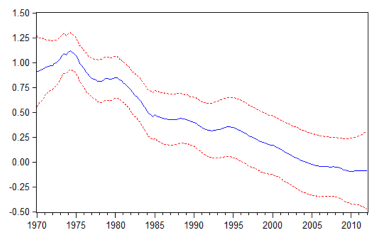
(a) Austria



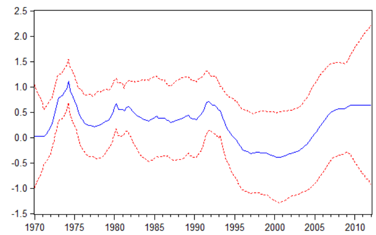
(b) Belgium



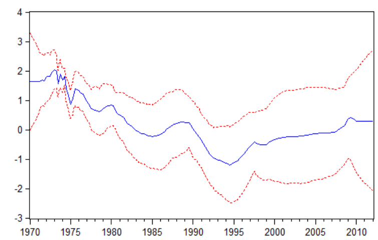
(c) Canada



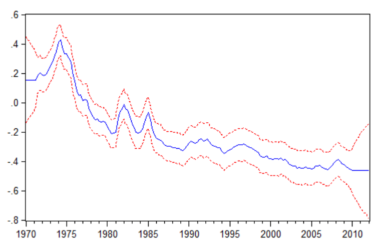
(d) Finland



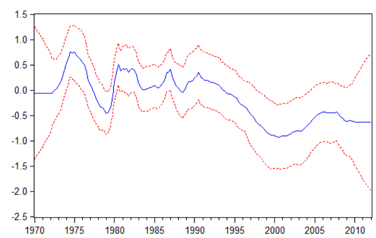
(e) Germany



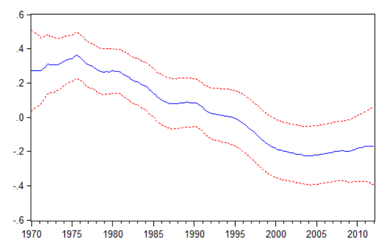
(f) Japan



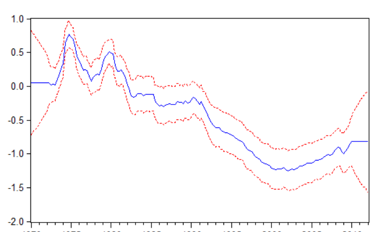
(g) Netherlands



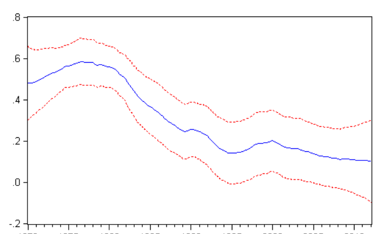
(h) Norway



(i) Sweden



(j) UK



(k) US

since the 1970s. This observation coupled with the observation that the trend inflation rates in these countries have been falling during the same period lends credence to the estimates.[20]

[20] Theory predicts a positive correlation between trend inflation and the degree of wage indexation.

For most of the sample period considered, the estimates for the time-varying degree of wage indexation for Austria, Germany, Japan, Norway and the UK are not statistically significant at 5%. The highest degree of wage indexation during the high inflation episode of the 1970s occurred in Belgium.[21] This result is not surprising, as this country has an automatic wage indexation policy which is applicable to all of its workers.

Having established that the estimated degrees of wage indexation (γ_t) under the TV-NKWPC do reasonably capture the degree of wage indexation, we now investigate the economic and institutional variables that explain the evolution of the degree of wage indexation. Gray (1976), Ragan and Bratsberg (2000), and Attey and de Vries (2011), among others, posit a number of variables as the factors influencing the level and distribution of the degree of wage indexation. Some of these variables are the following: real (productivity) shocks, monetary shocks (inflation uncertainty), bargaining power of unions and the number of independent unions involved in collective bargaining. In particular, the readily available estimates for time-varying wage indexation permit one to derive a test of the 'Gray hypothesis' (after Gray (1976)) which is captured in the following equation:

$$\gamma_t = f(\sigma_m^2, \sigma_z^2) \qquad \frac{\partial f}{\partial \sigma_m^2} > 0, \quad \frac{\partial f}{\partial \sigma_z^2} < 0, \tag{2.17}$$

where σ_m^2 denotes the variance of monetary shocks and σ_z^2 denotes the variance of real (productivity) shocks. The intuition behind this hypothesis lies in the fact that wage indexation insulates an economy from the effects of monetary shocks, while exacerbating those of real shocks. An optimal degree of indexation should therefore be close to a full indexation when monetary shocks are relatively dominant and close to zero when real shocks are relatively dominant. The aforementioned test of the Gray hypothesis can best be described as ad hoc since the original result on which the hypothesis is based describes the relationship between wage index-

[21]There is a general misconception that the Belgian wage legislation implies $\gamma_t = 1$ for all the time periods. However, one has to bear in mind that this full indexation represents the minimum extent of wage adjustment which cannot be undercut. Thus, it is possible to have a degree of wage indexation above 1 as observed in the late 1970s. Also, legislation put in place in 1989 imposed a maximum wage increase to be around the level of wage increase in Belgium's largest trading partners,(see Mongourdin-Denoix and Wolf (2010)). This might explain the general declining trend in wage indexation in Netherlands, Germany and France since the 1990s.

ation under optimal monetary policy and the variances of monetary and real shocks. Therefore, any formal test of the hypothesis requires the assumption on the use of wage indexation as a policy tool in the conduct of monetary policy. However, the estimations performed in order to obtain the time-varying degrees of wage indexation variables require no such assumption. This implies that the observed time variation in wage indexation could either result from the actions of a policy maker or be an optimal outcome from bargaining between agents (for example the employers and workers unions as described in Attey and de Vries (2013)).

The wage indexation regression employs four sets of explanatory variables, namely: variances of monetary policy shocks, variances of productivity shocks, variables indicating the bargaining power of unions, and variables indicating the independence of unions. The quarter-on-quarter change in trend inflation is used as a proxy variable for the variance monetary shocks.[22] The data for quarterly trend inflation is obtained by applying the HP filter on quarterly inflation data. In order to obtain a proxy variable for the variance of productivity shocks, a GARCH(1,1) estimation is performed on the quarterly growth in output per hour with the mean equation modeled as an AR(4) process.

The variables employed as proxies for bargaining power are the quarterly changes in union density ($\Delta unden$) and the quarterly growth rate of the number of strikes ($\Delta unstr$). Finally, to get a rough gauge of the independence of unions engaged in a wage bargaining process, we use three institutional variables namely: coordination of wage setting (crd), the predominant level at which wage bargaining occurs (lvl), and the mandatory extension of collective agreements by law to non-organized labour (ext). A high coordination of wage setting among unions, a centralized level of wage bargaining nationwide and an existence of a mandatory extension of collective agreement in one sector to other sectors generally reflect higher levels of interdependence (or lower levels of independence) among unions.

Annual data on union density and strike variables were obtained from the OECD and ILO

[22] The motivation behind the use of this variable stems from the observation that higher levels of trend inflation are generally associated with higher inflation volatility (variance). Also, the use of trend inflation permits the test of whether variations in trend inflation affect the negotiations with regards to wage indexation. Finally, the use of trend inflation as a proxy for the variance of monetary policy shocks enables us to sidestep the problem of common monetary policy in Eurozone member countries.

statistics databases respectively. The annual union density data spans the period from 1960 to 2013 while the annual strike data (when available for a specific country) spans the period from early 1970s to 2013. In order to convert these to quarterly data, it is assumed that the annual data figures correspond to the last quarter of each year. We then obtain the figures for the other three quarters by the use of spline interpolation. Finally the three institutional variables used as proxies for independence of union are available from the ICTWSS database in annual frequency spanning the period from 1960 to 2011. In order to convert these variables to quarterly data, it is again assumed that the annual variables correspond to the last quarter of each year and the same figure is repeated for the previous quarters in the year. This is not only convenient but also reasonable given the fact institutional variables do not change much over time.

The country specific regression equation estimated is given below:

$$\Delta\hat{\gamma}_t = \alpha_0 + \alpha_1 \Delta\pi^{\tau}_{t-1} + \alpha_2 \sigma^2_{z,t-1} + \sum_{i=1}^{p} \beta_i barg_{i,t-1} + \sum_{j=1}^{q} \theta_j ind_{j,t} + \epsilon_{\gamma t}, \tag{2.18}$$

where $\epsilon_{\gamma t} \sim N(0, \sigma^2_{\gamma})$. The variables $\Delta\hat{\gamma}_t$ and σ^2_z are the quarterly changes in degree of wage indexation and quarterly variance of productivity shocks. The sets of variables denoted by $barg_i$ and ind_j represent proxies for the bargaining power of unions and independence of unions respectively. With the exception of variables used as proxies for independence of unions, all explanatory variables introduced in equation (2.18) are lagged. This is to account for the informational constraints faced by either policy makers or other optimizing agents (unions) when deciding the wage indexation outcome. However, these constraints do not apply to the labour market instutional variables employed in this regression due to their considerable lack of time variation.

It is expected that there is a positive relationship between trend inflation and wage indexation irrespective of whether wage indexation is derived from the conduct of optimal monetary policy or is an outcome determined by bargaining agents. This posited relationship is implied in equation (2.17). The aforementioned equation also predicts a negative relationship between the variance of productivity shocks and wage indexation.

In the latter case wherein wage indexation is a bargaining outcome, it is conceivable that workers index their wages to inflation in order to correct for any perceived erosion in the values of their real wages that inflation might cause. A rising trend inflation therefore increases the incidences and the extent of wage indexation. The bargaining power of unions is expected to have a positive effect on wage indexation. The independence of unions engaged in bargaining (or independence of negotiations) regarding wage indexation can have both positive and negative effects on the wage indexation outcome. For instance, the presence of mandatory extension of negotiated outcomes to all other unions might result in lower aggregate wage indexation if one bargaining process results in a lower wage indexation outcome when compared to aggregate wage indexation resulting from independent bargaining processes.

The following table gives a summary definition of the explanatory variables and the expected signs of their coefficients:

Table 2.6: Expected signs of coefficients

$\Delta\pi^{\tau}$	change in trend inflation	+
σ_z^2	variance of productivity shocks	-
bargaining power		
$\Delta unden$	change in union density	+
$\Delta unstri$	growth rate of union strikes	+
independence of unions		
ext	mandatory extension of settlement terms to other sectors	+/-
lvl	predominant level at which collective bargaining takes place	+/-
crd	presence of coordination in collective wage bargaining	+/-

Table (2.10) in the appendix contains the results of the country specific estimations of equation (2.18). The table indicates that in most cases, variations in the proxies for independence of unions do not explains variations in wage indexation. The only exceptions to this result are in the cases of Austria and Finland whereby coordination between negotiating unions significantly explains variations in wage indexation. The table also shows that generally, bargaining power of unions does not significantly influence the degree of wage indexation. The exceptions are the cases of Finland and Norway, in which the effects of bargaining power of unions are statistically significant in the hypothesized direction. The estimates for Belgium indicate a significant correlation between union bargaining power (as measured by union density) and wage indexation but in a direction contrary to that hypothesized.

The table provides evidence, albeit a weak one, in support of the Gray hypothesis, implying that wage indexation is decreasing in the variance of productivity shocks. The estimates have correct signs in a majority of the countries. For Austria, Canada and Finland, variance of productivity has a significant negative effect on wage indexation. In the case of Belgium and the US, however, the variance of productivity shocks have significant positive effects on wage indexation. The fact that wage indexation is automatic (given high levels of inflation resulting from stagflation or inflationary gaps) may account for the positive correlation between the lag of the variance of productivity shocks and wage indexation.

The estimated coefficient for lagged variance of productivity is significant and positive, albeit of a negligible magnitude for the US. Among all the variables consequential to explaining the time variation in wage indexation considered, variations in trend inflation is the most significant explanatory factor. The coefficients are mostly positive with the exception of those of the Netherlands and Norway. The results in the case of the Netherlands can be explained by the 'Wassenaar Agreement', which in effect moderated wages during the early 1980s when inflation was observed to be historically high. This explains the negative correlation between the lag of inflation and the degree of wage indexation.

In this section, the TV-NKWPC was estimated for 11 OECD countries. There is evidence in support of the existence of a time-varying wage indexation. The country-specific time-varying degrees of wage indexation estimated indicate the prevalence of high levels of wage indexation from the 1970s to early 1980s, and a steady decline thereafter. It can also be concluded from the estimates that variations in trend inflation significantly affect the variations in time-varying wage indexation. While there is weak evidence in support of the hypothesis that the degree of wage indexation is decreasing in the variance of productivity shocks in some countries, there is no conclusive evidence supporting the significance of labour market institutional variables in explaining wage indexation. The next section includes tests on the robustness of the results obtained to alternative specifications.

2.4.4 Robustness: alternative specifications to wage indexation

The relatively better empirical fit of the TV-NKWPC contained in Equation (2.14a) has been established by the estimations performed so far. However, this specification of the TV-NKWPC relies on the rather simple assumption of constant indexation to productivity. In order to investigate how robust the findings in the previous section are to alternative specifications, we investigate the empirical fit of the TV-NKWPC under two alternative rules for wage indexation below:

$$x_{t+k|t} = \begin{cases} 0 & k = 0 \\ \sum_{s=0}^{k-1} \left(\gamma_{t+s+1}(\bar{\pi}^p_{t+s} + \pi^z_{t+s+1}) + (1-\gamma_{t+s+1})(\pi^p + \pi^z)\right) & k \geq 1 \end{cases} \tag{2.19a}$$

$$x_{t+k|t} = \begin{cases} 0 & k = 0 \\ \sum_{s=0}^{k-1} \left(\gamma_{t+s+1}\bar{\pi}^p_{t+s} + (1-\gamma_{t+s+1})\pi^p + \phi_{t+s+1}\pi^z_{t+s+1} + (1-\phi_{t+s+1})\pi^z\right) & k \geq 1. \end{cases} \tag{2.19b}$$

The first indexation rule suggests that wages are indexed at time-varying degrees to the sum of inflation and productivity growth, while the second indexation rule implies that wages are indexed to both inflation and productivity growth at their respective time-varying degrees (γ_t and ϕ_t). The TV-NKWPC in the case of each of the wage indexation rules presented are derived in the same manner as those in the earlier sections of this paper. The reduced-form TV-NKWPC in the case of the indexation Equation (2.19a) is given below:

$$\pi^w_t = \alpha'_t + \gamma_t(\bar{\pi}^p_{t-1} + \pi^z_t) + \psi_0 u_t + \psi_1 u_{t-1} + \xi_t, \tag{2.20a}$$

where $\alpha'_t \equiv (1-\gamma_t)(\pi^p + \pi^z) - (\psi_0 + \psi_1)u^n$ and all the other parameters retain their definitions as in the TV-NKWPC expression (2.14a) in the main derivation. The TV-NKWPC associated with the indexation rule (2.19b) is:

$$\pi_t^w = \alpha_t' + \gamma_t \bar{\pi}_{t-1}^p + \phi_t \pi_t^z + \psi_0 u_t + \psi_1 u_{t-1} + \xi_t, \tag{2.20b}$$

where $\alpha_t' \equiv (1-\gamma_t)\pi^p + (1-\phi_t)\pi^z - (\psi_0 + \psi_1)u^n$ and all other coefficients retain their definitions as under (2.14a). In expressing the TV-NKWPC contained in (2.20) in its state-space form , we assume that the random variable ξ_t is an iid random variable. This reflects the findings in the previous section that reject the hypothesis that ξ is an AR process. The time-varying degree of wage indexation is again assumed to behave like a random walk process in both cases of wage indexation expressions in (2.19). Estimating the equation (2.20b) requires one to specify the process for the time-varying degree of indexation of wages to productivity growth (i.e. ϕ_t). Whereas a number of studies exist that lend credence to the assertion that the degree of indexed wages to inflation (γ_t) is a function of a random walk process (for instance trend inflation), nothing in any of the available studies suggests the random walk process for the degree of wage indexation to productivity growth. Furthermore, suggesting a random walk process for ϕ_t requires one to economically justify why this variable might be non-stationary. An AR(1) process with a non-zero stationary value is therefore suggested for ϕ_t. Thus, the state-space versions of 2.20 are given in the following two equations:

$$\begin{cases} \pi_t^w &= \varphi_1 u_t + \mu_t + \varphi_2 \gamma_t + \gamma_t(\bar{\pi}_{t-1}^p + \pi_t^z) \\ \mu_t &= \varphi_4 + \xi_t \\ \gamma_t &= \gamma_{t-1} + \eta_t \end{cases} \tag{2.21a}$$

$$\begin{cases} \pi_t^w &= \varphi_1 u_t + \mu_t + \varphi_2 \gamma_t + \varphi_3 \phi_t + \gamma_t \bar{\pi}_{t-1}^p + \phi_t \pi_t^z \\ \mu_t &= \varphi_4 + \xi_t \\ \gamma_t &= \gamma_{t-1} + \eta_t \\ \phi_t &= (1-\rho_\phi)\phi + \rho_\phi \phi_{t-1} + v_t. \end{cases} \tag{2.21b}$$

where $\xi_t \sim \ N(0, \sigma_\xi^2)$, $\eta_t \sim \ N(0, \sigma_\eta^2)$, and $v_t \sim \ N(0, \sigma_v^2)$. The variable u_{t-1} is omitted among the list of regressors due to its lack of significance in explaining wage inflation as seen from Table 2.4. Thus, the second AR coefficient in the unemployment equation is set to zero ($\phi_2 = 0$). Also, we only consider the case where $\bar{\pi}^p_{t-1} = \pi_{t-1}$ due to the better plausibility of the estimated degree of wage indexation under this assumption compared to $\bar{\pi}^p_{t-1} = \pi^{(4)}_{t-1}$. Finally, the following definition for the other coefficients in (2.21) in terms of the structural parameters contained in Section 2.2 are given as follows:

$$\begin{aligned}
\varphi_1 &= -\frac{\lambda\varphi}{1-\beta\phi_1} \\
\varphi_2 &= -\pi^p \\
\varphi_3 &= -\pi^z \\
\varphi_4 &= \pi^p + \pi^z - \varphi_1 u^n.
\end{aligned}$$

The estimates of the two versions of the TV-NKWPC indicated in equation (2.21a) and equation (2.21b) are respectively presented in Table 2.11 and Table 2.12. Comparing the estimates obtained in Table 2.5 to those obtained in the two aforementioned tables reveal similarities of the AIC values of the three versions of the TV-NKWPC. This implies that the relative fit of the three versions of the TV-NKWPC to data are not considerably dissimilar. Furthermore, the country specific coefficients of unemployment tend to be roughly similar under the three versions of the TV-NKWPC. Finally, the figures (2.6), (2.7) and (2.8) do reveal generally similar trends in the variations in wage indexation.

However, it can be seen from comparing the tables (2.5), (2.11) and (2.12) that the AIC values given in the second of the aforementioned tables are the highest in a majority of the countries. Additionally, the magnitude of the time-varying wage estimates under the TV-NKWPC in (2.20a) as shown in Figure 2.7 differ from those under the other two models as presented in Figure 2.6 and Figure 2.8. For the US, the estimated time-varying degree of wage indexation under equation (2.21a) at its peak is less than half the estimates obtained under the other versions (equation (2.16) and equation (2.21b)). It should be noted that the estimates for the degree of wage indexation are closer to the figures suggested by percentage COLA coverage under the latter two models than the first model. The version of the TV-NKWPC in Equation (2.20a) and its estimated output will therefore be dropped from further analysis in the subsequent part of this section due to the preceding observations given in this paragraph.

Table 2.12 shows that incorporating a time-varying indexation to productivity growth dampens the evidence that supports the existence of time variation in wage indexation to inflation for Sweden and the UK. Moreover, the estimates in this table suggest that in the majority of the countries, the log variance of shocks to the wage indexation to productivity growth is not significant at 5% level, implying an absence of evidence for a time-varying process for indexation to productivity. The estimated coefficients of the time-varying process of wage indexation to productivity growth are statistically significant (not shown). One can therefore conclude that the specification of the TV-NKWPC captured in equation (2.16) does adequately describe the dynamics of wage inflation as well as the time-varying wage indexation process.

2.5 Conclusion and discussion

This study seeks to answer three main research questions. First, is there empirical evidence supporting the existence of time variation in wage indexation and second? Second,is there a way of estimating time-varying wage indexation using available data? Finally, what variables best explain the time variations in the degree of wage indexation? In response to the first question, this study provides ample empirical evidence to back the claim of time variation in the degree of wage indexation in 11 OECD countries. To this end, it first demonstrates the possible existence of a specification bias in the estimations carried out in Gali (2011) which are based on the assumption of a constant degree of wage indexation. A structural model incorporating time variation in the degree of wage indexation (the TV-NKWPC) is then used to estimate the degree of wage indexation. The time-varying degree of wage indexation estimates derived for the US are very similar to estimates suggested by the percentage of COLA coverage figures, a widely accepted proxy for the time variation in wage indexation. The estimates also show a common trend of higher levels of indexation from the 1970s to early 1980s and a steady decline afterwards in the OECD countries. Furthermore, there is evidence backing the presence of ‘over-indexation’, i.e. when the degree of wage indexation exceeds 1, in some of the countries during the 1970s .

Subsequent analysis in the study suggests that variations in trend inflation significantly explain the variations in wage indexation in all countries. This finding is supported by Ascari et al. (2011), among others. The theoretical prediction in Attey and de Vries (2013) suggests the importance of labour market institutional variables such as independence of unions and bargaining power in explaining the level of aggregate wage indexation. However, this study yields no evidence in support of this claim in most of the countries. The estimated time-varying degree of wage indexation obtained provides us with ample opportunity to test the Gray-hypothesis (after Gray (1976)) that wage indexation is negatively correlated with the variance of productivity shocks. We uncover some evidence in support of this hypothesis. Given that no assumptions have been made concerning the derivation of wage indexation, one can interpret this result as evidence for wage indexation as possibly being the result of some optimization process which takes the stochastic structure of the economy into account.

While ours is not the first attempt to estimate the time variation in the degree of wage indexation, our estimates are more similar to the percentage COLA coverage figures than the estimates found in existing studies. The results obtained in this paper also contrasts with those obtained in Holland (1986). That paper models wage indexation as an AR(1) process.[23] The results obtained in this study imply that time variations in wage indexation are explained by variations in trend inflation. The fact that trend inflation is often empirically modeled as a random walk process supports the process proposed for wage indexation adopted in this study.

Given the empirical documentation of the time variation in wage indexation for at least the past three decades, one may wonder why the assumption of constant wage indexation seems to be the norm in macro modeling. Perhaps, the decline in trend inflation over the past two decades, and the consequent decline in the degree of wage indexation has led the attention of policy makers away from the consequences of the time variation in the degree of wage indexation. It

[23]The AR coefficient was estimated at 0.62 for annual data. This should translate to about 0.88 for quarterly data.

is however still puzzling that current models that investigate the effects of rising trend inflation neglect rising levels of wage indexation since the two are often observed together. Furthermore, recent inflationary demand side policies engaged by the European Central Bank (ECB) and the FED imply that the time variation in wage indexation as a result of these policies may become of importance once again..

The TV-NKWPC model derived and estimated in this study is by no means perfect. It is possible that variations in trend inflation might not only affect the degree of wage indexation, but also how wage inflation reacts to unemployment. A possible extension of this model might adopt an approach similar to that used in Cogley and Sbordone (2008) to derive a version of NKWPC with all parameters being functions of trend inflation. With such a model, one may be able to better describe the wage dynamics in OECD countries.

2.A Intratemporal decision by household members

Given the wage rate $W_t(i)$, a household member i maximizes labour income subject to the constraint implied by the aggregate labour. The Lagrangian formulation of this intratemporal problem is given as follows:

$$\max_{N_t(i)} W_t(i)N_t(i)di - \lambda\left(N_t - \left[\int_0^1 N_t(i)^{1-\frac{1}{\epsilon}}di\right]^{\frac{\epsilon}{\epsilon-1}}\right). \tag{2.22}$$

We note that λ is a constant since this is a simple intratemporal (static) optimization problem. Noting that the constraint implied by the aggregate labour is binding permits one to write the first order conditions associated with this problem in addition to other implied derivations as in the following expressions:

$$\begin{aligned}
W_t(i) &= \lambda N_t(i)^{-\frac{1}{\epsilon}} N_t^{\frac{1}{\epsilon}} \\
W_t(j) &= \lambda N_t(j)^{-\frac{1}{\epsilon}} N_t^{\frac{1}{\epsilon}} \\
\frac{W_t(i)}{W_t(j)} &= \left(\frac{N_t(i)}{N_t(j)}\right)^{-\frac{1}{\epsilon}}.
\end{aligned}$$

The final expression is derived by dividing the first expression by the second. Assuming that $N_t(j) = N_t$, then $W_t(j) = W_t$. Noting this allows one to derive the demand for individual labour type as follows:

$$N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon} N_t. \tag{2.23}$$

As an intermediate step, both sides of the expression (2.23) are raised to the power $\frac{\epsilon}{\epsilon-1}$. The expression for aggregate wages W_t is then derived in the following expressions:

$$\begin{aligned}
N_t(i)^{\frac{\epsilon-1}{\epsilon}} &= N_t^{\frac{\epsilon-1}{\epsilon}} W_t^{\epsilon-1} W_t(i)^{1-\epsilon} \\
\int_0^1 N_t(i)^{1-\frac{1}{\epsilon}}di &= N_t^{\frac{\epsilon-1}{\epsilon}} W_t^{\epsilon-1} \int_0^1 W_t(i)^{1-\epsilon}di.
\end{aligned}$$

To proceed further, we begin by making the following substitution as implied by the aggregate labour index: $N_t^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 N_t(i)^{1-\frac{1}{\epsilon}} di$. This permits us to derive the following expression for aggregate wages.

$$W_t^{1-\epsilon} = \int_0^1 W_t(i)^{1-\epsilon} di.$$

The final expression can be rearranged to give the definition for aggregate wages W_t as follows:

$$W_t = \left[\int_0^1 W_t(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}. \tag{2.24}$$

2.B The Time-Varying New Keynesian Wage Phillips Curve

2.B.1 Deriving the structural TV-NKWPC

The problem of a worker optimizing in the current period is to choose the optimal wage rate (W_t^*) in order to maximize their utility subject to their budget constraints and their labour demand schedules. In algebraic terms, the problem of the re-optmizing household is to maximize:

$$E_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k U(C_{t+k|t}, N_{t+k|k})\right] \tag{2.25}$$

subject to the aggregate labour demand constraint and the budget constraint given respectively below:

$$N_{t+k|t} = \left(\frac{W_t^* X_{t+k|t}}{W_{t+k}}\right)^{-\epsilon} N_{t+k} \tag{2.26}$$

$$P_{t+k}C_{t+k|t} + E_{t+k}\{Q_{t+k,t+k+1}B_{t+k+1|t} \leq B_{t+k|t} + W_t^* X_{t+k|t}N_{t+k|t} - T_{t+k}. \tag{2.27}$$

Noting that $C_{t+k|t}$ and $N_{t+k|k}$ are both functions of W_t^*, one can derive the first order condition associated with this problem as follows:

$$\begin{aligned}
0 =& E_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k\left(\frac{\partial U}{\partial C_{t+k|t}}\frac{\partial C_{t+k|t}}{\partial W_t^*}+\frac{\partial U}{\partial N_{t+k|t}}\frac{\partial N_{t+k|t}}{\partial W_t^*}\right)\right]\\
=& E_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k\left((1-\epsilon)N_{t+k|t}U_C(\cdot)\frac{X_{t+k|t}}{P_{t+k}}-\epsilon N_{t+k|t}U_N(\cdot)\frac{1}{W_t^*}\right)\right]\\
=& E_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k N_{t+k|t}U_C(\cdot)\left(W_t^*\frac{X_{t+k|t}}{P_{t+k}}-\frac{\epsilon}{1-\epsilon}\frac{U_N}{U_C}\right)\right].
\end{aligned}$$

Let the marginal rate of substitution for any household member that resets its wages in time t be defined as $MRS_{t+k|t} = -U_N/U_C$, and let $\mathcal{M} = \epsilon/(\epsilon-1)$. The last expression then becomes:

$$\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[N_{t+k|t}U_c(\cdot)\left(\frac{W_t^* X_{t+k|t}}{P_{t+k}}-\mathcal{M}MRS_{t+k|t}\right)\right] = 0. \tag{2.28}$$

There are a couple of points worth noting about the non stochastic steady state version of 2.28 which will be useful for the derivation of the loglinearized version of this equation.

- While prices (P) and wages (W) may be non stationary even in the steady state, real wages (W/P) are stationary since consumption (C) and labour (N) are stationary in the non-stochastic steady state. This further implies that the marginal rate of substitution is also stationary.

- The steady-state value of the indexed part of wages is $X_{t+1|t}$ is 1. Also the definition of the steady state (absence of any form of nominal rigidity) implies that there is no indexation ($X = 1$or $x = 0$).

- The non-stochastic steady-state version of this equation implies that the following holds:

$$\frac{W}{P} = \mathcal{M}MRS$$

Let $\mu = log\mathcal{M}$. In terms of log variables the last expression can be written as follows:

$$w - p = \mu + mrs$$
$$\mu = w - p - mrs.$$

One can then loglinearize equation (2.28) as shown in the following steps.

$$\begin{aligned}
0 =& \sum_{k=0}^{\infty}(\beta\theta)^k E_t \left(\frac{W}{P}(w_t^* - w) + \frac{W}{P}x_{t+k|t} - \frac{W}{P}(p_{t+k} - p) - \mathcal{M}MRS(mrs_{t+k|t} - mrs) \right) \\
=& \sum_{k=0}^{\infty}(\beta\theta)^k E_t \left(\frac{W}{P}(w_t^* - w) + \frac{W}{P}x_{t+k|t} - \frac{W}{P}(p_{t+k} - p) - \frac{W}{P}(mrs_{t+k|t} - mrs) \right) \\
=& \sum_{k=0}^{\infty}(\beta\theta)^k E_t \left((w_t^* - w) + x_{t+k|t} - (p_{t+k} - p) - (mrs_{t+k|t} - mrs) \right) \\
=& \sum_{k=0}^{\infty}(\beta\theta)^k E_t \left(w_t^* + x_{t+k|t} - p_{t+k} - mrs_{t+k|t} - (w + p - mrs) \right) \\
=& \sum_{k=0}^{\infty}(\beta\theta)^k E_t \left(w_t^* + x_{t+k|t} - p_{t+k} - mrs_{t+k|t} - \mu \right) \\
=& w_t^*/(1 - \beta\theta) + \sum_{k=0}^{\infty}(\beta\theta)^k E_t \left(x_{t+k|t} - p_{t+k} - mrs_{t+k|t} - \mu \right).
\end{aligned}$$

The final expression implies the following expression for optimal wages set by members of the household who have the opportunity to set wages:

$$w_t^* = (1 - \beta\theta) \sum_{k=0}^{\infty}(\beta\theta)^k E_t \left(mrs_{t+k|k} + p_{t+k} - x_{t+k|t} + \mu \right).$$

The next step is to express marginal rate of substitution in terms of wages and the indexed part of wages. We begin by noting that due to perfect risk sharing by members of the household, all members have identical marginal utility hence identical consumption $C_{t+k} = C_{t+k|k}$. Let the utility function of a representative household be:

$$U(C_t, N_t(i), \chi_t) = \log C_t - \chi_t \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} di.$$

The derivation of the expression of marginal rate of substitution of a household member in period $t+k$ given that they last set their optimal wage rate in period t is given in the following steps:

$$\begin{aligned}
MRS_{t+k|t} &= -U_N/U_C \\
&= \chi_{t+k} C_{t+k|t} N_{t+k|t}^{\varphi} \\
&= \chi_{t+k} C_{t+k} N_{t+k|t}^{\varphi} \\
mrs_{t+k|t} &= \log(\chi) + c_{t+k} + \varphi n_{t+k|t} \\
&= \log(\chi) + c_{t+k} + \varphi n_{t+k} + (\varphi n_{t+k|t} - \varphi n_{t+k}) \\
&= mrs_{t+k} + \varphi(n_{t+k|t} - n_{t+k}).
\end{aligned}$$

The loglinearized version of (2.26) implies $\varphi(n_{t+k|t} - n_{t+k}) = -\varphi\epsilon(w_t^* + x_{t+k|t} - w_{t+k})$. Making this substitution permits the last expression for $mrs_{t+k|t}$ to be written as follows:

$$mrs_{t+k|t} = mrs_{t+k} - \epsilon\varphi(w_t^* + x_{t+k|t} - w_{t+k}). \tag{2.29}$$

We proceed further by expressing optimal wages by wage setting household members as a function of aggregate marginal rate of substitution and other variables.

$$\begin{aligned}
w_t^* =&(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[mrs_{t+k|k}+p_{t+k}-x_{t+k|t}+\mu\right]\\
=&(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[mrs_{t+k}-\epsilon\varphi(w_t^*+x_{t+k|t}-w_{t+k})+p_{t+k}-x_{t+k|t}+\mu\right]\\
=&-\epsilon\varphi w_t^*+(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[mrs_{t+k}-w_{t+k}+p_{t+k}+\mu+(1+\epsilon\varphi)(w_{t+k}-x_{t+k|t})\right]\\
w_t^* =& (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[(mrs_{t+k}-w_{t+k}+p_{t+k}+\mu)/(1+\epsilon\varphi)+w_{t+k}-x_{t+k|t}\right].
\end{aligned}$$

Let $w_t - p_t - mrs_t = \mu_t$ and $\mu_t - \mu = \hat{\mu}_t$. Then

$$w_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\left(w_{t+k}-x_{t+k|t}-\frac{1}{1+\epsilon\varphi}\hat{\mu}_{t+k}\right). \tag{2.30}$$

Noting that $w_t^* = w_{t|t}$ and that $x_{t+1|t} = (x_{t+k+1|t} - x_{t+k+1|t+1})$, a step by step derivation of an intermediate version of the structural NKWPC can be given as follows:

$$\begin{aligned}
w_{t+1|t+1} =&(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_{t+1}\left(w_{t+k+1}-x_{t+k+1|t}-\frac{1}{1+\epsilon\varphi}\hat{\mu}_{t+k+1}+(x_{t+k+1|t}-x_{t+k+1|t+1})\right)\\
=&(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_{t+1}\left(w_{t+k+1}-x_{t+k+1|t}-\frac{1}{1+\epsilon\varphi}\hat{\mu}_{t+k+1}\right)+(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\nu_{t+1}\\
=&(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_{t+1}\left(w_{t+k+1}-x_{t+k+1|t}-\frac{1}{1+\epsilon\varphi}\hat{\mu}_{t+k+1}\right)+\nu_{t+1},
\end{aligned}$$

where $\nu_{t+1} = x_{t+1|t}$. Multiplying both sides of the last expression by $\beta\theta$ and subsequently taking expectation conditional on information available at time t, we get the following:

$$\begin{aligned}
(\beta\theta)w_{t+1}^* =&(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^{k+1} E_{t+1}\left(w_{t+k+1}-x_{t+k+1|t}-\frac{1}{1+\epsilon\varphi}\hat{\mu}_{t+k+1}\right)+(\beta\theta)\nu_{t+1}\\
(\beta\theta)E_t w_{t+1}^* =&(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^{k+1} E_t\left(w_{t+k+1}-x_{t+k+1|t}-\frac{1}{1+\epsilon\varphi}\hat{\mu}_{t+k+1}\right)+(\beta\theta)E_t\nu_{t+1}.
\end{aligned}$$

Next, we replace the time index by making the substitution $s = k+1$. This implies the previous expression can be alternatively rendered as:

$$(\beta\theta)E_t w^*_{t+1} = (1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^s E_t\left(w_{t+s} - x_{t+s|t} - \frac{1}{1+\epsilon\varphi}\hat{\mu}_{t+s}\right) + (\beta\theta)E_t\nu_{t+1}.$$

Finally, the equation for optimal wages for wage setting household members at time t given in (2.30) implies the previous equation can be recast as follows:

$$(\beta\theta)E_t w^*_{t+1} = w^*_t - (1-\beta\theta)\left(w_t - x_{t|t} - \frac{1}{1+\epsilon\varphi}\hat{\mu}_t\right) + (\beta\theta)E_t\nu_{t+1}.$$

We recall from the wage indexation expression in (2.4) given in the main part of this work that$x_{t|t} = 0$. Noting this, the last expression can be rearranged to result in the following expression:

$$w^*_t = \beta\theta E_t(w^*_{t+1} - \nu_{t+1}) + (1-\beta\theta)(w_t - (1+\epsilon\varphi)^{-1}\hat{\mu}_t). \tag{2.31}$$

Aggregate wages in the economy is assumed to be a weighted average of reset wages and indexed wages (those not derived from optimizing). The expression for aggregate wages and the loglinearized version is presented below:

$$\begin{aligned}
W_t &= \left[\theta\left(W_{t-1}X_{t|t-1}\right)^{1-\epsilon} + (1-\theta)(W^*_t)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\
W_t^{1-\epsilon} &= \theta(W_{t-1}X_{t|t-1})^{1-\epsilon} + (1-\theta)(W^*_t)^{1-\epsilon} \\
(1-\epsilon)W^{1-\epsilon}(w_t - w) &= (1-\epsilon)\left[W^{1-\epsilon}\theta[(w_{t-1}-w) + x_{t|t-1}] + (1-\theta)W^{1-\epsilon}(w^*_t - w)\right] \\
w_t - w &= \theta[(w_{t-1}-w) + x_{t|t-1}] + (1-\theta)(w^*_t - w)
\end{aligned}$$

which after making the substitution $\nu_t = x_{t|t-1}$ leads us to this expression

$$w_t = \theta(w_{t-1} + \nu_t) + (1-\theta)w^*_t. \tag{2.32}$$

Substitution of the above expression into that in 2.31 gives the following version of the NKWPC:

$$\pi_t^w - \nu_t = \beta E_t(\pi_{t+1}^w - \nu_{t+1}) - \lambda \hat{\mu}_t \qquad \lambda = \frac{(1-\theta)(1-\beta\theta)}{(1+\epsilon\varphi)\theta}. \tag{2.33}$$

2.B.2 Reduced-form TV-NKWPC

Next we derive the reduced form TV-NKWPC. To do this, we introduce unemployment into equation (2.33) by noting that $\hat{\mu}_t = \varphi \hat{u}_t$ as explained in the main part of this text. We assume unemployment is an AR(2) process given as follows:

$$\hat{u}_t = \phi_1 \hat{u}_{t-1} + \phi_2 \hat{u}_{t-2} + v_t \qquad v_t \sim N(0, \sigma_v^2).$$

Let $V_t = \pi_t^w - \nu_t$, and $\delta = \lambda\varphi$. We rewrite the expression (2.33) as follows:

$$V_t = \beta E_t V_{t+1} - \delta \hat{u}_t.$$

To solve the difference equation we make an initial guess. We guess that V_t will be a function of unemployment and its lag. Thus,

$$V_t = \psi_0 \hat{u}_t + \psi_1 \hat{u}_{t-1}.$$

We lead V_t by one time period and take expectation of the resulting expression. This derives the following sets of equations:

$$\begin{aligned} \beta E_t V_{t+1} &= \beta \psi_0 E_t(\hat{u}_{t+1}) + \beta \psi_1 \hat{u}_t \\ &= \beta \psi_0 (\phi_1 \hat{u}_t + \phi_2 \hat{u}_{t-1}) + \beta \psi_1 \hat{u}_t \\ &= (\beta \psi_0 \phi_1 + \beta \psi_1) \hat{u}_t + \beta \psi_0 \phi_2 \hat{u}_{t-1}. \end{aligned}$$

We substitute the previous expression for $\beta E_t V_{t+1}$ into our initial guess $V_t = \beta E_t V_{t+1} - \delta \hat{u}_t$ to obtain the following:

$$V_t = (\beta\psi_0\phi_1 + \beta\psi_1 - \delta)\hat{u}_t + \beta\psi_0\phi_2\hat{u}_{t-1}.$$

Equating this expression to the initial guess $V_t = \psi_0\hat{u}_t + \psi_1\hat{u}_{t-1}$ results in the following simultaneous equation for the coefficients ψ_0 and ψ_1:

$$\begin{aligned}\psi_1 &= (\beta\psi_0\phi_2)\\ \psi_0 &= (\beta\psi_0\phi_1 + \beta\psi_1 - \delta).\end{aligned}$$

Solving the simultaneous equations above yield the following expressions for ψ_0 and ψ_1:

$$\begin{aligned}\psi_0 &= -\frac{\delta}{1-\beta(\phi_1+\beta\phi_2)}\\ \psi_1 &= -\frac{\beta\phi_2\delta}{1-\beta(\phi_1+\beta\phi_2)}.\end{aligned}$$

After making the substitutions $\hat{u}_t = (u_t - u^n)$ and assuming the presence of a measurement error ξ_t, the reduced form TV-NKWPC can be written as:

$$\pi_t^w = (1-\gamma_t)\pi^p + (1-\phi)\pi^z + \gamma_t\bar{\pi}_{t-1}^p + \phi\pi_t^z + \psi_0(u_t - u^n) + \psi_1(u_{t-1} - u^n) + \xi_t. \quad (2.34)$$

2.C Tables and figures

2.C.1 Tables

Table 2.7: AR(2) process for unemployment (u_t)

parameter	const	ϕ_1	ϕ_2	Adj R^2
	0.299**	1.470**	-0.520**	0.953
std err	(0.082)	(0.054)	(0.054)	

* $p < 0.05$, **$p < 0.01$

Table 2.8: Estimating $\hat{\alpha}_t = \beta_0 + \beta_1 \hat{\gamma}_t + \varepsilon_t$

	$\bar{\pi}^p_{t-1} = \pi^{(4)}_{t-1}$			$\bar{\pi}^p_{t-1} = \pi_{t-1}$		
	β_0	β_1	R^2	β_0	β_1	R^2
estimate	1.375 **	-0.217 **	0.167	1.566 **	-0.497 **	0.584
std err	0.02	0.03		0.0179	0.0351	

[1] This table estimates the correlation between the time-varying parameters $\hat{\alpha}_t$ and $\hat{\gamma}_t$ obtained from rolling regression estimates of Equation (2.14a).
[2] * $p < 0.05$, ** $p < 0.01$

Table 2.9: Autocorrelation in $\hat{\epsilon}$

	(1)	(2)	(3)	(4)
$\hat{\epsilon}_{t-1}$	0.18**	0.21**	0.14*	0.20**
	(0.062)	(0.062)	(0.062)	(0.062
const	0.002	0.003	0.002	0.003
	(0.037)	(0.036)	(0.037)	(0.036)
Adj-R^2	0.029	0.042	0.016	0.036
AIC	1.815	1.792	1.77	1.752

Estimation of $\hat{\epsilon}_t = \zeta_0 + \zeta_1 \hat{\epsilon}_{t-1} + v_t$

Standard errors of estimates are indicated in parenthesis.

* $p < 0.05$, ** $p < 0.01$.

Table 2.10: Determinants of wage indexation $\hat{\gamma}_t$ Equation (2.18)

	Estimated Coefficients								
				Union Bargaining Power		Union Independence			
	$const$	$\Delta\pi^{\tau}_{t-1}$	$\sigma^2_{z,t-1}$	$\Delta unden_{t-1}$	$\Delta unstr_{t-1}$	crd_t	ext_t	lvl_t	$Adj\ R^2$
Austria	0.146**	0.180**	-0.907**	0.005		-0.031**		-0.0003	0.48
	(0.019)	(0.0.061)	(0.249)	(0.005)		(0.004)		(0.004)	
Belgium	0.007	0.499**	0.309*	-0.017*		-0.005		0.001	0.33
	(0.017)	(0.065)	(0.144)	(0.007)		(0.004)		(0.004)	
Canada	-0.004	0.267**	-0.663**	0.004	-0.005	-0.002		0.007	0.50
	(0.004)	(0.034)	(0.170)	(0.003)	(0.005)	(0.001)		(0.003)	
Finland	0.001	0.091**	0.015	0.008**		0.004**		-0.005**	0.24
	(0.003)	(0.027)	(0.013)	(0.002)		(0.001)		(0.001)	
Germany	-0.023	1.144**	-0.081	0.02		0.01			0.24
	(0.033)	(0.167)	(0.065)	(0.013)		(0.009)			
Japan	0.085	0.347*	-0.821	-0.001	-0.259	-0.018			0.03
	(0.085)	(0.215)	(0.602)	(0.081)	(0.155)	(0.169)			
Netherlands	-0.008	-0.172**	-0.413	0.005		-0.003		0.005	0.09
	(0.014)	(0.053)	(0.219)	(0.008)		(0.003)		(0.004)	
Norway	0.117	-0.401*	-3.605**	0.09**		-0.009	-0.012	-0.006	0.245
	(0.044)	(0.155)	(0.995)	(0.017)		(0.008)	(0.013)	(0.009)	
Sweden	0.014*	0.179**	0.06	0.005		-0.004		-0.001	0.16
	(0.007)	(0.043)	(0.133)	(0.004)		(0.002)		(0.002)	
UK	0.001	0.327**	1.056	0.026	0.06	0.013		-0.013	0.22
	(0.008)	(0.1)	(0.656)	(0.013)	(0.068)	(0.006)		(0.006)	
the US	-0.005**	0.113**	0.006**			-0.0001			0.52e
	(0.001)	(0.014)	(0.002)			(0.0004)			

1 Estimation of Equation (2.18).
2 Explanations regarding the explanatory variables are found in Table 2.6 in the main text.
3 $*p < 0.05$, $**p < 0.01$., Standard errors of estimates are indicated in parenthesis

Table 2.11: Robustness: Estimating version (2.21a) of the TV-NKWPC

	Estimated Coefficients					
	u_t	γ_t	φ_4	$\ln(\sigma_\varepsilon^2)$	$\ln(\sigma_\eta^2)$	AIC
Austria	-0.422**	-1.253**	4.062**	0.568**	-5.249**	3.633
	(0.098)	(0.377)	(0.502)	(0.089)	(1.257)	
Belgium	-0.313**	1.236	4.005**	-0.372*	-4.291**	3.020
	(0.100)	(0.717)	(0.807)	(0.150)	(0.379)	
Canada	-0.303**	1.683*	3.252**	0.266 **	-5.758**	3.432
	(0.088)	(0.766)	(0.829)	(0.095)	(0.654)	
Finland	-0.121*	0.504	2.463**	0.665**	-5.494**	3.806
	(0.073)	(1.127)	(0.727)	(0.059)	(0.783)	
Germany	-0.211**	-0.880**	3.085**	-0.614**	-4.549**	2.513
	(0.036)	(0.288)	(0.319)	(0.107)	(0.645)	
Japan	-0.701**	-0.345**	3.252**	0.742**	-1.772**	4.085
	(0.184)	(0.096)	(0.687)	(0.119)	(0.510)	
Netherlands	-0.3373**	3.565*	4.728**	-0.321*	-5.112**	3.145
	(0.144)	(1.700)	(0.957)	(0.141)	(0.652)	
Norway	-0.593*	0.280	4.051**	1.001**	-3.247**	4.261
	(0.302)	(0.668)	(0.958)	(0.096)	(0.564)	
Sweden	-0.187*	2.787	3.156**	0.029	-6.598**	3.261
	(0.085)	(2.362)	(0.680)	(0.137)	(1.226)	
UK	-0.106	2.33*	4.737**	0.418**	-4.527**	3.800
	(0.132)	(0.968)	(0.835)	(0.143)	(0.617)	
the US	-0.11*	9.782	0.281	-1.159**	-9.616**	2.046
	(0.043)	(8.569)	(1.359)	(0.101)	(1.662)	

[1] Standard errors of estimates are indicated in parenthesis

[2] * $p < 0.05$, ** $p < 0.01$

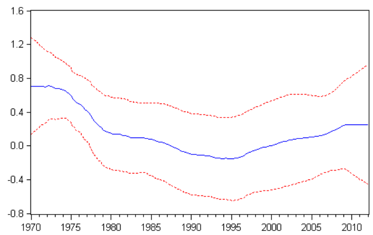
Table 2.12: Robustness: Estimating version (2.21b) of the TV-NKWPC

	Estimated Coefficients									
	u_t	γ_t	ϕ_t	φ_4	ρ_ϕ	ϕ	$\ln(\sigma^2_\varepsilon)$	$\ln(\sigma^2_\eta)$	$\ln(\sigma^2_v)$	AIC
Austria	-0.497**	1.444	0.270	4.323**	-0.231*	0.017	-0.591	-5.321**	0.003	3.561
	(0.161)	(1.503)	(0.247)	(1.075)	(0.092)	(0.252)	(0.365)	(1.320)	0.481	
Belgium	-0.256*	1.614 *	5.808	3.085**	-0.174**	0.007	25.165	-4.686**	-4.013	2.925
	(0.064)	(0.736)	(3.693)	(1.1363)	(0.061)	(0.156)	($3e9$)	(0.560)	(1.156)	
Canada	-0.268*	1.683*	-8.567	4.687	0.027	0.212	0.270	(5.682**	-25.229	3.476
	(0.106)	(0.766)	(113.15)	(24.597)	($44e4$)	(0.391)	(1.157)	(0.791)	$1.92e9$	
Finland	-0.090	1.311	5.272	1.928	-0.274	-0.061	-17.831	-5.895**	-2.984*	3.77
	(0.061)	(1.614)	(6.719)	(1.663)	(0.231)	(0.208)	(1.09e8)	(1.0756)	(1.417)	
Germany	-0.195**	-0.349** ‘	0.619	2.376**	-0.185	0.47**	-1.248**	-3.575**	-2.026	2.499
	(0.031)	(0.147)	(0.862)	(0.545)	(0.221)	(0.151)	(0.407)	(0.645)	(1.051)	
Japan	-0.607**	0.251	27.001	-2.137	-0.144*	0.179	-25.021	-3.189**	-5.747	4.096
	(0.161)	(0.247)	(44.512)	(13.466)	(0.057)	(0.258)	(2.2e9)	(0.471)	(3.213) ‘	
Netherlands	-0.358**	8.479	0.046	6.087**	0.269	0.093	-0.671**	-7.25**	-0.152	3.136
	(0.128)	(6.705)	(0.277)	(1.981)	(0.138)	(0.337)	(0.178)	(1.406)	(0.651)	
Norway	-0.789**	0.757	5.133	-0.21	-0.324	0.976**	-21.117	-2.37**	-2.804	4.23
	(0.295)	(0.412)	(6.988)	(7.244)	(0.287)	(0.323)	3e9	(0.306)	(1.52)	
Sweden	-0.173	2.709	-19.281	3.525	–0.157	0.038	-22.718	-5.959	-5.909	3.313
	(0.085)	(2.202)	(63.638)	(6.863)	(0.142)	(0.326)	(1e9)	(1.296)	(6.758)	
UK	-0.162	2.836**	-0.63**	5.384**	-0.466**	0.447	-0.178	-4.246	0.646	3.753
	(0.132)	(1.026)	(0.077)	(1.004)	(0.174)	(0.357)	(0.257)	(0.529)	(0.52)	
the US	-0.097*	0.925	-0.934**	-1.288**	0	0.15*	-1.4*	-7.43**	-2.25**	2.068
	(0.048)	(1.038)	(0.351)	(0.403)	(0.032)	(0.073)	(0.207)	(1.214)	(0.618)	

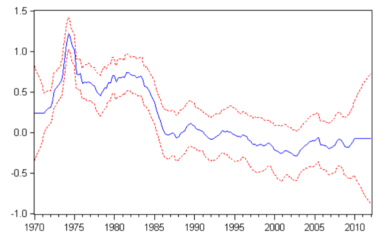
[1] Standard errors of estimates are indicated in parenthesis
[2] * $p < 0.05$, ** $p < 0.01$

2.C.2 Figures

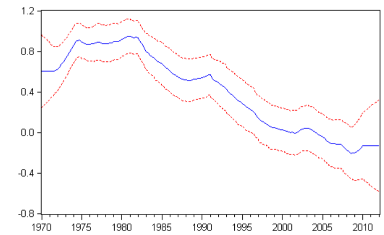
Figure 2.7: Smoothed estimates for γ_t: Model (2.21a)



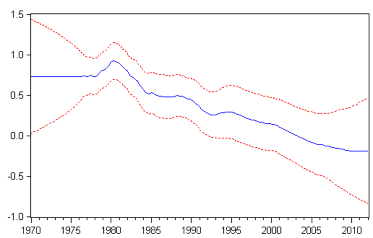
(a) Austria



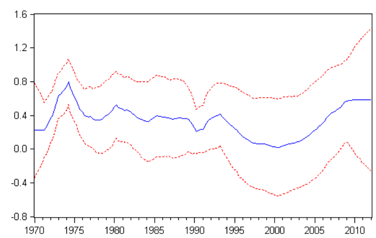
(b) Belgium



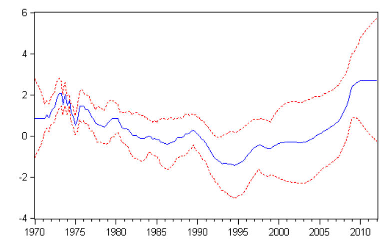
(c) Canada



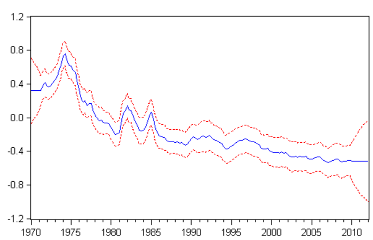
(d) Finland



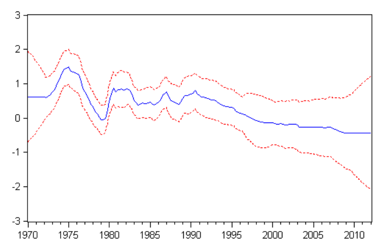
(e) Germany



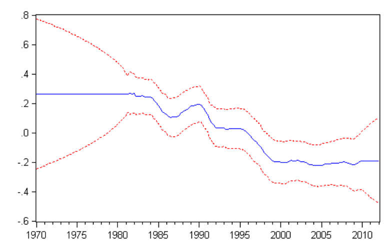
(f) Japan



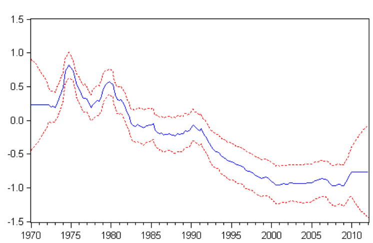
(g) Netherlands



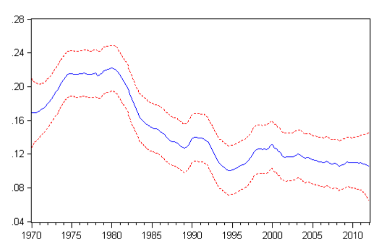
(h) Norway



(i) Sweden

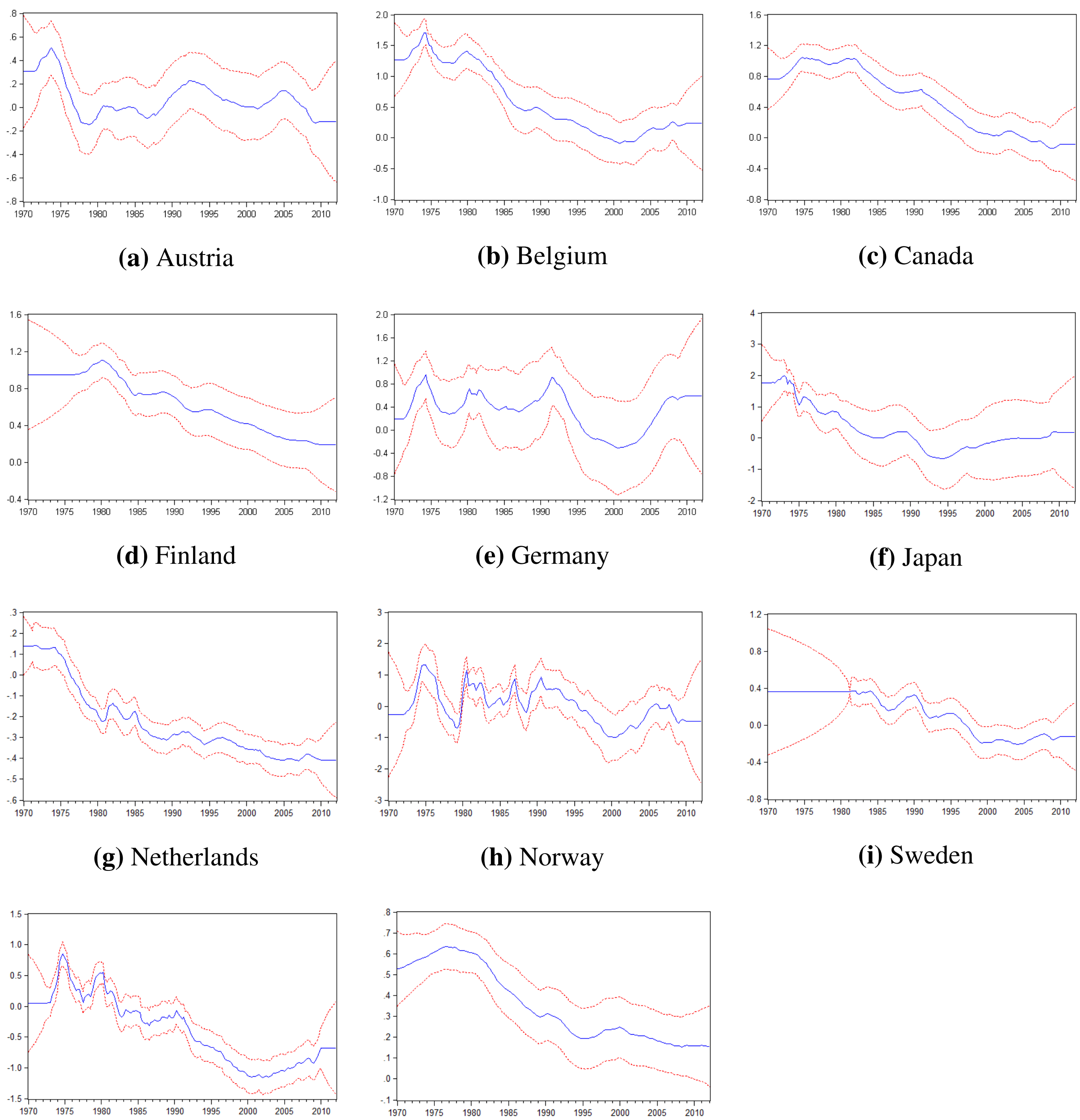


(j) UK



(k) US

Figure 2.8: Smoothed estimates for γ_t: Model (2.21b)



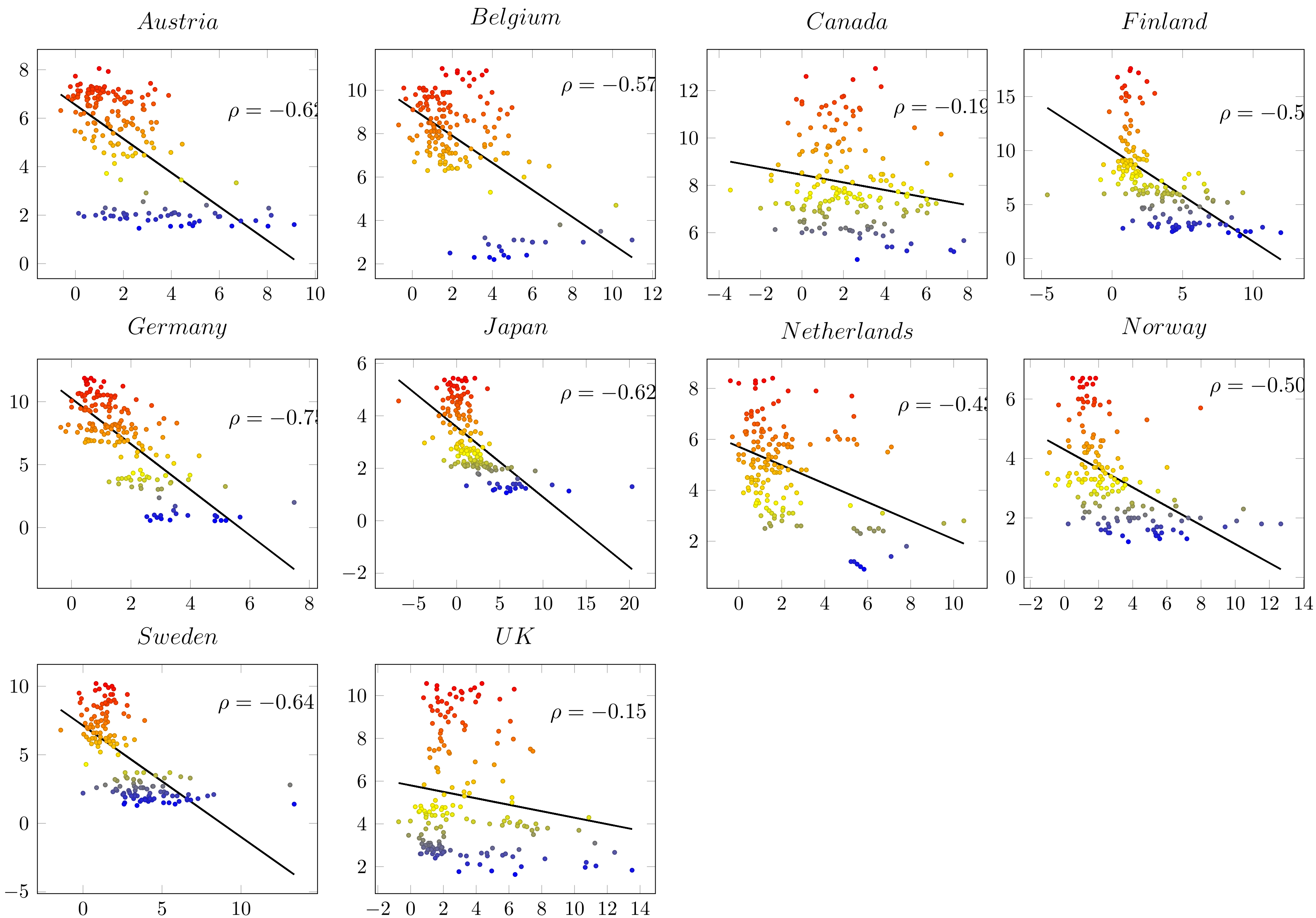
(a) Austria **(b)** Belgium **(c)** Canada

(d) Finland **(e)** Germany **(f)** Japan

(g) Netherlands **(h)** Norway **(i)** Sweden

(j) UK **(k)** US

Figure 2.9: Correlations between unemployment (y $axis$) and wage inflation (x $axis$)



Chapter 3

Implications of Random Wage Indexation Outcome

'One man's wage increase is another man's price increase'.

–Harold Wilson

3.1 Introduction

Wage indexation links wages to the evolution of some underlying variables which are typically unobservable when the base contracts are negotiated. The main motivation behind indexing wages is to reduce the incentive for monetary authorities to create surprise inflation. Indexation to cost of living (inflation) is widespread in the US. It is implemented in various ways and at different levels across EU member countries. In terms of coverage for instance, all workers in Belgium, Cyprus, Luxembourg and Malta are entitled by law to have wages indexed to inflation. In Spain, wage indexation is present in every collective agreement, even though not required by law (see Mongourdin-Denoix and Wolf (2010)).

Given the widespread nature of wage indexation, it is not surprising to find a substantial portion of literature devoted to this topic. Two fundamental assumptions made in theoretical studies on wage indexation and its effect on optimal monetary policy are that the elasticity of indexed wages to prices is constant and that policy makers can directly influence this elasticity. Under these assumptions, results in the seminal paper of Gray (1976), among others, conclude that optimal wage indexation attenuates the effects of nominal shocks while exacerbating the effects of real shocks. Empirical evidence documented in Holland (1986) and Ascari et al. (2011) among others however suggest a time-varying process for the degree of wage indexation. Furthermore, there is no direct empirical evidence in support of the use of wage indexation by policy makers as a policy tool.

The first of the two objectives of this paper is to attempt to give a theoretical explanation for the time variation in the degree of wage indexation. In pursuit of this objective, this study adopts a different approach to wage indexation than that implied by the assumptions mentioned above. It is assumed that wage indexation is a bargaining or negotiation outcome between two unions rather than a potential instrument used by a policy maker in the conduct of optimal monetary policy.

The main motivation for this approach is based on the following result from Caju et al. (2008): '...With regard to elements entering wage negotiations, prices are the most important determining factor...'. In other words, wage indexation to prices constitutes the foremost reason for wage negotiations. Given that mixed strategies are part of the generalized solution sets to bargaining problems and other games, wage indexation outcome does not necessarily need to be constant. An implication of non-constant wage indexation as a bargaining outcome is the difficulty policy makers might face in its use as a policy instrument. Therefore any optimal monetary policy can only take into account the properties of the distribution of wage indexation. This is especially the case when policies are formulated before the realization of the wage indexation outcome. The bargaining outcomes derived in this study imply continuous distributions for the degree of wage indexation.

The second objective of this study is to investigate within a small theoretical macro model the implications of the bargaining outcomes regarding wage indexation. To this end, we derive an AS curve that differs from the standard Lucas supply curve in that it exhibits a Brainard type multiplicative uncertainty. It is subsequently shown that the popular linearized solution around the deterministic steady state can differ wildly from the truly stochastic stationary solution. As Woodford (2003, p.142) aptly writes: 'But my interest in the present study is in the identification of better monetary policies within the class of policies under which inflation is never great. In fact, I make extensive use of approximations that are expected to be accurate only for the analysis of policies of that kind'. We show that even in our simple macro model the usual linearization dramatically alters the stochastic properties of the macro variables. This result echoes that of Babus and de Vries (2010). Even though it is standard practice in macroeconomics to use linear approximations, we show that it is also very important to consider the effects of linearization on the stochastic properties of the model solution.

This study is organized as follows. Section 3.2 derives wage indexation as a bargaining or negotiation outcome. Section 3.3 derives the AS curve based on the random wage indexing outcome derived. Section 3.4 investigates the implications of random wage indexation for the conduct of optimal monetary policy and Section 3.5 concludes.

3.2 Wage indexation as a bargaining outcome

Early applications of game theory to bilateral bargaining have been motivated by the alternating-offer game, first suggested by Stahl (1972). Kennan and Wilson (1988) compare the theoretical characteristics of wage bargaining games to the empirical properties of strike data. They conclude that while attrition models appear to fit Canadian data better, the properties of US data are best described by screening models. In this work we consider two main settings under which wage indexation bargaining occurs: one requires the presence of an arbitration party while the other does not.

The number of independent wage indexation negotiations in an economy can vary, depending on how centralized and regulated the wage bargaining system is. The aggregate wage indexation outcome will be a weighted average of all the outcomes from the individual negotiations. Let x_i be the wage indexation outcome from an individual negotiation. Assume there are m bilateral negotiations conducted. Further assume that aggregate wage indexation is simply the average of all individual indexations. The aggregate indexation is as follows:

$$x = \frac{1}{m}\sum_{i=1}^{m} x_i$$

The aggregate indexation is in itself, a random variable with its expected value the expectation of the individual indexation. Its variance is given as follows:

$$Var(x) = \frac{1}{m}Var(x_i)$$

From the previous expression it can be seen that the variance of aggregate wage indexation is decreasing in number of negotiations. In other words, a relatively centralized and regulated wage bargaining system has a larger variance in wage indexation outcome.

In each of the bargaining processes considered in this study, we assume that the value of the object is public information. For instance, if bargaining is done over the deviation of inflation from its expected value, the value of this differential is known and is the same to both bargaining parties .

Bargaining with arbitration

Theoretical and empirical studies on wage negotiations in the presence of arbitration reveal two main sources of uncertainty in the negotiation outcomes. One source of uncertainty is the random element of an arbitrator's preferred settlement as found in Ashenfelter and Bloom (1984) and Bloom (1988). The other source is the inherently random offers and counteroffers from the two parties involved in the negotiation. The bargaining with arbitration model in this part of our work falls under the latter class of models. We describe the bargaining setting in the presence of arbitration in what follows.

Consider the bargaining setting under which workers unions and employers unions bargain over the degree of indexation. The workers union prefers a full indexation (i.e. when wage indexation is 1). The employers union prefers a zero wage indexation. Thus, the outcome of the bargaining process lies within the interval $[0, 1]$. Both bargaining unions have to inform the arbitrator of their respective offers to each other.

Let ω and ϵ represent the offers of the workers union and employers union respectively. Furthermore, let the payoff from the bargaining process be p_w and p_e for the workers union and the employers union respectively. Based on offers made by each of the negotiating parties, the arbitrator awards the following payoff to the workers union (p_w)[1]:

$$p_w(\omega, \epsilon) = \begin{cases} 1 - (\omega - \epsilon) \text{ if } \omega > \epsilon \\ -(\omega - \epsilon) \quad \text{if } \omega < \epsilon \end{cases}. \tag{3.1}$$

Thus, if the employers union is more generous than the workers union, the latter receives the differential between the former's bid ϵ and the its own bid ω. In other words, the degree of wage indexation in this case becomes $(\epsilon - \omega)$. By implication, the employers get $1 - (\epsilon - \omega)$. Conversely, if the workers union is the more generous party, then it accomplishes a $1 - (\omega - \epsilon)$ degree of wage indexation. The employers retain $(\omega - \epsilon)$.

This payoff structure ensures that neither party can be too generous with its offer since the resulting payoff might be very little. Also, neither party can offer too little to the other since doing so might make them worse off. These two repelling forces give rise to a mixed strategy equilibrium under which the parties draw from the following uniform distribution:[2]

$$F(\omega) = \omega \qquad\qquad \omega \in [0, 1]. \tag{3.2}$$

Here, ω is the symmetric mixed strategy bid offered by a bargaining party. Let A be the bargaining outcome (payoff) for the workers union (i.e. $A = p_w(\omega, \epsilon)$). It is is shown in Appendix

[1]A symmetric payoff structure applies to p_e.

[2]See Appendix 3.A.1 for the derivation of the mixed strategy equilibrium.

3.A.1 that

$$F_A(a) = a.$$

Averaging the bargaining outcome across the m number of independent negotiations, one derives the following distribution for the aggregate bargaining outcome.

$$Pr[\bar{A}_m \leq a] = \frac{1}{m!} \sum_i (-1)^j \binom{m}{j} (ma - j)_+^m. \tag{3.3}$$

The above class of distributions have expected value 1/2. The special case $m = 1$ yields the uniform distribution, which has the highest variance.

War of attrition

Strikes and other forms of industrial actions are known to occur when labour negotiations break down. War of attrition models are frequently used to describe strike data (see Geraghty and Wiseman (2008) for example). In this game, we assume that strikes are used by unions to force concessions from each other. We briefly describe the setting of the second bargaining setup in what follows in this section.

Consider a setting under which the workers union resorts to strikes in order to achieve any degree of wage indexation. Assume each party bears costs proportional to the duration of the strike. Further assume that each union's cost is private information. The duration of holdouts for each of the unions involved depends on the maximum costs it is willing to incur, hereinafter referred to as the union's bid. The union that bids the highest wins the value of the differential between 1 and the losing union's bid. The losing union loses the value of its bid (cost).

Let the bids of workers union and employers unions be respectively ω and ϵ. The payoff structure of a striking workers union is:

$$p_w(\omega, \epsilon) = \begin{cases} 1 - \epsilon \text{ if } \omega > \epsilon \\ -\omega \quad \text{if } \omega < \epsilon \end{cases} . \tag{3.4}$$

The payoff structure in (3.4) implies each union has the incentive not to concede given that it does not concede right at the beginning (when bid is 0). However, the strike cannot continue indefinitely since the bid (cost) eventually outweighs the value contested (i.e. 1). This gives rise to a mixed strategy equilibrium concerning the amount of bid (cost) to lose. While some pure equilibrium strategies exist for this bargaining setup, our interest lies in the mixed strategy equilibrium.

It is worth noting that the payoff structure in (3.4) allows for negative payoffs even for the winning union. Imagine a situation in which both unions have a common but incorrect valuation of the inflation differential (which has been normalized to 1) during the strike. In particular, if the actual value of the inflation differential is much less than the perceived value, a union's bid can exceed the actual inflation differential. This implies that there is a possibility of negative wage indexation even if the workers union wins concessions.

It is shown in Appendix 3.A.2 that the solution to this game involves a mixed strategy over an exponential distribution and is as follows:

$$F(\omega) = 1 - e^{-\omega}. \tag{3.5}$$

Let B be the payoff to the workers union under war-of-attrition type bargaining (i.e. $B = p_w(\omega, \epsilon)$). In Appendix 3.A.2, we derive the following distribution of B:

$$F_B(b) = \left[\frac{1}{2} + \frac{e^{2b-2}}{2}\right] \mathbb{1}_{(0<b\leq 1)} + \left[\frac{e^{2b-2}}{2} + \frac{e^{2b}}{2}\right] \mathbb{1}_{(b\leq 0)}$$

The above distribution is a mixture distribution with a mean of 0. The support suggests that a negative wage indexation bargaining outcome is possible. Also, this outcome is bounded from above. In the absence of over-indexation, the upper limit to the wage indexation bargaining outcome is 1. This value is obtained when the employers union immediately concedes to the workers unions, such that $\epsilon = 0$.

Algebraically deriving the distribution of the average bargaining outcome of m independent negotiations ($\bar{B}_m$) is rather tedious. However, simulations show that a normal distribution approximates it as as m increases.

From bargaining outcome to wage indexation

In the subsequent part of this study, it is assumed that wages are indexed to inflation. If indexation results from bargaining in the presence of an arbitrator, then the outcome is between 0 and 1. However the possibility of indexing to output could cause wages to change more than the change in inflation from its expected value upon which contracts were previously negotiated. Furthermore, high observed inflation could serve as a trigger for workers to demand wage increases more than the deviations of inflation from their expected values. To account for these possibilities, we define aggregate wage indexation in the two cases of bargaining considered above as follows:

$$x_t = \kappa_a \bar{A}_m \qquad \kappa_a \geq 1 \tag{3.6}$$

$$x_t = 1 + \kappa_w - \bar{B}_m \qquad \kappa_w \geq 0, \tag{3.7}$$

The random outcome of wage indexation in our model only partially explains the origins of the randomness in the wage indexation parameter, but does not explain its persistence as would be suggested by Holland (1986). Again, according to our model, only exogenous forces drive the randomness, which might not be fully representative of real world observations. For instance, it is known that workers agitate for indexed wage contracts when inflation is observed to be persistently high in the economy. In addition to acknowledging the just mentioned observations, we maintain that this study offers an exploratory glimpse into the effects of the random component of wage indexation. In the subsequent sections, we examine the implications of random wage indexation on optimal monetary policy.

3.3 Macroeconomic model

Indexed wage contracts allow wages to be automatically adjusted in the event that actual inflation differs from the expected. Wage indexation was modeled as a bargaining or negotiation outcome in the previous section. Following this result, we assume it is an iid random process. The adaptive indexation rule is as follows:

$$w_t = E_{t-1}w_t^* + x_t(p_t - E_{t-1}p_t), \tag{3.8}$$

where w_t, w_t^*, p_t and E_{t-1} denote the log levels of the indexed wage rate, the labour market clearing wage rate, the price level and the expectation operator respectively. The x_t denotes the iid random degree of wage indexation with mean x. This variable reflects the uncertain outcome of the bargaining process, which is here taken exogenous to the model. Thus all labour contracts earn a wage rate equal to the expected market clearing rate $E_{t-1}w_t^*$ plus an extra compensation proportional to the difference between the actual inflation rate and expected inflation. The indexation rule (3.8) is a slightly modified version of that found Gray (1976) in that it is time varying instead of constant.

Assume that output Y is produced by a fixed coefficient Ricardian technology $Y = ZN^a$, where $a < 1$ reflects diminishing marginal returns to scale. Productivity shocks are captured by the iid random variable Z, where $z = \ln Z$ has a zero mean. Further assume that industry is perfectly competitive, so that profits are zero. Labour demand then derives from the profit optimization condition equating the real wage with the marginal productivity of labour:

$$\frac{W_t}{P_t} = aZ_tN_t^{a-1}.$$

Let small case letters denote the log values of their upper case counterparts. Solving for labour demand gives

$$\begin{aligned} n_t^d &= \frac{\ln a}{1-a} - \frac{1}{1-a}(w_t - p_t) + \frac{1}{1-a} z_t \\ &= \delta_0 - \delta_1 (w_t - p_t) + \delta_1 z_t, \end{aligned} \tag{3.9}$$

where $\delta_0 = (\ln a) / (1-a)$ and $\delta_1 = 1/(1-a)$. The labour supply function derives from household's optimization problem. Without further ado we assume that labour supply is upward sloping in the real wage rate:

$$n_t^s = \beta_0 + \beta_1 (w_t - p_t). \tag{3.10}$$

In the absence of any nominal rigidity, solving (3.9) and (3.10) gives the labour market clearing wage rate w_t^* and equilibrium employment n_t^*. These are written below:

$$w_t^* = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t + \frac{\delta_1}{\delta_1 + \beta_1} z_t \tag{3.11}$$

$$n_t^* = \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \frac{\delta_1 \beta_1}{\delta_1 + \beta_1} z_t. \tag{3.12}$$

Equilibrium log output y_t^* is then derived by substituting(3.12) into the following expression for log output: $y_t = a n_t^d + z_t$. The resulting expression for the market clearing equilibrium output is

$$y_t^* = a \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \left(\frac{a \delta_1 \beta_1}{\delta_1 + \beta_1} + 1 \right) z_t. \tag{3.13}$$

Due to wage indexation, the labour market may not clear. The aggregate supply function can then be obtained by substituting the expression for the indexed wage into the labour demand function and defining the output gap as $y_t - y_t^*$. Appendix 3.B gives a detailed derivation of the following aggregate supply function under wage indexation:

$$y_t - y_t^* = A_t (\pi_t - E_{t-1} \pi_t) + c z_t \tag{3.14}$$

where $c = a\delta_1^2/(\delta_1 + \beta_1)$ and the slope of the 'Lucas style' supply curve is

$$A_t = a\delta_1(1 - x_t) = \frac{a}{1-a}(1 - x_t). \tag{3.15}$$

Note that the slope in (3.15) is not fixed but random due to the random degree of wage indexation x. Also, it is possible to have a negatively sloping AS curve when there is over-indexation (i.e. $\kappa_a > 1$).

The aggregate supply curve with a random coefficient has less restrictions than the one with a constant coefficient.[3] One implication of expression (3.15) is the possibility that aggregate supply could become vertical in the short run, mimicking the long run aggregate supply curve, when $x_t = 1$. For example, if wages are fully indexed, $x_t = 1$, as can be the case in Belgium, then the real wage rate equals the expected labour market clearing real wage rate. The expected output gap in this case is zero and does not depend on the level of inflation.

3.4 Wage indexation and optimal monetary policy

This section investigates the conduct of monetary policy in the presence of an AS curve with a random slope. Brainard (1967) conducts a similar analysis on the effects of the random slope parameter on the conduct of policy. The analysis contained in this section differs from that in the aforementioned study in one main respect: there is no uncertainty in the dependence of the output gap on our policy instrument (interest rate). We however view the analysis contained in this section as complementary to that by Brainard (1967).

3.4.1 FED versus ECB

The result from the analysis performed in this part of the study is not a direct consequence of time variation or the random nature of wage indexation. Nevertheless, we find it interesting to

[3]The idea of a time-varying Phillips coefficient is not new. Aside the investigation of the effect of time-varying Phillips coefficient by Brainard (1967), Cogley and Sbordone (2008), Swamy and Tavlas (2007) and Hondroyiannis et al. (2009) also estimate the New Keynesian Phillips Curve with time-varying coefficients.

investigate optimal monetary policy in the presence of both forward looking expectations and lagged expectations. In the previous section we derived the following AS schedule with lagged expectations for the output gap g_t:

$$g_t = y_t - y_t^* = A_t(\pi_t - E_{t-1}\pi_t) + \mu_t \tag{3.16}$$

where $\mu = cz_t$. We complete the macro model with the following NK type IS curve:

$$g_t = -B(i_t - E_t\pi_{t+1}) + \gamma g_{t-1} + \varepsilon_t, \tag{3.17}$$

where i is the nominal interest rate. Note that the NK version of the IS curve, in contrast to the supply schedule, has forward looking expectations. Except for the random coefficient A_t and the lagged expectations $E_{t-1}\pi_t$ in the AS schedule, our model is a textbook NK macro model. We turn to discussing policy rules that are generated by alternative objectives.

The Maastricht treaty dictates that the ECB targets an inflation level $\hat{\pi}$. A second order approximation to welfare makes the policy loss function quadratic in missing the target. Thus, the ECB loss function is given by

$$L = E_{t-1}\left[(\pi_t - \widehat{\pi})^2\right].$$

The treaty gave discretion to the ECB and left $\widehat{\pi}$ open. The ECB has decided $\widehat{\pi} = 2\%$. We assume that interest rates are used as intermediate targets to stabilize inflation at its optimal level in the case of inflation targeting. Quite a different objective applies to the FED. By law the FED is required to minimize a weighted average of the squared deviation of output and inflation from their respective targets. Assuming equal weights on inflation stabilization and output stabilization, the loss function can be written down as in the following:

$$L = E_{t-1}\left[(g_t - \widehat{g})^2 + (\pi_t - \widehat{\pi})^2\right].$$

These are the standard types of objective functions found in most macro textbooks. Interestingly, while one often finds a treatment with the inflation objective being given zero weight, the other extreme of the ECB objective with zero weight on the output gap is much less common.

Before we can solve for the optimal rule, we state the assumptions on the distributions of the innovations μ_t, ε_t and A_t. All three are iid innovations that are also cross sectionally independent. The additive terms μ_t and ε_t have zero mean. For now it suffices to assume that $E\left[A\right] > 0$ and finite, while it also holds that $E\left[1/A\right] \neq 0$ and $E\left[|1/A|\right] < \infty$, $E\left[1/A^2\right] < \infty$, i.e. the inverse also has a finite first and second moment.

We derive the optimal interest targeting rate rule under the FED type objective and ECB inflation objective. Consider the following Taylor type interest rate rule:

$$i_t = \widehat{\pi} + \frac{\gamma}{B} g_{t-1}. \tag{3.18}$$

By the IS curve (3.17) and AS curve (3.16), this rule implies that

$$\begin{aligned}
E_{t-1}\left[\frac{g_t}{A_t}\right] &= E_{t-1}\left[\frac{-B(\widehat{\pi} - E_t\pi_{t+1}) + \varepsilon_t}{A_t}\right] \\
&= E_{t-1}\left[\frac{1}{A_t}\right] B\left(E_{t-1}\pi_{t+1} - \widehat{\pi}\right) \\
&= E_{t-1}\left[\frac{A_t(\pi_t - E_{t-1}\pi_t) + \mu_t}{A_t}\right] \\
&= 0.
\end{aligned}$$

Given the above expression for the expected output gap and the assumption made regarding distribution of the noise terms, we know the following holds:

$$E_{t-1}\pi_{t+1} = \widehat{\pi}$$

Moreover, Equation (3.17) implies that

$$\begin{aligned} E_{t-1}\left[g_t\right] &= E_{t-1}\left[-B(\widehat{\pi} - E_t\pi_{t+1}) + \varepsilon_t\right] \\ &= B\left(E_{t-1}\pi_{t+1} - \widehat{\pi}\right) = 0. \end{aligned}$$

Thus, the output gap is a pure white noise which is the following:

$$g_t = \varepsilon_t.$$

The time shifted IS curve implies

$$\pi_{t+1} = E_t\pi_{t+1} + \frac{\varepsilon_{t+1} - \mu_{t+1}}{A_{t+1}}.$$

Given that $E_{t-1}\pi_{t+1} = \widehat{\pi}$, it follows that $E_{t-1}\pi_t = \widehat{\pi}$ is model consistent, i.e.

$$\pi_t = \widehat{\pi} + \frac{\varepsilon_t - \mu_t}{A_t}. \tag{3.19}$$

With these expressions at hand, the loss function of the FED can be written as

$$\begin{aligned} L &= E_{t-1}\left[\left(g_t - \widehat{g}\right)^2 + \left(\pi_t - \widehat{\pi}\right)^2\right] \\ &= E_{t-1}\left[\varepsilon_t^2 + 2\widehat{g}\varepsilon_t + \widehat{}^{2}\right] + E_{t-1}\left[\left(\pi_t - E_{t-1}\pi_t\right)^2\right] + \left(E_{t-1}\pi_t - \widehat{\pi}\right)^2 \\ &= \sigma_\varepsilon^2 + \widehat{g}^2 + \left(\sigma_\varepsilon^2 + \sigma_\mu^2\right)\left(\sigma_{1/A}^2 + E[1/A]^2\right) \end{aligned}$$

One shows that no other targeting rule can lower this loss level.

Interestingly, it directly follows from (3.19) that the same targeting rule also minimizes the ECB objective. The two targeting rules coincide due to the fact that under the rule (3.18), the output gap is pure white noise. The deeper reason is that the IS curve has forward looking expectations so that current shocks and policy actions are taken into account by the public, eliminating the scope for discretionary policy actions. Thus while the Maastricht treaty has provided a commitment device against dovish central bankers, it serves no purpose since the public is forward looking on the demand side.

3.4.2 Stochastic properties of equilibrium inflation

We showed in the foregoing analysis that interest targeting under the ECB objective or the hybrid FED objective requires the same targeting rule and leads to the same equilibrium inflation rate in (3.19). We repeat this equation below:

$$\pi_t = \widehat{\pi} + \frac{\varepsilon_t - \mu_t}{A_t}.$$

We now proceed to investigate the stochastic properties of this solution. But first we note that in case of the more traditional fixed coefficient specification for the AS curve, with $A_t = \bar{A}$ for instance, the same targeting rule (3.18) is optimal and implies

$$\pi_t = \widehat{\pi} + \frac{\varepsilon_t - \mu_t}{\overline{A}}. \tag{3.20}$$

Under the ECB objective it is apparent that $i_t = \widehat{\pi} + (\gamma/B)\, g_{t-1}$ is optimal as $L = \left(\sigma_\varepsilon^2 + \sigma_\mu^2\right)/\overline{A}^2$. The fixed coefficient specification also follows from the linearization of the wage indexation rule (3.8) around the deterministic steady state. The latter practice is commonly used in solving more complex DSGE models. However, this practice is not innocuous as we now intend to argue.

Consider the first moment of (3.19) and (3.20). Taking expectations directly reveals that $E_{t-1}\pi_t = \widehat{\pi}$ under either specification. Moreover, for the variance

$$\sigma_\pi^2 = \left(\sigma_\varepsilon^2 + \sigma_\mu^2\right)\left(\sigma_{1/A}^2 + E[1/A]^2\right)$$

in the case of (3.19), while (3.20) implies

$$\sigma_\pi^2 = \left(\sigma_\varepsilon^2 + \sigma_\mu^2\right)\frac{1}{\overline{A}^2}$$

which is not much of a material difference except for the fact that we expect the variance of inflation under the random aggregate supply slope to be larger than the linearized version.

Nevertheless, the fluctuations in inflation are not necessarily well captured by the first two moments. To see this, assume a specific distribution for A_t. Suppose that A_t follows a beta distribution on $[0,1]$[4] such that

$$P\{A_t \leq x\} = x^{\alpha},\ \alpha > 2.$$

The slope of the AS curve varies between the diagonal and a vertically positioned curve. The fact that zero is in the support makes it possible that the AS curve assumes its long run position. Furthermore, note that the m-th central moment of A equals $(\alpha+1)/(m+\alpha+1)$.

The distribution of the inverse of A_t can then be derived as follows:

$$\begin{aligned}
\Pr\left\{\frac{1}{A_t} \leq q\right\} &= \Pr\left\{A_t \geq \frac{1}{q}\right\} \\
&= 1 - \Pr\left\{A_t \leq \frac{1}{q}\right\} \\
&= 1 - \frac{1}{q^{\alpha}}.
\end{aligned}$$

The last expression is a Pareto distribution with support $[1,\infty)$. By assuming that $\alpha > 2$, it readily follows that the moment conditions assumed earlier are satisfied, i.e. $E\left[1/A\right] = \alpha/\left(\alpha-1\right)$ and $E\left[1/A^2\right] = \alpha/\left(\alpha-1\right)^2\left(\alpha-2\right)$. This not withstanding, all moments $m > \alpha$ are unbounded. Thus $1/A$ has fat tails and can easily take on large values.

Since inflation is a function of the inverse of A_t, inflation also follows a fat-tailed distribution. This is different for the output gap, which was shown to equal white noise ε_t; as long as ε_t is well behaved, this is transferred to g_t. But even though A_t itself is nicely behaved, in the sense that it has all moments bounded, this is different for the inverse. A fat-tailed distribution for inflation

[4] While this assumption might appear somewhat arbitrary, it should noted that a beta distribution approximates the distribution of the wage indexation outcome derived in 3.3 under some conditions. For instance, consider aggregate wage indexation as an outcome of the bargaining with arbitration process outlined in Section 3.2. Further assume that $\kappa_a = 1$ and $a = 1/2$. The resulting distribution for A_t for $m \to \infty$ is symmetric around $1/2$ and resembles a $beta(m,m)$ distribution. Furthermore, it is a bounded distribution on the unit interval.

implies that the economy experiences relatively high inflation more often than is predicted under the assumption that inflation is for instance normally distributed (which most linearized models implicitly do). Moreover, similar to Babus and de Vries (2010), it follows that the behavior of inflation derived from the linearized version (3.20) is quite different as only the stochastic properties of ε_t and μ_t would matter.

3.5 Conclusion

In spite of empirical evidence pointing to the contrary, most studies on wage indexation are based on the assumption that wage indexation is constant. In this study, we derive wage indexation as a mixed strategy bargaining outcome. It is subsequently shown that both the exponential distribution and the uniform distribution are possible outcomes regarding the distribution of wage indexation. The resulting AS schedule derived differs from the standard ones in that it has a random slope coefficient. Using this AS relation, we subsequently investigate the effects the random nature of wage indexation has on the conduct of policy.

We also show how a FED based hybrid objective and an ECB type inflation goal imply the same interest rate targeting rule in the context of our model. Strictly speaking, this is not a direct consequence of the random nature of wage indexation. Rather, it is a consequence of having both forward looking expectations and lagged expectations in the macro model we constructed. We also show how the stochastic properties of inflation implied by a linearized AS schedule differ dramatically from those of the true stationary solution. While the stationary distribution of inflation from the true model implies quite extreme fluctuations in inflation, the linearized model smoothens this feature away. As the recent financial crisis has shown us, the effects of extreme fluctuations in economic variables cannot be ignored. However, as shown in this work, this is precisely what is implicitly done by linearizing models.

Even though this study is to the best of our opinion the first to attempt to explain the source of the randomness in wage indexation, it has some shortcomings. The study models wage indexation as an exogenous process whereas it is empirically established that wage indexation (as proxied by COLA adjusted contracts) depends on the level of inflation. Furthermore, it is plausible that variables pertaining to the labour market institutions do affect the distribution of wage indexation. The derivation in this study however does not incorporate this fact. We hope to pick these issues up in a future research.

3.A Derivation of mixed strategy equilibrium

3.A.1 Bargaining under arbitration

Consider the following payoff structure under bargaining in the presence of an arbitrator.

$$p_w(\omega, \epsilon) = \begin{cases} 1 - (\omega - \epsilon) \text{ if } \omega > \epsilon \\ -(\omega - \epsilon) \quad \text{if } \omega < \epsilon. \end{cases}$$

Suppose F is the mixed-strategy equilibrium with density $f(\omega)$, the expected payoff of a party who offers ω against the other party's mixed strategy is[5]

$$EU(\omega) = \int_0^\omega p_w(\omega, \epsilon) dF(\epsilon) + \int_\omega^1 p_w(\omega, \epsilon) dF(\epsilon).$$

It holds that the offer ω maximizes the above expression given that F is a mixed strategy equilibrium. Hence, the following expression should hold:

$$\frac{\partial EU(\omega)}{\partial \omega} = 0.$$

After substituting the expressions for the payoff functions into the expected payoff function, we get the following expression:

$$EU(\omega) = \int_0^\omega (1 - \omega + \epsilon) dF(\epsilon) + \int_\omega^1 (-\omega + \epsilon) dF(\epsilon).$$

[5]See Baye et al. (2009) for a formal definition of symmetric mixed strategy equilibrium.

By noting that $F(0) = 0$ and $F(1) = 1$ and by the use of integration by parts, the above expression can be easily rendered as follows:

$$EU(\omega) = F(\omega) + (1-\omega) + \int_0^1 F(\epsilon)d\epsilon.$$

Maximizing this expression with respect to ω yields the simple differential equation

$$f(\omega) = 1,$$

which after considering the boundary conditions $F(0) = 0$ and $F(1) = 1$ has the same solution as indicated in Section 1 of the text:

$$F(\omega) = \omega. \qquad (3.21)$$

Distribution of payoff

The payoff p_w is dependent on the difference between the bids offered by the bargaining parties. Let Z be the difference between the workers union's bid and that of the employers union. In other words,

$$Z = \omega - \epsilon$$

Given the unit uniform distributions for ω and ϵ, it follows that Z has the following density function:

$$f_Z(z) = (1+z)\mathbb{1}_{\{-1 \leq Z \leq 0\}} + (1-z)\mathbb{1}_{\{0 < Z \leq 1\}}.$$

The symbol $\mathbb{1}$ denotes the indicator function which takes on a value of 1 if the statement in the subscript holds true and 0 otherwise.

Let A be the payoff of the workers union from the bargaining game. It should be noted that the payoff structure implies that A is always positive and lies within the unit interval. Therefore, in the absence of over-indexation, this payoff is the individual wage indexation outcome. A can be expressed in terms of Z as follows:

$$A = (-Z)\mathbb{1}_{\{-1 \leq Z \leq 0\}} + (1 - Z)\mathbb{1}_{\{0 < Z \leq 1\}}.$$

The distribution of A can be derived from the distribution of Z since the former variable is a function of the latter variable. A step-by-step derivation of the distribution of A is as follows:

$$\begin{aligned} Pr(A \leq a) &= Pr(-Z \leq a)\mathbb{1}_{\{-1 \leq Z \leq 0\}} + Pr(1 - Z \leq a)\mathbb{1}_{\{0 < Z \leq 1\}} \\ &= Pr(Z \geq -a)\mathbb{1}_{\{-1 \leq Z \leq 0\}} + Pr(Z \geq 1 - a)\mathbb{1}_{\{0 < Z \leq 1\}} \\ &= \int_{-a}^{0} (1 + z)dz + \int_{1-a}^{1} (1 - z)dz \\ &= a - \frac{a^2}{2} + \frac{a^2}{2} \\ F_A(a) &= a. \end{aligned}$$

Thus, it can be concluded that the individual wage indexation bargaining outcome is uniformly distributed on the interval $[0, 1]$.

3.A.2 War of attrition

Assume that the negotiating parties (a and b) bid ω and ϵ respectively. The payoff structure to this war of attrition game can be written as follows:

$$p_w(\omega, \epsilon) = \begin{cases} 1 - \epsilon \text{ if } \omega > \epsilon \\ -\omega \quad \text{if } \omega < \epsilon. \end{cases}$$

The expected utility under a mixed strategy equilibrium with distribution F is given by the following expression:

$$\begin{aligned} EU &= \int_0^{\omega} f(\epsilon)d\epsilon - \omega \int_{\omega}^{\infty} f(\epsilon)d\epsilon \\ &= \int_0^{\omega} f(\epsilon)d\epsilon - \omega[1 - F(\omega)]. \end{aligned}$$

Maximizing this with respect to ω and equating to 0 allows us to derive the first order condition as follows:

$$\frac{f(\omega)}{1 - F(\omega)} = 1.$$

Solving the above equation while taking into consideration that $F(0) = 0$, we arrive at the following solution:

$$F(\omega) = 1 - e^{-\omega}. \tag{3.22}$$

Distribution of payoff

The payoff for the workers union under the war-of-attrition type bargain is distributed on the support $-\infty \leq p_w \leq 1$. In deriving the distribution of the payoff, it is worth to note that a particular realization of the payoff depends on the difference between the workers union's bid and that of the employers union ($\omega - \epsilon$). Let B be the payoff to workers. The expression for B is as follows:

$$B = (1 - \epsilon)\mathbb{1}_{\omega > \epsilon} + (-\omega)\mathbb{1}_{\epsilon > \omega}$$

In order to derive the probability distribution function for B, we first derive the distributions under the following case: $\omega > \epsilon$.

$$\begin{aligned}
Pr(B \leq b) &= Pr(1 - \epsilon \leq b, \omega \geq \epsilon) \\
&= Pr(\epsilon \geq (1 - b), \omega \geq \epsilon) \\
&= \int_{1-b}^{\infty} e^{-\epsilon} \left(\int_{\epsilon}^{\infty} e^{-\omega} \ d\omega \right) \ d\epsilon.
\end{aligned}$$

Solving the integral above yields the following distribution function of B on the support $[B \leq 1]$:

$$F_B(b) = \frac{e^{2b-2}}{2}$$

Similarly, a distribution function for B when $\epsilon \leq \omega$ is derived in the following equations.

$$\begin{aligned}
Pr(B \leq b) &= Pr(-\omega \leq b, \epsilon \leq \omega) \\
&= Pr(\omega \geq -b, \epsilon \leq \omega) \\
&= \int_{-b}^{\infty} e^{-\omega} \left(\int_{\omega}^{\infty} e^{-\epsilon} \ d\epsilon \right) \ d\epsilon.
\end{aligned}$$

Solving the integral above yields the following distribution of B on the support $[B \leq 0]$.

$$F_B(b) = \frac{e^{2b}}{2}$$

Summing up the respective distributions under the two cases, one derives the following mixture distribution for the random wage indexation bargaining outcome (B):

$$F_B(b) = \frac{1}{2}[1 + e^{2b-2}]\mathbb{1}_{0<b\leq 1} + \frac{1}{2}[e^{2b-2} + e^{2b}]\mathbb{1}_{b\leq 0}$$

3.B The Aggregate supply schedule

We derive the AS schedule (3.14) from the main text. The expected market clearing wage rate follows from (3.11) in the main text. It is as follows:

$$w_t^* = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t + \frac{\delta_1}{\delta_1 + \beta_1} z_t.$$

By taking expectations and recalling the zero mean assumption regarding productivity shocks z_t, we obtain the following expression:

$$E_{t-1} w_t^* = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + E_{t-1} p_t.$$

Substitute this into the expression for wage indexation rule (3.8) to get:

$$\begin{aligned} w_t &= E_{t-1} w_t^* + x_t (p_t - E_{t-1} p_t) \\ &= \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + E_{t-1} p_t + x_t (p_t - E_{t-1} p_t). \end{aligned}$$

Hence the real wage rate is:

$$w_t - p_t = (x_t - 1)(p_t - E_{t-1} p_t) + \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1}.$$

Given that the labour market does not clear at this real wage rate, the rationed level of employment follows from substitution into the labour demand schedule (3.9)

$$\begin{aligned} n_t^d &= \delta_0 - \delta_1 (w_t - p_t) + \delta_1 z_t \\ &= \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} - \delta_1 (x_t - 1)(p_t - E_{t-1} p_t) + \delta_1 z_t. \end{aligned}$$

The output gap then follows from the above and the expression for market clearing output (3.13)

$$y_t^* = a \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \left(\frac{a \delta_1 \beta_1}{\delta_1 + \beta_1} + 1 \right) z_t.$$

as follows

$$\begin{aligned}
y_t - y_t^* &= g_t = an_t^d + z_t - y_t^* \\
&= a\frac{\beta_1\delta_0 - \beta_0\delta_1}{\delta_1 + \beta_1} - a\delta_1(x_t - 1)(p_t - E_{t-1}p_t) + a\delta_1 z_t + z_t - y_t^* \\
&= -a\delta_1(x_t - 1)(p_t - E_{t-1}p_t) + a\delta_1 z_t - \left(\frac{a\delta_1\beta_1}{\delta_1 + \beta_1}\right) z_t \\
&= a\delta_1(1 - x_t)(p_t - E_{t-1}p_t) + a\frac{\delta_1^2}{\delta_1 + \beta_1} z_t.
\end{aligned}$$

The final expression gives us the expression for the aggregate supply relation:

$$y_t - y_t^* = A_t(p_t - E_{t-1}p_t) + cz_t,$$

where $A_t = a\delta_1(1 - x_t)$, and $c = a\delta_1^2/\left(\delta_1 + \beta_1\right)$.

Chapter 4

Random Wage Indexation and Monetary Policy

'Accepting that the world is full of uncertainty and ambiguity does not and should not stop people from being pretty sure about a lot of things'.

–Julian Baggini

4.1 Introduction

The bulk of the literature on wage indexation assumes a constant degree of indexation. While this assumption might describe the empirical reality during the recent periods of low and stable inflation, the degree of wage indexation has been observed to exhibit a substantial amount of time variation. For instance, the percentage of contracts in the US with cost-of-living-adjustment (COLA) clauses has been observed to rise from 31% in the mid 1960s to 61% in the mid 1970s (see Weiner (1996)). Given that percentage COLA coverage is a widely accepted proxy for the degree of wage indexation, one can conclude that the degree of wage indexation is not constant. Furthermore, results from recent studies provide evidence in support of the time variation in the degree of wage indexation. Analysis by Holland (1986) and Ascari et al. (2011) show that the degree of wage indexation is positively correlated to inflation uncertainty. Empirical studies documenting substantial time variation in inflation uncertainty imply a substantial time variation in the degree of wage indexation.

Even though the assumption of a constant degree of wage indexation may describe the behaviour of wage indexation only for recent times, the effects of wage indexation under this assumption have nevertheless been shown to be quite consequential. The seminal study on wage indexation, Gray (1976) examines the effect of wage indexation on the conduct of monetary policy. Results of this paper show that wage indexation insulates the real sector of the economy from nominal or monetary shocks, but tends to make the effects of real shocks worse. Jadresic (1998) also investigates the effects of constant wage indexation. The indexation rule employed in the aforementioned study differs from that of previous studies in that it assumes an indexation to lagged inflation scheme. It is shown that indexation to lagged inflation destabilizes output.

It is conceivable that the time variation in the degree of wage indexation adds another dimension to the implications of wage indexation for macroeconomic stability. The purpose of this study is to theoretically investigate the additional implications that come with time variation in wage indexation. In particular, we investigate the macroeconomic consequences of independent and identically distributed (iid) shocks to the degree of wage indexation. Empirical estimates either imply an autoregressive (AR) process or a near random-walk process for the degree of wage indexation which contrasts with the iid assumption regarding wage indexation we make in this study. We nevertheless work with iid shocks to wage indexation in order to obtain preliminary insights into the effects of time variation in the degree of wage indexation. Attey (2015) estimates the time-varying degrees of wage indexation for 11 OECD countries. Figure 4.1 presents the country specific estimates and their 95% confidence bounds. The estimates reveal three main properties of the degree of wage indexation.

First, there is a substantial time variation in the degree of wage indexation in all countries. This observation provides further evidence for the time-varying nature of wage indexation. Second, the empirical estimations do not give a conclusive view on whether the distribution of wage indexation is bounded or not. The process assumed for the degree of wage indexation implies an unbounded distribution for this variable. However, it can be seen from Figure 4.1 that estimates of the degree of wage indexation do not generally stray much from the unit interval. Thus, one cannot conclusively rule out the possibility of bounded distributions for wage indexation. Finally, the estimates of the degree of wage indexation can lie outside the unit interval. For instance, the estimates of the degree of wage indexation were significantly less than 0 for the Netherlands since the beginning of the 1980s. Also, the estimates show that the degree of wage indexation for Belgium was above 1 during the mid 1970s. It is also worth noting that wage moderation was sometimes agreed upon during periods of stagflation, thus resulting in the negative correlation between lagged inflation and wage inflation.[1] These two observations stand in contrast to conventional wisdom that wage indexation should be on the unit interval.

Key among the results of this study is that the unconditional distributions of inflation, the output gap and the interest rates can potentially exhibit heavy-tailed characteristics. This result relies on the assumption that wage indexation is random and can lie outside the unit interval. Thus it is implied that countries with full indexation schemes are more likely to have heavy-tailed distributions of variables than countries with a degree of wage indexation which lies within the unit interval. Also, a Taylor rule targeting current inflation outperforms a rule that targets past inflation regarding the minimization of the loss function. The analysis employed in deriving this study's results assumes that wage indexation is iid uniformly distributed. While this implies taking a definite stand on the boundedness of the distribution of wage indexation, assuming otherwise does not qualitatively alter the main results.

Recent empirical studies including Grier and Perry (1996), Chang (2012) and Caporale et al. (2012) employ the use of various versions of GARCH models to estimate inflation and inflation uncertainty. The relatively good fit of these models imply that the unconditional distribution of inflation exhibits tails heavier than that of a normal distribution. Furthermore, Fagiolo et al. (2008) conclude that in the majority of OECD countries, the distribution of output growth exhibits tails heavier than those of the Gaussian distribution. Contrary to this empirical evidence, the class of new Keynesian models commonly used for macroeconomic analysis typically imply that inflation and the output gap have normal unconditional distributions.

[1] The so-called Wassenaar Agreement in the Netherlands in 1982 is a widely known example of this case.

This study can be seen as an attempt to theoretically explain the source of the heavy tails in the aforementioned macroeconomic variables. Other approaches to explaining the presence of heavy tails involve the assumption of Student-t distributed error terms (see Curdia et al. (2012) and Chib and Ramamurthy (2011) for example). De Grauwe (2012) criticizes this exogenous approach of introducing the fat tail, maintaining that it does not shed light on how endogenous clustered volatility can be generated. The approach in this study involves a multiplicative shock similar to that first espoused by Brainard (1967) and later adopted by Attey and de Vries (2011). Following the latter study, it is assumed that the random degree of wage indexation is the source of the multiplicative shocks. The role of these multiplicative shocks is to amplify extreme realizations of the lag of inflation. This results in tails heavier in the unconditional distribution of inflation than would be expected under the normal distribution.[2] These heavy tails are passed on to the distribution of the output gap and the interest rate.

The remainder of this study is organized as follows. Section 4.2 derives the Phillips curve under the assumption of random degree of wage indexation. Section 4.3 investigates optimal monetary policy. Section 4.4 investigates monetary policy under two alternative policy rules. The performances of these policy rules are subsequently compared to that of optimal monetary policy under random wage indexation. Finally, Section 4.5 concludes.

4.2 Wage indexation and the Phillips curve

The representative firm has a fixed coefficient Ricardian production technology with labour as the sole input.[3] Assuming diminishing marginal returns to labour, the expression for output, Y_t, is:

$$Y_t = A_t N_t^{\alpha} \qquad\qquad 0 < \alpha < 1,$$

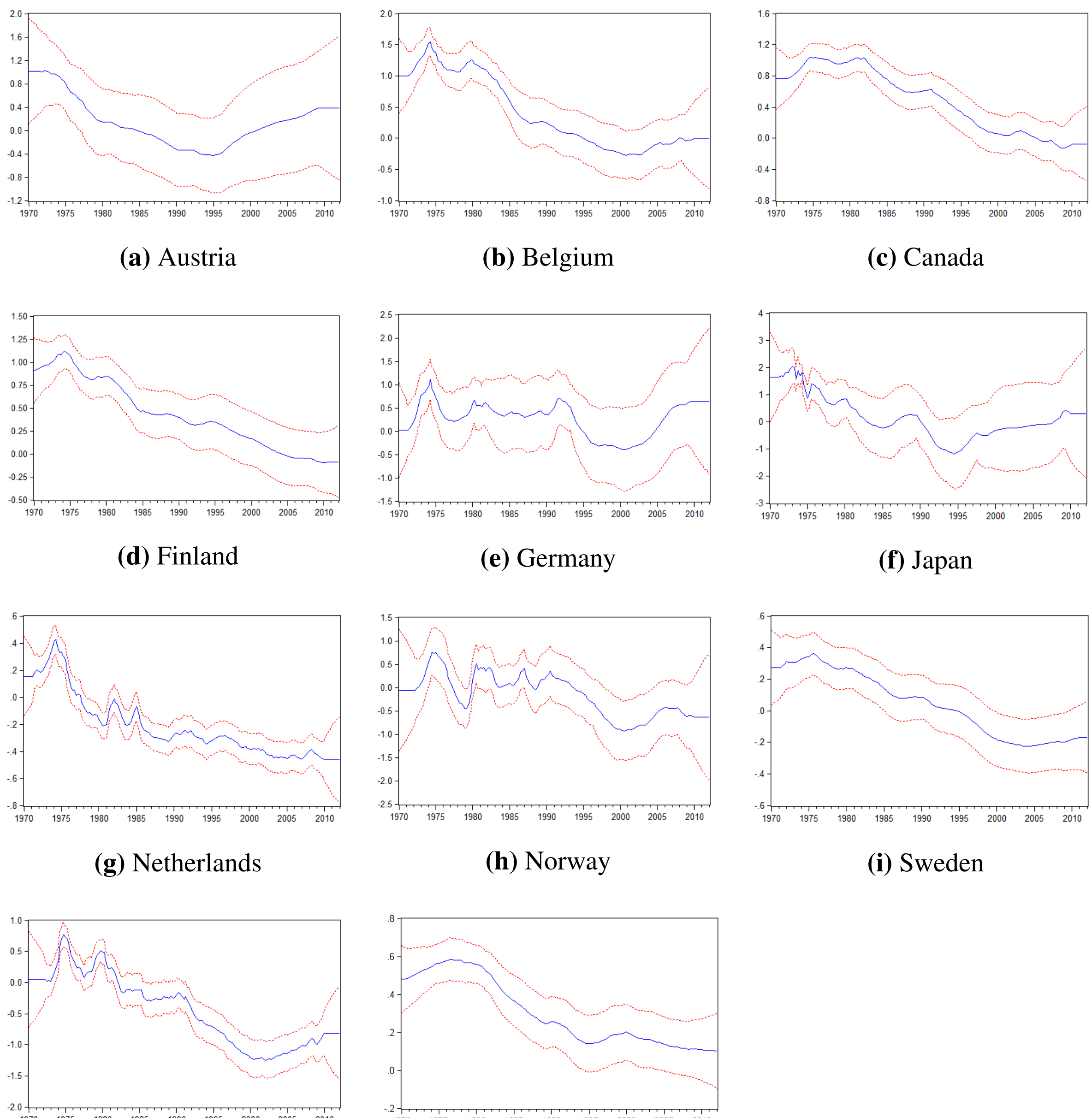
[2]The inflation process derived under random wage indexation exhibits a random AR coefficient.

[3] McCallum and Nelson (1999) argue that modeling variations in capital stock as exogenous is largely consistent with empirical observation.

where N_t is the amount of labour employed in production. The logarithmic level of total factor productivity, A_t, follows a stationary AR process. The process is given below:

$$\log A_t = a_t = \rho_a a_{t-1} + \varepsilon_{at},$$

Figure 4.1: Degree of wage indexation in selected OECD countries. Source: Attey (2015)



(a) Austria **(b)** Belgium **(c)** Canada

(d) Finland **(e)** Germany **(f)** Japan

(g) Netherlands **(h)** Norway **(i)** Sweden

(j) UK **(k)** US

where the iid random variable ε_{at}, has the following distribution: $N(0, \sigma_a^2)$. Firms maximize profit with respect to labour inputs. Thus, marginal productivity of labour should be equal to real wages. Let $\delta_0 = \log \alpha/(1-\alpha)$ and $\delta_1 = 1/(1-\alpha)$. The following equation gives the labour demand in log values:

$$n_t = \delta_0 - \delta_1(w_t - p_t) + \delta_1 a_t. \tag{4.1}$$

Labour supply in micro-founded models is typically derived from the optimization conditions of the representative household. Let the labour supply relation be given as follows:

$$n_t = \beta_0 + \beta_1(w_t - p_t) \qquad\qquad \beta_1 > 0,$$

where the parameters β_0 and β_1 are functions of the parameters governing household preferences. The market clearing wage implied by the labour supply and labour demand relations is:

$$w_t^* = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t + \frac{\delta_1}{\delta_1 + \beta 1} a_t. \tag{4.2}$$

The corresponding market clearing output is obtained by substituting this expression into the labour demand (or the labour supply) relation and again substituting the resulting expression in the expression for aggregate output. This gives the log market clearing output (y^*) as:

$$y_t^* = \alpha \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \left(1 + \alpha \frac{\delta_1 \beta_1}{\delta_1 + \beta_1}\right) a_t.$$

4.2.1 Wage indexation

Fischer (1988) among others argues that informational lags make it impossible to index wages to current inflation. At a particular point in time, any available information concerning inflation relates to either inflation forecasts or lagged inflation, and not current inflation. In view of this, we consider an indexation scheme with indexation to a period's lagged inflation as follows:

$$w_t = w_t^{*e} + x_t(\pi_{t-1} - \hat{\pi}), \tag{4.3}$$

where w_t, w_t^*, and x_t are respectively the nominal wages, market clearing nominal wages and time-varying wage indexation respectively. Inflation is denoted by the variable π_t. The superscript ^{e} in the model denotes the expectations of private agents. It is assumed that the inflation target announced by the policy maker ($\hat{\pi}$) effectively captures the expected inflation on which basis wage contracts are set a period in advance. While this assumption may come across as ad hoc, the European Central Bank's (ECB) constant target of 2% can be cited as evidence in support of our assumption.

The wage indexation variable x_t effectively captures the elasticity of wages to lag of prices. Some of the country-specific estimates for x_t provided in Figure 4.1 suggest the possibility of over-indexation (when $x_t > 1$). We therefore assume that $x_t \sim U(0, \kappa_a)$, where $\kappa_a > 1$, in order to allow for this possibility.

Wage indexation under a rule given by (4.3) may even exacerbate the destabilizing effects of monetary shocks. This runs contrary to the finding in Gray (1976) that wage indexation insulates the economy from monetary shocks. The reason behind the differing results lies in the way wages are indexed. Gray (1976) considers an indexation scheme under which wages are indexed to current inflation while we assume that wages are indexed to lag of inflation. This implies that real wages are flexible most of the time.

Take expectations of the market clearing wage in (4.2) and substitute the resulting expression into (4.3). This gives the following expression for real wages in the presence of wage indexation:

$$w_t - p_t = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e - (p_t - p_t^e) + x_t(\pi_{t-1} - \hat{\pi}). \tag{4.4a}$$

It is worth noting that $(p_t - p_t^e) = (\pi_t - \pi_t^e)$. Following our earlier assumption, the expected inflation is equal to the target inflation, i.e. $\pi_t^e = \hat{\pi}$. This implies that (4.4a) can be rewritten as:

$$w_t - p_t = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e - (\pi_t - \hat{\pi}) + x_t(\pi_{t-1} - \hat{\pi}). \tag{4.4b}$$

4.2.2 Aggregate supply or the Phillips curve

The aggregate supply is derived by substituting out real wages in (4.1) with (4.4b). This gives the log labour demand as a function of inflation, lagged inflation and productivity. The output is subsequently computed by noting that $y_t = \alpha n_t^d + a_t$. Section 4.A.1 gives a more detailed derivation of the aggregate supply equation.

Let $\lambda_{1t} = \alpha\delta_1 x_t$ and $\lambda_2 = \alpha\delta_1$. Further assume that the output gap is defined as the deviation of log output under wage indexation from the log market clearing output level , i.e. $g_t = (y_t - y_t^*)$. The expression for the AS curve is:

$$g_t = -\lambda_{1t}\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t + u_t, \tag{4.5}$$

where $u_t = [\alpha\delta_1^2/(\delta_1 + \beta_1)]\varepsilon_{at}$. The variable $\tilde{\pi}_t$ is the deviation of inflation from target inflation, i.e. $\pi_t - \hat{\pi}$.

The aggregate supply relation in (4.5) implies a time-varying response of the output gap to the lag of inflation. This is due to the assumption that wages are indexed to lag of inflation. If the degree of wage indexation is positive ($x_t > 0$), the lag of inflation has a negative effect on the output gap. In other words, indexation just increases the labour cost thereby decreasing output. A negative indexation resulting from a wage moderation response to high levels of lagged inflation implies a positive effect of the lag of inflation on output. This suggests that wage moderation as a response to high levels of lagged inflation increases output beyond the level determined by total factor productivity shocks (u_t) and current inflation.

4.3 Monetary policy

This section derives inflation, the interest rate and the output gap under optimal monetary policy and two interest rate rate rules. The set-up adopted in solving for optimal monetary policy is similar to that of Clarke et al. (1999). A major distinction between our model and Clarke et al. (1999) lies in the slope parameter of the Phillips curve. The Phillips curve in our model has a random slope coefficient as opposed to the conventional constant slope Phillips curve employed in the the study by Clarke et al. (1999).

We earlier on assumed that the degree of wage indexation is an iid random variable distributed as follows: $x \sim U[0, \kappa_a]$, where $\kappa_a > 1$. While the uniform distribution suggested as the distribution of the degree of wage indexation might seem ad hoc, Attey and de Vries (2013) show that it can be a mixed equilibrium outcome of wage indexation bargaining under arbitration.

4.3.1 Optimal monetary policy

We now investigate the effect of random wage indexation to lagged inflation. The interest rate is introduced into the model by incorporating the aggregate demand or the IS curve. The aggregate demand relation is given as follows:

$$g_t = y_t - y^* = -\phi(i_t - \tilde{\pi}_t^e - r) + v_t \qquad v_t \sim iid\ \mathcal{N}(0, \sigma_v), \tag{4.6}$$

where r corresponds to the natural rate of interest which is assumed to be constant and v_t is a demand shock uncorrelated with productivity and the random wage indexation.

In deriving the optimal monetary policy, we make the following assumptions: the policy maker uses the interest rate (i_t) as an instrument, all bargaining with regards to wage indexation in the economy are concluded at the beginning of the current time period, and private agents do not observe the aggregate wage indexation outcome. For all purposes, the aggregate wage indexation outcome can also be viewed as a supply shock, uncorrelated to productivity shocks.

The use of the interest rate as the instrument requires the policy maker to observe the supply and demand shocks in order to react before the private sector does. The expected inflation can be derived from the expressions (4.5) and (4.6) to obtain the following:

$$\tilde{\pi}_t^e = -\frac{\phi}{\lambda_2 - \phi}(i_t^e - r) + \frac{\lambda_1}{\lambda_2 - \phi}\tilde{\pi}_{t-1} \qquad \lambda_1 = \lambda_{1t}^e. \tag{4.7}$$

Alternative forms of the expression (4.6) can be derived by expressing inflation and the output gap in terms of the control variable i_t and state variables (π_{t-1} and the random shocks), as well as substituting in (4.7) as follows:

$$g_t = -\phi(i_t - i_t^e) - \frac{\phi\lambda_2}{\lambda_2 - \phi}(i_t^e - r) + \frac{\phi\lambda_1}{\lambda_2 - \phi}\tilde{\pi}_{t-1} + v_t \tag{4.8a}$$

$$g_t = g_t^e - \phi(i_t - i_t^e) + v_t. \tag{4.8b}$$

Similarly, analogous expressions can be derived for the aggregate supply relation as follows:

$$\tilde{\pi}_t = -\frac{\phi}{\lambda_2}(i_t - i_t^e) - \frac{\phi}{\lambda_2 - \phi}(i_t^e - r) + \lambda_{3t}\tilde{\pi}_{t-1} + \frac{1}{\lambda_2}(v_t - u_t) \tag{4.9a}$$

$$\tilde{\pi}_t = \tilde{\pi}_t^e - \frac{\phi}{\lambda_2}(i_t - i_t^e) + \frac{\eta_t}{\lambda_2}\tilde{\pi}_{t-1} + \frac{1}{\lambda_2}(v_t - u_t), \tag{4.9b}$$

where $\lambda_{3t} = \phi\lambda_1/[\lambda_2(\lambda_2 - \phi)] + \lambda_{1t}/\lambda_2$ and $\eta_t = \lambda_{1t} - \lambda_1$.

Optimization problem of the policy maker

It is assumed that the policy maker targets both inflation and the output gap. In particular, they seek to stabilize both g_t and $\tilde{\pi}_t$ at 0, albeit without necessarily placing equal weights on both objectives. Let θ be the weight the policy maker places on inflation stabilization. The loss function of the policy maker is:

$$\mathcal{L}_t = g_t^2 + \theta\tilde{\pi}_t^2 \qquad \theta \geq 0.$$

We consider the case of optimal monetary policy under commitment, thus requiring the policy maker to take into account the effect of its policy on the expectations of agents in the economy. This requires the presence of another constraint in addition to (4.8a) and (4.9a) (or alternatively (4.8b) and (4.9b)) as follows:

$$i_t^e = E_{t-1} i_t. \tag{4.10}$$

The policy maker aims at minimizing all current and future losses stemming from deviations of the output gap and inflation from their respective targets. Let β be the discount factor and E_{t-1} be the expectation of the policy maker. The optimization problem of the policy maker is given as follows:

$$\begin{aligned} &\max_{i_t, i_t^e} \;\; E_{t-1} - \sum_{t=1}^{\infty} \beta^t \mathcal{L}_t \\ &s.t. \;\; \text{(4.8a), (4.9a) and (4.10).} \end{aligned} \tag{4.11}$$

The constraint (4.9a) is dynamic in $\tilde{\pi}$. In stabilizing current inflation and the output gap, one has to be mindful of the intertemporal effects of one's actions on the subsequent period's inflation. Thus, we can conclude that the optimization problem is a dynamic one with $\tilde{\pi}_t$ as the endogenous state variable. The Bellman formulation of the expression (4.11) is given as:

$$\begin{aligned} V(\pi_{t-1}) = &\max_{i_t, i_t^e} E_{t-1} \left[-g_t^2 - \theta \tilde{\pi}_t^2 + \beta V(\tilde{\pi}_t) \right] \\ &s.t. \;\; \text{(4.8a), (4.9a) and (4.10).} \end{aligned} \tag{4.12}$$

Following Clarke et al. (1999), we argue that since the problem is of linear-quadratic nature as far as the endogenous state variable is concerned, and owing to the independence of the exogenous state variables u_t and v_t, the value function must also be quadratic. Thus, we conjecture the following value function:

$$V(\tilde{\pi}_{t-1}) = \gamma_0 + 2\gamma_1\tilde{\pi}_{t-1} + \gamma_2\tilde{\pi}_{t-1}^2.$$

Let the variable Λ_{t-1} be the Lagrangian multiplier associated with the commitment constraint indicated by (4.10). We write down the first order conditions associated with the problem as follows:

$$0 = 2\phi[g_t + \tilde{\pi}_t(\theta/\lambda_2) - (\gamma_1 + \gamma_2\tilde{\pi}_t)(\beta/\lambda_2)] - \Lambda_{t-1}$$
$$0 = -2\phi[g_t^e(1 - \lambda_2/(\lambda_2 - \phi)) + \tilde{\pi}_t^e(\theta/\lambda_2 - \theta/(\lambda_2 - \phi)) + (\gamma_1 + \gamma_2\tilde{\pi}_t^e)(\beta/\lambda_2 - \beta/(\lambda_2 - \phi))] + \Lambda_{t-1}.$$

The sum of the last two expressions derives the following expression:

$$0 = 2\phi[(g_t - g_t^e) + (\tilde{\pi}_t - \tilde{\pi}_t^e)(\theta - \beta\gamma_2)/\lambda_2] + 2\phi[\lambda_2 g_t^e + (\theta - \beta\gamma_2)\tilde{\pi}_t^e - \beta\gamma_1]/(\lambda_2 - \phi). \tag{4.13}$$

Taking expectation of the above expression yields

$$0 = 2\phi[\lambda_2 g_t^e + (\theta - \beta\gamma_2)\tilde{\pi}_t^e - \beta\gamma_1]/(\lambda_2 - \phi). \tag{4.14}$$

Substitute the expressions (4.8b) and (4.9b) into (4.13) to obtain an expression in terms of the control variables. The derived optimal feedback rule after imposing (4.14) and subsequently simplifying is given below:

$$0 = [-\phi(i_t - i_t^e) + v_t]\left(1 + \frac{\theta - \beta\gamma_2}{\lambda_2^2}\right) - \left(\frac{\theta - \beta\gamma_2}{\lambda_2^2}\right)(u_t - \eta_t\tilde{\pi}_{t-1}). \tag{4.15}$$

For the value function to be concave in $\tilde{\pi}$, we require that $\gamma_2 < 0$. Therefore, we know that $1 + (\theta - \beta\gamma_2)/\lambda_2^2 \neq 0$. This implies that under optimal control, $[-\phi(i_t - i_t^e) + v_t]$ is a function of u_t and $\eta_t\tilde{\pi}_{t-1}$. We also know that $i_t^e - r$ (and $\tilde{\pi}_t^e$) under optimal control must be a function of the endogenous state variable , $\tilde{\pi}_{t-1}$. Thus it follows from (4.9a) that under optimal policy, we can conjecture the following for the inflation process:

$$\tilde{\pi}_t = a + b\tilde{\pi}_{t-1} + \delta\eta_t\tilde{\pi}_{t-1} + cu_t, \tag{4.16}$$

where a, b, δ and c are parameters to be determined. For the value function to be concave, and thus, for the existence of a solution to the maximization problem, it is required that $\beta(b^2 + \delta^2\sigma_\eta^2) < 1$. Appendix 4.A.2 derives the process for equilibrium inflation under optimal control, which is

$$\tilde{\pi}_t = b\left(1 + \frac{\eta_t}{\lambda_1}\right)\tilde{\pi}_{t-1} - \frac{b}{\lambda_1}u_t, \tag{4.17}$$

where

$$b = \frac{[(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2] - \sqrt{[(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2 - 4\beta(\lambda_1\lambda_2)^2(1 + \sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1 + \sigma_\eta^2/\lambda_1^2)}. \tag{4.18}$$

There are a few points worth noting about the behavior of the value representing the mean persistence of inflation (b) under optimal monetary policy. First, this parameter is always positive and it is bounded from above by $\bar{x}$. This implies that under random wage indexation to lagged inflation, the mean persistence in equilibrium inflation is at most the mean of the aggregate wage indexation. This maximum occurs when the weight of inflation stabilization in the policy maker's loss function is 0 ($\theta = 0$). To see this, define $a = (\lambda_2^2 + \theta)$ and $y = \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2$ thus permitting the mean persistence to be written down as follows:

$$b = \frac{(a + y) - \sqrt{(a + y)^2 - 4\lambda_2^2 y}}{4y}\kappa_a,$$

whereby we made the substitution $\lambda_2 = 2\lambda_1/\kappa_a$. Taking all other parameters as given, this function assumes its extremum value when the derivative with respect to the variable y equals zero ($\partial b/\partial y = 0$). The expression for this derivative is

$$\frac{\partial b}{\partial y} = \frac{-a\sqrt{(a+y)^2 - 4\lambda_2^2 y} + a(a+y) - 2\lambda_2^2 y}{4y\sqrt{(a+y)^2 - 4\lambda_2^2 y}}\kappa_a.$$

Imposing the first order maximization condition and simplifying the above expression further yields $y^2\lambda_2^2(\lambda_2^2 - a) = 0$. The necessary condition for maximization is therefore satisfied if any combination of the following expressions holds: $\lambda_2 = 0$ and $a = \lambda_2^2$. Reasonable estimates of the output elasticity to labour input in a Cobb-Douglass production function[4] imply that $\alpha > 0$, thus ruling out the condition $\lambda_2 = \alpha/(1-\alpha) = 0$.[5] The remaining condition for maximization implies $\theta = 0$ at which b is at its maximum irrespective of the value of the variance of wage indexation. We now show that the extremum value of b is indeed the maximum if $\theta = 0$. In order to do this, we show that for $\theta > 0$ the following must hold: $\partial b/\partial y < 0$. This requires that either any or all of the following expressions must hold: $\lambda_2 < 0$, $y < 0$, and $\lambda_2^2 < a$. As indicated earlier, all reasonable estimates in earlier studies imply that $\lambda_2 > 0$. This rules out the first condition. We know that the second condition is also ruled out since $y = \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2 > 0$. This leaves us with the condition $\lambda_2^2 < a$ which implies $\theta > 0$.

Second, b is strictly decreasing in θ. This can be seen from the partial derivative of b with respect to θ. The derivative is

$$\partial b/\partial\theta = (\partial b/\partial a)(\partial a/\partial\theta) = \frac{\sqrt{(a+y)^2 - 4\lambda_2^2 y} - (a+y)}{4y\sqrt{(a+y)^2 - 4\lambda_2^2 y}} < 0.$$

The average persistence of inflation is therefore smaller when the policy maker attaches more weight to inflation stabilization in their loss function and it is zero in the extreme case when the policy maker targets only inflation (i.e. $\theta = \infty$).

Third, there are two cases in which the mean persistence of inflation assumes the highest value: when the production function exhibits constant marginal returns to labour ($\alpha = 1$) and

[4]The production function in this study can be considered as a Cobb-Douglass function with capital normalized to 1.

[5] Estimates from Christiano and Eichenbaum (1992) give the value of $1-\alpha$ to be between 0.339 and 0.35 while values widely used in literature on Real Business Cycle range from 1/3 to 0.4.

when the policy maker does not put any weight on stabilizing inflation ($\theta = 0$). In the former case, the effects of productivity and the output gap on inflation are zero, implying that the persistence in inflation is solely determined by wage indexation. With regards to the latter case, an intuitive explanation can be given as follows: in the absence of any commitment to inflation stabilization, the expected persistence in equilibrium inflation is solely determined by how much, on the average, economic agents index to past inflation. Therefore, in order to disinflate an economy characterized by high persistent inflation, monetary authorities need to be committed to an inflation stabilization policy. This is in line with the empirical observation that inflation is less persistent under inflation targeting than under the absence of any form of commitment to stabilizing inflation.[6]

Equilibrium inflation under optimal monetary policy

That inflation is a persistent phenomenon is a well known observation. Most new Keynesian models incorporate inflation persistence by assuming that prices are indexed to lagged inflation. Jadresic (1998) and Perez (2003), among others, introduce inflation persistence by indexing wages to lagged inflation. The latter study concludes that persistence in inflation is higher, the higher the proportion of labour contracts that include indexation clauses. There is one fundamental difference between our study and the last two studies cited: wage indexation in our model is a random outcome rather than a given constant. The variance of the aggregate wage indexation outcome also affects the mean persistence of inflation in the economy. Consider the expression for expected equilibrium inflation:

$$\tilde{\pi}_t^e = b\tilde{\pi}_{t-1}.$$

The expression for $\partial b/\partial y$ in the preceding section implies that the average persistence of inflation (b) is a decreasing function of the variance of wage indexation, $\sigma_x^2 = \sigma_\eta^2/\lambda_2^2$. In other words, on the average, past inflation is less important in explaining current inflation the higher

[6] Literature that report this finding include Gerlach and Tillman (2012) and Kuttner and Posen (2001).

the variance of aggregate wage indexation. The conditional variance of inflation under optimal control can be derived from (4.17) as follows:

$$\sigma_\pi^2 = \frac{b^2}{\lambda_1^2}(\sigma_\eta^2 \tilde{\pi}_{t-1}^2 + \sigma_u^2). \tag{4.19}$$

The expression above reveals that the conditional variance in inflation depends on three variables: the variance of wage indexation, lagged inflation and the variance of productivity shocks. The effects of lagged inflation and the variance of productivity shocks are unambiguous; they increase the conditional variance of inflation. However, no concrete conclusion can be drawn with regards to the effect of the variance of wage indexation on the conditional variance of inflation under general conditions. Under the rather specific assumption that the lagged inflation is at its target (i.e. $\tilde{\pi}_{t-1} = 0$), it can then be concluded that the variance of wage indexation has a decreasing effect on the variance of inflation. To see this, one must first note that the higher the variance in wage indexation (captured by the variable σ_η^2), the lower the average persistence in inflation (b), and thus the lower the variance of inflation holding all other variables constant.

Interest rate under optimal monetary policy

The expression (4.56) substituted into (4.57) (both found in Appendix 4.A.2) gives an interest rate rule to which a policy maker has to adhere when conducting optimal monetary policy. After making the substitution $a = 0$ and further simplifications, the interest rate rule under optimal monetary policy is given below:

$$i_t = r + b\tilde{\pi}_{t-1} + \frac{\lambda_1 - b\lambda_2}{\lambda_1 \phi}(\lambda_{1t}\tilde{\pi}_{t-1} - u_t) + \frac{1}{\phi}v_t. \tag{4.20}$$

As will be shown later, the expression above is reminiscent of the Taylor rule in that it contains a sort of reaction function to inflation and the output gap. The expression for inflation under optimal control as given in (4.17) and the implied output gap derived from (4.5) can be written as follows:

$$\tilde{\pi}_t = (b/\lambda_1)[\lambda_{1t}\tilde{\pi}_{t-1} - u_t]$$
$$g_t = -(1 - b\lambda_2/\lambda_1)[\lambda_{1t}\tilde{\pi}_{t-1} - u_t].$$

A substitution of the former of the above two expressions into (4.20) permits the rendition of the interest rate rule under optimal monetary policy into a more recognizable form as follows:

$$i_t = r + b\tilde{\pi}_{t-1} + \frac{\lambda_1 - b\lambda_2}{b\phi}\tilde{\pi}_t + \frac{1}{\phi}v_t. \tag{4.21}$$

The last expression indicates a reaction function of the interest rate to lagged inflation and current inflation. In addition to the variables just mentioned, it also reacts to demand shocks v_t as per the assumptions made when solving the optimal control problem in Appendix 4.A.2. Given reasonable values for the model's structural parameters, the coefficients of $\tilde{\pi}_t$ and v_t are all greater than 1. This suggests an aggressive reaction to deviation of these variables from 0, thus ensuring determinacy of the model under this rule.

4.3.2 Two simple interest rate rules

In what follows in this part, we examine monetary policy under two types of Taylor rules: one that targets current inflation (hereafter denoted by CTR) and the backward looking Taylor rule (hereafter denoted by BTR). These rules are given in the following expressions:

$$i_t = r + \omega_c\tilde{\pi}_t \tag{4.22}$$
$$i_t = r + \omega_b\tilde{\pi}_{t-1}. \tag{4.23}$$

The rules considered above are similar to those considered in Gali and Monacelli (2005). Besides, as can be seen in the preceding paragraphs, the equilibrium output gap (g_t) under optimal control is a linear function of inflation thus permitting the interest rate rule to be expressed in terms of shocks and inflation only. In the case of the BTR, the policy maker reacts to lagged inflation. A motivation for considering this version of the Taylor rule can be drawn from the same reasoning as to why one should consider wage indexation to lagged inflation: policy makers may not have information on current shocks during policy formulation and implementation. The other expressions needed for the analysis are the aggregate demand or the IS curve and the aggregate supply or the Phillips curve equations. They are repeated here below.

$$\lambda_2\tilde{\pi}_t = \lambda_{1t}\tilde{\pi}_{t-1} + g_t - u_t$$
$$g_t = -\phi(i_t - \tilde{\pi}^e_t - r) + v_t.$$

After substituting the Taylor rule (4.22) into the IS equation, a compact representation of the linear system is given below:

$$\mathbf{A_c}\begin{pmatrix}\tilde{\pi}_t \\ g_t\end{pmatrix} = \mathbf{B_{c,t}}\begin{pmatrix}\tilde{\pi}_{t-1} \\ g_{t-1}\end{pmatrix} + \mathbf{C_c}\begin{pmatrix}\tilde{\pi}^e_t \\ g^e_t\end{pmatrix} + \mathbf{D_c}\begin{pmatrix}u_t \\ v_t\end{pmatrix}, \tag{4.24}$$

where $\mathbf{A_c} = \begin{pmatrix}\lambda_2 & -1 \\ \phi\omega_c & 1\end{pmatrix}$; $\mathbf{B_{c,t}} = \begin{pmatrix}\lambda_{1t} & 0 \\ 0 & 0\end{pmatrix}$; $\mathbf{C_c} = \begin{pmatrix}0 & 0 \\ \phi & 0\end{pmatrix}$ and $\mathbf{D_c} = \begin{pmatrix}-1 & 0 \\ 0 & 1\end{pmatrix}$. Let the vector X be defined as $[\tilde{\pi}_t g_t]'$ and vector ϵ_t be defined as $[u_t v_t]'$. The above representation can further be simplified to get the following

$$X_t = \mathbf{F_{c,t}}X_{t-1} + \mathbf{G_c}E_{t-1}X_t + \mathbf{H_c}\epsilon_t,$$

where $\mathbf{F_{c,t}} = \mathbf{A_c^{-1}B_{c,t}}$, $\mathbf{G_c} = \mathbf{A_c^{-1}C_c}$ and $\mathbf{H_c} = \mathbf{A_c^{-1}D_c}$. It is shown in Appendix 4.B that the solution to the above system of equations is

$$X_t = \mathbf{P_{c,t}}X_{t-1} + \mathbf{H_c}\epsilon_t, \tag{4.25}$$

where $\mathbf{P_{c,t}} = [\mathbf{F_{c,t}} + \mathbf{G_c}(\mathbf{I} - \mathbf{G_c})^{-1}\mathbf{F_c}]$. The solution is basically an autoregressive system with time-varying coefficients. Unlike its counterpart in extant literature investigating determinacy under a Taylor rule, the eigenvalue criterion for determinacy is not applicable. If the coefficient matrix $P_{c,t}$ were constant, then the obvious requirement for such a system to be determinate will be that both eigenvalues of the matrix must lie within the unit circle. Given the random nature of the coefficient matrix, the Kesten conditions are used to verify the existence of a stable asymptotic unconditional distribution of both the output gap and inflation. Algebraic verification of the Kesten conditions in the case of monetary policy under the two Taylor rules are rather tedious. We therefore resort to numerical computations using the MATLAB programme to verify the conditions.

The solution derived in (4.25) implies the following expressions for equilibrium inflation and the output gap under the CTR:

$$\tilde{\pi}_t = [\lambda_{1t}/\Delta + \phi\lambda_1/(\Delta^2 - \phi\Delta)]\tilde{\pi}_{t-1} + [1/\Delta]v_t - [1/\Delta]u_t \tag{4.26}$$

$$g_t = [-\lambda_{1t}(\phi\omega_c)/\Delta + (\phi\lambda_1\lambda_2)/(\Delta^2 - \phi\Delta)]\tilde{\pi}_{t-1} + [\lambda_2/\Delta]v_t + [\phi\omega_c/\Delta]u_t, \tag{4.27}$$

where $\Delta = \lambda_2 + \phi\omega_c$. Let $\Lambda = \phi(\lambda_1 - \phi\omega_b)/(\lambda_2 - \phi)$. A similar derivation procedure in the case of the BTR permits us to derive the equilibrium process for inflation and the output gap as follows:

$$\tilde{\pi}_t = [\lambda_{1t}/\lambda_2 + \Lambda/\lambda_2]\tilde{\pi}_{t-1} + [1/\lambda_2]v_t - [1/\lambda_2]u_t \tag{4.28}$$

$$g_t = \Lambda\tilde{\pi}_{t-1} + v_t. \tag{4.29}$$

Comparing the equilibrium output gap under the CTR (4.27) with its counterpart under the BTR (4.29) reveals a difference in the conditional distributions of the output gap under the two rules: while the conditional distribution under the CTR is not normal, that under the BTR is normally distributed if one assumes a normal distribution for v_t.[7] The CTR therefore comes closer to mimicking the optimal monetary policy as far as conditional distribution of the output gap is concerned. If the Kesten conditions are satisfied, the unconditional distribution of all variables

[7]To see this, note that Λ is a function of constant parameters. Thus the conditional distribution of g_t under the BTR depends only on the distribution of v_t.

are heavy tailed under both Taylor rules.

4.4 Evaluation of alternative policy rules

This section carries out a quantitative analysis of the two policy rules and compares the equilibrium dynamics of inflation, the output gap and the interest rate obtained under these rules to those obtained under optimal monetary policy in the previous section. The loss function used in deriving the optimal monetary policy in Section 4.3 reveals a hybrid stabilization policy that targets both inflation and the output gap. In the new Keynesian literature, a similar welfare function is derived as a second order approximation of the representative consumer's utility function.[8] In such a case, the relative weight placed on inflation stabilization is a function of structural parameters in the new Keynesian model. We restate the objective function of the policy maker below:

$$\mathcal{W} = -\sum_{t=0}^{\infty} \beta^t E_0 \left(g_t^2 + \theta \tilde{\pi}_t^2 \right). \tag{4.30}$$

In Gali and Monacelli (2005), it is noted that for $\beta \to 1$, the loss function can be rewritten in terms of the unconditional variances of the output gap and inflation. The logic behind the expression of (4.30) in terms of these variances differs from that of Gali and Monacelli (2005). Assume $\beta \to 1$ and that the loss function can be approximated as a sum of instantaneous losses over a finite time horizon. A step by step approximation of (4.30) is given below:

$$\begin{aligned}\mathcal{W} &\approx -E_0 \sum_{t=0}^{T} \beta^t \left(g_t^2 + \theta \tilde{\pi}_t^2 \right) \\ &= -T E_0 \sum_{t=0}^{T} \beta^t \frac{1}{T} \left(g_t^2 + \theta \tilde{\pi}_t^2 \right) \\ &\approx -T[var(g_t) + \theta var(\pi_t)],\end{aligned}$$

[8] Woodford (2003) contains such derivations.

where T is an arbitrarily large number. Given the ordinal nature of the measurement of the loss of the policy maker, any monotonic transformation of the last expression should be an adequate measure for the loss of the policy maker. We therefore express the loss function as follows:

$$\mathcal{V} = -[var(g_t) + \theta var(\pi_t)]. \tag{4.31}$$

The existence of stationary distributions for the output gap and inflation once Kesten conditions are satisfied guarantees a finite variance. The version of the loss function contained in (4.31) will be used to rank the rules and the performance of optimal monetary policy. The calibration in this section is carried out with respect to the dynamics of the output gap, inflation and the interest rate in the economy of the Euro area.

Recap of the calibrated models

We compare the dynamics of inflation, the output gap and interest rate under the optimal monetary policy and the two Taylor rules. In order to get a lucid comparison of the distributions, we include a version of the model under which inflation under optimal policy has a non random persistence (OCW). In other words, we assume that $\lambda_{1t} = \lambda_1$ in the OCW model. Each of the calibrated models can be summarized by the following three expressions: the aggregate supply or the Phillips curve, the aggregate demand curve and the interest rate rule. Let $\varphi = (\lambda_1 - b\lambda_2)/(b\phi)$. Table 4.1 gives a summary of the models employed in the calibrations.

It should be noted that the definition of the coefficient b given in (4.18) changes under the OCW. Recall that wage indexation is assumed constant at its mean under the OCW. Thus, the variance of wage indexation and by implication σ_η^2 are both 0. The average persistence under the OCW ($\bar{b}$) and the corresponding value under the ORW (b) are stated below:

$$\bar{b} = \frac{[(\lambda_2^2 + \theta + \beta\lambda_1^2] - \sqrt{(\lambda_2^2 + \theta + \beta\lambda_1^2)^2 - 4\beta(\lambda_1\lambda_2)^2}}{2\beta\lambda_1\lambda_2}$$

$$b = \frac{[(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2] - \sqrt{[(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2 - 4\beta(\lambda_1\lambda_2)^2(1 + \sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1 + \sigma_\eta^2/\lambda_1^2)}.$$

Parameter values

We derive the values of the parameters used in the calibration exercise from three main sources. They are Amisano and Tristiani (2010), Gali and Monacelli (2005) and our own estimations. There are some cases in which directly corresponding values of certain parameters in the source literature are not available. In these cases, we construct values based on a set of related parameters obtained from the literature. The next three paragraphs give a more detailed explanation on how some parameter values are set for the calibration.

The constant in the labour supply equation (β_0) is set to 0. Using a different value does not change our results in any significant way. Besides, there is no constant term in most micro founded derivation of the labour supply curve found in literature.[9] It is assumed that the policy maker places twice as much weight on output stabilization as they place on inflation stabilization. Thus, we assume that $\theta = 0.5$. Following Gali and Monacelli (2005), we set the coefficients of inflation in both Taylor rules at 1.5 ($\omega_c = \omega_b = 1.5$).

The values of the interest rate elasticity of aggregate demand (ϕ), the standard deviation of the aggregate-demand shocks (σ_v), and the wage elasticity of labour supply (β_1) are not directly available from the estimates in Amisano and Tristiani (2010). We express these parameters as functions of available estimates under some plausible assumptions. Assume a power utility function which is separable in both consumption and labour (or leisure). A micro founded derivation of the aggregate demand (or the IS curve) implies that the interest rate elasticity is

[9] In these models, labour supply is typically given by the following $(w_t - p_t) = \log(MRS_t) = \log(-(U_n)) - \log(U_c)$. Assuming a power utility function then implies that real wages are increasing in labour hours and productivity after imposing equilibrium conditions. The log of the latter variable is typically assumed to be a stationary AR(1) process around a 0 unconditional mean.

the inverse of the constant relative risk aversion (CRRA) parameter. We therefore set $\phi = 1/\gamma$, where γ is the CRRA estimate from Amisano and Tristiani (2010). Under the same assumption, it can be shown that the real wage elasticity of labour supply is a function of the labour share of production (α), the disutility of labour , and the constant relative risk aversion parameter when one assumes a power utility function. In particular, $\beta_1 = 1/(\phi + \alpha\gamma)$, where ϕ captures the disutiltiy of labour in the model of the study just cited. Finally, we assume that demand shock is the sum of the inflation target shock and the interest rate shock found in the literature. This permits us to set $\sigma_v = \sqrt{\sigma_{\bar{\pi}}^2 + \sigma_i^2}$ where $\sigma_{\bar{\pi}}^2$, and σ_i^2 are respectively the variances of inflation target shocks and the interest rate shocks.

The wage parameter indicating the extent of over-indexation (κ_a) is fixed at 1.5. This value is motivated by the estimates obtained from Attey (2015) for the case of Belgium. While there are estimates found in other literature, those estimates are derived under the rather restrictive assumption of a time-invariant degree of wage indexation. We carry out our own estimations to estimate the parameters α, σ_a and ρ_a. Details concerning the estimation procedure are given in section 4.C of the appendix. Table 4.2 gives a summary on the parameters and their corresponding values used in the calibration exercise.

Table 4.1: Summary of models

Optimal policy	
Constant index (OCW)	Random index (ORW)
$\begin{cases} \lambda_2\tilde{\pi}_t & = g_t + \lambda_1\tilde{\pi}_{t-1} - u_t \\ g_t & = -\phi(i_t - \tilde{\pi}^e_t - r) + v_t \\ i_t & = r + \bar{b}\tilde{\pi}_{t-1} + \varphi\tilde{\pi}_t + [1/\phi]v_t \end{cases}$	$\begin{cases} \lambda_2\tilde{\pi}_t & = g_t + \lambda_{1t}\tilde{\pi}_{t-1} - u_t \\ g_t & = -\phi(i_t - \tilde{\pi}^e_t - r) + v_t \\ i_t & = r + b\tilde{\pi}_{t-1} + \varphi\tilde{\pi}_t + [1/\phi]v_t \end{cases}$
Taylor rules	
Current inflation (CTR)	Lagged inflation (BTR)
$\begin{cases} \lambda_2\tilde{\pi}_t & = g_t + \lambda_{1t}\tilde{\pi}_{t-1} - u_t \\ g_t & = -\phi(i_t - \tilde{\pi}^e_t - r) + v_t \\ i_t & = r + \omega_c\tilde{\pi}_t \end{cases}$	$\begin{cases} \lambda_2\tilde{\pi}_t & = g_t + \lambda_{1t}\tilde{\pi}_{t-1} - u_t \\ g_t & = -\phi(i_t - \tilde{\pi}^e_t - r) + v_t \\ i_t & = r + \omega_b\tilde{\pi}_{t-1} \end{cases}$

Table 4.2: Parameter values

Parameter	Description	Source	Source parameters	Value
β_0	labour supply constant			0
θ	relative weight of inflation			0.5
β	time preference	Amisano and Tristiani (2010)	β	0.99
r	steady-state interest rate	Amisano and Tristiani (2010)	$1/\beta - 1$	0.01
ϕ	interest elasticity of agg demand	Amisano and Tristiani (2010)	$1/\gamma$	0.421
σ_v	std dev agg demand shock	Amisano and Tristiani (2010)	$\sqrt{\sigma_{\bar{\pi}}^2 + \sigma_i^2}$	0.003
β_1	labour supply elasticity	Amisano and Tristiani (2010)	$1/(\phi + \alpha\gamma)$	0.201
κ_a	mean wage indexation	Attey (2015)		1.5
α	labour share	own estimates: Appendix (4.C.1)		0.64
ρ_a	persistence in productivity	own estimates: Appendix (4.C.1)		0.9
σ_a	std dev productivity shock	own estimates: Appendix (4.C.1)		0.013
ω_c	Taylor rule conventional	Gali and Monacelli (2005)	ϕ_π	1.5
ω_b	Taylor rule backwards	Gali and Monacelli (2005)	ϕ_π	1.5

The existence of a stationary unconditional distribution

We conduct tests on the inflation processes presented in (4.17), (4.27), and (4.29) for the existence of heavy-tailed distributions. We do not need to conduct tests on the processes of the output gap and interest rate since they are functions of inflation. Any heavy-tailed property of the unconditional distribution of inflation is automatically passed on to the other variables. The expressions for equilibrium inflation obtained under optimal monetary policy (4.17), the CTR (4.26) and the BTR (4.28) imply that inflation can generally be represented by the following univariate AR(1) process:

$$X_t = V_t + B_t X_{t-1}, \tag{4.32}$$

where (V_t, B_t) are iid with absolutely continuous distribution functions. Equation (4.32) is an AR process with random coefficient B_t. The Kesten conditions give the general conditions for such a process under which the unconditional distributions of inflation, the output gap and the interest rate under optimal monetary policy and the two interest rate rules are stationary.

Kesten Conditions: Consider a time-varying autoregressive process as in (4.32) above. If there exists a $\kappa > 0$ such that the following conditions are satisfied:

- $E \log |B_1| < 0$
- $E|B_1|^\kappa = 1$
- $E|B_1|^\kappa \log^+ |B_1| < \infty$
- $0 < E|V_1|^\kappa < \infty$,

then a stationary distribution exists for the process X irrespective of how it is initialized. The distribution is heavy tailed. For an AR(1) univariate process to have a heavy-tailed unconditional distribution, it suffices to check only the second condition. In what follows in this section, we investigate the conditions for the existence of stationary distributions of the inflation processes under optimal monetary policy and the two Taylor rules.

Optimal monetary policy

As already mentioned in Section 4.2 of this study, we assume a uniform distribution for the degree of wage indexation. In particular, we assumed that $x_t \sim U(0, \kappa_a)$ where $\kappa_a > 1$. The implied process for inflation under both the OCW and the ORW are:

$$\tilde{\pi}_t = \bar{b}\tilde{\pi}_{t-1} - (\bar{b}/\lambda_1)u_t \tag{4.33a}$$

$$\tilde{\pi}_t = 2bA_t\tilde{\pi}_{t-1} - (b/\lambda_1)u_t, \tag{4.33b}$$

where $A_t \sim U(0, 1)$.

The inflation process under the OCW is an AR(1) process with a constant coefficient $\bar{b}$. The existence of a stationary unconditional distribution hinges on the following assumption: $|\bar{b}| < 1$. Given the parameters in Table 4.2, this condition is satisfied since $\bar{b} = 0.5859$.[10] Earlier on, we assume that the productivity shock term (u_t) is normally distributed. This implies that the unconditional distribution of inflation under the OCW (4.33a) is normal.

Concerning inflation under the ORW, the first Kesten condition requires that the following holds: $b < e/2$. If b is at its maximum ($\theta = 0$), this condition translates to $\kappa_a < \sqrt{e}$. The second condition implies solving for a κ which satisfies $(2b)^\kappa = \kappa + 1$. A solution exists for any $b \in (1/2, e/2)$. The last two conditions can easily be verified, given that there exists a κ that satisfies the second condition. For our set of parameters, $b = 0.5551$. This implies that the Kesten conditions are satisfied since $b = 0.5551 \in (1/2, e/2)$. This guarantees the existence of a stationary heavy-tailed distribution for inflation, the output gap and the interest rate.

Remarkably, the mean persistence of inflation under the OCW is larger than that under the ORW (i.e. $\bar{b} > b$). However, inflation under the latter model rather exhibits heavy-tailed properties. This observation proves the importance of multiplicative shocks such as random wage indexation in generating heavy-tailed distributions.

[10]The computations for the calibration exercise were carried out in MATLAB.

Monetary policy under CTR and BTR

We test whether the coefficients under the CTR and BTR satisfy the Kesten conditions using MATLAB. The process for inflation under CTR as found in (4.26) in the main derivation can respectively be expressed as follows:

$$\tilde{\pi}_t = [c_{cmin} + c_{cext}A_t]\tilde{\pi}_{t-1} + [1/\Delta]v_t - [1/\Delta]u_t, \tag{4.34a}$$

where $c_{cmin} = (\phi\lambda_1)/(\Delta^2 - \phi\Delta)$ and $c_{cext} = \lambda_2\kappa_a/\Delta$. As in (4.26), the parameter $\Delta = \lambda_2 + \phi\omega_c$. Here again, A_t is a random variable uniformly distributed on the unit interval. Similarly, the process for inflation under a BTR as found in (4.28) can be expressed as follows:

$$\tilde{\pi}_t = [c_{bmin} + c_{bext}A_t]\tilde{\pi}_{t-1} + [1/\lambda_2]v_t - [1/\lambda_2]u_t, \tag{4.34b}$$

where $c_{bmin} = \Lambda/\lambda_2$ and $c_{bext} = \kappa_a$.

From (4.33) and (4.34), the inflation processes can be given the following generic representation:

$$\tilde{\pi}_t = [c_{min} + c_{ext}A_t]\tilde{\pi}_{t-1} + \mu_t. \tag{4.35}$$

Earlier on, we asserted that one needs to only check the second of the Kesten conditions for the existence of a heavy-tailed unconditional distribution. We nevertheless check both the first and second of the conditions in our computations. As we will explain later, the first condition reveals information about the average persistence in the inflation process. Given the inflation process (4.35), the first two conditions can be derived using the following:

$$\int_{c_{min}}^{c_{max}} \frac{\log(|x|)}{c_{ext}} dx < 0$$

$$\int_{c_{min}}^{c_{max}} \frac{|x|^{\kappa}}{c_{ext}} dx = 1,$$

where $c_{max} = c_{min} + c_{ext}$.

The Table 4.3 gives the results of the tests regarding the Kesten conditions. From the table, the value of κ in the case of the CTR is 12.307 while that for the case of the BTR is 2.313. Thus, it can be concluded that the unconditional distributions of inflation under both types of Taylor rules are stationary and heavy tailed.

Table 4.3: Results from the Kesten tests

Condition	OCW	ORW	CTR	BTR
$E \log \lvert B_1 \rvert$	N/A	-0.896	-0.549	-0.305
κ	N/A	34.038	12.307	2.314
Distribution	normal	heavy tailed	heavy tailed	heavy tailed

[1] Table gives results of the Kesten tests from calibrations N/A denotes that the test is not applicable. Test conducted on the process $X_t = B_t X_{t-1} + V_t$.
[2] Calibrations were conducted in MATLAB

Interpreting $E \log |B_1|$ and the parameter κ

The first Kesten condition requires $E \log |B_1| < 0$ for the existence of a stationary unconditional distribution of inflation. This condition can be seen as analogous to the condition that $|\bar{b}| < 1$ under the OCW. We can therefore deduce preliminary insights into the persistence of the inflation process from the condition. It can be concluded from Table (4.3) that a stationary distribution exists for the inflation process under each of the models. Also, one can again conclude that the persistence in the inflation process is the highest under the BTR and the lowest under the ORW. The persistence of this process under the CTR falls between those of the BTR and the ORW.

The presence of a heavy-tailed unconditional distribution of inflation depends on the existence of a $\kappa > 0$ that satisfies the second condition. The magnitude of this parameter indicates how heavy the tails of the distribution are. In particular, κ is inversely related to the heaviness of the tail: a higher κ denotes that the tails of the particular distribution in question are relatively less heavy. The intuition behind this inverse relationship is as follows. A distribution which does not meet the second Kesten condition may have $\kappa = \infty$. Thus the further away κ is, the more likely it is that that distribution will fail the requirements for the presence of heavy tails.

From Table 4.3, it can be concluded that the distribution of inflation under the BTR has the highest persistence and the heaviest tails. We may prematurely conclude that variations of variables under that model are most extreme and most undesirable. The inflation rate, output gap, and interest rate have the least variances under the ORW (not counting the OCW). One may therefore give the following ranking of the models based on loss minimization: ORW, CTR, and BTR.

4.4.1 Time paths of variables

Figure 4.2 gives the time path of the various macroeconomic variables in this section based on a simulation of $10,000$ observations. Noting that the unconditional distributions of variables under OCW are normally distributed, the figure shows that extreme observations are more frequent under the ORW model than those under the OCW model. It can therefore be concluded that inflation, the output gap and the interest rate under the ORW model have heavy-tailed unconditional distributions.

Figure 4.2 also shows the dynamics of the three variables under the two Taylor rules. The unconditional distributions of both variables are heavy tailed since the processes of inflation under these two Taylor rules satisfy the Kesten conditions. The figure also shows that the distributions under the BTR are more heavy tailed than those under the CTR. This result was already implied by the κ values.

4.4.2 Impulse responses to productivity shocks

The dynamic effects of productivity shocks are displayed in Figure 4.3 in the appendix. The figure suggests that the three variables converge back to their steady states faster under the

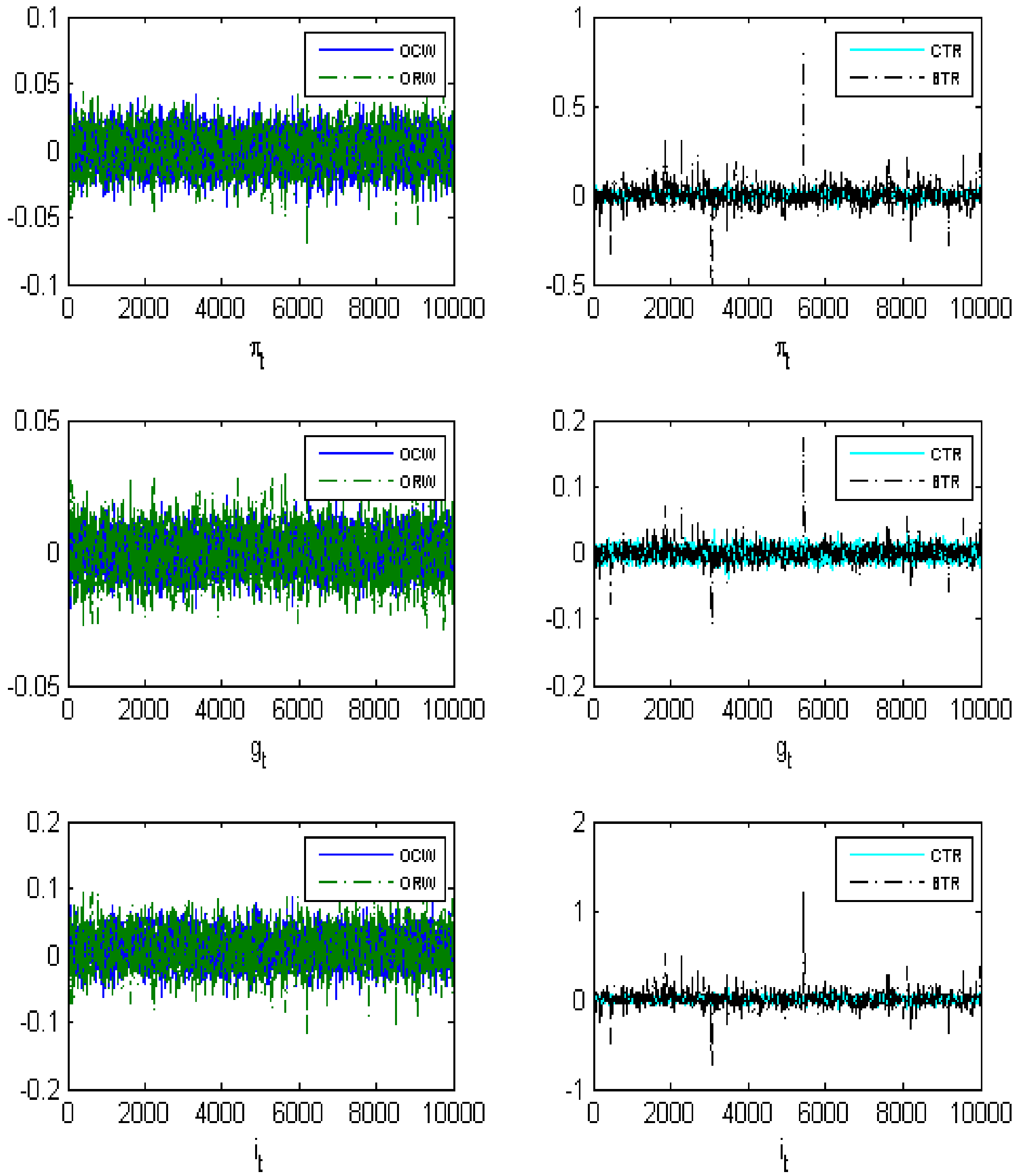


Figure 4.2: Time paths of variables

ORW model than the OCW model. However, this is the case only because of the particular set of draws of the random wage indexation parameter λ_{1t}. For other sets of draws, the variables converge back to their steady states faster under the OCW than the ORW. Repeated simulations show that on the average, it takes 10 periods to converge back to the steady state after an initial productivity shock. This is the same number of periods it takes for variables to converge back to their respective steady states under the OCW. Thus, random wage indexation induces uncertainty in the amount of time it takes for the three variables to converge back to their steady states.

Inflation has a higher initial response to productivity shocks under the CTR than under the BTR. Given the same draws of λ_{1t}, all variables converge faster to their respective steady states under the CTR than under the BTR. The reason for this result lies in the implied processes of inflation under the two models. From the expressions (4.26) and (4.28), the mean persistence parameters prevailing under these models are:

$$\begin{aligned} E[\lambda_{1t}/\Delta + \phi\lambda_1/(\Delta^2 - \phi\Delta)] = \lambda_1/(\Delta - \phi) &\qquad = 0.6706 \qquad (CTR) \\ E[\lambda_{1t}/\lambda_2 + \Lambda/\lambda_2] = (\lambda_1 + \Lambda)/\lambda_2 &\qquad = 0.8725 \qquad (BTR). \end{aligned}$$

The computations above imply that on the average, the inflation process is more persistent under the BTR than under the CTR.[11] We therefore expect inflation to converge back to its steady state faster under the CTR. Since the output gap and the interest rate are functions of the inflation rate, they also converge back to their steady states quicker under the latter model.

4.4.3 Impulse responses to demand shocks

Table 4.4 presents the responses of the various variables to a one standard deviation shock in demand v_t. Under optimal monetary policy, inflation and the output gap are not impacted by demand shocks. This is due to the assumption that the policy maker observes the demand shocks and moves to offset their likely effects. The interest rate initially rises in response to demand shocks, but converges back to its steady state in the subsequent period.

The responses of the output gap and inflation to a one-time demand shock are larger under the BTR than under the CTR. The interest rate has a delayed response to demand shocks under the BTR. All three variables converge back to their respective steady states faster under the

[11] The results from the tests of the first of the Kesten conditions already implied this.

CTR than under the BTR. The reason for this is identical to the one provided for the case of productivity shocks.

One can therefore conclude that compared to optimal monetary policy, a Taylor rule targeting only the inflation rate performs poorly when the economy is subject to demand shocks. This conclusion hinges on the assumption that a policy maker can observe demand shocks immediately in order to react to them.

4.4.4 Losses from alternative policy rules

Table 4.4 presents the standard deviations and the implied loss under each type of monetary policy considered in this work. Inflation is less volatile under the ORW than under the CTR while the output gap is less volatile under the latter than the former. This contrast concerning the volatility of the output gap and inflation under these two policies stems from their respective interest rate rules. The ORW interest rate in (4.21) reacts to demand shocks and current inflation, while the CTR interest rate in (4.22) targets only current inflation. Therefore, the excessive volatility in the interest rate under ORW is transferred to the output gap under this policy regime.

However, it should be noted that the volatility of inflation increases when the interest rate does not respond to current shocks. The following observations can be made about the various interest rate policy rules: the BTR interest rate targets none of the current shocks, the CTR interest rate targets only productivity shocks (embedded in current inflation), and the ORW interest rate targets both productivity and demand shocks. As a result from the nature of the interest rate rules, inflation is most volatile under the BTR and least volatile under the ORW.

Not surprisingly, the optimal monetary policy generates the lowest loss among the three types of monetary policy considered, although the losses from the ORW and the CTR do not differ that much in magnitude. Thus, given the parameters, a Taylor rule targeting current inflation almost replicates optimal monetary policy. Of the two types of Taylor rules considered, the one targeting current inflation (CTR) outperforms the lagged inflation targeting Taylor rule (BTR). This comes as no surprise as it is already known that the CTR comes closest to mimicking the interest rate rule under optimal monetary policy (see Woodford (2001)).

Table 4.4: Standard deviations and loss

	OCW	ORW	CTR	BTR
	$sd\%$	$sd\%$	$sd\%$	$sd\%$
Inflation (π_t)	1.16	1.16	1.34	4.67
Output gap (g_t)	0.58	0.73	0.69	1.06
Interest rate (i_t)	1.99	2.27	2.01	7
	Variance of variables in $\%$			
Inflation	0.0135	0.0135	0.0180	0.2472
Output gap	0.0033	0.0052	0.0047	0.0126
Loss ($\mathcal{V}$)	0.0101	0.0120	0.0137	0.1362

4.5 Conclusion

This study investigates the effect of random wage indexation on monetary policy. Most of the extant literature on wage indexation and its role in monetary policy is based on the assumption that the degree of wage indexation is constant. However, recent empirical estimates suggest a time-varying process for the degree of wage indexation. Drawing on the empirical properties of the degree of wage indexation, this study investigates the conduct of monetary policy in the presence of random wage indexation. In particular, we investigate the conduct of monetary policy under three interest rate rules: the rule implied by optimal monetary policy under commitment, a current inflation targeting Taylor rule and a lagged or expected inflation targeting Taylor rule.

Our findings reveal that under the plausible scenario of wages being overly indexed to inflation, the unconditional stationary distribution of inflation, the interest rate and the output gap do exhibit heavy-tailed characteristics under all of the three types of monetary policies considered. This implies that extreme observations in these variables are more likely to occur than as would be predicted under current standard theoretical models. Also, inflation exhibits volatility clustering with expected or lagged inflation having a positive effect on the conditional variance of inflation. Finally, it is better to commit to a Taylor rule targeting current inflation rather than one targeting lagged inflation.

4.A Aggregate supply and optimal monetary policy

4.A.1 Deriving the aggregate supply (Phillips) curve

It is assumed that the representative firm is perfectly competitive. The real wage is therefore equal to the marginal productivity of labour. With the production technology assumed in the main text, the expression for real wages is:[12]

$$\frac{W_t}{P_t} = \alpha A_t N_t^{\alpha-1}.$$

Let $\delta_0 = (\ln \alpha))(1-\alpha)$ and $\delta_1 = 1/(1-\alpha)$. The labour demand expression can be derived by taking the log of the real wage expression just previously given. This is given below:

$$n_t^d = \delta_0 - \delta_1(w_t - p_t) + \delta_1 a_t. \tag{4.36}$$

The expression for labour supply can be derived from a representative household's optimizing behaviour. For the purposes of this study, we make use of the following ad hoc labour supply relation:

$$n_t^s = \beta_0 + \beta_1(w_t - p_t). \tag{4.37}$$

By equating (4.36) to (4.37) , one derives the following expressions for equilibrium nominal wage rate (w_t^*) and equilibrium labour (n_t^*):

$$w_t^* = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t + \frac{\delta_1}{\delta_1 + \beta_1} a_t \tag{4.38}$$

$$n_t^* = \frac{\beta_1\delta_0 + \beta_0\delta_1}{\delta_1 + \beta_1} + \frac{\beta_1\delta_1}{\delta_1 + \beta_1} a_t. \tag{4.39}$$

[12]Lower cases of variables denote their log values. In discussing these variables, we omit the word' 'log' for convenience.

The production function was already given in the main part of this study as follows: $Y_t = A_t N_t^{\alpha}$. Taking the log of this function permits us to derive an expression in terms of log variables as follows:

$$y_t = \alpha n_t + a_t. \tag{4.40}$$

The expression for equilibrium output is then derived by substituting (4.39) into (4.40). We give the equation for equilibrium output below:

$$y_t^* = \alpha \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \left(\frac{\alpha \beta_1 \delta_1}{\delta_1 + \beta_1} + 1 \right) a_t. \tag{4.41}$$

The (log) productivity shock term a_t is assumed in the main part of this text to follow the stationary AR(1) process given below:

$$a_t = \rho_a a_{t-1} + \varepsilon_{at},$$

where ε_{at} is iid normal with a zero mean. The AR coefficient ρ_a is assumed to lie within the unit internal to ensure stationarity of the AR process.

The wage indexation rule given in Equation (4.3) stipulates for wages to be adjusted if previously observed inflation deviates from the target inflation. The rule is repeated below:

$$w_t = w_t^{*e} + x_t(\pi_{t-1} - \hat{\pi}).$$

From the expression (4.38), we can derive the expression for the expectation of the wage rate prevailing at the competitive equilibrium. Let $a_t^e = a_t \equiv \rho_a a_{t-1}$. The expectation of the equilibrium wage rate is:

$$w_t^{*e} = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t^e + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e.$$

Substituting the expression above into the expression for wage indexation we get the following:

$$w_t = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e + p_t^e + x_t(\pi_{t-1} - \hat{\pi}).$$

The presence of indexation introduces nominal rigidity into the model. A trade-off between inflation and the output gap can therefore be realized in the presence of wage indexation. Subtracting prices from both sides of the equation, one derives the following expression for real wages under wage indexation:

$$w_t - p_t = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e - (p_t - p_t^e) + x_t(\pi_{t-1} - \hat{\pi}).$$

We note that $(p_t - p_t^e) = \pi_t - \pi_t^e$, where $\pi_t = p_t - p_{t-1}$. Substitute the expression for the real wage under wage indexation into the labour demand expression (4.36) to obtain the following:

$$n_t = \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \delta_1(\pi_t - \pi_t^e) - \delta_1 x_t(\pi_{t-1} - \hat{\pi}) + \delta_1 a_t - \frac{\delta_1^2}{\delta_1 + \beta_1} a_t^e.$$

We note that $\delta_1 a_t = \delta_1 a_t^e + \delta_1 \varepsilon_{at}$ and also that $\pi_t^e = \hat{\pi}$ as per the assumption made in the main text. Thus making this substitution into the labour demand equation previously written down results in the following equation:

$$n_t = \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \delta_1(\pi_t - \hat{\pi}) - \delta_1 x_t(\pi_{t-1} - \hat{\pi}) + \frac{\delta_1 \beta_1}{\delta_1 + \beta_1} a_t^e + \delta_1 \varepsilon_{at}.$$

We can derive the output under wage indexation by using the log form of the production technology: $y_t = \alpha n_t + a_t$. The output is given as follows:

$$y_t = \alpha \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \alpha \delta_1(\pi_t - \hat{\pi}) - \alpha \delta_1 x_t(\pi_{t-1} - \hat{\pi}) + \alpha \frac{\delta_1 \beta_1}{\delta_1 + \beta_1} a_t^e + \alpha \delta_1 \varepsilon_{at} + a_t.$$

With the help of equation (4.41), we express the output under wage indexation as a function of equilibrium output prevailing under flexible wages (y_t^*). The resulting expression is as follows:

$$y_t = y_t^* + \alpha\delta_1(\pi_t - \hat{\pi}) - \alpha\delta_1 x_t(\pi_{t-1} - \hat{\pi}) + \frac{\alpha\delta^2}{\delta_1 + \beta_1}\varepsilon_{at}.$$

Let the output gap(g_t) be defined as the deviation of output from the output prevailing under flexible wage equilibrium. Further assume the following: $(\pi_t - \hat{\pi}) = \tilde{\pi}_t$, $\alpha\delta_1 = \lambda_2$, and $\lambda_2 x_t = \lambda_{1t}$. The aggregate supply relation is given as follows:

$$g_t = -\lambda_{1t}\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t + u_t,$$

where $u_t = \alpha\delta_1^2/(\delta_1 + \beta_1)\varepsilon_{at}$.

4.A.2 Optimal monetary policy

We assume that wages are indexed to lagged inflation. We assume that agents in the economy fix their expectations equal to a target inflation which does not necessarily need to be 0. We give the IS and the Phillips Curve as follows:

$$\begin{aligned} g_t &= -\lambda_{1t}\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t + u_t \\ g_t &= -\phi(i_t - \tilde{\pi}_t^e - r) + v_t. \end{aligned}$$

We again assume that in conducting optimal monetary policy, the central bank uses the interest rate and the expected interest rate (i_t and i_t^e) as instruments. Alternative ways of expressing the Phillips and the IS expressions which will be useful for our optimization purposes are given below:

$$\tilde{\pi}_t = -\frac{\phi}{\lambda_2}(i_t - i_t^e) - \frac{\phi}{\lambda_2 - \phi}(i_t^e - r) + \lambda_{3,t}\tilde{\pi}_{t-1} + \frac{1}{\lambda_2}(v_t - u_t) \tag{4.42a}$$

$$\tilde{\pi}_t = \tilde{\pi}_t^e - \frac{\phi}{\lambda_2}(i_t - i_t^e) + \frac{\eta_t}{\lambda_2}\tilde{\pi}_{t-1} + \frac{1}{\lambda_2}(v_t - u_t), \tag{4.42b}$$

where $\lambda_{3,t} = \frac{\phi\bar{\lambda_1}}{\lambda_2(\lambda_2-\phi)} + \frac{\lambda_{1t}}{\lambda_2}$ and $\eta_t = \lambda_{1t} - \lambda_1$. The expected inflation can easily be obtained by taking expectation of the equation (4.42a). The expected inflation is:

$$\tilde{\pi}_t^e = -\frac{\phi}{\lambda_2 - \phi}(i_t^e - r) + \frac{\lambda_1}{\lambda_2 - \phi}\tilde{\pi}_{t-1} \qquad \lambda_1 = E[\lambda_{1t}] \tag{4.43}$$

$$g_t = -\phi(i_t - i_t^e) - \frac{\phi\lambda_2}{\lambda_2 - \phi}(i_t^e - r) + \frac{\phi\lambda_1}{\lambda_2 - \phi}\tilde{\pi}_{t-1} + v_t \tag{4.44a}$$

$$g_t = g_t^e - \phi(i_t - i_t^e) + v_t, \tag{4.44b}$$

The (endogenous) state variable in this model is inflation $\tilde{\pi}_t$. Thus, we can write the value function, assuming a zero output gap target as follows:

$$V(\tilde{\pi}_{t-1}) = \max_{i_t, i_t^e} E_{t-1}\left[-g_t^2 - \theta\tilde{\pi}_t^2 + \beta V(\tilde{\pi}_t)\right]. \tag{4.45}$$

This is maximized subject to the constraints in (4.42a) and (4.44a) in addition to the expression which must hold under commitment:

$$i_t^e = E_{t-1}[i_t]. \tag{4.46}$$

Since the loss function is quadratic, the value function must be quadratic in the state variable. We therefore conjecture the following expression for the value function:

$$V(\tilde{\pi}_t) = \gamma_0 + 2\gamma_1\tilde{\pi}_t + \gamma_2\tilde{\pi}_t^2. \tag{4.47}$$

where the parameters γ_0, γ_1 and γ_2 are parameters which are assumed to be functions of the parameters in (4.42a) and (4.44a). Let Λ_{t-1} be the Lagrangian multiplier associated with the commitment constraint (4.46). By the chain rule of differentiation, we can write down the first order conditions as follows:

$$0 = -2g_t \frac{\partial g_t}{\partial i_t} - 2\theta\tilde{\pi}_t \frac{\partial \tilde{\pi}_t}{\partial i_t} + \beta \frac{\partial V(\tilde{\pi}_t)}{\partial \tilde{\pi}_t} \frac{\partial \tilde{\pi}_t}{\partial i_t} - \Lambda_{t-1} \tag{4.48}$$

$$0 = E_{t-1}\left[-2g_t \frac{\partial g_t}{\partial i_t^e} - 2\theta\tilde{\pi}_t \frac{\partial \tilde{\pi}_t}{\partial i_t^e} + \beta \frac{\partial V(\tilde{\pi}_t)}{\partial \tilde{\pi}_t} \frac{\partial \tilde{\pi}_t}{\partial i_t^e} + \Lambda_{t-1}\right]. \tag{4.49}$$

The expectation sign appears in the second of the first order conditions because the policy maker does not directly control i_t^e, but rather influences it through policy instrument i_t. From (4.42a) and (4.44a), we derive the following:

$$\partial g_t/\partial i_t = -\phi$$
$$\partial \tilde{\pi}_t/\partial i_t = -\phi/\lambda_2.$$

The conjectured value function in (4.47) implies that the derivative of the value function with respect to inflation is:

$$\partial V(\tilde{\pi}_t)/\partial \tilde{\pi}_t = 2(\gamma_1 + \gamma_2\tilde{\pi}_t).$$

We obtain the following derivatives of g_t and $\tilde{\pi}_t^e$ with regards to i_t^e:

$$\partial g_t/\partial i_t^e = \phi - \phi\lambda_2/(\lambda_2 - \phi)$$
$$\partial \tilde{\pi}_t/\partial i_t^e = \phi/\lambda_2 - \phi/(\lambda_2 - \phi).$$

Substitute these expressions into the FOCs derived with respect to i_t and i_t^e as given by (4.48) and (4.49) to obtain the following equations:

$$0 = 2\phi[g_t + \tilde{\pi}_t(\theta/\lambda_2) - (\gamma_1 + \gamma_2\tilde{\pi}_t)(\beta/\lambda_2)] - \Lambda_{t-1}$$
$$0 = -2\phi[g_t^e(1 - \lambda_2/(\lambda_2 - \phi)) + \tilde{\pi}_t^e(\theta/\lambda_2 - \theta/(\lambda_2 - \phi)) + (\gamma_1 + \gamma_2\tilde{\pi}_t^e)(\beta/\lambda_2 - \beta/(\lambda_2 - \phi))] + \Lambda_{t-1}.$$

Adding the two equations just listed above derives an intermediate version of the optimal feedback rule. This expression and a version derived by taking expectations are given below:

$$0 = 2\phi[(g_t - g_t^e) + (\tilde{\pi}_t - \tilde{\pi}_t^e)(\theta - \beta\gamma_2)/\lambda_2] + 2\phi[\lambda_2 g_t^e + (\theta - \beta\gamma_2)\tilde{\pi}_t^e - \beta\gamma_1]/(\lambda_2 - \phi) \tag{4.50}$$

$$0 = 2\phi[\lambda_2 g_t^e + (\theta - \beta\gamma_2)\tilde{\pi}_t^e - \beta\gamma_1]/(\lambda_2 - \phi). \tag{4.51}$$

We substitute the expressions (4.42b) and (4.44b) into (4.50) to obtain an expression in terms of the control variables. The derived optimal feedback rule after imposing (4.51) and some simplifications is as follows:

$$0 = [-\phi(i_t - i_t^e) + v_t]\left(1 + \frac{\theta - \beta\gamma_2}{\lambda_2^2}\right) - \left(\frac{\theta - \beta\gamma_2}{\lambda_2^2}\right)(u_t - \eta_t\tilde{\pi}_{t-1}). \tag{4.52}$$

The value function needs to be concave in the state variable to ensure the existence of a solution to the dynamic optimization problem. It will later be shown that a necessary condition for the value function to be concave in the state variable is the following:

$$\beta(b^2 + \delta^2\sigma_\eta^2) < 1,$$

where b and δ are coefficients (to be later determined) governing the process of inflation under optimal control. The other variables, β and σ_η^2 are the discount rate and the variance of η_t respectively. Given that the necessary conditions for concavity are satisfied, we know that $1 + (\theta - \beta\gamma_2)/\lambda_2^2 \neq 0$. This implies that under optimal control, $[-\phi(i_t - i_t^e) + v_t]$ is a function of u_t and $\eta_t\tilde{\pi}_{t-1}$.[13]

Thus g_t is a function of u_t and $\eta_t\tilde{\pi}_{t-1}$. This observation coupled with the Phillips curve expression $g_t = -\lambda_{1t}\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t + u_t$, implies that inflation under optimal control assumes the

[13] It is assumed that the policy maker observes and reacts to the shocks in an interim period within which private agents can neither observe those shocks nor react to them. The shocks are not observed by both parties ex-ante. See Clarke et al. (1999) for detailed discussion on the implication of this assumption.

following general form

$$\tilde{\pi}_t = a + b\tilde{\pi}_{t-1} + \delta\eta_t\tilde{\pi}_{t-1} + cu_t, \tag{4.53}$$

where a, b, δ and c are parameters to be determined. Noting that $\eta_t = \lambda_{1t} - \lambda_1$ is a zero mean iid random variable, the expected inflation under this guess can easily be derived as follows:

$$\tilde{\pi}^e = a + b\tilde{\pi}_{t-1}.$$

However, noting that the original specification of the AS (Phillips curve) relation implies $g_t^e = \lambda_1\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t^e$ and substituting this expression into (4.51), we get the following expression for expected inflation:

$$\tilde{\pi}_t^e = \frac{\beta\gamma_1}{\lambda_2^2 + \theta - \beta\gamma_2} + \frac{\lambda_1\lambda_2}{\lambda_2^2 + \theta - \beta\gamma_2}\tilde{\pi}_{t-1}. \tag{4.54}$$

We can identify the parameters a and b in terms of value function parameters and the structural parameters after comparing (4.54) to the expectation of (4.53) as follows.

$$a = \frac{\beta\gamma_1}{\lambda_2^2 + \theta - \beta\gamma_2} \qquad\qquad b = \frac{\lambda_1\lambda_2}{\lambda_2^2 + \theta - \beta\gamma_2}. \tag{4.55}$$

From the expression given for b, we can rule out that $\lambda_2^2[1 + (\theta - \beta\gamma_2)/\lambda_2^2] = 0$ as earlier on claimed.[14] This is a necessary condition for a stable inflation under optimal control process since b is an AR coefficient. We substitute the expression for expected inflation into the expression (4.43) to obtain the expected interest rate expression:

$$i_t^e = r + [(\lambda_1 - b(\lambda_2 - \phi))\tilde{\pi}_{t-1} - a(\lambda_2 - \phi)]/\phi. \tag{4.56}$$

The guess we made for equilibrium inflation under (4.53) implies that $\tilde{\pi}_t - \tilde{\pi}_t^e = \delta\eta_t\tilde{\pi}_{t-1} + cu_t$. Substituting this into the expression (4.42b) implies the following expression for the interest rate

[14]The fact that $\lambda_1, \lambda_2 \neq 0$ reinforces this claim.

rule under optimal control.

$$i_t = i_t^e - \frac{1 + c\lambda_2}{\phi} u_t + \frac{1 - \delta\lambda_2}{\phi} \eta_t \tilde{\pi}_{t-1} + \frac{1}{\phi} v_t. \tag{4.57}$$

This expression substituted into (4.52) implies that the parameters δ and c can be identified as follows:

$$\delta = \frac{\lambda_2}{\lambda_2^2 + \theta - \beta\gamma_2} \qquad c = \frac{-\lambda_2}{\lambda_2^2 + \theta - \beta\gamma_2}. \tag{4.58}$$

To proceed further, we note once again that $\tilde{\pi}_t^e = a + b\tilde{\pi}_{t-1}$. One can derive the following expression for the deviation of expected real interest rate from the natural rate of interest as follows:

$$i_t^e - \tilde{\pi}_t^e - r = [(\lambda_1 b - \lambda_2)\tilde{\pi}_{t-1} - a\lambda_2]/\phi. \tag{4.59}$$

The interest rate equation in (4.57) implies the following expression for the deviation of the interest rate from its expected value $i_t - i_t^e = [-(1 + c\lambda_2)u_t + (1 - \delta\lambda_e)\eta_t\tilde{\pi}_{t-1} + v_t]/\phi$. Add $(i_t - i_t^e)$ to both sides of Equation (4.59). Using (4.57), make the necessary substitution at the RHS of the resulting equation, to obtain the following:

$$i_t - \tilde{\pi}_t^e - r = [(\lambda_1 - \lambda_2 b)\tilde{\pi}_{t-1} - a\lambda_2 - (1 + c\lambda_2)u_t + (1 - \delta\lambda_2)\eta_t\tilde{\pi}_{t-1} + v_t]/\phi. \tag{4.60}$$

The last expression implies that the output gap can then be expressed as a function of only the state variables. The output gap given by (4.6) in the main part of this text can then be rewritten as follows:

$$g_t = -\phi(i_t - \tilde{\pi}_t^e - r) + v_t = a\lambda_2 - (\lambda_1 - \lambda_2 b)\tilde{\pi}_{t-1} + (1 + c\lambda_2)u_t - (1 - \delta\lambda_2)\eta_t\tilde{\pi}_{t-1}. \tag{4.61}$$

Deriving parameters of the value function

We have expressed both inflation and the output gap in terms of the state variables. These are contained in equations (4.53) and (4.61) respectively. We now proceed to express the various components of the value function in terms of the state variables. From (4.53) and (4.61), we make the following derivations:

$$
\begin{aligned}
E_{t-1}g_t^2 &= a^2\lambda_2^2 - 2a\lambda_2(\lambda_1-\lambda_2 b)\tilde{\pi}_{t-1} + (\lambda_1-\lambda_2 b)^2\tilde{\pi}_{t-1}^2 + (1+c\lambda_2)^2\sigma_u^2 + (1-\delta\lambda_2)^2\sigma_\eta^2\tilde{\pi}_{t-1}^2 \\
E_{t-1}\tilde{\pi}^2 &= a^2 + 2ab\tilde{\pi}_{t-1} + b^2\tilde{\pi}_{t-1}^2 + \delta^2\sigma_\eta^2\tilde{\pi}_{t-1}^2 + c^2\sigma_u^2 \\
E_{t-1}[V(\tilde{\pi}_t)] &= \gamma_0 + 2\gamma_1 a + \gamma_2(a^2+c^2\sigma_u^2) + 2b(\gamma_1 + a\gamma_2)\tilde{\pi}_{t-1} + \gamma_2(b^2+\delta^2\sigma_\eta^2)\tilde{\pi}_{t-1}^2.
\end{aligned}
$$

Substitute the three expressions above into (4.45) to obtain the following:

$$
\begin{aligned}
V(\tilde{\pi}_{t-1}) = {} & \beta\gamma_0 + 2\beta\gamma_1 a + (\beta\gamma_2-\theta)(a^2+c^2\sigma_u^2) - a^2\lambda_2^2 - (1+c\lambda_2)^2\sigma_u^2 \\
& + 2[\beta b\gamma_1 + (\beta\gamma_2-\theta)ab + a\lambda_2(\lambda_1 - b\lambda_2)]\tilde{\pi}_{t-1} \\
& + \left[(\beta\gamma_2-\theta)(b^2+\delta^2\sigma_\eta^2) - [(\lambda_1-b\lambda_2)^2 + (1-\delta\lambda_2)^2\sigma_\eta^2]\right]\tilde{\pi}_{t-1}^2.
\end{aligned}
$$

Equating the coefficients to the ones in the expressions $V(\tilde{\pi}_{t-1}) = \gamma_0 + 2\gamma_1\tilde{\pi}_{t-1} + \gamma_2\tilde{\pi}_{t-1}^2$, we obtain the following systems of equations :

$$
\gamma_2 = -\left[\frac{\theta(b^2+\delta^2\sigma_\eta^2) + \lambda_1^2(1-\delta\lambda_2)^2 + (1-\delta\lambda_2)^2\sigma_\eta^2}{1-\beta(b^2+\delta^2\sigma_\eta^2)}\right] \tag{4.62}
$$

$$
\gamma_1 = a\left[\frac{\lambda_2\lambda_1 - (\lambda_2^2+\theta-\beta\gamma_2)b}{1-\beta b}\right] \tag{4.63}
$$

$$
\gamma_0 = \left[\frac{2\beta\gamma_1 a + (\beta\gamma_2-\theta)(a^2+c^2\sigma_u^2) - a^2\lambda_2^2 - (1+c\lambda_2)^2\sigma_u^2}{1-\beta}\right]. \tag{4.64}
$$

Since the loss function, $\mathcal{L} = -g_t^2 - \theta\tilde{\pi}_t^2$, is concave in $\tilde{\pi}_{t-1}$, it holds that the value function must necessarily be concave in that state variable. This implies that $\gamma_2 < 0$, which holds only if $\beta(b^2+\delta^2\sigma_\eta^2) < 1$.

Solving for policy function parameters

The value for b as given by (4.55) implies that the numerator of (4.63) is 0. We can therefore conclude that $\gamma_1 = 0$. This implies the following:

$$a = \frac{\beta\gamma_1}{\lambda_2^2 + \theta - \beta\gamma_2} = 0. \tag{4.65}$$

In order to solve for b, we begin by noting that $\delta = b/\lambda_1$ from (4.55) and (4.58). Substituting out the δ in (4.62) and substituting (4.55) into (4.63) gives a quadratic equation for b. In order to perform a step by step derivation of this quadratic equation, we begin by noting that an alternative rendition of (4.62) is the following:

$$\begin{aligned}\gamma_2 &= -\left[\frac{[\theta b^2 + (\lambda_1 - b\lambda_2)^2](1 + \sigma_\eta^2/\lambda_1^2)}{1 - \beta b^2(1 + \sigma_\eta^2/\lambda_1^2)}\right] \\ &= -\left[\frac{[b^2(\lambda_2^2 + \theta) - 2\lambda_2\lambda_1 b + \lambda_1^2](1 + \sigma_\eta^2/\lambda_1^2)}{1 - \beta b^2(1 + \sigma_\eta^2/\lambda_1^2)}\right].\end{aligned}$$

The next step is to derive an expression for $(\lambda_2^2 + \theta - \beta\gamma_2)$ and note (4.55) implies that $(\lambda_2^2 + \theta - \beta\gamma_2) = (\lambda_1\lambda_2)/b$. The derivations corresponding to this step are given below:

$$\begin{aligned}-\beta\gamma_2 &= \left[\frac{[\beta b^2(\lambda_2^2 + \theta) - 2\beta\lambda_2\lambda_1 b + \beta\lambda_1^2](1 + \sigma_\eta^2/\lambda_1^2)}{1 - \beta b^2(1 + \sigma_\eta^2/\lambda_1^2)}\right] \\ \lambda_2^2 + \theta - \beta\gamma_2 &= \left[\frac{[\beta b^2(\lambda_2^2 + \theta) - 2\beta\lambda_2\lambda_1 b + \beta\lambda_1^2](1 + \sigma_\eta^2/\lambda_1^2)}{1 - \beta b^2(1 + \sigma_\eta^2/\lambda_1^2)}\right] + \lambda_2^2 + \theta \\ \frac{\lambda_1\lambda_2}{b} &= \left[\frac{(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2 - 2\beta\lambda_1\lambda_2(1 + \sigma_\eta^2/\lambda_1^2)b}{1 - \beta b^2(1 + \sigma_\eta^2/\lambda_1^2)}\right].\end{aligned}$$

The final of the previous expressions can be rearranged to obtain the following equation which is quadratic in b:

$$0 = [\beta(\lambda_1\lambda_2)(1 + \sigma_\eta^2/\lambda_1^2)]b^2 - [(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2]b + (\lambda_1\lambda_2).$$

This equation has two roots on whose values the stability of the system of equations depends.

The root that satisfies the condition $\beta(b^2+\delta^2\sigma_\eta^2)\leq 1$ is

$$b=\frac{[(\lambda_2^2+\theta)+\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]-\sqrt{[(\lambda_2^2+\theta)+\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2-4\beta(\lambda_1\lambda_2)^2(1+\sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1+\sigma_\eta^2/\lambda_1^2)}. \tag{4.66}$$

In what follows, we show that $0\leq b\leq\bar{x}$. It is clear from (4.66) that $b\geq 0$ since $[(\lambda_2^2+\theta)+\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]>\sqrt{[(\lambda_2^2+\theta)+\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2-4\beta(\lambda_1\lambda_2)^2(1+\sigma_\eta^2/\lambda_1^2)}$. The derivation of the upper bound on this parameter is given below:

$$\begin{aligned}
b&=\frac{[(\lambda_2^2+\theta)+\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]-\sqrt{[(\lambda_2^2+\theta)+\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2-4\beta(\lambda_1\lambda_2)^2(1+\sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1+\sigma_\eta^2/\lambda_1^2)}\\
&=\frac{[(\lambda_2^2+\theta)+\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]-\sqrt{[(\lambda_2^2+\theta)-\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2+4\beta\theta(\lambda_1^2)(1+\sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1+\sigma_\eta^2/\lambda_1^2)}\\
&\leq\frac{[(\lambda_2^2+\theta)+\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]-\sqrt{[(\lambda_2^2+\theta)-\beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2}}{2\beta\lambda_1\lambda_2(1+\sigma_\eta^2/\lambda_1^2)}\\
&=\frac{\lambda_1}{\lambda_2}\\
&=\bar{x}.
\end{aligned}$$

From (4.58) it is obvious that $c=-\delta$. We are therefore able to solve for the remaining policy function parameters as follows:

$$\delta=\frac{b}{\lambda_1} \tag{4.67}$$
$$c=-\frac{b}{\lambda_1}. \tag{4.68}$$

4.B Solution to a linear system with rational expectations

Consider the linear system given below as given in Section 4.3 of the main text:

$$X_t = \mathbf{F_t} X_{t-1} + \mathbf{G} E_{t-1} X_t + \mathbf{H} \epsilon_t. \tag{4.69}$$

We guess the solution is of the form $X_t = \mathbf{P_t} X_{t-1} + \mathbf{Q} \epsilon_t$ which implies that $\mathbf{G} E_{t-1} X_t = \mathbf{GP} X_{t-1}$ where $E_{t-1} \mathbf{P_t} = \mathbf{P}$. A substitution of this guess into 4.69 allows us to solve the system by the method of undetermined coefficients. This is illustrated in a step by step manner below:

$$\begin{aligned} \mathbf{P_t} X_{t-1} + \mathbf{Q} &= \mathbf{F_t} X_{t-1} + \mathbf{GP} X_{t-1} + \mathbf{H} \epsilon_t \\ &= (\mathbf{F_t} + \mathbf{GP}) X_{t-1} + \mathbf{H} \epsilon_t. \end{aligned}$$

By comparing the coefficients, we know that that the following should hold true:

$$\mathbf{Q} = \mathbf{H} \tag{4.70}$$

$$\mathbf{P_t} = \mathbf{F_t} + \mathbf{GP}. \tag{4.71}$$

Let $E_{t-1} \mathbf{F_t} = \mathbf{F}$. Taking expectation of the second equation, we obtain the following:

$$\begin{aligned} \mathbf{P} &= \mathbf{F} + \mathbf{GP} \\ (\mathbf{I} - \mathbf{G}) \mathbf{P} &= \mathbf{F} \\ \mathbf{P} &= (\mathbf{I} - \mathbf{G})^{-1} \mathbf{F}. \end{aligned}$$

Substitute the last expression into 4.71 to obtain the following expression for $\mathbf{P_t}$:

$$\mathbf{P_t} = \mathbf{F_t} + \mathbf{G} (\mathbf{I} - \mathbf{G})^{-1} \mathbf{F}. \tag{4.72}$$

4.C Productivity parameters

In this section, we derive alternative values for the parameters regarding productivity. We do this by first computing the Solow residual for 3 countries, namely, France, Germany and the UK. We then estimate the AR coefficient of productivity and the standard deviation of the productivity shocks.

We obtained the real income growth and growth in labour hours data from the OECD data base. Data on capital stock was obtained from the Federal Reserve Economic Data on FRED St.Louis website. We now proceed to discuss our estimations in detail. Consider the following Cobb-Douglass function

$$Y_t = Z_t N_t^{\alpha} K_t^{1-\alpha}. \tag{4.73}$$

where Z_t is productivity, N_t is capital and K_t is labour supplied. To allow for growth in the long-run, we assume that productivity has two components: one that follows a deterministic trend and the other which is stationary. In other words,

$$Z_t = A_t^{\tau} A_t \qquad\qquad A_t^{\tau} = A_0 \exp^{\upsilon t}. \tag{4.74}$$

Let log values of the variables be represented by small case versions of the relevant letters. Take the natural log of (4.73) to obtain the following:

$$y_t = a_0 + \upsilon t + \alpha n_t + (1-\alpha)k_t + a_t. \tag{4.75}$$

We proceed by first noting that a differenced version of (4.75) gives the growth version of (4.73). The difference version of (4.75) is

$$\Delta y_t = \upsilon + \alpha \Delta n_t + (1-\alpha)\Delta k_t + \Delta a_t. \tag{4.76}$$

The equation (4.76) can be easily estimated by a constrained OLS if one assumes Δa_t is the error term. This error term should be stationary, albeit, possibly serially correlated.

The next procedure is to obtain the estimated residual $\hat{\epsilon}_t = \Delta\hat{a}_t$ from the first estimation. Now, assume the following AR(1) structure for a_t:

$$a_t = \rho_a a_{t-1} + \varepsilon_{at}, \tag{4.77}$$

where $\varepsilon_{at} \sim \mathcal{N}(0, \sigma_a^2)$. It follows that both the AR coefficient ρ_a and the variance of the productivity shock σ_a^2 can be estimated using the following state-space specification:

$$\begin{cases} \hat{\epsilon}_t & = a_t - a_{t-1} \\ a_t & = \rho_a a_{t-1} + \varepsilon_{at} \\ \varepsilon_{at} & \sim \mathcal{N}(0, \sigma_a^2). \end{cases} \tag{4.78}$$

We used the version 8 of the EVIEWS statistical package to estimate equation (4.78). Table 4.5 provides the estimates of (4.76) for the three countries and Table (4.6) estimates for ρ_a and σ_a^2 for the same countries.

The estimates of α for Germany and the UK are similar to the estimates obtained from other literature. That of the UK however is outside the generally accepted range for α. In the main part of this study, we set $\alpha = 0.64$ for the calibration exercise to reflect a notional average of the estimates for α. It can be seen from Table (4.6) that the country specific estimates for both ρ_a and σ_a^2 do not differ that much. The estimates suggest that productivity shocks are highly persistent, albeit stationary. We will therefore set ρ_a at 0.9 for the calibration. Finally, from the estimates, the country specific standard deviation of productivity shocks σ_a lies between 0.0121 and 0.0151. We will set $\sigma_a = 0.013$ for the calibration.

4.C.1 Tables

Table 4.5: Cobb-Douglass (4.76)

Country	$\hat{\upsilon}$	$\hat{\alpha}$	R^2
France	0.005	0.366**	0.50
	(0.004)	(0.098)	
Germany	0.013**	0.675**	0.57
	(0.003)	(0.107)	
UK	0.015**	0.64**	0.50
	(0.004)	(0.124)	

[1] Standard errors in parenthesis
[2] * $p > 0.05$, **$p > 0.01$

Table 4.6: Productivity (4.78)

Country	$\hat{\rho}_a$	$\ln(\hat{\sigma}_a^2)$	AIC
France	0.922**	-8.818**	-5.86
	(0.12)	(0.194)	
Germany	0.978**	-8.713**	-5.77
	(0.099)	(0.134)	
Uk	0.945**	-8.379**	-5.42
	(0.129)	(0.221)	

[1] Standard errors in parenthesis
[2] * $p > 0.05$, **$p > 0.01$

4.D Figures

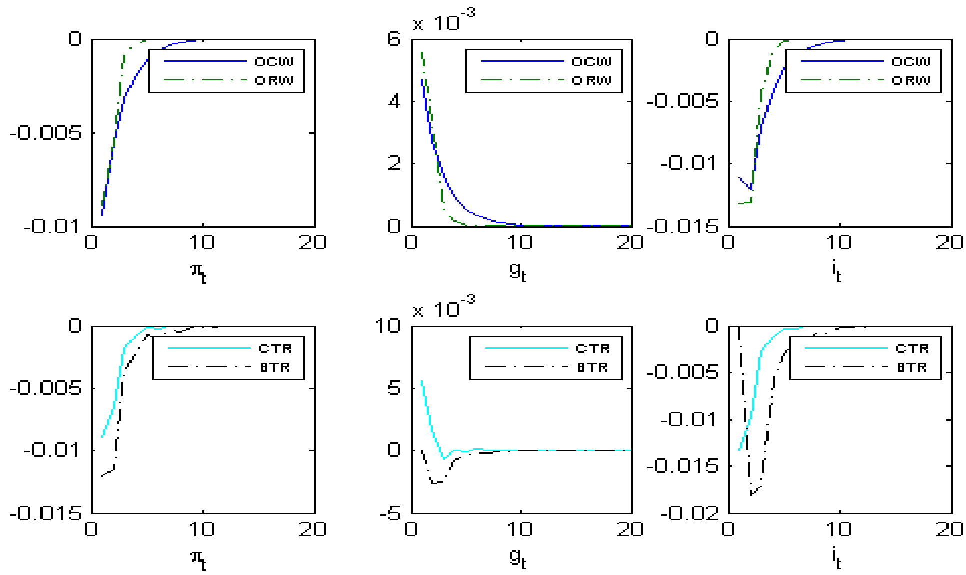


Figure 4.3: Impulse response to productivity shocks

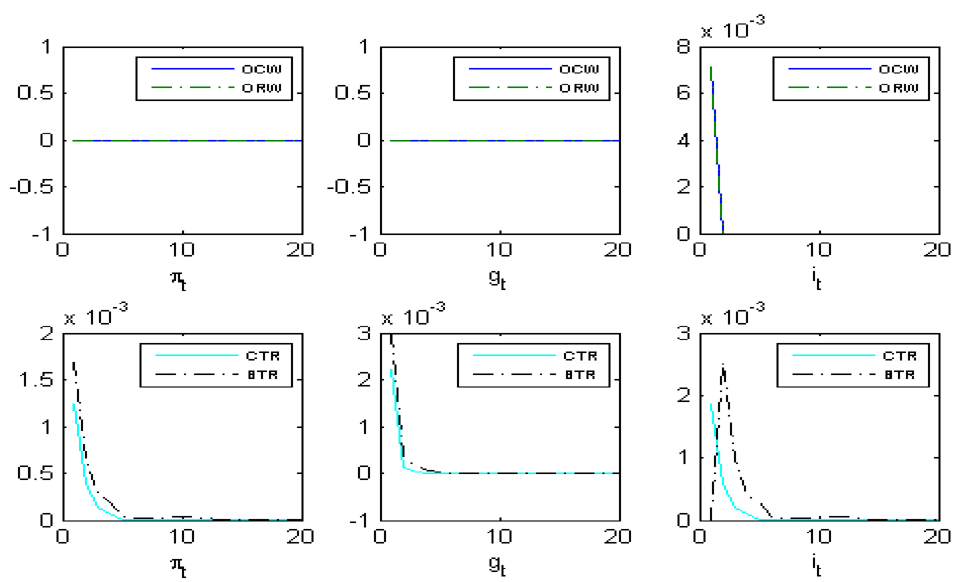


Figure 4.4: Impulse response to demand shocks

Chapter 5

Wage Indexation Negotiations and Inflation Volatility

'The methods by which a trade union can act alone, are necessarily destructive; its organization is necessarily tyrannical'.

– Henry George

5.1 Introduction

That the volatility of inflation exhibits time variation is a fact first established by Engle (1982) and later by Bollerslev (1986). Since then, a number of theoretical and empirical studies have investigated the causes of the time variation in inflation volatility. The aim of this study is to investigate the role of labour market institutions in explaining the volatility of inflation. For the purposes of this study, the terms inflation variance and inflation volatility are used interchangeably.

Three main categories of causes of the time variation in inflation volatility can be identified from the existing literature. The first category of causal variables can be linked to the macroeconomic policy actions on the part of policy authorities. A considerable amount of studies have shown that fiscal and monetary policy actions affect the volatility or uncertainty of inflation.

Examples of such studies include those by Ball (1992) and Rother (2004). Ball (1992) gives a theoretical explanation as to how an inflationary policy raises the level of inflation uncertainty, while Rother (2004) finds a positive relationship between inflation volatility and the standard deviation of changes in fiscal stance.

The second category of causal variables that explain inflation volatility are exogenous shocks such as Total Factor Productivity (TFP) shocks and oil price shocks. Standard models employed in studies typically imply that equilibrium inflation is a function of these shocks. It therefore follows that the variance of inflation is a function of the variance of these shocks.

The final category of variables that influence the volatility of inflation are labour market institutional variables and other political economic variables. The relative importance of the labour market institutional variables in stabilizing the economy is conveyed by the following quote: '...wage setting may be as important as government policy for macroeconomic performance. Today it is commonplace to explain the diverse experiences of countries with reference to differences in wage-setting institutions' (Calmfors and Driffill (1988)).

Our interest lies in the third category of variables that explain inflation volatility: the labour market institutional variables. Among this set of variables, the degree of wage centralization of wage bargaining (hereinafter referred to as centralization) has received the most attention from researchers. Perhaps, the contradicting hypotheses concerning the effect of centralization on inflation make it an interesting academic topic. One hypothesis posits a negative relationship between centralization and wage increases. This hypothesis hinges on the view that centralization guarantees that wage setters will recognize broader interests. The other hypothesis posits more restrained wage increases if wage bargaining is decentralized.[1]

Empirical evidence by Calmfors and Driffill (1988) and Daniels et al. (2006) basically describe an inverse U-shaped relationship between centralization (or coordination) of wage bargaining and wage inflation (inflation). This result is subject to two interpretations concerning the implications of centralization on inflation volatility. The first interpretation, which is the widely held one in empirical literature, posits a negative relationship between the degree of

[1] See Calmfors and Driffill (1988) for a detailed discussion of these hypotheses.

centralization and inflation volatility. The intuition behind this position is as follows: a highly centralized bargaining process will recognize a broader range of issues and will therefore be prone to less volatile wage increases. The results of the analysis by Rumler and Scharler (2011) and Barbier-Gauchard et al. (2014) confirm this interpretation. The other interpretation implies an increased volatility of inflation as the wage bargaining process becomes more centralized. This implication is consistent with the view that the democratization of the wage bargaining process leads to more restricted increases in wages on the average. The results of Campolmi and Faia (2011) confirm this view.

We agree with the assertion that centralization has implications for inflation volatility. However, there are some caveats to be taken into consideration when extending the analysis in Calmfors and Driffill (1988) to making predictions regarding the relationship between centralization and inflation volatility. Firstly, an increase in wages does not necessarily imply an increase in inflation volatility. Secondly, any analysis on the effects of centralization on inflation volatility should account for the fact that government is more likely to intervene in a centralized bargaining process. Thus, any decrease in inflation volatility associated with increased centralization might rather reflect the effect of government intervention if the effect of the latter variable is not controlled for. Finally, any negative correlation between centralization and inflation volatility may be spurious if one does not control for the bargaining power of parties involved in the negotiation. A centralized bargaining process will most likely yield restricted wage increases if the labour unions have weak bargaining power. Thus, a decreased inflation volatility might rather reflect weak bargaining power on the part of unions.

Due to the aforementioned caveats, the conclusion by Barbier-Gauchard et al. (2014) that strengthening the power of unions helps stabilize inflation might be misleading. In their analysis, they assume the presence of a policy maker whose goal is to stabilize inflation. However, the stabilization of inflation may be the result of the presence of the policy maker rather than the bargaining power of unions. Therefore, their conclusion might be flawed. In another related study, Rumler and Scharler (2011) conclude that increased coordination of the wage bargaining process (hereinafter referred to as coordination) stabilizes inflation. This implies that increased centralization stabilizes inflation. In their analysis, they control for union bargaining power. However, they leave out government intervention. Also, the variable used as a proxy for bargaining power only indicates the bargaining power of labour unions. It is conceivable that employers with strong bargaining powers might drive down wages without the intervention of the government. One can therefore argue that employers' bargaining power also increases inflation volatility. Thus, proxy variables for bargaining power should include measures of the bargaining power of employers as well.

Given the conflicting theories and evidence on the effect of centralization on inflation volatility, one may wonder whether a theory that unambiguously predicts the effect of centralization on inflation volatility exists. In this study, we attempt to show that under some conditions one can derive an unambiguous effect of centralization and other labour market institutions on inflation volatility. The novelty of our approach lies in the fact that we are able to derive and test a hypothesis concerning other labour market institutional variables as well. The crucial assumption we make in deriving this theory is that the aggregate wage indexation is a simple average over all independent wage indexation outcomes. In this case, we can show that the variance of aggregate wage indexation is decreasing in the number of independent negotiations or bilateral bargaining processes that result in wage indexation.

Testing this hypothesis presents a challenge as the number of negotiations or bilateral bargaining that result in wage indexation are typically unobservable. To deal with this problem, we break down the number of independent negotiations into two conceptual dimensions. These are the number of negotiations and the independence of negotiations. The theoretical prediction proposed in this study unambiguously implies a negative correlation between the number of negotiations and the independence of negotiations on the one hand and inflation volatility on the other hand. It also predicts a positive relationship between bargaining power and inflation volatility.

In order to test the empirical validity of this prediction, we use data of the following selected OECD countries: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, Sweden, the UK and the US. These countries are included in the panel based on the availability of relevant data. While most other related studies propose only one proxy variable for bargaining power and independence of negotiations, we propose two for each of the categories of variables. Centralization (or coordination) is included in the set of variables employed as proxies for independence of negotiations. In particular, the other proxy variable proposed for bargaining power captures bargaining power of both employers and labour unions. The effects of variables depicting deliberate policy actions of government, exogenous shocks, government intervention and bargaining power on inflation volatility were controlled for when testing our hypotheses. We find some evidence for the negative correlation between independence and number of negotiations on the one hand and inflation volatility on the other hand. Our results also indicate a positive correlation between the two proxies for bargaining power and inflation volatility.

The crucial assumption made in deriving our hypotheses is that wage indexation is a random outcome from a bargaining or a negotiation process. While this assumption permits us to set a clear hypothesis concerning the effect of labour market institutional variables on inflation, it might come across as arbitrary. However, it is pointed out by Caju et al. (2008) that prices (or inflation rates) are the most important factors determining elements that enter wage negotiations (or wage bargaining). Also, previous studies have pointed out the time-varying nature of wage indexation (see Holland (1986) and Ascari et al. (2011)). We view these observations as evidence in support of the assumption that aggregate wage indexation is an outcome of negotiation or bilateral bargaining processes.

The rest of this study is organized as follows. Section 5.2 presents a theoretical model from which the hypothesis on the relationship between labour market institutional variables and inflation is derived. Section 5.3 gives a description of the data used to test the hypothesis and discusses the result of the estimation performed. Finally, Section 5.4 concludes.

5.2 Theoretical model

This section provides a summary of the model used in Attey and de Vries (2011). The key result in this model is then formulated in a way so as to clarify the relationship between inflation volatility and labour market institutional variables.

Consider a setting under which there are several simultaneous negotiations regarding wage indexation. Let the wage indexation outcome associated with each negotiation i be x_{it}. Assuming that there are m negotiations conducted independently and that the aggregate wage indexation outcome is a simple average of all independent[2] outcomes, the aggregate wage indexation x_t is

$$x_t = \frac{1}{m} \sum_{i=1}^{m} x_{it}. \tag{5.1}$$

For the purpose of this paper, we assume that each individual wage indexation outcome is an independent and identically distributed (iid) random variable. No further assumptions regarding the distribution of the random wage indexation variable are necessary. Since individual negotiation outcomes are time-varying, it follows that the aggregate wage indexation is also time varying. The variance of the aggregate wage indexation given the number of independent negotiating units m can then be expressed as follows:

$$var(x_t) = \frac{var(x_{it})}{m}. \tag{5.2}$$

It is important to note that this variance is decreasing in the number of independent negotiations or bargaining units that engage in wage indexation bargaining. Thus, whenever m varies, the variance of wage indexation also varies as well. Writing $var(x_t) = \sigma_x^2$, the following expression is implied to hold when the number of independent negotiations or bilateral bargaining

[2] In this work, *independence* should be interpreted as independence conditional on common factors such as macroeconomic volatility that drive negotiations.

m is allowed to vary:

$$\sigma_x^2 = f(m, \xi) \qquad\qquad \frac{\partial f}{\partial m} < 0. \tag{5.3}$$

The variable ξ contains a set of variables specific to the individual unions involved in the bargaining processes. We also assume that it contains other variables that affect the distribution of the individual bargaining or negotiation outcome. For example, a higher bargaining power of one of the negotiating parties might result in a higher variance of the individual negotiation outcome. Furthermore, higher levels of inflation following periods of lower inflation will most likely reinforce the bargaining power of unions when negotiating for the degree of wage indexation thus resulting in a higher variance in aggregate wage indexation.[3]

5.2.1 Random wage indexation and equilibrium inflation

Following Attey and de Vries (2011) we assume that wage indexation is the only source of nominal rigidity in the economy. Let the expectation of private agents at time $t-1$ be represented by the superscript ^{e}. We consider the following indexation rule:

$$w_t = w_t^{*e} + x_t(p_t - p_t^e), \tag{5.4}$$

where w_t, π_t and p_t are log of nominal wages, inflation and log of prices prevailing in the economy. The log of the flexible equilibrium wage rate is indicated by w_t^*. The variable x_t is the aggregate wage indexation given in (5.1). The expression for wage indexation in (5.4) allows for a time-varying degree of wage indexation.

Output Y_t is produced by a fixed coefficient Ricardian technology with labour (N_t) as a sole input $Y_t = Z_t N_t^a$. Here, $a < 1$ reflects diminishing marginal returns to scale and Z represents technological shocks, which are assumed to be iid normal distributed.[4] Labour supply is an

[3]The effect of inflation in this context is to create some level of dependence among bargaining unions. Therefore, we can work with conditional independence in the context of this study.

[4]Allowing for $\log(Z_t)$ to follow an AR process as normally assumed in the literature does not qualitatively change our results in any way.

increasing function of the real wage. The expression for labour supply is

$$n_t^s = \beta_0 + \beta_1(w_t - p_t).$$

It can be shown that the expressions for wage indexation, output and labour supply imply the following expression for the Phillips curve (see Appendix 5.A.1 for a detailed derivation):

$$g_t = \lambda_t(\hat{\pi}_t - \hat{\pi}_t^e) + u_t, \tag{5.5}$$

where $u_t = \log(Z_t)$, and $u_t \sim N(0, \sigma_u^2)$. The time-varying parameter $\lambda_t = (a/(1-a))(1-x_t)$ captures the time-varying response of the output gap (g_t) to changes in inflation (or in this case, the deviation of inflation from its target, $\hat{\pi}_t$). The variable $\hat{\pi}_t$ ($\hat{\pi}_t^e$) denotes the deviation of inflation (expected inflation) from the target inflation rate. This target inflation is assumed constant (i.e. $\hat{\pi}_t = \pi_t - \pi^*$). The Phillips curve in (5.5) above differs from the conventional curves found in other literature in that it allows for time variation in the slope coefficient.

It is assumed that the monetary policy authority uses the interest rate as an instrument in the conduct of optimal policy.[5] The use of the interest rate as an instrument necessitates the introduction of the IS relation. This relation is given below:

$$g_t = -\phi(i_t - \hat{\pi}_t^e - r - \pi^*) + v_t, \tag{5.6}$$

where i_t is the nominal interest rate and r is the natural interest rate. The random variable v_t captures the demand shock which is assumed to be iid distributed as follows $v_t \sim N(0, \sigma_v^2)$.

A crucial assumption made is that monetary policy is conducted under rational expectations. One therefore expects the expectations concerning all variables of all agents within the model to be identical. The objective of the monetary policy authority is to minimize the expected squared deviation of inflation from its target. Let the expectations operator of the policy maker be E_{t-1}.

[5]Assuming the use of money supply as an instrument does not qualitatively change our results.

The objective function of the policy maker is therefore the following:

$$\min_{i} E_{t-1}[\hat{\pi}_t^2]. \tag{5.7}$$

The detailed optimal solution to this monetary problem is contained in Section 5.A.2. The expression for equilibrium inflation under optimal monetary policy in the presence of the random degree of wage indexation scheme considered in (5.4) is as follows:

$$\hat{\pi}_t = \frac{v_t - u_t}{\lambda_t}. \tag{5.8}$$

The expression (5.8) above indicates that equilibrium inflation also depends on the random degree of wage indexation variable. Noting that the variable $\lambda_t = [a/(1-a)](1-x_t)]$, one notices that equilibrium inflation rate explodes when one approaches full indexation $x_t = 1$. Also, (5.8) implies that the variance of equilibrium inflation is influenced by the variance of the random degree of wage indexation. It can be shown that the distribution of the equilibrium inflation resulting from this optimal monetary policy exhibits heavy tail characteristics under some realistic assumptions.[6] It turns out that the variance of the equilibrium inflation as given in (5.8) is

$$\sigma_\pi^2 = (\sigma_v^2 + \sigma_u^2)(\sigma_{1/\lambda_t}^2 + E[1/\lambda_t]^2). \tag{5.9}$$

The equilibrium inflation process in (5.8) depends on the degree of wage indexation. It follows that the variance of the equilibrium inflation process should also depend on the variance of the random wage indexation process. To see this, first note that the random Phillips curve slope parameter λ_t is a function of aggregate wage indexation x_t. Thus, moments of $[1/\lambda_t]$ should be functions of the moments of x_t.

[6]See Attey and de Vries (2011) and Attey and de Vries (2013) for more detailed discussions.

5.2.2 Number of independent negotiations and inflation volatility

For the purpose of this paper, the volatility of inflation is proxied by the conditional variance of inflation. We derive an approximation to the variance of equilibrium inflation in Appendix 5.A.3. The resulting expression is

$$\sigma_\pi^2 \approx \left(\frac{1-a}{a}\right)^2 \left[\frac{1}{(1-\bar{x})^2} + 3\frac{\sigma_x^2}{(1-\bar{x})^4} + \frac{\sigma_x^4}{(1-\bar{x})^6} + ...\right] (\sigma_v^2 + \sigma_u^2). \tag{5.10}$$

That the volatility of equilibrium inflation increases in the variance of demand and supply shocks is a result which is readily obtainable from conventional monetary models. From expression (5.10) above, another variable that influences the volatility of inflation is the variance of the degree of wage indexation. The apparent neglect of this variable in models explaining inflation volatility stems from the assumption of a constant degree of wage indexation made in these models. However, we will show later that the variance of wage indexation plays a significant role in explaining inflation volatility.

It is worth noting that since (σ_π^2) is increasing in (σ_x^2) and (σ_x^2) is decreasing in total number of independent negotiations m (as indicated by Equation (5.2)) , σ_π^2 should be decreasing in m. Differently put, the more the number of independent negotiations, the lower the variance of aggregate (weighted average of) wage indexation and the lower the variance of inflation.

In order to establish the link between inflation volatility and industrial relations variables, it is worthwhile to reiterate that the variance of aggregate wage indexation is dependent on the number of independent negotiations (m) and other variables that affect the individual negotiation outcomes regarding wage indexation (i.e. $\sigma_x^2 = f(m, \xi)$). Thus, the equation for the variance of wage indexation in (5.10) can be summarily recast as follows:

$$\sigma_\pi^2 = f(\sigma_v^2, \sigma_u^2, \sigma_x^2) = f(\sigma_v^2, \sigma_u^2, m, \xi),$$

where $(\partial\sigma_\pi^2/\partial\sigma_v^2) \geq 0$, $(\partial\sigma_\pi^2/\partial\sigma_u^2) \geq 0$ and $(\partial\sigma_\pi^2/\partial m) \leq 0$.[7]

[7] Given that $\sigma_v, \ \sigma_u \ \sigma_\pi \ and \ \ \in \mathcal{R}^+$, these conditions can be derived: $(\partial\sigma_\pi/\partial\sigma_v) \geq 0$, $(\partial\sigma_\pi/\partial\sigma_u) \geq 0$ and $(\partial\sigma_\pi/\partial m) \leq 0$.

The last inequality stems from the observation that the volatility of inflation is increasing in the variance of the degree of wage indexation which is decreasing in the number of independent wage indexation negotiations conducted. An intuitive reason as to why an increase in the number of independent negotiations (m) should lower the variance or volatility of equilibrium inflation is as follows. The wage indexation outcome is more volatile if it is a bargaining outcome of only one negotiation. However, if it is the average over several independently conducted negotiations, the variance of wage indexation decreases due to the law of large numbers.

5.2.3 Empirical specification

In order to make our model easily applicable to the panel data, we assume that ξ also includes country specific variables that explain the variations in the variance of inflation. Assuming that a linear equation approximates the function for variance of inflation, one derives the following regression when there exist time variations in the explanatory variables:

$$\sigma_{\pi it} = \delta_0 + \delta_1 m_{it} + \sum_{j=1}^{n} \alpha_j bpow_{j,it} + \sum_{j=1}^{k} \gamma_j cont_{j,it} + \epsilon_{it}, \tag{5.11}$$

where the $bpow_j$'s are variables serving as proxies for the bargaining power of negotiating units and $cont_j$'s are other control variables. The variable ϵ_t may be a sum of country specific effects which affect inflation volatility and approximation errors.

While we do agree that the constant term δ_0 is arbitrarily imposed, it can be argued that this parameter captures the time and cross-section invariant aspects of the variance of demand and supply shocks. Since the countries contained in the panel are Western European OECD countries, it is plausible to assume the existence of constant mean time invariant volatility of these shocks. The letter i indexes the countries included in the panel. The parameter α_j is the coefficient of the j^{th} bargaining power variable while γ_j captures the effect of the j^{th} control variable. If the number of independent negotiations m_t is readily observed, the regression equation in

(5.11) will give rise to the readily testable hypothesis as follows:

$$\alpha_j > 0 \ \ \forall j \tag{5.12a}$$

$$\delta_1 < 0. \tag{5.12b}$$

The major difficulty one faces in testing the hypothesis in (5.12) above is the fact that the number of independent negotiations is typically unobservable. Thus, one cannot easily measure the variable m_t. To deal with this problem, we 'break down' the number of independent negotiating pairs (m_t) into *number* of negotiations and *independence* of negotiations.

Here, we attempt to distinguish between the concepts *number* of negotiations and *independence* of negotiations. Consider the following two types of labour regulations: one guaranteeing freedom of negotiations to unions and the other mandating all unions to abide by the terms of the settlement from the first successful negotiation. It is likely that the first regulation increases the number of negotiations, *ceteris paribus*. However, it does not give any indication of the dependence between the negotiation outcomes. The second regulation will most likely decrease the independence between the negotiation outcomes, *ceteris paribus*. A conclusion regarding the number of negotiations will be difficult to reach on the basis of the second regulation alone. The two concepts considered together may roughly indicate changes in the number of independent negotiations.

The total number of independent negotiations is increasing in total number of negotiations and independence of negotiations. Let the variables ind_{jt} and num_{jt} be the independence of negotiations and number of negotiations respectively for country j at time t. Incorporating the just mentioned variables into the regression model, the main specification is:

$$\sigma_{\pi it} = \alpha_0 + \sum_{j=1}^{n} \alpha_j bpow_{j,it} + \sum_{j=1}^{p} \theta ind_{j,it} + \sum_{j=1}^{q} \beta_j num_{j,it} + \sum_{j=1}^{r} \gamma_j cont_{j,it} + \varepsilon_{it}, \tag{5.13}$$

where ε_t is a random variable capturing the effects of possible measurement and approximation errors, and is iid distributed as follows: $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. It is assumed that an increase in one of the variables indicating the two dimensions in the equation (5.13) holding the other constant increases m_t. Thus, one would expect the following conditions to hold:

$$\alpha_j > 0 \quad \forall j \tag{5.14a}$$

$$\theta_j < 0 \quad \forall j \tag{5.14b}$$

$$\beta_j < 0 \quad \forall j. \tag{5.14c}$$

The hypothesis in equations (5.14) will be tested by using a panel data estimation methodology. The next section gives details on the estimation method and the type of data used for the analysis of this work, after which the results obtained are discussed.

5.3 Data and empirical analysis

This section conducts empirical tests on the alternative versions of the hypothesis in (5.14) using a panel data methodology. The panel consists of the following 15 OECD countries: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, Sweden, the UK and the US. The panel data spans the period from 1960 to 2011. The countries were chosen on the basis of availability of relevant data for the purposes of this study. For instance, data for post-Soviet countries begin from 1990. Also, variables for some countries do not vary over the 50 year period which the data spans. Data regarding the countries for which the above situations are applicable are omitted from the panel.

The analysis in this study uses three datasets namely: inflation, productivity and labour market institutional data. The monthly inflation data is obtained from the OECD's statistical database. The Purchasing Power Parity Converted GDP Laspeyres per hour worked by employees is used as a proxy for productivity for all countries but Germany. The proxy used in Germany's case is the manufacturing output per hour. The two measures of productivity are taken from the Federal Bank of Saint Louis Database. Finally, the Amsterdam Institute for Advanced Labour Studies (AIAS) compiles data on industrial relations spanning 1960 to 2011 for OECD member countries. The data on industrial relation variables are obtained from the AIAS' ICTWSS database.

5.3.1 Data

The average monthly inflation volatility is the dependent variable. A substantial number of explanatory variables are labour market institutional variables. These variables are grouped into three categories. The first category includes proxy variables indicating bargaining power of employers and labour unions involved in wage (indexation) negotiations. The second category includes proxy variables that indicate the independence of negotiations regarding wage or wage indexation. The final category comprises proxy variables that indicate the number of wage negotiations. The three categories of the labour market institutional variables can at best give only a rough idea about the three observations that they are supposed to indicate. This is due to the fact that direct information about the number of negotiations concerning wage indexation in each year is unobservable. We provide further details on all variables used in the analysis.

Inflation volatility

We employ two alternative measures of inflation volatility. The first measure is the annual average of monthly volatility values obtained from GARCH(1,1) estimates. The mean equations are modeled as AR(1) processes. The second measure uses standard deviations of monthly inflation figures over a calendar year. Rother (2004) in a closely related study uses volatility figures derived from GARCH(1,1) estimates on annual inflation. This measure might not adequately capture the effects that labour institutional variables have on inflation volatility. In other words, a higher frequency measure of inflation volatility might more readily pick up extreme volatility caused by bargaining power, for instance. The lag of (log) inflation volatility will be employed as an explanatory variable in the dynamic panel versions.

Bargaining power

Two variables are included in the analysis to serve as proxies for bargaining power category. The first variable, union density (ud) is defined as the net union membership as a proportion of wage and salary earners in employment. It ranges from 0 to 100%. The second variable, $sect$ measures the strength of sectoral institutions representing employment relations. It takes the value of 0 when there are weak or no institutions, 1 when there is a strong institution on one side and 2 when there are strong institutions on both sides. In order to give an interpretation of this variable which is more consistent with increasing bargaining power, we reorder the values as follows: 0 when there are weak or no institutions, 1 when there are strong institutions on both sides and 2 when there are strong institutions on only one side. We find this ordering more plausible since weak institutions on both sides are likely to maintain the status quo while a dominant institution on only one side is more likely to drive bargaining outcome into the extremes. As already mentioned, we expect bargaining power to have a positive effect on the variance of wage indexation.

Number of negotiations

We identify two proxy variables for the relative number of negotiations. The first of these variables, $bart$, indicates whether there are legal restrictions placed on wage bargaining. Its values range from 1 (when there are no constraints on bargaining) to 5 (when there are severe limitations on additional bargaining on wages). A value of 0 denotes the absence or near absence of negotiations of any form. In order to render the interpretation of this variable more consistent with increasing number of negotiations, we reverse-order the values when they are between 1 and 5, while 0 still denotes the absence of any form of sectoral bargaining. The second variable $wage$ indicates the presence of a social pact concerning wages in a particular year. It takes the value of 1 when there is a social pact and 0 when there is none.[8]

[8]There are other variables such as number of independent unions and total number of unions that could be interpreted as falling under this category. We abstract from the use of these variables due the substantial amount of missing observations.

Independence of negotiations

The two proxy variables used for this category are coordination of wage setting (crd) and autonomous negotiation of wages (auw). The values of crd range from 1 when there is fragmented wage bargaining at plant level to 5 when there is a centralized nationwide bargaining. The values are reordered (in reverse order) to permit an interpretation more consistent with increasing independence of negotiations. The variable auw is binary taking on the value 1 when there is autonomous wage negotiations and 0 when there is none.

Control variables

It is plausible that labour market institutional variables may have indirect effects on inflation volatility through inflation and variance of productivity shocks. In order to further isolate the direct effects, we introduce inflation (π_t) and variance of productivity shocks ($\sigma_{u,t}$) as control variables. The latter of the aforementioned variables is estimated by use of GARCH(1,1) on the productivity data. Following both theoretical and empirical findings in the literature on inflation volatility, we hypothesize that inflation has a positive effect on inflation volatility. We also hypothesize that the volatility of productivity shocks also has a positive effect on volatility of inflation.

The variable indicating government intervention in wage bargaining ($gvint$) is also included in the regression model as a control variable. The assigned values of this variable reflect an increasing severity of intervention. The values range from 1 when there is no intervention to 5 when the government imposes wage settlements in the private sector. This variable can be perceived as being negatively correlated with independence of negotiations, in which case it is expected to have a positive effect on inflation volatility. However, an alternative hypothesis could be derived from the observation that governments are more likely to intervene to stabilize the economy. Thus one would expect the volatility of inflation to decrease with increasing severity of government intervention.

Finally, we introduce a set of dummies to control for the effects of crises (d_{cr}) and Eurozone accession of the member countries included in the panel (d_{eu}) on the volatility of inflation. The following years are considered as crises years: 1973, 1974, 2008 and 2009. The former two years are included to reflect the first oil crises while the latter two years are included to reflect the global financial crises. The second oil crisis years are excluded due to the fact that their impact was to a large extent limited to the US economy. We further include an interaction of the dummies with the volatility of productivity shocks. This is because it is likely that the correlation between volatility of inflation and variance of productivity shocks might be different under the events indicated by the dummies. We expect a positive relationship between the volatility of inflation and the crises dummy.

5.3.2 Empirical model

Various specifications are used to test the empirical validity of the hypothesis in (5.12). First, a pooled OLS estimation is carried out on the panel data. Subsequently, panel data estimations are conducted. For the purposes of the estimations carried out in the immediate part of this study, inflation volatility ($\sigma_{\pi t}$) is defined as the standard deviation of monthly inflation figures. The empirical model is summarized as follows:

$$y_{it} = \alpha_0 + \varphi' \mathbf{x}_{it} + \gamma' \mathbf{c}_{it} + \nu_i + \varepsilon_{it}, \tag{5.15a}$$

where $y = \ln(\sigma_\pi)$, **x**=[$\Delta ud\ sect\ bart\ wage\ crd\ auw$]′, and φ is a 6×1 vector containing the respective coefficients of the variables contained in **x**. The vector **c** contains the set of control variables previously mentioned whose coefficients are the respective elements of the vector γ. The variable ε_{it} is the error term assumed to be iid distributed as follows: $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$ and the term ν_i is the country specific effect. The Δ is the first difference operator. We use the first difference of ud_t in our regressions due to the persistent fall in union density observed in most of the countries in the panel.[9] Two versions of (5.15a) are estimated in this study. The first version estimated is basically a pooled OLS regression which assumes no country specific

[9] See Western (1995) for a detailed discussion of this issue.

effects (i.e. $\nu_i = 0 \ \ \forall i$). The second version estimates the country specific effects in addition to other coefficients.

There are two problems one could potentially encounter in estimating the specification given in (5.15a) above. First, it can be argued that the inflation level and the variance of productivity are potentially endogenous. Second, given the low level of time variation in the institutional variables, it can be argued that the correlation between the said variables and inflation volatility might be spurious if volatility is autoregressive. We therefore introduce a specification which includes the lag of volatility of inflation in the list of explanatory variables to control for persistence. The resulting regression equation estimated is:

$$y_{it} = \alpha_0 + \rho y_{it-1} + \varphi' \mathbf{x}_{it} + \gamma' \mathbf{c}_{it} + \nu_i + \varepsilon_{it}.$$

However, introducing the AR term results in a dynamic panel bias as y_{it} is endogenous to the country specific effect ν_i. We make use of an additional regression equation employing the Arellano-Bond GMM (difference) estimator to deal with both the dynamic panel bias and the potential endogeneity of inflation and productivity volatility. The differenced version of the last previous equation is:

$$\Delta y_{it} = \Delta \rho y_{it-1} + \varphi' \Delta \mathbf{x}_{it} + \gamma' \Delta \mathbf{c}_{it} + \Delta \varepsilon_{it}. \tag{5.15b}$$

It follows that the error term in the above equation, $\Delta \varepsilon_{it}$, is an AR(1) process. Table 5.1 gives a summary of the explanatory variables and the expected signs of their coefficients.

5.3.3 Results

Table 5.2 gives the results obtained under the pooled OLS estimation (columns (1) and (2)), the panel with fixed effects estimation (columns (2) and (4)) and Arellano-Bond sytem GMM estimation (columns (5) and (6)). We include pooled OLS estimations in our results in order to get a preliminary idea about the effects of the labour market institutional variables on the volatility of inflation.

Results shown in Table 5.2 indicate that five of the explanatory variables have significant

Table 5.1: Categories and expected signs of coefficients

Variable	Category	Sign
ud	bargaining power	$+$
sect	bargaining power	$+$
bart	number of bilateral bargaining/ negotiations	$-$
wage	number of bilateral bargaining/negotiations	$-$
auw	independence of bilateral bargaining/negotiations	$-$
crd	independence of bilateral bargaining/negotiations	$-$
gvint	control variable	$+/-$
$\ln(\sigma_{\pi,t-1})$	control variable	$+$
$\ln(\sigma_u)$	control variable	$+$
$\ln(\pi)$	control variable	$+$
$d_{cr}\ln(\sigma_u)$	control variable	$-$
$d_{eu}\ln(\sigma_u)$	control variable	$-$
d_{cr}	control variable	$+$
d_{eu}	control variable	$+/-$

[1] Expected signs of the various explanatory variables. d_{cr} denotes dummy for crisis periods while d_{eu} denotes the dummy for Eurozone accession.

effects on the volatility of inflation under all the estimations presented. The variables are, ud, $sect$, $\ln(\sigma_\pi)$, $\ln(\sigma_u)$ and d_{cr}. The results suggest that indeed, inflation and variance of productivity shocks do consistently have relatively high significant effects on inflation volatility in the direction as hypothesized. These results still hold even after correcting for possible endogeneity emanating from reverse causality. Also, the table indicates a significant autocorrelation in inflation volatility. As hypothesized, inflation volatility increased during the crisis periods 1973, 1974, 2008 and 2009. Our results also suggest that Eurozone member countries experience higher inflation volatility when other variables are corrected for. At first, this may appear counter-intuitive as one would expect a better stability in inflation levels, as is the goal of the Maastricht treaty. However, the fact that maintaining stability in annual inflation might imply relatively higher volatility in monthly inflation can explain this result.

It can be seen from the results that the correlation between the variance of productivity and the volatility of inflation significantly reduced during the periods of crises and Eurozone accession. An explanation for this result stems from the fact that the structural dependence between

Table 5.2: $\ln(\sigma_{\pi t})$: standard deviation of monthly inflation

	Pooled OLS		*Panel (Fixed Effects)*		*Arellano-Bond GMM*	
	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\sigma_{\pi,t-1})$					0.219***	0.233***
					(0.037)	(0.036)
Δud_t	0.0367***	0.034***	0.038***	0.036***	0.029**	0.025**
	(0.014)	(0.014)	(0.012)	(0.054)	(0.011)	(0.011)
$sect_t$	-0.114***	-112***	0.377***	0.382***	0.156*	0.168*
	(0.031)	(0.031)	(0.053)	(0.054)	(0.094)	(0.093)
$bart_t$	-0.018	-0.011	-0.08***	-0.078***	-0.056**	-0.052***
	(0.013)	(0.013)	(0.018)	(0.018)	(0.024)	(0.024)
$wage_t$	0.021	0.003	-0.048	-0.059	-0.052	-0.057
	(0.065)	(0.065)	(0.057)	(0.057)	(0.05)	(0.05)
crd_t	-0.041***	-0.041***	0.022	0.023	-0.032	-0.03
	(0.014)	(0.014)	(0.022)	(0.022)	(0.022)	(0.022)
auw_t	0.208***	-0.19***	-0.06	-0.055	-0.25***	-0.259***
	(0.071)	(0.071)	(0.074)	(0.073)	(0.082)	(0.082)
$gvint_t$	-0.019	-0.012	-0.017	-0.019	-0.004	-0.009
	(0.016)	(0.016)	(0.019)	(0.019)	(0.021)	(0.021)
$\ln(\pi_t)$	0.246***	0.226***	0.209***	0.211***	0.19***	0.199***
	(0.02)	(0.022)	(0.018)	(0.019)	(0.019)	(0.019)
$\ln(\sigma_{u,t})$	0.179***	0.2***	0.232***	0.254***	0.177***	0.226***
	(0.041)	(0.046)	(0.042)	(0.046)	(0.044)	(0.049)
$d_{eu} \cdot \ln(\sigma_{u,t})$		0.003		-0.046		-0.172**
		(0.102)		(0.089)		(0.084)
$d_{cr} \cdot \ln(\sigma_{u,t})$		-0.189		-0.251		-0.189*
		(0.148)		(0.125)		(0.106)
d_{eu}		-0.158*		0.025		0.258***
		(0.082)		(0.074)		(0.072)
d_{cr}		0.214**		0.249***		0.194***
		(0.108)		(0.092)		(0.078)
$const$	-0.881***	-0.9***	-1.425***	-1.455***		
	(0.087)	0.087)	(0.101)	0.104		

[1] Arellano Bond GMM estimates for dynamic panels in columns (5) and (6) . Sample period 1960-2011

[2] $ln(\pi)$, $\ln(\sigma_u)$, $d_{eu} \cdot \ln(\sigma_u)$ and $d_{cr} \cdot \ln(\sigma_{u,t})$ are treated as endogenous variables. Two to five periods lagged values of the differences of the endogenous variables were used as instruments. All labour market institutional variables as well as dummies or Eurozone accession and crises are treated as exogenous.

[3] * $p > 0.10$, **$p > 0.05$ and ***> 0.01

[4] Figures in parenthesis indicate the standard errors of the coefficients.

productivity and inflation might have been attenuated during crisis periods. Also, accession to the Eurozone requires member countries to place relatively more emphasis on inflation stabilization. This places limitations on the use of inflationary policy to stabilize output hence the decreased correlation between productivity volatility and inflation volatility.

Earlier in this paper, we hypothesized a positive effect of bargaining power of parties involved in bilateral negotiations or bargaining on inflation volatility. The results obtained in Table 5.2 support this hypothesis. Of the two proxy variables used to measure bargaining power, the coefficient of $sect$ has the higher magnitude. This is not a surprising result since the $sect$ gives a measure of bargaining power of both the labour and employers unions. In contrast, union density ud measures the labour unions' bargaining power. One would therefore expect variations in the relatively general measure $sect$, rather than the relatively specific measure (ud), to better explain variations in inflation volatility.

Our results seem to stand in contrast to that of Barbier-Gauchard et al. (2014) who argue that strengthening unions improves the efficacy of the monetary authority in stabilizing inflation. However, that study assumes the presence of government intervention in addition to a high degree of centralization and coordination among unions. It might be difficult to observe the independent effects of bargaining power without controlling for the aforementioned variables. We are therefore of the opinion that bargaining power of both employers and labour unions (as proxied by ud and $sect$) considered alone destabilizes inflation.

Concerning the number of independent negotiations, our results indicate a negative relationship between measures of independence and inflation volatility. For both measures ($bart$ and $wage$), the pooled OLS estimates do not yield significant estimates. However, the sign of $bart$ is as hypothesized. The effect of articulation of sectoral bargaining becomes significant when one employs panel regression or the Arellano-Bond GMM estimation. It can therefore be concluded that there is partial evidence in support of the hypothesis that inflation volatility is decreasing in the number of bilateral negotiations when one controls for other variables.

The estimates for the coefficients of autonomous wage negotiation (auw) and coordination of wage setting (crd) do support our hypothesis on the independence of negotiations, albeit weakly. After correcting for the effects of the lag effects of volatility, the estimates for the effects of auw become significant. The magnitude of these estimates are among the highest, indicating the relative importance of independence of negotiations for inflation volatility. Previous estimations performed in Daniels et al. (2006) seem to confirm our results regarding independence of negotiations. The authors show that inflation initially increases in centralization of wage bargaining at low levels of centralization, then decrease as centralization increases. This result implies that increasing centralization of negotiations (decreasing independence) results in an increase in inflation volatility.

Rumler and Scharler (2011) show that highly coordinated wage bargaining systems have a dampening impact on inflation volatility. Differently put, a higher level of independence in wage bargaining or negotiation systems do have a positive impact on inflation volatility. This apparent contradiction can be resolved when one notes that it is highly possible that highly coordinated bargaining/negotiation systems might be subject to more government intervention. Any policy maker will not be oblivious to the potentially destabilizing consequences of a coordinated bargaining system and may be forced to intervene. Thus, any analysis on the impact of coordination on inflation volatility should control for government intervention. This is done in this study. We are therefore of the opinion that the destabilizing effects of interdependence in bargaining/negotiation systems shown in this study better portray empirical reality.

In this section, we test the hypothesis on labour market institutional variables given in (5.14). The hypothesis predicts a positive relationship between bargaining power of unions and inflation volatility. It also predicts a negative relationship between number of negotiations/bilateral bargaining and independence of negotiations/bilateral bargaining on the one hand and inflation volatility on the other hand. Inflation volatility is defined as the annual log standard deviation of monthly inflation. The results generally confirm the direction of correlation hypothesized. Among the three categories of labour market variables considered, bargaining power has the most unequivocal impact in terms of significance and magnitude. There is some evidence to support the negative impact of independence and number of negotiations/bilateral bargaining.

In what follows in this section, we investigate how robust our findings are to another measure of inflation volatility. Under this measure, inflation volatility is defined as the annual average of log monthly volatility derived from GARCH estimations.

5.3.4 Robustness

Table 5.3 shows results of estimations performed using annual average of monthly GARCH volatility as the dependent variable. The results are comparable to those of the estimations using annual standard deviation of monthly inflation as a measure of inflation volatility in many regards. Once again, the positive relationship between bargaining power of negotiating parties and inflation volatility is significant under the panel with fixed effects estimations and the Arellano-Bond GMM estimations. As before, the lag of log inflation volatility, inflation levels and variance of productivity do significantly explain variations in inflation volatility. The results also indicate a decreased correlation between volatility of productivity and inflation volatility during crises and the period of being an Eurozone member. Finally, there is some evidence in support of the hypothesis that the independence of negotiations or bilateral bargaining does negatively impact inflation volatility.

However, a few differences exist between the two estimations. First, the lag of inflation volatility plays a bigger role in explaining inflation volatility when the dependent variable is derived from GARCH estimations. The fact that GARCH variances are modeled to be persistent does explain this result. Also, none of the measures used for number of negotiations are statistically significant in explaining variations in inflation volatility under the Arellano-Bond GMM estimations. The variable $bart$ does retain some significant explanatory power as hypothesized under the panel-fixed-effects estimation and one of the pooled OLS estimations. Also, crd becomes the significant variable explaining inflation volatility (under the GMM estimation) when one switches to using GARCH volatility as the measure of inflation volatility. These differences do not change our results materially. From Table 5.3, we can conclude that there is some evidence in support of the hypothesis that inflation volatility varies positively with bargaining power and negatively with number and independence of negotiations.

5.4 Conclusion

In this paper we investigate the role of labour market institutional variables in explaining the volatility of inflation. We group the labour market institutional variables into three categories: bargaining power, number of negotiations and independence of negotiations. Results by Attey and de Vries (2011) imply a positive relationship between the variance of wage indexation and the variance of inflation. By exploiting the relationship between labour market institutional variation and the variance of wage indexation, we derive a readily testable hypothesis on the link between volatility of inflation and labour market institutional factors. The hypothesis was tested on the data of 15 OECD countries. The methodologies used include pooled OLS estimations, panel estimation with fixed effects, and Arellano-Bond GMM estimation.

Similar to the findings in Rumler and Scharler (2011) we consistently find evidence to support the positive correlation between bargaining power and inflation volatility as measured by annual standard deviation of monthly inflation. Union density (ud) and strength of sectoral institutions ($sect$) are used as proxies for bargaining power. The use of these proxies permits us to measure not only the labour unions' bargaining power (ud) but also that of the employers unions. The magnitude of the coefficients of $sect$, a proxy for bargaining power is always higher than that of the other proxy ud under the panel fixed effects and Arrelano-Bond GMM estimations. This result still holds in the estimations performed to check the robustness of our results. This suggests that the variable $sect$ contains more information on bargaining power of both labour and employers unions than ud.

Contrary to Rumler and Scharler (2011) who point to the stabilizing effects of a higher degree of coordination, our results show that increased coordination and more generally increased interdependence of negotiations or bargaining increase inflation volatility. Our results are robust to alternative definitions of inflation volatility. We argue that the apparent contradiction stems from the fact that the just cited work does not correct for government intervention which more likely comes with coordination. It is also found in this study that the variable $bart$, which is a proxy for the number of negotiations has significant power in explaining variations in inflation volatility as hypothesized. This finding is robust under the panel with fixed effect estimation when one uses GARCH-derived volatility estimates as the dependent variable.

Finally, our results also suggest the importance of variables such as lag of inflation volatility, inflation level, variance of productivity shocks, Eurozone accession and crises in explaining variations in inflation volatility. The lag of inflation volatility has a bigger effect when inflation volatility is derived from GARCH estimations than when it is derived from standard deviation calculations. In particular, we find that the correlation between the variance of productivity and inflation volatility decreases during the crisis periods. It also decreases when a country becomes and remains a member of the Eurozone. As one would expect, inflation is more volatile during crisis periods. A more surprising result is the fact that being a member of the Eurozone leads to more inflation volatility. However, the fact that stabilizing annual inflation might be at the expense of more volatile monthly inflation can explain this result.

5.A The Phillips curve and optimal monetary policy

5.A.1 Deriving the Phillips curve

Given the output technology described in the main text, the labour demand can be derived from the otpimizing behaviour of the representative firm. Under this behaviour, the first order conditions dictate that wages be equal to the marginal product of labour, as indicated in the equation below:

$$\frac{W_t}{P_t} = aZ_t N_t^{a-1}. \tag{5.16}$$

Let the smaller cases represent the natural $\log$ values of the upper cases. The expression for labour demand is derived by taking the log of 5.16 and expressing the resulting equation in terms of $n_t = \log N_t$. Let $\delta_0 = (\ln a) / (1-a)$ and $\delta_1 = 1/(1-a)$. The expression below gives the labour demand:

$$n_t^d = \delta_0 - \delta_1(w_t - p_t) + \delta_1 z_t. \tag{5.17}$$

The expression for labour supply can be derived from a representative household's optimizing behaviour. For the purposes of this study, we make use of the following ad hoc labour supply relation:

$$n_t^s = \beta_0 + \beta_1(w_t - p_t). \tag{5.18}$$

The production function was already given in the main part of this study as follows: $Y_t = Z_t N_t^a$. Taking the log of this function permits us to derive an expression in terms of log variables as follows:

$$y_t = an_t + z_t. \tag{5.19}$$

By equating (5.17) to (5.18), one derives the following expressions for equilibrium nominal wage rate (w_t^*) and equilibrium labour (n_t^*). These expressions are as follows:

$$w_t^* = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t + \frac{\delta_1}{\delta_1 + \beta_1} z_t \tag{5.20}$$

$$n_t^* = \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \frac{\beta_1 \delta_1}{\delta_1 + \beta_1} z_t. \tag{5.21}$$

The expression for equilibrium output is then derived by substituting (5.21) into (5.19). We give the equation for equilibrium output below:

$$y_t^* = a \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \left(\frac{a \beta_1 \delta_1}{\delta_1 + \beta_1} + 1 \right) z_t. \tag{5.22}$$

Assume z_t is an iid process distributed as follows $z_t \sim N(0, \sigma_z^2)$. Taking expectations of equation (5.20) derives the expected prevailing equilibrium wage rate given below:

$$E_{t-1} w_t^* = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t^e. \tag{5.23}$$

To proceed with the derivation of the Phillips curve (aggregate supply) relation, we need to make an assumption about the source of nominal rigidity. We assume that wage indexation indicates the extent of nominal rigidity in the economy. The expression for wage indexation has already been given in equation (5.4) as follows:

$$w_t = E_{t-1} w_t^* + x_t (p_t - p_t^e), \tag{5.24}$$

where x_t is the aggregate wage indexation. It is defined as an average of all individual indexation outcomes. Making the substitution of expected equilibrium wages into the wage indexation expression given in (5.24) and subtracting p_t from both sides of the equation gives the following expression for real wages:

$$w_t - p_t = (x_t - 1)(p_t - p_t^e) + \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1}. \tag{5.25}$$

The presence of nominal rigidities (as governed by wage indexation rules) prevents the labour market from clearing. The level of employment and output is therefore determined by labour demand. Noting this, one derives labour demand in the presence of wage indexation by substituting equation(5.25) into (5.17) to obtain the following:

$$n_t^d = \frac{\beta_1\delta_0 - \beta_0\delta_1}{\delta_1 + \beta_1} - \delta_1(x_t - 1)(p_t - p_t^e) + \delta_1 z_t. \tag{5.26}$$

Recalling the expression $y_t = an_t + z_t$ permits one to derive the output prevailing in the presence of wage indexation. The step-by-step derivation is given below:

$$y_t = a\frac{\beta_1\delta_0 + \beta_0\delta_1}{\delta_1 + \beta_1} - a\delta_1(x_t - 1)(p_t - p_t^e) + a\delta_1 z_t + z_t$$

$$y_t = a\frac{\beta_1\delta_0 + \beta_0\delta_1}{\delta_1 + \beta_1} + \left(\frac{a\beta_1\delta_1}{\delta_1 + \beta_1} + 1\right) z_t - \left(\frac{a\beta_1\delta_1}{\delta_1 + \beta_1} + 1\right) z_t + (a\delta_1 + 1)z_t - a\delta_1(x_t - 1)(p_t - p_t^e).$$

Recognizing that the sum of the first two addends is simply the flexible price (or potential) output, we can simply rewrite the final expression as follows:

$$y_t = y_t^* + a\delta_1(1 - x_t)(p_t - p_t^e) + \frac{a\delta_1^2}{\delta_1 + \beta_1} z_t. \tag{5.27}$$

To proceed with the derivation of the Phillips curve, we make use of the following definitions for the output gap (g_t), Phillips slope parameter (λ_t) and shock term (u_t). In particular, let

$$g_t = y_t - y_t^*$$

$$\lambda_t = a\delta_1(1 - x_t)$$

$$u_t = \frac{a\delta_1^2}{\delta_1 + \beta_1} z_t.$$

It is worth noting that the difference between actual log price level (p_t) and a lag period's expectation of the log price level (p_t^e) can be expressed in terms of inflation levels. Let the target inflation level of the monetary authority be (π^*).

$$\begin{aligned} p_t - p_t^e &= (p_t - p_{t-1}) - (p_t^e - p_{t-1}) \\ &= \pi_t - \pi_t^e \\ &= (\pi_t - \pi^*) - (\pi_t^e - \pi^*). \end{aligned}$$

Finally, define $\hat{\pi} = \pi_t - \pi^*$. After making relevant substitutions into the expression (5.27), we obtain the following expression for the Phillips curve.

$$g_t = \lambda_t(\hat{\pi}_t - \hat{\pi}_t^e) + u_t. \tag{5.28}$$

5.A.2 Optimal monetary policy

Consider a monetary authority whose policy goal is to stabilize inflation only at a specified rate (π^*) using the interest rate as an instrument. The optimizing problem faced by the monetary authority is as follows:

$$\begin{aligned} \min_{i_t} E_{t-1}(\pi_t - \pi^*)^2 &= E_{t-1}(\pi_t - E_{t-1}\pi_t)^2 + (E_{t-1}\pi_t - \pi^*)^2 \\ &= \sigma_\pi^2 + E_{t-1}(\pi_t - \pi^*)^2 \\ &= \sigma_\pi^2 + [E_{t-1}\hat{\pi}_t]^2. \end{aligned}$$

The preceding optimizing problem requires the monetary authority to set interest rates at a level that corresponds to expected inflation ($E_{t-1}\pi_t$ or π_t^e)[10] being maintained at a level equal to the

[10]Investigating the conduct of monetary policy under rational expectations implies that the monetary authority's expectations coincide with those of private agents.

target inflation (π^*). Consequently, the following expression is expected to hold in all periods.

$$E_{t-1}\hat{\pi}_t = 0. \tag{5.29}$$

To investigate the level at which interest rate should be set, we recall the aggregate demand function. The expression is given as follows:

$$g_t = -\phi(i_t - \hat{\pi}_t^e - r - \pi^*) + v_t.$$

Equating this expression to that of the Phillips curve in (5.28) results in the following relationship between the interest rate and equilibrium inflation:

$$(\hat{\pi}_t - \hat{\pi}_t^e) = \frac{v_t - u_t}{\lambda_t} - \frac{\phi}{\lambda_t}(i_t - \hat{\pi}_t^e - r - \pi^*). \tag{5.30}$$

The interest rate i_t is an instrument set by the monetary policy authority and thus always equal to its expectation. Noting this and imposing the condition set by (5.29), one derives an interest rate rule by taking expectation of the equation (5.30) above. The resulting interest rate rule is as follows:

$$i_t = r + \pi^*. \tag{5.31}$$

Finally substituting the first order condition and the interest rate rule in (5.31) into (5.30) derives the equilibrium inflation rate under optimal monetary policy as follows:

$$\hat{\pi}_t = \frac{v_t - u_t}{\lambda_t}. \tag{5.32}$$

5.A.3 Inflation variance under optimal monetary policy

It follows from (5.32) that the variance of equilibrium under optimal monetary policy is as follows:

$$\sigma_\pi^2 = (\sigma_v^2 + \sigma_u^2)(\sigma_{1/\lambda_t}^2 + E[1/\lambda_t]^2). \tag{5.33}$$

Recalling that $\lambda_t = a\delta_1(1 - x_t)$, we can conclude that the variance of inflation (σ_π^2) should be a function of the variance of wage indexation (σ_x^2). However, finding a closed form expression of σ_π^2 in terms of the variance of wage indexation σ_x^2 is difficult. We therefore resort to the use of Taylor approximations of the function $[1/\lambda_t]$ around the mean wage indexation $\bar{x}$. The step by step derivation of the approximation is as follows:

$$\begin{aligned}\frac{1}{\lambda_t} =& \frac{1}{a\delta_1}\left(\frac{1}{1-x_t}\right)\\ \approx& \frac{1}{a\delta_1}\left(\frac{1}{1-\bar{x}} + \frac{1}{(1-\bar{x})^2}(x_t - \bar{x}) + \frac{1}{(1-\bar{x})^3}(x_t - \bar{x})^2 + ...\right).\end{aligned}$$

By taking expectations of the last expression, one derives an approximation to the mean of $(1/\lambda_t)$. This is given in the equation that follows.

$$E\left[\frac{1}{\lambda_t}\right] \approx \frac{1}{a\delta_1}\left(\frac{1}{1-\bar{x}} + \frac{1}{(1-\bar{x})^3}\sigma_x^2 + ...\right). \tag{5.34}$$

If one ignores the terms above the first order in the approximation for $1/\lambda_t$ deriving the variance is a matter of straight-forward computations. The equation below gives the variance.

$$\sigma_{1/\lambda_t}^2 \approx \frac{1}{a^2\delta_1^2}\left(\frac{\sigma_x^2}{(1-\bar{x})^4}\right). \tag{5.35}$$

Substituting (5.34) and (5.35) into the expression for the variance of inflation as given in (5.33) gives the following:

$$\sigma_\pi^2 = \frac{1}{a^2\delta_1^2}\left[\frac{1}{(1-\bar{x})^2} + 3\frac{\sigma_x^2}{(1-\bar{x})^4} + \frac{\sigma_x^4}{(1-\bar{x})^6}\right](\sigma_v^2 + \sigma_u^2). \tag{5.36}$$

5.B Tables

Table 5.3: $\ln(\sigma_{\pi t})$: annual average of monthly GARCH volatility

	Pooled OLS		*Panel (Fixed Effects)*		*Arellano-Bond GMM*	
	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\sigma_{\pi,t-1})$					0.807***	0.813***
					(0.017)	(0.017)
Δud_t	0.0368***	0.036***	0.033***	0.033***	0.008**	0.007**
	(0.011)	(0.011)	(0.008)	(0.008)	(0.003)	(0.003)
$sect_t$	-0.057**	-0.057**	0.411***	0.398***	0.083***	0.071**
	(0.025)	(0.024)	(0.038)	(0.039)	(0.026)	(0.026)
$bart_t$	-0.018*	-0.01	-0.085***	-0.078***	-0.005	-0.003
	(0.013)	(0.01)	(0.012)	(0.013)	(0.006)	(0.006)
$wage_t$	0.038	0.025	-0.038	-0.048	0.018	0.012
	(0.053)	(0.052)	(0.041)	(0.041)	(0.015)	(0.014)
crd_t	-0.022*	-0.023**	0.024	0.022	-0.017***	-0.017***
	(0.011)	(0.011)	(0.016)	(0.016)	(0.006)	(0.006)
auw_t	0.255***	-0.232***	-0.063	-0.055	-0.01	-0.004
	(0.057)	(0.056)	(0.053)	(0.053)	(0.024)	(0.024)
$gvint_t$	-0.018	-0.009	-0.014	-0.013	-0.015**	-0.016***
	(0.013)	(0.013)	(0.014)	(0.014)	(0.006)	(0.006)
$\ln(\pi_t)$	0.24 ***	0.213***	0.21***	0.188***	0.045***	0.046***
	(0.016)	(0.017)	(0.013)	(0.013)	(0.006)	(0.006)
$\ln(\sigma_{u,t})$	0.129***	0.155***	0.211***	0.233***	0.047***	0.066***
	(0.033)	(0.037)	(0.03)	(0.033)	(0.013)	(0.014)
$d_{eu} \cdot \ln(\sigma_{u,t})$		0.027		-0.042		-0.071***
		(0.081)		(0.064)		(0.025)
$d_{cr} \cdot \ln(\sigma_{u,t})$		-0.098		-0.156*		-0.09***
		(0.118)		(0.09)		(0.03)
d_{eu}		-0.19***		0.071		0.071***
		(0.065)		(0.053)		(0.021)
d_{cr}		0.032		0.066***		0.084***
		(0.087)		(0.066)		(0.022)
$const$	-0.695***	-0.704***	-1.241***	-1.231***		
	(0.069)	0.07)	(0.074)	(0.075)		

[1] System GMM estimates for dynamic panels in columns (5) and (6) . Sample period 1960-2011

[2] All labour market institutional variables are treated as exogenous. $ln(\pi)$, $\ln(\sigma_u)$, $d_{eu}\cdot\ln(\sigma_u)$ and $d_{cr}\cdot\ln(\sigma_{u,t})$ are treated as endogenous.variables. Two to five periods lagged values of the differences of the endogenous variables were used as instruments.

[3] * $p > 0.10$, **$p > 0.05$ and ***> 0.01

[4] Figures in parenthesis indicate the standard errors of the coefficients

Table 5.4: Miscellaneous Tests on the Arrelano-Bond GMM Estimations in Table (5.2)

Tests	(5)	(6)
Autocorrelation Tests for $\Delta\varepsilon_{it}$ (H_0: No Autocorrelation)		
$p - value$: $\Delta\varepsilon_{it-1}$	0.000	0.000
$p - value$: $\Delta\varepsilon_{it-2}$	0.307	0 .289
Total number of instruments	565	575
Sargan test for overidentifying restrictions ($p - value$)	0.016	0.048
Exogeneity of Instrument Subsets: Difference-in-Sargan Tests		
Sargan test excluding group :$p - value$	0.013	0.048
Difference (null H = exogenous):$p - value$	0.670	0.371

Table 5.5: Miscellaneous Tests on the Arrelano-Bond GMM Estimations in Table (5.3)

Tests	(5)	(6)
Autocorrelation Tests for $\Delta\varepsilon_{it}$ (H_0: No Autocorrelation)		
$p - value$: $\Delta\varepsilon_{it-1}$	0.000	0.000
$p - value$: $\Delta\varepsilon_{it-2}$	0.000	0 .000
Miscellaneous		
Total number of instruments	565	575
Sargan test for overidentifying restrictions ($p - value$)	0.000	0.000
Exogeneity of Instrument Subsets: Difference-in-Sargan Tests		
Sargan test excluding group :$p - value$	0.000	0.000
Difference (null H = exogenous):$p - value$	0.039	0.033

Table 5.6: Descriptive statistics

Variable	Description	Obs	Mean	Std Dev	Min	Max
auw	autonomous negotiations and implementation of agreements	779	0.929	0.256	0	1
bart	articulation of sectoral bargaining	778	1.812	1.475	0	5
crd	coordination of wage setting	780	0 .823	0.841	0	2
gvint	government intervention in wage bargaining	780	2.419	2.581	0	4
sect	strength of sectoral institutions	780	0.837	0.645	0	2
typ	type of coordination	780	3.109	1.653	0	5
ud	union density	744	44.151	18.744	7.576	87.442
wage	pact or agreement about wage issues negotiated	779	0.0706	0.256	0	1
$\ln(\sigma_\pi)$(gch)	log average monthly volatility from GARCH estimations	752	-0.957	0.505	-2.343	0 .993
$\ln(\sigma_\pi)$(std)	log standard deviation of monthly inflation	755	-0.763	0 .426	-1.75	1.335
$\ln(\sigma_u)$	log volatility of productivity from GARCH estimations	751	0 .519	0.451	-2.261	2.48
$\ln(\pi)$	log annual inflation	736	1.211	0.853	-2.303	3.186

[1] Each of the variables above have been placed under one of the following categories: bargaining power, number of negotiations and independence of negotiations. Appropriate transformations are applied to some variables to facilitate their interpretation with regards to the categories under which those variables fall.

[3] A value of 2 has been observed for $negp$ in the case of the Netherlands. This has been corrected as 1 given that $negp$ is a binary variable.

Chapter 6

Summary

In this dissertation, I investigate the causes and the effects of time variations in the degree of wage indexation. I address the following three questions. First, is there any empirical evidence in support of the time-varying nature of the degree of wage indexation? Second, what are the factors that account for the time variation in the degree of wage indexation? Finally, what are the consequences of the time variations in the degree of wage indexation? The contribution of this thesis to the literature lies in the three major questions it addresses. While there are a few other studies on time-varying wage indexation, none has explored the topic to such a depth as this dissertation. By developing and using a simple but effective methodology, Chapter 2 provides evidence and available measures for the time variation in wage indexation in 11 OECD countries. The dissertation further provides explanations as to the causes of the time variations in wage indexation in Chapter 2 and Chapter 3. Finally, the consequences of wage indexation are explored in Chapter 3, Chapter 4 and Chapter 5. The subsequent paragraphs of this summary contain details on what each of the chapters entails.

Chapter 2 addresses the first question. In this chapter, I provide preliminary evidence in support of the time-varying nature of wage indexation. I also provide a novel methodology to measure the time variation in wage indexation. Estimates for wage indexation obtained from this methodology are very similar to the values of the proportion of Cost-Of-Living Adjustment (COLA) coverage in the US. This result gives a rough indication of the accuracy of the method-

ology. I further estimate the time-varying wage indexation for 10 other OECD countries. The results show that the degree of wage indexation in most of these countries rose during the mid 1970s to early 1980s and decreased thereafter. Chapter 2 also addresses the second question, but mainly from an empirical standpoint. It is shown that trend inflation is the most important factor influencing wage indexation in almost all of the countries. There is some statistical evidence in support of the negative effects of productivity shock variances on wage indexation.

Chapter 3 addresses the second and third questions. In addressing the second question, this chapter assumes that wage indexation is an outcome of negotiations or bargaining between employers and labour unions. Under this assumption, I show that a random wage indexation outcome arises as a result of mixed equilibrium strategies pursued by the bargaining parties. While the number of independent negotiations does not affect the average wage indexation outcome, it certainly influences the variance of wage indexation. Regarding the third question, I show that under optimal monetary policy, inflation has a fat-tailed distribution. The key assumptions made when obtaining this result are the following: wages are indexed randomly to inflation and the interest rate is used as an instrument in the conduct of monetary policy. The fat-tail property does not extend to the output gap under the assumptions made.

Chapter 4 investigates the effects of time variation in wage indexation on the conduct of monetary policy. To this end, I consider a dynamic version of the Barro-Gordon model under which wages are randomly indexed to the lag of inflation. The model is calibrated to the Euro area using plausible parameter values. The results indicate that the distributions of inflation, the output gap and the interest rate are heavy-tailed. Also, in the presence of a random wage-indexation scheme, a Taylor rule targeting current inflation performs better than that targeting the lag of inflation.

The final chapter addresses whether labour market institutions have effects on the volatility of inflation. It draws on the theoretical framework presented in Chapter 3 to derive a testable hypothesis concerning the topic. The derived hypothesis is subsequently tested using panel data of 15 OECD countries. The study in this chapter can be viewed as an indirect test on the following result from chapter 3: wage indexation is a random bargaining or negotiation outcome.

The results indicate that labour market institutions have significant effects on inflation volatility. Inflation volatility rises when the bargaining power of negotiating parties rises. It falls when the number of independent wage (indexation) negotiations rises. In particular, I show that when one controls for bargaining power and government intervention, coordination of wage negotiations increases inflation volatility. This result runs in contrast to findings in the related literature which do not incorporate the above mentioned control variables.

Chapter 7

Nederlandse samenvatting (Summary in Dutch)

In dit proefschrift onderzoek ik de oorzaken en gevolgen van tijdsvariatie in de mate van loonindexatie. Ik stel de volgende drie vragen: Is er er enig empirisch bewijsmateriaal dat het tijdsvarirend karakter van de loonindexatie ondersteunt? Wat zijn de factoren die de tijdsvariatie in de loonindexatie bepalen? Wat zijn de directe en indirecte gevolgen van tijdsvariatie in de loonindexatie? De bijdrage aan de literatuur ligt in de drie hoofdvragen die het proefschrift behandelt. Hoewel er eerdere studies zijn gedaan naar tijdsvariatie in de loonindexatie, is het onderwerp nog niet eerder zo uitputtend behandeld. Door het ontwikkelen en toepassen van een simpele maar effectieve methodiek geeft hoofdstuk 2 een bewijs van en maatstaven voor tijdsvariatie in de loonindexatie in 11 OESO landen. De oorzaken van de tijdsvariatie in de loonindexatie worden vervolgens nader toegelicht in hoofdstuk 2 en 3, terwijl de gevolgen worden besproken in hoofdstuk 3, 4 en 5. De hier volgende paragrafen bevatten details over de inhoud van elk afzonderlijk hoofdstuk.

Hoofstuk 2 richt zich voornamelijk op de vraag of er enig empirisch bewijs is dat het tijdsvarirend karakter van loonindexatie ondersteunt. In dit hoofdstuk wordt niet alleen het preliminair bewijs naar voren gebracht, maar tevens toon ik een nieuwe methodiek aan om de tijdsvariatie in de loonindexatie te meten. De schattingen voor loonindexatie die uit deze methodologie worden verkregen, zijn vergelijkbaar met de waarden voor de prijsindexatie. Dit resultaat geeft een globale indicatie van de nauwkeurigheid van de methodologie. Hiernaast schat ik de tijdsvarirende loonindexatie voor 10 andere OESO landen. Hieruit blijkt dat de loonindexatie in de meeste landen steeg gedurende het midden van de jaren zeventig tot het begin van de jaren tachtig en daarna weer daalde. Het laatste gedeelte van het hoofdstuk richt zich op de factoren die de tijdsvariatie in de loonindexatie bepalen. Dit gebeurt voornamelijk vanuit een empirisch perspectief. Hierbij wordt aangetoond dat de inflatietrend in bijna alle landen de meest belangrijke factor is die de loonindexatie benvloedt. Er is enig statistisch bewijs ter ondersteuning van de negatieve effecten van de variantie van productiviteitsschokken op de loonindexatie.

Hoofdstuk 3 richt zich op de tweede en derde vraag. Bij het behandelen van de tweede vraag veronderstelt dit hoofdstuk dat de loonindexatie het resultaat is van onderhandelingen tussen twee vakbonden. Onder deze veronderstelling toon ik aan dat er een willekeurige loonindexatie ontstaat als gevolg van de gemengde evenwichtsstrategien die door de onderhandelende partijen worden gehanteerd. Hoewel het aantal onafhankelijke onderhandelingen geen invloed heeft op de gemiddelde loonindexatie, is dit zeker van invloed op de variantie van loonindexatie. Wat betreft de derde vraag laat ik zien dat onder een optimaal monetair beleid de inflatie een kansverdeling met een dikke staart heeft. De belangrijkste veronderstellingen die gemaakt zijn bij het verkrijgen van dit resultaat zijn de volgende: de lonen zijn willekeurig gendexeerd op basis van de inflatie en de rente is het monetaire beleidsinstrument. Het kenmerk van de dikke staarten strekt zich onder de gemaakte veronderstellingen niet uit tot de output gap.

Hoofdstuk 4 onderzoekt de effecten van tijdsvariatie in de loonindexatie op het monetair beleid. Hierbij gebruik ik een dynamische versie van het Barro-Gordon model waarin de lonen willekeurig gendexeerd zijn op basis van de inflatie in de voorgaande periode. Het model is gekalibreerd met voor de eurozone plausibele parameterwaarden. De resultaten tonen aan dat de kansverdeling van de inflatie, de output gap en de rente dikke staarten hebben. In het geval van willekeurige loonindexatie presteert een Taylor regel, die is gebaseerd op de inflatie in de huidige periode, beter dan een model dat is gebaseerd op de inflatie in de voorgaande periode.

Het laatste hoofdstuk, hoofdstuk 5, houdt zich bezig met de vraag of arbeidsmarktinstituties invloed hebben op de volatiliteit van de inflatie. Dit hoofdstuk is gebaseerd op het theoretisch kader dat in hoofdstuk drie werd gepresenteerd met het doel een toetsbare hypothesis met betrekking tot het onderwerp te formuleren. Het onderzoek in dit hoofdstuk kan worden beschouwd als een indirecte test van het resultaat uit hoofdstuk drie: de loonindexatie is een willekeurige onderhandelingsuitkomst. De resultaten geven aan dat de arbeidsmarktinstituties een significante invloed hebben op de volatiliteit van inflatie. De volatiliteit van de inflatie stijgt wanneer de onderhandelingspositie van de onderhandelende partijen sterker is. Echter, deze daalt zodra het aantal onafhankelijke onderhandelingen over loon(indexatie) toeneemt. In het bijzonder laat ik zien dat, wanneer er gecontroleerd wordt voor onderhandelingsmacht en overheidsingrijpen, de volatiliteit van inflatie stijgt als gevolg van de coördinatie van de loononderhandelingen. Dit resultaat is in strijd met de bevindingen in de bestaande literatuur waarin de hierboven vermelde controlevariabelen niet worden gebruikt.

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