The Many Decompositions of Total Factor Productivity Change

Bert M. Balk*
Rotterdam School of Management
Erasmus University
E-mail bbalk@rsm.nl

José L. Zofío
Departamento de Análisis Económico
Universidad Autónoma de Madrid
E-mail jose.zofio@uam.es

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Abstract

Total factor productivity change, here defined as output quantity change divided by input quantity change, is the combined result of (technical) efficiency change, technological change, a scale effect, and input and output mix effects. Sometimes allocative efficiency change is supposed to also play a role. Given a certain functional form for the productivity index, the problem is how to decompose such an index into factors corresponding to these five or six components. A basic insight offered in the present paper is that meaningful decompositions of productivity indices can only be obtained for indices which are transitive in the main variables. Using a unified approach, we obtain decompositions for Malmquist, Moorsteen-Bjurek, price-weighted, and share-weighted productivity indices. A unique feature of this paper is that all the decompositions are applied to the same dataset of a real-life panel of decision-making units so that the extent of the differences between the various decompositions can be judged.

Keywords: Total factor productivity; index; decomposition; Malmquist; Moorsteen; Bjurek; Fisher; Törnqvist.

JEL Classification Codes: C43, D24, D61.

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1 Introduction

In this paper we posit that there is no unique measure of productivity change, and no unique way of decomposing any measure of productivity change either.[1]

In an environment where input and output prices are available – either because there is a market or by imputation – productivity change in ratio form naturally materializes as the real (i. e., quantity) component of profitability change (see Balk 2003, 2010, 2016). There are, however, many ways of decomposing profitability change (i.e., the ratio of total revenue and total cost change) into price and quantity components, and *a fortiori* many ways of calculating productivity change. When data limitations do not dictate the choice, axiomatic index theory may be helpful, but at the end of the day we are still not certain whether to choose, say, a Fisher or a Törnqvist productivity index. Alternatively, we could choose a Malmquist productivity index, which is anyhow the only option available in an environment where output prices are non-existent. But again, there are a large number of possibilities here, and axiomatic considerations appear to be of limited value.

The embarrassment is exacerbated when it comes to decomposing productivity indices to get some insight into the components of productivity change. This subject continues to attract attention, as review papers by Lovell (2003) and Grosskopf (2003) are still consulted. The present paper is another contribution to this area.

It is well known that productivity change is the combined result of (technical) efficiency change, technological change, a scale effect, and input and output mix effects. However, it is less clear how allocative efficiency change should be accounted for. Given a certain functional form for the productivity index, the problem is how to decompose such an index into factors corresponding to the five or six components mentioned. Every mathematical expression a can, given any other expression b, be decomposed as $a = (a/b)b$. However, not all such decompositions are meaningful. At the very least, the two factors a/b and b should be independent of each other and admit a clear economic interpretation to be meaningful. The basic insight offered in the present paper is that meaningful decompositions of productivity indices can only be obtained for indices which are transitive in the main variables, input and output quantities.

Such decompositions can be obtained in a systematic way by considering the various hypothetical paths in input and output quantity space that connect a firm's base period position to its comparison period position. For example, the Malmquist index which conditions on the base period cone technology admits six different decompositions, as does the index which conditions on the comparison period cone technology. Their geometric mean even admits eighteen different decompositions. By merging either the input or the output mix effect with the scale effect, it is possible to obtain two different decompositions which are symmetric in all of their variables. The Moorsteen-Bjurek (MB) productivity index is defined as a ratio of Malmquist output and input quantity indices and hence contains a number of conditioning variables. If and only if these are specified independently of the main variables, the MB index is transitive and can be decomposed. Our decompositions are compared to those provided by Nemoto and Goto (2005), Peyrache (2014),

[1]In this paper 'productivity' is to be understood as 'total factor productivity' (TFP).

Grifell-Tatjé and Lovell (2015), and Diewert and Fox (2017). The last part of the paper reports on the decomposition issue for price-weighted and share-weighted productivity indices, and discusses the incorporation of allocative efficiency change.

The lay-out of this paper is as follows. Section 2 reviews basic definitions from production theory. Sections 3 and 4 discuss the problem of decomposing Malmquist productivity indices, first using the output orientation and then using the input orientation. Section 5 provides a brief intermediate conclusion. Section 6 considers the class of Moorsteen-Bjurek indices. Sections 7 and 8 study the decomposition problem for conventional productivity indices which are either price-weighted or share-weighted. The outcome of these sections bears on the decomposition of Fisher and Törnqvist indices, respectively. Section 9 is devoted to the problem how allocative efficiency change could be incorporated. Section 10 contains a number of general conclusions. Throughout the paper, we apply the decompositions obtained to a real-life dataset of a panel of individual production units.

2 Basic definitions

We consider a single production unit, for simplicity called a firm, which is observed during time periods of equal length.[2] Such a firm is considered here as an entity transforming inputs into outputs. The input quantities are represented by an N-dimensional vector of non-negative real values $x \equiv (x_1, ..., x_N) \in \Re^N_+ - \{0_N\}$. The output quantities are represented by an M-dimensional vector of non-negative real values $y \equiv (y_1, ..., y_M) \in \Re^M_+ - \{0_M\}$. Thus there is always at least one positive input and output quantity. Vectors without superscripts, with or without primes, are used as generic variables, whereas vectors with superscripts represent observations. Thus, for instance, (x^t, y^t) denotes the input and output quantities of our firm at period t.

2.1 Technologies and distance functions

We assume that this firm has access to a certain technology. The technology at period t is given by the set $S^t \subset \Re^N_+ \times \Re^M_+$ of all feasible input-output quantity combinations.[3] As in Balk (1998), we assume that the (usual) Färe and Primont (1995) axioms hold.[4]

The (direct) output distance function is defined by

$$D^t_o(x, y) \equiv \inf\{\delta \mid \delta > 0, (x, y/\delta) \in S^t\}. \quad (1)$$

Thus, $(x, y/D^t_o(x, y))$ is the point on the frontier of the period t technology that is obtained by holding the input quantity vector x constant while radially expanding the output quantity vector y. Put otherwise, the point $(x, y/D^t_o(x, y))$ could be called the projection of (x, y) on the frontier in the direction of y. The output distance

[2] A translation of the theory to spatial comparisons is simple. Instead of a single firm in two time periods, two firms at different locations are considered.

[3] According to Førsund (2015, 198), this is the micro-unit *ex ante* viewpoint.

[4] Diewert and Fox (2017) provide a story without convexity assumptions.

function is positive, nonincreasing in x, and nondecreasing and linearly homogeneous in y. When $M = 1$ (the case of a single output), $F^t(x) \equiv y/D_o^t(x, y) = 1/D_o^t(x, 1)$ is the familiar production function.

The (direct) input distance function is defined by

$$D_i^t(x, y) \equiv \sup\{\delta \mid \delta > 0, (x/\delta, y) \in S^t\}. \tag{2}$$

Thus, $(x/D_i^t(x, y), y)$ is the point on the frontier of the period t technology that is obtained by holding the output quantity vector y constant while radially contracting the input quantity vector x. Put otherwise, the point $(x/D_i^t(x, y), y)$ could be called the projection of (x, y) on the frontier in the direction of x. The input distance function is positive, nondecreasing and linearly homogeneous in x, and nonincreasing in y.

Both functions are measures of technical efficiency. The output distance function, $D_o^t(x, y)$, measures output orientated technical efficiency with values between 0 and 1, and the inverse of the input distance function, $1/D_i^t(x, y)$, measures input orientated technical efficiency with values between 0 and 1. Both belong to the class of path-based measures as defined by Russell and Schworm (2018).

The period t technology is said to exhibit global constant returns to scale (global CRS) if for all $\theta > 0$, $(\theta x, \theta y) \in S^t$ whenever $(x, y) \in S^t$. This property can also be expressed as

$$S^t = \theta S^t \ (\theta > 0).$$

Two equivalent conditions for global CRS are

$D_o^t(x, y)$ is homogeneous of degree -1 in x

and

$D_i^t(x, y)$ is homogeneous of degree -1 in y.

Associated with the (actual) technology is the cone technology, which is the virtual technology defined as the conical envelopment of S^t,

$$\check{S}^t \equiv \{(\lambda x, \lambda y) \mid (x, y) \in S^t, \lambda > 0\}. \tag{3}$$

It is thereby assumed that $\check{S}^t$ is a proper subset of $\Re_+^N \times \Re_+^M$, which means that globally increasing returns-to-scale of the period t technology is excluded.

The output distance function of the cone technology is denoted by $\check{D}_o^t(x, y)$, the input distance function by $\check{D}_i^t(x, y)$, and (when $M = 1$) the production function by $\check{F}^t(x)$. Their definitions are the same as the foregoing, except that S^t is replaced by $\check{S}^t$. It is immediately clear that $\check{S}^t$ exhibits global CRS, and that S^t exhibits global CRS if and only if $S^t = \check{S}^t$, i.e., if the actual technology coincides with the associated cone technology. It is straightforward to show that $\check{D}_i^t(x, y) = 1/\check{D}_o^t(x, y)$.

Since $S^t \subset \check{S}^t$, $\check{D}_o^t(x, y) \leq D_o^t(x, y)$. The ratio

$$OSE^t(x, y) \equiv \frac{\check{D}_o^t(x, y)}{D_o^t(x, y)} \tag{4}$$

is called output orientated scale efficiency. Notice that $OSE^t(x,y)$ is homogeneous of degree 0 in y (thus depends only on the output mix), is always less than or equal to 1, and attains the value 1 for all x and y if and only if the technology exhibits global CRS.

Likewise, since $S^t \subset \check{S}^t$, $\check{D}_i^t(x,y) \geq D_i^t(x,y)$. The ratio

$$ISE^t(x,y) \equiv \frac{D_i^t(x,y)}{\check{D}_i^t(x,y)} \tag{5}$$

is called input orientated scale efficiency. Notice that $ISE^t(x,y)$ is homogeneous of degree 0 in x (thus depends only on the input mix), is always less than or equal to 1, and attains the value 1 for all x and y if and only if the technology exhibits global CRS. Both measures of scale efficiency are extensively discussed in Balk (2001).

2.2 Measuring productivity change and level

Productivity change between the input-output situation (x,y) and the input-output situation (x',y') is measured by some positive, finite function $F : ((\Re^N_+ - \{0_N\}) \times (\Re^M_+ - \{0_M\}))^2 \rightarrow \Re_{++} - \{\infty\}$.[5] This function, with values $F(x',y',x,y)$, should be nonincreasing in x', nondecreasing in y', nondecreasing in x, and nonincreasing in y.[6] Moreover, this function should exhibit proportionality in input and output quantities; i.e.,

$$F(\lambda x, \mu y, x, y) = \mu/\lambda \ (\lambda, \mu > 0). \tag{6}$$

In particular, property (6) implies that $F(x,y,x,y) = 1$; that is, $F(x',y',x,y)$ satisfies the Identity Test. Taken together, the function $F(x',y',x,y)$ should be such that by fixing input quantities $x = x' = \bar{x}$ the function $F(\bar{x},y',\bar{x},y)$ behaves as an output quantity index, and by fixing output quantities $y = y' = \bar{y}$ the function $F(x',\bar{y},x,\bar{y})$ behaves as the reciprocal of an input quantity index. See Balk (2008) for requirements for quantity indices.

A function $F(x',y',x,y)$ is called transitive in (x,y) if it satisfies the equality

$$F(x'',y'',x,y) = F(x'',y'',x',y')F(x',y',x,y) \tag{7}$$

for any (x,y), (x',y') and (x'',y''). Transitivity implies that

$$F(x',y',x,y) = G(x',y')/G(x,y) \tag{8}$$

for a certain function $G(x,y)$. Reversely, any function $F(x',y',x,y)$ that has the form (8) is transitive. Property (6) then implies that the function $G(x,y)$ must be linearly homogeneous in y and homogeneous of degree -1 in x.[7] Put otherwise, if $F(x',y',x,y)$ is a transitive measure of productivity *change*, then $G(x,y)$ measures the productivity *level* at the input-output situation (x,y), up to a certain scalar normalization.

[5] Formally stated, $F(x',y',x,y)$ satisfies the Determinateness Test.

[6] These monotonicity properties were considered to be fundamental by Agrell and West (2001).

[7] A further specification, $G(x,y) = Y(y)/X(x)$, leads to functions considered by O'Donnell in various articles. Here $X(x)$ and $Y(y)$ are aggregator functions which are nonnegative, nondecreasing, and linearly homogeneous.

3 Decomposing a Malmquist productivity index by output distance functions

Well-known candidates for measuring productivity change are those from the class of Malmquist indices. We start by selecting a certain benchmark (or reference) technology, which must be conical in view of the required properties.[8] The output orientated Malmquist productivity index, conditional on the period t cone technology, is defined by

$$\check{M}_o^t(x', y', x, y) \equiv \frac{\check{D}_o^t(x', y')}{\check{D}_o^t(x, y)}. \tag{9}$$

Notice that numerator and denominator are always finite. This index has indeed the required monotonicity and proportionality properties, and is by construction transitive in (x, y). Thus, the output distance function $\check{D}_o^t(x, y)$ measures the productivity level at the input-output situation (x, y).

Consider now the movement of our firm from a base period situation (x^0, y^0) to a (later) comparison period situation (x^1, y^1). These periods may or may not be adjacent. Which cone technology should then be selected for the Malmquist productivity index defined by expression (9)? Although, in principle, no relation needs to exist between the benchmark technology time period t and the observation periods 0 and 1, it is natural to identify t with one of those periods.[9] Selecting the base period technology then leads to $\check{M}_o^0(x^1, y^1, x^0, y^0)$ and selecting the comparison period technology leads to $\check{M}_o^1(x^1, y^1, x^0, y^0)$. We also consider their geometric mean. Let us start with the first option.

3.1 The base period viewpoint

How do we decompose

$$\check{M}_o^0(x^1, y^1, x^0, y^0) = \frac{\check{D}_o^0(x^1, y^1)}{\check{D}_o^0(x^0, y^0)} \tag{10}$$

into meaningful, independent factors?[10] It appears that this problem can be solved by breaking up the movement of the firm into hypothetical, independent segments. Figure 1 shows a single-input/single-output situation. The base period technology set S^0 is pictured by the area between the horizontal axis and the lower curve, whereas the comparison period technology set S^1 is pictured by the area between the horizontal axis and the upper curve. It is assumed that $S^0 \neq S^1$. The figure suggests

[8] Notice that "using a CRS frontier as a reference does not mean that we assume CRS, it just serves as a reference for TFP measures." (Førsund 2015, 214)

[9] Natural but not necessary. For instance, Førsund (2016) considers the conical envelopment of the pooled technologies of the two periods, $S^0 \cup S^1$. This is a special case of the "global Malmquist productivity index" as defined by Pastor and Lovell (2005).

[10] The type of decomposition considered here differs from that studied by Färe *et al.* (2001). These authors considered a decomposition into components corresponding to subvectors of x and y.

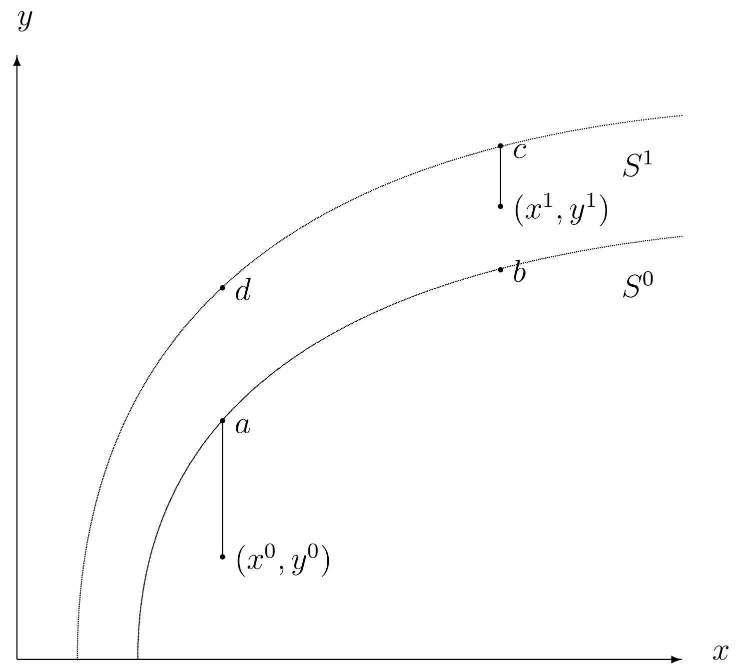

Figure 1: Decomposing productivity change (1)

uniform technological progress and inefficient firm behaviour in both periods. The figure also suggests that $D_o^0(x^1, y^1)$ is finite.

We first break up the firm's journey into four segments. The first segment stretches from the actual base period position to its projection in the y^0-direction on the base period technology frontier. This point is represented by a in Figure 1. Thus the first segment is formally defined by

$$(x^0, y^0) \longrightarrow (x^0, y^0/D_o^0(x^0, y^0)). \tag{11}$$

The second segment stretches along the base period frontier from a to the point represented by b, which is the projection of the firm's comparison period position on the base period frontier, on the assumption that $D_o^0(x^1, y^1)$ is finite,

$$(x^0, y^0/D_o^0(x^0, y^0)) \longrightarrow (x^1, y^1/D_o^0(x^1, y^1)). \tag{12}$$

The third segment stretches from the base period frontier at b to the comparison period frontier at c, which is the projection of the firm's comparison period position on the comparison period frontier,

$$(x^1, y^1/D_o^0(x^1, y^1)) \longrightarrow (x^1, y^1/D_o^1(x^1, y^1)). \tag{13}$$

The fourth segment stretches from point c back to the firm's comparison period position,

$$(x^1, y^1/D_o^1(x^1, y^1)) \longrightarrow (x^1, y^1). \tag{14}$$

Assuming that $D_o^0(x^1, y^0)$ is finite, Balk (2001) proposed to split the segment from a to b into two parts, namely a part corresponding to the change in input quantity space,

$$(x^0, y^0/D_o^0(x^0, y^0)) \longrightarrow (x^1, y^0/D_o^0(x^1, y^0)), \tag{15}$$

and a part corresponding to the change in output quantity space,

$$(x^1, y^0/D_o^0(x^1, y^0)) \longrightarrow (x^1, y^1/D_o^0(x^1, y^1)). \tag{16}$$

Assuming that $D_o^0(\lambda x^0, y^0)$ is finite, Lovell (2003) proposed to split the first of these two parts, given by expression (15), into two more parts, namely a part corresponding to radial change in input quantity space,

$$(x^0, y^0/D_o^0(x^0, y^0)) \longrightarrow (\lambda x^0, y^0/D_o^0(\lambda x^0, y^0)), \tag{17}$$

and a remainder part,

$$(\lambda x^0, y^0/D_o^0(\lambda x^0, y^0)) \longrightarrow (x^1, y^0/D_o^0(x^1, y^0)), \tag{18}$$

where λ is some positive scalar. Notice that by virtue of the positive linear homogeneity in y of the output distance function, $(\lambda x^0, y^0/D_o^0(\lambda x^0, y^0)) = (\lambda x^0, \mu y^0/D_o^0(\lambda x^0, \mu y^0))$ for any positive scalar μ.

Thus, summarizing, the entire journey from (x^0, y^0) to (x^1, y^1) is broken up into six segments, respectively defined by expressions (11), (17), (18), (16), (13), and (14), as pictured in the following frame.

Path A: $(x^0, y^0) \longrightarrow (x^0, y^0/D_o^0(x^0, y^0)) \longrightarrow (\lambda x^0, y^0/D_o^0(\lambda x^0, y^0)) \longrightarrow (x^1, y^0/D_o^0(x^1, y^0)) \longrightarrow (x^1, y^1/D_o^0(x^1, y^1)) \longrightarrow (x^1, y^1/D_o^1(x^1, y^1)) \longrightarrow (x^1, y^1)$

Along each segment the index $\check{M}_o^0(x', y', x, y)$ can be computed. Respectively this produces the following results:

$$\frac{\check{D}_o^0(x^0, y^0/D_o^0(x^0, y^0))}{\check{D}_o^0(x^0, y^0)} = \frac{1}{D_o^0(x^0, y^0)}, \tag{19}$$

$$\begin{aligned} \frac{\check{D}_o^0(\lambda x^0, y^0/D_o^0(\lambda x^0, y^0))}{\check{D}_o^0(x^0, y^0/D_o^0(x^0, y^0))} &= \frac{\check{D}_o^0(\lambda x^0, y^0)}{D_o^0(\lambda x^0, y^0)} \frac{D_o^0(x^0, y^0)}{\check{D}_o^0(x^0, y^0)} \\ &= \frac{OSE^0(\lambda x^0, y^0)}{OSE^0(x^0, y^0)} = SEC_{o,M}^0(\lambda x^0, x^0, y^0), \end{aligned} \tag{20}$$

$$\begin{aligned} \frac{\check{D}_o^0(x^1, y^0/D_o^0(x^1, y^0))}{\check{D}_o^0(\lambda x^0, y^0/D_o^0(\lambda x^0, y^0))} &= \frac{\check{D}_o^0(x^1, y^0)}{D_o^0(x^1, y^0)} \frac{D_o^0(\lambda x^0, y^0)}{\check{D}_o^0(\lambda x^0, y^0)} \\ &= \frac{OSE^0(x^1, y^0)}{OSE^0(\lambda x^0, y^0)} = SEC_{o,M}^0(x^1, \lambda x^0, y^0), \end{aligned} \tag{21}$$

$$\begin{aligned}\frac{\check{D}_o^0(x^1,y^1/D_o^0(x^1,y^1))}{\check{D}_o^0(x^1,y^0/D_o^0(x^1,y^0))} &= \frac{\check{D}_o^0(x^1,y^1)}{D_o^0(x^1,y^1)}\frac{D_o^0(x^1,y^0)}{\check{D}_o^0(x^1,y^0)} \\ &= \frac{OSE^0(x^1,y^1)}{OSE^0(x^1,y^0)} = OME_M^0(x^1,y^1,y^0),\end{aligned} \tag{22}$$

$$\frac{\check{D}_o^0(x^1,y^1/D_o^1(x^1,y^1))}{\check{D}_o^0(x^1,y^1/D_o^0(x^1,y^1))} = \frac{D_o^0(x^1,y^1)}{D_o^1(x^1,y^1)} = TC_o^{1,0}(x^1,y^1), \tag{23}$$

$$\frac{\check{D}_o^0(x^1,y^1)}{\check{D}_o^0(x^1,y^1/D_o^1(x^1,y^1))} = D_o^1(x^1,y^1), \tag{24}$$

where the notation introduced by Balk (2001) was used.[11] By virtue of transitivity, multiplying the left-hand sides of these six equations delivers precisely $\check{M}_o^0(x^1,y^1,x^0, y^0)$. Then, joining expressions (19) and (24), defining $EC_o(x^1,y^1,x^0,y^0) \equiv D_o^1(x^1,y^1) /D_o^0(x^0,y^0)$, and multiplying the right-hand sides provides a decomposition which can be summarized as

$$\begin{aligned}\check{M}_o^0(x^1,y^1,x^0,y^0) = EC_o(x^1,y^1,x^0,y^0) \times TC_o^{1,0}(x^1,y^1)\times \\ SEC_{o,M}^0(\lambda x^0,x^0,y^0) \times SEC_{o,M}^0(x^1,\lambda x^0,y^0) \times OME_M^0(x^1,y^1,y^0).\end{aligned} \tag{25}$$

There are thus five factors, respectively corresponding to technical efficiency change, technological change, a radial scale effect – recall that $SEC_{o,M}^0(\lambda x^0,x^0, y^0) = SEC_{o,M}^0(\lambda x^0,x^0,\mu y^0)$ for any positive μ –, an input mix effect, and an output mix effect.

These five factors are indeed independent, as can be verified easily. First, if there is no technological change, i.e., $S^1 = S^0$, then $TC_o^{1,0}(x,y) = 1$ for all x,y. Second, if the firm is technically efficient in both periods, then $D_o^0(x^0,y^0) = 1 = D_o^1(x^1,y^1)$, and thus $EC_o(x^1,y^1,x^0,y^0) = 1$. Third, if $x^1 = \lambda x^0$ for some $\lambda > 0$, then the input mix effect vanishes. Fourth, if $y^1 = \mu y^0$ for some $\mu > 0$, then the output mix effect vanishes. (Notice that in the single-output case the output mix effect always vanishes.) If all these conditions are fulfilled, the only remaining part at the right-hand side of expression (25) is the radial scale effect $SEC_{o,M}^0(\lambda x^0,x^0,y^0)$. Using the linear homogeneity of the distance functions a number of times, we see that

$$SEC_{o,M}^0(\lambda x^0,x^0,y^0) = \frac{1}{\lambda D_o^0(\lambda x^0,y^0)} = \frac{\mu}{\lambda D_o^0(\lambda x^0,\mu y^0)} = \frac{\mu}{\lambda D_o^1(x^1,y^1)} = \frac{\mu}{\lambda}, \tag{26}$$

as it should be.

Two important observations must be made:

- Although the left-hand side of expression (25) and the efficiency change factor on the right-hand side are always well-determined, this is not necessarily the case for the other four factors on the right-hand side.

[11] Specifically, $SEC_{o,M}^t(x^1,x^0,\bar{y}) \equiv OSE^t(x^1,\bar{y})/OSE^t(x^0,\bar{y})$ and $OME_M^t(\bar{x},y^1,y^0) \equiv OSE^t(\bar{x},y^1)/OSE^t(\bar{x},y^0)$. The additional subscript M, standing for Malmquist, serves to distinguish these factors from their counterparts in other productivity index decompositions.

- If the base period technology exhibits global CRS (i.e., $S^0 = \check{S}^0$), then the last three factors on the right-hand side of expression (25) (i.e., radial scale, input mix, and output mix effect) vanish. This can easily be checked by the various definitions.

It is interesting to relate the decomposition in expression (25) to a number of alternative decompositions occurring in the literature. By merging the radial scale effect and the input mix effect, we obtain

$$\begin{aligned}\check{M}_o^0(x^1, y^1, x^0, y^0) = EC_o(x^1, y^1, x^0, y^0) \times TC_o^{1,0}(x^1, y^1) \times \\ SEC_{o,M}^0(x^1, x^0, y^0) \times OME_M^0(x^1, y^1, y^0). \end{aligned} \quad (27)$$

This is the decomposition proposed by Balk (2001). By merging the radial scale effect, the input mix effect, and the output mix effect, we obtain

$$\check{M}_o^0(x^1, y^1, x^0, y^0) = EC_o(x^1, y^1, x^0, y^0) \times TC_o^{1,0}(x^1, y^1) \times \frac{OSE^0(x^1, y^1)}{OSE^0(x^0, y^0)}. \quad (28)$$

This is the decomposition proposed by Ray and Desli (1997).

Another way of writing expression (25) is as

$$\begin{aligned}\check{M}_o^0(x^1, y^1, x^0, y^0) = M_o^0(x^1, y^1, x^0, y^0) \times SEC_{o,M}^0(\lambda x^0, x^0, y^0) \times \\ SEC_{o,M}^0(x^1, \lambda x^0, y^0) \times OME_M^0(x^1, y^1, y^0), \end{aligned} \quad (29)$$

where

$$M_o^0(x^1, y^1, x^0, y^0) \equiv EC_o(x^1, y^1, x^0, y^0) \times TC_o^{1,0}(x^1, y^1) = \frac{D_o^0(x^1, y^1)}{D_o^0(x^0, y^0)}. \quad (30)$$

Recall that if the base period technology exhibits global CRS, then the other three factors on the right-hand side of expression (29) become equal to 1, and we find that $\check{M}_o^0(x^1, y^1, x^0, y^0) = M_o^0(x^1, y^1, x^0, y^0)$. Expression (30) defines the base period output orientated *CCD index*. This function, generically defined as

$$M_o^t(x', y', x, y) \equiv \frac{D_o^t(x', y')}{D_o^t(x, y)}, \quad (31)$$

was introduced by Caves, Christensen and Diewert (1982) and then believed to be a productivity index. However, it does not possess the proportionality property (6) unless the benchmark technology exhibits global CRS. Nevertheless, following established practice, we refer to $M_o^t(.)$ as an index. We encounter the comparison period counterpart in expression (47) below.

By substituting expression (30) into expression (28) we obtain

$$\check{M}_o^0(x^1, y^1, x^0, y^0) = M_o^0(x^1, y^1, x^0, y^0) \times \frac{OSE^0(x^1, y^1)}{OSE^0(x^0, y^0)}, \quad (32)$$

which is another way of writing the Ray and Desli decomposition. The last factor was called 'returns to scale effect' by Lovell (2003).

The oldest decomposition of $\check{M}_o^0(x^1, y^1, x^0, y^0)$ was provided by Färe *et al.* (1989, 1994), and later expanded by Färe *et al.* (1994) as

$$\check{M}_o^0(x^1, y^1, x^0, y^0) = EC_o(x^1, y^1, x^0, y^0) \times \check{TC}_o^{1,0}(x^1, y^1) \times \frac{OSE^1(x^1, y^1)}{OSE^0(x^0, y^0)}. \tag{33}$$

The first factor on the right-hand side measures technical efficiency change. The second factor measures technological change. However, it does not refer to the actual technologies but to the encompassing cone technologies. The third factor, called 'scale efficiency change' by Färe *et al.* (1994) and Färe *et al.* (1997), conflates scale efficiency effects with technological change. As argued by Balk (2001), scale efficiency is a measure pertaining to points at an actual technology frontier, and the scale effect comes from the curvature of such a frontier, going from a base period position x^0 to a comparison position x^1, conditional on a certain output mix. As such, this has nothing to do with technological change (which is movement of the frontier itself).

To compare the decomposition in expression (33) with the Ray and Desli decomposition (28), Zofío (2007) proposed to split the second factor of the latter decomposition into two components, resulting in

$$\begin{aligned}\check{M}_o^0(x^1, y^1, x^0, y^0) = EC_o(x^1, y^1, x^0, y^0) \times TC_o^{1,0}(x^1, y^1) \times \\ \frac{OSE^1(x^1, y^1)}{OSE^0(x^0, y^0)} \times \frac{OSE^0(x^1, y^1)}{OSE^1(x^1, y^1)}.\end{aligned} \tag{34}$$

The last factor was called scale bias of technological change. This interpretation hinges on the fact that, by using the definition of OSE in expression (4), this factor can be written as

$$\frac{OSE^0(x^1, y^1)}{OSE^1(x^1, y^1)} = \frac{\check{TC}_o^{1,0}(x^1, y^1)}{TC_o^{1,0}(x^1, y^1)}; \tag{35}$$

i.e., technological change of the (virtual) cone technology divided by technological change of the actual technology. Hence, scale bias is not independent of (actual) technological change itself, as measured by $TC_o^{1,0}(x^1, y^1)$. We conclude that the components of the Ray and Desli decomposition (28) are independent, but that the components of the Färe *et al.* decomposition (33) are not.

Let us now return to expression (25). As we have seen that the choice of μ is immaterial, the remaining task is to choose a suitable value for λ. Our choice would be the solution $\lambda^{(1)}$ of

$$D_o^0(\lambda x^0, y^0) = D_o^0(x^1, y^0), \tag{36}$$

which means that λx^0 and x^1 are on the same output isoquant of the base period technology.[12] Lovell (2003) suggests $\mu = 1/D_o^0(x^1, y^0)$ and $\lambda = 1/D_i^0(x^0, \mu y^0)$, or

[12] The same expression materializes in Peyrache (2014) and in Diewert and Fox (2017). A

$$D_i^0(\lambda x^0, y^0/D_o^0(x^1, y^0)) = 1. \tag{37}$$

Provided that some mild regularity conditions are met (see Färe 1988, Lemma 2.3.10), $D_i^t(x, y) = 1$ if and only if $D_o^t(x, y) = 1$, and thus equation (37) appears to be equivalent to

$$D_o^0(\lambda x^0, y^0/D_o^0(x^1, y^0)) = 1, \tag{38}$$

which brings us back to expression (36). We could also take the solution of

$$\check{D}_o^0(x^1, y^0/D_o^0(x^1, y^0)) = \check{D}_o^0(\lambda x^0, y^0/D_o^0(\lambda x^0, y^0)). \tag{39}$$

We can easily verify that this implies that $SEC_{o,M}^0(x^1, \lambda x^0, y^0) = 1$; i.e., the input mix effect vanishes.

Recall that the segment from a to b was split into three parts, respectively given by expressions (17), (18), and (16). Reversing the order in which changes in input and output space take place, and assuming that $D_o^0(x^0, y^1)$ and $D_o^0(\lambda x^0, y^1)$ are finite, we get an alternative decomposition of this segment. The entire journey from (x^0, y^0) to (x^1, y^1) is now pictured in the next frame.

Path B: $(x^0, y^0) \longrightarrow (x^0, y^0/D_o^0(x^0, y^0)) \longrightarrow (x^0, y^1/D_o^0(x^0, y^1)) \longrightarrow (\lambda x^0, y^1/D_o^0(\lambda x^0, y^1)) \longrightarrow (x^1, y^1/D_o^0(x^1, y^1)) \longrightarrow (x^1, y^1/D_o^1(x^1, y^1)) \longrightarrow (x^1, y^1)$

In the same way as demonstrated earlier, Path B leads to the following decomposition of the productivity index:

$$\begin{aligned}\check{M}_o^0(x^1, y^1, x^0, y^0) = M_o^0(x^1, y^1, x^0, y^0) \times SEC_{o,M}^0(\lambda x^0, x^0, y^1)\times \\ SEC_{o,M}^0(x^1, \lambda x^0, y^1) \times OME_M^0(x^0, y^1, y^0).\end{aligned} \tag{40}$$

The differences between this decomposition and the earlier one, expression (29), are subtle but noteworthy. The parts capturing efficiency change and technological change are identical. In expression (29) the radial scale effect and the input mix effect are conditional on y^0, but in expression (40) they are conditional on y^1. In a certain sense, the reverse happens with the output mix effect; in expression (29) this effect is conditional on x^1 but in expression (40) it is conditional on x^0. As in the previous case, if the base period technology exhibits global CRS (i.e., $S^0 = \check{S}^0$), then the last three factors on the right-hand side of expression (40) (i.e., radial scale, input mix, and output mix effect) vanish.

By merging the radial scale effect and the input mix effect, we now obtain

$$\check{M}_o^0(x^1, y^1, x^0, y^0) = M_o^0(x^1, y^1, x^0, y^0) \times SEC_{o,M}^0(x^1, x^0, y^1) \times OME_M^0(x^0, y^1, y^0), \tag{41}$$

sufficient condition for the existence of such a solution is that the distance function $D_o^0(x, y)$ is continuously differentiable. If the underlying technology is approximated by DEA (see Appendix B), a solution may not exist.

which should be compared to expression (27) to see the differences in the conditioning variables. By merging the radial scale effect, the input mix effect, and the output mix effect, we obtain again the Ray and Desli decomposition of expression (28).

The obvious choice for λ is now the solution $\lambda^{(2)}$ of

$$D_o^0(\lambda x^0, y^1) = D_o^0(x^1, y^1). \tag{42}$$

Notice that in general $\lambda^{(2)} \neq \lambda^{(1)}$. A sufficient condition for equality is that $y^1 = \mu y^0$ for some $\mu > 0$. This, however, would mean that the output mix effect vanishes. We must introduce the concept of output homotheticity for the formulation of a necessary and sufficient condition The period t technology is said to exhibit output homotheticity if $D_o^t(x, y) = D_o^t(1_N, y)G^t(x)$, where $G^t(x)$ is some nonincreasing function which is consistent with the axioms, and 1_N is a vector of N ones. Essentially, output homotheticity means that all the output sets $P^t(x)$ are radial expansions of $P^t(1_N)$.

Theorem 1 $\lambda^{(1)} = \lambda^{(2)}$ *if and only if the base period technology exhibits output homotheticity.*

Proof: The sufficiency follows immediately. For the necessity part, we notice that equations (36) and (42) imply that $D_o^0(x, y^1)/D_o^0(x, y^0)$ is independent of x. Thus

$$\frac{D_o^0(x, y^1)}{D_o^0(x, y^0)} = \frac{g^0(y^1)}{g^0(y^0)}$$

for some function $g^0(y)$. Thus, $D_o^0(x, y^1) = D_o^0(x, y^0)g^0(y^1)/g^0(y^0)$, and since the left-hand side is independent of y^0, the right-hand side must also be independent of y^0, which implies that $D_o^0(x, y^0)/g^0(y^0) = h^0(x)$ for some function $h^0(x)$. Thus

$$D_o^0(x, y^1) = h^0(x)g^0(y^1).$$

In particular

$$D_o^0(1_N, y^1) = h^0(1_N)g^0(y^1),$$

which upon substitution in the foregoing expression leads to

$$D_o^0(x, y^1) = D_o^0(1_N, y^1)h^0(x)/h^0(1_N).$$

But this means that the base period technology exhibits output homotheticity. ■

At this point we may conclude that there are two, equally meaningful, decompositions of the Malmquist productivity index $\check{M}_o^0(x^1, y^1, x^0, y^0)$. They differ with respect to the radial scale effect, the input mix effect and the output mix effect. By taking the geometric mean of expressions (29) and (40), we obtain the third decomposition

$$\check{M}_o^0(x^1, y^1, x^0, y^0) = M_o^0(x^1, y^1, x^0, y^0)\times$$
$$[SEC_{o,M}^0(\lambda^{(1)}x^0, x^0, y^0)SEC_{o,M}^0(\lambda^{(2)}x^0, x^0, y^1)]^{1/2}\times$$

$$[SEC^0_{o,M}(x^1,\lambda^{(1)}x^0,y^0)SEC^0_{o,M}(x^1,\lambda^{(2)}x^0,y^1)]^{1/2}\times$$
$$[OME^0_M(x^0,y^1,y^0)OME^0_M(x^1,y^1,y^0)]^{1/2}. \quad (43)$$

The first factor captures technological change and efficiency change, the second factor captures the radial scale effect, the third factor captures the input mix effect, and the fourth factor captures the output mix effect. By merging the radial scale effect and the input mix effect, we obtain

$$\check{M}^0_o(x^1,y^1,x^0,y^0) = M^0_o(x^1,y^1,x^0,y^0)\times$$
$$[SEC^0_{o,M}(x^1,x^0,y^0)SEC^0_{o,M}(x^1,x^0,y^1)]^{1/2}\times$$
$$[OME^0_M(x^1,y^1,y^0)OME^0_M(x^0,y^1,y^0)]^{1/2}, \quad (44)$$

whereas by merging all the three effects expression (43) reduces to the Ray and Desli decomposition, given by expression (28). If the base period technology exhibits global CRS (i.e., $S^0 = \check{S}^0$), then only the first factor remains.

3.2 The comparison period viewpoint

The second candidate productivity index is quite naturally given by the output orientated Malmquist productivity index conditional on the comparison period cone technology:

$$\check{M}^1_o(x^1,y^1,x^0,y^0) = \frac{\check{D}^1_o(x^1,y^1)}{\check{D}^1_o(x^0,y^0)}. \quad (45)$$

To decompose this index into meaningful factors, we consider the following path from (x^0,y^0) to (x^1,y^1):

Path C: $(x^0,y^0) \longrightarrow (x^0,y^0/D^0_o(x^0,y^0)) \longrightarrow (x^0,y^0/D^1_o(x^0,y^0)) \longrightarrow (\lambda x^0,y^0/D^1_o(\lambda x^0,y^0)) \longrightarrow (x^1,y^0/D^1_o(x^1,y^0)) \longrightarrow (x^1,y^1/D^1_o(x^1,y^1)) \longrightarrow (x^1,y^1)$

in which λ is as yet an undetermined positive scalar. Referring back to Figure 1, we see that the first segment connects the firm's base period position to its projection on the base period frontier (point a). The second segment connects this point to the projection of the firm's base period position on the comparison period frontier, which is depicted as point d. Next, we travel from point d to point c, which depicts the projection of the firm's comparison period position on the comparison period frontier. This segment is divided into three subsegments, respectively corresponding to a radial movement in x-space, a remainder movement in x-space, and a movement in y-space. The final segment connects point c to the firm's comparison period position. It is thereby assumed that $D^1_o(x^0,y^0)$, $D^1_o(\lambda x^0,y^0)$ and $D^1_o(x^1,y^0)$ are finite.

This leads to the following decomposition:

$$\check{M}^1_o(x^1,y^1,x^0,y^0) = M^1_o(x^1,y^1,x^0,y^0)\times SEC^1_{o,M}(\lambda x^0,x^0,y^0)\times$$
$$SEC^1_{o,M}(x^1,\lambda x^0,y^0)\times OME^1_M(x^1,y^1,y^0), \quad (46)$$

where

$$M_o^1(x^1, y^1, x^0, y^0) \equiv EC_o(x^1, y^1, x^0, y^0) \times TC_o^{1,0}(x^0, y^0) = \frac{D_o^1(x^1, y^1)}{D_o^1(x^0, y^0)} \tag{47}$$

defines the comparison period output orientated *CCD index.* It is straightforward to check from the various definitions that if the comparison period technology exhibits global CRS (i.e., $S^1 = \check{S}^1$), then the last three factors on the right-hand side of expression (46) (i.e., radial scale, input mix, and output mix effect) vanish. The obvious choice for λ is now the solution $\lambda^{(3)}$ of[13]

$$D_o^1(\lambda x^0, y^0) = D_o^1(x^1, y^0). \tag{48}$$

Grifell-Tatjé and Lovell (1999) considered the same path, but with $\lambda = 1/D_i^1(x^0, y^0/D_o^1(x^1, y^0))$. However, under the regularity conditions mentioned before, this equality is equivalent to expression (48).

By merging the radial scale effect $SEC_{o,M}^1(\lambda x^0, x^0, y^0)$ with the input mix effect $SEC_{o,M}^1\ (x^1, \lambda x^0, y^0)$, we obtain

$$\check{M}_o^1(x^1, y^1, x^0, y^0) = M_o^1(x^1, y^1, x^0, y^0) \times SEC_{o,M}^1(x^1, x^0, y^0) \times OME_M^1(x^1, y^1, y^0). \tag{49}$$

By also merging with the output mix effect $OME_M^1(x^1, y^1, y^0)$, we obtain

$$\check{M}_o^0(x^1, y^1, x^0, y^0) = M_o^1(x^1, y^1, x^0, y^0) \times \frac{OSE^1(x^1, y^1)}{OSE^1(x^0, y^0)}, \tag{50}$$

which is another instance of the Ray and Desli (1997) decomposition.

The alternative path, assuming now that $D_o^1(x^0, y^0)$, $D_o^1(\lambda x^0, y^1)$ and $D_o^1(x^0, y^1)$ are finite, is defined by the following sequence:

Path D: $(x^0, y^0) \longrightarrow (x^0, y^0/D_o^0(x^0, y^0)) \longrightarrow (x^0, y^0/D_o^1(x^0, y^0)) \longrightarrow (x^0, y^1/D_o^1(x^0, y^1)) \longrightarrow (\lambda x^0, y^1/D_o^1(\lambda x^0, y^1)) \longrightarrow (x^1, y^1/D_o^1(x^1, y^1)) \longrightarrow (x^1, y^1)$

This leads to the second decomposition of the productivity index (45), namely as

$$\begin{aligned} \check{M}_o^1(x^1, y^1, x^0, y^0) = M_o^1(x^1, y^1, x^0, y^0) \times SEC_{o,M}^1(\lambda x^0, x^0, y^1) \times \\ SEC_{o,M}^1(x^1, \lambda x^0, y^1) \times OME_M^1(x^0, y^1, y^0), \end{aligned} \tag{51}$$

the obvious choice for λ now being the solution $\lambda^{(4)}$ of

$$D_o^1(\lambda x^0, y^1) = D_o^1(x^1, y^1). \tag{52}$$

[13] A sufficient condition for the existence of such a solution is that the distance function $D_o^1(x, y)$ is continuously differentiable. If the underlying technology is approximated by DEA (see Appendix B), a solution may not exist.

Notice the subtle differences between expressions (46) and (51). Again, if the comparison period technology exhibits global CRS (i.e., $S^1 = \check{S}^1$), then the last three factors at the right-hand side of expression (51) vanish.

By merging in expression (51) the radial scale effect $SEC^1_{o,M}(\lambda x^0, x^0, y^1)$ with the input mix effect $SEC^1_{o,M}(x^1, \lambda x^0, y^1)$, we obtain

$$\check{M}^1_o(x^1, y^1, x^0, y^0) = M^1_o(x^1, y^1, x^0, y^0) \times SEC^1_{o,M}(x^1, x^0, y^1) \times OME^1_M(x^0, y^1, y^0), \tag{53}$$

a decomposition also obtained by Balk (2001); notice the subtle differences with expression (49). By merging also with the output mix effect $OME^1_M(x^0, y^1, y^0)$, we obtain again expression (50).

Notice that in general $\lambda^{(4)} \neq \lambda^{(3)}$ unless $y^1 = \mu y^0$ for some $\mu > 0$, which, however, would mean that the output mix effect vanishes. Similar to the earlier theorem, one can prove that

Theorem 2 $\lambda^{(3)} = \lambda^{(4)}$ *if and only if the comparison period technology exhibits output homotheticity.*

As before, the third decomposition of the productivity index (45) is obtained by taking the geometric mean of expressions (46) and (51), resulting in

$$\begin{aligned} \check{M}^1_o(x^1, y^1, x^0, y^0) = M^1_o(x^1, y^1, x^0, y^0) \times \\ [SEC^1_{o,M}(\lambda^{(3)} x^0, x^0, y^0) SEC^1_{o,M}(\lambda^{(4)} x^0, x^0, y^1)]^{1/2} \times \\ [SEC^1_{o,M}(x^1, \lambda^{(3)} x^0, y^0) SEC^1_{o,M}(x^1, \lambda^{(4)} x^0, y^1)]^{1/2} \times \\ [OME^1_M(x^0, y^1, y^0) OME^1_M(x^1, y^1, y^0)]^{1/2}. \end{aligned} \tag{54}$$

The first factor captures technological change and efficiency change, the second factor captures the radial scale effect, the third factor captures the input mix effect, and the fourth factor captures the output mix effect.

By merging the radial scale effect and the input mix effect, we obtain

$$\begin{aligned} \check{M}^1_o(x^1, y^1, x^0, y^0) = M^1_o(x^1, y^1, x^0, y^0) \times \\ [SEC^1_{o,M}(x^1, x^0, y^0) SEC^1_{o,M}(x^1, x^0, y^1)]^{1/2} \times \\ [OME^1_M(x^0, y^1, y^0) OME^1_M(x^1, y^1, y^0)]^{1/2}, \end{aligned} \tag{55}$$

whereas merging all the three effects reduces expression (55) to the Ray and Desli decomposition, given by expression (50). If the comparison period technology exhibits global CRS (i.e., $S^1 = \check{S}^1$), then only the first factor remains.

3.3 The 'geometric mean' viewpoint

Our third candidate productivity index is defined as the geometric mean of the two one-sided indices; that is,

$$\check{M}_o(x^1, y^1, x^0, y^0) \equiv [\check{M}_o^0(x^1, y^1, x^0, y^0) \times \check{M}_o^1(x^1, y^1, x^0, y^0)]^{1/2} = \left[\frac{\check{D}_o^0(x^1, y^1)}{\check{D}_o^0(x^0, y^0)} \frac{\check{D}_o^1(x^1, y^1)}{\check{D}_o^1(x^0, y^0)}\right]^{1/2}. \tag{56}$$

As can be verified easily, there are nine possible decompositions, which can be obtained by combining respectively expression (29) with (46), (29) with (51), (29) with (54); (40) with (46), (40) with (51), (40) with (54); (43) with (46), (43) with (51), and (43) with (54). The last combination is given by

$$\begin{aligned}
&\check{M}_o(x^1, y^1, x^0, y^0) = \\
&[M_o^0(x^1, y^1, x^0, y^0) M_o^1(x^1, y^1, x^0, y^0)]^{1/2} \times \\
&[SEC_{o,M}^0(\lambda^{(1)} x^0, x^0, y^0) SEC_{o,M}^0(\lambda^{(2)} x^0, x^0, y^1) \times \\
&SEC_{o,M}^1(\lambda^{(3)} x^0, x^0, y^0) SEC_{o,M}^1(\lambda^{(4)} x^0, x^0, y^1)]^{1/4} \times \\
&[SEC_{o,M}^0(x^1, \lambda^{(1)} x^0, y^0) SEC_{o,M}^0(x^1, \lambda^{(2)} x^0, y^1) \times \\
&SEC_{o,M}^1(x^1, \lambda^{(3)} x^0, y^0) SEC_{o,M}^1(x^1, \lambda^{(4)} x^0, y^1)]^{1/4} \times \\
&[OME_M^0(x^0, y^1, y^0) OME_M^0(x^1, y^1, y^0) OME_M^1(x^0, y^1, y^0) OME_M^1(x^1, y^1, y^0)]^{1/4}.
\end{aligned} \tag{57}$$

This decomposition would be symmetric in all its variables if $\lambda^{(1)} = \lambda^{(2)} = \lambda^{(3)} = \lambda^{(4)}$. In general, however, this is unlikely to happen. For the next result, we introduce the concept of implicit Hicks input neutral technological change. This type of technological change holds if $D_o^1(x, y) = D_o^0(x, y)A(y)$ for some function $A(y)$ which is consistent with the axioms.

Theorem 3 $\lambda^{(1)} = \lambda^{(2)} = \lambda^{(3)} = \lambda^{(4)}$ *if and only if the technologies* S^0 *and* S^1 *exhibit output homotheticity and technological change exhibits implicit Hicks input neutrality.*

Proof: The sufficiency part is obvious. For the necessity part, we notice that the former two theorems imply the property of output homotheticity for both technologies. Then, using the definition of output homotheticity, we see that equations (36) and (42) imply that $G^0(\lambda x^0) = G^0(x^1)$, and that equations (48) and (52) imply that $G^1(\lambda x^0) = G^1(x^1)$. Since these equations are assumed to hold for all x^0, x^1, the ratio $G^1(x)/G^0(x)$ must be independent of x. Thus, $G^1(x) = \alpha G^0(x)$ for some positive scalar α. But then we can infer by simple substitution that

$$D_o^1(x, y) = D_o^1(1_N, y)\alpha G^0(x) = D_o^0(x, y)\alpha \frac{D_o^1(1_N, y)}{D_o^0(1_N, y)},$$

which means that technological change exhibits implicit Hicks input neutrality. ■

Of course, we could select any of the solutions of equations (36), (42), (48), or (52) and set $\lambda^{(1)} = \lambda^{(2)} = \lambda^{(3)} = \lambda^{(4)}$. This, however, would introduce an essential element of arbitrariness into the decomposition (57).

Full symmetry of the productivity index decomposition can only be obtained by merging the radial scale effect and the input mix effect, so that we obtain the following decomposition instead of expression (57):

$$\begin{gathered}\check{M}_o(x^1,y^1,x^0,y^0) = \\ [M_o^0(x^1,y^1,x^0,y^0)M_o^1(x^1,y^1,x^0,y^0)]^{1/2}\times \\ [SEC_{o,M}^0(x^1,x^0,y^0)SEC_{o,M}^0(x^1,x^0,y^1)SEC_{o,M}^1(x^1,x^0,y^0)SEC_{o,M}^1(x^1,x^0,y^1)]^{1/4}\times \\ [OME_M^0(x^0,y^1,y^0)OME_M^0(x^1,y^1,y^0)OME_M^1(x^0,y^1,y^0)OME_M^1(x^1,y^1,y^0)]^{1/4} = \\ [M_o^0(x^1,y^1,x^0,y^0)M_o^1(x^1,y^1,x^0,y^0)]^{1/2}\times \\ \left[[SEC_{o,M}^0(x^1,x^0,y^0)SEC_{o,M}^1(x^1,x^0,y^0)]^{1/2}[SEC_{o,M}^0(x^1,x^0,y^1)SEC_{o,M}^1(x^1,x^0,y^1)]^{1/2}\right]^{1/2}\times \\ \left[[OME_M^0(x^0,y^1,y^0)OME_M^1(x^0,y^1,y^0)]^{1/2}[OME_M^0(x^1,y^1,y^0)OME_M^1(x^1,y^1,y^0)]^{1/2}\right]^{1/2},\end{gathered} \tag{58}$$

where the second decomposition is obtained by simply rearranging the first. This second decomposition shows what happens if we first combine decompositions along paths A and C, B and D, and next combine these two combinations.

By merging all the three effects, expression (58) further reduces to

$$\begin{gathered}\check{M}_o(x^1,y^1,x^0,y^0) = \\ [M_o^0(x^1,y^1,x^0,y^0)M_o^1(x^1,y^1,x^0,y^0)]^{1/2}\times \\ \left[\frac{OSE^0(x^1,y^1)}{OSE^0(x^0,y^0)}\frac{OSE^1(x^1,y^1)}{OSE^1(x^0,y^0)}\right]^{1/2},\end{gathered} \tag{59}$$

which is again a Ray and Desli (1997) type decomposition. This decomposition was used by Chen and Yang (2011) in an extension to meta-frontiers. Notice that if the base and comparison period technologies exhibit global CRS (i.e., $S^0 = \check{S}^0$ and $S^1 = \check{S}^1$), then the scale and mix effects vanish.

The geometric mean index proposed by Balk (2001) corresponds to the combination of expressions (29) and (51) whereby radial scale and input mix effects are merged. Expression (59) can be expanded as

$$\begin{gathered}\check{M}_o(x^1,y^1,x^0,y^0) = \\ [M_o^0(x^1,y^1,x^0,y^0)M_o^1(x^1,y^1,x^0,y^0)]^{1/2}\times \\ \frac{OSE^1(x^1,y^1)}{OSE^0(x^0,y^0)}\left[\frac{OSE^0(x^0,y^0)}{OSE^1(x^0,y^0)}\frac{OSE^0(x^1,y^1)}{OSE^1(x^1,y^1)}\right]^{1/2},\end{gathered} \tag{60}$$

which plays a fundamental role in the forecasting exercise of Daskovska, Simar, and Van Bellegem (2010).

Notice that $[M_o^0(x^1,y^1,x^0,y^0)M_o^1(x^1,y^1,x^0,y^0)]^{1/2}$, occurring in expressions (57), (58), (59), and (60), is the geometric mean output orientated *CCD index*. In the extant literature on productivity measurement, this index frequently figures under the name of ‘the (output orientated) Malmquist productivity index’. Notice, however, that a CCD index does not satisfy the proportionality requirement.

Following Färe, Grosskopf, and Margaritis (2008, 572-574), the technological change component of the geometric mean CCD index, $[TC_o^{1,0}(x^1,y^1)TC_o^{1,0}(x^0,y^0)]^{1/2}$, can be decomposed multiplicatively into three components, respectively measuring the output bias, the input bias, and the magnitude. The magnitude is thereby defined as $TC_o^{1,0}(x^0,y^0)$. Magnitude and bias, however, are badly defined concepts. It is unclear why a particular input-output combination is singled out to provide the magnitude of technological change, and the other combination only its input- or output bias. We could as well measure the magnitude by $TC_o^{1,0}(x^1,y^1)$ and define the bias components accordingly. The point is simply that unless there is output neutrality (Balk 1998, 98), these two magnitudes differ. Then it makes sense to use their (geometric) mean as the overall magnitude of technological change and their spread as a measure of the extent of non-neutrality.[14]

An alternative decomposition of the geometric mean output orientated Malmquist productivity index figuring in the literature (see Färe *et al.* (1994), Färe, Grosskopf, and Margaritis (2008, 576)) is

$$\check{M}_o(x^1,y^1,x^0,y^0) = EC_o(x^1,y^1,x^0,y^0) \times \left[\check{TC}_o^{1,0}(x^1,y^1)\check{TC}_o^{1,0}(x^0,y^0)\right]^{1/2} \times \frac{OSE^1(x^1,y^1)}{OSE^0(x^0,y^0)}. \quad (61)$$

The first right-hand side factor measures efficiency change. The second factor measures technological change as exhibited by the (virtual) cone technologies. We have already discussed the third factor immediately after expression (33) as a conflation of scale efficiency effects and technological change. Apart from the fact that the last two factors are not independent of each other, it is not so clear why technological feasibilities should be determined by a virtual rather than an actual technology.

3.4 Empirical application

3.4.1 Data and DEA approach

This section provides an empirical illustration of the various Malmquist productivity index decompositions discussed so far. The data come from a balanced panel of 31 Taiwanese banks over the period 2006-2010, previously studied by Juo *et al.* (2015).[15] A complete discussion of the statistical sources, variables specification, and summary statistics can be found there. Regarding the technology and interrelations between inputs and outputs, the variables reflect the intermediation approach suggested by Sealey and Lindley (1977), whereby financial institutions, through labour and capital, collect deposits from savers to produce loans and other earning assets for borrowers. Inputs are financial funds (x_1), labour (x_2), and physical capital (x_3). The output vector includes financial investments (y_1) and loans (y_2).

[14]See Balk and Althin (1996) for a generalisation to a situation with multiple production units and multiple time periods.

[15]We are grateful to these authors for sharing the data.

We rely on Data Envelopment Analysis (DEA) techniques for the estimation of technologies. The main features and computational aspects are discussed in Appendix B. Relevant for the expressions presented in this section is the fact that the extended five-factor decomposition as in expression (25), accounting for efficiency change, technical change, a radial scale effect, and input and output mix effects, requires calculating the scaling parameter λ as suggested in expression (36). However, under the DEA approximation such a parameter may neither exist, nor be unique. Moreover, Aparicio, Pastor, and Zofío (2015, 887) showed that the common DEA models are not able to characterize homothetic technologies. Thus Theorems 1-3 are not applicable. Therefore, the most extended decompositions might be infeasible, or so involved that they become impractical from an applied perspective. Consequently, in this paper we focus on four-factor decompositions in which radial scale and input oor output mix effects are merged.

3.4.2 Results

Tables 1, 2, and 3 present the main descriptive statistics of the results for the output orientated Malmquist productivity index (MPI), according to the base period viewpoint, which is updated in each successive year-to-year comparison.[16] The index $\breve{M}_o^0(x^1, y^1, x^0, y^0)$ and its components efficiency change $EC_o(x^1, y^1, x^0, y^0)$ and technological change $TC_o^{1,0}(x^1, y^1)$, common to paths A–B, are shown in Table 1 (expressions (27) and (41)). The scale effect $SEC_{o,M}^0(x^1, x^0, y^0)$ and the output mix effect $OME_M^0(x^1, y^1, y^0)$ corresponding to path A are in Table 2. In this table, the returns-to-scale effect (RTS), $OSE_o^0(x^1, y^1)/OSE_o^0(x^0, y^0)$, as defined in expression (28), is reported in the last four columns. The decomposition involving the alternative measures of the scale effect $SEC_{o,M}^0(x^1, x^0, y^1)$ and the output mix effect $OME_M^0(x^0, y^1, y^0)$ along path B is in Table 3, where the returns-to-scale component is repeated to emphasize that its magnitudes are the same of whether path A or B was chosen. However, scale and mix effects are very similar on these two paths, suggesting that it is rather innocuous which one is chosen for managerial and policy decisions. The correlations are high and significant: $\rho(SEC_{o,M}^0(x^1, x^0, y^0), SEC_{o,M}^0(x^1, x^0, y^1)) = 0.968$ and $\rho(OME_M^0(x^1, y^1, y^0), OME_M^0 (x^0, y^1, y^0)) = 0.964$.[17]

Tables 4, 5, and 6 exhibit the decomposition from the comparison period viewpoint, corresponding to expressions (49) and (53), and represented by paths C–D.[18] These tables have the same structure as the previous ones, with MPI and the common efficiency change $EC_o(x^1, y^1, x^0, y^0)$ and technological change $TC_o^{1,0}(x^0, y^0)$ components shown in Table 4, and the specific components, along with the returns-

[16] Here and in the remainder of the paper, all individual outcomes are in the supplemental appendix, such that Table 1 is the summary table corresponding with Table S.1, etc. Tables S.1, S.2, and S.3 contain $\breve{M}_o^t(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt})$ for $k = 1, ..., 31$, $t = 2006, 2007, 2008, 2009$, along paths A and B.

[17] Unless otherwise specified, all correlation coefficients calculated in the empirical sections are statistically significant at the standard 0.05 % level, thereby rejecting the null hypothesis that there is no linear relationship between the indices.

[18] To be precise, these tables contain $\breve{M}_o^{t+1}(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt})$ for $k = 1, ..., 31$, $t = 2006, 2007, 2008, 2009$, along paths C and D.

to-scale term in expression (50), in Tables 5 and 6. In Table 5 the scale and output mix effects are $SEC^1_{o,M}(x^1, x^0, y^0)$ and $OME^1_M(x^1, y^1, y^0)$, whereas in Table 6 they are $SEC^1_{o,M}(x^1, x^0, y^1)$ and $OME^1_M(x^0, y^1, y^0)$. As in the previous case, the differences between the scale and output mix effects depending on each path are negligible.

However, comparing the MPI indices and their factors with respect to choice of benchmark period (base or comparison) yields striking differences. Though the correlation between the productivity indices is rather high, $\rho(\check{M}^0_o(x^1, y^1, x^0, y^0), \check{M}^1_o(x^1, y^1, x^0, y^0)) = 0.897$, this does not extend to technological change since $\rho(TC^{1,0}_o(x^1, y^1), TC^{1,0}_o(x^0, y^0)) = 0.213$. This can also be seen by comparing their average values in Tables 1 and 4. While $TC^{1,0}_o(x^1, y^1)$ systematically reports annual values greater than one, its comparison period counterpart $TC^{1,0}_o(x^0, y^0)$ is three out of four times less than one. The differences between the scale (including input mix) and output mix factors are also large, particulary in the latter case. Comparing paths A and C, $\rho(SEC^0_{o,M}(x^1, x^0, y^0), SEC^1_{o,M}(x^1, x^0, y^0)) = 0.710$, but $\rho(OME^0_M(x^1, y^1, y^0), OME^1_M(x^1, y^1, y^0)) = 0.186$.

We further explore the disparity between the base and comparison period indices and determine whether their distributions differ significantly. As solving each model under both approaches yields paired samples, we rely on the t-test and the Wilcoxon signed-rank test. The null hypotheses of the t-test and the Wilcoxon test are that both samples come from distributions with equal means and medians, respectively. The results of the t-tests for $\check{M}^0_o(x^1, y^1, x^0, y^0)$ and $\check{M}^1_o(x^1, y^1, x^0, y^0)$ and $SEC^0_{o,M}(x^1, x^0, y^0)$ and $SEC^1_{o,M}(x^1, x^0, y^0)$ do not reject the null hypotheses of equal means at the 5% significance level, but do reject it for the technological change components $TC^{1,0}_o(x^1, y^1)$ and $TC^{1,0}_o(x^0, y^0)$, as expected from their low correlation, and for $OME^0_M(x^1, y^1, y^0)$ and $OME^1_M(x^1, y^1, y^0)$, but only in the 2008-2009 period. The Wilcoxon tests yield the same results for the MPIs and technological change components, equal and different medians, respectively. However, the distributions of the scale (including input mix) and output mix effects present different medians in most periods, except for 2007-2008.[19]

Focusing now on individual results, a pairwise bilateral comparison of all these tables, starting with the MPI, shows that there are significant differences between the base and comparison period MPIs at the firm level. Taking the first bank as an example, we see that productivity grew by 20.27% from 2006 to 2007 if the first year is taken as viewpoint, whereas this growth reduces to a mere 3.86% if the second year is taken (Tables S.1 and S.4). This is also the only efficient bank for which $TC^{1,0}_o(.)$ cannot be computed due to unattainability of the data. It is also interesting to consider bank #16 with a productivity growth of 30.26% in 2006-2007, and bank #14 with a growth of 107.55% in 2009-2010 (Table S.1). However, the growth percentages are less extreme for the majority of the banks.

On average, the Taiwanese banks saw their productivity increase by 4.38% from 2006 to 2007, and by similar magnitudes in the next two years. However, a remark-

[19]In the remaining empirical sections, if the Wilcoxon test is in accordance with the t-test regarding the equivalence of the bilateral distributions, we do not report this. Also, if results are similar we do not discuss bilateral comparisons for alternative decompositions following symmetric paths in different periods, e.g., paths B and D on this occasion.

able 20.65% spike occurred in 2010. The source of the last productivity increase is mainly technological progress, driven once again by the extraordinary local technological change of bank #14, to the tune of 112.70%. Indeed, technological change appears to be the component mainly responsible for the aggregate result, rather than efficiency change (reported also in Table 1) or the scale and output mix effects (reported in Tables 2 or 3, depending on whether path A or B is followed). Scale contributes positively from the base period viewpoint decomposition for paths A or B, except in the first period. This is a robust result since from the comparison period viewpoint for paths C–D in Tables 5 and 6, we find magnitudes that are always greater than one. Incontrast, the change in the output mix yields conflicting results, as previously confirmed by their low correlation and different distributions. It reduces productivity growth along paths A–B (Tables 2 and 3), except for the last period, but increases it along paths C–D, except for the second period (Tables 5 and 6). Both effects tend to compensate each other, and their product, being RTS, is always close to one.

Finally, Tables 7 and 8 show the geometric mean of the base and comparison viewpoints, as defined in expressions (58) and (59).[20] We can conclude from these outcomes that the banking industry in Taiwan experienced relevant productivity growth from 2006 to 2010, particularly in the last period. The main components are technological progress and the scale effect. Technical efficiency change and output mix change are hardly contributed to this growth.

[20] To be precise, these tables contain $[\check{M}_o^t(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt})\check{M}_o^{t+1}(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt})]^{1/2}$ for $k = 1, ..., 31$, $t = 2006, 2007, 2008, 2009$, combining decompositions along paths A, B, C, and D.

Output orientated Malmquist Productivity Indices (MPI)

Table 1: MPI decomposition along paths A and B: Output orientation, base period viewpoint, common components.

All banks	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0438	1.0355	1.0313	1.2065	1.0221	0.9845	0.9749	1.0442	1.0337	1.0425	1.0740	1.1241
Std. Dev.	0.1010	0.1210	0.0989	0.2661	0.0750	0.0923	0.0547	0.0790	0.0857	0.0927	0.1646	0.2411
Max.	1.3026	1.3220	1.1747	2.0755	1.1834	1.2414	1.0770	1.2335	1.3066	1.3128	1.6347	2.1270
Min.	0.8260	0.8273	0.7816	0.8466	0.8758	0.8233	0.8242	0.9304	0.9535	0.8416	0.7942	0.9909

Table 2: MPI decomposition along path A: Output orientation, base period viewpoint, specific components, RTS.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	0.9928	1.0375	1.0272	1.0185	0.9935	0.9769	0.9671	1.0165	0.9863	1.0125	0.9942	1.0363
Std. Dev.	0.0543	0.1006	0.0565	0.0632	0.0456	0.0536	0.0763	0.0780	0.0709	0.1010	0.1012	0.1117
Max.	1.0940	1.4642	1.2505	1.1398	1.0754	1.0427	1.1223	1.2056	1.1765	1.3812	1.2490	1.2591
Min.	0.8265	0.9139	0.9329	0.8081	0.8165	0.7822	0.7584	0.8723	0.7830	0.7934	0.7142	0.8220

Table 3: MPI decomposition along path B: Output orientation, base period viewpoint, specific components, RTS.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	0.9931	1.0385	1.0287	1.0136	0.9932	0.9695	0.9668	1.0212	0.9863	1.0125	0.9942	1.0363
Std. Dev.	0.0442	0.0954	0.0560	0.0636	0.0508	0.0614	0.0763	0.0833	0.0709	0.1010	0.1012	0.1117
Max	1.0841	1.4283	1.2600	1.1398	1.1620	1.0427	1.1193	1.2540	1.1765	1.3812	1.2490	1.2591
Min	0.8968	0.9268	0.9588	0.8233	0.8147	0.7822	0.7449	0.8620	0.7830	0.7934	0.7142	0.8220

Table 4: MPI decomposition along paths C and D: Output orientation, comparison period viewpoint, common components.

All banks	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0270	1.0202	1.0257	1.1509	1.0221	0.9845	0.9749	1.0442	0.9762	0.9905	0.9893	1.0047
Std. Dev.	0.0957	0.1418	0.1008	0.1945	0.0750	0.0923	0.0547	0.0790	0.0847	0.1024	0.0757	0.0679
Max.	1.2953	1.3127	1.1846	1.5712	1.1834	1.2414	1.0770	1.2335	1.0790	1.0918	1.1550	1.2254
Min.	0.8322	0.5820	0.7458	0.8521	0.8758	0.8233	0.8242	0.9304	0.5714	0.5038	0.6509	0.7669

Table 5: MPI decomposition along path C: Output orientation, comparison period viewpoint, specific components, RTS.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	1.0250	1.0507	1.0521	1.0390	1.1025	0.9995	1.0146	1.0590	1.0367	1.0483	1.0663	1.0999
Std. Dev.	0.0966	0.0920	0.0584	0.0668	0.5024	0.0516	0.0695	0.1088	0.1117	0.0827	0.0781	0.1150
Max	1.4768	1.4105	1.2680	1.1428	3.8433	1.1651	1.1953	1.4325	1.4977	1.2897	1.2570	1.4523
Min	0.9001	0.9556	0.9511	0.8188	0.9030	0.8872	0.8543	0.9374	0.8753	0.8867	0.8576	0.8770

Table 6: MPI decomposition along path D: Output orientation, comparison period viewpoint, specific components, RTS.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	1.0312	1.0571	1.0548	1.0434	1.0055	0.9930	1.0117	1.0555	1.0367	1.0483	1.0663	1.0999
Std. Dev.	0.1060	0.0884	0.0583	0.0643	0.0440	0.0478	0.0649	0.1068	0.1117	0.0827	0.0781	0.1150
Max.	1.5375	1.3925	1.2688	1.1461	1.1375	1.1356	1.1740	1.4327	1.4977	1.2897	1.2570	1.4523
Min.	0.9001	0.9556	0.9511	0.8281	0.9030	0.8872	0.8541	0.9379	0.8753	0.8867	0.8576	0.8770

Table 7: MPI decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

All banks	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0347	1.0274	1.0284	1.1772	1.0221	0.9845	0.9749	1.0442	1.0019	1.0148	1.0277	1.0581
Std. Dev.	0.0906	0.1302	0.0985	0.2232	0.0750	0.0923	0.0547	0.0790	0.0537	0.0870	0.0949	0.1176
Max.	1.2989	1.3029	1.1796	1.7774	1.1834	1.2414	1.0770	1.2335	1.1273	1.1967	1.2484	1.5152
Min.	0.8291	0.6939	0.7643	0.8493	0.8758	0.8233	0.8242	0.9304	0.8509	0.6512	0.7190	0.9732

Table 8: MPI decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	1.0094	1.0457	1.0413	1.0282	1.0003	0.9857	0.9880	1.0367	1.0099	1.0293	1.0280	1.0656
Std. Dev.	0.0596	0.0917	0.0528	0.0591	0.0376	0.0411	0.0547	0.0703	0.0740	0.0821	0.0665	0.0897
Max.	1.2187	1.4236	1.2618	1.1413	1.1018	1.0482	1.1275	1.1917	1.2150	1.3347	1.2530	1.2200
Min.	0.9010	0.9606	0.9637	0.8195	0.9030	0.8735	0.8603	0.9379	0.8762	0.8990	0.8636	0.8491

4 Decomposing a Malmquist productivity index by input distance functions

Since a cone technology exhibits global CRS, $\check{D}_o^t(x,y) = 1/\check{D}_i^t(x,y)$, the productivity index defined by expression (9) can also be written as

$$\check{M}_o^t(x',y',x,y) = \frac{\check{D}_i^t(x,y)}{\check{D}_i^t(x',y')} \equiv \check{M}_i^t(x',y',x,y); \tag{62}$$

that is, as an input orientated Malmquist index conditional on the period t cone technology. The productivity level at the input-output situation (x,y) is now measured by $1/\check{D}_i^t(x,y)$. The productivity index at the right-hand side of expression (62) has the desired monotonicity and proportionality properties, is by construction transitive in (x,y), and its numerator and denominator are always finite.

As before, we consider a number of options. The discussion is brief as the theoretical development runs parallel to the previous section

4.1 The base period viewpoint

Let us first consider the index which conditions on the base period cone technology; that is,

$$\check{M}_i^0(x^1,y^1,x^0,y^0) = \frac{\check{D}_i^0(x^0,y^0)}{\check{D}_i^0(x^1,y^1)}. \tag{63}$$

The imagination will now be guided by Figure 2. To start with, we consider the following path from the firm's base period position to its comparison period position:

Path E: $(x^0,y^0) \longrightarrow (x^0/D_i^0(x^0,y^0),y^0) \longrightarrow (x^1/D_i^0(x^1,y^0),y^0) \longrightarrow (x^1/D_i^0(x^1,y^1),y^1) \longrightarrow (x^1/D_i^1(x^1,y^1),y^1) \longrightarrow (x^1,y^1)$

The first segment connects the firm's base period position to its projection, now in the x^0-direction, on the base period technology frontier (point a). We next travel along this frontier to point b, which represents the projection of the firm's comparison period position on the base period frontier. This movement is split into two parts, respectively corresponding to the change in input quantity space and the change in output quantity space. The fourth segment bridges the distance between the two technology frontiers at point b, to arrive at point c. The final segment connects this point, i.e., the projection of the firm's comparison period position on the comparison period frontier to the firm's comparison period position itself.

By using the same technique as before, and assuming that $D_i^0(x^1,y^0)$ and $D_i^0(x^1,y^1)$ are finite, this leads to the following decomposition:

$$\begin{aligned}\check{M}_i^0(x^1,y^1,x^0,y^0) = M_i^0(x^1,y^1,x^0,y^0)\times \\ IME_M^0(x^1,x^0,y^0) \times SEC_{i,M}^0(x^1,y^1,y^0),\end{aligned} \tag{64}$$

where

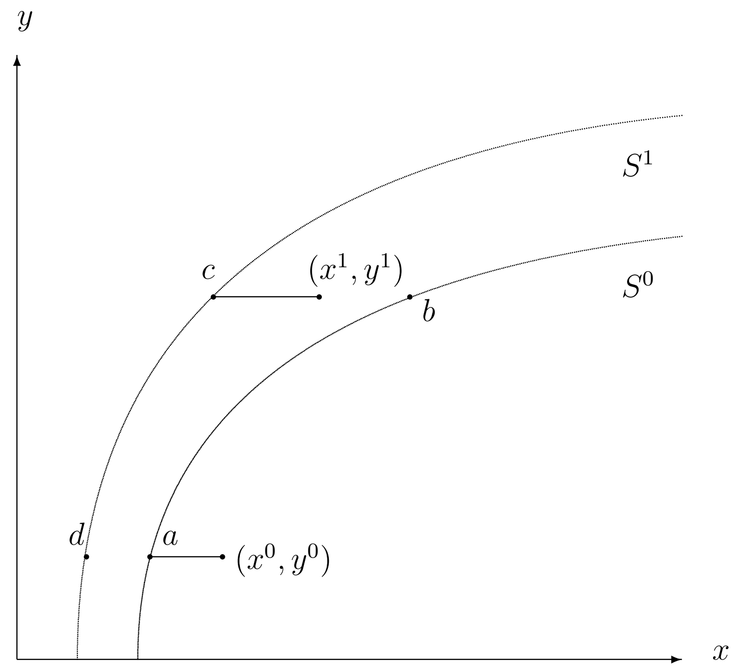

Figure 2: Decomposing productivity change (2)

$$M_i^0(x^1, y^1, x^0, y^0) \equiv EC_i(x^1, y^1, x^0, y^0) \times TC_i^{1,0}(x^1, y^1) = \frac{D_i^0(x^0, y^0)}{D_i^0(x^1, y^1)} \tag{65}$$

defines the base period input orientated *CCD index*, like its output orientated counterpart introduced by Caves, Christensen, and Diewert (1982). The generic definition reads

$$M_i^t(x', y', x, y) \equiv \frac{D_i^t(x, y)}{D_i^t(x', y')}. \tag{66}$$

Notice that in general this function does not exhibit the proportionality property required for a genuine productivity index. In expression (65) we see that the CCD index $M_i^0(x^1, y^1, x^0, y^0)$ comprises efficiency change, defined as $EC_i(x^1, y^1, x^0, y^0) \equiv D_i^0(x^0, y^0)/D_i^1(x^1, y^1)$, and technological change, defined as $TC_i^{1,0}(x^1, y^1) \equiv D_i^1(x^1, y^1) /D_i^0(x^1, y^1)$. Further,

$$IME_M^0(x^1, x^0, y^0) = \frac{ISE^0(x^1, y^0)}{ISE^0(x^0, y^0)} \tag{67}$$

is the input mix effect, and

$$SEC_{i,M}^0(x^1, y^1, y^0) = \frac{ISE^0(x^1, y^1)}{ISE^0(x^1, y^0)} \tag{68}$$

is the scale (including output mix) effect.[21] The definitions of the last two concepts were explained in Balk (2001).

The alternative path is obtained by reversing the order in which, along the segment from a to b, changes in input and output quantity space are accounted for. Thus, assuming that $D_i^0(x^0, y^1)$ and $D_i^0(x^1, y^1)$ are finite,

Path F: $(x^0, y^0) \longrightarrow (x^0/D_i^0(x^0, y^0), y^0) \longrightarrow (x^0/D_i^0(x^0, y^1), y^1) \longrightarrow (x^1/D_i^0(x^1, y^1), y^1) \longrightarrow (x^1/D_i^1(x^1, y^1), y^1) \longrightarrow (x^1, y^1)$

This leads to the second decomposition, which reads

$$\check{M}_i^0(x^1, y^1, x^0, y^0) = M_i^0(x^1, y^1, x^0, y^0) \times \\ IME_M^0(x^1, x^0, y^1) \times SEC_{i,M}^0(x^0, y^1, y^0). \tag{69}$$

This decomposition was obtained by Balk (2001) and used in the empirical work of Pantzios, Karagiannis, and Tzouvelekas (2011).

The third decomposition is obtained by taking the geometric mean of the previous two decompositions, expressions (64) and (69), resulting in

$$\check{M}_i^0(x^1, y^1, x^0, y^0) = M_i^0(x^1, y^1, x^0, y^0) \times \\ [IME_M^0(x^1, x^0, y^0) IME_M^0(x^1, x^0, y^1)]^{1/2} \times \\ [SEC_{i,M}^0(x^0, y^1, y^0) SEC_{i,M}^0(x^1, y^1, y^0)]^{1/2}. \tag{70}$$

The first factor at the right-hand side of the equality sign captures technological change and efficiency change, the second factor captures the scale effect, and the third factor captures the input mix effect. Using the various definitions, it is straightforward to check that if the base period technology exhibits global CRS (i.e., $S^0 = \check{S}^0$), then the scale and mix effects vanish.

4.2 The comparison period viewpoint

Next, we consider the index which conditions on the comparison period cone technology; that is,

$$\check{M}_i^1(x^1, y^1, x^0, y^0) = \frac{\check{D}_i^1(x^0, y^0)}{\check{D}_i^1(x^1, y^1)}, \tag{71}$$

and the following path connecting the firm's base period position with its comparison period position:

Path G: $(x^0, y^0) \longrightarrow (x^0/D_i^0(x^0, y^0), y^0) \longrightarrow (x^0/D_i^1(x^0, y^0), y^0) \longrightarrow (x^1/D_i^1(x^1, y^0), y^0) \longrightarrow (x^1/D_i^1(x^1, y^1), y^1) \longrightarrow (x^1, y^1)$

Recall Figure 2. The first part takes us from the firm's base period position to its projection on the base period frontier (point a). At this point we cross over to

[21] This effect could of course be decomposed into a radial scale effect (in output quantity space) and an output mix effect by introducing a scalar μ that plays a similar role as λ in the previous section. This complication is not pursued here.

point d, which is the projection of the firm's base period position on the comparison period frontier. Then we move along this frontier to point c, which is the projection of the firm's comparison period position on the comparison period frontier. This movement is split into two parts, respectively corresponding to a movement in input quantity space and a movement in output quantity space. Finally, we connect point c to the firm's comparison period position.

Assuming that $D_i^1(x^0, y^0)$ and $D_i^1(x^1, y^0)$ are finite, we obtain the following decomposition:

$$\begin{aligned} \check{M}_i^1(x^1, y^1, x^0, y^0) = M_i^1(x^1, y^1, x^0, y^0) \times \\ IME_M^1(x^1, x^0, y^0) \times SEC_{i,M}^1(x^1, y^1, y^0), \end{aligned} \tag{72}$$

where

$$M_i^1(x^1, y^1, x^0, y^0) \equiv EC_i(x^1, y^1, x^0, y^0) \times TC_i^{1,0}(x^0, y^0) = \frac{D_i^1(x^0, y^0)}{D_i^1(x^1, y^1)} \tag{73}$$

defines the comparison period input orientated *CCD index*, comprising efficiency change and technological change. Further, $IME_M^1(x^1, x^0, y^0)$ is the input mix effect, and $SEC_{i,M}^1(x^1, y^1, y^0)$ is the scale (including output mix) effect. The decomposition in expression (72) was obtained by Balk (2001).

The alternative path is obtained by reversing the order in which, along the segment from d to c, changes in input and output quantity space are accounted for. Thus,

Path H: $(x^0, y^0) \longrightarrow (x^0/D_i^0(x^0, y^0), y^0) \longrightarrow (x^0/D_i^1(x^0, y^0), y^0) \longrightarrow$ $(x^0/D_i^1(x^0, y^1), y^1) \longrightarrow (x^1/D_i^1(x^1, y^1), y^1) \longrightarrow (x^1, y^1)$

Assuming that $D_i^1(x^0, y^0)$ and $D_i^1(x^0, y^1)$ are finite, we obtain the second decomposition, which reads

$$\begin{aligned} \check{M}_i^1(x^1, y^1, x^0, y^0) = M_i^1(x^1, y^1, x^0, y^0) \times \\ IME_M^1(x^1, x^0, y^1) \times SEC_{i,M}^1(x^0, y^1, y^0). \end{aligned} \tag{74}$$

Like before, the third decomposition is obtained by taking the geometric mean of these two decompositions, expressions (72) and (74), resulting in

$$\begin{aligned} \check{M}_i^1(x^1, y^1, x^0, y^0) = M_i^1(x^1, y^1, x^0, y^0) \times \\ [IME_M^1(x^1, x^0, y^0) IME_M^1(x^1, x^0, y^1)]^{1/2} \times \\ [SEC_{i,M}^1(x^0, y^1, y^0) SEC_{i,M}^1(x^1, y^1, y^0)]^{1/2}. \end{aligned} \tag{75}$$

The first factor at the right-hand side of the equality sign captures technological change and efficiency change, the second factor captures the scale effect, and the third factor captures the input mix effect. As before, it is straightforward to check that if the comparison period technology exhibits global CRS (i.e., $S^1 = \check{S}^1$), then the scale and mix effects vanish.

4.3 The 'geometric mean' viewpoint

Finally, we consider the geometric mean of the two input-distance-function-based Malmquist productivity indices $\check{M}_i^0(x^1, y^1, x^0, y^0)$ and $\check{M}_i^1(x^1, y^1, x^0, y^0)$. Again there are nine possible decompositions. The completely symmetric one is obtained by combining the decompositions (70) and (75), resulting in

$$
\begin{gathered}
\check{M}_i(x^1, y^1, x^0, y^0) \equiv \\
[M_i^0(x^1, y^1, x^0, y^0) M_i^1(x^1, y^1, x^0, y^0)]^{1/2} \times \\
[IME_M^0(x^1, x^0, y^0) IME_M^0(x^1, x^0, y^1) IME_M^1(x^1, x^0, y^0) IME_M^1(x^1, x^0, y^1)]^{1/4} \times \\
[SEC_{i,M}^0(x^0, y^1, y^0) SEC_{i,M}^0(x^1, y^1, y^0) SEC_{i,M}^1(x^0, y^1, y^0) SEC_{i,M}^1(x^1, y^1, y^0)]^{1/4} = \qquad (76) \\
[M_i^0(x^1, y^1, x^0, y^0) M_i^1(x^1, y^1, x^0, y^0)]^{1/2} \times \\
\left[[IME_M^0(x^1, x^0, y^0) IME_M^1(x^1, x^0, y^0)]^{1/2} [IME_M^0(x^1, x^0, y^1) IME_M^1(x^1, x^0, y^1)]^{1/2}\right]^{1/2} \times \\
\left[[SEC_{i,M}^0(x^0, y^1, y^0) SEC_{i,M}^1(x^0, y^1, y^0)]^{1/2} [SEC_{i,M}^0(x^1, y^1, y^0) SEC_{i,M}^1(x^1, y^1, y^0)]^{1/2}\right]^{1/2},
\end{gathered}
$$

where the part after the last equality sign is a simple rearrangement of the foregoing. This second decomposition shows what happens when we first combine decompositions along paths E and G, F and H, and next combine these two combinations. In the light of the theoretical framework developed here, we see that the geometric mean index proposed by Balk (2001) corresponds to the combination of expressions (69) and (72).

Notice that $[M_i^0(x^1, y^1, x^0, y^0) M_i^1(x^1, y^1, x^0, y^0)]^{1/2}$ is the geometric mean input orientated *CCD index*. In the extant literature on productivity measurement, this index frequently figures under the name 'the (input orientated) Malmquist productivity index'. Notice, however, that a CCD index does not satisfy the proportionality requirement. If the base and comparion period technologies both exhibit global CRS (i.e., $S^0 = \check{S}^0$ and $S^1 = \check{S}^1$), then the scale and mix effects in expression (76) vanish.

4.4 Empirical application

We calculate the input orientated Malmquist productivity index (MPI) and its components, considering the base, comparison, and 'mean' period viewpoints respectively, according to the structure laid out in section 3.4.2.[22] Thus, the first set of three tables corresponds to the base period viewpoint. Table 9 contains $\check{M}_i^0(x^1, y^1, x^0, y^0)$, as well as its efficiency and technological change components, $EC_i(x^1, y^1, x^0, y^0)$ and $TC_i^{1,0}(x^1, y^1)$, along paths E and F. Tables 10 and 11 present the specific components for the scale (including input mix) and output mix effects corresponding to both paths. Similarly, Tables 12, 13 and 14 report the results for the comparison period MPI, $\check{M}_i^1(x^1, y^1, x^0, y^0)$. The pairwise comparisons between paths E–G and F–H yield the same conclusions as those between paths A–C and

[22] To be precise, the tables contain $\check{M}_i^t(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt})$, $\check{M}_i^{t+1}(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt})$, and $[\check{M}_i^t(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}) \check{M}_i^{t+1}(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt})]^{1/2}$, respectively, for $k = 1, ..., 31$, $t = 2006, 2007, 2008, 2009$, along paths E, F, G, H, and their combination.

B–D. Consequently, here we focus mainly on the differences between the input and output orientated indices.

As $\check{M}_i^0(x^1, y^1, x^0, y^0) = \check{M}_o^0(x^1, y^1, x^0, y^0)$ (recall expression (62)) columns 2-5 of Table 9 are identical to those of Table 1. This is not the case for the efficiency change and technological change components. However, the average values for the input and output orientated efficiency change, $EC_i(x^1, y^1, x^0, y^0)$ and $EC_o(x^1, y^1, x^0, y^0)$ are similar for all the biennial periods, showing efficiency increases in the first and last, and decreases in the middle years, with a higher standard deviation for 2007-2008. Although there are large differences between the maximum and minimum values, the correlation between the two is rather high: $\rho(EC_i(x^1, y^1, x^0, y^0), EC_o(x^1, y^1, x^0, y^0))$ $= 0.876$. Again, we tested for the equality of means relying on t-tests for all the biennial periods, and the null hypothesis of equal means is not rejected.

However, the technological change component $TC_i^{1,0}(x^1,\ y^1)$, corresponding to paths E–F, differs from its output orientated counterpart $TC_o^{1,0}(x^1, y^1)$, on paths A–B. The largest difference is observed for 2006-2007, when the average input orientated $TC_i^{1,0}(x^1, y^1)$ reached 15.62%, while average output orientated $TC_o^{1,0}(x^1, y^1)$ was only 3.37%. Although these differences are large, they are still smaller than those between $TC_i^{1,0}(x^1, y^1)$ and its comparison period counterpart $TC_i^{1,0}\ (x^0, y^0)$, corresponding to paths G–H. Indeed, $\rho(TC_i^{1,0}(x^1, y^1), TC_o^{1,0}(x^1, y^1)) = 0.783$, while $\rho(TC_i^{1,0}(x^1, y^1),\ TC_i^{1,0}(x^0, y^0)) = 0.036$. In the former case, this is due to estimating the underlying technologies by variable-returns-to-scale DEA. In the latter case, the differences are due to the use of different benchmark periods. Moreover, the t-tests show that the distributions do not have the same means in half of the biennial periods. Therefore, based on the input and output orientated results, we conclude that the differences between orientations tend to be smaller than between reference periods.

Notice that infeasibilities are found for different banks (e.g., bank #2 in Table S.9 instead of bank #1 in Table S.1). Focusing on technological change, the maximum value is achieved by the efficient bank #26 with a technological progress of 235.48%, rather than the 26.71% from the output orientation. However, such large individual differences are not observed at the aggregate level, as previously shown.

Table 10 reports the input mix effect $IME_M^0(x^1, x^0, y^0)$ and the scale effect $SEC_{i,M}^0(x^1, y^1, y^0)$ corresponding to path E (expression (64)), as well as the returns-to-scale effect $ISE^0(x^1, y^1)/ISE^0(x^0, y^0)$. Here, as expected, the information provided by the scale and mix effects differs substantially from the information coming from the output side of the production process. The weak relationship is summarized by the correlation coefficients: $\rho(IME_M^0(x^1, x^0, y^0), OME_M^0(x^1, y^1, y^0)) =$ 0.044, and $\rho(SEC_{i,M}^0(x^1, y^1, y^0), SEC_{o,M}^0(x^1, x^0, y^0)) = 0.113$. In the first case, however, t-tests do not reject the hypothesis of equality of means except for the period 2007-2008, while in the second case, the hypothesis is not rejected for the first and last biennial periods only. However, merging both effects into the returns-to-scale factor increases the correlation between the input and output orientated measures to 0.590, and results in the equality of means for all periods. This illustrates the loss of information when components of productivity change are merged. Consequently, even if, as previously mentioned, these scale, mix and returns-to-scale factors play a limited role in the productivity change of the Taiwanese banking industry, their differences raise concerns about the need to examine both sides of the production process to get a full understanding of productivity change and the implications for managerial decision-making and economic policy analysis.

Finally, from the base period perspective, the alternative scale and input mix effects along path F are presented in Table 11. The factors in expression (69), $IME_M^0(x^1, x^0, y^1)$ and $SEC_{i,M}^0(x^0, y^1, y^0)$, hardly differ from their path E counterparts as shown by the correlation coefficients: $\rho(IME_M^0(x^1, x^0, y^1), IME_M^0(x^1, x^0, y^0)) = 0.842$ and $\rho(SEC_{i,M}^0(x^0, y^1, y^0), SEC_{i,M}^0(x^1, y^1, y^0)) = 0.993$. The t-tests confirm the pairwise equality of means. But they are also uncorrelated and differ in their distributions with their output orientated counterparts in Table 3.

Tables 12, 13, and 14 show the decomposition from the comparison period viewpoint, corresponding to expressions (72) and (74), along paths G–H, respectively. Notice that by definition, columns 2-5 of Tables 12 and 4 are identical. The common efficiency change $EC_i(x^1, y^1, x^0, y^0)$ and technological change $TC_i^{1,0}(x^0, y^0)$ are reported in Table 12. Tables 13 and 14 show the scale and input mix effects corresponding to each path: $IME_M^1(x^1, x^0, y^0)$ and $SEC_{i,M}^1(x^1, y^1, y^0)$ in Table 13, and $IME_M^1(x^1, x^0, y^1)$ and $SEC_{i,M}^1(x^0, y^1, y^0)$ in Table 14. In this case, the pairwise correlations with the output orientated factors shown in Tables 4, 5 and 6 are low, and with distributions characterized by different means, as previously shown for the base period viewpoint. The fact that infeasibilities affect different banks, driving differences in the average, standard deviations and maximum and minimum values can be examined by the interested reader. We only highlight that these differences mainly affect the technological change factor, with their maximum and minimum values across the different years.

Finally, Tables 15 and 16 show the geometric mean viewpoint as presented in expression (76), along with the input orientated counterpart to expression (59). Notice that by definition, columns 2-5 of Tables 15 and 7 are identical. Table 16 presents the geometric mean of all input mix effects and the geometric mean of all scale (including output mix) effects, corresponding to the last part of expression (76). Again, given the different values that are obtained from the base and comparison period viewpoints, the 'mean' period values represent a middle ground for the analyst. As with their output counterparts, we can conclude that the large productivity growth experienced by the Taiwanese banking industry is mainly driven by technological progress, and to a lesser extent by scale effects. However, over the entire four year period the input mix effect generally contributes to productivity growth but the output mix effect reduces it. Efficiency change, however, remains neutral, with increasing and decreasing biennial variations compensating each other.

Input orientated Malmquist Productivity Indices (MPI)

Table 9: MPI decomposition along paths E and F: Input orientation, base period viewpoint, common components.

All banks	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0438	1.0355	1.0313	1.2065	1.0174	0.9622	0.9537	1.0729	1.1562	1.0838	1.0490	1.1528
Std. Dev.	0.1010	0.1210	0.0989	0.2661	0.1177	0.1270	0.0905	0.1037	0.4486	0.1193	0.0845	0.4454
Max.	1.3026	1.3220	1.1747	2.0755	1.4838	1.2266	1.0827	1.3134	3.3548	1.3088	1.3056	3.3612
Min.	0.8260	0.8273	0.7816	0.8466	0.8257	0.6174	0.5833	0.8762	0.9325	0.8335	0.8844	0.9189

Table 10: MPI decomposition along path E: Input orientation, base period viewpoint, specific components, RTS.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	0.9902	1.0247	1.0231	1.0179	0.9508	0.9809	1.0009	0.9786	0.9454	1.0087	1.0328	0.9990
Std. Dev.	0.0416	0.0654	0.0607	0.0546	0.1541	0.0864	0.0963	0.1196	0.1585	0.1037	0.1054	0.1374
Max.	1.0658	1.1892	1.2733	1.1142	1.1227	1.1795	1.3926	1.1610	1.1227	1.1795	1.3926	1.1905
Min.	0.8857	0.8692	0.9307	0.8121	0.3723	0.7624	0.8842	0.5743	0.3723	0.7933	0.8842	0.5743

Table 11: MPI decomposition along path F: Input orientation, base period viewpoint, specific components, RTS.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	1.0076	1.0336	1.0338	1.0254	0.9457	0.9771	0.9999	0.9757	0.9454	1.0087	1.0328	0.9990
Std. Dev.	0.0725	0.0674	0.0615	0.0619	0.1672	0.0923	0.0966	0.1308	0.1585	0.1037	0.1054	0.1374
Max.	1.3258	1.1948	1.2737	1.1655	1.1227	1.1795	1.3926	1.1663	1.1227	1.1795	1.3926	1.1905
Min.	0.8857	0.8864	0.9914	0.8250	0.2808	0.7747	0.8842	0.5743	0.3723	0.7933	0.8842	0.5743

Table 12: MPI decomposition along paths G and H: Input orientation, comparison period viewpoint, common components.

All banks	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0270	1.0202	1.0257	1.1509	1.0174	0.9622	0.9537	1.0729	1.0046	1.0041	1.0079	0.9874
Std. Dev.	0.0957	0.1418	0.1008	0.1945	0.1177	0.1270	0.0905	0.1037	0.1104	0.1608	0.0797	0.0619
Max.	1.2953	1.3127	1.1846	1.5712	1.4838	1.2266	1.0827	1.3134	1.3215	1.2284	1.1693	1.1399
Min.	0.8322	0.5820	0.7458	0.8521	0.8257	0.6174	0.5833	0.8762	0.7369	0.2507	0.8347	0.8444

Table 13: MPI decomposition along path G: Input orientation, comparison period viewpoint, specific components, RTS.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	1.0090	1.0337	1.0429	1.0282	1.0129	1.0721	1.0331	1.0564	1.0213	1.1009	1.0750	1.0838
Std. Dev.	0.0371	0.0587	0.0657	0.0632	0.1105	0.2829	0.1004	0.1119	0.1124	0.2421	0.0976	0.1109
Max.	1.0773	1.1900	1.2804	1.1497	1.4251	2.5280	1.3247	1.4208	1.4491	2.3219	1.3247	1.4208
Min.	0.9074	0.9185	0.9170	0.8264	0.7417	0.8427	0.9009	0.9044	0.7800	0.9118	0.9009	0.8959

Table 14: MPI decomposition along path H: Input orientation, comparison period viewpoint, specific components, RTS.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	1.0124	1.0450	1.0492	1.0364	1.0092	1.0574	1.0264	1.0473	1.0213	1.1009	1.0750	1.0838
Std. Dev.	0.0428	0.0647	0.0662	0.0597	0.1084	0.2513	0.0924	0.1068	0.1124	0.2421	0.0976	0.1109
Max.	1.1451	1.2089	1.2804	1.1558	1.4043	2.3219	1.3247	1.4094	1.4491	2.3219	1.3247	1.4208
Min.	0.9074	0.9731	0.9169	0.8351	0.7468	0.7996	0.9009	0.9044	0.7800	0.9118	0.9009	0.8959

Table 15: MPI decomposition along paths EFGH: Input orientation, 'mean' period viewpoint.

All banks	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0347	1.0274	1.0284	1.1772	1.0174	0.9622	0.9537	1.0729	1.0629	1.0376	1.0314	1.0587
Std. Dev.	0.0906	0.1302	0.0985	0.2232	0.1177	0.1270	0.0905	0.1037	0.1500	0.1350	0.0726	0.1804
Max.	1.2989	1.3029	1.1796	1.7774	1.4838	1.2266	1.0827	1.3134	1.6359	1.2482	1.1804	1.9133
Min.	0.8291	0.6939	0.7643	0.8493	0.8257	0.6174	0.5833	0.8762	0.8776	0.4571	0.8630	0.9189

Table 16: MPI decomposition along paths EFGH: Input orientation, 'mean' period viewpoint.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
Average	1.0078	1.0330	1.0356	1.0278	0.9717	1.0182	1.0099	1.0023	0.9778	1.0493	1.0444	1.0295
Std. Dev.	0.0360	0.0593	0.0629	0.0566	0.1196	0.1263	0.0898	0.0789	0.1175	0.1190	0.0946	0.0924
Max.	1.0730	1.1908	1.2769	1.1332	1.1973	1.5495	1.3582	1.1645	1.2168	1.5181	1.3582	1.2075
Min.	0.9074	0.9672	0.9576	0.8246	0.5889	0.8225	0.8925	0.8590	0.6320	0.8883	0.8925	0.8565

5 Conclusion on the decomposition of Malmquist productivity indices

Due to the fact that any Malmquist productivity index has two appearances, namely as an input or an output orientated index, a large number of decompositions exist. The index which conditions on the base period technology was shown to admit six decompositions: expressions (29), (40), and (43) using output distance functions, and expressions (64), (69), and (70) using input distance functions. Likewise, the index which conditions on the comparison period technology admits six decompositions: expressions (46), (51), and (54) using output distance functions, and expressions (72), (74), and (75) using input distance functions.

The geometric mean of these two indices admits eighteen decompositions, nine of which use input distance functions, and the other nine use output distance functions. Versions that are symmetric in all the variables can be obtained by merging either the input or the output mix effect with the radial scale effect. Of these, expression (58) uses output distance functions, and expression (76) uses input distance functions. Hence, there is no unique decomposition of productivity change, unless the technologies involved are severely restricted.

6 Decomposing a Moorsteen-Bjurek productivity index

Recall that a productivity index is a function $F(x', y', x, y)$ which behaves as an output quantity index for $x = x' = \bar{x}$, and as the reciprocal of an input quantity index for $y = y' = \bar{y}$. Thus it seems rather natural to *define* a productivity index as an output quantity index divided by an input quantity index. This is the basic idea behind the family of Moorsteen-Bjurek (MB) productivity indices.

For any period t technology, a Malmquist output quantity index, comparing output quantities y' to y, conditional on certain input quantities $\bar{x}$, can be defined as $Q_o^t(y', y, \bar{x}) \equiv D_o^t(\bar{x}, y')/D_o^t(\bar{x}, y)$. Similarly, a Malmquist input quantity index, comparing input quantities x' to x, conditional on certain output quantities $\bar{y}$, can be defined as $Q_i^t(x', x, \bar{y}) \equiv D_i^t(x', \bar{y})/D_i^t(x, \bar{y})$. Both indices can be traced back to suggestions by Moorsteen (1961), and their properties were extensively discussed in Balk (1998, Sections 3.4 and 4.3).[23]

The family of Moorsteen-Bjurek (MB) productivity indices is then defined by

$$MB^t(x', y', x, y; \bar{x}, \bar{y}) \equiv \frac{Q_o^t(y', y, \bar{x})}{Q_i^t(x', x, \bar{y})}; \tag{77}$$

that is, the ratio of a period t Malmquist output quantity index and a period t Malmquist input quantity index, conditional on a vector of input quantities $\bar{x}$ and a vector of output quantities $\bar{y}$, respectively. Notice that there does not need to be any relation between the benchmark technology period t and the timing of the input-output combinations (x', y'), (x, y), or $(\bar{x}, \bar{y})$. Typically, however, in an empirical

[23]See also Diewert and Fox (2017) who used slightly weaker regularity conditions.

application involving many production units, $\bar{x}$ and $\bar{y}$ would be chosen as vectors of sample means.

Notice that expression (77) defines a *family* of indices. Each specific selection of t and $(\bar{x}, \bar{y})$ generates a member of this family. We can easily check that any specific MB index exhibits the monotonicity and proportionality properties which are essential for a productivity index, and that for $\bar{x} \neq x, x'$ and $\bar{y} \neq y, y'$ the index is transitive in (x, y). Determinatess requires that $(\bar{x}, y) \in S^t$, $y \in \Re^M_{++}$, $(x, \bar{y}) \in S^t$, $x' \in \Re^N_{++}$.[24]

Actually, by substituting the two quantity index definitions, we see that an MB productivity index can be written in the form

$$MB^t(x', y', x, y; \bar{x}, \bar{y}) = \frac{D^t_o(\bar{x}, y')/D^t_i(x', \bar{y})}{D^t_o(\bar{x}, y)/D^t_i(x, \bar{y})}; \tag{78}$$

that is, as a ratio of two productivity levels. Up to a scalar normalization, and conditional on certain $\bar{x}$ and $\bar{y}$, the productivity level at the input-output situation (x, y) is thereby measured as $D^t_o(\bar{x}, y)/D^t_i(x, \bar{y})$. Thus, the family of MB indices belongs to the class of "multiplicatively complete" TFP indices, as defined by O'Donnell (2012).[25]

It is interesting to relate the MB indices to the CCD productivity indices introduced in the previous chapters. Going back to expressions (31) and (66), we can verify that

$$MB^t(x', y', x, y; \bar{x}, \bar{y}) = M^t_o(\bar{x}, y', \bar{x}, y) \times M^t_i(x', \bar{y}, x, \bar{y}). \tag{79}$$

The first factor at the right-hand side of the equality sign is an output orientated CCD index comparing the input-output combination $(\bar{x}, y')$ to $(\bar{x}, y)$. The second factor is an input orientated CCD index comparing the input-output combination $(x', \bar{y})$ to $(x, \bar{y})$. It is clear that if $x' = x$, then the MB index reduces to an output orientated CCD index, and if $y' = y$, then the MB index reduces to an input orientated CCD index.[26] Expression (79) also throws light on the fact that MB and CCD indices share the monotonicity properties but not the proportionality property. This should settle the "healthy debate on the relative merits of the orientated [*i.e.*,

[24]Proof: For the numerator, consider $\lambda \equiv \min_m\{y_m/y'_m \mid y'_m > 0\}$. Then $(\bar{x}, \lambda y') \leq (\bar{x}, y)$ and thus, by free disposability of outputs, $(\bar{x}, \lambda y') \in S^t$. Then $D^t_o(\bar{x}, \lambda y') \leq 1$ and, by linear homogeneity of the output distance function, $D^t_o(\bar{x}, y') \leq 1/\lambda < \infty$. For the denominator, consider $\lambda' \equiv \max_n\{x_n/x'_n \mid x'_n > 0\}$. Then $(\lambda' x', \bar{y}) \geq (x, \bar{y})$ and thus, by free disposability of inputs, $(\lambda' x', \bar{y}) \in S^t$. Then $D^t_i(\lambda(x', \bar{y}) \geq 1$ and, by linear homogeneity of the input distance function, $D^t_i(x', \bar{y}) \geq 1/\lambda > 0$.■ This proof generalizes the proof provided by Briec and Kerstens (2011), which was concerned with the specific cases $MB^0(x^1, y^1, x^0, y^0; x^0, y^0)$ and $MB^1(x^1, y^1, x^0, y^0; x^1, y^1)$.

[25]O'Donnell (2014) called the indices defined by expression (77) after Färe and Primont because the component output and input quantity indices were discussed in their 1995 book. The indices were called Bjurek productivity indices by Diewert and Fox (2017). To continue footnote 7, $X(x) \equiv D^t_i(x, \bar{y})$ and $Y(y) \equiv D^t_o(\bar{x}, y)$.

[26]Moreover, if $x \in \Re^1_+$ (single input) and constant and the benchmark technology S^t is equal to its DEA approximation, then $M^t_o(\bar{x}, y', \bar{x}, y) = \check{M}^t_o(\bar{x}, y', \bar{x}, y) = \check{M}^t_i(\bar{x}, y', \bar{x}, y)$, and the scale, input and output mix effects vanish, as noticed by Karagiannis and Lovell (2016). Of course, a similar result holds for the single-output case.

CCD] and non-orientated [*i.e.*, MB] Malmquist productivity indices", called for by Lovell (2016).

We obtain the indices as originally considered by Bjurek (1996) by selecting in the generic definition of the MB index $\bar{x} = x$ and $\bar{y} = y$ or $\bar{x} = x'$ and $\bar{y} = y'$. The first employs the 'Laspeyres' perspective,

$$MB^t(x', y', x, y; x, y) = \frac{Q_o^t(y', y, x)}{Q_i^t(x', x, y)}; \tag{80}$$

the second employs the 'Paasche' perspective,

$$MB^t(x', y', x, y; x', y') = \frac{Q_o^t(y', y, x')}{Q_i^t(x', x, y')}; \tag{81}$$

and the third is their geometric mean. Bjurek (1996) called these indices Malmquist Total Factor Productivity indices, although the fact that they take all the inputs into account is not a feature that distinguishes them from the Malmquist indices discussed in the previous sections.[27]

We can easily check that neither the MB indices defined by expression (80) nor those defined by expression (81) nor their geometric means can be written as a ratio of productivity levels, so that these indices are *not* transitive. This has implications for the way in which these and other indices belonging to the MB family are decomposed into meaningful components, as will be shown in the next subsections.

A further specification concerns the benchmark technology figuring in expressions (80) and (81). The base period, Laspeyres-perspective MB index for period 1 relative to period 0 is then given by $MB^0(x^1, y^1, x^0, y^0; x^0, y^0)$, and the comparison period, Paasche-perspective MB index by $MB^1(x^1, y^1, x^0, y^0; x^1, y^1)$. Conditions under which their geometric mean coincides with the geometric mean CCD indices materializing in expressions (58) and (76), $[M_o^0(x^1, y^1, x^0, y^0)M_o^1\ (x^1, y^1, x^0, y^0)]^{1/2}$ and $[M_i^0(x^1, y^1, x^0, y^0)M_i^1(x^1, y^1, x^0, y^0)]^{1/2}$, respectively, were analyzed in Balk (1998, Section 4.6).[28]

6.1 Satisfactory decompositions

We now consider the general MB productivity index $MB^t(x', y', x, y; \bar{x}, \bar{y})$ for fixed values of the conditioning variables t, $\bar{x}$ and $\bar{y}$. Consider path A and let $\lambda = 1$. The six segments then reduce to five. The MB index is evaluated along each segment.

[27]The geometric mean index was called the Hicks-Moorsteen TFP index by Färe, Grosskopf, and Margaritis (2008), and by O'Donnell (2012) with a reference to footnote 4 in Hicks (1961). It was called Malmquist productivity index by Grifell-Tatjé and Lovell (2015). On closer scrutiny, however, there appears to be insufficient evidence for ascribing a partial fathership to Hicks. Though Hicks definitely discussed the concepts of Malmquist output and input quantity indices in a qualitative, thus not formal, way, there is no hint that he took their ratio as a measure of productivity change. See Epure, Kerstens, and Prior (2011) on the use of the MB indices in benchmarking.

[28]The conditions appear to be 1) that both technologies exhibit global CRS and output or input homotheticity, or 2) that both technologies exhibit global CRS and technological change is implicit Hicks output neutral or input neutral. The second set of conditions generalizes a result obtained by Mizobuchi (2017).

Using its transitivity, the parts can be joined and the following decomposition is obtained:

$$MB^t(x^1,y^1,x^0,y^0;\bar{x},\bar{y}) = EC_o(x^1,y^1,x^0,y^0)\times TC_o^{1,0}(x^1,y^1)\times \left(\frac{D_o^0(x^0,y^0)}{D_o^0(x^1,y^0)}\frac{D_i^t(x^0,\bar{y})}{D_i^t(x^1,\bar{y})}\right)\times\left(\frac{D_o^t(\bar{x},y^1)}{D_o^t(\bar{x},y^0)}\frac{D_o^0(x^1,y^0)}{D_o^0(x^1,y^1)}\right). \tag{82}$$

The factors at the right-hand side of the equality sign represent, respectively, technical efficiency change (corresponding to the first and last segment of the path), technological change (corresponding to the fourth segment), radial scale and input mix effect (corresponding to the second segment, and conditional on y^0), and output mix effect (corresponding to the third segment, and conditional on x^1).

The first two factors are familiar. Together they constitute the base period, output orientated CCD index as defined by expression (30). To facilitate comparisons with the components of the Malmquist indices, we denote the scale (including input mix) effect by

$$SEC_{o,MB}^{0t}(x^1,x^0,y^0;\bar{y}) \equiv \frac{D_o^0(x^0,y^0)}{D_o^0(x^1,y^0)}\frac{D_i^t(x^0,\bar{y})}{D_i^t(x^1,\bar{y})},$$

and the output mix effect by

$$OME_{MB}^{0t}(x^1,y^1,y^0;\bar{x}) \equiv \frac{D_o^t(\bar{x},y^1)}{D_o^t(\bar{x},y^0)}\frac{D_o^0(x^1,y^0)}{D_o^0(x^1,y^1)}.$$

Expression (82) can then be written as

$$MB^t(x^1,y^1,x^0,y^0;\bar{x},\bar{y}) =$$

$$M_o^0(x^1,y^1,x^0,y^0)\times SEC_{o,MB}^{0t}(x^1,x^0,y^0;\bar{y})\times OME_{MB}^{0t}(x^1,y^1,y^0;\bar{x}). \tag{83}$$

The third factor in expression (83) indeed measures the output mix effect. This can be seen by substituting $y^1=\mu y^0$ ($\mu>0$); this factor then becomes equal to 1. To interpret the second factor, we assume that there is no technological change, that the firm is technically efficient in both periods, that $x^1=\lambda x^0$ for some positive scalar λ, and that $y^1=\mu y^0$ for some positive scalar μ. Then expression (83) reduces to

$$MB^t(x^1,y^1,x^0,y^0;\bar{x},\bar{y}) = \frac{D_o^0(x^0,y^0)}{\lambda D_o^0(\lambda x^0,y^0)} = \frac{1}{\lambda D_o^0(\lambda x^0,y^0)}, \tag{84}$$

since the firm is efficient in period 0. By multiplying numerator and denominator by μ, by using the linear homogeneity of the output distance function, and assuming period 1 efficiency, respectively, we obtain

$$MB^t(x^1,y^1,x^0,y^0;\bar{x},\bar{y}) = \frac{\mu}{\lambda D_o^0(\lambda x^0,\mu y^0)} = \frac{\mu}{\lambda D_o^1(x^1,y^1)} = \frac{\mu}{\lambda}, \tag{85}$$

which is the outcome we expect.

If the base period technology exhibits global CRS (i.e., $S^0 = \check{S}^0$), then its output distance function is identically equal to the inverse of its input distance function and the second factor reduces to

$$SEC^{0t}_{o,MB}(x^1, x^0, y^0; \bar{y}) = \frac{D^0_i(x^1, y^0)}{D^0_i(x^0, y^0)} \frac{D^t_i(x^0, \bar{y})}{D^t_i(x^1, \bar{y})}.$$

By using the linear homogeneity of the two input distance functions we obtain

$$SEC^{0t}_{o,MB}(x^1, x^0, y^0; \bar{y}) = \frac{D^0_i(x^1/\|x^1\|, y^0)}{D^0_i(x^0/\|x^0\|, y^0)} \frac{D^t_i(x^0/\|x^0\|, \bar{y})}{D^t_i(x^1/\|x^1\|, \bar{y})},$$

which means that the scale effect has vanished and only the input mix effect remains.

It is interesting to consider under which conditions the scale (including input mix) and output mix effects vanish entirely from expression (82). We can easily check that a sufficient set of conditions is that 1) the base period technology is chosen as benchmark, 2) the base period technology exhibits global CRS, and 3) the conditioning variables are chosen as $(\bar{x}, \bar{y}) = (x^1, y^0)$. Then $MB^0(x^1, y^1, x^0, y^0; x^1, y^0) = M^0_o(x^1, y^1, x^0, y^0)$.

Along path B ($\lambda = 1$) we obtain

$$\begin{aligned} MB^t(x^1, y^1, x^0, y^0; \bar{x}, \bar{y}) = M^0_o(x^1, y^1, x^0, y^0) \times \\ \left(\frac{D^0_o(x^0, y^1)}{D^0_o(x^1, y^1)} \frac{D^t_i(x^0, \bar{y})}{D^t_i(x^1, \bar{y})} \right) \times \left(\frac{D^t_o(\bar{x}, y^1)}{D^t_o(\bar{x}, y^0)} \frac{D^0_o(x^0, y^0)}{D^0_o(x^0, y^1)} \right). \end{aligned} \quad (86)$$

The scale (including input mix) and output mix effects vanish from expression (86) if 1) the base period technology is chosen as benchmark, 2) the base period technology exhibits global CRS, and 3) the conditioning variables are chosen as $(\bar{x}, \bar{y}) = (x^0, y^1)$. Then $MB^0(x^1, y^1, x^0, y^0; x^0, y^1) = M^0_o(x^1, y^1, x^0, y^0)$.

Along path C ($\lambda = 1$) we obtain

$$\begin{aligned} MB^t(x^1, y^1, x^0, y^0; \bar{x}, \bar{y}) = M^1_o(x^1, y^1, x^0, y^0) \times \\ \left(\frac{D^1_o(x^0, y^0)}{D^1_o(x^1, y^0)} \frac{D^t_i(x^0, \bar{y})}{D^t_i(x^1, \bar{y})} \right) \times \left(\frac{D^t_o(\bar{x}, y^1)}{D^t_o(\bar{x}, y^0)} \frac{D^1_o(x^1, y^0)}{D^1_o(x^1, y^1)} \right). \end{aligned} \quad (87)$$

The scale (including input mix) and output mix effects vanish from expression (87) if 1) the comparison period technology is chosen as benchmark, 2) the comparison period technology exhibits global CRS, and 3) the conditioning variables are chosen as $(\bar{x}, \bar{y}) = (x^1, y^0)$. Then $MB^1(x^1, y^1, x^0, y^0; x^1, y^0) = M^1_o(x^1, y^1, x^0, y^0)$.

Along path D ($\lambda = 1$) we obtain

$$\begin{aligned} MB^t(x^1, y^1, x^0, y^0; \bar{x}, \bar{y}) = M^1_o(x^1, y^1, x^0, y^0) \times \\ \left(\frac{D^1_o(x^0, y^1)}{D^1_o(x^1, y^1)} \frac{D^t_i(x^0, \bar{y})}{D^t_i(x^1, \bar{y})} \right) \times \left(\frac{D^t_o(\bar{x}, y^1)}{D^t_o(\bar{x}, y^0)} \frac{D^1_o(x^0, y^0)}{D^1_o(x^0, y^1)} \right). \end{aligned} \quad (88)$$

The scale (including input mix) and output mix effects vanish from expression (88) if 1) the comparison period technology is chosen as benchmark, 2) the commparison period technology exhibits global CRS, and 3) the conditioning variables are chosen as $(\bar{x}, \bar{y}) = (x^0, y^1)$. Then $MB^1(x^1, y^1, x^0, y^0; x^0, y^1) = M^1_o(x^1, y^1, x^0, y^0)$.

In the next four decompositions, output distance functions are replaced by input distance functions. Thus, along path E,

$$MB^t(x^1,y^1,x^0,y^0;\bar{x},\bar{y}) = M_i^0(x^1,y^1,x^0,y^0)\times \left(\frac{D_i^0(x^1,y^0)}{D_i^0(x^0,y^0)}\frac{D_i^t(x^0,\bar{y})}{D_i^t(x^1,\bar{y})}\right)\times\left(\frac{D_o^t(\bar{x},y^1)}{D_o^t(\bar{x},y^0)}\frac{D_i^0(x^1,y^1)}{D_i^0(x^1,y^0)}\right). \quad (89)$$

At the right-hand side of this expression we see, respectively, the base period, input orientated CCD index as defined by expression (65), the input mix effect (conditional on y^0), and the radial scale and output mix effect (conditional on x^1).

The input mix and scale (including output mix) effects vanish from expression (89) if 1) the base period technology is chosen as benchmark, 2) the base period technology exhibits global CRS, and 3) the conditioning variables are chosen as $(\bar{x},\bar{y}) = (x^1,y^0)$. Then $MB^0(x^1,y^1,x^0,y^0;x^1,y^0) = M_i^0(x^1,y^1,x^0,y^0)$.

Path F delivers similarly

$$MB^t(x^1,y^1,x^0,y^0;\bar{x},\bar{y}) = M_i^0(x^1,y^1,x^0,y^0)\times \left(\frac{D_i^0(x^1,y^1)}{D_i^0(x^0,y^1)}\frac{D_i^t(x^0,\bar{y})}{D_i^t(x^1,\bar{y})}\right)\times\left(\frac{D_o^t(\bar{x},y^1)}{D_o^t(\bar{x},y^0)}\frac{D_i^0(x^0,y^1)}{D_i^0(x^0,y^0)}\right). \quad (90)$$

The input mix and scale (including output mix) effects vanish from expression (90) if 1) the base period technology is chosen as benchmark, 2) the base period technology exhibits global CRS, and 3) the conditioning variables are chosen as $(\bar{x},\bar{y}) = (x^0,y^1)$. Then $MB^0(x^1,y^1,x^0,y^0;x^0,y^1) = M_i^0(x^1,y^1,x^0,y^0)$.

Along path G we obtain

$$MB^t(x^1,y^1,x^0,y^0;\bar{x},\bar{y}) = M_i^1(x^1,y^1,x^0,y^0)\times \left(\frac{D_i^1(x^1,y^0)}{D_i^1(x^0,y^0)}\frac{D_i^t(x^0,\bar{y})}{D_i^t(x^1,\bar{y})}\right)\times\left(\frac{D_o^t(\bar{x},y^1)}{D_o^t(\bar{x},y^0)}\frac{D_i^1(x^1,y^1)}{D_i^1(x^1,y^0)}\right). \quad (91)$$

The input mix and scale (including output mix) effects vanish from expression (91) if 1) the comparison period technology is chosen as benchmark, 2) the comparison period technology exhibits global CRS, and 3) the conditioning variables are chosen as $(\bar{x},\bar{y}) = (x^1,y^0)$. Then $MB^1(x^1,y^1,x^0,y^0;x^1,y^0) = M_i^1(x^1,y^1,x^0,y^0)$.

Along path H we obtain

$$MB^t(x^1,y^1,x^0,y^0;\bar{x},\bar{y}) = M_i^1(x^1,y^1,x^0,y^0)\times \left(\frac{D_i^1(x^1,y^1)}{D_i^1(x^0,y^1)}\frac{D_i^t(x^0,\bar{y})}{D_i^t(x^1,\bar{y})}\right)\times\left(\frac{D_o^t(\bar{x},y^1)}{D_o^t(\bar{x},y^0)}\frac{D_i^1(x^0,y^1)}{D_i^1(x^0,y^0)}\right). \quad (92)$$

The input mix and scale (including output mix) effects vanish from expression (92) if 1) the comparison period technology is chosen as benchmark, 2) the comparison period technology exhibits global CRS, and 3) the conditioning variables are chosen as $(\bar{x},\bar{y}) = (x^0,y^1)$. Then $MB^1(x^1,y^1,x^0,y^0;x^0,y^1) = M_i^1(x^1,y^1,x^0,y^0)$.

Thus we have eight decompositions of the productivity index $MB^t(x^1,y^1,x^0,y^0;\bar{x},\bar{y})$. By specifying $t=0$ in expressions (83) and (86), we obtain two decompositions of $MB^0(x^1,y^1,x^0,y^0;\bar{x},\bar{y})$. Similarly, by specifying $t=1$ in expressions

(87) and (88), we obtain two decompositions of $MB^1(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$. By taking their geometric mean, we obtain a decomposition of the geometric mean MB index $[MB^0(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})MB^1(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})]^{1/2}$. All these decompositions make use of output distance functions.

By replacing these by input distance functions we obtain alternative decompositions. Thus, by specifying $t = 0$ in expressions (89) and (90), we obtain two decompositions of $MB^0(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$. Similarly, by specifying $t = 1$ in expressions (91) and (92), we obtain two decompositions of $MB^1(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$. By taking their geometric mean, we obtain a decomposition of the geometric mean MB index $[MB^0(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})MB^1(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})]^{1/2}$.

By further specifying the conditioning variables $\bar{x}$ and $\bar{y}$, we can establish linkages with decompositions proposed in the literature. For instance, by setting $t = 0$ and $(\bar{x}, \bar{y}) = (x^0, y^0)$ in expression (83), we obtain the decomposition contemplated by Grifell-Tatjé and Lovell (2015, 136-137). Before doing this, however, we consider the (in-)transitivity issue in more detail.

6.2 The effect of intransitivity

As mentioned, the specific MB indices in expressions (80) and (81) are not transitive. What does this imply? To see this, we consider the base period, Laspeyres-perspective MB index $MB^0(x', y', x, y; x, y)$ along path A with $\lambda = 1$. It is straightforward to verify that along the first segment the index reduces to

$$1/D_o^0(x^0, y^0). \tag{93}$$

Along the second segment the index reduces to the expression

$$\frac{D_o^0(x^0, y^0)/D_o^0(x^1, y^0)}{D_i^0(x^1, y^0/D_o^0(x^0, y^0))/D_i^0(x^0, y^0/D_o^0(x^0, y^0))}, \tag{94}$$

which represents the radial scale and input mix effect. Along the third segment the index is identically equal to 1, which means that the output mix effect vanishes. Along the fourth segment the index reduces to

$$\frac{D_o^0(x^1, y^1)}{D_o^1(x^1, y^1)} = TC_o^{1,0}(x^1, y^1), \tag{95}$$

which measures technological change. Finally, along the last segment the index reduces to

$$D_o^1(x^1, y^1). \tag{96}$$

Multiplying expressions (93) and (96), one obtains

$$\frac{D_o^1(x^1, y^1)}{D_o^0(x^0, y^0)} = EC_o(x^1, y^1, x^0, y^0), \tag{97}$$

which measures technical efficiency change. By multiplying the four factors: technical efficiency change, technological change, radial scale and input mix effect, and output mix effect, we obtain

$$\frac{D_o^0(x^1,y^1)/D_o^0(x^1,y^0)}{D_i^0(x^1,y^0/D_o^0(x^0,y^0))/D_i^0(x^0,y^0/D_o^0(x^0,y^0))} \tag{98}$$
$$= MB^0(x^1,y^1,x^0,y^0;x^1,y^0/D_o^0(x^0,y^0)).$$

This is an MB index, but clearly not identical to $MB^0(x^1,y^1,x^0,y^0;x^0,y^0)$, which would be our target.

We can easily check that the alternative path where the order of the changes in input and output space are reversed, that is, path B with $\lambda = 1$, leads to a similar result, namely

$$\frac{D_o^0(x^0,y^1)/D_o^0(x^0,y^0)}{D_i^0(x^1,y^1/D_o^0(x^0,y^1))/D_i^0(x^0,y^1/D_o^0(x^0,y^1))}. \tag{99}$$
$$= MB^0(x^1,y^1,x^0,y^0;x^0,y^1/D_o^0(x^0,y^1)),$$

which is also not identical to $MB^0(x^1,y^1,x^0,y^0;x^0,y^0)$. Hence, these two results show the effect of the intransitivity of the base period Laspeyres-perspective MB index.

6.3 Decompositions in the literature

The literature on productivity analysis documents several attempts to decompose an MB index into meaningful components. To start with, we return to the decomposition in expression (83) and specify $t = 0$ and $(\bar{x},\bar{y}) = (x^0,y^0)$. Then expression (83) reduces to

$$MB^0(x^1,y^1,x^0,y^0;x^0,y^0) = M_o^0(x^1,y^1,x^0,y^0)\times$$
$$\frac{D_i^0(x^0,y^0)}{D_i^0(x^1,y^0)} \times \frac{D_o^0(x^0,y^1)}{D_o^0(x^1,y^1)}. \tag{100}$$

This is the decomposition of Grifell-Tatjé and Lovell (2015, 136-137). Of course, similar decompositions hold for the comparison period Paasche-perspective MB index $MB^1(x^1,y^1,x^0,y^0;x^1,y^1)$, and the geometric mean of these two indices.[29]

The product of the last two factors was interpreted as "contribution of size change". From the foregoing, however, we know that a better interpretation is the joint contribution of radial scale, input mix, and output mix. To get another idea of what the last two factors of expression (100) possibly mean, let $x^1 = \lambda x^0$. Then

$$\frac{D_o^0(x^0,y^1)}{D_o^0(x^1,y^1)} \times \frac{D_i^0(x^0,y^0)}{D_i^0(x^1,y^0)} = \frac{D_o^0(x^0,y^1/\|y^1\|)}{\lambda D_o^0(\lambda x^0,y^1/\|y^1\|)},$$

by virtue of the linear homogeneity of output and input distance functions. The right-hand side ratio measures the curvature of the base period technology frontier

[29] The attempt to decompose the product of the last two factors into three components, respectively measuring the radial scale effect, the output mix effect, and the input mix effect, as in Lovell (2003, eq. (26)), unfortunately foundered by a mathematical mistake. Reparation comes at the cost of destroying the interpretation of the component supposedly measuring the radial scale effect.

at the input-output combination $(x^0, y^1/\|y^1\|)$ rather than the size change going from x^0 to x^1.

Nemoto and Goto (2005) provided a decomposition of the geometric mean MB index $[MB^0(x^1, y^1, x^0, y^0; x^0, y^0)MB^1(x^1, y^1, x^0, y^0; x^1, y^1)]^{1/2}$. This proposal can best be understood by first considering the first component of the mean. Let us start with the following identity,

$$MB^0(x^1, y^1, x^0, y^0; x^0, y^0) = M_o^0(x^1, y^1, x^0, y^0) \times \frac{D_o^0(x^0, y^0)}{D_o^0(\lambda^0 x^0, y^0)} \times \frac{D_o^0(\lambda^0 x^0, y^0)}{D_o^0(x^1, y^1)} \times MB^0(x^1, y^1, x^0, y^0; x^0, y^0), \quad (101)$$

where $M_o^0(x^1, y^1, x^0, y^0)$ is the output orientated CCD index, defined by expression (30), and λ^0 is some positive scalar. Using the definition of $MB^0(x^1, y^1, x^0, y^0; x^0, y^0)$ and the linear homogeneity of the output distance function, expression (101) can simply be rearranged into the form

$$MB^0(x^1, y^1, x^0, y^0; x^0, y^0) = M_o^0(x^1, y^1, x^0, y^0) \times \left(\frac{D_o^0(x^0, y^0)}{D_o^0(\lambda^0 x^0, y^0)} \frac{1}{Q_i^0(x^1, x^0, y^0)}\right) \times \frac{D_o^0(\lambda^0 x^0, Q_o^0(y^1, y^0, x^0)y^0)}{D_o^0(x^1, y^1)}. \quad (102)$$

Nemoto and Goto selected $\lambda^0 = Q_i^0(x^1, x^0, y^0)$. The first factor on the right-hand side of expression (102) captures technical efficiency change and technological change. The second factor reflects the scale effect, and the third factor reflects the simultaneous input and output mix effect.

Similarly, one obtains

$$MB^1(x^1, y^1, x^0, y^0; x^1, y^1) = M_o^1(x^1, y^1, x^0, y^0) \times \left(\frac{D_o^1(x^1/\lambda^1, y^1)}{D_o^1(x^1, y^1)} \frac{1}{Q_i^1(x^1, x^0, y^1)}\right) \times \frac{D_o^1(x^0, y^0)}{D_o^1(x^1/\lambda^1, y^1/Q_o^1(y^1, y^0, x^1))}, \quad (103)$$

where $M_o^1(x^1, y^1, x^0, y^0)$ is the output orientated CCD index defined by expression (47), and λ^1 is again some positive scalar. Here Nemoto and Goto selected $\lambda^1 = Q_i^1(x^1, x^0, y^1)$. We can easily check that taking the geometric mean of expressions (102) and (103) delivers the decomposition favoured by Nemoto and Goto.

The foregoing reconstruction, however, reveals the weak points of this decomposition. First, expression (101) is just a restatement of the definition of the CCD index, and thus cannot be considered as a sort of decomposition. Second, the decompositions (102) and (103) are valid for all values of λ^0 and λ^1 respectively. This introduces a high degree of arbitrariness, as in the case of Lovell's decomposition. The choices made by Nemoto and Goto are just two out of an infinity of alternatives.

Diewert and Fox (2017) also considered the geometric mean MB index, $[MB^0(x^1, y^1, x^0, y^0; x^0, y^0)MB^1(x^1, y^1, x^0, y^0; x^1, y^1)]^{1/2}$. They showed that the ratio of this index and the geometric mean *output* orientated CCD index $[M_o^0(x^1, y^1, x^0, y^0)M_o^1(x^1, y^1, x^0,$

$y^0)]^{1/2}$, which captures efficiency change and technological change, can be interpreted as the joint contribution of scale, input and output mix. A similar interpretation holds for the ratio of the geometric mean MB index and the geometric mean *input* orientated CCD index $[M_i^0(x^1, y^1, x^0, y^0)M_i^1(x^1, y^1, x^0, y^0)]^{1/2}$. As shown by Balk (1998, Section 4.6), if the technologies of the two periods exhibit global CRS and input or output homotheticity, respectively, then these two ratios reduce to unity.

Peyrache (2014) considered two special cases of the subfamily of indices[30] $MB^0(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$, namely $(\bar{x}, \bar{y}) = (x^0, y^1/D_o^0(x^0, y^1))$ and $(\bar{x}, \bar{y}) = (x^1, y^0/D_o^0(x^1, y^0))$. He contends that in both cases $MB^0(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$ divided by the base period output orientated CCD index $M_o^0(x^1, y^1, x^0, y^0)$ could be interpreted as (radial) scale change. Indeed, suppose that the base period technology exhibits global CRS, then its input distance function is homogeneous of degree -1 in y and its output distance function is the inverse of its input distance function. One then easily checks that

$$\frac{MB^0(x^1, y^1, x^0, y^0; x^0, y^1/D_o^0(x^0, y^1))}{M_o^0(x^1, y^1, x^0, y^0)} = \frac{D_o^0(x^0, y^1)}{D_o^0(x^1, y^1)} \frac{D_i^0(x^0, y^1)}{D_i^0(x^1, y^1)} = 1, \tag{104}$$

and

$$\frac{MB^0(x^1, y^1, x^0, y^0; x^1, y^0/D_o^0(x^1, y^0))}{M_o^0(x^1, y^1, x^0, y^0)} = \frac{D_o^0(x^0, y^0)}{D_o^0(x^1, y^0)} \frac{D_i^0(x^0, y^0)}{D_i^0(x^1, y^0)} = 1, \tag{105}$$

irrespective of input or output mix effects. These results hold only for the two very special choices of the conditioning variables $(\bar{x}, \bar{y})$.

However, if we do not make the assumption of CRS and let $x^1 = \lambda x^0$ for some positive scalar λ, so that there is no input mix effect, then

$$\frac{MB^0(x^1, y^1, x^0, y^0; x^0, y^1/D_o^0(x^0, y^1))}{M_o^0(x^1, y^1, x^0, y^0)} = \frac{D_o^0(x^0, y^1/\|y^1\|)}{\lambda D_o^0(\lambda x^0, y^1/\|y^1\|)}, \tag{106}$$

and

$$\frac{MB^0(x^1, y^1, x^0, y^0; x^1, y^0/D_o^0(x^1, y^0))}{M_o^0(x^1, y^1, x^0, y^0)} = \frac{D_o^0(x^0, y^0/\|y^0\|)}{\lambda D_o^0(\lambda x^0, y^0/\|y^0\|)}. \tag{107}$$

Thus it turns out that there is no output mix effect at all, and the two final expressions measure the curvature of the base period technology frontier at its intersection with the ray through the input-output combinations $(x^0, y^1/\|y^1\|)$ and $(x^0, y^0/\|y^0\|)$, respectively.

6.4 Empirical application

For each specification of the base and comparison period in $MB^t(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$, the elements of the vector $(\bar{x}, \bar{y})$ are the means of the observed input and output quantities of the sample of production units in the two periods. Tables 17–32 present

[30] This subfamily was called the generalized Hicks-Moorsteen indices.

the main descriptive statistics of the various Moorsteen-Bjurek productivity indices (MBPI) and their decompositions corresponding to expressions (83), (86) - (92), and their geometric means. Tables S.17–S.32 show the outcomes for individual banks.[31]

Recall that MB indices along each pair of paths, A–B, C–D, E–F, and G–H, share the same CCD index and, therefore, the scale, input, and/or output mix factors determine the differences between the pairs of MB indices. For example, as presented in expression (83), the base period MB productivity index $MB^0(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$ is related to the base period output orientated CCD index, $M_o^0(x^1, y^1, x^0, y^0)$. Consequently, the efficiency and technological change components, $EC_o(x^1, y^1, x^0, y^0)$ and $TC_o^{1,0}(x^1, y^1)$, reported in Table 17 are identical to those already reported in Table 1.

As an MB index simultaneously considers the two sides (input and output) of the production process, rather than the single side considered by a CCD index, it comes as no surprise that we find more cases in which an MB index cannot be calculated because one of its components becomes infinite. Indeed, when a CCD index appears infeasible, this normally translates to the corresponding MB index.

The second factor in expression (83) merges the radial scale and input mix effects, and the third factor captures the output mix effect – both factors are counterparts to $SEC_{o,M}^0(x^1, x^0, y^0)$ and $OME_M^0(x^1, y^1, y^0)$ in expression (27). Table 18 shows the results for path A, and Table 19 shows the analogous results for path B, as presented in expression (86).

On average, productivity growth in the Taiwanese banking industry computed according to the base period MB index is a few percentage points lower than the growth computed according to the base period viewpoint Malmquist index. For example, the percentage changes were 3.55% and 4.38%, respectively, from 2006 to 2007, and were similar for the other periods. However, both indices consistently yield productivity growth throughout the period, and exhibit a relatively high correlation: $\rho(MB^0(x^1, y^1, x^0, y^0; \bar{x}, \bar{y}), \check{M}_o^0(x^1, y^1, x^0, y^0)) = 0.810$. The t-test does not reject the equality of means, while the Wilcoxon test rejects the equality of medians only in last biennial period 2009-2010. This confirms expectations since, the indices share the efficiency and technological change components. A pairwise comparison of the scale and mix components in Tables 2 and 18 for path A, and Tables 3 and 19 for path B, reveals that on average the differences are relatively small, with the latter values showing less variability. Specifically $\rho(SEC_{MB}^0(x^1, x^0, y^0; \bar{y}), SEC_{o,M}^0(x^1, x^0, y^0)) = 0.498$, and the t-test does not reject the equality of means except in the third biennial period 2008-2009. However, the Wilcoxon test rejects the equality of medians except in the first period 2006-2007. The correlation of the output mix factors is even negative, although not significant: $\rho(OME_{MB}^0(x^1, y^1, y^0; \bar{x}), OME_M^0(x^1, y^1, y^0)) = -0.081$. The t-test rejects the equality of means in the two middle biennial periods 2007-2008 and 2008-09. The same result is obtained for the medians based on the Wilcoxon test.

It is worth remarking that the MPI and the MBPI may yield exactly the same

[31] To be precise, the tables contain $MB^t(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; \bar{x}^t, \bar{y}^t)$, $MB^{t+1}(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; \bar{x}^t, \bar{y}^t)$, and $[MB^t(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; \bar{x}^t, \bar{y}^t)MB^{t+1}(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; \bar{x}^t, \bar{y}^t)]^{1/2}$, where $\bar{x}^t \equiv \sum_{k=1}^{K}(x^{k,t+1} + x^{kt})/2K$ and $\bar{y}^t \equiv \sum_{k=1}^{K}(y^{k,t+1} + y^{kt})/2K$, for $k = 1, ..., K(= 31)$, $t =$ 2006, 2007, 2008, 2009, along the various paths.

outcomes in some cases. For example, consider the base period viewpoint indices for bank #16 in Tables S.1 and S.17. However, individual differences can be rather large, as seen in the supplemental appendix tables. When comparing the output and input orientated Malmquist indices, relevant disparities or even conflicting results can be found for individual banks because an MB index considers both sides of the production process. For example, although bank #26 increased productivity by 24.88% between 2006 and 2007 according to the MPI, productivity declined by –4.33% according to the MBPI. This result is remarkable because it shows that when substantial scale and mix changes take place at the input and output sides of the production process, one may obtain a partial misrepresentation of productivity change if only one side is taken into account. We can investigate this issue by looking at the input side in Table S.10, reporting results for $\breve{M}_i^0(x^1, y^1, x^0, y^0)$. Here, we observe that bank #26 suffered a remarkable -62.77% productivity decline due to scale and output mix effects; i.e., $SEC_{i,M}^0(x^1, y^1, y^0) = 0.3723$, whereas the input mix effect did not contribute, as $IME_M^0(x^1, x^0, y^0) = 1$. This result also turns up in the third factor of the output-distance-function based decomposition of the base-period-viewpoint MB index with a -24.50% reduction, as shown in Table S.18. Moreover, since the factor capturing the output mix effect is equal to one, and making use of the information on the output- and input-orientated Malmquist index decompositions, we learn that this productivity decline can be solely attributed to the change in the scale of the inputs employed.

Another individual result previously commented in sections 3.4.2 and 4.4 is the sharp increase in the MPI of bank #14 by 107.55% in the 2009-2010 period, mostly driven by technological change to the tune of 112.70%. The MBPI yields almost the same value, 107.02%, and therefore the scale and mix terms both from the output and input sides are also very similar. For the same bank, MB productivity growth was 9.43% in the period 2008-2009, while the MPI amounted to 17.47%. Looking at the differences in Tables S.2–S.3, and S.18–S.19, this is mostly due to the disparity between the output mix effect factors in both decompositions.

Following the structure of the first set of results, Tables 20, 21, and 22 show the results for the comparison-period-viewpoint MB productivity index $MB^1(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$, which is related to the output-orientated comparison-period-viewpoint CCD index, $M_o^1(x^1, y^1, x^0, y^0)$, as presented in expression (87) and (88) and corresponding to paths C and D. Thus, notice that the $EC_o(x^1, y^1, x^0, y^0)$ and $TC_o^{0,1}(x^0, y^0)$ parts of Tables 20 and 4 are identical. The pairwise correlation $\rho(MB^1(x^1, y^1, x^0, y^0; \bar{x}, \bar{y}), \breve{M}_o^1 (x^1, y^1, x^0, y^0)) = 0.777$. The t-test rejects the equality of means for the last two biennial periods 2008-2009 and 2009-2010. The Wilcoxon test rejects the equality of means for the periods 2007-2008, 2008-2009, and 2009-2010. Table 20 shows the MBPI outcomes and the common efficiency and technical change components, $EC_o(x^1, y^1, x^0, y^0)$ and $TC_o^{1,0}(x^0, y^0)$, respectively. It is interesting to note that in this case the average MB index not only shows lower productivity increases than its Malmquist index counterpart $\breve{M}_o^1(x^1, y^1, x^0, y^0)$ in Table 4, but even shows decreases instead of increases for the middle periods 2007-2008 and 2008-2009. Tables 21 and 22 present the contributions of the scale (including input mix) and output mix effects. These should be compared to Tables 5 and 6.

Tables 23 and 24 show the output-distance-function based decomposition of the geometric mean MB index $[MB^0(x^1,\, y^1, x^0, y^0; \bar{x}, \bar{y})MB^1(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})]^{1/2}$, based on paths A–B–C–D. This index comprises the geometric mean CCD index, capturing efficiency change and technological change, as reported in Table 7 as part of the output-distance-function based decomposition of the 'mean' period Malmquist productivity index in expression (58). Thus, the EC and TC parts of Tables 23 and 7 are identical. The correlation between the geometric mean of the MPI and MBPI is higher than for their single period components: $\rho([MB^0(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})MB^1(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})]^{1/2}, [\check{M}_o^0(x^1, y^1, x^0, y^0)\check{M}_0^1(x^1, y^1, x^0, y^0)]^{1/2}) = 0.853$. The t-test does not reject the equality of means for any biennial period, and the Wilcoxon test rejects equal medians for the last two periods 2008-2009 and 2009-2010.

The next eight tables follow the same structure as the previous eight, but now considering the input distance function based decompositions of the MB productivity index; that is, those in expressions (89) - (92), corresponding to paths E–F–G–H. Recall that as an MB index only depends on the benchmark period technology, its outcomes are independent of the type of distance function chosen to decompose the index. Therefore the MB index numbers in columns 2-5 of Table 25, $MB^0(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$, are identical to those in columns 2-5 of Table 17, but their decompositions differ along paths E–F, and A–B, respectively. However, notice that the EC and TC parts of Table 25 are by definition identical to those of Table 9. Similarly, the MB index numbers in columns 2–5 of Table 28, $MB^1(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})$, are identical to those in columns 2–5 of Table 20. The EC and TC parts of Table 28 are by definition identical to those of Table 12. The geometric mean MB index numbers in columns 2-5 of Tables 31 and 23, combining paths E–F–G–H, and A–B–C–D, respectively, are of course identical when their distance functions are not , although . However, their decompositions are different. Notice that the EC and TC parts of Table 31 are identical to those of Table 15.

Finally, it is worth remarking that although the MBPI and the MPI and their decompositions may coincide completely, as already reported for bank #16 in the supplemental appendix, this will normally not be the case. By examining the MB decompositions, we learn about the importance of considering the input and output orientation together when assessing productivity change. Our results illustrate the relevance of evaluating the productivity effects of changes in the scale of inputs and outputs, as well as in their mixes. Such an insight can only be provided by the four- or five-components decompositions discussed above.

This conclusion is reinforced at the industry level by looking at the average index numbers and the other statistics. Again, results depend on the type of index, benchmark technology period, and orientation of decomposition. For the case of the Taiwanese banking industry, the Malmquist and the MB productivity indices show conflicting outcomes, particularly for the comparison period viewpoint. This result is explained by the difference in the scale of the inputs and outputs used by the banks as well as their mixes.

Moorsteen-Bjurek Productivity Indices (MBPI)

Table 17: MBPI decomposition along paths A and B: Output orientation, base period viewpoint, common components.

All banks	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0355	1.0210	1.0205	1.1446	1.0221	0.9845	0.9749	1.0442	1.0337	1.0425	1.0740	1.1241
Std. Dev.	0.0853	0.1605	0.1623	0.2214	0.0750	0.0923	0.0547	0.0790	0.0857	0.0927	0.1646	0.2411
Max.	1.3026	1.6530	1.6620	2.0702	1.1834	1.2414	1.0770	1.2335	1.3066	1.3128	1.6347	2.1270
Min.	0.8828	0.8417	0.7906	0.9105	0.8758	0.8233	0.8242	0.9304	0.9535	0.8416	0.7942	0.9909

Table 18: MBPI decomposition along path A: Output orientation, base period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	0.9964	1.0049	1.0022	0.9935	0.9997	1.0016	1.0010	0.9984
Std. Dev.	0.0528	0.0587	0.0200	0.0408	0.0093	0.0110	0.0030	0.0097
Max.	1.0940	1.2787	1.0607	1.0909	1.0287	1.0589	1.0143	1.0135
Min.	0.7550	0.9388	0.9728	0.8766	0.9578	0.9843	0.9964	0.9573

Table 19: MBPI decomposition along path B: Output orientation, base period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	0.9938	1.0097	1.0046	0.9859	0.9989	0.9974	0.9987	1.0039
Std. Dev.	0.0494	0.0612	0.0192	0.0470	0.0124	0.0145	0.0034	0.0135
Max.	1.0841	1.2787	1.0607	1.0909	1.0349	1.0182	1.0062	1.0524
Min.	0.7550	0.9468	0.9728	0.8722	0.9416	0.9226	0.9869	0.9848

Table 20: MBPI decomposition along paths C and D: Output orientation, comparison period viewpoint, common components.

All banks	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0125	0.9860	0.9724	1.0596	1.0221	0.9845	0.9749	1.0442	0.9762	0.9905	0.9893	1.0047
Std. Dev.	0.0872	0.1375	0.0863	0.1058	0.0750	0.0923	0.0547	0.0790	0.0847	0.1024	0.0757	0.0679
Max.	1.3026	1.3446	1.1197	1.3245	1.1834	1.2414	1.0770	1.2335	1.0790	1.0918	1.1550	1.2254
Min.	0.8786	0.6217	0.6458	0.8840	0.8758	0.8233	0.8242	0.9304	0.5714	0.5038	0.6509	0.7669

Table 21: MBPI decomposition along path C: Output orientation, comparison period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	1.0187	1.0120	1.0077	1.0073	1.0029	1.0043	1.0014	1.0026
Std. Dev.	0.0912	0.0830	0.0261	0.0410	0.0107	0.0120	0.0055	0.0107
Max.	1.4768	1.3631	1.0824	1.1349	1.0411	1.0580	1.0247	1.0334
Min.	0.9233	0.9173	0.9632	0.9702	0.9791	0.9923	0.9952	0.9703

Table 22: MBPI decomposition along path D: Output orientation, comparison period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	1.0245	1.0189	1.0104	1.0124	0.9977	0.9967	0.9988	0.9978
Std. Dev.	0.0997	0.0840	0.0272	0.0477	0.0068	0.0098	0.0047	0.0095
Max.	1.5375	1.3726	1.0824	1.1527	1.0002	1.0019	1.0026	1.0176
Min.	0.9539	0.9445	0.9639	0.9446	0.9728	0.9488	0.9744	0.9677

Table 23: MBPI decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

All banks	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0252	1.0015	0.9908	1.1022	1.0221	0.9845	0.9749	1.0442	1.0019	1.0148	1.0277	1.0581
Std. Dev.	0.0850	0.1434	0.1044	0.1419	0.0750	0.0923	0.0547	0.0790	0.0537	0.0870	0.0949	0.1176
Max.	1.3026	1.4909	1.2648	1.5357	1.1834	1.2414	1.0770	1.2335	1.1273	1.1967	1.2484	1.5152
Min.	0.8837	0.7279	0.7145	0.9105	0.8758	0.8233	0.8242	0.9304	0.8509	0.6512	0.7190	0.9732

Table 24: MBPI decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	1.0079	1.0129	1.0071	0.9992	0.9998	0.9999	1.0000	1.0006
Std. Dev.	0.0260	0.0685	0.0210	0.0347	0.0037	0.0016	0.0007	0.0020
Max.	1.0960	1.3226	1.0640	1.0942	1.0101	1.0047	1.0034	1.0074
Min.	0.9630	0.9459	0.9701	0.9394	0.9857	0.9942	0.9989	0.9975

Table 25: MBPI decomposition along paths E and F: Input orientation, base period viewpoint, common components.

All banks	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0355	1.0210	1.0205	1.1446	1.0174	0.9622	0.9537	1.0729	1.1562	1.0838	1.0490	1.1528
Std. Dev.	0.0853	0.1605	0.1623	0.2214	0.1177	0.1270	0.0905	0.1037	0.4486	0.1193	0.0845	0.4454
Max.	1.3026	1.6530	1.6620	2.0702	1.4838	1.2266	1.0827	1.3134	3.3548	1.3088	1.3056	3.3612
Min.	0.8828	0.8417	0.7906	0.9105	0.8257	0.6174	0.5833	0.8762	0.9325	0.8335	0.8844	0.9189

Table 26: MBPI decomposition along path E: Input orientation, base period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	0.9903	0.9942	0.9993	0.9942	0.9545	1.0092	1.0137	0.9527
Std. Dev.	0.0427	0.0120	0.0048	0.0182	0.1466	0.1070	0.0860	0.0991
Max.	1.0000	1.0008	1.0123	1.0030	1.1274	1.4070	1.3926	1.1175
Min.	0.7661	0.9475	0.9791	0.9038	0.3723	0.8315	0.8944	0.5728

Table 27: MBPI decomposition along path F: Input orientation, base period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	1.0012	1.0018	1.0005	0.9984	0.9499	1.0075	1.0096	0.9467
Std. Dev.	0.0036	0.0075	0.0012	0.0150	0.1611	0.1053	0.0860	0.1084
Max.	1.0156	1.0310	1.0047	1.0534	1.1274	1.4070	1.3926	1.1175
Min.	1.0000	0.9841	1.0000	0.9596	0.2808	0.8399	0.8944	0.5728

Table 28: MBPI decomposition along paths G and H: Input orientation, comparison period viewpoint, common components.

All banks	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0127	0.9860	0.9724	1.0596	1.0174	0.9622	0.9537	1.0729	1.0046	1.0041	1.0079	0.9874
Std. Dev.	0.0886	0.1375	0.0863	0.1058	0.1177	0.1270	0.0905	0.1037	0.1104	0.1608	0.0797	0.0619
Max.	1.3026	1.3446	1.1197	1.3245	1.4838	1.2266	1.0827	1.3134	1.3215	1.2284	1.1693	1.1399
Min.	0.8786	0.6217	0.6458	0.8840	0.8257	0.6174	0.5833	0.8762	0.7369	0.2507	0.8347	0.8444

Table 29: MBPI decomposition along path G: Input orientation, comparison period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	0.9986	0.9940	0.9982	0.9972	0.9997	1.0783	1.0210	1.0005
Std. Dev.	0.0072	0.0162	0.0067	0.0050	0.0794	0.2825	0.0838	0.0536
Max.	1.0129	1.0013	1.0033	1.0003	1.2528	2.5280	1.3247	1.1609
Min.	0.9649	0.9264	0.9719	0.9845	0.8213	0.9343	0.7674	0.9153]

Table 30: MBPI decomposition along path H: Input orientation, comparison period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	1.0059	1.0065	1.0043	1.0060	0.9965	1.0614	1.0147	0.9912
Std. Dev.	0.0229	0.0191	0.0142	0.0227	0.0794	0.2491	0.0821	0.0422
Max.	1.1288	1.0810	1.0593	1.1161	1.2345	2.3219	1.3247	1.1047
Min.	1.0000	0.9967	0.9960	0.9642	0.8270	0.8835	0.7674	0.9157

Table 31: MBPI decomposition along paths EFGH: Input orientation, 'mean' period viewpoint

All banks	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0252	1.0015	0.9908	1.1022	1.0174	0.9622	0.9537	1.0729	1.0629	1.0376	1.0314	1.0563
Std. Dev.	0.0850	0.1434	0.1044	0.1419	0.1177	0.1270	0.0905	0.1037	0.1500	0.1350	0.0726	0.1776
Max.	1.3026	1.4909	1.2648	1.5357	1.4838	1.2266	1.0827	1.3134	1.6359	1.2482	1.1804	1.9133
Min.	0.8837	0.7279	0.7145	0.9105	0.8257	0.6174	0.5833	0.8762	0.8776	0.4571	0.8630	0.9189

Table 32: MBPI decomposition along paths EFGH: Input orientation, 'mean' period viewpoint.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	0.9977	0.9990	0.8926	0.9642	0.9697	1.0352	0.9019	0.9356
Std. Dev.	0.0113	0.0049	0.3092	0.1824	0.1056	0.1343	0.3289	0.1917
Max.	1.0019	1.0131	1.0001	1.0238	1.0973	1.5718	1.3582	1.1110
Min.	0.9392	0.9801	0.0000	0.0000	0.5967	0.8977	0.0000	0.0000

7 Decomposing a price-weighted productivity index

In the case of a single input and a single output, productivity is naturally measured by the ratio of output quantity over input quantity, that is, y/x. The productivity index, measuring the change through time, is then unequivocally given by $(y'/x')/(y/x) = (y'/y)/(x'/x)$. This simplicity is lost in the multi-input-multi-output case. The natural generalization is to weigh output quantities y by certain output prices p, and input quantities x by certain input prices w, and to define the productivity level by $p \cdot y/w \cdot x$.[32]

Hence, a price-weighted productivity index is defined as

$$PROD(x', y', x, y; w, p) \equiv \frac{p \cdot y'/w \cdot x'}{p \cdot y/w \cdot x} = \frac{p \cdot y'/p \cdot y}{w \cdot x'/w \cdot x}, \tag{108}$$

where $p \equiv (p_1, ..., p_M) \in \Re^M_{++}$ denotes a vector of output prices and $w \equiv (w_1, ..., w_N) \in \Re^N_{++}$ denotes a vector of input prices[33]. We can verify that $PROD(x', y', x, y; w, p)$ satisfies the fundamental monotonicity and proportionality requirements and, for fixed w and p, exhibits transitivity in (x, y). As the last part of expression (108) shows, the productivity index can be written as the ratio of a Lowe output quantity index and a Lowe input quantity index; see Balk (2008) for their axiomatic properties.

A simple transformation of a price-weighted productivity index produces an instance of the well-known Solow residual. Consider

$$\begin{aligned} &\ln PROD(x', y', x, y; w, p) \\ &\quad = \ln (p \cdot y'/p \cdot y) - \ln (w \cdot x'/w \cdot x) \\ &\quad = \ln \left(1 + \frac{p \cdot (y' - y)}{p \cdot y}\right) - \ln \left(1 + \frac{w \cdot (x' - x)}{w \cdot x}\right). \end{aligned} \tag{109}$$

If $\| y' - y \|$ and $\| x' - x \|$ are small, then the following approximation holds:

$$\ln PROD(x', y', x, y; w, p) \approx \frac{p \cdot (y' - y)}{p \cdot y} - \frac{w \cdot (x' - x)}{w \cdot x}. \tag{110}$$

If we define revenue shares by $u_m \equiv p_m y_m / p \cdot y$ $(m = 1, ..., M)$ and cost shares by $s_n \equiv w_n x_n / w \cdot x$ $(n = 1, ..., N)$, then expression (110) can be written as

$$\ln PROD(x', y', x, y; w, p) \approx \sum_{m=1}^{M} u_m \frac{y'_m - y_m}{y_m} - \sum_{n=1}^{N} s_n \frac{x'_n - x_n}{x_n}, \tag{111}$$

which in continuous time would be written as

$$\ln PROD(x', y', x, y; w, p) \approx \sum_{m=1}^{M} u_m d \ln y_m - \sum_{n=1}^{N} s_n d \ln x_n. \tag{112}$$

[32]Thus, to continue footnote 7, $X(x) \equiv w \cdot x$ and $Y(y) \equiv p \cdot y$.

[33]These prices might be shadow prices. See for example Coelli *et al.* (2003).

Thus the logarithm of a price-weighted productivity index is approximately equal to a weighted mean of output quantity growth rates minus a weighted mean of input quantity growth rates, where the weights are revenue shares and cost shares, respectively.

7.1 Choices and consequences

It is straightforward to verify that path A with $\lambda = 1$ leads to the following decomposition:

$$PROD(x^1, y^1, x^0, y^0; w, p) = \frac{D_o^1(x^1, y^1)}{D_o^0(x^0, y^0)} \times \frac{D_o^0(x^1, y^1)}{D_o^1(x^1, y^1)} \times \left[\frac{D_o^0(x^0, y^0)}{D_o^0(x^1, y^0)} \frac{w \cdot x^0}{w \cdot x^1}\right] \times \left[\frac{D_o^0(x^1, y^0)}{D_o^0(x^1, y^1)} \frac{p \cdot y^1}{p \cdot y^0}\right]. \tag{113}$$

The first factor measures technical efficiency change, and the second factor measures technological change. The third factor corresponds to the scale (including input mix) effect, since, under CRS, this factor reduces to 1 if $x^1 = \lambda x^0$ ($\lambda > 0$). The fourth factor is a Lowe output quantity index number divided by a Malmquist output quantity index number. The factor is homogeneous of degree 0 in (y^0, y^1). This factor also reduces to 1 if $y^1 = \mu y^0$ ($\mu > 0$). Hence, the fourth factor corresponds to the output mix effect.

The first two factors are familiar; together they constitute the base period, output orientated CCD index as defined by expression (30). To facilitate comparisons with the components of the Malmquist indices, we denote the scale (including input mix) effect by

$$SEC^0_{o,PROD}(x^1, x^0, y^0; w) \equiv \frac{D_o^0(x^0, y^0)}{D_o^0(x^1, y^0)} \frac{w \cdot x^0}{w \cdot x^1},$$

and the output mix effect by

$$OME^0_{PROD}(x^1, y^1, y^0; p) \equiv \frac{D_o^0(x^1, y^0)}{D_o^0(x^1, y^1)} \frac{p \cdot y^1}{p \cdot y^0}.$$

Expression (113) can then be written as

$$PROD(x^1, y^1, x^0, y^0; w, p) =$$

$$M_o^0(x^1, y^1, x^0, y^0) \times SEC^0_{o,PROD}(x^1, x^0, y^0; w) \times OME^0_{PROD}(x^1, y^1, y^0; p). \tag{114}$$

The alternative path B delivers

$$PROD(x^1, y^1, x^0, y^0; w, p) = M_o^0(x^1, y^1, x^0, y^0) \times \left[\frac{D_o^0(x^0, y^1)}{D_o^0(x^1, y^1)} \frac{w \cdot x^0}{w \cdot x^1}\right] \times$$

$$\left[\frac{D_o^0(x^0,y^0)}{D_o^0(x^0,y^1)}\frac{p\cdot y^1}{p\cdot y^0}\right]. \quad (115)$$

Taking the geometric mean of these two expressions yields

$$PROD(x^1,y^1,x^0,y^0;w,p) = M_o^0(x^1,y^1,x^0,y^0)\times$$
$$\left[\left(\frac{D_o^0(x^0,y^0)}{D_o^0(x^1,y^0)}\frac{D_o^0(x^0,y^1)}{D_o^0(x^1,y^1)}\right)^{1/2}\frac{w\cdot x^0}{w\cdot x^1}\right]\times\left[\left(\frac{D_o^0(x^1,y^0)}{D_o^0(x^1,y^1)}\frac{D_o^0(x^0,y^0)}{D_o^0(x^0,y^1)}\right)^{1/2}\frac{p\cdot y^1}{p\cdot y^0}\right]. \quad (116)$$

If we repeat this sequence of steps for the paths C and D, both with $\lambda = 1$, two decompositions are obtained which can be averaged as

$$PROD(x^1,y^1,x^0,y^0;w,p) = M_o^1(x^1,y^1,x^0,y^0)\times$$
$$\left[\left(\frac{D_o^1(x^0,y^0)}{D_o^1(x^1,y^0)}\frac{D_o^1(x^0,y^1)}{D_o^1(x^1,y^1)}\right)^{1/2}\frac{w\cdot x^0}{w\cdot x^1}\right]\times\left[\left(\frac{D_o^1(x^1,y^0)}{D_o^1(x^1,y^1)}\frac{D_o^1(x^0,y^0)}{D_o^1(x^0,y^1)}\right)^{1/2}\frac{p\cdot y^1}{p\cdot y^0}\right]. \quad (117)$$

Here $M_o^1(x^1,y^1,x^0,y^0)$ is the comparison period based, output orientated CCD index, which was defined by expression (47). In turn, expressions (116) and (117) can be averaged to get as final decomposition

$$PROD(x^1,y^1,x^0,y^0;w,p) = [M_o^0(x^1,y^1,x^0,y^0)M_o^1(x^1,y^1,x^0,y^0)]^{1/2}\times$$
$$\left[\left(\frac{D_o^0(x^0,y^0)}{D_o^0(x^1,y^0)}\frac{D_o^0(x^0,y^1)}{D_o^0(x^1,y^1)}\frac{D_o^1(x^0,y^0)}{D_o^1(x^1,y^0)}\frac{D_o^1(x^0,y^1)}{D_o^1(x^1,y^1)}\right)^{1/4}\frac{w\cdot x^0}{w\cdot x^1}\right]\times$$
$$\left[\left(\frac{D_o^0(x^1,y^0)}{D_o^0(x^1,y^1)}\frac{D_o^0(x^0,y^0)}{D_o^0(x^0,y^1)}\frac{D_o^1(x^1,y^0)}{D_o^1(x^1,y^1)}\frac{D_o^1(x^0,y^0)}{D_o^1(x^0,y^1)}\right)^{1/4}\frac{p\cdot y^1}{p\cdot y^0}\right], \quad (118)$$

which is symmetric in all variables. This is the counterpart of expression (58). The first major factor at the right-hand side of the equality sign is the geometric mean output orientated CCD index, the second factor concerns the scale (including input mix) effect, and the third factor concerns the output mix effect.

The whole exercise can be repeated with the paths specified in Section 4. For example, along path E we obtain, by using the definition of the input orientated CCD index conditional on the base period technology – see expression (65) –,

$$PROD(x^1,y^1,x^0,y^0;w,p) = M_i^0(x^1,y^1,x^0,y^0)\times\left[\frac{D_i^0(x^1,y^0)}{D_i^0(x^0,y^0)}\frac{w\cdot x^0}{w\cdot x^1}\right]\times$$
$$\left[\frac{D_i^0(x^1,y^1)}{D_i^0(x^1,y^0)}\frac{p\cdot y^1}{p\cdot y^0}\right]. \quad (119)$$

The first factor captures technical efficiency change and technological change. The second factor is a Malmquist input quantity index number divided by a price-weighted input quantity index number. The factor is homogeneous of degree 0 in (x^0,x^1) and reduces to 1 if $x^1 = \lambda x^0$ $(\lambda > 0)$. The factor thus measures the input mix effect. The third factor measures the scale (including output mix) effect: under CRS, the factor reduces to 1 if $y^1 = \mu y^0$ $(\mu > 0)$.

The paths F, G, and H are left to the reader. The completely symmetric decomposition reads

$$PROD(x^1, y^1, x^0, y^0; w, p) = [M_i^0(x^1, y^1, x^0, y^0) M_i^1(x^1, y^1, x^0, y^0)]^{1/2} \times$$
$$\left[\left(\frac{D_i^0(x^1, y^0)}{D_i^0(x^0, y^0)} \frac{D_i^0(x^1, y^1)}{D_i^0(x^0, y^1)} \frac{D_i^1(x^1, y^0)}{D_i^1(x^0, y^0)} \frac{D_i^1(x^1, y^1)}{D_i^1(x^0, y^1)} \right)^{1/4} \frac{w \cdot x^0}{w \cdot x^1} \right] \times$$
$$\left[\left(\frac{D_i^0(x^1, y^1)}{D_i^0(x^1, y^0)} \frac{D_i^0(x^0, y^1)}{D_i^0(x^0, y^0)} \frac{D_i^1(x^1, y^1)}{D_i^1(x^1, y^0)} \frac{D_i^1(x^0, y^1)}{D_i^1(x^0, y^0)} \right)^{1/4} \frac{p \cdot y^1}{p \cdot y^0} \right]. \quad (120)$$

This is the counterpart of expression (76). The first major factor at the right-hand side of the equality sign is the geometric mean input orientated CCD index, the second factor concerns the input mix effect, and the third factor concerns the scale (including output mix) effect.

Summarizing, we again have a large number of decompositions of the productivity index. Conditioning on the base period technology, there are six decompositions, three of which use output distance functions and three use input distance functions. Conditioning on the comparison period technology, there are also six decompositions. Taking the 'average' viewpoint, there are nine decompositions that use output distance functions, and nine that use input distance functions. The two completely symmetric decompositions are given by expressions (118) and (120). They differ by being based on either output distance functions or input distance functions. Moreover, the decompositions as such differ: apart from technical efficiency change, technological change, and the scale effect, expression (118) includes the output mix effect, whereas expression (120) includes the input mix effect.

The number of different decompositions is still larger because we conditioned on the price variables (w, p). Every choice gives rise to a different productivity index number and a different decomposition. Notice, however, that only the scale and (output or input) mix components are affected, since the CCD indices are independent of prices.

Natural choices in the case of our firm are the base and comparison period prices. The corresponding productivity index numbers are then given by $PROD(x^1, y^1, x^0, y^0; w^0, p^0)$ and $PROD(x^1, y^1, x^0, y^0; w^1, p^1)$. The first can be called a Laspeyres productivity index number, which is the ratio of a Laspeyres output quantity index number and a Laspeyres input quantity index number. The second can be called a Paasche productivity index number, which is the ratio of a Paasche output quantity index number and a Paasche input quantity index number.

We can again take the 'mean' viewpoint, and compute the geometric mean of these two index numbers, which is then called a *Fisher* productivity index number. Using expression (118), this index number can be decomposed as

$$\frac{Q^F(p^1, y^1, p^0, y^0)}{Q^F(w^1, x^1, w^0, x^0)} = [M_o^0(x^1, y^1, x^0, y^0) M_o^1(x^1, y^1, x^0, y^0)]^{1/2} \times$$

$$\left[\left(\frac{D_o^0(x^0,y^0)}{D_o^0(x^1,y^0)}\frac{D_o^0(x^0,y^1)}{D_o^0(x^1,y^1)}\frac{D_o^1(x^0,y^0)}{D_o^1(x^1,y^0)}\frac{D_o^1(x^0,y^1)}{D_o^1(x^1,y^1)}\right)^{1/4}\frac{1}{Q^F(w^1,x^1,w^0,x^0)}\right]\times$$
$$\left[\left(\frac{D_o^0(x^1,y^0)}{D_o^0(x^1,y^1)}\frac{D_o^0(x^0,y^0)}{D_o^0(x^0,y^1)}\frac{D_o^1(x^1,y^0)}{D_o^1(x^1,y^1)}\frac{D_o^1(x^0,y^0)}{D_o^1(x^0,y^1)}\right)^{1/4}Q^F(p^1,y^1,p^0,y^0)\right], \quad (121)$$

where $Q^F(p^1,y^1,p^0,y^0) \equiv [(p^0\cdot y^1/p^0\cdot y^0)(p^1\cdot y^1/p^1\cdot y^0)]^{1/2}$ denotes the Fisher output quantity index number, and $Q^F(w^1,x^1,w^0,x^0) \equiv [(w^0\cdot x^1/w^0\cdot x^0)(w^1\cdot x^1/w^1\cdot x^0)]^{1/2}$ the Fisher input quantity index number.[34]

Alternatively, by using expression (120), the Fisher productivity index number can be decomposed as

$$\frac{Q^F(p^1,y^1,p^0,y^0)}{Q^F(w^1,x^1,w^0,x^0)} = [M_i^0(x^1,y^1,x^0,y^0)M_i^1(x^1,y^1,x^0,y^0)]^{1/2}\times$$
$$\left[\left(\frac{D_i^0(x^1,y^0)}{D_i^0(x^0,y^0)}\frac{D_i^0(x^1,y^1)}{D_i^0(x^0,y^1)}\frac{D_i^1(x^1,y^0)}{D_i^1(x^0,y^0)}\frac{D_i^1(x^1,y^1)}{D_i^1(x^0,y^1)}\right)^{1/4}\frac{1}{Q^F(w^1,x^1,w^0,x^0)}\right]\times$$
$$\left[\left(\frac{D_i^0(x^1,y^1)}{D_i^0(x^1,y^0)}\frac{D_i^0(x^0,y^1)}{D_i^0(x^0,y^0)}\frac{D_i^1(x^1,y^1)}{D_i^1(x^1,y^0)}\frac{D_i^1(x^0,y^1)}{D_i^1(x^0,y^0)}\right)^{1/4}Q^F(p^1,y^1,p^0,y^0)\right]. \quad (122)$$

Grifell-Tatjé and Lovell (2015, 140-144), (2016) and Lovell (2016) related the Fisher productivity index number to the geometric mean MB index number, as follows:

$$\frac{Q^F(p^1,y^1,p^0,y^0)}{Q^F(w^1,x^1,w^0,x^0)} = [MB^0(x^1,y^1,x^0,y^0;x^0,y^0)MB^1(x^1,y^1,x^0,y^0;x^1,y^1)]^{1/2}$$
$$\times\left(\frac{p^0\cdot y^1/D_o^0(x^0,y^1)}{p^0\cdot y^0/D_o^0(x^0,y^0)}\frac{p^1\cdot y^1/D_o^1(x^1,y^1)}{p^1\cdot y^0/D_o^1(x^1,y^0)}\right)^{1/2}$$
$$\times\left(\frac{w^0\cdot x^1/D_i^0(x^1,y^0)}{w^0\cdot x^0/D_i^0(x^0,y^0)}\frac{w^1\cdot x^1/D_i^1(x^1,y^1)}{w^1\cdot x^0/D_i^1(x^0,y^1)}\right)^{-1/2}. \quad (123)$$

Consider the factor on the second line of this expression. The output vector in the first numerator, $y^1/D_o^0(x^0,y^1)$, lies at the same output isoquant as the output vector in the first denominator, $y^0/D_o^0(x^0,y^0)$. The vectors $y^1/D_o^1(x^1,y^1)$ and $y^0/D_o^1(x^1,y^0)$ in the second ratio also lie at the same, but different, output isoquant. The whole factor is interpreted as measuring the mean effect of output mix change, going from a ray through y^0 to a ray through y^1 in output space. If $y^1 = \mu y^0$ ($\mu > 0$), then this factor reduces to 1.

In the same way the factor on the third line of expression (123) is interpreted as measuring the mean effect of input mix change, going from a ray through x^0 to a ray through x^1 in input space. If $x^1 = \lambda x^0$ ($\lambda > 0$), then this factor reduces to 1.

This interpretation does not come as a surprise. From their definitions it immediately follows that if $x^1 = \lambda x^0$ and $y^1 = \mu y^0$, then the Fisher productivity index number as well as any MB index number attains the value μ/λ.

[34] See Diewert (1992) or Balk (2008) for the axiomatic properties of Fisher indices.

7.2 Empirical application

For the empirical application of the price-weighted index (108), we follow the classic definitions and choose the base or the comparison period prices, corresponding to the Laspeyres and Paasche productivity indices, respectively. Taking the geometric mean of the two yields the Fisher productivity index.[35]

Tables 33–35 report the descriptive statistics corresponding to the output orientated base period viewpoint decomposition of $PROD(x^1, y^1, x^0, y^0; w^0, p^0)$ along paths A–B. Tables 36–38 show the comparison period counterparts for $PROD(x^1, y^1, x^0, y^0; w^1, p^1)$ along paths C–D, and Tables 39–40 show the Fisher index decomposition according to expression (121). Focusing on the latter index, using market prices as aggregators results in lower productivity growth than indicated by the Malmquist and Moorsteen-Bjurek productivity indices for 2006-2007. The Fisher index indicates a 1.32% increase in productivity on average, while its counterpart MPI indicates 3.47%, and the MBPI 2.52% (Tables 7 and 23). The differences are larger for the base period Laspeyres and MPI indices, and smaller for the comparison period, but always in favor of the quantity-only productivity definitions. Indeed the correlation coefficients are relatively low: $\rho(Q^F(p^1, y^1, p^0, y^0)/Q^F(w^1, x^1, w^0, x^0), \check{M}_o(x^1, y^1, x^0, y^0)) = 0.773$, $\rho(Q^F(p^1, y^1, p^0, y^0)/Q^F(w^1, x^1, w^0, x^0), MB(x^1, y^1, x^0, y^0; \bar{x}, \bar{y})) = 0.624$. The t-test does not reject equality of means for the former pair, whereas the Wilcoxon test rejects equality of medians in the last two biennial periods 2008-2009 and 2009-2010. Neither of the tests reject the null hypotheses for any period for the latter pair.

Since the price-weighted indices comprise the same efficiency change $EC_o(x^1, y^1, x^0, y^0)$ and technological change $TC_o^{1,0}(x^1, y^1)$ factors as their MPI and MBPI counterparts, this difference in productivity change must be reflected by smaller scale (including input mix) effects and output mix effects. Indeed the magnitudes of these factors in Table 40 are generally smaller than their MPI and MBPI counterparts in Table 8 and Table 24. Comparing the Laspeyres and MPI factors along paths A–B: $\rho([SEC^0_{o,PROD}(x^1, x^0, y^0; w^0) SEC^0_{o,PROD}(x^1, x^0, y^1; w^0)]^{1/2}, [SEC^0_{o,M}(x^1, x^0, y^0) SEC^0_{o,M}(x^1, x^0, y^1)]^{1/2}) = 0.360$. However, the t-test shows that the equality of means cannot be rejected in the first two biennial periods 2006-2007 and 2007-2008, while the Wilcoxon test does not reject the equality of medians in any period. The results for the output mix terms yield higher correlations and similarity in the distributions: $\rho([OME^0_{PROD}(x^1, y^1, y^0; p^0) OME^0_{PROD}(x^0, y^1, y^0; p^0)]^{1/2}, [OME_M{}^0(x^1, y^1, y^0) OME_M{}^0(x^0, y^1, y^0)]^{1/2}) = 0.803$, while the t-test and Wilcoxon test do not reject the equality of means and medians. Similar results are obtained between the Laspeyres and MBPI factors, as well as for the comparison periods (Paasche) and geometric means (Fisher).

However, although these productivity differentials hold on average, we find significant reversals at the individual level. Detailed outcomes for all banks are shown in the supplemental appendix Tables S.33–S.40. The already discussed banks #14 and #16 report much lower productivity values for 2006-2007 using the price-weighted

[35] To be precise, the tables contain $PROD(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; w^{kt}, p^{kt})$, $PROD(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; w^{k,t+1}, p^{k,t+1})$, and $[PROD(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; w^{kt}, p^{kt}) PROD (x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; w^{k,t+1}, p^{k,t+1})]^{1/2}$, respectively, for $k = 1, ..., K(= 31)$, t = 2006, 2007, 2008, 2009, along the various paths. Notice that our dataset contains firm-specific prices.

Fisher index than the quantity-only MPI or MBPI indices. The difference is so stark: the Fisher index shows a decline of –7.80% and –17.72% (Table S.39), whereas the MPI shows a decline of –1.04% and an increase of 29.89%, respectively (Table S.7). The MBPI results in 0.06% and 30.26%, respectively (Table S.23). Besides conflicting results, the gap for bank #16 illustrates the enormous differences that can be obtained between the price-weighted and quantity-only indices. The differences could be reversed with the Fisher indices reflecting productivity growth, and the MPI and MBPI showing stagnation or even decline. Looking at bank #31, the Fisher productivity index reports an increase of 8.21% for 2007-2008, whereas the MPI and the MBPI show a reduction of –2.51% and –4.61%, respectively. Bank #21 presents a similar pattern although the gap is less marked. Again, since the efficiency change and technological change factors are the same for these alternative indices, the scale, input and output mix effects factors capture all the differences.

Similar results are observed from the input orientation perspective for the base and comparison period viewpoint decompositions of the Laspeyres, Paasche and Fisher productivity indices. Laspeyres and Paasche are shown in Tables 41 through 46, along paths E–F and G–H, and Fisher as defined in expression (122) in Tables 47 and 48.

These results illustrate that the choice of productivity index is not neutral, leading to significant discrepancies when studying individual observations. These disparities are more pronounced between the price-weighted and quantity-only productivity indices, because of the differences between market and shadow prices. Academics, officials, and professionals must be aware of the effect that the different prices may have on the results; e.g., for industry benchmarking and policy analysis.

Price-weighted Productivity Indices (PROD)

Table 33: PROD decomposition along paths A and B: Output orientation, base period viewpoint, common components.

All banks	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0196	0.9989	0.9905	1.1885	1.0221	0.9845	0.9749	1.0442	1.0337	1.0425	1.0740	1.1241
Std. Dev.	0.0948	0.0944	0.1158	0.3480	0.0750	0.0923	0.0547	0.0790	0.0857	0.0927	0.1646	0.2411
Max.	1.3131	1.1509	1.4066	2.6366	1.1834	1.2414	1.0770	1.2335	1.3066	1.3128	1.6347	2.1270
Min.	0.8874	0.7693	0.6819	0.8636	0.8758	0.8233	0.8242	0.9304	0.9535	0.8416	0.7942	0.9909

Table 34: PROD decomposition along path A: Output orientation, base period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	0.9726	0.9974	0.9872	0.9813	0.9930	0.9816	0.9689	1.0355
Std. Dev.	0.0758	0.0593	0.0441	0.0646	0.0611	0.0756	0.1303	0.1253
Max.	1.1965	1.0831	1.0717	1.0736	1.1684	1.1253	1.4346	1.3977
Min.	0.7855	0.8563	0.8810	0.7547	0.8693	0.7252	0.7033	0.7969

Table 35: PROD decomposition along path B: Output orientation, base period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	0.9734	1.0012	0.9894	0.9758	0.9923	0.9773	0.9669	1.0423
Std. Dev.	0.0765	0.0527	0.0430	0.0619	0.0633	0.0738	0.1311	0.1359
Max.	1.1965	1.0831	1.0795	1.0450	1.1684	1.1253	1.4346	1.4426
Min.	0.7855	0.8636	0.9055	0.7547	0.8674	0.7252	0.7024	0.7969

Table 36: PROD decomposition along paths C and D: Output orientation, comparison period viewpoint, common components.

All banks	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0074	0.9846	0.9759	1.0865	1.0221	0.9845	0.9749	1.0442	0.9762	0.9905	0.9893	1.0047
Std. Dev.	0.1098	0.1101	0.1150	0.1055	0.0750	0.0923	0.0547	0.0790	0.0847	0.1024	0.0757	0.0679
Max.	1.3415	1.1356	1.1751	1.3196	1.1834	1.2414	1.0770	1.2335	1.0790	1.0918	1.1550	1.2254
Min.	0.7073	0.6829	0.5037	0.8356	0.8758	0.8233	0.8242	0.9304	0.5714	0.5038	0.6509	0.7669

Table 37: PROD decomposition along path C: Output orientation, comparison period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	1.0203	1.0255	1.0247	1.0247	1.0983	0.9937	0.9914	1.0201
Std. Dev.	0.1317	0.0711	0.0377	0.0912	0.5702	0.0915	0.1077	0.0683
Max.	1.6663	1.2948	1.1359	1.4407	4.1881	1.1881	1.2632	1.1912
Min.	0.8880	0.8914	0.9631	0.8264	0.6629	0.6585	0.5667	0.7694

Table 38: PROD decomposition along path D: Output orientation, comparison period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	1.0264	1.0329	1.0273	1.0299	0.9901	0.9859	0.9887	1.0152
Std. Dev.	0.1422	0.0643	0.0373	0.0956	0.0830	0.0931	0.1065	0.0670
Max.	1.7348	1.2948	1.1366	1.4633	1.1436	1.1851	1.2632	1.1912
Min.	0.8880	0.9269	0.9631	0.8264	0.6629	0.6585	0.5666	0.7698

Table 39: PROD decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

All banks	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0147	0.9926	0.9814	1.1199	1.0221	0.9845	0.9749	1.0442	1.0019	1.0148	1.0277	1.0581
Std. Dev.	0.0977	0.0985	0.1084	0.1716	0.0750	0.0923	0.0547	0.0790	0.0537	0.0870	0.0949	0.1176
Max	1.3263	1.1320	1.2209	1.6410	1.1834	1.2414	1.0770	1.2335	1.1273	1.1967	1.2484	1.5152
Min	0.8732	0.7713	0.5862	0.8495	0.8758	0.8233	0.8242	0.9304	0.8509	0.6512	0.7190	0.9732

Table 40: PROD decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	0.9950	1.0132	1.0061	1.0015	0.9936	0.9850	0.9754	1.0182
Std. Dev.	0.0656	0.0475	0.0348	0.0586	0.0627	0.0713	0.1034	0.0739
Max	1.2088	1.0899	1.1055	1.1370	1.1485	1.1014	1.2365	1.2110
Min	0.8496	0.8859	0.9560	0.7886	0.8197	0.7809	0.6494	0.7831

Table 41: PROD decomposition along paths E and F: Input orientation, base period viewpoint, common components.

All banks	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0196	0.9989	0.9905	1.1885	1.0174	0.9622	0.9537	1.0729	1.1562	1.0838	1.0490	1.1528
Std. Dev.	0.0948	0.0944	0.1158	0.3480	0.1177	0.1270	0.0905	0.1037	0.4486	0.1193	0.0845	0.4454
Max.	1.3131	1.1509	1.4066	2.6366	1.4838	1.2266	1.0827	1.3134	3.3548	1.3088	1.3056	3.3612
Min.	0.8874	0.7693	0.6819	0.8636	0.8257	0.6174	0.5833	0.8762	0.9325	0.8335	0.8844	0.9189

Table 42: PROD decomposition along path E: Input orientation, base period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	0.9714	0.9884	0.9838	0.9809	0.9494	0.9850	1.0099	0.9978
Std. Dev.	0.0800	0.0693	0.0479	0.0727	0.1616	0.0972	0.1408	0.1432
Max.	1.2122	1.0819	1.0913	1.1196	1.1697	1.2435	1.4335	1.3324
Min.	0.7970	0.8301	0.8789	0.7212	0.3605	0.7913	0.7928	0.6516

Table 43: PROD decomposition along path F: Input orientation, base period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	0.9836	1.0024	0.9991	0.9853	0.9445	0.9779	1.0082	0.9958
Std. Dev.	0.0760	0.0627	0.0410	0.0767	0.1740	0.1011	0.1415	0.1565
Max.	1.2122	1.1040	1.0913	1.1196	1.1697	1.2435	1.4335	1.3744
Min.	0.8153	0.8431	0.9086	0.7212	0.2719	0.8041	0.7928	0.6469

Table 44: PROD decomposition along paths G and H: Input orientation, comparison period viewpoint, common components.

All banks	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0074	0.9846	0.9759	1.0865	1.0174	0.9622	0.9537	1.0729	1.0046	1.0041	1.0079	0.9874
Std. Dev.	0.1098	0.1101	0.1150	0.1055	0.1177	0.1270	0.0905	0.1037	0.1104	0.1608	0.0797	0.0619
Max.	1.3415	1.1356	1.1751	1.3196	1.4838	1.2266	1.0827	1.3134	1.3215	1.2284	1.1693	1.1399
Min.	0.7073	0.6829	0.5037	0.8356	0.8257	0.6174	0.5833	0.8762	0.7369	0.2507	0.8347	0.8444

Table 45: PROD decomposition along path G: Input orientation, comparison period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	1.0027	1.0114	1.0157	1.0159	0.9999	1.0742	1.0080	1.0186
Std. Dev.	0.0732	0.0642	0.0457	0.0807	0.1347	0.3724	0.1197	0.0899
Max.	1.2438	1.1181	1.1470	1.2616	1.4328	2.9958	1.3546	1.2573
Min.	0.8192	0.7956	0.9245	0.7632	0.7264	0.6608	0.6160	0.7424

Table 46: PROD decomposition along path H: Input orientation, comparison period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	1.0083	1.0235	1.0218	1.0244	0.9966	1.0569	1.0016	1.0091
Std. Dev.	0.0717	0.0632	0.0448	0.0785	0.1334	0.3335	0.1161	0.0770
Max.	1.2438	1.1264	1.1470	1.2767	1.4119	2.7516	1.3546	1.1719
Min.	0.8192	0.7934	0.9245	0.7705	0.7275	0.6620	0.6160	0.7424

Table 47: PROD decomposition along paths EFGH: Input orientation, 'mean' period viewpoint

All banks	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0132	0.9914	0.9827	1.1313	1.0174	0.9622	0.9537	1.0729	1.0629	1.0376	1.0314	1.0563
Std. Dev.	0.1005	0.1010	0.1115	0.1980	0.1177	0.1270	0.0905	0.1037	0.1500	0.1350	0.0726	0.1776
Max.	1.3272	1.1328	1.2579	1.8094	1.4838	1.2266	1.0827	1.3134	1.6359	1.2482	1.1804	1.9133
Min.	0.8228	0.7712	0.5861	0.8495	0.8257	0.6174	0.5833	0.8762	0.8776	0.4571	0.8630	0.9189

Table 48: PROD decomposition along paths EFGH: Input orientation, 'mean' period viewpoint.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	0.9902	1.0076	1.0063	1.0018	0.9641	1.0177	1.0055	0.9969
Std. Dev.	0.0716	0.0575	0.0412	0.0734	0.1312	0.1621	0.1236	0.0977
Max.	1.2279	1.0892	1.1188	1.1205	1.2075	1.7179	1.3935	1.1706
Min.	0.8178	0.8153	0.9306	0.7437	0.5791	0.7572	0.6988	0.7511

8 Decomposing a share-weighted geometric productivity index

A second quite natural generalization of the single-input-single-output productivity index is given by the share-weighted geometric index, defined by

$$GPROD(x', y', x, y; s, u) \equiv \frac{\prod_{m=1}^{M}(y'_m/y_m)^{u_m}}{\prod_{n=1}^{N}(x'_n/x_n)^{s_n}}, \tag{124}$$

where $s_n > 0$ $(n = 1, ..., N)$, $\sum_{n=1}^{N} s_n = 1$, $u_m > 0$ $(m = 1, ..., M)$, $\sum_{m=1}^{M} u_m = 1$, and it is supposed that all the quantities are positive. The index defined by expression (124) satisfies the fundamental monotonicity and proportionality requirements and, for fixed s and u, exhibits transitivity in (x, y). The output and input quantity indices in numerator and denominator, respectively, are known as Cobb-Douglas indices; see Balk (2008) for their axiomatic properties.[36]

The relation between a share-weighted geometric productivity index and the Solow residual is immediate, as expression (124) is equivalent to

$$\ln GPROD(x', y', x, y; s, u) = \sum_{m=1}^{M} u_m(\ln y'_m - \ln y_m) - \sum_{n=1}^{N} s_n(\ln x'_n - \ln x_n), \tag{125}$$

which in continuous time would be written as

$$\ln GPROD(x', y', x, y; s, u) = \sum_{m=1}^{M} u_m d \ln y_m - \sum_{n=1}^{N} s_n d \ln x_n. \tag{126}$$

If the weights u_m $(m = 1, ..., M)$ are revenue shares and s_n $(n = 1, ..., N)$ are cost shares, then the right-hand side of expression (126) is the Solow residual.

8.1 Choices and consequences

It is straightforward to verify that path A with $\lambda = 1$ leads to the following decomposition:

$$GPROD(x^1, y^1, x^0, y^0; s, u) = M_o^0(x^1, y^1, x^0, y^0) \times \left[\frac{D_o^0(x^0, y^0)}{D_o^0(x^1, y^0)} \prod_{n=1}^{N}(x_n^0/x_n^1)^{s_n}\right] \times \left[\frac{D_o^0(x^1, y^0)}{D_o^0(x^1, y^1)} \prod_{m=1}^{M}(y_m^1/y_m^0)^{u_m}\right], \tag{127}$$

where $M_o^0(x^1, y^1, x^0, y^0)$ is the output orientated CCD index conditional on the base period technology, capturing technical efficiency change and technological change. The second factor on the right-hand side,

[36] To continue footnote 7, now $X(x) \equiv \prod_{n=1}^{N} x_n^{s_n}$ and $Y(y) \equiv \prod_{m=1}^{M} y_m^{u_m}$.

$$SEC^0_{o,GPROD}(x^1, x^0, y^0; s) \equiv \left[\frac{D^0_o(x^0, y^0)}{D^0_o(x^1, y^0)} \prod_{n=1}^{N} (x^0_n/x^1_n)^{s_n}\right],$$

measures the scale (including input mix) effect, since, under CRS, this factor reduces to 1 if $x^1 = \lambda x^0$ ($\lambda > 0$). The third factor,

$$OME^0_{GPROD}(x^1, y^1, y^0; u) \equiv \left[\frac{D^0_o(x^1, y^0)}{D^0_o(x^1, y^1)} \prod_{m=1}^{M} (y^1_m/y^0_m)^{u_m}\right],$$

is a Cobb-Douglas output quantity index number divided by a Malmquist output quantity index number. The factor is homogeneous of degree 0 in (y^0, y^1), and reduces to 1 if $y^1 = \mu y^0$ ($\mu > 0$). Hence, it corresponds to the output mix effect.

The alternative path B, also with $\lambda = 1$, gives

$$GPROD(x^1, y^1, x^0, y^0; s, u) = M^0_o(x^1, y^1, x^0, y^0) \times \left[\frac{D^0_o(x^0, y^1)}{D^0_o(x^1, y^1)} \prod_{n=1}^{N} (x^0_n/x^1_n)^{s_n}\right] \times \left[\frac{D^0_o(x^0, y^0)}{D^0_o(x^0, y^1)} \prod_{m=1}^{M} (y^1_m/y^0_m)^{u_m}\right]. \quad (128)$$

Taking the geometric mean of the last two expressions yields

$$GPROD(x^1, y^1, x^0, y^0; s, u) = M^0_o(x^1, y^1, x^0, y^0) \times \left[\left(\frac{D^0_o(x^0, y^0)}{D^0_o(x^1, y^0)} \frac{D^0_o(x^0, y^1)}{D^0_o(x^1, y^1)}\right)^{1/2} \prod_{n=1}^{N} (x^0_n/x^1_n)^{s_n}\right] \times \left[\left(\frac{D^0_o(x^1, y^0)}{D^0_o(x^1, y^1)} \frac{D^0_o(x^0, y^0)}{D^0_o(x^0, y^1)}\right)^{1/2} \prod_{m=1}^{M} (y^1_m/y^0_m)^{u_m}\right]. \quad (129)$$

If we repeat this sequence of steps for the paths C and D with $\lambda = 1$, we obtain two decompositions which can be averaged to

$$GPROD(x^1, y^1, x^0, y^0; s, u) = M^1_o(x^1, y^1, x^0, y^0) \times \left[\left(\frac{D^1_o(x^0, y^0)}{D^1_o(x^1, y^0)} \frac{D^1_o(x^0, y^1)}{D^1_o(x^1, y^1)}\right)^{1/2} \prod_{n=1}^{N} (x^0_n/x^1_n)^{s_n}\right] \times \left[\left(\frac{D^1_o(x^1, y^0)}{D^1_o(x^1, y^1)} \frac{D^1_o(x^0, y^0)}{D^1_o(x^0, y^1)}\right)^{1/2} \prod_{m=1}^{M} (y^1_m/y^0_m)^{u_m}\right], \quad (130)$$

in which $M^1_o(x^1, y^1, x^0, y^0)$ is the comparison period, output orientated CCD index. In turn the last two expressions can be averaged to obtain as final decomposition

$$GPROD(x^1, y^1, x^0, y^0; s, u) = [M^0_o(x^1, y^1, x^0, y^0) M^1_o(x^1, y^1, x^0, y^0)]^{1/2} \times$$

$$\left[\left(\frac{D_o^0(x^0,y^0)}{D_o^0(x^1,y^0)}\frac{D_o^0(x^0,y^1)}{D_o^0(x^1,y^1)}\frac{D_o^1(x^0,y^0)}{D_o^1(x^1,y^0)}\frac{D_o^1(x^0,y^1)}{D_o^1(x^1,y^1)}\right)^{1/4}\prod_{n=1}^{N}(x_n^0/x_n^1)^{s_n}\right]\times$$
$$\left[\left(\frac{D_o^0(x^1,y^0)}{D_o^0(x^1,y^1)}\frac{D_o^0(x^0,y^0)}{D_o^0(x^0,y^1)}\frac{D_o^1(x^1,y^0)}{D_o^1(x^1,y^1)}\frac{D_o^1(x^0,y^0)}{D_o^1(x^0,y^1)}\right)^{1/4}\prod_{m=1}^{M}(y_m^1/y_m^0)^{u_m}\right], \tag{131}$$

which is symmetric in all variables. This is the counterpart of expressions (58) and (118). The first major factor on the right-hand side of the equality sign is the geometric mean output orientated CCD index, the second factor concerns the scale (including input mix) effect, and the third factor concerns the output mix effect.

The entire exercise can be repeated on the paths E, F, G, and H, specified in Section 4. The completely symmetric decomposition then reads

$$GPROD(x^1,y^1,x^0,y^0;s,u) = [M_i^0(x^1,y^1,x^0,y^0)M_i^1(x^1,y^1,x^0,y^0)]^{1/2}\times$$
$$\left[\left(\frac{D_i^0(x^1,y^0)}{D_i^0(x^0,y^0)}\frac{D_i^0(x^1,y^1)}{D_i^0(x^0,y^1)}\frac{D_i^1(x^1,y^0)}{D_i^1(x^0,y^0)}\frac{D_i^1(x^1,y^1)}{D_i^1(x^0,y^1)}\right)^{1/4}\prod_{n=1}^{N}(x_n^0/x_n^1)^{s_n}\right]\times$$
$$\left[\left(\frac{D_i^0(x^1,y^1)}{D_i^0(x^1,y^0)}\frac{D_i^0(x^0,y^1)}{D_i^0(x^0,y^0)}\frac{D_i^1(x^1,y^1)}{D_i^1(x^1,y^0)}\frac{D_i^1(x^0,y^1)}{D_i^1(x^0,y^0)}\right)^{1/4}\prod_{m=1}^{M}(y_m^1/y_m^0)^{u_m}\right]. \tag{132}$$

This is the counterpart of expressions (76) and (120). The first major factor on the right-hand side of the equality sign, $[M_i^0(x^1,y^1,x^0,y^0)M_i^1(x^1,y^1,x^0,y^0)]^{1/2}$, is the geometric mean input orientated CCD index which captures technical efficiency change and technological change, the second factor measures the input mix effect, and the third factor the scale (including output mix) effect.

Hence, the message is the same as in the previous section. Conditioning on the base period technology, there are six decompositions, three of which use output distance functions and three use input distance functions. Conditioning on the comparison period technology, there are also six decompositions. Taking the 'average' viewpoint, there are nine decompositions that use output distance functions, and nine that use input distance functions. The two completely symmetric decompositions are given by expressions (131) and (132). They differ by being based on either output distance functions or input distance functions. Moreover, the decompositions as such differ: apart from technical efficiency change, technological change, and the scale effect, expression (131) includes the output mix effect whereas expression (132) includes the input mix effect.

The number of different decompositions is still larger because we conditioned on the share variables (s,u). Every choice gives rise to a different productivity index number and a different decomposition. Notice, however, that only the scale and (output or input) mix components are affected, since the CCD indices are independent of these shares.

Natural choices in the case of our firm are the base and comparison period cost and revenue shares, which are defined by $s_n^t \equiv w_n^t x_n^t / w^t \cdot x^t$ $(n=1,...,N)$ and $u_m^t \equiv p_m^t y_m^t / p^t \cdot y^t$ $(m=1,...,M)$, respectively $(t=0,1)$. The corresponding productivity index numbers are then given by $GPROD(x^1,y^1,x^0,y^0;s^0,u^0)$ and

$GPROD(x^1, y^1, x^0, y^0; s^1, u^1)$. The first can be called a Geometric Laspeyres productivity index number, which is the ratio of a Geometric Laspeyres output quantity index number and a Geometric Laspeyres input quantity index number. The second can be called a Geometric Paasche productivity index number, which is the ratio of a Geometric Paasche output quantity index number and a Geometric Paasche input quantity index number.

We again take the 'mean' viewpoint, and compute $GPROD(x^1, y^1, x^0, y^0; (s^0 + s^1)/2, (u^0 + u^1)/2)$, which is the Törnqvist productivity index number, defined as ratio of Törnqvist output and input quantity index numbers. Using expression (131), this index number can be decomposed as

$$\frac{Q^T(p^1, y^1, p^0, y^0)}{Q^T(w^1, x^1, w^0, x^0)} = [M_o^0(x^1, y^1, x^0, y^0) M_o^1(x^1, y^1, x^0, y^0)]^{1/2} \times$$
$$\left[\left(\frac{D_o^0(x^0, y^0)}{D_o^0(x^1, y^0)} \frac{D_o^0(x^0, y^1)}{D_o^0(x^1, y^1)} \frac{D_o^1(x^0, y^0)}{D_o^1(x^1, y^0)} \frac{D_o^1(x^0, y^1)}{D_o^1(x^1, y^1)} \right)^{1/4} \frac{1}{Q^T(w^1, x^1, w^0, x^0)} \right] \times$$
$$\left[\left(\frac{D_o^0(x^1, y^0)}{D_o^0(x^1, y^1)} \frac{D_o^0(x^0, y^0)}{D_o^0(x^0, y^1)} \frac{D_o^1(x^1, y^0)}{D_o^1(x^1, y^1)} \frac{D_o^1(x^0, y^0)}{D_o^1(x^0, y^1)} \right)^{1/4} Q^T(p^1, y^1, p^0, y^0) \right], \tag{133}$$

where $Q^T(p^1, y^1, p^0, y^0) \equiv \prod_{m=1}^M (y_m^1/y_m^0)^{(u_m^0+u_m^1)/2}$ denotes the Törnqvist output quantity index number, and $Q^T(w^1, x^1, w^0, x^0) \equiv \prod_{n=1}^N (x_n^1/x_n^0)^{(s_n^0+s_n^1)/2}$ the Törnqvist input quantity index number.[37] Notice that $GPROD(x^1, y^1, x^0, y^0; (s^0 + s^1)/2, (u^0 + u^1)/2)$ is equal to the geometric mean of $GPROD(x^1, y^1, x^0, y^0; s^0, u^0)$ and $GPROD(x^1, y^1, x^0, y^0; s^1, u^1)$.

Alternatively, by using expression (132), the Törnqvist productivity index number can be decomposed as

$$\frac{Q^T(p^1, y^1, p^0, y^0)}{Q^T(w^1, x^1, w^0, x^0)} = [M_i^0(x^1, y^1, x^0, y^0) M_i^1(x^1, y^1, x^0, y^0)]^{1/2} \times$$
$$\left[\left(\frac{D_i^0(x^1, y^0)}{D_i^0(x^0, y^0)} \frac{D_i^0(x^1, y^1)}{D_i^0(x^0, y^1)} \frac{D_i^1(x^1, y^0)}{D_i^1(x^0, y^0)} \frac{D_i^1(x^1, y^1)}{D_i^1(x^0, y^1)} \right)^{1/4} \frac{1}{Q^T(w^1, x^1, w^0, x^0)} \right] \times$$
$$\left[\left(\frac{D_i^0(x^1, y^1)}{D_i^0(x^1, y^0)} \frac{D_i^0(x^0, y^1)}{D_i^0(x^0, y^0)} \frac{D_i^1(x^1, y^1)}{D_i^1(x^1, y^0)} \frac{D_i^1(x^0, y^1)}{D_i^1(x^0, y^0)} \right)^{1/4} Q^T(p^1, y^1, p^0, y^0) \right]. \tag{134}$$

Notice that expressions (121) and (133), as well as expressions (122) and (134), have the same structure.

8.2 A digression on Orea (2002)

It is interesting to connect expression (131) to a certain result obtained by Orea (2002). Orea specified u and s as the firm's mean shadow revenue shares and shadow cost shares respectively; that is

$$u^{01*} = [\nabla_{\ln y} D_o^0(x^0, y^0) + \nabla_{\ln y} D_o^1(x^1, y^1)]/2, \tag{135}$$

[37] See Balk (2008) for the axiomatic properties of Törnqvist indices.

$$s^{01*} = [\frac{\nabla_{\ln x} D_o^0(x^0, y^0)}{1_N \cdot \nabla_{\ln x} D_o^0(x^0, y^0)} + \frac{\nabla_{\ln x} D_o^1(x^1, y^1)}{1_N \cdot \nabla_{\ln x} D_o^1(x^1, y^1)}]/2. \quad (136)$$

Orea's final result (which is his equation (13)), bears a strong resemblance to expression (131) in the sense that there are factors corresponding to technical efficiency change, technological change, and the scale effect. However, why is there no factor corresponding to the output mix effect? This appears to be an implication of Orea's technology specification, as will be shown now.

Orea specified the output distance functions as being of the translog form with time-invariant second-order coefficients; that is,

$$\ln D_o^t(x, y) = \alpha_0^t + \sum_{n=1}^{N} \alpha_n^t \ln x_n + \sum_{m=1}^{M} \beta_m^t \ln y_m + \quad (137)$$

$$\frac{1}{2} \sum_{n=1}^{N} \sum_{n'=1}^{N} \alpha_{nn'} \ln x_n \ln x_{n'} + \frac{1}{2} \sum_{m=1}^{M} \sum_{m'=1}^{M} \beta_{mm'} \ln y_m \ln y_{m'} +$$

$$\sum_{n=1}^{N} \sum_{m=1}^{M} \gamma_{nm} \ln x_n \ln y_m \; (x \in \Re_{++}^N, y \in \Re_{++}^M) \; (t = 0, 1)$$

with the usual restrictions ensuring linear homogeneity in output quantities.[38] Applying the Quadratic Identity Lemma[39] to the first part of the output mix factor in expression (131) yields

$$\begin{aligned} &\ln \left(\frac{D_o^0(x^1, y^0)}{D_o^0(x^1, y^1)} \frac{D_o^0(x^0, y^0)}{D_o^0(x^0, y^1)} \frac{D_o^1(x^1, y^0)}{D_o^1(x^1, y^1)} \frac{D_o^1(x^0, y^0)}{D_o^1(x^0, y^1)} \right)^{1/4} \\ &= (1/8)[\nabla_{\ln y} \ln D_o^0(x^1, y^0) + \nabla_{\ln y} \ln D_o^0(x^1, y^1) + \\ &\quad \nabla_{\ln y} \ln D_o^0(x^0, y^0) + \nabla_{\ln y} \ln D_o^0(x^0, y^1) + \\ &\quad \nabla_{\ln y} \ln D_o^1(x^1, y^0) + \nabla_{\ln y} \ln D_o^1(x^1, y^1) + \\ &\quad \nabla_{\ln y} \ln D_o^1(x^0, y^0) + \nabla_{\ln y} \ln D_o^1(x^0, y^1)] \cdot \ln(y^0/y^1) \\ &= (1/2)[\nabla_{\ln y} \ln D_o^0(x^0, y^0) + \nabla_{\ln y} \ln D_o^1(x^1, y^1)] \cdot \ln(y^0/y^1) \\ &= u^{01*} \cdot \ln(y^0/y^1), \end{aligned} \quad (138)$$

where the second equality is based on evaluating the derivatives, the third on expression (135), and $\ln y \equiv (\ln y_1, ..., \ln y_M)$. Exponentiating both sides of expression (138) delivers

$$\left(\frac{D_o^0(x^1, y^0)}{D_o^0(x^1, y^1)} \frac{D_o^0(x^0, y^0)}{D_o^0(x^0, y^1)} \frac{D_o^1(x^1, y^0)}{D_o^1(x^1, y^1)} \frac{D_o^1(x^0, y^0)}{D_o^1(x^0, y^1)} \right)^{1/4} = \prod_{m=1}^{M} (y_m^0/y_m^1)^{u_m^{01*}} \quad (139)$$

As the second part of the output mix factor was specified as $\prod_{m=1}^{M} (y_m^1/y_m^0)^{u_m^{01*}}$ the entire factor vanishes.

[38]In particular, α_0^t was specified as a quadratic function of t, and α_n^t $(n = 1, ..., N)$ and β_m^t $(m = 1, ..., M)$ as linear functions.

[39]See Balk (1998, Appendix A) for a general formulation.

Concluding, the output mix effect vanishes if the two technologies are characterized by a translog output distance function with time-invariant second-order coefficients, and the output shares of the share-weighted geometric productivity index are equal to the mean shadow revenue shares. It is straightforward to verify that, the application of the same technique to the first part of the scale-effect term in expression (131) leads to an expression for the entire term which matches exactly with the expression derived by Orea (see his equation (13)). We leave this exercise to the reader. Hence, our finding limits the general applicability of Orea's decomposition result, since this appears to depend on a particular technology specification.

8.3 Empirical application

As in the previous section, for the empirical application of the share-weighted index (124) we follow the classic definitions and weigh the individual input quantity relatives, x_n^1/x_n^0, and output quantity relatives, y_m^1/y_m^0, by cost and revenue shares in the base or comparison period. For $t = 0$ we obtain the GeoLaspeyres index, and for $t = 1$ the GeoPaasche index. Taking the geometric mean of the two yields the Törnqvist productivity index.[40]

In the Taiwanese banking industry in the 2006-2010 period, financial funds (x_1) accounted for the largest average cost share with 50.37%, followed by labour (x_2) with 28.68%, and physical capital (x_3) with 20.95%. The output side is more unbalanced, with loans (y_2) accounting for 81.75%, and financial investments (y_1) for the remaining 18.25% of total revenue. These weights differ yearly. On the cost side the variations are relevant since the cost share for financial funds (x_1) fell from 55.79% in 2006 to 34.35% in 2010. A twenty percent loss against the other two inputs, particularly labour (x_2), whose share increased from 24.61% to 39.49%, while physical capital (x_3) also increased but to a lower extent, from 19.60% to 26.16%. In contrast, the output side was more stable with output shares barely changing.

Tables 49–51 show the descriptive statistics for the GeoLaspeyres productivity index $GPROD(x^1, y^1, x^0, y^0; s^0, u^0)$ including its output orientated decomposition along paths A–B. Tables 52–54 show the GeoPaasche $GPROD(x^1, y^1, x^0, y^0; s^1, u^1)$ results along paths C–D, and Tables 55 and 56 the Törnqvist index results. Similar decompositions based on the input orientation are presented in Tables 57–59 (GeoLaspeyres, paths E–F), Tables 60–62 (GeoPaasche, paths G–H), and Tables 63–64 (Törnqvist, paths E–F–G–H).

The similarity of results with the price-weighted Laspeyres, Paasche and Fisher counterparts is high, with correlation coefficients above 0.90 in all three pairwise comparisons: $\rho(PROD(x^1, y^1, x^0, y^0; w^0, p^0), GPROD(x^1, y^1, x^0, y^0; s^0, u^0)) = 0.923$, $\rho(PROD(x^1, y^1, x^0, y^0; w^1, p^1), GPROD(x^1, y^1, x^0, y^0; s^1, u^1)) = 0.931$, and $\rho(Q^F(p^1, y^1, p^0, y^0)/Q^F(w^1, x^1, w^0, x^0), Q^T(p^1, y^1, p^0, y^0)/Q^T(w^1, x^1, w^0, x^0)) = 0.996$. Interestingly enough, although these results suggest the existence of a highly linear relationship, there is conflicting evidence regarding the similarity of the bilateral

[40] To be precise, the tables contain $GPROD(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; s^{kt}, u^{kt})$, $GPROD(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; s^{k,t+1}, u^{k,t+1})$, and $[GPROD(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; s^{kt}, u^{kt}) \times GPROD(x^{k,t+1}, y^{k,t+1}, x^{kt}, y^{kt}; s^{k,t+1}, u^{k,t+1})]^{1/2}$, respectively, for $k = 1, ..., K(= 31)$, $t = 2006$, 2007, 2008, 2009, along the various paths. Recall that our dataset contains firm-specific prices so that firm-specific cost and revenue shares can be computed.

index number distributions. Whereas the t-test does not reject the hypothesis of equality of means, the Wilcoxon test rejects the equality of medians for the base and comparison period viewpoint comparisons. This discrepancy is quite robust as it is observed for all biennial periods. However, as suspected by looking at their high correlation coefficient, the Fisher and Törnqvist index number distributions exhibit the same means and medians. This is further confirmed by inspecting the productivity index numbers themselves.

The largest difference between the price-weighted and share-weighted indices is observed in the last biennial period with the Laspeyres and GeoLaspeyres indices signaling a 18.85% and 11.06% increase, respectively. The base period viewpoint price-weighted indices report larger increases in general, while the opposite occurs in the case of the comparison period viewpoint. Thus the choice of reference period matters. Ultimately, this is why the geometric mean indices, Fisher and Törnqvist, are so relevant. Similar conclusions can be drawn for the scale (including input mix) and output mix effects, where the difference between the price-weighted and share-weighted indices lies, as both indices share the CCD index. The factors of the former tend to be larger than those of the latter, particularly in the case of the output mix effect.

If we compare the output orientated and input orientated decompositions of the base and comparison period viewpoint share-weighted indices, we find similar differences as reported in the previous empirical Section 7.2. The differences are noticeable for all the factors, but not that marked, tending to be larger in case of the scale effects. All the differences tend to vanish when the geometric decompositions are compared. In all cases the t-test does not reject the null hypotheses of equality of means for all biennial periods. Hence, while the choice of orientation and, most particularly, the reference period is relevant when studying productivity change and its components, the choice between price-weighted or share-weighted indices is less important.

Finally, we do not pursue the comparison between the share-weighted indices and their quantity-only counterparts since it follows the same patterns as reported in Section 7.2 for the price-weighted indices.

Share-weighted Productivity Indices (GPROD)

Table 49: GPROD decomposition along paths A and B: Output orientation, base period viewpoint, common components.

All banks	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0131	0.9926	0.9764	1.1106	1.0221	0.9845	0.9749	1.0442	1.0337	1.0425	1.0740	1.1241
Std. Dev.	0.1005	0.0992	0.1053	0.1682	0.0750	0.0923	0.0547	0.0790	0.0857	0.0927	0.1646	0.2411
Max.	1.3475	1.1394	1.2311	1.6733	1.1834	1.2414	1.0770	1.2335	1.3066	1.3128	1.6347	2.1270
Min.	0.8669	0.7667	0.6038	0.8388	0.8758	0.8233	0.8242	0.9304	0.9535	0.8416	0.7942	0.9909

Table 50: GPROD decomposition along path A: Output orientation, base period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	0.9751	1.0007	0.9885	0.9831	0.9840	0.9720	0.9531	0.9836
Std. Dev.	0.0778	0.0595	0.0446	0.0634	0.0677	0.0826	0.1180	0.1025
Max.	1.2298	1.0899	1.0779	1.0767	1.1538	1.1130	1.2535	1.1561
Min.	0.7917	0.8613	0.8834	0.7643	0.8070	0.7200	0.6587	0.6891

Table 51: GPROD decomposition along path B: Output orientation, base period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	0.9759	1.0045	0.9907	0.9775	0.9839	0.9618	0.9527	0.9888
Std. Dev.	0.0784	0.0531	0.0436	0.0606	0.0684	0.0860	0.1171	0.0970
Max.	1.2298	1.0899	1.0858	1.0471	1.1538	1.1130	1.2535	1.1540
Min.	0.7917	0.8665	0.9079	0.7643	0.8070	0.7200	0.6585	0.6879

Table 52: GPROD decomposition along paths C and D: Output orientation, comparison period viewpoint, common components.

All banks	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0164	0.9926	0.9866	1.1298	1.0221	0.9845	0.9749	1.0442	0.9762	0.9905	0.9893	1.0047
Std. Dev.	0.0954	0.0981	0.1123	0.1785	0.0750	0.0923	0.0547	0.0790	0.0847	0.1024	0.0757	0.0679
Max.	1.3054	1.1371	1.2108	1.6093	1.1834	1.2414	1.0770	1.2335	1.0790	1.0918	1.1550	1.2254
Min.	0.8781	0.7758	0.5691	0.8603	0.8758	0.8233	0.8242	0.9304	0.5714	0.5038	0.6509	0.7669

Table 53: GPROD decomposition along path C: Output orientation, comparison period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	1.0175	1.0215	1.0232	1.0229	1.1096	1.0053	1.0039	1.0588
Std. Dev.	0.1284	0.0684	0.0368	0.0916	0.5664	0.0756	0.1050	0.1119
Max.	1.6524	1.2873	1.1293	1.4360	4.1904	1.1964	1.2750	1.3671
Min.	0.8810	0.8883	0.9627	0.8137	0.8326	0.7789	0.6404	0.7923

Table 54: GPROD decomposition along path D: Output orientation, comparison period viewpoint, specific components.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	1.0236	1.0289	1.0259	1.0281	1.0017	0.9979	1.0011	1.0550
Std. Dev.	0.1390	0.0616	0.0364	0.0959	0.0664	0.0776	0.1038	0.1117
Max.	1.7203	1.2873	1.1301	1.4585	1.1461	1.1962	1.2750	1.3671
Min.	0.8810	0.9236	0.9627	0.8137	0.8326	0.7789	0.6402	0.7927

Table 55: GPROD decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

All banks	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0132	0.9914	0.9827	1.1313	1.0221	0.9845	0.9749	1.0442	1.0019	1.0148	1.0277	1.0581
Std. Dev.	0.1005	0.1010	0.1115	0.1980	0.0750	0.0923	0.0547	0.0790	0.0537	0.0870	0.0949	0.1176
Max	1.3272	1.1328	1.2579	1.8094	1.1834	1.2414	1.0770	1.2335	1.1273	1.1967	1.2484	1.5152
Min	0.8228	0.7712	0.5861	0.8495	0.8758	0.8233	0.8242	0.9304	0.8509	0.6512	0.7190	0.9732

Table 56: GPROD decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

All banks	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
Average	0.9950	1.0134	1.0061	1.0015	0.9921	0.9836	0.9766	1.0265
Std. Dev.	0.0658	0.0475	0.0348	0.0583	0.0673	0.0747	0.1068	0.0808
Max	1.2095	1.0899	1.1055	1.1359	1.1486	1.1022	1.2740	1.2364
Min	0.8492	0.8860	0.9561	0.7897	0.7724	0.7543	0.6492	0.7831

Table 57: GPROD decomposition along paths E and F: Input orientation, base period viewpoint, common components.

All banks	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0131	0.9926	0.9764	1.1106	1.0174	0.9622	0.9537	1.0729	1.1562	1.0838	1.0490	1.1528
Std. Dev.	0.1005	0.0992	0.1053	0.1682	0.1177	0.1270	0.0905	0.1037	0.4486	0.1193	0.0845	0.4454
Max.	1.3475	1.1394	1.2311	1.6733	1.4838	1.2266	1.0827	1.3134	3.3548	1.3088	1.3056	3.3612
Min.	0.8669	0.7667	0.6038	0.8388	0.8257	0.6174	0.5833	0.8762	0.9325	0.8335	0.8844	0.9189

Table 58: GPROD decomposition along path E: Input orientation, base period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	0.9738	0.9915	0.9851	0.9826	0.9413	0.9748	0.9926	0.9547
Std. Dev.	0.0819	0.0681	0.0483	0.0717	0.1605	0.1018	0.1290	0.1376
Max.	1.2460	1.0864	1.0976	1.1228	1.1552	1.2320	1.4275	1.1352
Min.	0.8033	0.8412	0.8813	0.7304	0.3604	0.7772	0.7020	0.4123

Table 59: GPROD decomposition along path F: Input orientation, base period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	0.9862	1.0057	1.0004	0.9872	0.9364	0.9678	0.9909	0.9521
Std. Dev.	0.0786	0.0614	0.0415	0.0759	0.1727	0.1053	0.1296	0.1456
Max.	1.2460	1.1155	1.0976	1.1228	1.1552	1.2320	1.4275	1.1352
Min.	0.8225	0.8544	0.9100	0.7304	0.2719	0.7861	0.7020	0.4123

Table 60: GPROD decomposition along paths G and H: Input orientation, comparison period viewpoint, common components.

All banks	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0164	0.9926	0.9866	1.1298	1.0174	0.9622	0.9537	1.0729	1.0046	1.0041	1.0079	0.9874
Std. Dev.	0.0954	0.0981	0.1123	0.1785	0.1177	0.1270	0.0905	0.1037	0.1104	0.1608	0.0797	0.0619
Max.	1.3054	1.1371	1.2108	1.6093	1.4838	1.2266	1.0827	1.3134	1.3215	1.2284	1.1693	1.1399
Min.	0.8781	0.7758	0.5691	0.8603	0.8257	0.6174	0.5833	0.8762	0.7369	0.2507	0.8347	0.8444

Table 61: GPROD decomposition along path G: Input orientation, comparison period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	1.0001	1.0076	1.0143	1.0141	1.0120	1.0870	1.0207	1.0563
Std. Dev.	0.0703	0.0634	0.0451	0.0816	0.1265	0.3717	0.1160	0.1157
Max.	1.2086	1.0838	1.1404	1.2575	1.4359	3.0239	1.3596	1.3560
Min.	0.8110	0.7840	0.9223	0.7514	0.7280	0.7815	0.6961	0.7644

Table 62: GPROD decomposition along path H: Input orientation, comparison period viewpoint, specific components.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	1.0057	1.0198	1.0203	1.0226	1.0087	1.0696	1.0142	1.0468
Std. Dev.	0.0689	0.0619	0.0441	0.0794	0.1252	0.3323	0.1122	0.1099
Max.	1.2086	1.1143	1.1404	1.2726	1.4150	2.7774	1.3596	1.3449
Min.	0.8110	0.7818	0.9223	0.7586	0.7331	0.7829	0.6961	0.7644

Table 63: GPROD decomposition along paths EFGH: Input orientation, 'mean' period viewpoint

All banks	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
Average	1.0147	0.9926	0.9814	1.1199	1.0174	0.9622	0.9537	1.0729	1.0629	1.0376	1.0314	1.0563
Std. Dev.	0.0977	0.0985	0.1084	0.1716	0.1177	0.1270	0.0905	0.1037	0.1500	0.1350	0.0726	0.1776
Max.	1.3263	1.1320	1.2209	1.6410	1.4838	1.2266	1.0827	1.3134	1.6359	1.2482	1.1804	1.9133
Min.	0.8732	0.7713	0.5862	0.8495	0.8257	0.6174	0.5833	0.8762	0.8776	0.4571	0.8630	0.9189

Table 64: GPROD decomposition along paths EFGH: Input orientation, 'mean' period viewpoint.

All banks	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
Average	0.9902	1.0074	1.0063	1.0018	0.9660	1.0192	1.0042	0.9893
Std. Dev.	0.0715	0.0576	0.0412	0.0736	0.1299	0.1603	0.1203	0.1002
Max.	1.2272	1.0886	1.1188	1.1215	1.2077	1.7181	1.3931	1.1231
Min.	0.8178	0.8147	0.9307	0.7426	0.5791	0.7839	0.6990	0.7152

9 Incorporating allocative efficiency change

How does allocative efficiency change fit into a decomposition of productivity change? Coelli *et al.* (2003) basically conceived the ratio

$$\frac{GPROD(x^1, y^1, x^0, y^0; (s^0 + s^1)/2, (u^0 + u^1)/2)}{GPROD(x^1, y^1, x^0, y^0; s^{01*}, u^{01*})} \tag{140}$$

as being the contribution of allocative efficiency change to productivity change as measured by the *numerator* of expression (140). Clearly, this interpretation is unsatisfactory. The ratio in expression (140) just is what it is: the ratio of two productivity index numbers, one weighing with average actual cost and revenue shares, and the other with average shadow cost and revenue shares. Both index numbers admit decompositions into factors corresponding to technical efficiency change, technological change, input or output mix effect, and scale (including output or input mix) effect. The only message which the ratio conveys is that (in general) the weights do matter.

To measure the contribution of allocative efficiency change to overall productivity change we must take a different approach. Consider a decomposition of productivity change in terms of output distance functions. Then the natural measure to use is output allocative efficiency, which is defined by[41]

$$OAE^t(x, p, y) \equiv \frac{p \cdot y}{R^t(x, p) D_o^t(x, y)}, \tag{141}$$

where $R^t(x, p) \equiv \max_y \{p \cdot y \mid (x, y) \in S^t\}$ is the (direct) revenue function. Notice that $OAE^t(x, p, y) \leq 1$. A firm with input-output quantity combination (x, y) and output prices p is called output allocatively efficient if $OAE^t(x, p, y) = 1$. Given input quantities x and output prices p, a firm is always output allocatively efficient at (x, y^*) whenever y^* is such that $R^t(x, p) = p \cdot y^*$; note that $D_o^t(x, y^*) = 1$.

To incorporate output allocative efficiency change in the productivity index we must construct the hypothetical path from (x^0, y^0) to (x^1, y^1) such that it runs through the base and comparison period allocatively efficient points. Specifically, consider the following path (which is a modification of path C with $\lambda = 1$):

Path C*: (x^0, y^0)	$\longrightarrow$	$(x^0, y^0/D_o^0(x^0, y^0))$	$\longrightarrow$
$(x^0, y^{0*}/D_o^0(x^0, y^{0*}))$	$\longrightarrow$	$(x^0, y^{0*}/D_o^1(x^0, y^{0*}))$	$\longrightarrow$
$(x^1, y^{0*}/D_o^1(x^1, y^{0*}))$	$\longrightarrow$	$(x^1, y^{1*}/D_o^1(x^1, y^{1*}))$	$\longrightarrow$
$(x^1, y^1/D_o^1(x^1, y^1)) \longrightarrow (x^1, y^1)$			

where $R^t(x^t, p^t) = p^t \cdot y^{t*}$, and recall that $D_o^t(x^t, y^{t*}) = 1$ $(t = 0, 1)$. Thus, the first segment brings us from the base period observation to its radial projection on the frontier of the base period technology. The second segment connects the technically efficient point with the allocatively efficient point. The third segment connects the allocatively efficient point on the base period frontier to its projection on the comparison period frontier. The fourth and fifth segment concern the move in input and output quantity space, respectively. The endpoint here is the allocatively efficient point on the comparison period frontier, which is determined by comparison

[41] See Balk (1998, Section 4.1). O'Donnell would call this output mix efficiency.

period output prices and input quantities. The sixth segment connects this point with the radial projection of the comparison period observation. The last segment links the radial projection and the actual observation.

A second path is obtained by interchanging the move in input and output quantity space. Two other paths are obtained by first moving in input or output quantity space along the base period frontier before moving to the projection on the comparison period frontier.

Along each path and each of its segments the productivity index $GPROD\,(x', y', x, y; s, u)$ can be computed.[42] By transitivity, we obtain four different decompositions, which can be averaged to

$$
\begin{aligned}
&GPROD(x^1, y^1, x^0, y^0; s, u) = \\
&\left(\frac{D_o^1(x^1, y^1)}{D_o^0(x^0, y^0)}\right) \times \left(\prod_{m=1}^{M} (y_m^{0*}/y_m^0)^{u_m} \prod_{m=1}^{M} (y_m^1/y_m^{1*})^{u_m} \frac{D_o^0(x^0, y^0)}{D_o^1(x^1, y^1)}\right) \times \\
&\left(\frac{D_o^0(x^0, y^{0*})}{D_o^1(x^0, y^{0*})} \frac{D_o^0(x^1, y^{1*})}{D_o^1(x^1, y^{1*})}\right)^{1/2} \times \\
&\left[\left(\frac{D_o^0(x^0, y^{0*})}{D_o^0(x^1, y^{0*})} \frac{D_o^0(x^0, y^{1*})}{D_o^0(x^1, y^{1*})} \frac{D_o^1(x^0, y^{0*})}{D_o^1(x^1, y^{0*})} \frac{D_o^1(x^0, y^{1*})}{D_o^1(x^1, y^{1*})}\right)^{1/4} \prod_{n=1}^{N} (x_n^0/x_n^1)^{s_n}\right] \times \\
&\left[\left(\frac{D_o^0(x^1, y^{0*})}{D_o^0(x^1, y^{1*})} \frac{D_o^0(x^0, y^{0*})}{D_o^0(x^0, y^{1*})} \frac{D_o^1(x^1, y^{0*})}{D_o^1(x^1, y^{1*})} \frac{D_o^1(x^0, y^{0*})}{D_o^1(x^0, y^{1*})}\right)^{1/4} \prod_{m=1}^{M} (y_m^{1*}/y_m^{0*})^{u_m}\right]. \qquad (142)
\end{aligned}
$$

The first factor on the right-hand side of the equality sign measures technical efficiency change, the second measures allocative efficiency change, the third measures technological change, the fourth measures the scale (including input mix) effect, and the fifth measures the output mix effect.

It is interesting to compare this decomposition to the previous one, in expression (131), where it is useful to recall that

$$
\begin{aligned}
&[M_o^0(x^1, y^1, x^0, y^0) M_o^1(x^1, y^1, x^0, y^0)]^{1/2} = \\
&\left(\frac{D_o^1(x^1, y^1)}{D_o^0(x^0, y^0)}\right) \times \left(\frac{D_o^0(x^0, y^0)}{D_o^1(x^0, y^0)} \frac{D_o^0(x^1, y^1)}{D_o^1(x^1, y^1)}\right)^{1/2}. \qquad (143)
\end{aligned}
$$

The efficiency change terms in expressions (131) and (142) are identical. The terms measuring technological change, scale effect, and output mix effect have the same structure, whereby the actual output quantities y^t in expression (131) have been replaced by optimal (revenue maximizing) output quantities y^{t*} in expression (142). The additional term in expression (142) is the term measuring allocative efficiency change. This term basically measures the change of the distance between the technically efficient output quantity vector $y^t/D_o^t(x^t, y^t)$ and the optimal, that is, technically as well as allocatively efficient vector y^{t*} when t goes from 0 to 1. It is simple to verify that if $y^t/D_o^t(x^t, y^t) = y^{t*}$ $(t = 0, 1)$, which means that in both periods our

[42] The following can be repeated for $PROD(x', y', x, y; w, p)$ with obvious modifications.

firm is allocatively but not necessarily technically efficient, then the right-hand sides of expressions (131) and (142) become identical.

The conclusion is that unless our firm is supposed to be (output) allocatively efficient, the technological change, scale, and output mix effect terms in decomposition (131) all incorporate a part of the allocative efficiency change.

The whole story can be repeated for a decomposition of productivity change in terms of input distance functions. The details are left to the reader. There are again four decompositions, the average of which is

$$\begin{aligned} &GPROD(x^1, y^1, x^0, y^0; s, u) = \\ &\left(\frac{D_i^0(x^0, y^0)}{D_i^1(x^1, y^1)}\right) \times \left(\prod_{n=1}^{N}(x_n^0/x_n^{0*})^{s_n} \prod_{n=1}^{N}(x_n^{1*}/x_n^1)^{s_n} \frac{D_i^1(x^1, y^1)}{D_i^0(x^0, y^0)}\right) \times \\ &\left(\frac{D_i^1(x^{0*}, y^0)}{D_i^0(x^{0*}, y^0)} \frac{D_i^1(x^{1*}, y^1)}{D_i^0(x^{1*}, y^1)}\right)^{1/2} \times \\ &\left[\left(\frac{D_i^0(x^{1*}, y^0)}{D_i^0(x^{0*}, y^0)} \frac{D_i^0(x^{1*}, y^1)}{D_i^0(x^{0*}, y^1)} \frac{D_i^1(x^{1*}, y^0)}{D_i^1(x^{0*}, y^0)} \frac{D_i^1(x^{1*}, y^1)}{D_i^1(x^{0*}, y^1)}\right)^{1/4} \prod_{n=1}^{N}(x_n^{0*}/x_n^{1*})^{s_n}\right] \times \\ &\left[\left(\frac{D_i^0(x^{1*}, y^1)}{D_i^0(x^{1*}, y^0)} \frac{D_i^0(x^{0*}, y^1)}{D_i^0(x^{0*}, y^0)} \frac{D_i^1(x^{1*}, y^1)}{D_i^1(x^{1*}, y^0)} \frac{D_i^1(x^{0*}, y^1)}{D_i^1(x^{0*}, y^0)}\right)^{1/4} \prod_{m=1}^{M}(y_m^1/y_m^0)^{u_m}\right], \end{aligned} \tag{144}$$

where $C^t(w^t, y^t) = w^t \cdot x^{t*}$ and $C^t(w, y) \equiv \min_x\{w \cdot x \mid (x, y) \in S^t\}$ is the (direct) cost function ($t = 0, 1$). The first factor on the right-hand side of the equality sign measures technical efficiency change, the second measures allocative efficiency change, the third measures technological change, the fourth measures the input mix effect, and the fifth measures the scale (including output mix) effect.

This decomposition must be compared to the one in expression (132), where it is useful to recall that

$$\begin{aligned} &[M_i^0(x^1, y^1, x^0, y^0) M_i^1(x^1, y^1, x^0, y^0)]^{1/2} = \\ &\left(\frac{D_i^0(x^0, y^0)}{D_i^1(x^1, y^1)}\right) \times \left(\frac{D_i^1(x^0, y^0)}{D_i^0(x^0, y^0)} \frac{D_i^1(x^1, y^1)}{D_i^0(x^1, y^1)}\right)^{1/2}. \end{aligned} \tag{145}$$

The efficiency change terms in expressions (132) and (144) are identical. The terms measuring technological change, input mix effect, and scale effect have the same structure, whereby the actual input quantities x^t in expression (132) have been replaced by optimal (cost minimizing) input quantities x^{t*} in expression (144). The additional term in expression (144) is the term measuring allocative efficiency change. This term basically measures the change of the distance between the technically efficient input quantity vector $x^t/D_i^t(x^t, y^t)$ and the optimal, that is, technically as well as allocatively efficient vector x^{t*} when t goes from 0 to 1. It is simple to verify that if $x^t/D_i^t(x^t, y^t) = x^{t*}$ ($t = 0, 1$), which means that our firm is allocatively but not necessarily technically efficient in both periods, then the right-hand sides of expressions (132) and (144) become identical.

The conclusion is that unless our firm is supposed to be (input) allocatively efficient, the technological change, input mix, and scale effect terms in decomposition (132) all incorporate a part of the allocative efficiency change.

In Appendix A some attention is paid to recent contributions in which allocative efficiency change plays also a role.

10 Conclusion

This paper presented a broader view on total factor productivity (TFP) measurement than the usual neo-classical approach provides. We now recapitulate the main points and gather some lessons.

For any production unit, be it an enterprise, an industry, or a country, TFP change between two time periods is defined as the ratio of an output quantity index over an input quantity index, whereby 'total' means that all the inputs (aka factors) are taken into account. As we know, our economic-statistical toolbox provides several ways of defining such a TFP index. Assuming that there exists a production frontier satisfying mild regularity requirements in any time period considered, we can decompose any TFP index into mutually independent factors: efficiency change (aka catching-up), technological change (aka frontier shift), scale effect, input mix effect, and output mix effect. Such decompositions can be input-orientated or output-orientated. The technological change component is what mainstream neo-classical economists mean when they refer to TFP change.

We considered the four families of Malmquist, Moorsteen-Bjurek, price-weighted, and share-weighted indices. As for any index from these families a specific viewpoint must be chosen – are we using the past or the current technology as benchmark? – it does not come as a surprise that we end up with numerous theoretically different measures and decompositions of TFP change. The literature provides us with a number of empirical implementations. However, most of these implementations, sometimes on microdata, sometimes on macrodata, appear to be partial, usually concerning only one, two, or three decompositions at a time. The unique feature of this paper is that all the theoretically possible decompositions are applied to the same dataset of a real-life panel of production units. Thus, in the particular case of our dataset, we are able to judge the extent to which the various measures and decompositions are empirically different. Moreover, the paper is accompanied by a software toolbox so that researchers can replicate our work with their own dataset.

What are the main findings?

- The common core of all the TFP indices appears to be a CCD index, capturing efficiency change and technological change. Although a CCD index is not a proper productivity index, except in the case of global CRS, the remaining factors – scale, input mix, and output mix – are numerically less important.
- Since the reference period (i.e., the technology used as benchmark) matters empirically, we recommend researchers to use geometric means of base period and comparison period viewpoint indices, unless there are compelling reasons that a particular viewpoint should be used.

- Decompositions can be based on input or output distance functions. In a four-factor decomposition, the scale effect is either combined with the input mix effect or with the output mix effect, but is not separately available (as that would require great numerical effort, since DEA does not characterize homothetic technologies–theorems 1 and 2).

- The Malmquist and Moorsteen-Bjurek indices, based on quantities only, deliver higher productivity growth figures than the price-weighted and share-weighted indices which are based on quantities and (market) prices. To which extent this is due to the fact that the technologies were estimated by DEA in our empirical exercise remains to be seen. (Interestingly, Balk (1998) obtained a similar pattern.)

- The results of price-weighted (e.g., Fisher) and share-weighted (e.g., Törnqvist) indices are numerically very close. Thus the choice between the two families of indices is immaterial.

- In our empirical example (local) technological change generally appeared to be the main component of TFP change.

Appendix A: A dual productivity index

A recent attempt to incorporate allocative efficiency change into a measure of productivity change was undertaken by Maniadakis and Thanassoulis (2000), (2004).[43] Their generic productivity index (actually its inverse) was defined by

$$\check{CM}^t(x', y', x, y; w) \equiv \frac{\check{C}^t(w, y')/w \cdot x'}{\check{C}^t(w, y)/w \cdot x}, \tag{146}$$

where $\check{C}^t(w, y) \equiv \min_x\{w \cdot x|(x, y) \in \check{S}^t\}$ is the cost function corresponding to the period t cone technology. This function is therefore linearly homogeneous in y. The index in expression (146) was baptized 'cost Malmquist productivity index'. It has the required monotonicity and proportionality properties and exhibits transitivity in (x, y). It is interesting to compare this index to the price-weighted productivity index as defined by expression (108): $p \cdot y$ has been replaced by $\check{C}^t(w, y)$, and $p \cdot y'$ by $\check{C}^t(w, y')$.[44]

Define input allocative efficiency with respect to the cone technology by

$$I\check{AE}^t(w, x, y) \equiv \frac{\check{C}^t(w, y)\check{D}_i^t(x, y)}{w \cdot x}. \tag{147}$$

Notice that this concept is dual to output allocative efficiency as defined by expression (141), except for the technologies to which the two concepts are applied.

Consider now the productivity index $\check{CM}^0(x^1, y^1, x^0, y^0; w^0)$, which conditions on base period technology and input prices, or $\check{CM}^1(x^1, y^1, x^0, y^0; w^1)$, which conditions on comparison period technology and input prices, or their geometric mean. For the first, the decomposition proposed by Maniadakis and Thanassoulis reads as follows:

$$\begin{aligned} &\check{CM}^0(x^1, y^1, x^0, y^0; w^0) = \\ &\quad \frac{\check{D}_i^0(x^0, y^0)}{\check{D}_i^1(x^1, y^1)} \times \frac{I\check{AE}^1(w^1, x^1, y^1)}{I\check{AE}^0(w^0, x^0, y^0)} \times \frac{\check{D}_i^1(x^1, y^1)}{\check{D}_i^0(x^1, y^1)} \times \frac{I\check{AE}^0(w^0, x^1, y^1)}{I\check{AE}^1(w^1, x^1, y^1)}. \end{aligned} \tag{148}$$

---

[43] The generalization to a metafrontier setting was developed by Huang, Juo and Fu (2015).

[44] In the literature a number of related concepts occur. Färe, Grosskopf and Margaritis (FGM) (2008, 569) called

$$\frac{C^t(w', y')/w' \cdot x'}{C^t(w, y)/w \cdot x}$$

a 'cost-based Malmquist productivity index', whereas Ball, Färe, Grosskopf and Zaim (BFGZ) (2005) called it 'Malmquist cost productivity index' (and outputs were divided in good and bad). Notice that $C^t(w, y)/w \cdot x$ is the cost efficiency of the input-output combination (x, y) under the technology S^t when input prices are w. Thus the concept considered by BFGZ or FGM is a cost-efficiency index rather than a productivity index. The same concept was considered by Diewert (2014). Replacing the actual technology S^t by the associated cone technology $\check{S}^t$, and thus $C^t(.)$ by $\check{C}^t(.)$, leads to the measure considered by Granderson and Prior (2013), where outputs were also divided in good and bad. However, unless $w' = w$, this is still not a productivity index. Thanassoulis *et al.* (2015) meant to transfer the productivity index defined by expression (146) to the spatial domain (with groups of firms) but actually ended up with the spatial version of the Granderson and Prior construct. This was generalized by Walheer (2018) to a situation with output-specific input prices.

In the interpretation of Maniadakis and Thanassoulis, the first factor on the right-hand side of the equality sign measures technical efficiency change, the second factor measures allocative efficiency change,[45] the third factor measures technological change, and the fourth factor is a residual input price effect.

We consider this decomposition as unsatisfactory for two main reasons: 1) all these measures are relative to the cone technologies rather than to the actual technologies, and 2) the interpretation of the residual price effect is unclear. As its notation makes clear, the fourth factor is a function not only of input prices but also of the cone technologies of the two periods. It thus confounds input price change with technological change.

Appendix B: Computational aspects

Suppose we are given panel data $(w^{kt}, x^{kt}, p^{kt}, y^{kt})$ for firms $k = 1, ..., K$ and time periods $t = 0, 1, ..., T$. These data allow us to calculate any productivity index we wish, either cross-sectionally[46] or intertemporally, or in a combination of both viewpoints. If only quantity data are given, the choice is restricted to Malmquist indices. As we have shown in the foregoing sections, several options are available for a decomposition of productivity change.

Although expressions such as (58) look quite intimidating, their computation should not be much of a problem, if one has knowledge of the functions involved. For example, for the computation of the various parts of expression (58) only knowledge of the output distance functions $D_o^t(x, y)$ and $\check{D}_o^t(x, y)$ is required, and for the various parts of expression (76) only knowledge of the input distance functions $D_i^t(x, y)$ and $\check{D}_i^t(x, y)$. For the computation of the various parts of expressions (118), (121), (131), and (133) knowledge of $D_o^t(x, y)$ suffices, whereas for the computation of the parts of expressions (120), (122), (132), and (134) knowledge of $D_i^t(x, y)$ suffices. For the computation of parts of expression (142) knowledge of $D_o^t(x, y)$ and of the optimal output quantity vector y^* implied by the revenue function $R^t(x, p)$ is required. For parts of expression (144) knowledge of $D_i^t(x, y)$ and of the optimal input quantity vector x^* implied by the cost function $C^t(w, y)$ is required.

Using the technique of Data Envelopment Analysis the period t technology S^t is approximated by

$$S^t \approx \{(x, y) \mid \sum_{k=1}^{K} z_k x^{kt} \leq x, y \leq \sum_{k=1}^{K} z_k y^{kt}, \quad z_k \geq 0 \; (k = 1, ..., K), \sum_{k=1}^{K} z_k = 1\}, \tag{149}$$

whereas the associated cone technology $\check{S}^t$ is approximated by

[45] This factor has the same structure as expression (3.181) in Balk (1998).

[46] In a cross-section productivity *differences* between firms are considered. It is then usually assumed that these firms share the same technology, which implies that the cross-sectional analogue of technological change does not exist.

$$\check{S}^t \approx \{(x,y) \mid \sum_{k=1}^{K} z_k x^{kt} \leq x, y \leq \sum_{k=1}^{K} z_k y^{kt}, \quad z_k \geq 0 \, (k = 1, ..., K)\}. \tag{150}$$

If $x \in \Re^1_+$ (single input) and all x^{kt} are equal, or if $y \in \Re^1_+$ (single output) and all y^{kt} are equal, then these two approximations coincide (Lovell and Pastor 1999).

Based on these approximations, the computation of the required output or input distance functions, and the optimal quantity vectors implied by the revenue or cost functions for particular values of their variables is seen to be reduced to the solution of linear programming problems.[47] These problems can be solved by using MATLAB. A stand-alone *Total Factor Productivity Toolbox* contains all the software and functions for calculating all the different productivity indices (allowing for choice of orientation, viewpoint, etc.) and their decompositions. The toolbox can be downloaded from `www.tfptoolbox.com`. A guide is provided by Balk, Barbero, and Zofío (2018).

The parametric approach is much more demanding. It requires the specification of a functional form for the output distance function and estimation of its parameters from the data. Where necessary, the cone technology output distance function can be obtained as[48]

$$\check{D}^t_o(x,y) = \min_{\lambda} D^t_o(\lambda x, \lambda y). \tag{151}$$

Where necessary, the output quantity vector y^* must be obtained by (numerically) solving the system of equations that emerges from the first-order conditions for revenue maximization,

$$\begin{aligned} p/p \cdot y^* &= \nabla_y D^t_o(x, y^*) \\ 1 &= D^t_o(x, y^*). \end{aligned} \tag{152}$$

Alternatively, the specification of a functional form for the input distance function is required. The cone technology input distance function is then obtained as

$$\check{D}^t_i(x,y) = \max_{\lambda} D^t_i(\lambda x, \lambda y). \tag{153}$$

Where necessary, the input quantity vector x^* must be obtained by (numerically) solving the system of equations that emerges from the first order conditions for cost minimization,

[47] See also Kerstens and Van de Woestyne (2014) for an activity analysis based comparison of the geometric mean input orientated Malmquist productivity index $[\check{M}^0_i(x^1,y^1,x^0,y^0) \times \check{M}^1_i(x^1,y^1,x^0,y^0)]^{1/2}$, see expression (76), and the geometric mean MB index $[MB^0(x^1,y^1,x^0,y^0;x^0,y^0)MB^1(x^1,y^1,x^0,y^0;x^1,y^1)]^{1/2}$ on two datasets. Balk (1998, 191-196) conducted a similar comparison.

[48] See Balk (2001; 167) for the details.

$$
\begin{aligned}
w/w \cdot x^* &= \nabla_x D_i^t(x^*, y) \\
1 &= D_i^t(x^*, y).
\end{aligned} \tag{154}
$$

The discussion of Orea's specification in Section 8 shows that the specification of the functional forms and the associated assumptions concerning firm behaviour should be sufficiently rich to rule out the possibility that some components of productivity change vanish by definition. This specifically applies if we want to compute the allocative efficiency change component. The specification of a parametric model should then explicitly allow for the occurrence of allocative inefficiency on the part of the firms in the panel.

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Supplemental appendix S: Detailed outcomes

Malmquist Productivity Indices (MPI)

Table S.1: MPI decomposition along paths A and B: Output orientation, base period viewpoint, common components.

Bank	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.2027	1.1228	1.1621	1.0042	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
2	1.0220	1.0361	1.0220	1.5589	1.0000	1.0000	1.0000	1.0000	1.1673	1.0502	1.0358	1.7839
3	1.0810	1.1412	1.0958	1.6148	1.1352	0.9880	1.0770	1.1975	0.9558	1.0100	0.9846	1.1293
4	1.0437	1.0611	0.9934	1.1671	1.0540	1.0185	0.9875	1.0764	1.0142	1.0362	1.0093	1.0116
5	1.0159	1.0867	1.0107	1.0715	1.0000	1.0000	1.0000	1.0000	1.0590	1.1741	1.0474	1.1124
6	1.0231	1.0144	1.0558	1.0766	1.0000	1.0000	1.0000	1.0000	1.3066	1.0587	1.1959	1.1288
7	1.0679	1.0599	1.1499	1.0658	1.0000	1.0000	1.0000	1.0000	1.0051	1.0705	1.5139	1.0386
8	1.0725	1.1316	1.0979	1.0380	1.0460	1.1503	0.9475	1.0281	0.9768	1.0748	1.3448	0.9943
9	0.9487	1.0587	1.0937	1.0137	1.0124	1.0107	1.0106	0.9936	0.9536	0.9675	1.0048	1.0024
10	1.0222	1.0265	1.1674	0.9839	1.0000	1.0000	1.0000	1.0000	1.1130	1.0799	1.6347	1.1347
11	0.8588	1.0164	1.0526	1.0826	1.0268	1.0116	0.8736	1.0695	0.9535	1.0009	0.9647	1.0020
12	1.1380	1.1105	1.0547	1.0989	1.1834	1.0423	0.9686	0.9304	1.0722	1.0316	1.0829	1.1821
13	1.0666	0.8395	0.7832	0.9946	1.0741	0.8686	0.9064	1.0658	0.9745	0.9758	0.9937	1.0046
14	1.0006	0.9928	1.1747	2.0755	0.9914	0.9512	1.0001	1.0603	1.0097	1.0364	1.0838	2.1270
15	1.0882	0.9305	0.9220	1.6454	1.0980	0.9453	0.9184	1.2335	0.9577	0.9899	0.9666	1.1501
16	1.3026	0.8790	0.8957	0.9105	1.1801	0.8233	0.8242	0.9331	1.0182	1.0452	1.0246	0.9914
17	0.9973	0.9921	0.9341	1.3427	1.0468	0.9616	0.9680	1.0973	0.9592	1.0133	0.9780	1.0123
18	0.9401	1.0436	0.9950	1.6180	0.8758	1.0282	0.8908	1.1731	1.0925	1.0313	1.0207	1.0954
19	0.9379	0.8501	1.1205	1.2329	0.9424	0.8243	1.0517	0.9994	0.9950	1.0322	1.0089	0.9920
20	0.8260	1.2081	1.1234	1.5509	0.8799	0.9795	1.0111	1.2084	0.9907	1.0309	1.0582	1.0764
21	0.9274	1.1603	1.1295	0.8466	1.0000	1.0000	1.0000	1.0000	1.0099	1.3118	1.2150	1.0299
22	0.9603	0.9038	0.9636	1.0884	0.9609	0.9281	0.9992	1.0343	0.9913	1.0311	1.0020	1.0009
23	0.9634	0.9497	0.9633	0.9249	0.9368	0.9144	0.9449	0.9316	1.0124	1.0401	1.0128	0.9955
24	1.0557	1.0677	1.0505	1.0511	1.0858	0.9725	1.0221	0.9812	0.9557	1.0103	0.9699	1.0008
25	1.0783	1.0891	1.0819	1.2315	1.0455	0.9544	1.0334	1.0704	0.9905	1.0269	0.9743	1.0229
26	1.2488	0.8273	1.0550	1.1976	1.0000	1.0000	1.0000	1.0000	1.2671	0.8416	1.0838	1.2349
27	1.1102	0.8579	1.0737	1.1091	1.0629	0.8363	0.9881	1.0261	1.0487	1.0530	1.0280	0.9909
28	1.0451	1.3220	0.9853	1.2950	0.9916	0.9091	0.9486	1.1842	1.0403	1.0529	1.0315	1.0323
29	1.1220	1.2931	0.7816	1.1966	1.0875	1.2414	0.9985	1.0015	1.0403	1.3128	0.7942	1.3430
30	1.1437	1.0612	1.0455	1.2210	0.8796	1.2165	0.8722	1.0317	1.1052	0.8646	1.1678	1.0956
31	1.0486	0.9656	0.9376	1.0944	1.0884	0.9422	0.9781	1.0414	0.9761	1.0209	0.9868	1.0080
Average	1.0438	1.0355	1.0313	1.2065	1.0221	0.9845	0.9749	1.0442	1.0337	1.0425	1.0740	1.1241
Std. Dev.	0.1010	0.1210	0.0989	0.2661	0.0750	0.0923	0.0547	0.0790	0.0857	0.0927	0.1646	0.2411
Max.	1.3026	1.3220	1.1747	2.0755	1.1834	1.2414	1.0770	1.2335	1.3066	1.3128	1.6347	2.1270
Min.	0.8260	0.8273	0.7816	0.8466	0.8758	0.8233	0.8242	0.9304	0.9535	0.8416	0.7942	0.9909

Table S.2: MPI decomposition along path A: Output orientation, base period viewpoint, specific components, RTS.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	0.8265	0.9999	1.0019	1.0019	1.0593	0.9867	0.9849	0.8723	0.8755	0.9866	0.9867	0.8739
3	0.9982	1.1728	1.0887	1.0697	0.9980	0.9752	0.9491	1.1162	0.9962	1.1437	1.0333	1.1940
4	1.0212	1.0953	1.0285	1.0827	0.9561	0.9179	0.9690	0.9899	0.9764	1.0054	0.9966	1.0718
5	0.9925	0.9139	0.9737	1.0006	0.9666	1.0128	0.9910	0.9627	0.9593	0.9256	0.9649	0.9633
6	0.9590	0.9873	0.9891	1.0093	0.8165	0.9704	0.8925	0.9450	0.7830	0.9581	0.8828	0.9537
7	1.0030	0.9752	1.0015	0.9986	1.0592	1.0153	0.7584	1.0277	1.0625	0.9901	0.7596	1.0262
8	1.0341	1.0294	1.0048	1.0173	1.0151	0.8892	0.8575	0.9981	1.0498	0.9153	0.8616	1.0154
9	0.9577	1.0720	1.0717	1.0357	1.0261	1.0100	1.0049	0.9827	0.9827	1.0827	1.0770	1.0178
10	0.9233	0.9908	0.9329	0.9311	0.9947	0.9593	0.7656	0.9312	0.9184	0.9505	0.7142	0.8671
11	0.9019	0.9627	1.2505	1.0117	0.9725	1.0427	0.9988	0.9986	0.8771	1.0039	1.2490	1.0103
12	0.9933	1.0015	0.9923	1.0035	0.9030	1.0313	1.0135	0.9957	0.8969	1.0328	1.0057	0.9992
13	1.0071	0.9931	1.0034	0.9901	1.0118	0.9974	0.8666	0.9383	1.0190	0.9905	0.8696	0.9290
14	0.9996	1.0071	1.0105	0.9589	1.0000	0.9999	1.0725	0.9598	0.9996	1.0070	1.0837	0.9203
15	1.0583	1.0206	1.0860	1.1032	0.9779	0.9743	0.9564	1.0513	1.0349	0.9944	1.0386	1.1598
16	1.0841	1.0215	1.0607	0.9842	1.0000	1.0000	1.0000	1.0000	1.0841	1.0215	1.0607	0.9842
17	0.9967	1.0039	0.9988	1.0172	0.9966	1.0143	0.9879	1.1884	0.9933	1.0182	0.9868	1.2089
18	0.9946	1.0067	0.9751	1.0444	0.9878	0.9775	1.1223	1.2056	0.9825	0.9841	1.0943	1.2591
19	1.0015	1.0423	1.0553	1.0323	0.9987	0.9586	1.0007	1.2047	1.0002	0.9991	1.0560	1.2436
20	0.9476	1.2390	1.0087	1.0465	1.0000	0.9657	1.0410	1.1394	0.9476	1.1965	1.0500	1.1924
21	0.8946	1.0196	1.0052	0.8081	1.0265	0.8675	0.9248	1.0172	0.9183	0.8845	0.9296	0.8220
22	1.0022	0.9995	1.0097	1.0514	1.0060	0.9450	0.9532	1.0000	1.0082	0.9445	0.9624	1.0514
23	1.0158	0.9986	1.0065	0.9973	1.0000	1.0000	1.0000	1.0000	1.0158	0.9986	1.0065	0.9973
24	1.0595	1.0588	1.0498	1.0658	0.9602	1.0263	1.0094	1.0044	1.0173	1.0867	1.0597	1.0705
25	1.0414	1.1007	1.0942	1.1398	1.0000	1.0096	0.9820	0.9868	1.0414	1.1113	1.0744	1.1247
26	0.9856	0.9807	0.9735	0.9699	1.0000	1.0023	1.0000	1.0000	0.9856	0.9830	0.9735	0.9699
27	0.9959	0.9633	1.0661	1.0909	1.0000	1.0113	0.9916	1.0000	0.9959	0.9742	1.0571	1.0909
28	1.0124	1.4642	1.0113	1.0761	1.0007	0.9433	0.9958	0.9844	1.0131	1.3812	1.0071	1.0593
29	0.9960	1.0143	0.9933	0.9068	0.9958	0.7822	0.9923	0.9811	0.9918	0.7934	0.9856	0.8897
30	1.0940	0.9998	1.0081	1.0416	1.0754	1.0091	1.0182	1.0371	1.1765	1.0089	1.0264	1.0802
31	0.9870	0.9917	1.0634	1.0692	1.0000	1.0123	0.9135	0.9750	0.9870	1.0039	0.9714	1.0425
Average	0.9928	1.0375	1.0272	1.0185	0.9935	0.9769	0.9671	1.0165	0.9863	1.0125	0.9942	1.0363
Std. Dev.	0.0543	0.1006	0.0565	0.0632	0.0456	0.0536	0.0763	0.0780	0.0709	0.1010	0.1012	0.1117
Max.	1.0940	1.4642	1.2505	1.1398	1.0754	1.0427	1.1223	1.2056	1.1765	1.3812	1.2490	1.2591
Min.	0.8265	0.9139	0.9329	0.8081	0.8165	0.7822	0.7584	0.8723	0.7830	0.7934	0.7142	0.8220

Table S.3: MPI decomposition along path B: Output orientation, base period viewpoint, specific components, RTS.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	NaN	NaN	NaN	NaN	1.0000	0.7865	1.0000	1.0000	NaN	NaN	NaN	NaN
2	0.9029	0.9999	1.0019	1.0138	0.9696	0.9867	0.9849	0.8620	0.8755	0.9866	0.9867	0.8739
3	0.9982	1.1728	1.0887	1.0364	0.9980	0.9752	0.9491	1.1521	0.9962	1.1437	1.0333	1.1940
4	1.0212	1.0943	1.0283	1.0827	0.9561	0.9188	0.9692	0.9899	0.9764	1.0054	0.9966	1.0718
5	0.9928	0.9268	0.9893	1.0007	0.9663	0.9987	0.9753	0.9626	0.9593	0.9256	0.9649	0.9633
6	0.9611	0.9877	0.9916	1.0189	0.8147	0.9701	0.8902	0.9360	0.7830	0.9581	0.8828	0.9537
7	1.0102	0.9755	1.0029	1.0278	1.0517	1.0150	0.7574	0.9985	1.0625	0.9901	0.7596	1.0262
8	1.0341	0.9951	0.9950	1.0192	1.0151	0.9198	0.8660	0.9963	1.0498	0.9153	0.8616	1.0154
9	0.9573	1.0720	1.0717	1.0357	1.0265	1.0100	1.0049	0.9827	0.9827	1.0827	1.0770	1.0178
10	0.9240	0.9993	0.9588	0.9325	0.9940	0.9511	0.7449	0.9299	0.9184	0.9505	0.7142	0.8671
11	0.9019	0.9627	1.2600	1.0117	0.9725	1.0427	0.9913	0.9986	0.8771	1.0039	1.2490	1.0103
12	0.9933	1.0015	0.9954	1.0035	0.9030	1.0313	1.0103	0.9957	0.8969	1.0328	1.0057	0.9992
13	1.0071	0.9931	1.0036	0.9901	1.0118	0.9974	0.8664	0.9383	1.0190	0.9905	0.8696	0.9290
14	0.9996	1.0071	1.0087	0.8722	1.0000	0.9999	1.0743	1.0551	0.9996	1.0070	1.0837	0.9203
15	1.0583	1.0206	1.0985	1.0522	0.9779	0.9743	0.9455	1.1023	1.0349	0.9944	1.0386	1.1598
16	1.0841	1.0215	1.0607	0.9842	1.0000	1.0000	1.0000	1.0000	1.0841	1.0215	1.0607	0.9842
17	0.9967	1.0039	0.9988	1.0170	0.9966	1.0143	0.9879	1.1887	0.9933	1.0182	0.9868	1.2089
18	0.9946	1.0067	0.9777	1.0041	0.9878	0.9775	1.1193	1.2540	0.9825	0.9841	1.0943	1.2591
19	1.0015	1.0198	1.0553	1.0512	0.9987	0.9798	1.0007	1.1831	1.0002	0.9991	1.0560	1.2436
20	0.9476	1.2470	1.0087	1.0465	1.0000	0.9595	1.0410	1.1394	0.9476	1.1965	1.0500	1.1924
21	0.8968	1.0196	1.0062	0.8233	1.0240	0.8675	0.9239	0.9984	0.9183	0.8845	0.9296	0.8220
22	1.0022	0.9995	1.0097	1.0514	1.0060	0.9450	0.9532	1.0000	1.0082	0.9445	0.9624	1.0514
23	1.0158	0.9986	1.0065	0.9973	1.0000	1.0000	1.0000	1.0000	1.0158	0.9986	1.0065	0.9973
24	1.0595	1.0588	1.0498	1.0658	0.9602	1.0263	1.0094	1.0044	1.0173	1.0867	1.0597	1.0705
25	1.0414	1.1007	1.0942	1.1398	1.0000	1.0096	0.9820	0.9868	1.0414	1.1113	1.0744	1.1247
26	0.9856	1.0677	0.9735	0.9699	1.0000	0.9207	1.0000	1.0000	0.9856	0.9830	0.9735	0.9699
27	0.9959	0.9680	1.0643	1.0909	1.0000	1.0063	0.9932	1.0000	0.9959	0.9742	1.0571	1.0909
28	1.0124	1.4283	1.0113	1.0761	1.0007	0.9670	0.9958	0.9844	1.0131	1.3812	1.0071	1.0593
29	0.9960	1.0143	1.0006	0.9068	0.9958	0.7822	0.9850	0.9811	0.9918	0.7934	0.9856	0.8897
30	1.0124	0.9998	1.0081	1.0155	1.1620	1.0091	1.0182	1.0637	1.1765	1.0089	1.0264	1.0802
31	0.9870	0.9917	1.0428	1.0702	1.0000	1.0123	0.9315	0.9742	0.9870	1.0039	0.9714	1.0425
Average	0.9931	1.0385	1.0287	1.0136	0.9932	0.9695	0.9668	1.0212	0.9863	1.0125	0.9942	1.0363
Std. Dev.	0.0442	0.0954	0.0560	0.0636	0.0508	0.0614	0.0763	0.0833	0.0709	0.1010	0.1012	0.1117
Max	1.0841	1.4283	1.2600	1.1398	1.1620	1.0427	1.1193	1.2540	1.1765	1.3812	1.2490	1.2591
Min	0.8968	0.9268	0.9588	0.8233	0.8147	0.7822	0.7449	0.8620	0.7830	0.7934	0.7142	0.8220

Table S.4: MPI decomposition along paths C and D: Output orientation, comparison period viewpoint, common components.

Bank	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.0386	1.0920	1.0520	0.9083	1.0000	1.0000	1.0000	1.0000	NaN	NaN	1.0236	NaN
2	1.0167	1.0328	1.0157	1.4752	1.0000	1.0000	1.0000	1.0000	1.0301	0.9845	0.9982	1.0157
3	1.0814	1.1194	1.0906	1.4112	1.1352	0.9880	1.0770	1.1975	0.9556	1.0103	0.9647	1.0329
4	1.0437	1.0278	0.9902	1.1724	1.0540	1.0185	0.9875	1.0764	1.0138	1.0065	1.0019	0.9916
5	1.0159	1.0900	1.0094	1.0767	1.0000	1.0000	1.0000	1.0000	1.0065	1.0019	0.9831	0.9981
6	1.0530	0.9981	1.0608	1.0687	1.0000	1.0000	1.0000	1.0000	0.9726	0.9418	0.9553	0.9866
7	1.0678	1.0602	1.1619	1.0723	1.0000	1.0000	1.0000	1.0000	0.9249	1.0617	1.0295	0.9682
8	1.0725	1.1775	1.1027	1.0457	1.0460	1.1503	0.9475	1.0281	0.9751	0.9963	1.0414	1.0158
9	0.9486	1.0675	1.0959	1.0174	1.0124	1.0107	1.0106	0.9936	0.9799	0.9877	0.9795	1.0009
10	1.0222	0.9945	1.1748	0.9921	1.0000	1.0000	1.0000	1.0000	0.9520	0.8215	0.9367	0.9211
11	0.8588	1.0346	1.0571	1.0832	1.0268	1.0116	0.8736	1.0695	0.9554	1.0097	0.9626	1.0020
12	1.1381	1.1019	1.0695	1.0987	1.1834	1.0423	0.9686	0.9304	1.0752	1.0100	1.0535	1.1274
13	1.0922	0.8362	0.7458	0.9912	1.0741	0.8686	0.9064	1.0658	0.9788	0.9743	0.9594	1.0010
14	0.9788	0.9928	1.1846	1.5221	0.9914	0.9512	1.0001	1.0603	1.0095	1.0351	1.0581	1.0794
15	1.0882	0.9198	0.9190	1.5712	1.0980	0.9453	0.9184	1.2335	0.9577	0.9664	0.9585	1.0002
16	1.2953	0.8790	0.8957	0.9105	1.1801	0.8233	0.8242	0.9331	0.9961	1.0430	1.0183	0.9908
17	0.9854	1.0140	0.9746	1.3024	1.0468	0.9616	0.9680	1.0973	0.9602	1.0128	0.9766	1.0032
18	0.9147	1.0235	1.0056	1.4138	0.8758	1.0282	0.8908	1.1731	1.0790	1.0284	1.0464	1.0195
19	0.9215	0.8106	1.0277	1.1529	0.9424	0.8243	1.0517	0.9994	0.9946	1.0316	1.0023	0.9974
20	0.8322	1.1898	1.1300	1.4573	0.8799	0.9795	1.0111	1.2084	0.9879	1.0286	0.9797	1.0173
21	0.9274	1.1760	1.1413	0.8521	1.0000	1.0000	1.0000	1.0000	0.9366	1.0918	1.0169	0.9716
22	0.9723	0.8266	0.9483	1.0255	0.9609	0.9281	0.9992	1.0343	0.9934	1.0045	1.0008	1.0004
23	0.9634	0.9497	0.9774	0.9329	0.9368	0.9144	0.9449	0.9316	1.0082	1.0403	1.0121	0.9958
24	1.0510	1.0864	1.0520	1.0502	1.0858	0.9725	1.0221	0.9812	0.9568	1.0110	0.9736	1.0005
25	1.0957	1.0980	1.0800	1.2396	1.0455	0.9544	1.0334	1.0704	0.9894	1.0286	0.9955	1.0203
26	0.8558	0.5820	0.9828	0.8721	1.0000	1.0000	1.0000	1.0000	0.5714	0.5038	0.9119	0.7669
27	1.1102	0.8503	1.0870	1.1091	1.0629	0.8363	0.9881	1.0261	1.0490	1.0567	1.0277	0.9894
28	1.0833	1.2222	0.9849	1.3052	0.9916	0.9091	0.9486	1.1842	1.0369	1.0424	1.0067	0.9985
29	1.1220	1.3127	0.7582	1.1939	1.0875	1.2414	0.9985	1.0015	0.9207	1.0382	0.6509	1.2254
30	1.1359	1.0764	1.0443	1.1850	0.8796	1.2165	0.8722	1.0317	1.0374	0.9236	1.1550	0.9959
31	1.0543	0.9842	0.9783	1.1682	1.0884	0.9422	0.9781	1.0414	0.9801	1.0219	0.9879	1.0071
Average	1.0270	1.0202	1.0257	1.1509	1.0221	0.9845	0.9749	1.0442	0.9762	0.9905	0.9893	1.0047
Std. Dev.	0.0957	0.1418	0.1008	0.1945	0.0750	0.0923	0.0547	0.0790	0.0847	0.1024	0.0757	0.0679
Max.	1.2953	1.3127	1.1846	1.5712	1.1834	1.2414	1.0770	1.2335	1.0790	1.0918	1.1550	1.2254
Min.	0.8322	0.5820	0.7458	0.8521	0.8758	0.8233	0.8242	0.9304	0.5714	0.5038	0.6509	0.7669

Table S.5: MPI decomposition along path C: Output orientation, comparison period viewpoint, specific components, RTS.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	NaN	NaN	1.0000	NaN	3.8433	1.0000	1.0277	1.0157	NaN	NaN	1.0277	NaN
2	0.9233	1.0258	1.0181	1.0138	1.0690	1.0226	0.9994	1.4325	0.9870	1.0490	1.0175	1.4523
3	0.9984	1.1723	1.1113	1.0689	0.9984	0.9567	0.9446	1.0674	0.9968	1.1215	1.0497	1.1409
4	1.0173	1.1115	1.0274	1.1066	0.9601	0.9019	0.9742	0.9926	0.9767	1.0025	1.0008	1.0984
5	1.0058	1.0710	1.0114	1.0839	1.0036	1.0158	1.0151	0.9952	1.0093	1.0880	1.0267	1.0788
6	0.9948	0.9880	1.0176	1.0243	1.0883	1.0727	1.0913	1.0575	1.0826	1.0598	1.1105	1.0832
7	1.0150	0.9859	1.1049	1.0137	1.1376	1.0129	1.0214	1.0925	1.1546	0.9986	1.1286	1.1075
8	1.0359	1.0297	1.1008	1.0264	1.0151	0.9978	1.0152	0.9755	1.0515	1.0274	1.1175	1.0012
9	0.9577	1.0677	1.0739	1.0344	0.9985	1.0015	1.0309	0.9890	0.9563	1.0693	1.1071	1.0230
10	1.0459	1.0391	1.0494	0.9860	1.0266	1.1651	1.1953	1.0923	1.0737	1.2106	1.2543	1.0770
11	0.9001	0.9624	1.2680	1.0121	0.9725	1.0525	0.9913	0.9987	0.8753	1.0129	1.2570	1.0109
12	0.9905	1.0216	1.0164	1.0468	0.9030	1.0246	1.0313	1.0005	0.8944	1.0467	1.0482	1.0473
13	1.0091	0.9926	1.0039	0.9911	1.0295	0.9955	0.8543	0.9374	1.0388	0.9881	0.8576	0.9291
14	0.9998	1.0084	1.0105	0.9702	0.9783	0.9999	1.1078	1.3709	0.9780	1.0083	1.1194	1.3300
15	1.0560	1.0121	1.0987	1.1038	0.9800	0.9948	0.9503	1.1538	1.0349	1.0069	1.0440	1.2735
16	1.1081	1.0236	1.0673	0.9848	0.9944	1.0000	1.0000	1.0000	1.1019	1.0236	1.0673	0.9848
17	0.9956	1.0044	1.0466	0.9929	0.9847	1.0366	0.9852	1.1917	0.9804	1.0411	1.0311	1.1832
18	1.0071	1.0095	0.9511	1.0733	0.9611	0.9587	1.1343	1.1014	0.9679	0.9678	1.0788	1.1821
19	1.0020	1.0116	0.9741	0.9715	0.9813	0.9424	1.0009	1.1904	0.9832	0.9533	0.9750	1.1565
20	0.9425	1.2451	1.0895	1.1073	1.0157	0.9485	1.0471	1.0707	0.9574	1.1810	1.1407	1.1855
21	0.9504	1.0697	1.1203	0.8188	1.0418	1.0070	1.0018	1.0710	0.9901	1.0772	1.1223	0.8770
22	1.0029	0.9995	1.0109	0.9912	1.0158	0.8872	0.9380	1.0000	1.0187	0.8867	0.9482	0.9912
23	1.0200	0.9984	1.0220	1.0057	1.0000	1.0000	1.0000	1.0000	1.0200	0.9984	1.0220	1.0057
24	1.0583	1.0605	1.0504	1.0659	0.9559	1.0419	1.0065	1.0036	1.0116	1.1050	1.0572	1.0698
25	1.0425	1.0989	1.0709	1.1428	1.0161	1.0178	0.9802	0.9933	1.0593	1.1185	1.0498	1.1351
26	1.4768	1.1552	1.0220	1.1307	1.0141	1.0000	1.0545	1.0057	1.4977	1.1552	1.0777	1.1371
27	0.9957	0.9556	1.0824	1.0925	1.0000	1.0068	0.9889	1.0000	0.9957	0.9621	1.0704	1.0925
28	1.0157	1.4105	1.0363	1.1125	1.0373	0.9144	0.9954	0.9922	1.0536	1.2897	1.0315	1.1038
29	1.0628	1.0009	0.9937	0.9779	1.0544	1.0177	1.1740	0.9948	1.1206	1.0185	1.1666	0.9728
30	1.1372	0.9981	1.0487	1.0922	1.0946	0.9598	0.9885	1.0560	1.2448	0.9580	1.0366	1.1534
31	0.9830	0.9907	1.1168	1.1281	1.0055	1.0318	0.9066	0.9873	0.9884	1.0222	1.0125	1.1138
Average	1.0250	1.0507	1.0521	1.0390	1.1025	0.9995	1.0146	1.0590	1.0367	1.0483	1.0663	1.0999
Std. Dev.	0.0966	0.0920	0.0584	0.0668	0.5024	0.0516	0.0695	0.1088	0.1117	0.0827	0.0781	0.1150
Max	1.4768	1.4105	1.2680	1.1428	3.8433	1.1651	1.1953	1.4325	1.4977	1.2897	1.2570	1.4523
Min	0.9001	0.9556	0.9511	0.8188	0.9030	0.8872	0.8543	0.9374	0.8753	0.8867	0.8576	0.8770

Table S.6: MPI decomposition along path D: Output orientation, comparison period viewpoint, specific components, RTS.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	NaN	NaN	1.0000	NaN	NaN	NaN	1.0277	NaN	NaN	NaN	1.0277	NaN
2	0.9664	1.0666	1.0395	1.0137	1.0213	0.9835	0.9788	1.4327	0.9870	1.0490	1.0175	1.4523
3	0.9984	1.1716	1.1113	1.0186	0.9984	0.9572	0.9446	1.1201	0.9968	1.1215	1.0497	1.1409
4	1.0173	1.1115	1.0273	1.1044	0.9601	0.9019	0.9743	0.9946	0.9767	1.0025	1.0008	1.0984
5	1.0362	1.0710	1.0115	1.0970	0.9741	1.0158	1.0150	0.9834	1.0093	1.0880	1.0267	1.0788
6	0.9959	1.0218	1.0267	1.0477	1.0871	1.0373	1.0816	1.0339	1.0826	1.0598	1.1105	1.0832
7	1.0150	0.9916	1.0995	1.0412	1.1375	1.0071	1.0265	1.0637	1.1546	0.9986	1.1286	1.1075
8	1.0359	1.0197	1.0933	1.0264	1.0151	1.0075	1.0221	0.9755	1.0515	1.0274	1.1175	1.0012
9	0.9573	1.0677	1.0739	1.0344	0.9989	1.0015	1.0309	0.9890	0.9563	1.0693	1.1071	1.0230
10	1.0796	1.0661	1.1036	1.0503	0.9946	1.1356	1.1366	1.0254	1.0737	1.2106	1.2543	1.0770
11	0.9001	0.9691	1.2688	1.0121	0.9725	1.0452	0.9907	0.9987	0.8753	1.0129	1.2570	1.0109
12	0.9905	1.0314	1.0164	1.0468	0.9030	1.0148	1.0313	1.0005	0.8944	1.0467	1.0482	1.0473
13	1.0091	0.9926	1.0041	0.9906	1.0295	0.9955	0.8541	0.9379	1.0388	0.9881	0.8576	0.9291
14	0.9998	1.0084	1.0105	1.0286	0.9783	0.9999	1.1078	1.2930	0.9780	1.0083	1.1194	1.3300
15	1.0560	1.0230	1.0996	1.0525	0.9800	0.9843	0.9495	1.2100	1.0349	1.0069	1.0440	1.2735
16	1.1081	1.0236	1.0673	0.9848	0.9944	1.0000	1.0000	1.0000	1.1019	1.0236	1.0673	0.9848
17	0.9956	1.0044	1.0453	0.9929	0.9847	1.0366	0.9864	1.1917	0.9804	1.0411	1.0311	1.1832
18	1.0071	1.0095	0.9511	1.0873	0.9611	0.9587	1.1343	1.0872	0.9679	0.9678	1.0788	1.1821
19	1.0020	1.0062	0.9740	0.9729	0.9813	0.9475	1.0010	1.1887	0.9832	0.9533	0.9750	1.1565
20	0.9425	1.2464	1.0895	1.1073	1.0157	0.9475	1.0471	1.0707	0.9574	1.1810	1.1407	1.1855
21	0.9504	1.0713	1.1203	0.8281	1.0418	1.0055	1.0018	1.0591	0.9901	1.0772	1.1223	0.8770
22	1.0028	0.9995	1.0109	0.9912	1.0158	0.8872	0.9380	1.0000	1.0187	0.8867	0.9482	0.9912
23	1.0200	0.9984	1.0220	1.0057	1.0000	1.0000	1.0000	1.0000	1.0200	0.9984	1.0220	1.0057
24	1.0583	1.0605	1.0504	1.0659	0.9559	1.0419	1.0065	1.0036	1.0116	1.1050	1.0572	1.0698
25	1.0425	1.0989	1.0709	1.1428	1.0161	1.0178	0.9802	0.9933	1.0593	1.1185	1.0498	1.1351
26	1.5375	1.1552	1.0342	1.1461	0.9741	1.0000	1.0421	0.9922	1.4977	1.1552	1.0777	1.1371
27	0.9957	0.9556	1.0824	1.0925	1.0000	1.0068	0.9889	1.0000	0.9957	0.9621	1.0704	1.0925
28	1.0157	1.3925	1.0363	1.1125	1.0373	0.9262	0.9954	0.9922	1.0536	1.2897	1.0315	1.1038
29	1.0628	1.0895	0.9937	0.9779	1.0544	0.9349	1.1740	0.9948	1.1206	1.0185	1.1666	0.9728
30	1.1555	0.9981	1.0471	1.0922	1.0773	0.9598	0.9900	1.0560	1.2448	0.9580	1.0366	1.1534
31	0.9830	0.9907	1.1168	1.1390	1.0055	1.0318	0.9066	0.9779	0.9884	1.0222	1.0125	1.1138
Average	1.0312	1.0571	1.0548	1.0434	1.0055	0.9930	1.0117	1.0555	1.0367	1.0483	1.0663	1.0999
Std. Dev.	0.1060	0.0884	0.0583	0.0643	0.0440	0.0478	0.0649	0.1068	0.1117	0.0827	0.0781	0.1150
Max.	1.5375	1.3925	1.2688	1.1461	1.1375	1.1356	1.1740	1.4327	1.4977	1.2897	1.2570	1.4523
Min.	0.9001	0.9556	0.9511	0.8281	0.9030	0.8872	0.8541	0.9379	0.8753	0.8867	0.8576	0.8770

Table S.7: MPI decomposition along paths ABCD: Output orientation, ‘mean’ period viewpoint.

Bank	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1176	1.1073	1.1057	0.9551	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
2	1.0193	1.0344	1.0188	1.5164	1.0000	1.0000	1.0000	1.0000	1.0966	1.0168	1.0168	1.3461
3	1.0812	1.1302	1.0932	1.5096	1.1352	0.9880	1.0770	1.1975	0.9557	1.0101	0.9746	1.0800
4	1.0437	1.0443	0.9918	1.1697	1.0540	1.0185	0.9875	1.0764	1.0140	1.0213	1.0056	1.0016
5	1.0159	1.0884	1.0100	1.0741	1.0000	1.0000	1.0000	1.0000	1.0324	1.0846	1.0148	1.0537
6	1.0379	1.0062	1.0583	1.0726	1.0000	1.0000	1.0000	1.0000	1.1273	0.9985	1.0688	1.0553
7	1.0679	1.0600	1.1559	1.0691	1.0000	1.0000	1.0000	1.0000	0.9641	1.0661	1.2484	1.0028
8	1.0725	1.1543	1.1003	1.0418	1.0460	1.1503	0.9475	1.0281	0.9759	1.0348	1.1834	1.0050
9	0.9487	1.0631	1.0948	1.0156	1.0124	1.0107	1.0106	0.9936	0.9666	0.9776	0.9921	1.0017
10	1.0222	1.0104	1.1711	0.9880	1.0000	1.0000	1.0000	1.0000	1.0294	0.9419	1.2374	1.0223
11	0.8588	1.0254	1.0549	1.0829	1.0268	1.0116	0.8736	1.0695	0.9545	1.0053	0.9637	1.0020
12	1.1381	1.1062	1.0621	1.0988	1.1834	1.0423	0.9686	0.9304	1.0737	1.0207	1.0681	1.1544
13	1.0794	0.8378	0.7643	0.9929	1.0741	0.8686	0.9064	1.0658	0.9766	0.9750	0.9764	1.0028
14	0.9896	0.9928	1.1796	1.7774	0.9914	0.9512	1.0001	1.0603	1.0096	1.0358	1.0709	1.5152
15	1.0882	0.9251	0.9205	1.6079	1.0980	0.9453	0.9184	1.2335	0.9577	0.9781	0.9626	1.0726
16	1.2989	0.8790	0.8957	0.9105	1.1801	0.8233	0.8242	0.9331	1.0071	1.0441	1.0214	0.9911
17	0.9913	1.0030	0.9542	1.3224	1.0468	0.9616	0.9680	1.0973	0.9597	1.0131	0.9773	1.0077
18	0.9274	1.0335	1.0003	1.5125	0.8758	1.0282	0.8908	1.1731	1.0858	1.0299	1.0335	1.0568
19	0.9297	0.8301	1.0731	1.1922	0.9424	0.8243	1.0517	0.9994	0.9948	1.0319	1.0056	0.9947
20	0.8291	1.1989	1.1267	1.5034	0.8799	0.9795	1.0111	1.2084	0.9893	1.0297	1.0182	1.0464
21	0.9274	1.1681	1.1354	0.8493	1.0000	1.0000	1.0000	1.0000	0.9726	1.1967	1.1115	1.0003
22	0.9663	0.8644	0.9559	1.0565	0.9609	0.9281	0.9992	1.0343	0.9923	1.0177	1.0014	1.0006
23	0.9634	0.9497	0.9703	0.9289	0.9368	0.9144	0.9449	0.9316	1.0103	1.0402	1.0124	0.9956
24	1.0534	1.0770	1.0512	1.0507	1.0858	0.9725	1.0221	0.9812	0.9563	1.0106	0.9717	1.0006
25	1.0870	1.0935	1.0809	1.2356	1.0455	0.9544	1.0334	1.0704	0.9900	1.0277	0.9849	1.0216
26	1.0338	0.6939	1.0183	1.0220	1.0000	1.0000	1.0000	1.0000	0.8509	0.6512	0.9941	0.9732
27	1.1102	0.8541	1.0803	1.1091	1.0629	0.8363	0.9881	1.0261	1.0488	1.0548	1.0279	0.9901
28	1.0640	1.2711	0.9851	1.3001	0.9916	0.9091	0.9486	1.1842	1.0386	1.0476	1.0190	1.0153
29	1.1220	1.3029	0.7698	1.1953	1.0875	1.2414	0.9985	1.0015	0.9786	1.1675	0.7190	1.2829
30	1.1398	1.0688	1.0449	1.2029	0.8796	1.2165	0.8722	1.0317	1.0708	0.8936	1.1614	1.0446
31	1.0514	0.9749	0.9577	1.1307	1.0884	0.9422	0.9781	1.0414	0.9781	1.0214	0.9873	1.0075
Average	1.0347	1.0274	1.0284	1.1772	1.0221	0.9845	0.9749	1.0442	1.0019	1.0148	1.0277	1.0581
Std. Dev.	0.0906	0.1302	0.0985	0.2232	0.0750	0.0923	0.0547	0.0790	0.0537	0.0870	0.0949	0.1176
Max.	1.2989	1.3029	1.1796	1.7774	1.1834	1.2414	1.0770	1.2335	1.1273	1.1967	1.2484	1.5152
Min.	0.8291	0.6939	0.7643	0.8493	0.8758	0.8233	0.8242	0.9304	0.8509	0.6512	0.7190	0.9732

Table S.8: MPI decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	0.9033	1.0227	1.0152	1.0108	1.0291	0.9947	0.9870	1.1145	0.9296	1.0173	1.0020	1.1266
3	0.9983	1.1724	1.1000	1.0482	0.9982	0.9660	0.9468	1.1135	0.9965	1.1325	1.0415	1.1672
4	1.0193	1.1031	1.0278	1.0941	0.9581	0.9101	0.9717	0.9917	0.9766	1.0040	0.9987	1.0850
5	1.0066	0.9928	0.9964	1.0446	0.9775	1.0108	0.9990	0.9759	0.9840	1.0035	0.9953	1.0194
6	0.9776	0.9961	1.0061	1.0249	0.9419	1.0117	0.9841	0.9917	0.9207	1.0077	0.9901	1.0164
7	1.0108	0.9820	1.0510	1.0202	1.0957	1.0126	0.8809	1.0450	1.1076	0.9943	0.9259	1.0661
8	1.0350	1.0184	1.0473	1.0223	1.0151	0.9522	0.9369	0.9863	1.0507	0.9697	0.9812	1.0083
9	0.9575	1.0698	1.0728	1.0350	1.0124	1.0057	1.0178	0.9858	0.9694	1.0760	1.0919	1.0204
10	0.9907	1.0234	1.0088	0.9738	1.0024	1.0482	0.9382	0.9924	0.9930	1.0727	0.9464	0.9664
11	0.9010	0.9643	1.2618	1.0119	0.9725	1.0458	0.9930	0.9987	0.8762	1.0084	1.2530	1.0106
12	0.9919	1.0139	1.0051	1.0249	0.9030	1.0255	1.0215	0.9981	0.8957	1.0397	1.0267	1.0230
13	1.0081	0.9929	1.0038	0.9905	1.0206	0.9964	0.8603	0.9379	1.0289	0.9893	0.8636	0.9290
14	0.9997	1.0077	1.0100	0.9558	0.9891	0.9999	1.0905	1.1575	0.9888	1.0076	1.1014	1.1064
15	1.0571	1.0191	1.0957	1.0776	0.9789	0.9819	0.9504	1.1278	1.0349	1.0006	1.0413	1.2154
16	1.0960	1.0226	1.0640	0.9845	0.9972	1.0000	1.0000	1.0000	1.0930	1.0226	1.0640	0.9845
17	0.9962	1.0041	1.0221	1.0049	0.9906	1.0254	0.9869	1.1901	0.9868	1.0296	1.0087	1.1960
18	1.0008	1.0081	0.9637	1.0518	0.9744	0.9681	1.1275	1.1599	0.9752	0.9759	1.0865	1.2200
19	1.0018	1.0199	1.0138	1.0063	0.9899	0.9569	1.0008	1.1917	0.9917	0.9759	1.0147	1.1993
20	0.9450	1.2444	1.0483	1.0764	1.0078	0.9553	1.0440	1.1045	0.9525	1.1887	1.0944	1.1889
21	0.9226	1.0448	1.0614	0.8195	1.0335	0.9343	0.9623	1.0360	0.9535	0.9761	1.0214	0.8491
22	1.0025	0.9995	1.0103	1.0208	1.0109	0.9156	0.9455	1.0000	1.0134	0.9152	0.9553	1.0208
23	1.0179	0.9985	1.0143	1.0015	1.0000	1.0000	1.0000	1.0000	1.0179	0.9985	1.0143	1.0015
24	1.0589	1.0597	1.0501	1.0658	0.9580	1.0341	1.0080	1.0040	1.0145	1.0958	1.0584	1.0701
25	1.0419	1.0998	1.0825	1.1413	1.0080	1.0137	0.9811	0.9900	1.0503	1.1149	1.0620	1.1299
26	1.2187	1.0872	1.0004	1.0507	0.9970	0.9801	1.0239	0.9995	1.2150	1.0656	1.0243	1.0502
27	0.9958	0.9606	1.0738	1.0917	1.0000	1.0078	0.9906	1.0000	0.9958	0.9681	1.0637	1.0917
28	1.0140	1.4236	1.0237	1.0942	1.0188	0.9375	0.9956	0.9883	1.0331	1.3347	1.0192	1.0814
29	1.0289	1.0292	0.9953	0.9417	1.0247	0.8735	1.0773	0.9879	1.0543	0.8990	1.0723	0.9303
30	1.0984	0.9989	1.0278	1.0599	1.1018	0.9842	1.0036	1.0531	1.2102	0.9831	1.0315	1.1162
31	0.9850	0.9912	1.0845	1.1012	1.0027	1.0220	0.9145	0.9786	0.9877	1.0130	0.9918	1.0776
Average	1.0094	1.0457	1.0413	1.0282	1.0003	0.9857	0.9880	1.0367	1.0099	1.0293	1.0280	1.0656
Std. Dev.	0.0596	0.0917	0.0528	0.0591	0.0376	0.0411	0.0547	0.0703	0.0740	0.0821	0.0665	0.0897
Max.	1.2187	1.4236	1.2618	1.1413	1.1018	1.0482	1.1275	1.1917	1.2150	1.3347	1.2530	1.2200
Min.	0.9010	0.9606	0.9637	0.8195	0.9030	0.8735	0.8603	0.9379	0.8762	0.8990	0.8636	0.8491

Table S.9: MPI decomposition along paths E and F: Input orientation, base period viewpoint, common components.

Bank	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.2027	1.1228	1.1621	1.0042	1.0000	1.0000	1.0000	1.0000	1.4313	1.3067	1.3056	1.0149
2	1.0220	1.0361	1.0220	1.5589	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
3	1.0810	1.1412	1.0958	1.6148	1.1387	0.9873	1.0827	1.2124	0.9576	1.0129	0.9879	1.1671
4	1.0437	1.0611	0.9934	1.1671	1.0267	1.0571	0.9553	1.1672	1.0913	1.0687	1.0332	1.0027
5	1.0159	1.0867	1.0107	1.0715	1.0000	1.0000	1.0000	1.0000	1.0966	1.2845	1.1064	1.2940
6	1.0231	1.0144	1.0558	1.0766	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
7	1.0679	1.0599	1.1499	1.0658	1.0000	1.0000	1.0000	1.0000	1.0451	1.3088	NaN	NaN
8	1.0725	1.1316	1.0979	1.0380	1.0730	1.1437	0.9494	1.0349	0.9511	1.2472	NaN	0.9864
9	0.9487	1.0587	1.0937	1.0137	1.0452	1.0064	1.0157	0.9897	0.9527	0.9709	1.0023	1.0062
10	1.0222	1.0265	1.1674	0.9839	1.0000	1.0000	1.0000	1.0000	1.7585	1.1623	NaN	1.1948
11	0.8588	1.0164	1.0526	1.0826	1.0261	1.0107	0.8668	1.0766	0.9552	1.0034	0.9626	1.0009
12	1.1380	1.1105	1.0547	1.0989	1.4838	1.1371	0.9538	0.9296	1.2122	1.0909	1.1145	1.2349
13	1.0666	0.8395	0.7832	0.9946	1.0638	0.8253	0.8335	1.1185	0.9898	0.9788	1.0174	0.9939
14	1.0006	0.9928	1.1747	2.0755	0.9899	0.9400	0.9994	1.0753	1.0115	1.0460	1.1036	3.3612
15	1.0882	0.9305	0.9220	1.6454	1.1043	0.9403	0.9098	1.2038	0.9585	0.9907	0.9673	1.2252
16	1.3026	0.8790	0.8957	0.9105	1.1294	0.6381	0.5833	1.0457	1.0817	1.1678	1.1026	0.9189
17	0.9973	0.9921	0.9341	1.3427	1.0465	0.9539	0.9599	1.1158	0.9606	1.0178	0.9801	1.0108
18	0.9401	1.0436	0.9950	1.6180	0.8557	1.0253	0.8517	1.2369	1.1102	1.0439	1.0617	1.1215
19	0.9379	0.8501	1.1205	1.2329	0.8984	0.7245	0.9722	1.0785	1.0246	1.0852	1.0713	0.9728
20	0.8260	1.2081	1.1234	1.5509	0.8257	0.9943	1.0095	1.2760	1.0199	1.0552	1.0752	1.0826
21	0.9274	1.1603	1.1295	0.8466	1.0000	1.0000	1.0000	1.0000	1.0133	NaN	NaN	1.0340
22	0.9603	0.9038	0.9636	1.0884	0.9265	0.8814	0.9634	1.0742	1.0139	1.0648	1.0327	0.9904
23	0.9634	0.9497	0.9633	0.9249	0.8671	0.8356	0.8394	0.8762	1.0639	1.1046	1.0906	0.9528
24	1.0557	1.0677	1.0505	1.0511	1.0902	0.9708	1.0235	0.9801	0.9538	1.0112	0.9681	1.0007
25	1.0783	1.0891	1.0819	1.2315	1.0288	0.9344	1.0442	1.1069	1.0148	1.0499	0.9751	1.0430
26	1.2488	0.8273	1.0550	1.1976	1.0000	1.0000	1.0000	1.0000	3.3548	0.8335	1.0648	1.5604
27	1.1102	0.8579	1.0737	1.1091	0.9963	0.6174	0.9289	1.2474	1.2237	1.2683	1.1230	0.9537
28	1.0451	1.3220	0.9853	1.2950	0.8299	0.9853	0.9950	1.3134	1.2577	1.1771	1.0088	0.9413
29	1.1220	1.2931	0.7816	1.1966	1.0717	1.0665	0.9995	1.0005	0.9325	1.1017	0.8844	1.1081
30	1.1437	1.0612	1.0455	1.2210	0.9254	1.2266	0.8615	1.0368	1.1081	0.8595	1.1917	1.0982
31	1.0486	0.9656	0.9376	1.0944	1.0964	0.9260	0.9652	1.0643	0.9848	1.0336	0.9951	1.0059
Average	1.0438	1.0355	1.0313	1.2065	1.0174	0.9622	0.9537	1.0729	1.1562	1.0838	1.0490	1.1528
Std. Dev.	0.1010	0.1210	0.0989	0.2661	0.1177	0.1270	0.0905	0.1037	0.4486	0.1193	0.0845	0.4454
Max.	1.3026	1.3220	1.1747	2.0755	1.4838	1.2266	1.0827	1.3134	3.3548	1.3088	1.3056	3.3612
Min.	0.8260	0.8273	0.7816	0.8466	0.8257	0.6174	0.5833	0.8762	0.9325	0.8335	0.8844	0.9189

Table S.10: MPI decomposition along path E: Input orientation, base period viewpoint, specific components, RTS.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	1.0000	1.0000	1.0000	1.0000	0.8403	0.8593	0.8901	0.9895	0.8403	0.8593	0.8901	0.9895
2	0.9104	0.9857	0.9893	0.9896	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	0.9977	1.1820	1.0852	1.0725	0.9936	0.9655	0.9441	1.0641	0.9914	1.1412	1.0245	1.1412
4	1.0380	1.1240	1.0322	1.0809	0.8974	0.8356	0.9751	0.9225	0.9315	0.9393	1.0065	0.9972
5	0.9916	0.8692	0.9697	1.0000	0.9342	0.9735	0.9420	0.8281	0.9264	0.8461	0.9135	0.8281
6	0.9577	0.9922	0.9869	1.0000	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
7	1.0000	0.9595	0.9970	1.0000	1.0218	0.8440	NaN	NaN	1.0218	0.8098	NaN	NaN
8	1.0366	1.0405	0.9963	1.0299	1.0139	0.7624	NaN	0.9873	1.0510	0.7933	NaN	1.0169
9	0.9573	1.0770	1.0797	1.0373	0.9953	1.0060	0.9950	0.9814	0.9527	1.0835	1.0743	1.0180
10	0.9220	1.0000	0.9307	0.9296	0.6304	0.8832	NaN	0.8858	0.5813	0.8832	NaN	0.8235
11	0.9074	0.9652	1.2733	1.0131	0.9657	1.0384	0.9908	0.9917	0.8762	1.0023	1.2616	1.0047
12	1.0000	1.0179	0.9847	1.0138	0.6327	0.8794	1.0077	0.9442	0.6327	0.8951	0.9922	0.9572
13	1.0000	1.0000	1.0000	1.0000	1.0130	1.0393	0.9236	0.8947	1.0130	1.0393	0.9236	0.8947
14	1.0000	1.0000	1.0000	1.0000	0.9993	1.0097	1.0651	0.5743	0.9993	1.0097	1.0651	0.5743
15	1.0658	1.0287	1.1093	1.1142	0.9647	0.9710	0.9444	1.0013	1.0281	0.9989	1.0476	1.1157
16	1.0000	1.0000	1.0000	1.0000	1.0662	1.1795	1.3926	0.9475	1.0662	1.1795	1.3926	0.9475
17	1.0000	1.0000	1.0000	1.0254	0.9920	1.0219	0.9930	1.1610	0.9920	1.0219	0.9930	1.1905
18	0.9862	1.0000	1.0000	1.0166	1.0035	0.9750	1.1003	1.1473	0.9896	0.9750	1.1003	1.1663
19	1.0000	1.0365	1.0000	1.0640	1.0189	1.0431	1.0758	1.1045	1.0189	1.0812	1.0758	1.1752
20	0.9357	1.1892	1.0000	1.0000	1.0482	0.9682	1.0350	1.1227	0.9808	1.1514	1.0350	1.1227
21	0.8857	0.9778	0.9932	0.8121	1.0333	NaN	NaN	1.0082	0.9152	NaN	NaN	0.8188
22	1.0000	1.0000	1.0000	1.0612	1.0223	0.9631	0.9686	0.9640	1.0223	0.9631	0.9686	1.0231
23	1.0000	1.0000	1.0000	0.9914	1.0443	1.0289	1.0522	1.1175	1.0443	1.0289	1.0522	1.1078
24	1.0630	1.0637	1.0522	1.0683	0.9550	1.0226	1.0076	1.0032	1.0152	1.0877	1.0602	1.0717
25	1.0414	1.1171	1.1107	1.1098	0.9918	0.9938	0.9567	0.9611	1.0329	1.1102	1.0626	1.0666
26	1.0000	1.0000	1.0000	1.0000	0.3723	0.9925	0.9908	0.7675	0.3723	0.9925	0.9908	0.7675
27	0.9990	1.0045	1.0538	1.0000	0.9115	1.0906	0.9767	0.9322	0.9106	1.0956	1.0292	0.9322
28	0.9999	1.1365	1.0000	1.0000	1.0014	1.0030	0.9817	1.0475	1.0012	1.1399	0.9817	1.0475
29	1.0000	1.0000	1.0000	1.0000	1.1227	1.1006	0.8842	1.0793	1.1227	1.1006	0.8842	1.0793
30	1.0000	1.0000	1.0000	1.0392	1.1153	1.0065	1.0184	1.0319	1.1153	1.0065	1.0184	1.0723
31	1.0000	1.0000	1.0716	1.0859	0.9712	1.0089	0.9110	0.9414	0.9712	1.0089	0.9762	1.0222
Average	0.9902	1.0247	1.0231	1.0179	0.9508	0.9809	1.0009	0.9786	0.9454	1.0087	1.0328	0.9990
Std. Dev.	0.0416	0.0654	0.0607	0.0546	0.1541	0.0864	0.0963	0.1196	0.1585	0.1037	0.1054	0.1374
Max.	1.0658	1.1892	1.2733	1.1142	1.1227	1.1795	1.3926	1.1610	1.1227	1.1795	1.3926	1.1905
Min.	0.8857	0.8692	0.9307	0.8121	0.3723	0.7624	0.8842	0.5743	0.3723	0.7933	0.8842	0.5743

Table S.11: MPI decomposition along path F: Input orientation, base period viewpoint, specific components, RTS.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	1.0010	1.0201	1.0000	1.0000	0.8395	0.8424	0.8901	0.9895	0.8403	0.8593	0.8901	0.9895
2	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	0.9977	1.1820	1.0874	1.0397	0.9936	0.9655	0.9422	1.0976	0.9914	1.1412	1.0245	1.1412
4	1.0380	1.1557	1.0322	1.1119	0.8974	0.8127	0.9751	0.8968	0.9315	0.9393	1.0065	0.9972
5	1.0000	0.8864	0.9955	1.0096	0.9264	0.9546	0.9176	0.8202	0.9264	0.8461	0.9135	0.8281
6	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
7	1.0168	0.9666	NaN	NaN	1.0049	0.8378	NaN	NaN	1.0218	0.8098	NaN	NaN
8	1.0366	1.0240	NaN	1.0299	1.0139	0.7747	NaN	0.9873	1.0510	0.7933	NaN	1.0169
9	0.9573	1.0770	1.0797	1.0373	0.9953	1.0060	0.9950	0.9814	0.9527	1.0835	1.0743	1.0180
10	0.9220	1.0736	NaN	0.9296	0.6304	0.8227	NaN	0.8858	0.5813	0.8832	NaN	0.8235
11	0.9074	0.9652	1.2737	1.0131	0.9657	1.0384	0.9905	0.9917	0.8762	1.0023	1.2616	1.0047
12	1.0325	1.0335	0.9914	1.0138	0.6128	0.8661	1.0009	0.9442	0.6327	0.8951	0.9922	0.9572
13	1.0000	1.0000	1.0000	1.0000	1.0130	1.0393	0.9236	0.8947	1.0130	1.0393	0.9236	0.8947
14	1.0000	1.0000	1.0000	1.0000	0.9993	1.0097	1.0651	0.5743	0.9993	1.0097	1.0651	0.5743
15	1.0658	1.0287	1.1093	1.0814	0.9647	0.9710	0.9444	1.0317	1.0281	0.9989	1.0476	1.1157
16	1.0000	1.0000	1.0000	1.0000	1.0662	1.1795	1.3926	0.9475	1.0662	1.1795	1.3926	0.9475
17	1.0000	1.0000	1.0000	1.0254	0.9920	1.0219	0.9930	1.1610	0.9920	1.0219	0.9930	1.1905
18	0.9862	1.0000	1.0000	1.0000	1.0035	0.9750	1.1003	1.1663	0.9896	0.9750	1.1003	1.1663
19	1.0000	1.0141	1.0000	1.0815	1.0189	1.0662	1.0758	1.0867	1.0189	1.0812	1.0758	1.1752
20	0.9357	1.1948	1.0000	1.0000	1.0482	0.9637	1.0350	1.1227	0.9808	1.1514	1.0350	1.1227
21	0.8857	NaN	NaN	0.8250	1.0333	NaN	NaN	0.9926	0.9152	NaN	NaN	0.8188
22	1.0000	1.0000	1.0000	1.0612	1.0223	0.9631	0.9686	0.9640	1.0223	0.9631	0.9686	1.0231
23	1.0000	1.0000	1.0000	0.9914	1.0443	1.0289	1.0522	1.1175	1.0443	1.0289	1.0522	1.1078
24	1.0630	1.0637	1.0522	1.0683	0.9550	1.0226	1.0076	1.0032	1.0152	1.0877	1.0602	1.0717
25	1.0414	1.1171	1.1107	1.1241	0.9918	0.9938	0.9567	0.9488	1.0329	1.1102	1.0626	1.0666
26	1.3258	1.0030	1.0014	1.1655	0.2808	0.9896	0.9894	0.6585	0.3723	0.9925	0.9908	0.7675
27	0.9990	1.0094	1.0616	1.0000	0.9115	1.0853	0.9695	0.9322	0.9106	1.0956	1.0292	0.9322
28	0.9999	1.1260	1.0000	1.0000	1.0014	1.0123	0.9817	1.0475	1.0012	1.1399	0.9817	1.0475
29	1.0000	1.0000	1.0000	1.0000	1.1227	1.1006	0.8842	1.0793	1.1227	1.1006	0.8842	1.0793
30	1.0078	1.0000	1.0000	1.0167	1.1067	1.0065	1.0184	1.0548	1.1153	1.0065	1.0184	1.0723
31	1.0000	1.0000	1.0508	1.0868	0.9712	1.0089	0.9290	0.9405	0.9712	1.0089	0.9762	1.0222
Average	1.0076	1.0336	1.0338	1.0254	0.9457	0.9771	0.9999	0.9757	0.9454	1.0087	1.0328	0.9990
Std. Dev.	0.0725	0.0674	0.0615	0.0619	0.1672	0.0923	0.0966	0.1308	0.1585	0.1037	0.1054	0.1374
Max.	1.3258	1.1948	1.2737	1.1655	1.1227	1.1795	1.3926	1.1663	1.1227	1.1795	1.3926	1.1905
Min.	0.8857	0.8864	0.9914	0.8250	0.2808	0.7747	0.8842	0.5743	0.3723	0.7933	0.8842	0.5743

Table S.12: MPI decomposition along paths G and H: Input orientation, comparison period viewpoint, common components.

Bank	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.0386	1.0920	1.0520	0.9083	1.0000	1.0000	1.0000	1.0000	0.9057	0.9913	1.0000	0.8844
2	1.0167	1.0328	1.0157	1.4752	1.0000	1.0000	1.0000	1.0000	1.0552	0.9733	0.9968	1.0382
3	1.0814	1.1194	1.0906	1.4112	1.1387	0.9873	1.0827	1.2124	0.9588	1.0122	0.9634	1.0339
4	1.0437	1.0278	0.9902	1.1724	1.0267	1.0571	0.9553	1.1672	1.1210	1.0332	1.0314	0.9772
5	1.0159	1.0900	1.0094	1.0767	1.0000	1.0000	1.0000	1.0000	1.0101	1.0021	0.9824	0.9979
6	1.0530	0.9981	1.0608	1.0687	1.0000	1.0000	1.0000	1.0000	0.9670	NaN	0.8479	0.9458
7	1.0678	1.0602	1.1619	1.0723	1.0000	1.0000	1.0000	1.0000	0.7369	1.0881	1.0311	0.9400
8	1.0725	1.1775	1.1027	1.0457	1.0730	1.1437	0.9494	1.0349	0.9514	1.0045	1.0310	1.0140
9	0.9486	1.0675	1.0959	1.0174	1.0452	1.0064	1.0157	0.9897	0.9744	0.9926	0.9742	1.0044
10	1.0222	0.9945	1.1748	0.9921	1.0000	1.0000	1.0000	1.0000	0.9491	0.7519	0.9337	0.8691
11	0.8588	1.0346	1.0571	1.0832	1.0261	1.0107	0.8668	1.0766	0.9572	1.0128	0.9607	1.0006
12	1.1381	1.1019	1.0695	1.0987	1.4838	1.1371	0.9538	0.9296	0.9833	0.9566	1.0800	1.1399
13	1.0922	0.8362	0.7458	0.9912	1.0638	0.8253	0.8335	1.1185	0.9977	0.9737	0.9636	0.9799
14	0.9788	0.9928	1.1846	1.5221	0.9899	0.9400	0.9994	1.0753	1.0111	1.0431	1.0727	1.0891
15	1.0882	0.9198	0.9190	1.5712	1.1043	0.9403	0.9098	1.2038	0.9598	0.9805	0.9570	0.9988
16	1.2953	0.8790	0.8957	0.9105	1.1294	0.6381	0.5833	1.0457	1.0509	1.1117	1.1591	0.9189
17	0.9854	1.0140	0.9746	1.3024	1.0465	0.9539	0.9599	1.1158	0.9620	1.0166	0.9754	0.9993
18	0.9147	1.0235	1.0056	1.4138	0.8557	1.0253	0.8517	1.2369	1.0920	1.0446	1.0720	1.0169
19	0.9215	0.8106	1.0277	1.1529	0.8984	0.7245	0.9722	1.0785	1.0166	1.0647	1.0526	0.9716
20	0.8322	1.1898	1.1300	1.4573	0.8257	0.9943	1.0095	1.2760	1.0009	1.0565	1.0256	1.0546
21	0.9274	1.1760	1.1413	0.8521	1.0000	1.0000	1.0000	1.0000	NaN	1.1043	1.0188	0.9511
22	0.9723	0.8266	0.9483	1.0255	0.9265	0.8814	0.9634	1.0742	1.0127	1.0154	1.0293	0.9877
23	0.9634	0.9497	0.9774	0.9329	0.8671	0.8356	0.8394	0.8762	1.0441	1.0966	1.0759	0.9638
24	1.0510	1.0864	1.0520	1.0502	1.0902	0.9708	1.0235	0.9801	0.9554	1.0118	0.9719	1.0005
25	1.0957	1.0980	1.0800	1.2396	1.0288	0.9344	1.0442	1.1069	1.0173	1.0532	1.0129	1.0164
26	0.8558	0.5820	0.9828	0.8721	1.0000	1.0000	1.0000	1.0000	0.7977	0.2507	0.8347	0.8444
27	1.1102	0.8503	1.0870	1.1091	0.9963	0.6174	0.9289	1.2474	1.3215	1.2284	1.1603	0.9300
28	1.0833	1.2222	0.9849	1.3052	0.8299	0.9853	0.9950	1.3134	1.2742	1.1807	1.0266	0.9492
29	1.1220	1.3127	0.7582	1.1939	1.0717	1.0665	0.9995	1.0005	0.9297	1.0746	0.8420	1.0945
30	1.1359	1.0764	1.0443	1.1850	0.9254	1.2266	0.8615	1.0368	1.1297	0.9625	1.1693	0.9938
31	1.0543	0.9842	0.9783	1.1682	1.0964	0.9260	0.9652	1.0643	0.9948	1.0343	0.9924	1.0038
Average	1.0270	1.0202	1.0257	1.1509	1.0174	0.9622	0.9537	1.0729	1.0046	1.0041	1.0079	0.9874
Std. Dev.	0.0957	0.1418	0.1008	0.1945	0.1177	0.1270	0.0905	0.1037	0.1104	0.1608	0.0797	0.0619
Max.	1.2953	1.3127	1.1846	1.5712	1.4838	1.2266	1.0827	1.3134	1.3215	1.2284	1.1693	1.1399
Min.	0.8322	0.5820	0.7458	0.8521	0.8257	0.6174	0.5833	0.8762	0.7369	0.2507	0.8347	0.8444

Table S.13: MPI decomposition along path G: Input orientation, comparison period viewpoint, specific components, RTS.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	1.0000	1.0000	1.0000	1.0000	1.1467	1.1016	1.0520	1.0270	1.1467	1.1016	1.0520	1.0270
2	1.0000	1.0360	1.0292	1.0000	0.9635	1.0242	0.9900	1.4208	0.9635	1.0611	1.0189	1.4208
3	0.9977	1.1820	1.1155	1.0714	0.9927	0.9476	0.9373	1.0508	0.9904	1.1201	1.0456	1.1258
4	1.0393	1.1166	1.0301	1.1314	0.8725	0.8427	0.9757	0.9085	0.9068	0.9410	1.0050	1.0279
5	1.0085	1.0721	1.0125	1.0897	0.9972	1.0146	1.0147	0.9901	1.0057	1.0877	1.0274	1.0789
6	1.0017	NaN	0.9857	1.0119	1.0870	NaN	1.2693	1.1167	1.0889	NaN	1.2511	1.1300
7	1.0168	0.9818	1.1096	1.0094	1.4251	0.9924	1.0155	1.1301	1.4491	0.9744	1.1268	1.1407
8	1.0366	1.0391	1.1051	1.0296	1.0135	0.9864	1.0193	0.9678	1.0506	1.0249	1.1264	0.9965
9	0.9573	1.0803	1.0782	1.0373	0.9731	0.9892	1.0272	0.9867	0.9315	1.0687	1.1076	1.0235
10	1.0771	1.0684	1.0169	0.9303	0.9999	1.2379	1.2374	1.2270	1.0770	1.3226	1.2582	1.1415
11	0.9074	0.9652	1.2804	1.0134	0.9636	1.0472	0.9915	0.9923	0.8743	1.0108	1.2695	1.0055
12	1.0517	1.0335	1.0079	1.0572	0.7417	0.9802	1.0303	0.9808	0.7800	1.0130	1.0384	1.0368
13	1.0000	1.0000	1.0000	1.0000	1.0291	1.0406	0.9286	0.9044	1.0291	1.0406	0.9286	0.9044
14	1.0000	1.0000	1.0000	1.0000	0.9779	1.0124	1.1049	1.2997	0.9779	1.0124	1.1049	1.2997
15	1.0658	1.0317	1.1119	1.1142	0.9634	0.9670	0.9493	1.1729	1.0267	0.9977	1.0556	1.3069
16	1.0000	1.0000	1.0000	1.0000	1.0914	1.2391	1.3247	0.9475	1.0914	1.2391	1.3247	0.9475
17	1.0000	1.0000	1.0479	1.0000	0.9788	1.0456	0.9934	1.1680	0.9788	1.0456	1.0410	1.1680
18	1.0000	1.0000	0.9900	1.0770	0.9789	0.9556	1.1126	1.0436	0.9789	0.9556	1.1014	1.1240
19	1.0000	1.0054	0.9170	1.0000	1.0090	1.0452	1.0951	1.1002	1.0090	1.0508	1.0042	1.1002
20	0.9262	1.1900	1.0489	1.0226	1.0872	0.9518	1.0406	1.0591	1.0069	1.1326	1.0914	1.0830
21	NaN	1.0754	1.1279	0.8264	NaN	0.9902	0.9931	1.0841	NaN	1.0649	1.1202	0.8959
22	1.0000	1.0000	1.0000	1.0000	1.0364	0.9237	0.9563	0.9667	1.0364	0.9237	0.9563	0.9667
23	1.0000	1.0000	1.0146	1.0000	1.0641	1.0364	1.0666	1.1047	1.0641	1.0364	1.0822	1.1047
24	1.0630	1.0660	1.0529	1.0683	0.9492	1.0376	1.0044	1.0025	1.0090	1.1061	1.0575	1.0710
25	1.0428	1.1171	1.0714	1.1497	1.0040	0.9988	0.9531	0.9583	1.0470	1.1158	1.0211	1.1018
26	1.0000	0.9185	1.0000	0.9901	1.0729	2.5280	1.1774	1.0431	1.0729	2.3219	1.1774	1.0328
27	1.0000	1.0000	1.0000	1.0000	0.8433	1.1211	1.0085	0.9559	0.8433	1.1211	1.0085	0.9559
28	1.0000	1.0361	1.0000	1.0000	1.0244	1.0140	0.9643	1.0469	1.0244	1.0506	0.9643	1.0469
29	1.0000	0.9946	1.0000	1.0016	1.1260	1.1517	0.9009	1.0885	1.1260	1.1454	0.9009	1.0902
30	1.0773	1.0000	1.0504	1.0981	1.0086	0.9118	0.9870	1.0474	1.0865	0.9118	1.0367	1.1501
31	1.0000	1.0000	1.1265	1.1447	0.9667	1.0277	0.9066	0.9553	0.9667	1.0277	1.0213	1.0935
Average	1.0090	1.0337	1.0429	1.0282	1.0129	1.0721	1.0331	1.0564	1.0213	1.1009	1.0750	1.0838
Std. Dev.	0.0371	0.0587	0.0657	0.0632	0.1105	0.2829	0.1004	0.1119	0.1124	0.2421	0.0976	0.1109
Max.	1.0773	1.1900	1.2804	1.1497	1.4251	2.5280	1.3247	1.4208	1.4491	2.3219	1.3247	1.4208
Min.	0.9074	0.9185	0.9170	0.8264	0.7417	0.8427	0.9009	0.9044	0.7800	0.9118	0.9009	0.8959

Table S.14: MPI decomposition along path H: Input orientation, comparison period viewpoint, specific components, RTS.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	1.0000	1.0000	1.0000	1.0000	1.1467	1.1016	1.0520	1.0270	1.1467	1.1016	1.0520	1.0270
2	1.0000	1.2089	1.0906	1.0081	0.9635	0.8777	0.9343	1.4094	0.9635	1.0611	1.0189	1.4208
3	0.9977	1.1820	1.1155	1.0280	0.9927	0.9476	0.9373	1.0951	0.9904	1.1201	1.0456	1.1258
4	1.0393	1.1768	1.0260	1.1324	0.8725	0.7996	0.9795	0.9078	0.9068	0.9410	1.0050	1.0279
5	1.0382	1.0800	1.0129	1.0990	0.9687	1.0071	1.0143	0.9817	1.0057	1.0877	1.0274	1.0789
6	1.0025	1.0273	1.0330	1.0523	1.0862	NaN	1.2112	1.0738	1.0889	NaN	1.2511	1.1300
7	1.0319	0.9961	1.1096	1.0453	1.4043	0.9782	1.0155	1.0913	1.4491	0.9744	1.1268	1.1407
8	1.0366	1.0357	1.1051	1.0296	1.0135	0.9896	1.0193	0.9678	1.0506	1.0249	1.1264	0.9965
9	0.9573	1.0803	1.0782	1.0373	0.9731	0.9892	1.0272	0.9867	0.9315	1.0687	1.1076	1.0235
10	1.0850	1.0689	1.1084	1.0547	0.9926	1.2374	1.1352	1.0823	1.0770	1.3226	1.2582	1.1415
11	0.9074	0.9731	1.2804	1.0134	0.9636	1.0387	0.9915	0.9923	0.8743	1.0108	1.2695	1.0055
12	1.0445	1.0528	1.0079	1.0572	0.7468	0.9622	1.0303	0.9808	0.7800	1.0130	1.0384	1.0368
13	1.0000	1.0000	1.0000	1.0000	1.0291	1.0406	0.9286	0.9044	1.0291	1.0406	0.9286	0.9044
14	1.0000	1.0000	1.0000	1.0000	0.9779	1.0124	1.1049	1.2997	0.9779	1.0124	1.1049	1.2997
15	1.0658	1.0317	1.1119	1.1065	0.9634	0.9670	0.9493	1.1811	1.0267	0.9977	1.0556	1.3069
16	1.0000	1.0000	1.0000	1.0000	1.0914	1.2391	1.3247	0.9475	1.0914	1.2391	1.3247	0.9475
17	1.0000	1.0000	1.0468	1.0000	0.9788	1.0456	0.9945	1.1680	0.9788	1.0456	1.0410	1.1680
18	1.0000	1.0000	0.9837	1.0937	0.9789	0.9556	1.1196	1.0277	0.9789	0.9556	1.1014	1.1240
19	1.0000	1.0000	0.9169	1.0000	1.0090	1.0508	1.0952	1.1002	1.0090	1.0508	1.0042	1.1002
20	0.9262	1.1892	1.0523	1.0324	1.0872	0.9525	1.0371	1.0490	1.0069	1.1326	1.0914	1.0830
21	0.9998	1.0770	1.1279	0.8351	NaN	0.9888	0.9931	1.0728	NaN	1.0649	1.1202	0.8959
22	1.0000	1.0000	1.0000	1.0000	1.0364	0.9237	0.9563	0.9667	1.0364	0.9237	0.9563	0.9667
23	1.0000	1.0000	1.0146	1.0000	1.0641	1.0364	1.0666	1.1047	1.0641	1.0364	1.0822	1.1047
24	1.0630	1.0660	1.0529	1.0683	0.9492	1.0376	1.0044	1.0025	1.0090	1.1061	1.0575	1.0710
25	1.0428	1.1171	1.0743	1.1497	1.0040	0.9988	0.9505	0.9583	1.0470	1.1158	1.0211	1.1018
26	1.0000	1.0000	1.0000	1.0000	1.0729	2.3219	1.1774	1.0328	1.0729	2.3219	1.1774	1.0328
27	1.0000	1.0000	1.0000	1.0247	0.8433	1.1211	1.0085	0.9329	0.8433	1.1211	1.0085	0.9559
28	1.0000	1.0130	1.0000	1.0000	1.0244	1.0372	0.9643	1.0469	1.0244	1.0506	0.9643	1.0469
29	1.0000	1.0181	1.0000	1.0080	1.1260	1.1251	0.9009	1.0816	1.1260	1.1454	0.9009	1.0902
30	1.1451	1.0000	1.0498	1.0981	0.9488	0.9118	0.9876	1.0474	1.0865	0.9118	1.0367	1.1501
31	1.0000	1.0000	1.1265	1.1558	0.9667	1.0277	0.9066	0.9461	0.9667	1.0277	1.0213	1.0935
Average	1.0124	1.0450	1.0492	1.0364	1.0092	1.0574	1.0264	1.0473	1.0213	1.1009	1.0750	1.0838
Std. Dev.	0.0428	0.0647	0.0662	0.0597	0.1084	0.2513	0.0924	0.1068	0.1124	0.2421	0.0976	0.1109
Max.	1.1451	1.2089	1.2804	1.1558	1.4043	2.3219	1.3247	1.4094	1.4491	2.3219	1.3247	1.4208
Min.	0.9074	0.9731	0.9169	0.8351	0.7468	0.7996	0.9009	0.9044	0.7800	0.9118	0.9009	0.8959

Table S.15: MPI decomposition along paths EFGH: Input orientation, ‘mean’ period viewpoint.

Bank	M(07,06)	M(08,07)	M(09,08)	M(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1176	1.1073	1.1057	0.9551	1.0000	1.0000	1.0000	1.0000	1.1385	1.1381	1.1426	0.9474
2	1.0193	1.0344	1.0188	1.5164	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
3	1.0812	1.1302	1.0932	1.5096	1.1387	0.9873	1.0827	1.2124	0.9582	1.0125	0.9756	1.0985
4	1.0437	1.0443	0.9918	1.1697	1.0267	1.0571	0.9553	1.1672	1.1061	1.0508	1.0323	0.9899
5	1.0159	1.0884	1.0100	1.0741	1.0000	1.0000	1.0000	1.0000	1.0525	1.1345	1.0426	1.1363
6	1.0379	1.0062	1.0583	1.0726	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
7	1.0679	1.0600	1.1559	1.0691	1.0000	1.0000	1.0000	1.0000	0.8776	1.1933	NaN	NaN
8	1.0725	1.1543	1.1003	1.0418	1.0730	1.1437	0.9494	1.0349	0.9513	1.1193	NaN	1.0001
9	0.9487	1.0631	1.0948	1.0156	1.0452	1.0064	1.0157	0.9897	0.9635	0.9817	0.9881	1.0053
10	1.0222	1.0104	1.1711	0.9880	1.0000	1.0000	1.0000	1.0000	1.2919	0.9348	NaN	1.0190
11	0.8588	1.0254	1.0549	1.0829	1.0261	1.0107	0.8668	1.0766	0.9562	1.0081	0.9616	1.0007
12	1.1381	1.1062	1.0621	1.0988	1.4838	1.1371	0.9538	0.9296	1.0918	1.0215	1.0971	1.1864
13	1.0794	0.8378	0.7643	0.9929	1.0638	0.8253	0.8335	1.1185	0.9937	0.9763	0.9902	0.9869
14	0.9896	0.9928	1.1796	1.7774	0.9899	0.9400	0.9994	1.0753	1.0113	1.0446	1.0880	1.9133
15	1.0882	0.9251	0.9205	1.6079	1.1043	0.9403	0.9098	1.2038	0.9591	0.9856	0.9621	1.1062
16	1.2989	0.8790	0.8957	0.9105	1.1294	0.6381	0.5833	1.0457	1.0662	1.1394	1.1305	0.9189
17	0.9913	1.0030	0.9542	1.3224	1.0465	0.9539	0.9599	1.1158	0.9613	1.0172	0.9777	1.0050
18	0.9274	1.0335	1.0003	1.5125	0.8557	1.0253	0.8517	1.2369	1.1011	1.0443	1.0668	1.0679
19	0.9297	0.8301	1.0731	1.1922	0.8984	0.7245	0.9722	1.0785	1.0206	1.0749	1.0619	0.9722
20	0.8291	1.1989	1.1267	1.5034	0.8257	0.9943	1.0095	1.2760	1.0104	1.0559	1.0501	1.0685
21	0.9274	1.1681	1.1354	0.8493	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
22	0.9663	0.8644	0.9559	1.0565	0.9265	0.8814	0.9634	1.0742	1.0133	1.0398	1.0310	0.9890
23	0.9634	0.9497	0.9703	0.9289	0.8671	0.8356	0.8394	0.8762	1.0539	1.1006	1.0832	0.9583
24	1.0534	1.0770	1.0512	1.0507	1.0902	0.9708	1.0235	0.9801	0.9546	1.0115	0.9700	1.0006
25	1.0870	1.0935	1.0809	1.2356	1.0288	0.9344	1.0442	1.1069	1.0161	1.0515	0.9938	1.0296
26	1.0338	0.6939	1.0183	1.0220	1.0000	1.0000	1.0000	1.0000	1.6359	0.4571	0.9428	1.1479
27	1.1102	0.8541	1.0803	1.1091	0.9963	0.6174	0.9289	1.2474	1.2716	1.2482	1.1415	0.9418
28	1.0640	1.2711	0.9851	1.3001	0.8299	0.9853	0.9950	1.3134	1.2659	1.1789	1.0176	0.9452
29	1.1220	1.3029	0.7698	1.1953	1.0717	1.0665	0.9995	1.0005	0.9311	1.0881	0.8630	1.1013
30	1.1398	1.0688	1.0449	1.2029	0.9254	1.2266	0.8615	1.0368	1.1189	0.9095	1.1804	1.0447
31	1.0514	0.9749	0.9577	1.1307	1.0964	0.9260	0.9652	1.0643	0.9898	1.0340	0.9938	1.0048
Average	1.0347	1.0274	1.0284	1.1772	1.0174	0.9622	0.9537	1.0729	1.0629	1.0376	1.0314	1.0587
Std. Dev.	0.0906	0.1302	0.0985	0.2232	0.1177	0.1270	0.0905	0.1037	0.1500	0.1350	0.0726	0.1804
Max.	1.2989	1.3029	1.1796	1.7774	1.4838	1.2266	1.0827	1.3134	1.6359	1.2482	1.1804	1.9133
Min.	0.8291	0.6939	0.7643	0.8493	0.8257	0.6174	0.5833	0.8762	0.8776	0.4571	0.8630	0.9189

Table S.16: MPI decomposition along paths EFGH: Input orientation, 'mean' period viewpoint.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	RTS(07,06)	RTS(08,07)	RTS(09,08)	RTS(10,09)
1	1.0002	1.0050	1.0000	1.0000	0.9814	0.9681	0.9677	1.0081	0.9816	0.9729	0.9677	1.0081
2	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	0.9977	1.1820	1.1008	1.0527	0.9932	0.9565	0.9402	1.0767	0.9909	1.1306	1.0350	1.1335
4	1.0387	1.1430	1.0301	1.1139	0.8848	0.8225	0.9763	0.9089	0.9190	0.9401	1.0058	1.0124
5	1.0094	0.9718	0.9975	1.0486	0.9562	0.9871	0.9712	0.9014	0.9652	0.9593	0.9688	0.9452
6	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
7	1.0163	0.9759	NaN	NaN	1.1973	0.9102	NaN	NaN	1.2168	0.8883	NaN	NaN
8	1.0366	1.0348	NaN	1.0298	1.0137	0.8714	NaN	0.9775	1.0508	0.9017	NaN	1.0066
9	0.9573	1.0787	1.0790	1.0373	0.9841	0.9976	1.0110	0.9840	0.9421	1.0760	1.0908	1.0207
10	0.9984	1.0523	NaN	0.9596	0.7925	1.0271	NaN	1.0103	0.7912	1.0808	NaN	0.9695
11	0.9074	0.9672	1.2769	1.0132	0.9646	1.0407	0.9911	0.9920	0.8753	1.0065	1.2655	1.0051
12	1.0320	1.0343	0.9979	1.0353	0.6807	0.9206	1.0172	0.9623	0.7025	0.9522	1.0150	0.9962
13	1.0000	1.0000	1.0000	1.0000	1.0210	1.0399	0.9261	0.8995	1.0210	1.0399	0.9261	0.8995
14	1.0000	1.0000	1.0000	1.0000	0.9885	1.0110	1.0848	0.8640	0.9885	1.0110	1.0848	0.8640
15	1.0658	1.0302	1.1106	1.1040	0.9640	0.9690	0.9469	1.0937	1.0274	0.9983	1.0516	1.2075
16	1.0000	1.0000	1.0000	1.0000	1.0787	1.2089	1.3582	0.9475	1.0787	1.2089	1.3582	0.9475
17	1.0000	1.0000	1.0234	1.0126	0.9854	1.0336	0.9934	1.1645	0.9854	1.0336	1.0167	1.1792
18	0.9931	1.0000	0.9934	1.0461	0.9911	0.9653	1.1082	1.0946	0.9842	0.9653	1.1009	1.1450
19	1.0000	1.0139	0.9576	1.0357	1.0140	1.0513	1.0855	1.0979	1.0140	1.0659	1.0394	1.1371
20	0.9309	1.1908	1.0250	1.0136	1.0675	0.9590	1.0369	1.0878	0.9938	1.1420	1.0628	1.1027
21	NaN	NaN	NaN	0.8246	NaN	NaN	NaN	1.0386	NaN	NaN	NaN	0.8565
22	1.0000	1.0000	1.0000	1.0302	1.0293	0.9432	0.9624	0.9653	1.0293	0.9432	0.9624	0.9945
23	1.0000	1.0000	1.0073	0.9957	1.0542	1.0327	1.0594	1.1110	1.0542	1.0327	1.0671	1.1063
24	1.0630	1.0648	1.0526	1.0683	0.9521	1.0301	1.0060	1.0029	1.0121	1.0969	1.0589	1.0713
25	1.0421	1.1171	1.0916	1.1332	0.9979	0.9963	0.9542	0.9566	1.0399	1.1130	1.0416	1.0841
26	1.0730	0.9797	1.0003	1.0365	0.5889	1.5495	1.0797	0.8590	0.6320	1.5181	1.0801	0.8903
27	0.9995	1.0035	1.0285	1.0061	0.8767	1.1044	0.9906	0.9383	0.8763	1.1083	1.0188	0.9440
28	0.9999	1.0765	1.0000	1.0000	1.0128	1.0165	0.9730	1.0472	1.0127	1.0943	0.9730	1.0472
29	1.0000	1.0031	1.0000	1.0024	1.1244	1.1193	0.8925	1.0822	1.1244	1.1228	0.8925	1.0848
30	1.0559	1.0000	1.0247	1.0624	1.0425	0.9580	1.0027	1.0453	1.1009	0.9580	1.0275	1.1105
31	1.0000	1.0000	1.0933	1.1178	0.9689	1.0183	0.9133	0.9458	0.9689	1.0183	0.9985	1.0573
Average	1.0078	1.0330	1.0356	1.0278	0.9717	1.0182	1.0099	1.0023	0.9778	1.0493	1.0444	1.0295
Std. Dev.	0.0360	0.0593	0.0629	0.0566	0.1196	0.1263	0.0898	0.0789	0.1175	0.1190	0.0946	0.0924
Max.	1.0730	1.1908	1.2769	1.1332	1.1973	1.5495	1.3582	1.1645	1.2168	1.5181	1.3582	1.2075
Min.	0.9074	0.9672	0.9576	0.8246	0.5889	0.8225	0.8925	0.8590	0.6320	0.8883	0.8925	0.8565

Moorsteen-Bjurek Productivity Indices (MBPI)

Table S.17: MBPI decomposition along paths A and B: Output orientation, base period viewpoint, common components.

Bank	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
2	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	1.1673	1.0502	1.0358	1.7839
3	1.0857	0.9900	1.0628	1.3145	1.1352	0.9880	1.0770	1.1975	0.9558	1.0100	0.9846	1.1293
4	1.0517	0.9998	0.9930	1.0685	1.0540	1.0185	0.9875	1.0764	1.0142	1.0362	1.0093	1.0116
5	1.0512	1.2218	1.0355	1.1025	1.0000	1.0000	1.0000	1.0000	1.0590	1.1741	1.0474	1.1124
6	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	1.3066	1.0587	1.1959	1.1288
7	0.9945	1.0802	NaN	NaN	1.0000	1.0000	1.0000	1.0000	1.0051	1.0705	1.5139	1.0386
8	1.0193	1.2038	1.2962	1.0097	1.0460	1.1503	0.9475	1.0281	0.9768	1.0748	1.3448	0.9943
9	0.9656	0.9733	1.0080	0.9945	1.0124	1.0107	1.0106	0.9936	0.9536	0.9675	1.0048	1.0024
10	1.1149	1.0182	1.6620	1.1374	1.0000	1.0000	1.0000	1.0000	1.1130	1.0799	1.6347	1.1347
11	0.9732	1.0099	0.8290	1.0701	1.0268	1.0116	0.8736	1.0695	0.9535	1.0009	0.9647	1.0020
12	1.2334	1.0439	1.0533	1.0886	1.1834	1.0423	0.9686	0.9304	1.0722	1.0316	1.0829	1.1821
13	1.0542	0.8417	0.9039	1.0601	1.0741	0.8686	0.9064	1.0658	0.9745	0.9758	0.9937	1.0046
14	1.0006	0.9929	1.0943	2.0702	0.9914	0.9512	1.0001	1.0603	1.0097	1.0364	1.0838	2.1270
15	1.0441	0.9284	0.8742	1.3736	1.0980	0.9453	0.9184	1.2335	0.9577	0.9899	0.9666	1.1501
16	1.3026	0.8790	0.8957	0.9105	1.1801	0.8233	0.8242	0.9331	1.0182	1.0452	1.0246	0.9914
17	1.0007	0.9781	0.9455	1.1017	1.0468	0.9616	0.9680	1.0973	0.9592	1.0133	0.9780	1.0123
18	0.9651	1.0675	0.8869	1.3163	0.8758	1.0282	0.8908	1.1731	1.0925	1.0313	1.0207	1.0954
19	0.9391	0.8556	1.1197	0.9627	0.9424	0.8243	1.0517	0.9994	0.9950	1.0322	1.0089	0.9920
20	0.8828	1.0529	1.0792	1.3611	0.8799	0.9795	1.0111	1.2084	0.9907	1.0309	1.0582	1.0764
21	1.0200	NaN	NaN	1.0183	1.0000	1.0000	1.0000	1.0000	1.0099	1.3118	1.2150	1.0299
22	0.9546	0.9565	1.0110	1.0255	0.9609	0.9281	0.9992	1.0343	0.9913	1.0311	1.0020	1.0009
23	0.9634	0.9497	0.9633	0.9329	0.9368	0.9144	0.9449	0.9316	1.0124	1.0401	1.0128	0.9955
24	1.0343	0.9782	0.9890	0.9796	1.0858	0.9725	1.0221	0.9812	0.9557	1.0103	0.9699	1.0008
25	1.0354	0.9656	0.9920	1.1168	1.0455	0.9544	1.0334	1.0704	0.9905	1.0269	0.9743	1.0229
26	0.9567	0.8522	1.0543	1.0825	1.0000	1.0000	1.0000	1.0000	1.2671	0.8416	1.0838	1.2349
27	1.1113	0.8445	1.0209	1.1091	1.0629	0.8363	0.9881	1.0261	1.0487	1.0530	1.0280	0.9909
28	1.0445	1.2239	0.9895	1.3155	0.9916	0.9091	0.9486	1.1842	1.0403	1.0529	1.0315	1.0323
29	1.1267	1.6530	0.7906	1.2197	1.0875	1.2414	0.9985	1.0015	1.0403	1.3128	0.7942	1.3430
30	1.0186	1.0516	1.0268	1.1286	0.8796	1.2165	0.8722	1.0317	1.1052	0.8646	1.1678	1.0956
31	1.0486	0.9539	0.9578	1.0336	1.0884	0.9422	0.9781	1.0414	0.9761	1.0209	0.9868	1.0080
Average	1.0355	1.0210	1.0205	1.1446	1.0221	0.9845	0.9749	1.0442	1.0337	1.0425	1.0740	1.1241
Std. Dev.	0.0853	0.1605	0.1623	0.2214	0.0750	0.0923	0.0547	0.0790	0.0857	0.0927	0.1646	0.2411
Max.	1.3026	1.6530	1.6620	2.0702	1.1834	1.2414	1.0770	1.2335	1.3066	1.3128	1.6347	2.1270
Min.	0.8828	0.8417	0.7906	0.9105	0.8758	0.8233	0.8242	0.9304	0.9535	0.8416	0.7942	0.9909

Table S.18: MBPI decomposition along path A: Output orientation, base period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	NaN	NaN	NaN	NaN	1.0287	1.0000	1.0000	1.0060
3	1.0006	0.9922	1.0022	0.9873	1.0000	1.0000	1.0000	0.9845
4	0.9838	0.9477	0.9963	0.9813	1.0000	0.9996	0.9999	1.0000
5	0.9925	1.0406	0.9831	0.9911	1.0002	1.0000	1.0056	1.0001
6	NaN	NaN	NaN	NaN	1.0011	1.0002	1.0013	1.0048
7	0.9867	1.0089	NaN	NaN	1.0028	1.0002	1.0007	1.0135
8	0.9977	0.9893	1.0209	0.9877	1.0000	0.9843	0.9964	1.0000
9	1.0004	0.9953	0.9926	0.9985	0.9998	1.0000	1.0000	1.0000
10	1.0014	0.9388	1.0023	1.0016	1.0004	1.0043	1.0143	1.0007
11	0.9940	0.9975	0.9820	0.9986	1.0000	1.0000	1.0017	1.0000
12	0.9721	0.9709	1.0042	0.9898	1.0000	1.0000	1.0000	1.0000
13	1.0071	0.9931	1.0034	0.9901	1.0000	1.0000	1.0001	1.0000
14	0.9996	1.0071	1.0105	0.9589	1.0000	1.0000	0.9991	0.9573
15	0.9930	0.9922	0.9790	0.9902	1.0000	1.0000	1.0059	0.9778
16	1.0841	1.0215	1.0607	0.9842	1.0000	1.0000	1.0000	1.0000
17	0.9967	1.0039	0.9988	0.9919	1.0000	1.0000	1.0000	0.9999
18	1.0085	1.0067	0.9751	1.0305	1.0000	1.0000	1.0003	0.9940
19	1.0015	1.0056	1.0553	0.9702	1.0000	1.0000	1.0000	1.0009
20	1.0127	1.0427	1.0087	1.0465	1.0000	1.0000	1.0000	1.0000
21	1.0100	NaN	NaN	0.9796	1.0000	1.0000	1.0005	1.0093
22	1.0022	0.9995	1.0097	0.9907	1.0000	1.0000	1.0000	1.0000
23	1.0158	0.9986	1.0065	1.0060	1.0000	1.0000	1.0000	1.0000
24	0.9967	0.9955	0.9977	0.9977	1.0000	1.0000	1.0000	1.0000
25	0.9999	0.9853	0.9851	1.0200	1.0000	1.0000	1.0000	1.0000
26	0.7550	0.9563	0.9728	0.8766	1.0000	1.0589	1.0000	1.0000
27	0.9969	0.9590	1.0051	1.0909	1.0000	1.0000	1.0000	1.0000
28	1.0125	1.2787	1.0113	1.0761	1.0000	1.0000	1.0000	1.0000
29	0.9960	1.0143	0.9933	0.9068	1.0000	1.0000	1.0037	1.0000
30	1.0940	0.9998	1.0081	0.9965	0.9578	1.0000	1.0000	1.0020
31	0.9870	0.9917	0.9924	0.9847	1.0000	1.0000	1.0000	1.0000
Average	0.9964	1.0049	1.0022	0.9935	0.9997	1.0016	1.0010	0.9984
Std. Dev.	0.0528	0.0587	0.0200	0.0408	0.0093	0.0110	0.0030	0.0097
Max.	1.0940	1.2787	1.0607	1.0909	1.0287	1.0589	1.0143	1.0135
Min.	0.7550	0.9388	0.9728	0.8766	0.9578	0.9843	0.9964	0.9573

Table S.19: MBPI decomposition along path B: Output orientation, base period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	NaN	NaN	NaN	NaN	0.9416	1.0000	1.0000	0.9941
3	1.0006	0.9922	1.0022	0.9566	1.0000	1.0000	1.0000	1.0162
4	0.9838	0.9468	0.9961	0.9813	1.0000	1.0005	1.0001	1.0000
5	0.9928	1.0553	0.9984	0.9912	0.9998	0.9860	0.9902	0.9999
6	NaN	NaN	NaN	NaN	0.9989	0.9998	0.9987	0.9953
7	0.9938	1.0092	NaN	NaN	0.9956	0.9998	0.9994	0.9848
8	0.9977	0.9563	1.0110	0.9896	1.0000	1.0182	1.0062	0.9981
9	1.0001	0.9953	0.9926	0.9985	1.0002	1.0000	1.0000	1.0000
10	1.0021	0.9469	1.0302	1.0031	0.9996	0.9957	0.9869	0.9993
11	0.9940	0.9975	0.9892	0.9986	1.0000	1.0000	0.9944	1.0000
12	0.9721	0.9709	1.0074	0.9898	1.0000	1.0000	0.9969	1.0000
13	1.0071	0.9931	1.0036	0.9901	1.0000	1.0000	0.9999	1.0000
14	0.9996	1.0071	1.0087	0.8722	1.0000	1.0000	1.0008	1.0524
15	0.9930	0.9922	0.9903	0.9443	1.0000	1.0000	0.9944	1.0253
16	1.0841	1.0215	1.0607	0.9842	1.0000	1.0000	1.0000	1.0000
17	0.9967	1.0039	0.9988	0.9918	1.0000	1.0000	1.0000	1.0001
18	1.0085	1.0067	0.9777	0.9907	1.0000	1.0000	0.9977	1.0339
19	1.0015	1.0056	1.0553	0.9719	1.0000	1.0000	1.0000	0.9991
20	1.0127	1.0427	1.0087	1.0465	1.0000	1.0000	1.0000	1.0000
21	1.0126	NaN	NaN	0.9980	0.9975	1.0000	0.9995	0.9907
22	1.0022	0.9995	1.0097	0.9907	1.0000	1.0000	1.0000	1.0000
23	1.0158	0.9986	1.0065	1.0060	1.0000	1.0000	1.0000	1.0000
24	0.9967	0.9955	0.9977	0.9977	1.0000	1.0000	1.0000	1.0000
25	0.9999	0.9853	0.9851	1.0200	1.0000	1.0000	1.0000	1.0000
26	0.7550	1.0975	0.9728	0.8766	1.0000	0.9226	1.0000	1.0000
27	0.9969	0.9590	1.0051	1.0909	1.0000	1.0000	1.0000	1.0000
28	1.0125	1.2787	1.0113	1.0761	1.0000	1.0000	1.0000	1.0000
29	0.9960	1.0143	1.0006	0.9068	1.0000	1.0000	0.9963	1.0000
30	1.0124	0.9998	1.0081	0.9716	1.0349	1.0000	1.0000	1.0277
31	0.9870	0.9917	0.9924	0.9847	1.0000	1.0000	1.0000	1.0000
Average	0.9938	1.0097	1.0046	0.9859	0.9989	0.9974	0.9987	1.0039
Std. Dev.	0.0494	0.0612	0.0192	0.0470	0.0124	0.0145	0.0034	0.0135
Max.	1.0841	1.2787	1.0607	1.0909	1.0349	1.0182	1.0062	1.0524
Min.	0.7550	0.9468	0.9728	0.8722	0.9416	0.9226	0.9869	0.9848

Table S.20: MBPI decomposition along paths C and D: Output orientation, comparison period viewpoint, common components.

Bank	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	NaN	NaN	1.0236	NaN	1.0000	1.0000	1.0000	1.0000	NaN	NaN	1.0236	NaN
2	0.9716	0.9296	0.9782	1.0274	1.0000	1.0000	1.0000	1.0000	1.0301	0.9845	0.9982	1.0157
3	1.0857	0.9896	1.0350	1.1993	1.1352	0.9880	1.0770	1.1975	0.9556	1.0103	0.9647	1.0329
4	1.0459	0.9683	0.9868	1.0442	1.0540	1.0185	0.9875	1.0764	1.0138	1.0065	1.0019	0.9916
5	1.0049	0.9970	0.9819	0.9991	1.0000	1.0000	1.0000	1.0000	1.0065	1.0019	0.9831	0.9981
6	0.9661	NaN	0.9905	0.9972	1.0000	1.0000	1.0000	1.0000	0.9726	0.9418	0.9553	0.9866
7	0.9213	1.0664	1.0227	0.9765	1.0000	1.0000	1.0000	1.0000	0.9249	1.0617	1.0295	0.9682
8	1.0193	1.1269	0.9781	1.0411	1.0460	1.1503	0.9475	1.0281	0.9751	0.9963	1.0414	1.0158
9	0.9923	0.9866	0.9860	0.9917	1.0124	1.0107	1.0106	0.9936	0.9799	0.9877	0.9795	1.0009
10	0.9476	0.8089	0.9626	0.9933	1.0000	1.0000	1.0000	1.0000	0.9520	0.8215	0.9367	0.9211
11	0.9732	1.0171	0.8329	1.0703	1.0268	1.0116	0.8736	1.0695	0.9554	1.0097	0.9626	1.0020
12	1.2138	1.0358	1.0290	1.0387	1.1834	1.0423	0.9686	0.9304	1.0752	1.0100	1.0535	1.1274
13	1.0610	0.8400	0.8731	1.0571	1.0741	0.8686	0.9064	1.0658	0.9788	0.9743	0.9594	1.0010
14	1.0006	0.9929	1.0693	1.1392	0.9914	0.9512	1.0001	1.0603	1.0095	1.0351	1.0581	1.0794
15	1.0419	0.9007	0.8703	1.1859	1.0980	0.9453	0.9184	1.2335	0.9577	0.9664	0.9585	1.0002
16	1.3026	0.8790	0.8957	0.9105	1.1801	0.8233	0.8242	0.9331	0.9961	1.0430	1.0183	0.9908
17	1.0007	0.9781	0.9440	1.0929	1.0468	0.9616	0.9680	1.0973	0.9602	1.0128	0.9766	1.0032
18	0.9517	1.0675	0.8985	1.1813	0.8758	1.0282	0.8908	1.1731	1.0790	1.0284	1.0464	1.0195
19	0.9391	0.8556	1.1197	0.9691	0.9424	0.8243	1.0517	0.9994	0.9946	1.0316	1.0023	0.9974
20	0.8846	1.0550	1.0272	1.3245	0.8799	0.9795	1.0111	1.2084	0.9879	1.0286	0.9797	1.0173
21	1.0051	1.0859	1.0100	0.9680	1.0000	1.0000	1.0000	1.0000	0.9366	1.0918	1.0169	0.9716
22	0.9572	0.9317	1.0110	1.0255	0.9609	0.9281	0.9992	1.0343	0.9934	1.0045	1.0008	1.0004
23	0.9634	0.9497	0.9633	0.9329	0.9368	0.9144	0.9449	0.9316	1.0082	1.0403	1.0121	0.9958
24	1.0343	0.9782	0.9926	0.9795	1.0858	0.9725	1.0221	0.9812	0.9568	1.0110	0.9736	1.0005
25	1.0341	0.9656	1.0269	1.0855	1.0455	0.9544	1.0334	1.0704	0.9894	1.0286	0.9955	1.0203
26	0.8786	0.6217	0.9376	0.8840	1.0000	1.0000	1.0000	1.0000	0.5714	0.5038	0.9119	0.7669
27	1.1102	0.8445	1.0992	1.1091	1.0629	0.8363	0.9881	1.0261	1.0490	1.0567	1.0277	0.9894
28	1.0443	1.3011	0.9895	1.3155	0.9916	0.9091	0.9486	1.1842	1.0369	1.0424	1.0067	0.9985
29	1.0641	1.3446	0.6458	1.1943	1.0875	1.2414	0.9985	1.0015	0.9207	1.0382	0.6509	1.2254
30	0.9100	1.1215	1.0043	1.0220	0.8796	1.2165	0.8722	1.0317	1.0374	0.9236	1.1550	0.9959
31	1.0486	0.9539	0.9578	1.0336	1.0884	0.9422	0.9781	1.0414	0.9801	1.0219	0.9879	1.0071
Average	1.0125	0.9860	0.9724	1.0596	1.0221	0.9845	0.9749	1.0442	0.9762	0.9905	0.9893	1.0047
Std. Dev.	0.0872	0.1375	0.0863	0.1058	0.0750	0.0923	0.0547	0.0790	0.0847	0.1024	0.0757	0.0679
Max.	1.3026	1.3446	1.1197	1.3245	1.1834	1.2414	1.0770	1.2335	1.0790	1.0918	1.1550	1.2254
Min.	0.8786	0.6217	0.6458	0.8840	0.8758	0.8233	0.8242	0.9304	0.5714	0.5038	0.6509	0.7669

Table S.21: MBPI decomposition along path C: Output orientation, comparison period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	1.0000	NaN	NaN	NaN	1.0000	NaN
2	0.9233	0.9173	0.9632	1.0115	1.0215	1.0294	1.0173	1.0000
3	1.0007	0.9918	0.9962	0.9837	1.0000	0.9997	1.0000	0.9857
4	0.9788	0.9445	0.9974	0.9784	1.0000	1.0000	1.0000	1.0000
5	0.9831	0.9952	0.9987	0.9904	1.0156	1.0000	1.0001	1.0108
6	0.9927	NaN	1.0324	0.9993	1.0005	1.0168	1.0044	1.0114
7	0.9961	0.9995	0.9958	0.9953	1.0000	1.0050	0.9977	1.0133
8	0.9994	0.9909	0.9961	0.9968	1.0000	0.9923	0.9952	1.0000
9	1.0004	0.9884	0.9960	0.9972	0.9998	1.0000	1.0000	1.0000
10	0.9674	0.9726	1.0029	1.0434	1.0290	1.0125	1.0247	1.0334
11	0.9920	0.9972	0.9903	0.9988	1.0000	0.9986	1.0001	1.0000
12	0.9539	0.9793	1.0085	0.9902	1.0000	1.0047	1.0000	1.0000
13	1.0091	0.9926	1.0039	0.9911	1.0000	1.0000	1.0001	0.9998
14	0.9998	1.0084	1.0105	0.9702	1.0000	1.0000	1.0000	1.0260
15	0.9908	0.9810	0.9881	0.9906	1.0000	1.0051	1.0006	0.9703
16	1.1081	1.0236	1.0673	0.9848	1.0000	1.0000	1.0000	1.0000
17	0.9956	1.0044	0.9987	0.9929	1.0000	1.0000	0.9999	1.0000
18	1.0071	1.0095	0.9639	0.9814	1.0000	1.0000	1.0000	1.0064
19	1.0020	1.0062	1.0622	0.9715	1.0000	1.0000	1.0000	1.0007
20	1.0176	1.0472	1.0370	1.0775	1.0000	1.0000	1.0000	1.0000
21	1.0730	0.9947	0.9932	0.9907	1.0000	1.0000	1.0000	1.0056
22	1.0029	0.9995	1.0109	0.9912	1.0000	1.0000	1.0000	1.0000
23	1.0200	0.9984	1.0073	1.0057	1.0000	1.0000	1.0000	1.0000
24	0.9956	0.9949	0.9976	0.9978	1.0000	1.0000	1.0000	1.0000
25	0.9997	0.9837	0.9982	0.9940	1.0000	1.0000	1.0000	1.0000
26	1.4768	1.2339	1.0220	1.1349	1.0411	1.0000	1.0060	1.0157
27	0.9957	0.9556	1.0824	1.0925	1.0000	1.0000	1.0000	1.0000
28	1.0157	1.3631	1.0363	1.1125	1.0000	1.0073	1.0000	1.0000
29	1.0628	0.9861	0.9937	0.9732	1.0000	1.0580	1.0000	1.0000
30	1.0185	0.9981	0.9984	0.9947	0.9791	1.0000	0.9985	1.0000
31	0.9830	0.9907	0.9913	0.9855	1.0000	1.0000	1.0000	1.0000
Average	1.0187	1.0120	1.0077	1.0073	1.0029	1.0043	1.0014	1.0026
Std. Dev.	0.0912	0.0830	0.0261	0.0410	0.0107	0.0120	0.0055	0.0107
Max.	1.4768	1.3631	1.0824	1.1349	1.0411	1.0580	1.0247	1.0334
Min.	0.9233	0.9173	0.9632	0.9702	0.9791	0.9923	0.9952	0.9703

Table S.22: MBPI decomposition along path D: Output orientation, comparison period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	1.0000	NaN	NaN	NaN	1.0000	NaN
2	0.9664	0.9538	0.9835	1.0115	0.9760	0.9900	0.9963	1.0000
3	1.0007	0.9912	0.9962	0.9553	1.0000	1.0003	1.0000	1.0149
4	0.9788	0.9445	0.9972	0.9751	1.0000	1.0000	1.0001	1.0033
5	1.0128	0.9952	0.9988	1.0023	0.9858	1.0000	0.9999	0.9988
6	0.9938	NaN	1.0416	1.0222	0.9994	0.9832	0.9955	0.9888
7	0.9962	1.0052	0.9909	1.0223	1.0000	0.9992	1.0026	0.9865
8	0.9994	0.9814	0.9893	0.9968	1.0000	1.0019	1.0020	1.0000
9	1.0001	0.9884	0.9960	0.9972	1.0002	1.0000	1.0000	1.0000
10	0.9985	0.9978	1.0547	1.1115	0.9969	0.9868	0.9744	0.9701
11	0.9920	0.9959	0.9909	0.9988	1.0000	0.9999	0.9995	1.0000
12	0.9539	0.9887	1.0085	0.9902	1.0000	0.9951	1.0000	1.0000
13	1.0091	0.9926	1.0041	0.9906	1.0000	1.0000	0.9999	1.0003
14	0.9998	1.0084	1.0105	1.0286	1.0000	1.0000	1.0000	0.9677
15	0.9908	0.9915	0.9889	0.9446	1.0000	0.9944	0.9997	1.0176
16	1.1081	1.0236	1.0673	0.9848	1.0000	1.0000	1.0000	1.0000
17	0.9956	1.0044	0.9986	0.9929	1.0000	1.0000	1.0001	1.0000
18	1.0071	1.0095	0.9639	0.9942	1.0000	1.0000	1.0000	0.9935
19	1.0020	1.0062	1.0622	0.9729	1.0000	1.0000	1.0000	0.9993
20	1.0176	1.0472	1.0370	1.0775	1.0000	1.0000	1.0000	1.0000
21	1.0730	0.9947	0.9932	1.0019	1.0000	1.0000	1.0000	0.9943
22	1.0028	0.9995	1.0109	0.9912	1.0000	1.0000	1.0000	1.0000
23	1.0200	0.9984	1.0073	1.0057	1.0000	1.0000	1.0000	1.0000
24	0.9956	0.9949	0.9976	0.9978	1.0000	1.0000	1.0000	1.0000
25	0.9997	0.9837	0.9982	0.9940	1.0000	1.0000	1.0000	1.0000
26	1.5375	1.2339	1.0342	1.1527	1.0000	1.0000	0.9941	1.0000
27	0.9957	0.9556	1.0824	1.0925	1.0000	1.0000	1.0000	1.0000
28	1.0157	1.3726	1.0363	1.1125	1.0000	1.0003	1.0000	1.0000
29	1.0628	1.0996	0.9937	0.9732	1.0000	0.9488	1.0000	1.0000
30	1.0252	0.9981	0.9969	0.9947	0.9728	1.0000	1.0000	1.0000
31	0.9830	0.9907	0.9913	0.9855	1.0000	1.0000	1.0000	1.0000
Average	1.0245	1.0189	1.0104	1.0124	0.9977	0.9967	0.9988	0.9978
Std. Dev.	0.0997	0.0840	0.0272	0.0477	0.0068	0.0098	0.0047	0.0095
Max.	1.5375	1.3726	1.0824	1.1527	1.0002	1.0019	1.0026	1.0176
Min.	0.9539	0.9445	0.9639	0.9446	0.9728	0.9488	0.9744	0.9677

Table S.23: MBPI decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

Bank	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
2	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	1.0966	1.0168	1.0168	1.3461
3	1.0857	0.9898	1.0488	1.2556	1.1352	0.9880	1.0770	1.1975	0.9557	1.0101	0.9746	1.0800
4	1.0488	0.9839	0.9899	1.0563	1.0540	1.0185	0.9875	1.0764	1.0140	1.0213	1.0056	1.0016
5	1.0278	1.1037	1.0084	1.0496	1.0000	1.0000	1.0000	1.0000	1.0324	1.0846	1.0148	1.0537
6	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	1.1273	0.9985	1.0688	1.0553
7	0.9572	1.0733	NaN	NaN	1.0000	1.0000	1.0000	1.0000	0.9641	1.0661	1.2484	1.0028
8	1.0193	1.1647	1.1260	1.0253	1.0460	1.1503	0.9475	1.0281	0.9759	1.0348	1.1834	1.0050
9	0.9789	0.9799	0.9969	0.9931	1.0124	1.0107	1.0106	0.9936	0.9666	0.9776	0.9921	1.0017
10	1.0279	0.9075	1.2648	1.0629	1.0000	1.0000	1.0000	1.0000	1.0294	0.9419	1.2374	1.0223
11	0.9732	1.0135	0.8310	1.0702	1.0268	1.0116	0.8736	1.0695	0.9545	1.0053	0.9637	1.0020
12	1.2236	1.0399	1.0411	1.0634	1.1834	1.0423	0.9686	0.9304	1.0737	1.0207	1.0681	1.1544
13	1.0576	0.8408	0.8883	1.0586	1.0741	0.8686	0.9064	1.0658	0.9766	0.9750	0.9764	1.0028
14	1.0006	0.9929	1.0818	1.5357	0.9914	0.9512	1.0001	1.0603	1.0096	1.0358	1.0709	1.5152
15	1.0430	0.9145	0.8722	1.2763	1.0980	0.9453	0.9184	1.2335	0.9577	0.9781	0.9626	1.0726
16	1.3026	0.8790	0.8957	0.9105	1.1801	0.8233	0.8242	0.9331	1.0071	1.0441	1.0214	0.9911
17	1.0007	0.9781	0.9448	1.0973	1.0468	0.9616	0.9680	1.0973	0.9597	1.0131	0.9773	1.0077
18	0.9584	1.0675	0.8927	1.2470	0.8758	1.0282	0.8908	1.1731	1.0858	1.0299	1.0335	1.0568
19	0.9391	0.8556	1.1197	0.9659	0.9424	0.8243	1.0517	0.9994	0.9948	1.0319	1.0056	0.9947
20	0.8837	1.0540	1.0529	1.3427	0.8799	0.9795	1.0111	1.2084	0.9893	1.0297	1.0182	1.0464
21	1.0125	NaN	NaN	0.9928	1.0000	1.0000	1.0000	1.0000	0.9726	1.1967	1.1115	1.0003
22	0.9559	0.9440	1.0110	1.0255	0.9609	0.9281	0.9992	1.0343	0.9923	1.0177	1.0014	1.0006
23	0.9634	0.9497	0.9633	0.9329	0.9368	0.9144	0.9449	0.9316	1.0103	1.0402	1.0124	0.9956
24	1.0343	0.9782	0.9908	0.9796	1.0858	0.9725	1.0221	0.9812	0.9563	1.0106	0.9717	1.0006
25	1.0348	0.9656	1.0093	1.1010	1.0455	0.9544	1.0334	1.0704	0.9900	1.0277	0.9849	1.0216
26	0.9168	0.7279	0.9942	0.9782	1.0000	1.0000	1.0000	1.0000	0.8509	0.6512	0.9941	0.9732
27	1.1107	0.8445	1.0594	1.1091	1.0629	0.8363	0.9881	1.0261	1.0488	1.0548	1.0279	0.9901
28	1.0444	1.2619	0.9895	1.3155	0.9916	0.9091	0.9486	1.1842	1.0386	1.0476	1.0190	1.0153
29	1.0950	1.4909	0.7145	1.2069	1.0875	1.2414	0.9985	1.0015	0.9786	1.1675	0.7190	1.2829
30	0.9628	1.0859	1.0155	1.0740	0.8796	1.2165	0.8722	1.0317	1.0708	0.8936	1.1614	1.0446
31	1.0486	0.9539	0.9578	1.0336	1.0884	0.9422	0.9781	1.0414	0.9781	1.0214	0.9873	1.0075
Average	1.0252	1.0015	0.9908	1.1022	1.0221	0.9845	0.9749	1.0442	1.0019	1.0148	1.0277	1.0581
Std. Dev.	0.0850	0.1434	0.1044	0.1419	0.0750	0.0923	0.0547	0.0790	0.0537	0.0870	0.0949	0.1176
Max.	1.3026	1.4909	1.2648	1.5357	1.1834	1.2414	1.0770	1.2335	1.1273	1.1967	1.2484	1.5152
Min.	0.8837	0.7279	0.7145	0.9105	0.8758	0.8233	0.8242	0.9304	0.8509	0.6512	0.7190	0.9732

Table S.24: MBPI decomposition along paths ABCD: Output orientation, ‘mean’ period viewpoint.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	NaN	NaN	NaN	NaN	0.9913	1.0047	1.0034	1.0000
3	1.0006	0.9918	0.9992	0.9706	1.0000	1.0000	1.0000	1.0002
4	0.9813	0.9459	0.9968	0.9790	1.0000	1.0000	1.0000	1.0008
5	0.9953	1.0212	0.9947	0.9937	1.0003	0.9965	0.9989	1.0024
6	NaN	NaN	NaN	NaN	1.0000	0.9999	1.0000	1.0000
7	0.9932	1.0057	NaN	NaN	0.9996	1.0010	1.0001	0.9994
8	0.9985	0.9794	1.0042	0.9927	1.0000	0.9991	1.0000	0.9995
9	1.0002	0.9918	0.9943	0.9978	1.0000	1.0000	1.0000	1.0000
10	0.9922	0.9637	1.0223	1.0390	1.0064	0.9998	0.9999	1.0007
11	0.9930	0.9970	0.9881	0.9987	1.0000	0.9996	0.9989	1.0000
12	0.9630	0.9774	1.0071	0.9900	1.0000	0.9999	0.9992	1.0000
13	1.0081	0.9929	1.0038	0.9905	1.0000	1.0000	1.0000	1.0000
14	0.9997	1.0077	1.0100	0.9558	1.0000	1.0000	1.0000	1.0001
15	0.9919	0.9892	0.9866	0.9671	1.0000	0.9999	1.0001	0.9975
16	1.0960	1.0226	1.0640	0.9845	1.0000	1.0000	1.0000	1.0000
17	0.9962	1.0041	0.9987	0.9924	1.0000	1.0000	1.0000	1.0000
18	1.0078	1.0081	0.9701	0.9990	1.0000	1.0000	0.9995	1.0068
19	1.0018	1.0059	1.0587	0.9716	1.0000	1.0000	1.0000	1.0000
20	1.0151	1.0450	1.0227	1.0619	1.0000	1.0000	1.0000	1.0000
21	1.0417	NaN	NaN	0.9925	0.9994	1.0000	1.0000	1.0000
22	1.0025	0.9995	1.0103	0.9909	1.0000	1.0000	1.0000	1.0000
23	1.0179	0.9985	1.0069	1.0059	1.0000	1.0000	1.0000	1.0000
24	0.9962	0.9952	0.9976	0.9977	1.0000	1.0000	1.0000	1.0000
25	0.9998	0.9845	0.9916	1.0069	1.0000	1.0000	1.0000	1.0000
26	1.0667	1.1243	1.0001	1.0013	1.0101	0.9942	1.0000	1.0039
27	0.9963	0.9573	1.0430	1.0917	1.0000	1.0000	1.0000	1.0000
28	1.0141	1.3226	1.0237	1.0942	1.0000	1.0019	1.0000	1.0000
29	1.0289	1.0277	0.9953	0.9394	1.0000	1.0009	1.0000	1.0000
30	1.0370	0.9989	1.0028	0.9893	0.9857	1.0000	0.9996	1.0074
31	0.9850	0.9912	0.9919	0.9851	1.0000	1.0000	1.0000	1.0000
Average	1.0079	1.0129	1.0071	0.9992	0.9998	0.9999	1.0000	1.0006
Std. Dev.	0.0260	0.0685	0.0210	0.0347	0.0037	0.0016	0.0007	0.0020
Max.	1.0960	1.3226	1.0640	1.0942	1.0101	1.0047	1.0034	1.0074
Min.	0.9630	0.9459	0.9701	0.9394	0.9857	0.9942	0.9989	0.9975

Table S.25: MBPI decomposition along paths E and F: Input orientation, base period viewpoint, common components.

Bank	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	1.4313	1.3067	1.3056	1.0149
2	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
3	1.0857	0.9900	1.0628	1.3145	1.1387	0.9873	1.0827	1.2124	0.9576	1.0129	0.9879	1.1671
4	1.0517	0.9998	0.9930	1.0685	1.0267	1.0571	0.9553	1.1672	1.0913	1.0687	1.0332	1.0027
5	1.0512	1.2218	1.0355	1.1025	1.0000	1.0000	1.0000	1.0000	1.0966	1.2845	1.1064	1.2940
6	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
7	0.9945	1.0802	NaN	NaN	1.0000	1.0000	1.0000	1.0000	1.0451	1.3088	NaN	NaN
8	1.0193	1.2038	1.2962	1.0097	1.0730	1.1437	0.9494	1.0349	0.9511	1.2472	NaN	0.9864
9	0.9656	0.9733	1.0080	0.9945	1.0452	1.0064	1.0157	0.9897	0.9527	0.9709	1.0023	1.0062
10	1.1149	1.0182	1.6620	1.1374	1.0000	1.0000	1.0000	1.0000	1.7585	1.1623	NaN	1.1948
11	0.9732	1.0099	0.8290	1.0701	1.0261	1.0107	0.8668	1.0766	0.9552	1.0034	0.9626	1.0009
12	1.2334	1.0439	1.0533	1.0886	1.4838	1.1371	0.9538	0.9296	1.2122	1.0909	1.1145	1.2349
13	1.0542	0.8417	0.9039	1.0601	1.0638	0.8253	0.8335	1.1185	0.9898	0.9788	1.0174	0.9939
14	1.0006	0.9929	1.0943	2.0702	0.9899	0.9400	0.9994	1.0753	1.0115	1.0460	1.1036	3.3612
15	1.0441	0.9284	0.8742	1.3736	1.1043	0.9403	0.9098	1.2038	0.9585	0.9907	0.9673	1.2252
16	1.3026	0.8790	0.8957	0.9105	1.1294	0.6381	0.5833	1.0457	1.0817	1.1678	1.1026	0.9189
17	1.0007	0.9781	0.9455	1.1017	1.0465	0.9539	0.9599	1.1158	0.9606	1.0178	0.9801	1.0108
18	0.9651	1.0675	0.8869	1.3163	0.8557	1.0253	0.8517	1.2369	1.1102	1.0439	1.0617	1.1215
19	0.9391	0.8556	1.1197	0.9627	0.8984	0.7245	0.9722	1.0785	1.0246	1.0852	1.0713	0.9728
20	0.8828	1.0529	1.0792	1.3611	0.8257	0.9943	1.0095	1.2760	1.0199	1.0552	1.0752	1.0826
21	1.0200	NaN	NaN	1.0183	1.0000	1.0000	1.0000	1.0000	1.0133	NaN	NaN	1.0340
22	0.9546	0.9565	1.0110	1.0255	0.9265	0.8814	0.9634	1.0742	1.0139	1.0648	1.0327	0.9904
23	0.9634	0.9497	0.9633	0.9329	0.8671	0.8356	0.8394	0.8762	1.0639	1.1046	1.0906	0.9528
24	1.0343	0.9782	0.9890	0.9796	1.0902	0.9708	1.0235	0.9801	0.9538	1.0112	0.9681	1.0007
25	1.0354	0.9656	0.9920	1.1168	1.0288	0.9344	1.0442	1.1069	1.0148	1.0499	0.9751	1.0430
26	0.9567	0.8522	1.0543	1.0825	1.0000	1.0000	1.0000	1.0000	3.3548	0.8335	1.0648	1.5604
27	1.1113	0.8445	1.0209	1.1091	0.9963	0.6174	0.9289	1.2474	1.2237	1.2683	1.1230	0.9537
28	1.0445	1.2239	0.9895	1.3155	0.8299	0.9853	0.9950	1.3134	1.2577	1.1771	1.0088	0.9413
29	1.1267	1.6530	0.7906	1.2197	1.0717	1.0665	0.9995	1.0005	0.9325	1.1017	0.8844	1.1081
30	1.0186	1.0516	1.0268	1.1286	0.9254	1.2266	0.8615	1.0368	1.1081	0.8595	1.1917	1.0982
31	1.0486	0.9539	0.9578	1.0336	1.0964	0.9260	0.9652	1.0643	0.9848	1.0336	0.9951	1.0059
Average	1.0355	1.0210	1.0205	1.1446	1.0174	0.9622	0.9537	1.0729	1.1562	1.0838	1.0490	1.1528
Std. Dev.	0.0853	0.1605	0.1623	0.2214	0.1177	0.1270	0.0905	0.1037	0.4486	0.1193	0.0845	0.4454
Max.	1.3026	1.6530	1.6620	2.0702	1.4838	1.2266	1.0827	1.3134	3.3548	1.3088	1.3056	3.3612
Min.	0.8828	0.8417	0.7906	0.9105	0.8257	0.6174	0.5833	0.8762	0.9325	0.8335	0.8844	0.9189

Table S.26: MBPI decomposition along path E: Input orientation, base period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	0.9990	0.9791	1.0000	1.0000	NaN	NaN	NaN	NaN
2	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	1.0000	1.0000	0.9989	0.9899	0.9956	0.9901	0.9948	0.9385
4	1.0000	0.9725	1.0000	0.9796	0.9386	0.9099	1.0061	0.9319
5	0.9916	0.9896	0.9791	0.9905	0.9667	0.9612	0.9559	0.8602
6	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
7	0.9837	0.9926	NaN	NaN	0.9673	0.8315	NaN	NaN
8	1.0000	1.0000	1.0123	1.0000	0.9988	0.8439	NaN	0.9892
9	1.0000	1.0000	1.0000	1.0000	0.9698	0.9961	0.9901	0.9987
10	1.0000	0.9475	1.0000	1.0000	0.6340	0.9246	NaN	0.9520
11	1.0000	1.0000	1.0000	1.0000	0.9930	0.9959	0.9936	0.9931
12	0.9787	0.9868	0.9966	1.0000	0.7007	0.8527	0.9942	0.9482
13	1.0000	1.0000	1.0000	1.0000	1.0012	1.0420	1.0659	0.9536
14	1.0000	1.0000	1.0000	1.0000	0.9993	1.0097	0.9922	0.5728
15	1.0000	1.0000	1.0000	1.0000	0.9865	0.9966	0.9933	0.9313
16	1.0000	1.0000	1.0000	1.0000	1.0662	1.1795	1.3926	0.9475
17	1.0000	1.0000	1.0000	1.0000	0.9955	1.0074	1.0051	0.9768
18	1.0000	1.0000	1.0000	1.0030	1.0158	0.9974	0.9808	0.9460
19	1.0000	1.0000	1.0000	1.0000	1.0203	1.0882	1.0751	0.9176
20	1.0000	1.0008	1.0000	1.0000	1.0482	1.0026	0.9942	0.9853
21	1.0000	NaN	NaN	0.9845	1.0066	NaN	NaN	1.0004
22	1.0000	1.0000	1.0000	1.0000	1.0163	1.0192	1.0162	0.9640
23	1.0000	1.0000	1.0000	1.0000	1.0443	1.0289	1.0522	1.1175
24	1.0000	1.0000	1.0000	1.0000	0.9946	0.9964	0.9982	0.9988
25	1.0000	1.0000	1.0000	0.9931	0.9918	0.9844	0.9742	0.9740
26	0.7661	0.9751	0.9993	0.9038	0.3723	1.0486	0.9908	0.7675
27	1.0000	1.0000	0.9935	1.0000	0.9115	1.0785	0.9850	0.9322
28	1.0000	0.9925	1.0000	1.0000	1.0006	1.0633	0.9859	1.0641
29	1.0000	1.0000	1.0000	1.0000	1.1274	1.4070	0.8944	1.1001
30	1.0000	1.0000	1.0000	0.9942	0.9934	0.9974	1.0002	0.9969
31	1.0000	1.0000	1.0000	1.0000	0.9712	0.9966	0.9972	0.9655
Average	0.9903	0.9942	0.9993	0.9942	0.9545	1.0092	1.0137	0.9527
Std. Dev.	0.0427	0.0120	0.0048	0.0182	0.1466	0.1070	0.0860	0.0991
Max.	1.0000	1.0008	1.0123	1.0030	1.1274	1.4070	1.3926	1.1175
Min.	0.7661	0.9475	0.9791	0.9038	0.3723	0.8315	0.8944	0.5728

Table S.27: MBPI decomposition along path F: Input orientation, base period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
2	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	1.0000	1.0000	1.0010	0.9596	0.9956	0.9901	0.9927	0.9681
4	1.0000	1.0000	1.0000	1.0077	0.9386	0.8850	1.0061	0.9060
5	1.0000	1.0092	1.0047	1.0000	0.9586	0.9425	0.9316	0.8521
6	NaN	NaN	NaN	NaN	0.9667	0.9612	0.9559	0.8602
7	1.0003	1.0000	NaN	NaN	NaN	NaN	NaN	NaN
8	1.0000	0.9841	NaN	1.0000	0.9988	0.8576	NaN	0.9892
9	1.0000	1.0000	1.0000	1.0000	0.9698	0.9961	0.9901	0.9987
10	1.0000	1.0172	NaN	1.0000	0.6340	0.8612	NaN	0.9520
11	1.0000	1.0000	1.0000	1.0000	0.9930	0.9959	0.9936	0.9931
12	1.0105	1.0019	1.0033	1.0000	0.6786	0.8399	0.9875	0.9482
13	1.0000	1.0000	1.0000	1.0000	1.0012	1.0420	1.0659	0.9536
14	1.0000	1.0000	1.0000	1.0000	0.9993	1.0097	0.9922	0.5728
15	1.0000	1.0000	1.0000	0.9706	0.9865	0.9966	0.9933	0.9596
16	1.0000	1.0000	1.0000	1.0000	1.0662	1.1795	1.3926	0.9475
17	1.0000	1.0000	1.0000	1.0000	0.9955	1.0074	1.0051	0.9768
18	1.0000	1.0000	1.0000	0.9867	1.0158	0.9974	0.9808	0.9616
19	1.0000	1.0000	1.0000	1.0000	1.0203	1.0882	1.0751	0.9176
20	1.0000	0.9992	1.0000	1.0000	1.0482	1.0043	0.9942	0.9853
21	1.0000	NaN	NaN	1.0000	1.0066	NaN	NaN	0.9848
22	1.0000	1.0000	1.0000	1.0000	1.0163	1.0192	1.0162	0.9640
23	1.0000	1.0000	1.0000	1.0000	1.0443	1.0289	1.0522	1.1175
24	1.0000	1.0000	1.0000	1.0000	0.9946	0.9964	0.9982	0.9988
25	1.0000	1.0000	1.0000	1.0059	0.9918	0.9844	0.9742	0.9615
26	1.0156	1.0310	1.0007	1.0534	0.2808	0.9917	0.9894	0.6585
27	1.0000	1.0000	1.0026	1.0000	0.9115	1.0785	0.9761	0.9322
28	1.0000	1.0081	1.0000	1.0000	1.0006	1.0469	0.9859	1.0641
29	1.0000	1.0000	1.0000	1.0000	1.1274	1.4070	0.8944	1.1001
30	1.0078	1.0000	1.0000	0.9726	0.9857	0.9974	1.0002	1.0191
31	1.0000	1.0000	1.0000	1.0000	0.9712	0.9966	0.9972	0.9655
Average	1.0012	1.0018	1.0005	0.9984	0.9499	1.0075	1.0096	0.9467
Std. Dev.	0.0036	0.0075	0.0012	0.0150	0.1611	0.1053	0.0860	0.1084
Max.	1.0156	1.0310	1.0047	1.0534	1.1274	1.4070	1.3926	1.1175
Min.	1.0000	0.9841	1.0000	0.9596	0.2808	0.8399	0.8944	0.5728

Table S.28: MBPI decomposition along paths G and H: Input orientation, comparison period viewpoint, common components.

Bank	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	NaN	NaN	1.0236	NaN	1.0000	1.0000	1.0000	1.0000	0.9057	0.9913	1.0000	0.8844
2	0.9716	0.9296	0.9782	1.0274	1.0000	1.0000	1.0000	1.0000	1.0552	0.9733	0.9968	1.0382
3	1.0857	0.9896	1.0350	1.1993	1.1387	0.9873	1.0827	1.2124	0.9588	1.0122	0.9634	1.0339
4	1.0459	0.9683	0.9868	1.0442	1.0267	1.0571	0.9553	1.1672	1.1210	1.0332	1.0314	0.9772
5	1.0049	0.9970	0.9819	0.9991	1.0000	1.0000	1.0000	1.0000	1.0101	1.0021	0.9824	0.9979
6	0.9661	NaN	0.9905	0.9972	1.0000	1.0000	1.0000	1.0000	0.9670	NaN	0.8479	0.9458
7	0.9213	1.0664	1.0227	0.9765	1.0000	1.0000	1.0000	1.0000	0.7369	1.0881	1.0311	0.9400
8	1.0193	1.1269	0.9781	1.0411	1.0730	1.1437	0.9494	1.0349	0.9514	1.0045	1.0310	1.0140
9	0.9923	0.9866	0.9860	0.9917	1.0452	1.0064	1.0157	0.9897	0.9744	0.9926	0.9742	1.0044
10	0.9476	0.8089	0.9626	0.9933	1.0000	1.0000	1.0000	1.0000	0.9491	0.7519	0.9337	0.8691
11	0.9732	1.0171	0.8329	1.0703	1.0261	1.0107	0.8668	1.0766	0.9572	1.0128	0.9607	1.0006
12	1.2138	1.0358	1.0290	1.0387	1.4838	1.1371	0.9538	0.9296	0.9833	0.9566	1.0800	1.1399
13	1.0610	0.8400	0.8731	1.0571	1.0638	0.8253	0.8335	1.1185	0.9977	0.9737	0.9636	0.9799
14	1.0006	0.9929	1.0693	1.1392	0.9899	0.9400	0.9994	1.0753	1.0111	1.0431	1.0727	1.0891
15	1.0419	0.9007	0.8703	1.1859	1.1043	0.9403	0.9098	1.2038	0.9598	0.9805	0.9570	0.9988
16	1.3026	0.8790	0.8957	0.9105	1.1294	0.6381	0.5833	1.0457	1.0509	1.1117	1.1591	0.9189
17	1.0007	0.9781	0.9440	1.0929	1.0465	0.9539	0.9599	1.1158	0.9620	1.0166	0.9754	0.9993
18	0.9517	1.0675	0.8985	1.1813	0.8557	1.0253	0.8517	1.2369	1.0920	1.0446	1.0720	1.0169
19	0.9391	0.8556	1.1197	0.9691	0.8984	0.7245	0.9722	1.0785	1.0166	1.0647	1.0526	0.9716
20	0.8846	1.0550	1.0272	1.3245	0.8257	0.9943	1.0095	1.2760	1.0009	1.0565	1.0256	1.0546
21	1.0051	1.0859	1.0100	0.9680	1.0000	1.0000	1.0000	1.0000	NaN	1.1043	1.0188	0.9511
22	0.9572	0.9317	1.0110	1.0255	0.9265	0.8814	0.9634	1.0742	1.0127	1.0154	1.0293	0.9877
23	0.9634	0.9497	0.9633	0.9329	0.8671	0.8356	0.8394	0.8762	1.0441	1.0966	1.0759	0.9638
24	1.0343	0.9782	0.9926	0.9795	1.0902	0.9708	1.0235	0.9801	0.9554	1.0118	0.9719	1.0005
25	1.0341	0.9656	1.0269	1.0855	1.0288	0.9344	1.0442	1.1069	1.0173	1.0532	1.0129	1.0164
26	0.8786	0.6217	0.9376	0.8840	1.0000	1.0000	1.0000	1.0000	0.7977	0.2507	0.8347	0.8444
27	1.1102	0.8445	1.0992	1.1091	0.9963	0.6174	0.9289	1.2474	1.3215	1.2284	1.1603	0.9300
28	1.0443	1.3011	0.9895	1.3155	0.8299	0.9853	0.9950	1.3134	1.2742	1.1807	1.0266	0.9492
29	1.0641	1.3446	0.6458	1.1943	1.0717	1.0665	0.9995	1.0005	0.9297	1.0746	0.8420	1.0945
30	0.9100	1.1215	1.0043	1.0220	0.9254	1.2266	0.8615	1.0368	1.1297	0.9625	1.1693	0.9938
31	1.0486	0.9539	0.9578	1.0336	1.0964	0.9260	0.9652	1.0643	0.9948	1.0343	0.9924	1.0038
Average	1.0125	0.9860	0.9724	1.0596	1.0174	0.9622	0.9537	1.0729	1.0046	1.0041	1.0079	0.9874
Std. Dev.	0.0872	0.1375	0.0863	0.1058	0.1177	0.1270	0.0905	0.1037	0.1104	0.1608	0.0797	0.0619
Max.	1.3026	1.3446	1.1197	1.3245	1.4838	1.2266	1.0827	1.3134	1.3215	1.2284	1.1693	1.1399
Min.	0.8786	0.6217	0.6458	0.8840	0.8257	0.6174	0.5833	0.8762	0.7369	0.2507	0.8347	0.8444

Table S.29: MBPI decomposition along path G: Input orientation, comparison period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	1.0000	1.0000	1.0000	1.0000	NaN	NaN	1.0236	NaN
2	1.0000	0.9264	0.9738	0.9977	0.9207	1.0310	1.0078	0.9918
3	1.0000	1.0000	1.0000	0.9859	0.9943	0.9903	0.9923	0.9704
4	1.0000	0.9488	1.0000	1.0003	0.9087	0.9343	1.0016	0.9153
5	0.9858	0.9962	0.9998	0.9956	1.0091	0.9988	0.9997	1.0056
6	0.9996	NaN	1.0000	0.9873	0.9994	NaN	1.1682	1.0680
7	0.9979	0.9954	1.0000	0.9911	1.2528	0.9846	0.9919	1.0481
8	1.0000	1.0000	1.0000	1.0000	0.9985	0.9809	0.9993	0.9921
9	1.0000	1.0000	1.0000	1.0000	0.9744	0.9877	0.9964	0.9977
10	0.9962	1.0000	0.9719	0.9845	1.0023	1.0758	1.0608	1.1609
11	1.0000	1.0000	1.0000	1.0000	0.9909	0.9937	1.0003	0.9935
12	1.0129	0.9907	1.0000	1.0000	0.8213	0.9611	0.9990	0.9803
13	1.0000	1.0000	1.0000	1.0000	0.9996	1.0453	1.0871	0.9646
14	1.0000	1.0000	1.0000	1.0000	0.9996	1.0125	0.9974	0.9728
15	1.0000	1.0000	1.0000	1.0000	0.9830	0.9770	0.9996	0.9863
16	1.0000	1.0000	1.0000	1.0000	1.0975	1.2391	1.3247	0.9475
17	1.0000	1.0000	1.0000	1.0000	0.9941	1.0086	1.0083	0.9802
18	1.0000	1.0000	1.0033	0.9847	1.0185	0.9967	0.9808	0.9537
19	1.0000	1.0000	1.0000	1.0000	1.0283	1.1091	1.0941	0.9249
20	1.0000	1.0009	0.9983	0.9951	1.0703	1.0034	0.9938	0.9892
21	NaN	1.0000	1.0000	0.9999	NaN	0.9834	0.9913	1.0178
22	1.0000	1.0000	1.0000	1.0000	1.0202	1.0411	1.0195	0.9667
23	1.0000	1.0000	1.0000	1.0000	1.0641	1.0364	1.0666	1.1047
24	1.0000	1.0000	1.0000	1.0000	0.9930	0.9959	0.9979	0.9989
25	1.0000	1.0000	0.9986	1.0000	0.9881	0.9813	0.9723	0.9648
26	1.0000	0.9810	1.0000	0.9938	1.1014	2.5280	1.1232	1.0535
27	1.0000	1.0000	1.0000	1.0000	0.8433	1.1135	1.0198	0.9559
28	1.0000	1.0013	1.0000	1.0000	0.9876	1.1170	0.9688	1.0552
29	1.0000	0.9799	1.0000	0.9967	1.0680	1.1973	0.7674	1.0942
30	0.9649	1.0000	1.0000	1.0000	0.9022	0.9499	0.9970	0.9918
31	1.0000	1.0000	1.0000	1.0000	0.9614	0.9960	1.0000	0.9675
Average	0.9986	0.9940	0.9982	0.9972	0.9997	1.0783	1.0210	1.0005
Std. Dev.	0.0072	0.0162	0.0067	0.0050	0.0794	0.2825	0.0838	0.0536
Max.	1.0129	1.0013	1.0033	1.0003	1.2528	2.5280	1.3247	1.1609
Min.	0.9649	0.9264	0.9719	0.9845	0.8213	0.9343	0.7674	0.9153]

Table S.30: MBPI decomposition along path H: Input orientation, comparison period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	1.0000	1.0000	1.0000	1.0000	NaN	NaN	1.0236	NaN
2	1.0000	1.0810	1.0319	1.0060	0.9207	0.8835	0.9510	0.9837
3	1.0000	1.0000	1.0000	0.9642	0.9943	0.9903	0.9923	0.9923
4	1.0000	1.0000	0.9960	0.9998	0.9087	0.8865	1.0056	0.9157
5	1.0148	1.0035	1.0002	1.0042	0.9803	0.9914	0.9993	0.9971
6	1.0004	NaN	1.0480	1.0267	0.9987	NaN	1.1147	1.0270
7	1.0127	1.0098	1.0000	1.0264	1.2345	0.9705	0.9919	1.0121
8	1.0000	0.9967	1.0000	1.0000	0.9985	0.9841	0.9993	0.9921
9	1.0000	1.0000	1.0000	1.0000	0.9744	0.9877	0.9964	0.9977
10	1.0036	1.0004	1.0593	1.1161	0.9950	1.0753	0.9732	1.0240
11	1.0000	1.0000	1.0000	1.0000	0.9909	0.9937	1.0003	0.9935
12	1.0060	1.0093	1.0000	1.0000	0.8270	0.9435	0.9990	0.9803
13	1.0000	1.0000	1.0000	1.0000	0.9996	1.0453	1.0871	0.9646
14	1.0000	1.0000	1.0000	1.0000	0.9996	1.0125	0.9974	0.9728
15	1.0000	1.0000	1.0000	0.9931	0.9830	0.9770	0.9996	0.9932
16	1.0000	1.0000	1.0000	1.0000	1.0975	1.2391	1.3247	0.9475
17	1.0000	1.0000	1.0000	1.0000	0.9941	1.0086	1.0083	0.9802
18	1.0000	1.0000	0.9969	1.0000	1.0185	0.9967	0.9871	0.9391
19	1.0000	1.0000	1.0000	1.0000	1.0283	1.1091	1.0941	0.9249
20	1.0000	0.9991	1.0016	1.0046	1.0703	1.0052	0.9905	0.9798
21	1.1288	1.0000	1.0000	1.0105	NaN	0.9834	0.9913	1.0072
22	1.0000	1.0000	1.0000	1.0000	1.0202	1.0411	1.0195	0.9667
23	1.0000	1.0000	1.0000	1.0000	1.0641	1.0364	1.0666	1.1047
24	1.0000	1.0000	1.0000	1.0000	0.9930	0.9959	0.9979	0.9989
25	1.0000	1.0000	1.0013	1.0000	0.9881	0.9813	0.9696	0.9648
26	1.0000	1.0681	1.0000	1.0057	1.1014	2.3219	1.1232	1.0410
27	1.0000	1.0000	1.0000	1.0247	0.8433	1.1135	1.0198	0.9329
28	1.0000	0.9985	1.0000	1.0000	0.9876	1.1202	0.9688	1.0552
29	1.0000	1.0275	1.0000	1.0031	1.0680	1.1418	0.7674	1.0872
30	1.0160	1.0000	0.9994	1.0000	0.8568	0.9499	0.9976	0.9918
31	1.0000	1.0000	1.0000	1.0000	0.9614	0.9960	1.0000	0.9675
Average	1.0059	1.0065	1.0043	1.0060	0.9965	1.0614	1.0147	0.9912
Std. Dev.	0.0229	0.0191	0.0142	0.0227	0.0794	0.2491	0.0821	0.0422
Max.	1.1288	1.0810	1.0593	1.1161	1.2345	2.3219	1.3247	1.1047
Min.	1.0000	0.9967	0.9960	0.9642	0.8270	0.8835	0.7674	0.9157

Table S.31: MBPI decomposition along paths EFGH: Input orientation, 'mean' period viewpoint

Bank	MB(07,06)	MB(08,07)	MB(09,08)	MB(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	1.1385	1.1381	1.1426	0.9474
2	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
3	1.0857	0.9898	1.0488	1.2556	1.1387	0.9873	1.0827	1.2124	0.9582	1.0125	0.9756	1.0985
4	1.0488	0.9839	0.9899	1.0563	1.0267	1.0571	0.9553	1.1672	1.1061	1.0508	1.0323	0.9899
5	1.0278	1.1037	1.0084	1.0496	1.0000	1.0000	1.0000	1.0000	1.0525	1.1345	1.0426	1.1363
6	NaN	NaN	NaN	NaN	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
7	0.9572	1.0733	NaN	NaN	1.0000	1.0000	1.0000	1.0000	0.8776	1.1933	NaN	NaN
8	1.0193	1.1647	1.1260	1.0253	1.0730	1.1437	0.9494	1.0349	0.9513	1.1193	NaN	1.0001
9	0.9789	0.9799	0.9969	0.9931	1.0452	1.0064	1.0157	0.9897	0.9635	0.9817	0.9881	1.0053
10	1.0279	0.9075	1.2648	1.0629	1.0000	1.0000	1.0000	1.0000	1.2919	0.9348	NaN	1.0190
11	0.9732	1.0135	0.8310	1.0702	1.0261	1.0107	0.8668	1.0766	0.9562	1.0081	0.9616	1.0007
12	1.2236	1.0399	1.0411	1.0634	1.4838	1.1371	0.9538	0.9296	1.0918	1.0215	1.0971	1.1864
13	1.0576	0.8408	0.8883	1.0586	1.0638	0.8253	0.8335	1.1185	0.9937	0.9763	0.9902	0.9869
14	1.0006	0.9929	1.0818	1.5357	0.9899	0.9400	0.9994	1.0753	1.0113	1.0446	1.0880	1.9133
15	1.0430	0.9145	0.8722	1.2763	1.1043	0.9403	0.9098	1.2038	0.9591	0.9856	0.9621	1.1062
16	1.3026	0.8790	0.8957	0.9105	1.1294	0.6381	0.5833	1.0457	1.0662	1.1394	1.1305	0.9189
17	1.0007	0.9781	0.9448	1.0973	1.0465	0.9539	0.9599	1.1158	0.9613	1.0172	0.9777	1.0050
18	0.9584	1.0675	0.8927	1.2470	0.8557	1.0253	0.8517	1.2369	1.1011	1.0443	1.0668	1.0679
19	0.9391	0.8556	1.1197	0.9659	0.8984	0.7245	0.9722	1.0785	1.0206	1.0749	1.0619	0.9722
20	0.8837	1.0540	1.0529	1.3427	0.8257	0.9943	1.0095	1.2760	1.0104	1.0559	1.0501	1.0685
21	1.0125	NaN	NaN	0.9928	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	0.9917
22	0.9559	0.9440	1.0110	1.0255	0.9265	0.8814	0.9634	1.0742	1.0133	1.0398	1.0310	0.9890
23	0.9634	0.9497	0.9633	0.9329	0.8671	0.8356	0.8394	0.8762	1.0539	1.1006	1.0832	0.9583
24	1.0343	0.9782	0.9908	0.9796	1.0902	0.9708	1.0235	0.9801	0.9546	1.0115	0.9700	1.0006
25	1.0348	0.9656	1.0093	1.1010	1.0288	0.9344	1.0442	1.1069	1.0161	1.0515	0.9938	1.0296
26	0.9168	0.7279	0.9942	0.9782	1.0000	1.0000	1.0000	1.0000	1.6359	0.4571	0.9428	1.1479
27	1.1107	0.8445	1.0594	1.1091	0.9963	0.6174	0.9289	1.2474	1.2716	1.2482	1.1415	0.9418
28	1.0444	1.2619	0.9895	1.3155	0.8299	0.9853	0.9950	1.3134	1.2659	1.1789	1.0176	0.9452
29	1.0950	1.4909	0.7145	1.2069	1.0717	1.0665	0.9995	1.0005	0.9311	1.0881	0.8630	1.1013
30	0.9628	1.0859	1.0155	1.0740	0.9254	1.2266	0.8615	1.0368	1.1189	0.9095	1.1804	1.0447
31	1.0486	0.9539	0.9578	1.0336	1.0964	0.9260	0.9652	1.0643	0.9898	1.0340	0.9938	1.0048
Average	1.0257	1.0015	0.9737	1.1022	1.0174	0.9622	0.9537	1.0729	1.0629	1.0376	1.0314	1.0563
Std. Dev.	0.0865	0.1434	0.0873	0.1419	0.1177	0.1270	0.0905	0.1037	0.1500	0.1350	0.0726	0.1776
Max.	1.3026	1.4909	1.1197	1.5357	1.4838	1.2266	1.0827	1.3134	1.6359	1.2482	1.1804	1.9133
Min.	0.8837	0.7279	0.7145	0.9105	0.8257	0.6174	0.5833	0.8762	0.8776	0.4571	0.8630	0.9189

Table S.32: MBPI decomposition along paths EFGH: Input orientation, 'mean' period viewpoint.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	0.9998	0.9947	1.0000	1.0000	NaN	NaN	NaN	NaN
2	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	1.0000	1.0000	1.0000	0.9748	0.9950	0.9902	0.9930	0.9671
4	1.0000	0.9801	0.9990	0.9968	0.9235	0.9037	1.0048	0.9172
5	0.9980	0.9996	0.9959	0.9976	0.9785	0.9732	0.9712	0.9259
6	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
7	0.9986	0.9994	0.0000	0.0000	1.0922	0.8999	0.0000	0.0000
8	1.0000	0.9952	0.0000	1.0000	0.9986	0.9142	0.0000	0.9906
9	1.0000	1.0000	1.0000	1.0000	0.9721	0.9919	0.9933	0.9982
10	0.9999	0.9909	0.0000	1.0238	0.7957	0.9797	0.0000	1.0188
11	1.0000	1.0000	1.0000	1.0000	0.9919	0.9948	0.9969	0.9933
12	1.0019	0.9972	1.0000	1.0000	0.7538	0.8977	0.9950	0.9641
13	1.0000	1.0000	1.0000	1.0000	1.0004	1.0436	1.0764	0.9591
14	1.0000	1.0000	1.0000	1.0000	0.9995	1.0111	0.9948	0.7465
15	1.0000	1.0000	1.0000	0.9908	0.9848	0.9868	0.9964	0.9673
16	1.0000	1.0000	1.0000	1.0000	1.0817	1.2089	1.3582	0.9475
17	1.0000	1.0000	1.0000	1.0000	0.9948	1.0080	1.0067	0.9785
18	1.0000	1.0000	1.0001	0.9936	1.0172	0.9971	0.9824	0.9501
19	1.0000	1.0000	1.0000	1.0000	1.0243	1.0986	1.0845	0.9212
20	1.0000	1.0000	1.0000	0.9999	1.0592	1.0039	0.9932	0.9849
21	NaN	NaN	NaN	0.9987	NaN	NaN	NaN	1.0025
22	1.0000	1.0000	1.0000	1.0000	1.0182	1.0301	1.0179	0.9653
23	1.0000	1.0000	1.0000	1.0000	1.0542	1.0327	1.0594	1.1110
24	1.0000	1.0000	1.0000	1.0000	0.9938	0.9962	0.9980	0.9988
25	1.0000	1.0000	1.0000	0.9998	0.9899	0.9828	0.9726	0.9663
26	0.9392	1.0131	1.0000	0.9877	0.5967	1.5718	1.0546	0.8628
27	1.0000	1.0000	0.9990	1.0061	0.8767	1.0959	1.0000	0.9383
28	1.0000	1.0001	1.0000	1.0000	0.9941	1.0863	0.9773	1.0596
29	1.0000	1.0017	1.0000	1.0000	1.0973	1.2826	0.8284	1.0954
30	0.9970	1.0000	0.9999	0.9916	0.9327	0.9734	0.9988	0.9998
31	1.0000	1.0000	1.0000	1.0000	0.9663	0.9963	0.9986	0.9665
Average	0.9977	0.9990	0.8926	0.9642	0.9697	1.0352	0.9019	0.9356
Std. Dev.	0.0113	0.0049	0.3092	0.1824	0.1056	0.1343	0.3289	0.1917
Max.	1.0019	1.0131	1.0001	1.0238	1.0973	1.5718	1.3582	1.1110
Min.	0.9392	0.9801	0.0000	0.0000	0.5967	0.8977	0.0000	0.0000

Price-weighted Productivity Indices (PROD)

Table S.33: PROD decomposition along paths A and B: Output orientation, base period viewpoint, common components.

Bank	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1901	1.0960	1.1369	0.9378	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
2	1.0542	0.8917	0.9580	1.4822	1.0000	1.0000	1.0000	1.0000	1.1673	1.0502	1.0358	1.7839
3	1.0683	0.9892	0.9811	1.9026	1.1352	0.9880	1.0770	1.1975	0.9558	1.0100	0.9846	1.1293
4	1.0318	1.0072	0.9817	1.0877	1.0540	1.0185	0.9875	1.0764	1.0142	1.0362	1.0093	1.0116
5	1.0132	1.0765	0.9920	1.0376	1.0000	1.0000	1.0000	1.0000	1.0590	1.1741	1.0474	1.1124
6	1.0926	1.0058	0.9895	1.0065	1.0000	1.0000	1.0000	1.0000	1.3066	1.0587	1.1959	1.1288
7	1.0459	1.1000	0.9973	1.0829	1.0000	1.0000	1.0000	1.0000	1.0051	1.0705	1.5139	1.0386
8	1.0532	1.1429	1.0067	1.0449	1.0460	1.1503	0.9475	1.0281	0.9768	1.0748	1.3448	0.9943
9	0.9956	1.0127	1.0220	1.0154	1.0124	1.0107	1.0106	0.9936	0.9536	0.9675	1.0048	1.0024
10	0.9707	0.9726	1.0581	1.0074	1.0000	1.0000	1.0000	1.0000	1.1130	1.0799	1.6347	1.1347
11	0.8874	1.1509	0.8984	1.0928	1.0268	1.0116	0.8736	1.0695	0.9535	1.0009	0.9647	1.0020
12	1.0512	1.0562	1.0242	1.0632	1.1834	1.0423	0.9686	0.9304	1.0722	1.0316	1.0829	1.1821
13	1.2062	0.8703	0.6819	0.8636	1.0741	0.8686	0.9064	1.0658	0.9745	0.9758	0.9937	1.0046
14	0.9211	0.9254	1.1724	2.6366	0.9914	0.9512	1.0001	1.0603	1.0097	1.0364	1.0838	2.1270
15	1.0177	0.9136	0.8653	1.7832	1.0980	0.9453	0.9184	1.2335	0.9577	0.9899	0.9666	1.1501
16	0.9571	0.9825	0.9260	0.9599	1.1801	0.8233	0.8242	0.9331	1.0182	1.0452	1.0246	0.9914
17	0.9943	1.0578	0.9257	1.3525	1.0468	0.9616	0.9680	1.0973	0.9592	1.0133	0.9780	1.0123
18	0.9181	0.9511	1.0102	1.2468	0.8758	1.0282	0.8908	1.1731	1.0925	1.0313	1.0207	1.0954
19	0.9273	0.8367	1.0504	1.4035	0.9424	0.8243	1.0517	0.9994	0.9950	1.0322	1.0089	0.9920
20	0.9025	0.8817	1.4066	1.2006	0.8799	0.9795	1.0111	1.2084	0.9907	1.0309	1.0582	1.0764
21	1.0015	1.1235	1.0146	1.0052	1.0000	1.0000	1.0000	1.0000	1.0099	1.3118	1.2150	1.0299
22	0.9894	0.9410	0.9787	1.0547	0.9609	0.9281	0.9992	1.0343	0.9913	1.0311	1.0020	1.0009
23	0.9521	0.9325	0.9488	0.9300	0.9368	0.9144	0.9449	0.9316	1.0124	1.0401	1.0128	0.9955
24	1.0209	1.0386	1.0111	1.0145	1.0858	0.9725	1.0221	0.9812	0.9557	1.0103	0.9699	1.0008
25	1.0573	1.0335	1.0300	1.0943	1.0455	0.9544	1.0334	1.0704	0.9905	1.0269	0.9743	1.0229
26	0.9638	0.7693	0.9282	1.0781	1.0000	1.0000	1.0000	1.0000	1.2671	0.8416	1.0838	1.2349
27	1.1141	0.9327	1.0293	1.0387	1.0629	0.8363	0.9881	1.0261	1.0487	1.0530	1.0280	0.9909
28	1.0822	0.9720	0.9546	1.1134	0.9916	0.9091	0.9486	1.1842	1.0403	1.0529	1.0315	1.0323
29	0.9272	1.0832	0.8118	1.1293	1.0875	1.2414	0.9985	1.0015	1.0403	1.3128	0.7942	1.3430
30	0.8874	1.1232	0.9952	1.1318	0.8796	1.2165	0.8722	1.0317	1.1052	0.8646	1.1678	1.0956
31	1.3131	1.0941	0.9198	1.0451	1.0884	0.9422	0.9781	1.0414	0.9761	1.0209	0.9868	1.0080
Average	1.0196	0.9989	0.9905	1.1885	1.0221	0.9845	0.9749	1.0442	1.0337	1.0425	1.0740	1.1241
Std. Dev.	0.0948	0.0944	0.1158	0.3480	0.0750	0.0923	0.0547	0.0790	0.0857	0.0927	0.1646	0.2411
Max.	1.3131	1.1509	1.4066	2.6366	1.1834	1.2414	1.0770	1.2335	1.3066	1.3128	1.6347	2.1270
Min.	0.8874	0.7693	0.6819	0.8636	0.8758	0.8233	0.8242	0.9304	0.9535	0.8416	0.7942	0.9909

Table S.34: PROD decomposition along path A: Output orientation, base period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	0.8623	0.8636	0.9377	0.9978	1.0474	0.9832	0.9864	0.8328
3	0.9910	1.0497	0.9937	1.0065	0.9935	0.9445	0.9310	1.3977
4	1.0076	1.0463	1.0106	1.0056	0.9579	0.9121	0.9746	0.9934
5	0.9732	0.9097	0.9653	0.9444	0.9831	1.0079	0.9811	0.9878
6	0.9619	0.9707	0.9676	0.9812	0.8693	0.9788	0.8551	0.9087
7	0.9710	1.0115	0.9367	0.9783	1.0716	1.0159	0.7033	1.0658
8	1.0050	1.0016	0.9455	0.9905	1.0257	0.9229	0.8355	1.0319
9	0.9900	1.0244	1.0220	1.0203	1.0417	1.0110	0.9847	0.9992
10	0.8774	0.9333	0.8810	0.9152	0.9940	0.9651	0.7347	0.9701
11	0.9785	1.0102	1.0717	1.0224	0.9263	1.1253	0.9947	0.9975
12	0.9309	0.9463	0.9484	0.9733	0.8900	1.0381	1.0297	0.9932
13	0.9863	1.0376	1.0177	1.0122	1.1684	0.9897	0.7439	0.7969
14	0.9802	0.9388	0.9546	1.0736	0.9388	0.9998	1.1331	1.0890
15	1.0028	1.0122	1.0123	1.0552	0.9651	0.9646	0.9630	1.1912
16	0.8851	1.0831	1.0653	0.9976	0.9000	1.0542	1.0294	1.0401
17	1.0547	1.0237	0.9922	1.0367	0.9390	1.0605	0.9856	1.1746
18	1.0241	0.9274	1.0130	0.9387	0.9368	0.9671	1.0968	1.0336
19	1.0433	1.0610	0.9886	1.0380	0.9479	0.9269	1.0014	1.3639
20	0.9875	1.0107	0.9164	0.7547	1.0484	0.8640	1.4346	1.2231
21	0.9656	0.9944	0.9366	0.9265	1.0271	0.8613	0.8916	1.0535
22	1.0079	1.0437	1.0304	1.0257	1.0306	0.9423	0.9486	0.9933
23	0.9955	0.9851	1.0048	0.9891	1.0085	0.9953	0.9866	1.0139
24	1.0239	1.0238	1.0143	1.0319	0.9608	1.0325	1.0056	1.0012
25	0.9929	1.0376	1.0435	1.0053	1.0284	1.0164	0.9803	0.9941
26	0.7855	0.8563	0.9685	0.8886	0.9684	1.0674	0.8844	0.9824
27	1.0042	1.0375	1.0204	1.0190	0.9953	1.0209	0.9931	1.0026
28	0.9851	1.0695	0.9804	0.9228	1.0650	0.9495	0.9952	0.9870
29	0.8120	0.9165	0.9596	0.8625	1.0094	0.7252	1.0669	0.9734
30	0.8963	1.0246	0.9710	0.9807	1.0184	1.0422	1.0063	1.0210
31	1.1965	1.0704	1.0460	1.0450	1.0330	1.0627	0.9111	0.9528
Average	0.9726	0.9974	0.9872	0.9813	0.9930	0.9816	0.9689	1.0355
Std. Dev.	0.0758	0.0593	0.0441	0.0646	0.0611	0.0756	0.1303	0.1253
Max.	1.1965	1.0831	1.0717	1.0736	1.1684	1.1253	1.4346	1.3977
Min.	0.7855	0.8563	0.8810	0.7547	0.8693	0.7252	0.7033	0.7969

Table S.35: PROD decomposition along path B: Output orientation, base period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	NaN	NaN	0.9998	0.7803	0.9996	0.9997
2	0.9420	0.8636	0.9377	1.0097	0.9587	0.9832	0.9864	0.8229
3	0.9910	1.0497	0.9937	0.9752	0.9935	0.9445	0.9310	1.4426
4	1.0076	1.0453	1.0104	1.0056	0.9579	0.9130	0.9748	0.9934
5	0.9735	0.9226	0.9803	0.9445	0.9828	0.9938	0.9661	0.9877
6	0.9640	0.9710	0.9701	0.9906	0.8674	0.9784	0.8529	0.9002
7	0.9780	1.0118	0.9380	1.0069	1.0640	1.0156	0.7024	1.0356
8	1.0050	0.9682	0.9363	0.9923	1.0257	0.9548	0.8437	1.0300
9	0.9896	1.0244	1.0220	1.0203	1.0420	1.0110	0.9847	0.9992
10	0.8781	0.9413	0.9055	0.9165	0.9933	0.9568	0.7149	0.9687
11	0.9785	1.0102	1.0795	1.0224	0.9263	1.1253	0.9874	0.9975
12	0.9309	0.9463	0.9514	0.9733	0.8900	1.0381	1.0265	0.9932
13	0.9863	1.0376	1.0179	1.0122	1.1684	0.9897	0.7437	0.7969
14	0.9802	0.9388	0.9529	0.9765	0.9388	0.9998	1.1350	1.1972
15	1.0028	1.0122	1.0240	1.0063	0.9651	0.9646	0.9520	1.2490
16	0.8851	1.0831	1.0653	0.9976	0.9000	1.0542	1.0294	1.0401
17	1.0547	1.0237	0.9922	1.0365	0.9390	1.0605	0.9856	1.1748
18	1.0241	0.9274	1.0157	0.9025	0.9368	0.9671	1.0938	1.0750
19	1.0433	1.0610	0.9886	1.0399	0.9479	0.9269	1.0014	1.3614
20	0.9875	1.0107	0.9164	0.7547	1.0484	0.8640	1.4346	1.2231
21	0.9680	0.9944	0.9375	0.9439	1.0245	0.8613	0.8907	1.0340
22	1.0079	1.0436	1.0304	1.0257	1.0306	0.9423	0.9486	0.9933
23	0.9955	0.9851	1.0048	0.9891	1.0085	0.9953	0.9866	1.0139
24	1.0239	1.0238	1.0143	1.0319	0.9608	1.0325	1.0056	1.0012
25	0.9929	1.0376	1.0435	1.0053	1.0284	1.0164	0.9803	0.9941
26	0.7855	0.9828	0.9685	0.8886	0.9684	0.9300	0.8844	0.9824
27	1.0042	1.0375	1.0204	1.0190	0.9953	1.0209	0.9931	1.0026
28	0.9851	1.0695	0.9804	0.9228	1.0650	0.9495	0.9952	0.9870
29	0.8120	0.9165	0.9667	0.8625	1.0094	0.7252	1.0590	0.9734
30	0.8295	1.0246	0.9710	0.9561	1.1005	1.0422	1.0063	1.0472
31	1.1965	1.0704	1.0460	1.0450	1.0330	1.0627	0.9111	0.9528
Average	0.9734	1.0012	0.9894	0.9758	0.9923	0.9773	0.9669	1.0423
Std. Dev.	0.0765	0.0527	0.0430	0.0619	0.0633	0.0738	0.1311	0.1359
Max.	1.1965	1.0831	1.0795	1.0450	1.1684	1.1253	1.4346	1.4426
Min.	0.7855	0.8636	0.9055	0.7547	0.8674	0.7252	0.7024	0.7969

Table S.36: PROD decomposition along paths C and D: Output orientation, comparison period viewpoint, common components.

Bank	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1902	1.1000	1.1194	0.9476	1.0000	1.0000	1.0000	1.0000	NaN	NaN	1.0236	NaN
2	1.0483	0.8990	0.9652	1.2225	1.0000	1.0000	1.0000	1.0000	1.0301	0.9845	0.9982	1.0157
3	1.0726	0.9903	0.9924	1.2756	1.1352	0.9880	1.0770	1.1975	0.9556	1.0103	0.9647	1.0329
4	1.0278	0.9993	0.9905	1.1068	1.0540	1.0185	0.9875	1.0764	1.0138	1.0065	1.0019	0.9916
5	1.0073	1.1253	0.9973	1.0372	1.0000	1.0000	1.0000	1.0000	1.0065	1.0019	0.9831	0.9981
6	1.0310	1.0055	0.9894	1.0053	1.0000	1.0000	1.0000	1.0000	0.9726	0.9418	0.9553	0.9866
7	1.0384	1.0996	0.9931	1.0861	1.0000	1.0000	1.0000	1.0000	0.9249	1.0617	1.0295	0.9682
8	1.0501	1.1055	1.0150	1.0425	1.0460	1.1503	0.9475	1.0281	0.9751	0.9963	1.0414	1.0158
9	0.9760	1.0151	1.0330	1.0161	1.0124	1.0107	1.0106	0.9936	0.9799	0.9877	0.9795	1.0009
10	0.9740	0.9601	1.0257	1.0173	1.0000	1.0000	1.0000	1.0000	0.9520	0.8215	0.9367	0.9211
11	0.8826	1.1149	0.9403	1.0973	1.0268	1.0116	0.8736	1.0695	0.9554	1.0097	0.9626	1.0020
12	1.0425	1.0580	1.0345	1.0707	1.1834	1.0423	0.9686	0.9304	1.0752	1.0100	1.0535	1.1274
13	1.1796	0.8680	0.5037	0.8356	1.0741	0.8686	0.9064	1.0658	0.9788	0.9743	0.9594	1.0010
14	0.9229	0.9254	1.1751	1.2418	0.9914	0.9512	1.0001	1.0603	1.0095	1.0351	1.0581	1.0794
15	1.0230	0.9136	0.8450	1.3196	1.0980	0.9453	0.9184	1.2335	0.9577	0.9664	0.9585	1.0002
16	0.7073	0.9544	0.9405	0.9511	1.1801	0.8233	0.8242	0.9331	0.9961	1.0430	1.0183	0.9908
17	0.9897	1.0447	0.9246	1.2196	1.0468	0.9616	0.9680	1.0973	0.9602	1.0128	0.9766	1.0032
18	0.9261	0.9544	0.9833	1.2073	0.8758	1.0282	0.8908	1.1731	1.0790	1.0284	1.0464	1.0195
19	0.9163	0.8094	1.0369	1.1844	0.9424	0.8243	1.0517	0.9994	0.9946	1.0316	1.0023	0.9974
20	0.9006	0.6829	1.1249	1.0734	0.8799	0.9795	1.0111	1.2084	0.9879	1.0286	0.9797	1.0173
21	1.0068	1.1105	1.0303	0.9939	1.0000	1.0000	1.0000	1.0000	0.9366	1.0918	1.0169	0.9716
22	0.9792	0.8584	0.9802	1.0682	0.9609	0.9281	0.9992	1.0343	0.9934	1.0045	1.0008	1.0004
23	0.9565	0.9418	0.9392	0.9221	0.9368	0.9144	0.9449	0.9316	1.0082	1.0403	1.0121	0.9958
24	1.0047	1.0292	1.0204	1.0199	1.0858	0.9725	1.0221	0.9812	0.9568	1.0110	0.9736	1.0005
25	1.0570	1.0381	1.0581	1.1200	1.0455	0.9544	1.0334	1.0704	0.9894	1.0286	0.9955	1.0203
26	0.9640	0.7731	0.9260	1.1025	1.0000	1.0000	1.0000	1.0000	0.5714	0.5038	0.9119	0.7669
27	1.1133	0.9179	1.0335	1.0541	1.0629	0.8363	0.9881	1.0261	1.0490	1.0567	1.0277	0.9894
28	1.0798	0.9262	0.9638	1.1322	0.9916	0.9091	0.9486	1.1842	1.0369	1.0424	1.0067	0.9985
29	0.9510	1.0947	0.7906	1.1342	1.0875	1.2414	0.9985	1.0015	0.9207	1.0382	0.6509	1.2254
30	0.8698	1.1356	1.0143	1.1142	0.8796	1.2165	0.8722	1.0317	1.0374	0.9236	1.1550	0.9959
31	1.3415	1.0702	0.8678	1.0633	1.0884	0.9422	0.9781	1.0414	0.9801	1.0219	0.9879	1.0071
Average	1.0074	0.9846	0.9759	1.0865	1.0221	0.9845	0.9749	1.0442	0.9762	0.9905	0.9893	1.0047
Std. Dev.	0.1098	0.1101	0.1150	0.1055	0.0750	0.0923	0.0547	0.0790	0.0847	0.1024	0.0757	0.0679
Max.	1.3415	1.1356	1.1751	1.3196	1.1834	1.2414	1.0770	1.2335	1.0790	1.0918	1.1550	1.2254
Min.	0.7073	0.6829	0.5037	0.8356	0.8758	0.8233	0.8242	0.9304	0.5714	0.5038	0.6509	0.7669

Table S.37: PROD decomposition along path C: Output orientation, comparison period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	1.0625	NaN	4.1881	1.0031	1.0292	1.0162
2	0.9640	0.8914	0.9647	1.0104	1.0556	1.0243	1.0022	1.1912
3	0.9923	1.0532	1.0525	1.0139	0.9964	0.9421	0.9075	1.0171
4	1.0048	1.0566	1.0152	1.0464	0.9573	0.9224	0.9861	0.9910
5	0.9843	1.1170	1.0094	1.0309	1.0168	1.0056	1.0049	1.0080
6	0.9980	0.9707	1.0033	0.9991	1.0622	1.0999	1.0323	1.0198
7	0.9817	1.0195	1.0559	0.9978	1.1437	1.0159	0.9136	1.1243
8	1.0066	1.0038	1.0587	1.0010	1.0228	0.9610	0.9716	0.9971
9	0.9912	1.0217	1.0420	1.0238	0.9926	0.9953	1.0015	0.9979
10	0.9959	0.9837	1.0152	0.9868	1.0273	1.1881	1.0787	1.1192
11	0.9743	1.0118	1.1359	1.0265	0.9234	1.0788	0.9844	0.9975
12	0.9264	0.9688	0.9654	1.0222	0.8844	1.0375	1.0502	0.9985
13	0.9936	1.0351	1.0221	1.0180	1.1292	0.9910	0.5667	0.7694
14	0.9793	0.9401	0.9774	1.0778	0.9416	0.9998	1.1361	1.0067
15	1.0003	1.0016	1.0398	1.0696	0.9726	0.9985	0.9232	1.0000
16	0.9077	1.0830	1.0959	1.0002	0.6629	1.0263	1.0226	1.0285
17	1.0494	1.0273	0.9921	1.0483	0.9383	1.0442	0.9859	1.0569
18	1.0380	0.9333	0.9956	0.9693	0.9441	0.9671	1.0595	1.0414
19	1.0450	1.0568	0.9820	1.0583	0.9355	0.9007	1.0017	1.1227
20	0.9920	1.0292	1.0038	0.8264	1.0444	0.6585	1.1313	1.0565
21	1.0297	1.0585	1.0541	0.9348	1.0438	0.9609	0.9611	1.0943
22	1.0091	1.0412	1.0388	1.0386	1.0167	0.8844	0.9436	0.9941
23	1.0025	0.9944	1.0054	0.9856	1.0102	0.9957	0.9768	1.0086
24	1.0223	1.0219	1.0236	1.0381	0.9460	1.0244	1.0018	1.0008
25	0.9941	1.0373	1.0377	1.0295	1.0279	1.0195	0.9911	0.9961
26	1.6663	1.2948	1.0292	1.4407	1.0124	1.1851	0.9867	0.9979
27	1.0030	1.0295	1.0267	1.0366	0.9955	1.0089	0.9913	1.0017
28	0.9924	1.0831	1.0132	0.9694	1.0583	0.9024	0.9961	0.9878
29	0.8880	0.9074	0.9631	0.9353	1.0696	0.9360	1.2632	0.9882
30	0.9546	1.0253	1.0161	1.0477	0.9985	0.9858	0.9908	1.0350
31	1.2227	1.0670	1.0672	1.0589	1.0285	1.0417	0.8416	0.9574
Average	1.0203	1.0255	1.0247	1.0247	1.0983	0.9937	0.9914	1.0201
Std. Dev.	0.1317	0.0711	0.0377	0.0912	0.5702	0.0915	0.1077	0.0683
Max.	1.6663	1.2948	1.1359	1.4407	4.1881	1.1881	1.2632	1.1912
Min.	0.8880	0.8914	0.9631	0.8264	0.6629	0.6585	0.5667	0.7694

Table S.38: PROD decomposition along path D: Output orientation, comparison period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	1.0625	NaN	NaN	NaN	1.0291	NaN
2	1.0090	0.9269	0.9851	1.0104	1.0086	0.9851	0.9816	1.1912
3	0.9923	1.0526	1.0525	0.9847	0.9964	0.9426	0.9075	1.0473
4	1.0048	1.0566	1.0151	1.0429	0.9573	0.9224	0.9862	0.9943
5	1.0140	1.1170	1.0096	1.0433	0.9869	1.0056	1.0048	0.9960
6	0.9991	1.0039	1.0123	1.0220	1.0610	1.0635	1.0232	0.9970
7	0.9818	1.0254	1.0507	1.0249	1.1436	1.0101	0.9181	1.0946
8	1.0066	0.9941	1.0515	1.0010	1.0228	0.9704	0.9783	0.9971
9	0.9909	1.0217	1.0420	1.0238	0.9929	0.9953	1.0015	0.9979
10	1.0279	1.0093	1.0676	1.0512	0.9953	1.1581	1.0257	1.0507
11	0.9743	1.0105	1.1366	1.0265	0.9234	1.0802	0.9837	0.9975
12	0.9264	0.9781	0.9654	1.0222	0.8844	1.0276	1.0502	0.9985
13	0.9936	1.0351	1.0223	1.0175	1.1292	0.9910	0.5666	0.7698
14	0.9793	0.9401	0.9774	1.1427	0.9416	0.9998	1.1361	0.9495
15	1.0003	1.0124	1.0407	1.0199	0.9726	0.9879	0.9224	1.0487
16	0.9077	1.0830	1.0959	1.0002	0.6629	1.0263	1.0226	1.0285
17	1.0494	1.0273	0.9920	1.0483	0.9383	1.0442	0.9860	1.0569
18	1.0380	0.9333	0.9956	0.9820	0.9441	0.9671	1.0595	1.0280
19	1.0450	1.0568	0.9820	1.0598	0.9355	0.9007	1.0017	1.1211
20	0.9920	1.0292	1.0038	0.8264	1.0444	0.6585	1.1313	1.0565
21	1.0297	1.0585	1.0541	0.9453	1.0438	0.9609	0.9611	1.0821
22	1.0090	1.0411	1.0388	1.0386	1.0167	0.8844	0.9436	0.9941
23	1.0025	0.9944	1.0054	0.9856	1.0102	0.9957	0.9768	1.0086
24	1.0223	1.0219	1.0236	1.0381	0.9460	1.0244	1.0018	1.0008
25	0.9941	1.0373	1.0377	1.0295	1.0279	1.0195	0.9911	0.9961
26	1.7348	1.2948	1.0415	1.4633	0.9725	1.1851	0.9750	0.9825
27	1.0030	1.0295	1.0267	1.0366	0.9955	1.0089	0.9913	1.0017
28	0.9924	1.0907	1.0132	0.9694	1.0583	0.8961	0.9961	0.9878
29	0.8880	1.0118	0.9631	0.9353	1.0696	0.8394	1.2632	0.9882
30	0.9608	1.0253	1.0146	1.0477	0.9921	0.9858	0.9923	1.0350
31	1.2227	1.0670	1.0672	1.0589	1.0285	1.0417	0.8416	0.9574
Average	1.0264	1.0329	1.0273	1.0299	0.9901	0.9859	0.9887	1.0152
Std. Dev.	0.1422	0.0643	0.0373	0.0956	0.0830	0.0931	0.1065	0.0670
Max.	1.7348	1.2948	1.1366	1.4633	1.1436	1.1851	1.2632	1.1912
Min.	0.8880	0.9269	0.9631	0.8264	0.6629	0.6585	0.5666	0.7698

Table S.39: PROD decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

Bank	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1902	1.0980	1.1281	0.9427	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
2	1.0512	0.8953	0.9616	1.3461	1.0000	1.0000	1.0000	1.0000	1.0966	1.0168	1.0168	1.3461
3	1.0705	0.9898	0.9867	1.5579	1.1352	0.9880	1.0770	1.1975	0.9557	1.0101	0.9746	1.0800
4	1.0298	1.0032	0.9861	1.0972	1.0540	1.0185	0.9875	1.0764	1.0140	1.0213	1.0056	1.0016
5	1.0102	1.1006	0.9946	1.0374	1.0000	1.0000	1.0000	1.0000	1.0324	1.0846	1.0148	1.0537
6	1.0614	1.0056	0.9895	1.0059	1.0000	1.0000	1.0000	1.0000	1.1273	0.9985	1.0688	1.0553
7	1.0421	1.0998	0.9952	1.0845	1.0000	1.0000	1.0000	1.0000	0.9641	1.0661	1.2484	1.0028
8	1.0516	1.1241	1.0109	1.0437	1.0460	1.1503	0.9475	1.0281	0.9759	1.0348	1.1834	1.0050
9	0.9857	1.0139	1.0275	1.0157	1.0124	1.0107	1.0106	0.9936	0.9666	0.9776	0.9921	1.0017
10	0.9723	0.9664	1.0418	1.0123	1.0000	1.0000	1.0000	1.0000	1.0294	0.9419	1.2374	1.0223
11	0.8850	1.1328	0.9191	1.0950	1.0268	1.0116	0.8736	1.0695	0.9545	1.0053	0.9637	1.0020
12	1.0469	1.0571	1.0294	1.0669	1.1834	1.0423	0.9686	0.9304	1.0737	1.0207	1.0681	1.1544
13	1.1929	0.8692	0.5861	0.8495	1.0741	0.8686	0.9064	1.0658	0.9766	0.9750	0.9764	1.0028
14	0.9220	0.9254	1.1738	1.8094	0.9914	0.9512	1.0001	1.0603	1.0096	1.0358	1.0709	1.5152
15	1.0203	0.9136	0.8551	1.5340	1.0980	0.9453	0.9184	1.2335	0.9577	0.9781	0.9626	1.0726
16	0.8228	0.9684	0.9332	0.9555	1.1801	0.8233	0.8242	0.9331	1.0071	1.0441	1.0214	0.9911
17	0.9920	1.0512	0.9252	1.2843	1.0468	0.9616	0.9680	1.0973	0.9597	1.0131	0.9773	1.0077
18	0.9221	0.9528	0.9966	1.2269	0.8758	1.0282	0.8908	1.1731	1.0858	1.0299	1.0335	1.0568
19	0.9218	0.8229	1.0436	1.2893	0.9424	0.8243	1.0517	0.9994	0.9948	1.0319	1.0056	0.9947
20	0.9016	0.7759	1.2579	1.1352	0.8799	0.9795	1.0111	1.2084	0.9893	1.0297	1.0182	1.0464
21	1.0041	1.1170	1.0224	0.9995	1.0000	1.0000	1.0000	1.0000	0.9726	1.1967	1.1115	1.0003
22	0.9843	0.8987	0.9795	1.0614	0.9609	0.9281	0.9992	1.0343	0.9923	1.0177	1.0014	1.0006
23	0.9543	0.9372	0.9440	0.9261	0.9368	0.9144	0.9449	0.9316	1.0103	1.0402	1.0124	0.9956
24	1.0128	1.0339	1.0157	1.0172	1.0858	0.9725	1.0221	0.9812	0.9563	1.0106	0.9717	1.0006
25	1.0572	1.0358	1.0439	1.1071	1.0455	0.9544	1.0334	1.0704	0.9900	1.0277	0.9849	1.0216
26	0.9639	0.7712	0.9271	1.0902	1.0000	1.0000	1.0000	1.0000	0.8509	0.6512	0.9941	0.9732
27	1.1137	0.9253	1.0314	1.0464	1.0629	0.8363	0.9881	1.0261	1.0488	1.0548	1.0279	0.9901
28	1.0810	0.9488	0.9592	1.1228	0.9916	0.9091	0.9486	1.1842	1.0386	1.0476	1.0190	1.0153
29	0.9390	1.0889	0.8012	1.1318	1.0875	1.2414	0.9985	1.0015	0.9786	1.1675	0.7190	1.2829
30	0.8786	1.1294	1.0047	1.1230	0.8796	1.2165	0.8722	1.0317	1.0708	0.8936	1.1614	1.0446
31	1.3272	1.0821	0.8934	1.0542	1.0884	0.9422	0.9781	1.0414	0.9781	1.0214	0.9873	1.0075
Average	1.0132	0.9914	0.9827	1.1313	1.0221	0.9845	0.9749	1.0442	1.0019	1.0148	1.0277	1.0581
Std. Dev.	0.1005	0.1010	0.1115	0.1980	0.0750	0.0923	0.0547	0.0790	0.0537	0.0870	0.0949	0.1176
Max	1.3272	1.1328	1.2579	1.8094	1.1834	1.2414	1.0770	1.2335	1.1273	1.1967	1.2484	1.5152
Min	0.8228	0.7712	0.5861	0.8495	0.8758	0.8233	0.8242	0.9304	0.8509	0.6512	0.7190	0.9732

Table S.40: PROD decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	0.9428	0.8860	0.9561	1.0070	1.0168	0.9938	0.9891	0.9930
3	0.9917	1.0513	1.0227	0.9949	0.9949	0.9434	0.9192	1.2106
4	1.0062	1.0512	1.0128	1.0249	0.9576	0.9175	0.9804	0.9930
5	0.9861	1.0116	0.9910	0.9897	0.9923	1.0032	0.9891	0.9948
6	0.9806	0.9789	0.9881	0.9981	0.9602	1.0288	0.9369	0.9550
7	0.9781	1.0171	0.9936	1.0018	1.1051	1.0144	0.8023	1.0796
8	1.0058	0.9918	0.9964	0.9962	1.0243	0.9521	0.9048	1.0139
9	0.9904	1.0230	1.0320	1.0221	1.0170	1.0031	0.9931	0.9985
10	0.9424	0.9664	0.9643	0.9658	1.0024	1.0617	0.8731	1.0253
11	0.9764	1.0107	1.1055	1.0245	0.9248	1.1022	0.9875	0.9975
12	0.9287	0.9598	0.9576	0.9974	0.8872	1.0353	1.0391	0.9958
13	0.9899	1.0363	1.0200	1.0150	1.1486	0.9903	0.6492	0.7831
14	0.9797	0.9395	0.9655	1.0660	0.9402	0.9998	1.1351	1.0566
15	1.0016	1.0096	1.0291	1.0374	0.9689	0.9788	0.9400	1.1177
16	0.8964	1.0830	1.0805	0.9989	0.7724	1.0402	1.0260	1.0343
17	1.0520	1.0255	0.9921	1.0424	0.9386	1.0523	0.9858	1.1142
18	1.0310	0.9303	1.0049	0.9476	0.9404	0.9671	1.0772	1.0444
19	1.0441	1.0589	0.9853	1.0489	0.9417	0.9137	1.0015	1.2364
20	0.9898	1.0199	0.9591	0.7897	1.0464	0.7543	1.2740	1.1368
21	0.9978	1.0260	0.9939	0.9376	1.0348	0.9097	0.9255	1.0657
22	1.0085	1.0424	1.0346	1.0321	1.0236	0.9129	0.9461	0.9937
23	0.9990	0.9897	1.0051	0.9873	1.0093	0.9955	0.9817	1.0112
24	1.0231	1.0228	1.0189	1.0350	0.9534	1.0284	1.0037	1.0010
25	0.9935	1.0374	1.0406	1.0174	1.0281	1.0179	0.9857	0.9951
26	1.1557	1.0899	1.0014	1.1359	0.9803	1.0866	0.9313	0.9863
27	1.0036	1.0335	1.0236	1.0277	0.9954	1.0149	0.9922	1.0022
28	0.9887	1.0782	0.9967	0.9458	1.0616	0.9240	0.9957	0.9874
29	0.8492	0.9371	0.9631	0.8981	1.0391	0.8018	1.1588	0.9808
30	0.9087	1.0250	0.9929	1.0073	1.0265	1.0136	0.9989	1.0345
31	1.2095	1.0687	1.0565	1.0519	1.0308	1.0521	0.8757	0.9551
Average	0.9950	1.0134	1.0061	1.0015	0.9921	0.9836	0.9766	1.0265
Std. Dev.	0.0658	0.0475	0.0348	0.0583	0.0673	0.0747	0.1068	0.0808
Max	1.2095	1.0899	1.1055	1.1359	1.1486	1.1022	1.2740	1.2364
Min	0.8492	0.8860	0.9561	0.7897	0.7724	0.7543	0.6492	0.7831

Table S.41: PROD decomposition along paths E and F: Input orientation, base period viewpoint, common components.

Bank	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1901	1.0960	1.1369	0.9378	1.0000	1.0000	1.0000	1.0000	1.4313	1.3067	1.3056	1.0149
2	1.0542	0.8917	0.9580	1.4822	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
3	1.0683	0.9892	0.9811	1.9026	1.1387	0.9873	1.0827	1.2124	0.9576	1.0129	0.9879	1.1671
4	1.0318	1.0072	0.9817	1.0877	1.0267	1.0571	0.9553	1.1672	1.0913	1.0687	1.0332	1.0027
5	1.0132	1.0765	0.9920	1.0376	1.0000	1.0000	1.0000	1.0000	1.0966	1.2845	1.1064	1.2940
6	1.0926	1.0058	0.9895	1.0065	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
7	1.0459	1.1000	0.9973	1.0829	1.0000	1.0000	1.0000	1.0000	1.0451	1.3088	NaN	NaN
8	1.0532	1.1429	1.0067	1.0449	1.0730	1.1437	0.9494	1.0349	0.9511	1.2472	NaN	0.9864
9	0.9956	1.0127	1.0220	1.0154	1.0452	1.0064	1.0157	0.9897	0.9527	0.9709	1.0023	1.0062
10	0.9707	0.9726	1.0581	1.0074	1.0000	1.0000	1.0000	1.0000	1.7585	1.1623	NaN	1.1948
11	0.8874	1.1509	0.8984	1.0928	1.0261	1.0107	0.8668	1.0766	0.9552	1.0034	0.9626	1.0009
12	1.0512	1.0562	1.0242	1.0632	1.4838	1.1371	0.9538	0.9296	1.2122	1.0909	1.1145	1.2349
13	1.2062	0.8703	0.6819	0.8636	1.0638	0.8253	0.8335	1.1185	0.9898	0.9788	1.0174	0.9939
14	0.9211	0.9254	1.1724	2.6366	0.9899	0.9400	0.9994	1.0753	1.0115	1.0460	1.1036	3.3612
15	1.0177	0.9136	0.8653	1.7832	1.1043	0.9403	0.9098	1.2038	0.9585	0.9907	0.9673	1.2252
16	0.9571	0.9825	0.9260	0.9599	1.1294	0.6381	0.5833	1.0457	1.0817	1.1678	1.1026	0.9189
17	0.9943	1.0578	0.9257	1.3525	1.0465	0.9539	0.9599	1.1158	0.9606	1.0178	0.9801	1.0108
18	0.9181	0.9511	1.0102	1.2468	0.8557	1.0253	0.8517	1.2369	1.1102	1.0439	1.0617	1.1215
19	0.9273	0.8367	1.0504	1.4035	0.8984	0.7245	0.9722	1.0785	1.0246	1.0852	1.0713	0.9728
20	0.9025	0.8817	1.4066	1.2006	0.8257	0.9943	1.0095	1.2760	1.0199	1.0552	1.0752	1.0826
21	1.0015	1.1235	1.0146	1.0052	1.0000	1.0000	1.0000	1.0000	1.0133	NaN	NaN	1.0340
22	0.9894	0.9410	0.9787	1.0547	0.9265	0.8814	0.9634	1.0742	1.0139	1.0648	1.0327	0.9904
23	0.9521	0.9325	0.9488	0.9300	0.8671	0.8356	0.8394	0.8762	1.0639	1.1046	1.0906	0.9528
24	1.0209	1.0386	1.0111	1.0145	1.0902	0.9708	1.0235	0.9801	0.9538	1.0112	0.9681	1.0007
25	1.0573	1.0335	1.0300	1.0943	1.0288	0.9344	1.0442	1.1069	1.0148	1.0499	0.9751	1.0430
26	0.9638	0.7693	0.9282	1.0781	1.0000	1.0000	1.0000	1.0000	3.3548	0.8335	1.0648	1.5604
27	1.1141	0.9327	1.0293	1.0387	0.9963	0.6174	0.9289	1.2474	1.2237	1.2683	1.1230	0.9537
28	1.0822	0.9720	0.9546	1.1134	0.8299	0.9853	0.9950	1.3134	1.2577	1.1771	1.0088	0.9413
29	0.9272	1.0832	0.8118	1.1293	1.0717	1.0665	0.9995	1.0005	0.9325	1.1017	0.8844	1.1081
30	0.8874	1.1232	0.9952	1.1318	0.9254	1.2266	0.8615	1.0368	1.1081	0.8595	1.1917	1.0982
31	1.3131	1.0941	0.9198	1.0451	1.0964	0.9260	0.9652	1.0643	0.9848	1.0336	0.9951	1.0059
Average	1.0196	0.9989	0.9905	1.1885	1.0174	0.9622	0.9537	1.0729	1.1562	1.0838	1.0490	1.1528
Std. Dev.	0.0948	0.0944	0.1158	0.3480	0.1177	0.1270	0.0905	0.1037	0.4486	0.1193	0.0845	0.4454
Max.	1.3131	1.1509	1.4066	2.6366	1.4838	1.2266	1.0827	1.3134	3.3548	1.3088	1.3056	3.3612
Min.	0.8874	0.7693	0.6819	0.8636	0.8257	0.6174	0.5833	0.8762	0.9325	0.8335	0.8844	0.9189

Table S.42: PROD decomposition along path E: Input orientation, base period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	0.9898	0.9826	0.9787	0.9341	0.8401	0.8536	0.8898	0.9892
2	0.9498	0.8513	0.9259	0.9856	NaN	NaN	NaN	NaN
3	0.9905	1.0579	0.9905	1.0091	0.9891	0.9351	0.9261	1.3324
4	1.0242	1.0737	1.0143	1.0039	0.8991	0.8303	0.9806	0.9258
5	0.9723	0.8652	0.9614	0.9438	0.9502	0.9687	0.9326	0.8497
6	0.9606	0.9755	0.9654	0.9722	NaN	NaN	NaN	NaN
7	0.9681	0.9952	0.9325	0.9797	1.0337	0.8446	NaN	NaN
8	1.0073	1.0125	0.9376	1.0028	1.0245	0.7913	NaN	1.0208
9	0.9896	1.0292	1.0297	1.0218	1.0104	1.0070	0.9750	0.9978
10	0.8762	0.9419	0.8789	0.9137	0.6300	0.8885	NaN	0.9228
11	0.9844	1.0127	1.0913	1.0238	0.9198	1.1206	0.9867	0.9906
12	0.9372	0.9618	0.9411	0.9833	0.6236	0.8852	1.0238	0.9418
13	0.9793	1.0448	1.0142	1.0224	1.1697	1.0312	0.7928	0.7599
14	0.9805	0.9323	0.9447	1.1196	0.9381	1.0096	1.1252	0.6516
15	1.0099	1.0202	1.0340	1.0656	0.9520	0.9613	0.9509	1.1346
16	0.8165	1.0603	1.0043	1.0137	0.9596	1.2435	1.4335	0.9855
17	1.0581	1.0197	0.9934	1.0451	0.9348	1.0684	0.9906	1.1475
18	1.0155	0.9213	1.0389	0.9137	0.9516	0.9647	1.0753	0.9836
19	1.0417	1.0551	0.9368	1.0699	0.9671	1.0086	1.0766	1.2504
20	0.9752	0.9701	0.9086	0.7212	1.0990	0.8663	1.4262	1.2051
21	0.9560	0.9536	0.9253	0.9311	1.0339	NaN	NaN	1.0441
22	1.0057	1.0441	1.0205	1.0353	1.0474	0.9603	0.9640	0.9575
23	0.9800	0.9865	0.9983	0.9832	1.0531	1.0241	1.0381	1.1330
24	1.0273	1.0284	1.0166	1.0343	0.9556	1.0288	1.0038	1.0000
25	0.9930	1.0531	1.0592	0.9789	1.0199	1.0005	0.9551	0.9682
26	0.7970	0.8732	0.9949	0.9163	0.3605	1.0570	0.8762	0.7540
27	1.0073	1.0819	1.0087	0.9341	0.9072	1.1010	0.9782	0.9347
28	0.9729	0.8301	0.9694	0.8576	1.0657	1.0096	0.9812	1.0502
29	0.8153	0.9036	0.9661	0.9511	1.1380	1.0204	0.9506	1.0709
30	0.8193	1.0248	0.9632	0.9785	1.0563	1.0396	1.0065	1.0159
31	1.2122	1.0794	1.0539	1.0613	1.0033	1.0591	0.9086	0.9199
Average	0.9714	0.9884	0.9838	0.9809	0.9494	0.9850	1.0099	0.9978
Std. Dev.	0.0800	0.0693	0.0479	0.0727	0.1616	0.0972	0.1408	0.1432
Max.	1.2122	1.0819	1.0913	1.1196	1.1697	1.2435	1.4335	1.3324
Min.	0.7970	0.8301	0.8789	0.7212	0.3605	0.7913	0.7928	0.6516

Table S.43: PROD decomposition along path F: Input orientation, base period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	0.9907	1.0036	0.9787	0.9341	0.8393	0.8358	0.8898	0.9892
2	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	0.9905	1.0579	0.9925	0.9783	0.9891	0.9351	0.9242	1.3744
4	1.0242	1.1040	1.0143	1.0327	0.8991	0.8075	0.9806	0.9000
5	0.9806	0.8823	0.9864	0.9529	0.9422	0.9499	0.9089	0.8416
6	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
7	0.9844	1.0026	NaN	NaN	1.0166	0.8383	NaN	NaN
8	1.0073	0.9963	NaN	1.0028	1.0245	0.8041	NaN	1.0208
9	0.9896	1.0292	1.0297	1.0218	1.0104	1.0070	0.9750	0.9978
10	0.8762	1.0112	NaN	0.9137	0.6300	0.8276	NaN	0.9228
11	0.9844	1.0127	1.0913	1.0238	0.9198	1.1206	0.9867	0.9906
12	0.9677	0.9765	0.9475	0.9833	0.6040	0.8719	1.0169	0.9418
13	0.9793	1.0448	1.0142	1.0224	1.1697	1.0312	0.7928	0.7599
14	0.9805	0.9323	0.9447	1.1196	0.9381	1.0096	1.1252	0.6516
15	1.0099	1.0202	1.0340	1.0343	0.9520	0.9613	0.9509	1.1690
16	0.8165	1.0603	1.0043	1.0137	0.9596	1.2435	1.4335	0.9855
17	1.0581	1.0197	0.9934	1.0451	0.9348	1.0684	0.9906	1.1475
18	1.0155	0.9213	1.0389	0.8989	0.9516	0.9647	1.0753	0.9999
19	1.0417	1.0551	0.9368	1.0699	0.9671	1.0086	1.0766	1.2504
20	0.9752	0.9685	0.9086	0.7212	1.0990	0.8677	1.4262	1.2051
21	0.9560	NaN	NaN	0.9458	1.0339	NaN	NaN	1.0279
22	1.0057	1.0441	1.0205	1.0353	1.0474	0.9603	0.9640	0.9575
23	0.9800	0.9865	0.9983	0.9832	1.0531	1.0241	1.0381	1.1330
24	1.0273	1.0284	1.0166	1.0343	0.9556	1.0288	1.0038	1.0000
25	0.9930	1.0531	1.0592	0.9915	1.0199	1.0005	0.9551	0.9559
26	1.0566	0.9233	0.9963	1.0679	0.2719	0.9996	0.8750	0.6469
27	1.0073	1.0819	1.0179	0.9341	0.9072	1.1010	0.9693	0.9347
28	0.9729	0.8431	0.9694	0.8576	1.0657	0.9940	0.9812	1.0502
29	0.8153	0.9036	0.9661	0.9511	1.1380	1.0204	0.9506	1.0709
30	0.8257	1.0248	0.9632	0.9572	1.0481	1.0396	1.0065	1.0384
31	1.2122	1.0794	1.0539	1.0613	1.0033	1.0591	0.9086	0.9199
Average	0.9836	1.0024	0.9991	0.9853	0.9445	0.9779	1.0082	0.9958
Std. Dev.	0.0760	0.0627	0.0410	0.0767	0.1740	0.1011	0.1415	0.1565
Max.	1.2122	1.1040	1.0913	1.1196	1.1697	1.2435	1.4335	1.3744
Min.	0.8153	0.8431	0.9086	0.7212	0.2719	0.8041	0.7928	0.6469

Table S.44: PROD decomposition along paths G and H: Input orientation, comparison period viewpoint, common components.

Bank	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1902	1.1000	1.1194	0.9476	1.0000	1.0000	1.0000	1.0000	0.9057	0.9913	1.0000	0.8844
2	1.0483	0.8990	0.9652	1.2225	1.0000	1.0000	1.0000	1.0000	1.0552	0.9733	0.9968	1.0382
3	1.0726	0.9903	0.9924	1.2756	1.1387	0.9873	1.0827	1.2124	0.9588	1.0122	0.9634	1.0339
4	1.0278	0.9993	0.9905	1.1068	1.0267	1.0571	0.9553	1.1672	1.1210	1.0332	1.0314	0.9772
5	1.0073	1.1253	0.9973	1.0372	1.0000	1.0000	1.0000	1.0000	1.0101	1.0021	0.9824	0.9979
6	1.0310	1.0055	0.9894	1.0053	1.0000	1.0000	1.0000	1.0000	0.9670	NaN	0.8479	0.9458
7	1.0384	1.0996	0.9931	1.0861	1.0000	1.0000	1.0000	1.0000	0.7369	1.0881	1.0311	0.9400
8	1.0501	1.1055	1.0150	1.0425	1.0730	1.1437	0.9494	1.0349	0.9514	1.0045	1.0310	1.0140
9	0.9760	1.0151	1.0330	1.0161	1.0452	1.0064	1.0157	0.9897	0.9744	0.9926	0.9742	1.0044
10	0.9740	0.9601	1.0257	1.0173	1.0000	1.0000	1.0000	1.0000	0.9491	0.7519	0.9337	0.8691
11	0.8826	1.1149	0.9403	1.0973	1.0261	1.0107	0.8668	1.0766	0.9572	1.0128	0.9607	1.0006
12	1.0425	1.0580	1.0345	1.0707	1.4838	1.1371	0.9538	0.9296	0.9833	0.9566	1.0800	1.1399
13	1.1796	0.8680	0.5037	0.8356	1.0638	0.8253	0.8335	1.1185	0.9977	0.9737	0.9636	0.9799
14	0.9229	0.9254	1.1751	1.2418	0.9899	0.9400	0.9994	1.0753	1.0111	1.0431	1.0727	1.0891
15	1.0230	0.9136	0.8450	1.3196	1.1043	0.9403	0.9098	1.2038	0.9598	0.9805	0.9570	0.9988
16	0.7073	0.9544	0.9405	0.9511	1.1294	0.6381	0.5833	1.0457	1.0509	1.1117	1.1591	0.9189
17	0.9897	1.0447	0.9246	1.2196	1.0465	0.9539	0.9599	1.1158	0.9620	1.0166	0.9754	0.9993
18	0.9261	0.9544	0.9833	1.2073	0.8557	1.0253	0.8517	1.2369	1.0920	1.0446	1.0720	1.0169
19	0.9163	0.8094	1.0369	1.1844	0.8984	0.7245	0.9722	1.0785	1.0166	1.0647	1.0526	0.9716
20	0.9006	0.6829	1.1249	1.0734	0.8257	0.9943	1.0095	1.2760	1.0009	1.0565	1.0256	1.0546
21	1.0068	1.1105	1.0303	0.9939	1.0000	1.0000	1.0000	1.0000	NaN	1.1043	1.0188	0.9511
22	0.9792	0.8584	0.9802	1.0682	0.9265	0.8814	0.9634	1.0742	1.0127	1.0154	1.0293	0.9877
23	0.9565	0.9418	0.9392	0.9221	0.8671	0.8356	0.8394	0.8762	1.0441	1.0966	1.0759	0.9638
24	1.0047	1.0292	1.0204	1.0199	1.0902	0.9708	1.0235	0.9801	0.9554	1.0118	0.9719	1.0005
25	1.0570	1.0381	1.0581	1.1200	1.0288	0.9344	1.0442	1.1069	1.0173	1.0532	1.0129	1.0164
26	0.9640	0.7731	0.9260	1.1025	1.0000	1.0000	1.0000	1.0000	0.7977	0.2507	0.8347	0.8444
27	1.1133	0.9179	1.0335	1.0541	0.9963	0.6174	0.9289	1.2474	1.3215	1.2284	1.1603	0.9300
28	1.0798	0.9262	0.9638	1.1322	0.8299	0.9853	0.9950	1.3134	1.2742	1.1807	1.0266	0.9492
29	0.9510	1.0947	0.7906	1.1342	1.0717	1.0665	0.9995	1.0005	0.9297	1.0746	0.8420	1.0945
30	0.8698	1.1356	1.0143	1.1142	0.9254	1.2266	0.8615	1.0368	1.1297	0.9625	1.1693	0.9938
31	1.3415	1.0702	0.8678	1.0633	1.0964	0.9260	0.9652	1.0643	0.9948	1.0343	0.9924	1.0038
Average	1.0074	0.9846	0.9759	1.0865	1.0174	0.9622	0.9537	1.0729	1.0046	1.0041	1.0079	0.9874
Std. Dev.	0.1098	0.1101	0.1150	0.1055	0.1177	0.1270	0.0905	0.1037	0.1104	0.1608	0.0797	0.0619
Max.	1.3415	1.1356	1.1751	1.3196	1.4838	1.2266	1.0827	1.3134	1.3215	1.2284	1.1693	1.1399
Min.	0.7073	0.6829	0.5037	0.8356	0.8257	0.6174	0.5833	0.8762	0.7369	0.2507	0.8347	0.8444

Table S.45: PROD decomposition along path G: Input orientation, comparison period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	1.0517	1.0042	1.0625	1.0427	1.2496	1.1050	1.0535	1.0276
2	1.0441	0.9003	0.9753	0.9966	0.9514	1.0258	0.9929	1.1815
3	0.9916	1.0619	1.0565	1.0162	0.9907	0.9332	0.9005	1.0014
4	1.0265	1.0615	1.0179	1.0698	0.8699	0.8619	0.9877	0.9071
5	0.9870	1.1181	1.0105	1.0364	1.0103	1.0043	1.0046	1.0028
6	1.0049	NaN	0.9718	0.9871	1.0610	NaN	1.2007	1.0768
7	0.9835	1.0153	1.0604	0.9936	1.4328	0.9953	0.9083	1.1629
8	1.0072	1.0129	1.0629	1.0042	1.0212	0.9500	0.9756	0.9892
9	0.9908	1.0337	1.0462	1.0267	0.9673	0.9831	0.9979	0.9956
10	1.0256	1.0115	0.9838	0.9311	1.0006	1.2624	1.1167	1.2573
11	0.9821	1.0147	1.1470	1.0278	0.9149	1.0735	0.9845	0.9911
12	0.9837	0.9801	0.9573	1.0323	0.7264	0.9925	1.0492	0.9788
13	0.9846	1.0428	1.0181	1.0271	1.1288	1.0359	0.6160	0.7424
14	0.9796	0.9323	0.9673	1.1109	0.9413	1.0123	1.1332	0.9545
15	1.0095	1.0210	1.0523	1.0797	0.9561	0.9706	0.9223	1.0165
16	0.8192	1.0580	1.0268	1.0156	0.7275	1.2716	1.3546	0.9745
17	1.0540	1.0228	0.9934	1.0559	0.9327	1.0532	0.9941	1.0359
18	1.0307	0.9245	1.0363	0.9726	0.9615	0.9639	1.0392	0.9868
19	1.0429	1.0503	0.9245	1.0893	0.9620	0.9989	1.0959	1.0377
20	0.9749	0.9837	0.9664	0.7632	1.1178	0.6608	1.1243	1.0451
21	NaN	1.0642	1.0614	0.9434	NaN	0.9449	0.9528	1.1077
22	1.0062	1.0417	1.0276	1.0478	1.0373	0.9207	0.9620	0.9609
23	0.9828	0.9960	0.9982	0.9800	1.0750	1.0320	1.0419	1.1142
24	1.0268	1.0271	1.0261	1.0404	0.9394	1.0202	0.9997	0.9997
25	0.9945	1.0545	1.0381	1.0358	1.0156	1.0004	0.9637	0.9611
26	1.1283	1.0295	1.0070	1.2616	1.0711	2.9958	1.1017	1.0350
27	1.0073	1.0773	0.9485	0.9489	0.8395	1.1235	1.0109	0.9576
28	0.9770	0.7956	0.9778	0.8713	1.0451	1.0007	0.9650	1.0422
29	0.8355	0.9017	0.9692	0.9579	1.1423	1.0593	0.9693	1.0812
30	0.9043	1.0273	1.0178	1.0534	0.9200	0.9364	0.9894	1.0265
31	1.2438	1.0770	1.0765	1.0745	0.9888	1.0375	0.8416	0.9263
Average	1.0027	1.0114	1.0157	1.0159	0.9999	1.0742	1.0080	1.0186
Std. Dev.	0.0732	0.0642	0.0457	0.0807	0.1347	0.3724	0.1197	0.0899
Max.	1.2438	1.1181	1.1470	1.2616	1.4328	2.9958	1.3546	1.2573
Min.	0.8192	0.7956	0.9245	0.7632	0.7264	0.6608	0.6160	0.7424

Table S.46: PROD decomposition along path H: Input orientation, comparison period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	1.0517	1.0042	1.0625	1.0427	1.2496	1.1050	1.0535	1.0276
2	1.0441	1.0505	1.0335	1.0048	0.9514	0.8792	0.9369	1.1719
3	0.9916	1.0619	1.0565	0.9938	0.9907	0.9332	0.9005	1.0240
4	1.0265	1.1187	1.0139	1.0693	0.8699	0.8178	0.9916	0.9075
5	1.0161	1.1264	1.0110	1.0453	0.9814	0.9969	1.0041	0.9943
6	1.0056	1.0093	1.0185	1.0265	1.0602	NaN	1.1457	1.0355
7	0.9980	1.0301	1.0604	1.0289	1.4119	0.9811	0.9083	1.1230
8	1.0072	1.0096	1.0629	1.0042	1.0212	0.9531	0.9756	0.9892
9	0.9908	1.0337	1.0462	1.0267	0.9673	0.9831	0.9979	0.9956
10	1.0331	1.0119	1.0723	1.0556	0.9933	1.2619	1.0245	1.1090
11	0.9821	1.0147	1.1470	1.0278	0.9149	1.0735	0.9845	0.9911
12	0.9770	0.9984	0.9573	1.0323	0.7314	0.9742	1.0492	0.9788
13	0.9846	1.0428	1.0181	1.0271	1.1288	1.0359	0.6160	0.7424
14	0.9796	0.9323	0.9673	1.1109	0.9413	1.0123	1.1332	0.9545
15	1.0095	1.0210	1.0523	1.0722	0.9561	0.9706	0.9223	1.0236
16	0.8192	1.0580	1.0268	1.0156	0.7275	1.2716	1.3546	0.9745
17	1.0540	1.0228	0.9934	1.0559	0.9327	1.0532	0.9941	1.0359
18	1.0307	0.9245	1.0298	0.9877	0.9615	0.9639	1.0458	0.9718
19	1.0429	1.0503	0.9245	1.0893	0.9620	0.9989	1.0959	1.0377
20	0.9749	0.9819	0.9696	0.7705	1.1178	0.6620	1.1206	1.0352
21	1.0833	1.0642	1.0614	0.9534	NaN	0.9449	0.9528	1.0961
22	1.0062	1.0417	1.0276	1.0478	1.0373	0.9207	0.9620	0.9609
23	0.9828	0.9960	0.9982	0.9800	1.0750	1.0320	1.0419	1.1142
24	1.0268	1.0271	1.0261	1.0404	0.9394	1.0202	0.9997	0.9997
25	0.9945	1.0545	1.0410	1.0358	1.0156	1.0004	0.9610	0.9611
26	1.1283	1.1209	1.0070	1.2767	1.0711	2.7516	1.1017	1.0227
27	1.0073	1.0773	0.9485	0.9723	0.8395	1.1235	1.0109	0.9345
28	0.9770	0.7934	0.9778	0.8713	1.0451	1.0035	0.9650	1.0422
29	0.8355	0.9455	0.9692	0.9640	1.1423	1.0102	0.9693	1.0744
30	0.9522	1.0273	1.0172	1.0534	0.8738	0.9364	0.9899	1.0265
31	1.2438	1.0770	1.0765	1.0745	0.9888	1.0375	0.8416	0.9263
Average	1.0083	1.0235	1.0218	1.0244	0.9966	1.0569	1.0016	1.0091
Std. Dev.	0.0717	0.0632	0.0448	0.0785	0.1334	0.3335	0.1161	0.0770
Max.	1.2438	1.1264	1.1470	1.2767	1.4119	2.7516	1.3546	1.1719
Min.	0.8192	0.7934	0.9245	0.7705	0.7275	0.6620	0.6160	0.7424

Table S.47: PROD decomposition along paths EFGH: Input orientation, 'mean' period viewpoint

Bank	PROD(07,06)	PROD(08,07)	PROD(09,08)	PROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1902	1.0980	1.1281	0.9427	1.0000	1.0000	1.0000	1.0000	1.1385	1.1381	1.1426	0.9474
2	1.0512	0.8953	0.9616	1.3461	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
3	1.0705	0.9898	0.9867	1.5579	1.1387	0.9873	1.0827	1.2124	0.9582	1.0125	0.9756	1.0985
4	1.0298	1.0032	0.9861	1.0972	1.0267	1.0571	0.9553	1.1672	1.1061	1.0508	1.0323	0.9899
5	1.0102	1.1006	0.9946	1.0374	1.0000	1.0000	1.0000	1.0000	1.0525	1.1345	1.0426	1.1363
6	1.0614	1.0056	0.9895	1.0059	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
7	1.0421	1.0998	0.9952	1.0845	1.0000	1.0000	1.0000	1.0000	0.8776	1.1933	NaN	NaN
8	1.0516	1.1241	1.0109	1.0437	1.0730	1.1437	0.9494	1.0349	0.9513	1.1193	NaN	1.0001
9	0.9857	1.0139	1.0275	1.0157	1.0452	1.0064	1.0157	0.9897	0.9635	0.9817	0.9881	1.0053
10	0.9723	0.9664	1.0418	1.0123	1.0000	1.0000	1.0000	1.0000	1.2919	0.9348	NaN	1.0190
11	0.8850	1.1328	0.9191	1.0950	1.0261	1.0107	0.8668	1.0766	0.9562	1.0081	0.9616	1.0007
12	1.0469	1.0571	1.0294	1.0669	1.4838	1.1371	0.9538	0.9296	1.0918	1.0215	1.0971	1.1864
13	1.1929	0.8692	0.5861	0.8495	1.0638	0.8253	0.8335	1.1185	0.9937	0.9763	0.9902	0.9869
14	0.9220	0.9254	1.1738	1.8094	0.9899	0.9400	0.9994	1.0753	1.0113	1.0446	1.0880	1.9133
15	1.0203	0.9136	0.8551	1.5340	1.1043	0.9403	0.9098	1.2038	0.9591	0.9856	0.9621	1.1062
16	0.8228	0.9684	0.9332	0.9555	1.1294	0.6381	0.5833	1.0457	1.0662	1.1394	1.1305	0.9189
17	0.9920	1.0512	0.9252	1.2843	1.0465	0.9539	0.9599	1.1158	0.9613	1.0172	0.9777	1.0050
18	0.9221	0.9528	0.9966	1.2269	0.8557	1.0253	0.8517	1.2369	1.1011	1.0443	1.0668	1.0679
19	0.9218	0.8229	1.0436	1.2893	0.8984	0.7245	0.9722	1.0785	1.0206	1.0749	1.0619	0.9722
20	0.9016	0.7759	1.2579	1.1352	0.8257	0.9943	1.0095	1.2760	1.0104	1.0559	1.0501	1.0685
21	1.0041	1.1170	1.0224	0.9995	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	0.9917
22	0.9843	0.8987	0.9795	1.0614	0.9265	0.8814	0.9634	1.0742	1.0133	1.0398	1.0310	0.9890
23	0.9543	0.9372	0.9440	0.9261	0.8671	0.8356	0.8394	0.8762	1.0539	1.1006	1.0832	0.9583
24	1.0128	1.0339	1.0157	1.0172	1.0902	0.9708	1.0235	0.9801	0.9546	1.0115	0.9700	1.0006
25	1.0572	1.0358	1.0439	1.1071	1.0288	0.9344	1.0442	1.1069	1.0161	1.0515	0.9938	1.0296
26	0.9639	0.7712	0.9271	1.0902	1.0000	1.0000	1.0000	1.0000	1.6359	0.4571	0.9428	1.1479
27	1.1137	0.9253	1.0314	1.0464	0.9963	0.6174	0.9289	1.2474	1.2716	1.2482	1.1415	0.9418
28	1.0810	0.9488	0.9592	1.1228	0.8299	0.9853	0.9950	1.3134	1.2659	1.1789	1.0176	0.9452
29	0.9390	1.0889	0.8012	1.1318	1.0717	1.0665	0.9995	1.0005	0.9311	1.0881	0.8630	1.1013
30	0.8786	1.1294	1.0047	1.1230	0.9254	1.2266	0.8615	1.0368	1.1189	0.9095	1.1804	1.0447
31	1.3272	1.0821	0.8934	1.0542	1.0964	0.9260	0.9652	1.0643	0.9898	1.0340	0.9938	1.0048
Average	1.0132	0.9914	0.9827	1.1313	1.0174	0.9622	0.9537	1.0729	1.0629	1.0376	1.0314	1.0563
Std. Dev.	0.1005	0.1010	0.1115	0.1980	0.1177	0.1270	0.0905	0.1037	0.1500	0.1350	0.0726	0.1776
Max.	1.3272	1.1328	1.2579	1.8094	1.4838	1.2266	1.0827	1.3134	1.6359	1.2482	1.1804	1.9133
Min.	0.8228	0.7712	0.5861	0.8495	0.8257	0.6174	0.5833	0.8762	0.8776	0.4571	0.8630	0.9189

Table S.48: PROD decomposition along paths EFGH: Input orientation, 'mean' period viewpoint.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	1.0205	0.9986	1.0198	0.9869	1.0244	0.9661	0.9682	1.0082
2	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	0.9911	1.0599	1.0235	0.9993	0.9899	0.9341	0.9127	1.1706
4	1.0254	1.0892	1.0151	1.0436	0.8844	0.8291	0.9851	0.9100
5	0.9889	0.9902	0.9921	0.9935	0.9706	0.9797	0.9616	0.9189
6	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
7	0.9834	1.0107	NaN	NaN	1.2075	0.9119	NaN	NaN
8	1.0073	1.0078	NaN	1.0035	1.0229	0.8712	NaN	1.0049
9	0.9902	1.0315	1.0379	1.0243	0.9886	0.9950	0.9864	0.9967
10	0.9497	0.9937	NaN	0.9517	0.7925	1.0403	NaN	1.0438
11	0.9832	1.0137	1.1188	1.0258	0.9173	1.0968	0.9856	0.9908
12	0.9662	0.9791	0.9508	1.0075	0.6688	0.9294	1.0346	0.9601
13	0.9820	1.0438	1.0162	1.0247	1.1491	1.0336	0.6988	0.7511
14	0.9801	0.9323	0.9559	1.1152	0.9397	1.0109	1.1292	0.7887
15	1.0097	1.0206	1.0431	1.0628	0.9541	0.9660	0.9365	1.0839
16	0.8178	1.0591	1.0155	1.0147	0.8355	1.2575	1.3935	0.9800
17	1.0561	1.0213	0.9934	1.0505	0.9337	1.0608	0.9923	1.0903
18	1.0231	0.9229	1.0360	0.9425	0.9566	0.9643	1.0588	0.9855
19	1.0423	1.0527	0.9306	1.0796	0.9645	1.0038	1.0862	1.1391
20	0.9750	0.9760	0.9378	0.7437	1.1083	0.7572	1.2653	1.1196
21	NaN	NaN	NaN	0.9434	NaN	NaN	NaN	1.0684
22	1.0059	1.0429	1.0240	1.0416	1.0423	0.9403	0.9630	0.9592
23	0.9814	0.9912	0.9982	0.9816	1.0640	1.0280	1.0400	1.1235
24	1.0271	1.0278	1.0214	1.0373	0.9475	1.0245	1.0017	0.9999
25	0.9937	1.0538	1.0493	1.0102	1.0178	1.0005	0.9587	0.9615
26	1.0175	0.9821	1.0013	1.1205	0.5791	1.7179	0.9821	0.8477
27	1.0073	1.0796	0.9804	0.9472	0.8727	1.1122	0.9922	0.9403
28	0.9750	0.8153	0.9736	0.8644	1.0553	1.0019	0.9731	1.0462
29	0.8253	0.9134	0.9676	0.9560	1.1402	1.0274	0.9599	1.0743
30	0.8736	1.0260	0.9900	1.0097	0.9713	0.9867	0.9980	1.0268
31	1.2279	1.0782	1.0652	1.0679	0.9960	1.0482	0.8745	0.9231
Average	0.9902	1.0076	1.0063	1.0018	0.9641	1.0177	1.0055	0.9969
Std. Dev.	0.0716	0.0575	0.0412	0.0734	0.1312	0.1621	0.1236	0.0977
Max.	1.2279	1.0892	1.1188	1.1205	1.2075	1.7179	1.3935	1.1706
Min.	0.8178	0.8153	0.9306	0.7437	0.5791	0.7572	0.6988	0.7511

Share-weighted Productivity Indices (GPROD)

Table S.49: GPROD decomposition along paths A and B: Output orientation, base period viewpoint, common components.

Bank	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1902	1.0960	1.1378	0.9387	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
2	1.0437	0.8938	0.9578	1.2391	1.0000	1.0000	1.0000	1.0000	1.1673	1.0502	1.0358	1.7839
3	1.0684	0.9867	0.9746	1.4651	1.1352	0.9880	1.0770	1.1975	0.9558	1.0100	0.9846	1.1293
4	1.0284	0.9907	0.9794	1.0914	1.0540	1.0185	0.9875	1.0764	1.0142	1.0362	1.0093	1.0116
5	1.0113	1.1006	0.9919	1.0375	1.0000	1.0000	1.0000	1.0000	1.0590	1.1741	1.0474	1.1124
6	1.0575	1.0038	0.9841	1.0047	1.0000	1.0000	1.0000	1.0000	1.3066	1.0587	1.1959	1.1288
7	1.0441	1.1002	0.9804	1.0814	1.0000	1.0000	1.0000	1.0000	1.0051	1.0705	1.5139	1.0386
8	1.0523	1.1226	1.0020	1.0419	1.0460	1.1503	0.9475	1.0281	0.9768	1.0748	1.3448	0.9943
9	0.9891	1.0133	1.0206	1.0149	1.0124	1.0107	1.0106	0.9936	0.9536	0.9675	1.0048	1.0024
10	0.9721	0.9671	1.0318	1.0056	1.0000	1.0000	1.0000	1.0000	1.1130	1.0799	1.6347	1.1347
11	0.8833	1.1394	0.9032	1.0932	1.0268	1.0116	0.8736	1.0695	0.9535	1.0009	0.9647	1.0020
12	1.0510	1.0567	1.0251	1.0644	1.1834	1.0423	0.9685	0.9304	1.0722	1.0316	1.0829	1.1820
13	1.1919	0.8715	0.6038	0.8388	1.0741	0.8686	0.9064	1.0658	0.9745	0.9758	0.9937	1.0046
14	0.9147	0.9265	1.1390	1.6733	0.9914	0.9512	1.0001	1.0603	1.0097	1.0364	1.0838	2.1270
15	1.0168	0.9120	0.8587	1.4977	1.0980	0.9453	0.9184	1.2335	0.9577	0.9899	0.9666	1.1501
16	0.8669	0.9796	0.9264	0.9533	1.1801	0.8233	0.8242	0.9331	1.0182	1.0452	1.0246	0.9914
17	0.9888	1.0530	0.9263	1.2400	1.0468	0.9616	0.9680	1.0973	0.9592	1.0133	0.9780	1.0123
18	0.9054	0.9464	0.9781	1.1734	0.8758	1.0282	0.8908	1.1731	1.0925	1.0313	1.0207	1.0954
19	0.9198	0.8168	1.0530	1.1919	0.9424	0.8243	1.0517	0.9994	0.9950	1.0322	1.0089	0.9920
20	0.8987	0.8036	1.2311	1.1454	0.8799	0.9795	1.0111	1.2084	0.9907	1.0309	1.0582	1.0764
21	1.0023	1.1207	1.0137	1.0095	1.0000	1.0000	1.0000	1.0000	1.0099	1.3118	1.2150	1.0299
22	0.9870	0.9111	0.9585	1.0550	0.9609	0.9281	0.9992	1.0343	0.9913	1.0311	1.0020	1.0009
23	0.9523	0.9354	0.9450	0.9281	0.9368	0.9144	0.9449	0.9316	1.0124	1.0401	1.0128	0.9955
24	1.0145	1.0363	1.0112	1.0149	1.0858	0.9725	1.0221	0.9812	0.9557	1.0103	0.9699	1.0008
25	1.0529	1.0339	1.0307	1.0975	1.0455	0.9544	1.0334	1.0704	0.9905	1.0269	0.9743	1.0229
26	0.9712	0.7667	0.9239	1.0835	1.0000	1.0000	1.0000	1.0000	1.2671	0.8416	1.0838	1.2349
27	1.1137	0.9319	1.0282	1.0401	1.0629	0.8363	0.9881	1.0261	1.0487	1.0530	1.0280	0.9909
28	1.0558	0.9615	0.9551	1.1152	0.9916	0.9091	0.9486	1.1842	1.0403	1.0529	1.0315	1.0323
29	0.9352	1.0777	0.8062	1.1300	1.0875	1.2414	0.9985	1.0015	1.0403	1.3128	0.7942	1.3430
30	0.8792	1.1218	0.9950	1.1206	0.8796	1.2165	0.8722	1.0317	1.1052	0.8646	1.1678	1.0956
31	1.3475	1.0926	0.8955	1.0415	1.0884	0.9422	0.9781	1.0414	0.9761	1.0209	0.9868	1.0080
Average	1.0131	0.9926	0.9764	1.1106	1.0221	0.9845	0.9749	1.0442	1.0337	1.0425	1.0740	1.1241
Std. Dev.	0.1005	0.0992	0.1053	0.1682	0.0750	0.0923	0.0547	0.0790	0.0857	0.0927	0.1646	0.2411
Max.	1.3475	1.1394	1.2311	1.6733	1.1834	1.2414	1.0770	1.2335	1.3066	1.3128	1.6347	2.1270
Min.	0.8669	0.7667	0.6038	0.8388	0.8758	0.8233	0.8242	0.9304	0.9535	0.8416	0.7942	0.9909

Table S.50: GPROD decomposition along path A: Output orientation, base period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	0.8624	0.8665	0.9384	0.9978	1.0367	0.9822	0.9855	0.6961
3	0.9912	1.0523	0.9972	1.0074	0.9934	0.9397	0.9217	1.0754
4	1.0081	1.0572	1.0108	1.0092	0.9543	0.8879	0.9722	0.9932
5	0.9736	0.9304	0.9654	0.9452	0.9808	1.0075	0.9810	0.9867
6	0.9620	0.9707	0.9678	0.9815	0.8414	0.9767	0.8503	0.9068
7	0.9713	1.0117	0.9379	0.9785	1.0695	1.0159	0.6904	1.0641
8	1.0051	1.0017	0.9468	0.9908	1.0247	0.9064	0.8305	1.0288
9	0.9901	1.0253	1.0228	1.0205	1.0347	1.0107	0.9825	0.9985
10	0.8787	0.9336	0.8834	0.9163	0.9939	0.9593	0.7145	0.9672
11	0.9792	1.0111	1.0779	1.0228	0.9214	1.1130	0.9943	0.9975
12	0.9330	0.9469	0.9496	0.9744	0.8878	1.0378	1.0293	0.9932
13	0.9869	1.0391	1.0179	1.0125	1.1538	0.9896	0.6587	0.7738
14	0.9803	0.9399	0.9561	1.0767	0.9322	0.9998	1.0990	0.6891
15	1.0032	1.0126	1.0132	1.0569	0.9639	0.9626	0.9547	0.9989
16	0.8940	1.0899	1.0702	0.9977	0.8070	1.0445	1.0251	1.0329
17	1.0559	1.0247	0.9933	1.0380	0.9327	1.0546	0.9851	1.0755
18	1.0245	0.9291	1.0136	0.9409	0.9236	0.9606	1.0613	0.9704
19	1.0445	1.0627	0.9910	1.0399	0.9391	0.9034	1.0014	1.1561
20	0.9881	1.0165	0.9179	0.7643	1.0433	0.7829	1.2535	1.1521
21	0.9667	0.9960	0.9376	0.9317	1.0267	0.8578	0.8898	1.0520
22	1.0083	1.0452	1.0315	1.0265	1.0277	0.9110	0.9281	0.9928
23	0.9969	0.9885	1.0051	0.9894	1.0072	0.9950	0.9824	1.0114
24	1.0245	1.0245	1.0146	1.0324	0.9543	1.0294	1.0055	1.0012
25	0.9933	1.0393	1.0458	1.0087	1.0237	1.0151	0.9787	0.9937
26	0.7917	0.8613	0.9685	0.8933	0.9682	1.0577	0.8802	0.9823
27	1.0042	1.0418	1.0208	1.0206	0.9949	1.0158	0.9916	1.0025
28	0.9859	1.0838	0.9810	0.9255	1.0381	0.9269	0.9951	0.9857
29	0.8193	0.9184	0.9598	0.8632	1.0091	0.7200	1.0593	0.9733
30	0.8998	1.0251	0.9711	0.9820	1.0051	1.0404	1.0059	1.0096
31	1.2298	1.0749	1.0478	1.0471	1.0314	1.0568	0.8855	0.9475
Average	0.9751	1.0007	0.9885	0.9831	0.9840	0.9720	0.9531	0.9836
Std. Dev.	0.0778	0.0595	0.0446	0.0634	0.0677	0.0826	0.1180	0.1025
Max.	1.2298	1.0899	1.0779	1.0767	1.1538	1.1130	1.2535	1.1561
Min.	0.7917	0.8613	0.8834	0.7643	0.8070	0.7200	0.6587	0.6891

Table S.51: GPROD decomposition along path B: Output orientation, base period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	NaN	NaN	0.9996	0.7803	0.9996	0.9997
2	0.9423	0.8665	0.9384	1.0097	0.9489	0.9822	0.9855	0.6879
3	0.9912	1.0523	0.9972	0.9760	0.9934	0.9397	0.9217	1.1100
4	1.0081	1.0562	1.0106	1.0092	0.9543	0.8887	0.9724	0.9932
5	0.9739	0.9435	0.9804	0.9453	0.9805	0.9934	0.9659	0.9866
6	0.9641	0.9711	0.9702	0.9909	0.8395	0.9763	0.8482	0.8983
7	0.9782	1.0121	0.9392	1.0071	1.0619	1.0156	0.6895	1.0339
8	1.0051	0.9684	0.9376	0.9926	1.0247	0.9377	0.8387	1.0268
9	0.9898	1.0253	1.0228	1.0205	1.0351	1.0107	0.9825	0.9985
10	0.8794	0.9416	0.9079	0.9176	0.9932	0.9511	0.6952	0.9658
11	0.9792	1.0111	1.0858	1.0228	0.9214	1.1130	0.9870	0.9975
12	0.9330	0.9469	0.9526	0.9744	0.8878	1.0378	1.0261	0.9932
13	0.9869	1.0391	1.0181	1.0125	1.1538	0.9896	0.6585	0.7738
14	0.9803	0.9399	0.9545	0.9794	0.9322	0.9998	1.1009	0.7576
15	1.0032	1.0126	1.0249	1.0079	0.9639	0.9626	0.9438	1.0474
16	0.8940	1.0899	1.0702	0.9977	0.8070	1.0445	1.0251	1.0329
17	1.0559	1.0247	0.9933	1.0378	0.9327	1.0546	0.9851	1.0757
18	1.0245	0.9291	1.0163	0.9047	0.9236	0.9606	1.0585	1.0093
19	1.0445	1.0627	0.9910	1.0418	0.9391	0.9034	1.0014	1.1540
20	0.9881	1.0165	0.9179	0.7643	1.0433	0.7829	1.2535	1.1521
21	0.9691	0.9960	0.9386	0.9493	1.0242	0.8578	0.8889	1.0325
22	1.0083	1.0452	1.0315	1.0265	1.0277	0.9110	0.9281	0.9928
23	0.9969	0.9885	1.0051	0.9894	1.0072	0.9950	0.9824	1.0114
24	1.0245	1.0245	1.0146	1.0324	0.9543	1.0294	1.0055	1.0012
25	0.9933	1.0393	1.0458	1.0087	1.0237	1.0151	0.9787	0.9937
26	0.7917	0.9885	0.9685	0.8933	0.9682	0.9216	0.8802	0.9823
27	1.0042	1.0418	1.0208	1.0206	0.9949	1.0158	0.9916	1.0025
28	0.9859	1.0838	0.9810	0.9255	1.0381	0.9269	0.9951	0.9857
29	0.8193	0.9184	0.9669	0.8632	1.0091	0.7200	1.0515	0.9733
30	0.8327	1.0251	0.9711	0.9574	1.0860	1.0404	1.0059	1.0355
31	1.2298	1.0749	1.0478	1.0471	1.0314	1.0568	0.8855	0.9475
Average	0.9759	1.0045	0.9907	0.9775	0.9839	0.9618	0.9527	0.9888
Std. Dev.	0.0784	0.0531	0.0436	0.0606	0.0684	0.0860	0.1171	0.0970
Max.	1.2298	1.0899	1.0858	1.0471	1.1538	1.1130	1.2535	1.1540
Min.	0.7917	0.8665	0.9079	0.7643	0.8070	0.7200	0.6585	0.6879

Table S.52: GPROD decomposition along paths C and D: Output orientation, comparison period viewpoint, common components.

Bank	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1906	1.1004	1.1181	0.9466	1.0000	1.0000	1.0000	1.0000	NaN	NaN	1.0236	NaN
2	1.0588	0.8966	0.9654	1.4030	1.0000	1.0000	1.0000	1.0000	1.0301	0.9845	0.9982	1.0157
3	1.0725	0.9928	1.0006	1.5176	1.1352	0.9880	1.0770	1.1975	0.9556	1.0103	0.9647	1.0329
4	1.0314	1.0154	0.9924	1.1035	1.0540	1.0185	0.9875	1.0764	1.0138	1.0065	1.0019	0.9916
5	1.0095	1.0911	0.9974	1.0379	1.0000	1.0000	1.0000	1.0000	1.0065	1.0019	0.9831	0.9981
6	1.0537	1.0074	0.9944	1.0068	1.0000	1.0000	1.0000	1.0000	0.9726	0.9418	0.9553	0.9866
7	1.0404	1.0994	1.0081	1.0877	1.0000	1.0000	1.0000	1.0000	0.9249	1.0617	1.0295	0.9682
8	1.0509	1.1225	1.0191	1.0454	1.0460	1.1503	0.9475	1.0281	0.9751	0.9963	1.0414	1.0158
9	0.9841	1.0144	1.0342	1.0167	1.0124	1.0107	1.0106	0.9936	0.9799	0.9877	0.9795	1.0009
10	0.9725	0.9665	1.0451	1.0194	1.0000	1.0000	1.0000	1.0000	0.9520	0.8215	0.9367	0.9211
11	0.8866	1.1246	0.9353	1.0968	1.0268	1.0116	0.8736	1.0695	0.9554	1.0097	0.9626	1.0020
12	1.0426	1.0575	1.0330	1.0694	1.1834	1.0423	0.9685	0.9304	1.0752	1.0100	1.0535	1.1274
13	1.1935	0.8669	0.5691	0.8603	1.0741	0.8686	0.9064	1.0658	0.9788	0.9743	0.9594	1.0010
14	0.9288	0.9244	1.2092	1.6093	0.9914	0.9512	1.0001	1.0603	1.0095	1.0351	1.0581	1.0794
15	1.0238	0.9151	0.8534	1.5074	1.0980	0.9453	0.9184	1.2335	0.9577	0.9664	0.9585	1.0002
16	0.8796	0.9539	0.9392	0.9565	1.1801	0.8233	0.8242	0.9331	0.9961	1.0430	1.0183	0.9908
17	0.9949	1.0484	0.9238	1.3089	1.0468	0.9616	0.9680	1.0973	0.9602	1.0128	0.9766	1.0032
18	0.9363	0.9585	1.0099	1.2855	0.8758	1.0282	0.8908	1.1731	1.0790	1.0284	1.0464	1.0195
19	0.9246	0.8314	1.0344	1.3384	0.9424	0.8243	1.0517	0.9994	0.9946	1.0316	1.0023	0.9974
20	0.9044	0.8022	1.2108	1.0829	0.8799	0.9795	1.0111	1.2084	0.9879	1.0286	0.9797	1.0173
21	1.0060	1.1126	1.0311	0.9895	1.0000	1.0000	1.0000	1.0000	0.9366	1.0918	1.0169	0.9716
22	0.9809	0.9076	0.9978	1.0678	0.9609	0.9281	0.9992	1.0343	0.9934	1.0045	1.0008	1.0004
23	0.9568	0.9391	0.9446	0.9234	0.9368	0.9144	0.9449	0.9316	1.0082	1.0403	1.0121	0.9958
24	1.0124	1.0310	1.0202	1.0194	1.0858	0.9725	1.0221	0.9812	0.9568	1.0110	0.9736	1.0005
25	1.0620	1.0380	1.0565	1.1167	1.0455	0.9544	1.0334	1.0704	0.9894	1.0286	0.9955	1.0203
26	0.9562	0.7758	0.9303	1.0990	1.0000	1.0000	1.0000	1.0000	0.5714	0.5038	0.9119	0.7669
27	1.1136	0.9166	1.0348	1.0528	1.0629	0.8363	0.9881	1.0261	1.0490	1.0567	1.0277	0.9894
28	1.1131	0.9504	0.9633	1.1301	0.9916	0.9091	0.9486	1.1842	1.0369	1.0424	1.0067	0.9985
29	0.9438	1.1030	0.7977	1.1335	1.0875	1.2414	0.9985	1.0015	0.9207	1.0382	0.6509	1.2254
30	0.8781	1.1371	1.0143	1.1238	0.8796	1.2165	0.8722	1.0317	1.0374	0.9236	1.1550	0.9959
31	1.3054	1.0704	0.9015	1.0663	1.0884	0.9422	0.9781	1.0414	0.9801	1.0219	0.9879	1.0071
Average	1.0164	0.9926	0.9866	1.1298	1.0221	0.9845	0.9749	1.0442	0.9762	0.9905	0.9893	1.0047
Std. Dev.	0.0954	0.0981	0.1123	0.1785	0.0750	0.0923	0.0547	0.0790	0.0847	0.1024	0.0757	0.0679
Max.	1.3054	1.1371	1.2108	1.6093	1.1834	1.2414	1.0770	1.2335	1.0790	1.0918	1.1550	1.2254
Min.	0.8781	0.7758	0.5691	0.8603	0.8758	0.8233	0.8242	0.9304	0.5714	0.5038	0.6509	0.7669

Table S.53: GPROD decomposition along path C: Output orientation, comparison period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	1.0613	NaN	4.1904	1.0035	1.0292	1.0162
2	0.9638	0.8883	0.9638	1.0104	1.0664	1.0252	1.0034	1.3671
3	0.9922	1.0505	1.0490	1.0130	0.9964	0.9468	0.9181	1.2112
4	1.0042	1.0446	1.0151	1.0430	0.9612	0.9482	0.9882	0.9912
5	0.9839	1.0827	1.0093	1.0300	1.0194	1.0059	1.0051	1.0096
6	0.9979	0.9706	1.0031	0.9988	1.0857	1.1021	1.0377	1.0217
7	0.9815	1.0193	1.0545	0.9976	1.1462	1.0159	0.9287	1.1261
8	1.0065	1.0036	1.0573	1.0007	1.0237	0.9759	0.9769	1.0003
9	0.9911	1.0207	1.0411	1.0236	1.0009	0.9955	1.0034	0.9987
10	0.9944	0.9834	1.0127	0.9855	1.0273	1.1964	1.1018	1.1230
11	0.9735	1.0108	1.1293	1.0261	0.9282	1.0892	0.9848	0.9975
12	0.9244	0.9681	0.9637	1.0210	0.8864	1.0377	1.0505	0.9985
13	0.9931	1.0337	1.0219	1.0178	1.1431	0.9910	0.6404	0.7923
14	0.9792	0.9391	0.9757	1.0749	0.9477	0.9998	1.1711	1.3082
15	0.9999	1.0012	1.0388	1.0680	0.9737	1.0006	0.9333	1.1440
16	0.8987	1.0765	1.0904	1.0002	0.8326	1.0319	1.0264	1.0344
17	1.0482	1.0262	0.9908	1.0470	0.9443	1.0491	0.9864	1.1357
18	1.0376	0.9316	0.9949	0.9667	0.9549	0.9729	1.0890	1.1119
19	1.0437	1.0552	0.9797	1.0567	0.9452	0.9266	1.0017	1.2706
20	0.9914	1.0223	1.0021	0.8137	1.0495	0.7789	1.2197	1.0827
21	1.0286	1.0569	1.0529	0.9291	1.0442	0.9642	0.9631	1.0961
22	1.0087	1.0397	1.0375	1.0377	1.0187	0.9364	0.9617	0.9945
23	1.0012	0.9913	1.0051	0.9852	1.0119	0.9960	0.9828	1.0103
24	1.0217	1.0212	1.0233	1.0375	0.9538	1.0269	1.0019	1.0009
25	0.9937	1.0356	1.0354	1.0263	1.0331	1.0211	0.9918	0.9964
26	1.6524	1.2873	1.0292	1.4360	1.0127	1.1962	0.9913	0.9980
27	1.0030	1.0253	1.0263	1.0351	0.9958	1.0116	0.9929	1.0018
28	0.9916	1.0673	1.0126	0.9664	1.0917	0.9397	0.9962	0.9889
29	0.8810	0.9055	0.9627	0.9346	1.0699	0.9450	1.2750	0.9883
30	0.9508	1.0247	1.0159	1.0465	1.0121	0.9876	0.9911	1.0452
31	1.1881	1.0626	1.0653	1.0569	1.0300	1.0462	0.8758	0.9620
Average	1.0175	1.0215	1.0232	1.0229	1.1096	1.0053	1.0039	1.0588
Std. Dev.	0.1284	0.0684	0.0368	0.0916	0.5664	0.0756	0.1050	0.1119
Max.	1.6524	1.2873	1.1293	1.4360	4.1904	1.1964	1.2750	1.3671
Min.	0.8810	0.8883	0.9627	0.8137	0.8326	0.7789	0.6404	0.7923

Table S.54: GPROD decomposition along path D: Output orientation, comparison period viewpoint, specific components.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	1.0614	NaN	NaN	NaN	1.0292	NaN
2	1.0088	0.9236	0.9841	1.0104	1.0189	0.9860	0.9827	1.3671
3	0.9922	1.0499	1.0490	0.9838	0.9964	0.9474	0.9181	1.2472
4	1.0042	1.0446	1.0150	1.0396	0.9612	0.9482	0.9883	0.9945
5	1.0137	1.0827	1.0094	1.0424	0.9895	1.0059	1.0050	0.9976
6	0.9990	1.0038	1.0121	1.0217	1.0845	1.0657	1.0285	0.9988
7	0.9815	1.0252	1.0493	1.0246	1.1461	1.0101	0.9332	1.0964
8	1.0065	0.9939	1.0500	1.0007	1.0237	0.9854	0.9836	1.0003
9	0.9907	1.0207	1.0411	1.0236	1.0013	0.9955	1.0034	0.9987
10	1.0263	1.0089	1.0650	1.0497	0.9953	1.1661	1.0477	1.0543
11	0.9735	1.0096	1.1301	1.0261	0.9282	1.0906	0.9841	0.9975
12	0.9244	0.9774	0.9637	1.0210	0.8864	1.0278	1.0505	0.9985
13	0.9931	1.0337	1.0222	1.0172	1.1431	0.9910	0.6402	0.7927
14	0.9792	0.9391	0.9757	1.1396	0.9477	0.9998	1.1711	1.2339
15	0.9999	1.0119	1.0396	1.0184	0.9737	0.9899	0.9325	1.1998
16	0.8987	1.0765	1.0904	1.0002	0.8326	1.0319	1.0264	1.0344
17	1.0482	1.0262	0.9906	1.0470	0.9443	1.0491	0.9865	1.1357
18	1.0376	0.9316	0.9949	0.9793	0.9549	0.9729	1.0890	1.0976
19	1.0437	1.0552	0.9797	1.0581	0.9452	0.9266	1.0017	1.2688
20	0.9914	1.0223	1.0021	0.8137	1.0495	0.7789	1.2197	1.0827
21	1.0286	1.0569	1.0529	0.9396	1.0442	0.9642	0.9631	1.0839
22	1.0087	1.0397	1.0375	1.0377	1.0188	0.9364	0.9617	0.9945
23	1.0012	0.9913	1.0051	0.9852	1.0119	0.9960	0.9828	1.0103
24	1.0217	1.0212	1.0233	1.0375	0.9538	1.0269	1.0019	1.0009
25	0.9937	1.0356	1.0354	1.0263	1.0331	1.0211	0.9918	0.9964
26	1.7203	1.2873	1.0415	1.4585	0.9727	1.1962	0.9796	0.9826
27	1.0030	1.0253	1.0263	1.0351	0.9958	1.0116	0.9929	1.0018
28	0.9916	1.0748	1.0126	0.9664	1.0917	0.9332	0.9962	0.9889
29	0.8810	1.0097	0.9627	0.9346	1.0699	0.8475	1.2750	0.9883
30	0.9570	1.0247	1.0144	1.0465	1.0056	0.9876	0.9926	1.0452
31	1.1881	1.0626	1.0653	1.0569	1.0300	1.0462	0.8758	0.9620
Average	1.0236	1.0289	1.0259	1.0281	1.0017	0.9979	1.0011	1.0550
Std. Dev.	0.1390	0.0616	0.0364	0.0959	0.0664	0.0776	0.1038	0.1117
Max.	1.7203	1.2873	1.1301	1.4585	1.1461	1.1962	1.2750	1.3671
Min.	0.8810	0.9236	0.9627	0.8137	0.8326	0.7789	0.6402	0.7927

Table S.55: GPROD decomposition along paths ABCD: Output orientation, ‘mean’ period viewpoint.

Bank	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1904	1.0982	1.1279	0.9426	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
2	1.0512	0.8952	0.9616	1.3185	1.0000	1.0000	1.0000	1.0000	1.0966	1.0168	1.0168	1.3461
3	1.0705	0.9897	0.9875	1.4911	1.1352	0.9880	1.0770	1.1975	0.9557	1.0101	0.9746	1.0800
4	1.0299	1.0030	0.9859	1.0974	1.0540	1.0185	0.9875	1.0764	1.0140	1.0213	1.0056	1.0016
5	1.0104	1.0958	0.9946	1.0377	1.0000	1.0000	1.0000	1.0000	1.0324	1.0846	1.0148	1.0537
6	1.0556	1.0056	0.9892	1.0058	1.0000	1.0000	1.0000	1.0000	1.1273	0.9985	1.0688	1.0553
7	1.0423	1.0998	0.9942	1.0845	1.0000	1.0000	1.0000	1.0000	0.9641	1.0661	1.2484	1.0028
8	1.0516	1.1225	1.0105	1.0437	1.0460	1.1503	0.9475	1.0281	0.9759	1.0348	1.1834	1.0050
9	0.9866	1.0138	1.0274	1.0158	1.0124	1.0107	1.0106	0.9936	0.9666	0.9776	0.9921	1.0017
10	0.9723	0.9668	1.0384	1.0125	1.0000	1.0000	1.0000	1.0000	1.0294	0.9419	1.2374	1.0223
11	0.8849	1.1320	0.9191	1.0950	1.0268	1.0116	0.8736	1.0695	0.9545	1.0053	0.9637	1.0020
12	1.0468	1.0571	1.0291	1.0669	1.1834	1.0423	0.9685	0.9304	1.0737	1.0207	1.0681	1.1544
13	1.1927	0.8692	0.5862	0.8495	1.0741	0.8686	0.9064	1.0658	0.9766	0.9750	0.9764	1.0028
14	0.9217	0.9254	1.1736	1.6410	0.9914	0.9512	1.0001	1.0603	1.0096	1.0358	1.0709	1.5152
15	1.0203	0.9136	0.8560	1.5026	1.0980	0.9453	0.9184	1.2335	0.9577	0.9781	0.9626	1.0726
16	0.8732	0.9666	0.9327	0.9549	1.1801	0.8233	0.8242	0.9331	1.0071	1.0441	1.0214	0.9911
17	0.9919	1.0507	0.9250	1.2740	1.0468	0.9616	0.9680	1.0973	0.9597	1.0131	0.9773	1.0077
18	0.9208	0.9524	0.9939	1.2282	0.8758	1.0282	0.8908	1.1731	1.0858	1.0299	1.0335	1.0568
19	0.9222	0.8241	1.0437	1.2630	0.9424	0.8243	1.0517	0.9994	0.9948	1.0319	1.0056	0.9947
20	0.9015	0.8029	1.2209	1.1137	0.8799	0.9795	1.0111	1.2084	0.9893	1.0297	1.0182	1.0464
21	1.0041	1.1167	1.0224	0.9995	1.0000	1.0000	1.0000	1.0000	0.9726	1.1967	1.1115	1.0003
22	0.9839	0.9093	0.9780	1.0614	0.9609	0.9281	0.9992	1.0343	0.9923	1.0177	1.0014	1.0006
23	0.9546	0.9373	0.9448	0.9257	0.9368	0.9144	0.9449	0.9316	1.0103	1.0402	1.0124	0.9956
24	1.0135	1.0336	1.0157	1.0172	1.0858	0.9725	1.0221	0.9812	0.9563	1.0106	0.9717	1.0006
25	1.0574	1.0360	1.0435	1.1071	1.0455	0.9544	1.0334	1.0704	0.9900	1.0277	0.9849	1.0216
26	0.9637	0.7713	0.9271	1.0913	1.0000	1.0000	1.0000	1.0000	0.8509	0.6512	0.9941	0.9732
27	1.1137	0.9242	1.0315	1.0464	1.0629	0.8363	0.9881	1.0261	1.0488	1.0548	1.0279	0.9901
28	1.0841	0.9559	0.9592	1.1226	0.9916	0.9091	0.9486	1.1842	1.0386	1.0476	1.0190	1.0153
29	0.9395	1.0903	0.8020	1.1318	1.0875	1.2414	0.9985	1.0015	0.9786	1.1674	0.7190	1.2829
30	0.8786	1.1294	1.0046	1.1222	0.8796	1.2165	0.8722	1.0317	1.0708	0.8936	1.1614	1.0446
31	1.3263	1.0815	0.8985	1.0538	1.0884	0.9422	0.9781	1.0414	0.9781	1.0214	0.9873	1.0075
Average	1.0147	0.9926	0.9814	1.1199	1.0221	0.9845	0.9749	1.0442	1.0019	1.0148	1.0277	1.0581
Std. Dev.	0.0977	0.0985	0.1084	0.1716	0.0750	0.0923	0.0547	0.0790	0.0537	0.0870	0.0949	0.1176
Max	1.3263	1.1320	1.2209	1.6410	1.1834	1.2414	1.0770	1.2335	1.1273	1.1967	1.2484	1.5152
Min	0.8732	0.7713	0.5862	0.8495	0.8758	0.8233	0.8242	0.9304	0.8509	0.6512	0.7190	0.9732

Table S.56: GPROD decomposition along paths ABCD: Output orientation, 'mean' period viewpoint.

Bank	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)	OME(07,06)	OME(08,07)	OME(09,08)	OME(10,09)
1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	0.9428	0.8859	0.9560	1.0070	1.0168	0.9938	0.9892	0.9727
3	0.9917	1.0513	1.0227	0.9949	0.9949	0.9434	0.9199	1.1588
4	1.0062	1.0506	1.0128	1.0251	0.9577	0.9178	0.9802	0.9930
5	0.9861	1.0072	0.9909	0.9897	0.9924	1.0031	0.9891	0.9951
6	0.9806	0.9789	0.9881	0.9981	0.9550	1.0287	0.9367	0.9548
7	0.9781	1.0170	0.9936	1.0018	1.1052	1.0144	0.8015	1.0796
8	1.0058	0.9918	0.9964	0.9962	1.0242	0.9508	0.9045	1.0139
9	0.9904	1.0230	1.0319	1.0221	1.0179	1.0031	0.9929	0.9986
10	0.9423	0.9664	0.9644	0.9657	1.0024	1.0622	0.8702	1.0255
11	0.9763	1.0107	1.1055	1.0245	0.9248	1.1014	0.9876	0.9975
12	0.9287	0.9597	0.9574	0.9975	0.8871	1.0353	1.0391	0.9958
13	0.9900	1.0364	1.0200	1.0150	1.1485	0.9903	0.6494	0.7831
14	0.9797	0.9395	0.9655	1.0661	0.9399	0.9998	1.1350	0.9581
15	1.0016	1.0096	1.0291	1.0375	0.9688	0.9788	0.9410	1.0947
16	0.8963	1.0832	1.0802	0.9989	0.8197	1.0382	1.0257	1.0336
17	1.0520	1.0255	0.9920	1.0424	0.9385	1.0518	0.9858	1.1052
18	1.0310	0.9303	1.0049	0.9475	0.9391	0.9667	1.0744	1.0456
19	1.0441	1.0589	0.9853	1.0491	0.9421	0.9149	1.0015	1.2110
20	0.9898	1.0194	0.9591	0.7886	1.0464	0.7809	1.2365	1.1169
21	0.9978	1.0260	0.9938	0.9374	1.0348	0.9095	0.9255	1.0658
22	1.0085	1.0424	1.0345	1.0321	1.0232	0.9236	0.9448	0.9936
23	0.9990	0.9899	1.0051	0.9873	1.0095	0.9955	0.9826	1.0109
24	1.0231	1.0228	1.0189	1.0350	0.9540	1.0282	1.0037	1.0010
25	0.9935	1.0374	1.0406	1.0174	1.0284	1.0181	0.9853	0.9950
26	1.1554	1.0899	1.0014	1.1370	0.9803	1.0867	0.9313	0.9863
27	1.0036	1.0335	1.0235	1.0278	0.9954	1.0137	0.9922	1.0021
28	0.9888	1.0774	0.9967	0.9458	1.0646	0.9316	0.9957	0.9873
29	0.8496	0.9371	0.9630	0.8981	1.0390	0.8027	1.1600	0.9808
30	0.9087	1.0249	0.9929	1.0073	1.0267	1.0136	0.9989	1.0337
31	1.2088	1.0687	1.0565	1.0520	1.0307	1.0515	0.8806	0.9547
Average	0.9950	1.0132	1.0061	1.0015	0.9936	0.9850	0.9754	1.0182
Std. Dev.	0.0656	0.0475	0.0348	0.0586	0.0627	0.0713	0.1034	0.0739
Max	1.2088	1.0899	1.1055	1.1370	1.1485	1.1014	1.2365	1.2110
Min	0.8496	0.8859	0.9560	0.7886	0.8197	0.7809	0.6494	0.7831

Table S.57: GPROD decomposition along paths E and F: Input orientation, base period viewpoint, common components.

Bank	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1902	1.0960	1.1378	0.9387	1.0000	1.0000	1.0000	1.0000	1.4313	1.3067	1.3056	1.0149
2	1.0437	0.8938	0.9578	1.2391	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
3	1.0684	0.9867	0.9746	1.4651	1.1387	0.9873	1.0827	1.2124	0.9576	1.0129	0.9879	1.1671
4	1.0284	0.9907	0.9794	1.0914	1.0267	1.0571	0.9553	1.1672	1.0913	1.0687	1.0332	1.0027
5	1.0113	1.1006	0.9919	1.0375	1.0000	1.0000	1.0000	1.0000	1.0966	1.2845	1.1064	1.2940
6	1.0575	1.0038	0.9841	1.0047	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
7	1.0441	1.1002	0.9804	1.0814	1.0000	1.0000	1.0000	1.0000	1.0451	1.3088	NaN	NaN
8	1.0523	1.1226	1.0020	1.0419	1.0730	1.1437	0.9494	1.0349	0.9511	1.2472	NaN	0.9864
9	0.9891	1.0133	1.0206	1.0149	1.0452	1.0064	1.0157	0.9897	0.9527	0.9709	1.0023	1.0062
10	0.9721	0.9671	1.0318	1.0056	1.0000	1.0000	1.0000	1.0000	1.7585	1.1623	NaN	1.1948
11	0.8833	1.1394	0.9032	1.0932	1.0261	1.0107	0.8668	1.0766	0.9552	1.0034	0.9626	1.0009
12	1.0510	1.0567	1.0251	1.0644	1.4838	1.1371	0.9537	0.9296	1.2122	1.0909	1.1146	1.2349
13	1.1919	0.8715	0.6038	0.8388	1.0638	0.8253	0.8335	1.1185	0.9898	0.9788	1.0174	0.9939
14	0.9147	0.9265	1.1390	1.6733	0.9899	0.9400	0.9994	1.0753	1.0115	1.0460	1.1036	3.3612
15	1.0168	0.9120	0.8587	1.4977	1.1043	0.9403	0.9098	1.2038	0.9585	0.9907	0.9673	1.2252
16	0.8669	0.9796	0.9264	0.9533	1.1294	0.6381	0.5833	1.0457	1.0817	1.1678	1.1026	0.9189
17	0.9888	1.0530	0.9263	1.2400	1.0465	0.9539	0.9599	1.1158	0.9606	1.0178	0.9801	1.0108
18	0.9054	0.9464	0.9781	1.1734	0.8557	1.0253	0.8517	1.2369	1.1102	1.0439	1.0617	1.1215
19	0.9198	0.8168	1.0530	1.1919	0.8984	0.7245	0.9722	1.0785	1.0246	1.0852	1.0713	0.9728
20	0.8987	0.8036	1.2311	1.1454	0.8257	0.9943	1.0095	1.2760	1.0199	1.0552	1.0752	1.0826
21	1.0023	1.1207	1.0137	1.0095	1.0000	1.0000	1.0000	1.0000	1.0133	NaN	NaN	1.0340
22	0.9870	0.9111	0.9585	1.0550	0.9265	0.8814	0.9634	1.0742	1.0139	1.0648	1.0327	0.9904
23	0.9523	0.9354	0.9450	0.9281	0.8671	0.8356	0.8394	0.8762	1.0639	1.1046	1.0906	0.9528
24	1.0145	1.0363	1.0112	1.0149	1.0902	0.9708	1.0235	0.9801	0.9538	1.0112	0.9681	1.0007
25	1.0529	1.0339	1.0307	1.0975	1.0288	0.9344	1.0442	1.1069	1.0148	1.0499	0.9751	1.0430
26	0.9712	0.7667	0.9239	1.0835	1.0000	1.0000	1.0000	1.0000	3.3548	0.8335	1.0648	1.5604
27	1.1137	0.9319	1.0282	1.0401	0.9963	0.6174	0.9289	1.2474	1.2237	1.2683	1.1230	0.9537
28	1.0558	0.9615	0.9551	1.1152	0.8299	0.9853	0.9950	1.3134	1.2577	1.1771	1.0088	0.9413
29	0.9352	1.0777	0.8062	1.1300	1.0717	1.0665	0.9995	1.0005	0.9325	1.1017	0.8844	1.1081
30	0.8792	1.1218	0.9950	1.1206	0.9254	1.2266	0.8615	1.0368	1.1081	0.8595	1.1917	1.0982
31	1.3475	1.0926	0.8955	1.0415	1.0964	0.9260	0.9652	1.0643	0.9848	1.0336	0.9951	1.0059
Average	1.0131	0.9926	0.9764	1.1106	1.0174	0.9622	0.9537	1.0729	1.1562	1.0838	1.0490	1.1528
Std. Dev.	0.1005	0.0992	0.1053	0.1682	0.1177	0.1270	0.0905	0.1037	0.4486	0.1193	0.0845	0.4454
Max.	1.3475	1.1394	1.2311	1.6733	1.4838	1.2266	1.0827	1.3134	3.3548	1.3088	1.3056	3.3612
Min.	0.8669	0.7667	0.6038	0.8388	0.8257	0.6174	0.5833	0.8762	0.9325	0.8335	0.8844	0.9189

Table S.58: GPROD decomposition along path E: Input orientation, base period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	0.9900	0.9826	0.9795	0.9350	0.8400	0.8536	0.8898	0.9892
2	0.9501	0.8541	0.9266	0.9856	NaN	NaN	NaN	NaN
3	0.9906	1.0605	0.9939	1.0100	0.9891	0.9304	0.9168	1.0252
4	1.0247	1.0849	1.0145	1.0075	0.8957	0.8083	0.9782	0.9256
5	0.9727	0.8848	0.9615	0.9447	0.9480	0.9684	0.9325	0.8487
6	0.9607	0.9756	0.9656	0.9725	NaN	NaN	NaN	NaN
7	0.9683	0.9954	0.9336	0.9799	1.0317	0.8445	NaN	NaN
8	1.0075	1.0126	0.9388	1.0031	1.0235	0.7772	NaN	1.0176
9	0.9898	1.0301	1.0305	1.0220	1.0036	1.0067	0.9728	0.9972
10	0.8775	0.9422	0.8813	0.9148	0.6299	0.8831	NaN	0.9200
11	0.9850	1.0137	1.0976	1.0242	0.9149	1.1084	0.9863	0.9906
12	0.9393	0.9625	0.9423	0.9845	0.6220	0.8850	1.0234	0.9418
13	0.9799	1.0463	1.0144	1.0226	1.1552	1.0312	0.7020	0.7379
14	0.9807	0.9333	0.9462	1.1228	0.9315	1.0096	1.0914	0.4123
15	1.0103	1.0206	1.0350	1.0674	0.9509	0.9593	0.9427	0.9514
16	0.8247	1.0670	1.0090	1.0137	0.8605	1.2320	1.4275	0.9787
17	1.0594	1.0208	0.9945	1.0464	0.9285	1.0625	0.9901	1.0507
18	1.0158	0.9229	1.0395	0.9159	0.9382	0.9581	1.0406	0.9235
19	1.0429	1.0568	0.9391	1.0719	0.9581	0.9830	1.0766	1.0599
20	0.9758	0.9756	0.9100	0.7304	1.0936	0.7850	1.2463	1.1352
21	0.9571	0.9551	0.9264	0.9364	1.0335	NaN	NaN	1.0426
22	1.0061	1.0457	1.0216	1.0361	1.0444	0.9284	0.9431	0.9571
23	0.9815	0.9899	0.9986	0.9835	1.0518	1.0238	1.0337	1.1302
24	1.0279	1.0291	1.0169	1.0348	0.9491	1.0258	1.0036	1.0000
25	0.9933	1.0548	1.0616	0.9822	1.0153	0.9993	0.9535	0.9678
26	0.8033	0.8783	0.9949	0.9210	0.3604	1.0474	0.8721	0.7540
27	1.0073	1.0864	1.0091	0.9356	0.9069	1.0955	0.9768	0.9345
28	0.9737	0.8412	0.9700	0.8601	1.0388	0.9855	0.9810	1.0489
29	0.8225	0.9055	0.9663	0.9519	1.1377	1.0130	0.9439	1.0708
30	0.8225	1.0254	0.9633	0.9798	1.0424	1.0377	1.0061	1.0045
31	1.2460	1.0839	1.0558	1.0634	1.0017	1.0532	0.8831	0.9148
Average	0.9738	0.9915	0.9851	0.9826	0.9413	0.9748	0.9926	0.9547
Std. Dev.	0.0819	0.0681	0.0483	0.0717	0.1605	0.1018	0.1290	0.1376
Max.	1.2460	1.0864	1.0976	1.1228	1.1552	1.2320	1.4275	1.1352
Min.	0.8033	0.8412	0.8813	0.7304	0.3604	0.7772	0.7020	0.4123

Table S.59: GPROD decomposition along path F: Input orientation, base period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	0.9909	1.0036	0.9795	0.9350	0.8392	0.8358	0.8898	0.9892
2	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	0.9906	1.0605	0.9959	0.9791	0.9891	0.9304	0.9150	1.0575
4	1.0247	1.1155	1.0145	1.0363	0.8957	0.7861	0.9782	0.8998
5	0.9809	0.9024	0.9865	0.9537	0.9401	0.9495	0.9088	0.8407
6	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
7	0.9846	1.0028	NaN	NaN	1.0147	0.8383	NaN	NaN
8	1.0075	0.9965	NaN	1.0031	1.0235	0.7898	NaN	1.0176
9	0.9898	1.0301	1.0305	1.0220	1.0036	1.0067	0.9728	0.9972
10	0.8775	1.0116	NaN	0.9148	0.6299	0.8226	NaN	0.9200
11	0.9850	1.0137	1.0976	1.0242	0.9149	1.1084	0.9863	0.9906
12	0.9699	0.9772	0.9487	0.9845	0.6025	0.8717	1.0165	0.9418
13	0.9799	1.0463	1.0144	1.0226	1.1552	1.0312	0.7020	0.7379
14	0.9807	0.9333	0.9462	1.1228	0.9315	1.0096	1.0914	0.4123
15	1.0103	1.0206	1.0350	1.0359	0.9509	0.9593	0.9427	0.9803
16	0.8247	1.0670	1.0090	1.0137	0.8605	1.2320	1.4275	0.9787
17	1.0594	1.0208	0.9945	1.0464	0.9285	1.0625	0.9901	1.0507
18	1.0158	0.9229	1.0395	0.9010	0.9382	0.9581	1.0406	0.9388
19	1.0429	1.0568	0.9391	1.0719	0.9581	0.9830	1.0766	1.0599
20	0.9758	0.9740	0.9100	0.7304	1.0936	0.7863	1.2463	1.1352
21	0.9571	NaN	NaN	0.9512	1.0335	NaN	NaN	1.0265
22	1.0061	1.0457	1.0216	1.0361	1.0444	0.9284	0.9431	0.9571
23	0.9815	0.9899	0.9986	0.9835	1.0518	1.0238	1.0337	1.1302
24	1.0279	1.0291	1.0169	1.0348	0.9491	1.0258	1.0036	1.0000
25	0.9933	1.0548	1.0616	0.9948	1.0153	0.9993	0.9535	0.9555
26	1.0649	0.9287	0.9963	1.0735	0.2719	0.9906	0.8709	0.6469
27	1.0073	1.0864	1.0183	0.9356	0.9069	1.0955	0.9679	0.9345
28	0.9737	0.8544	0.9700	0.8601	1.0388	0.9703	0.9810	1.0489
29	0.8225	0.9055	0.9663	0.9519	1.1377	1.0130	0.9439	1.0708
30	0.8289	1.0254	0.9633	0.9585	1.0344	1.0377	1.0061	1.0268
31	1.2460	1.0839	1.0558	1.0634	1.0017	1.0532	0.8831	0.9148
Average	0.9862	1.0057	1.0004	0.9872	0.9364	0.9678	0.9909	0.9521
Std. Dev.	0.0786	0.0614	0.0415	0.0759	0.1727	0.1053	0.1296	0.1456
Max.	1.2460	1.1155	1.0976	1.1228	1.1552	1.2320	1.4275	1.1352
Min.	0.8225	0.8544	0.9100	0.7304	0.2719	0.7861	0.7020	0.4123

Table S.60: GPROD decomposition along paths G and H: Input orientation, comparison period viewpoint, common components.

Bank	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1906	1.1004	1.1181	0.9466	1.0000	1.0000	1.0000	1.0000	0.9057	0.9913	1.0000	0.8844
2	1.0588	0.8966	0.9654	1.4030	1.0000	1.0000	1.0000	1.0000	1.0552	0.9733	0.9968	1.0382
3	1.0725	0.9928	1.0006	1.5176	1.1387	0.9873	1.0827	1.2124	0.9588	1.0122	0.9634	1.0339
4	1.0314	1.0154	0.9924	1.1035	1.0267	1.0571	0.9553	1.1672	1.1210	1.0332	1.0314	0.9772
5	1.0095	1.0911	0.9974	1.0379	1.0000	1.0000	1.0000	1.0000	1.0101	1.0021	0.9824	0.9979
6	1.0537	1.0074	0.9944	1.0068	1.0000	1.0000	1.0000	1.0000	0.9670	NaN	0.8479	0.9458
7	1.0404	1.0994	1.0081	1.0877	1.0000	1.0000	1.0000	1.0000	0.7369	1.0881	1.0311	0.9400
8	1.0509	1.1225	1.0191	1.0454	1.0730	1.1437	0.9494	1.0349	0.9514	1.0045	1.0310	1.0140
9	0.9841	1.0144	1.0342	1.0167	1.0452	1.0064	1.0157	0.9897	0.9744	0.9926	0.9742	1.0044
10	0.9725	0.9665	1.0451	1.0194	1.0000	1.0000	1.0000	1.0000	0.9491	0.7519	0.9337	0.8691
11	0.8866	1.1246	0.9353	1.0968	1.0261	1.0107	0.8668	1.0766	0.9572	1.0128	0.9607	1.0006
12	1.0426	1.0575	1.0330	1.0694	1.4838	1.1371	0.9537	0.9296	0.9833	0.9566	1.0800	1.1399
13	1.1935	0.8669	0.5691	0.8603	1.0638	0.8253	0.8335	1.1185	0.9977	0.9737	0.9636	0.9799
14	0.9288	0.9244	1.2092	1.6093	0.9899	0.9400	0.9994	1.0753	1.0111	1.0431	1.0727	1.0891
15	1.0238	0.9151	0.8534	1.5074	1.1043	0.9403	0.9098	1.2038	0.9598	0.9805	0.9570	0.9988
16	0.8796	0.9539	0.9392	0.9565	1.1294	0.6381	0.5833	1.0457	1.0509	1.1117	1.1591	0.9189
17	0.9949	1.0484	0.9238	1.3089	1.0465	0.9539	0.9599	1.1158	0.9620	1.0166	0.9754	0.9993
18	0.9363	0.9585	1.0099	1.2855	0.8557	1.0253	0.8517	1.2369	1.0920	1.0446	1.0720	1.0169
19	0.9246	0.8314	1.0344	1.3384	0.8984	0.7245	0.9722	1.0785	1.0166	1.0647	1.0526	0.9716
20	0.9044	0.8022	1.2108	1.0829	0.8257	0.9943	1.0095	1.2760	1.0009	1.0565	1.0256	1.0546
21	1.0060	1.1126	1.0311	0.9895	1.0000	1.0000	1.0000	1.0000	NaN	1.1043	1.0188	0.9511
22	0.9809	0.9076	0.9978	1.0678	0.9265	0.8814	0.9634	1.0742	1.0127	1.0154	1.0293	0.9877
23	0.9568	0.9391	0.9446	0.9234	0.8671	0.8356	0.8394	0.8762	1.0441	1.0966	1.0759	0.9638
24	1.0124	1.0310	1.0202	1.0194	1.0902	0.9708	1.0235	0.9801	0.9554	1.0118	0.9719	1.0005
25	1.0620	1.0380	1.0565	1.1167	1.0288	0.9344	1.0442	1.1069	1.0173	1.0532	1.0129	1.0164
26	0.9562	0.7758	0.9303	1.0990	1.0000	1.0000	1.0000	1.0000	0.7977	0.2507	0.8347	0.8444
27	1.1136	0.9166	1.0348	1.0528	0.9963	0.6174	0.9289	1.2474	1.3215	1.2284	1.1603	0.9300
28	1.1131	0.9504	0.9633	1.1301	0.8299	0.9853	0.9950	1.3134	1.2742	1.1807	1.0266	0.9492
29	0.9438	1.1030	0.7977	1.1335	1.0717	1.0665	0.9995	1.0005	0.9297	1.0746	0.8420	1.0945
30	0.8781	1.1371	1.0143	1.1238	0.9254	1.2266	0.8615	1.0368	1.1297	0.9625	1.1693	0.9938
31	1.3054	1.0704	0.9015	1.0663	1.0964	0.9260	0.9652	1.0643	0.9948	1.0343	0.9924	1.0038
Average	1.0164	0.9926	0.9866	1.1298	1.0174	0.9622	0.9537	1.0729	1.0046	1.0041	1.0079	0.9874
Std. Dev.	0.0954	0.0981	0.1123	0.1785	0.1177	0.1270	0.0905	0.1037	0.1104	0.1608	0.0797	0.0619
Max.	1.3054	1.1371	1.2108	1.6093	1.4838	1.2266	1.0827	1.3134	1.3215	1.2284	1.1693	1.1399
Min.	0.8781	0.7758	0.5691	0.8603	0.8257	0.6174	0.5833	0.8762	0.7369	0.2507	0.8347	0.8444

Table S.61: GPROD decomposition along path G: Input orientation, comparison period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	1.0515	1.0042	1.0614	1.0416	1.2503	1.1055	1.0535	1.0276
2	1.0438	0.8971	0.9744	0.9966	0.9612	1.0268	0.9940	1.3560
3	0.9915	1.0593	1.0530	1.0153	0.9907	0.9379	0.9111	1.1924
4	1.0260	1.0494	1.0178	1.0663	0.8734	0.8859	0.9897	0.9073
5	0.9866	1.0838	1.0104	1.0355	1.0129	1.0046	1.0047	1.0044
6	1.0048	NaN	0.9717	0.9868	1.0845	NaN	1.2069	1.0788
7	0.9833	1.0151	1.0590	0.9934	1.4359	0.9954	0.9233	1.1648
8	1.0071	1.0128	1.0614	1.0039	1.0221	0.9647	0.9809	0.9924
9	0.9906	1.0327	1.0453	1.0265	0.9754	0.9833	0.9998	0.9963
10	1.0240	1.0112	0.9813	0.9298	1.0007	1.2712	1.1406	1.2616
11	0.9814	1.0137	1.1404	1.0274	0.9198	1.0838	0.9849	0.9911
12	0.9816	0.9794	0.9556	1.0311	0.7280	0.9927	1.0495	0.9788
13	0.9841	1.0414	1.0180	1.0269	1.1427	1.0360	0.6961	0.7644
14	0.9794	0.9312	0.9656	1.1079	0.9474	1.0123	1.1681	1.2403
15	1.0092	1.0206	1.0513	1.0782	0.9572	0.9726	0.9324	1.1629
16	0.8110	1.0517	1.0216	1.0156	0.9138	1.2786	1.3596	0.9802
17	1.0528	1.0217	0.9920	1.0545	0.9387	1.0581	0.9946	1.1132
18	1.0303	0.9228	1.0355	0.9700	0.9725	0.9697	1.0681	1.0536
19	1.0416	1.0487	0.9223	1.0876	0.9719	1.0278	1.0959	1.1743
20	0.9743	0.9771	0.9648	0.7514	1.1232	0.7815	1.2121	1.0709
21	NaN	1.0626	1.0601	0.9377	NaN	0.9482	0.9547	1.1095
22	1.0058	1.0402	1.0263	1.0470	1.0394	0.9749	0.9805	0.9613
23	0.9815	0.9929	0.9978	0.9796	1.0768	1.0322	1.0483	1.1161
24	1.0263	1.0264	1.0258	1.0399	0.9471	1.0227	0.9998	0.9998
25	0.9941	1.0528	1.0358	1.0325	1.0208	1.0019	0.9644	0.9613
26	1.1189	1.0235	1.0070	1.2575	1.0713	3.0239	1.1068	1.0351
27	1.0073	1.0730	0.9481	0.9475	0.8397	1.1264	1.0126	0.9577
28	0.9763	0.7840	0.9772	0.8687	1.0782	1.0421	0.9651	1.0435
29	0.8289	0.8999	0.9688	0.9572	1.1426	1.0695	0.9784	1.0813
30	0.9007	1.0267	1.0176	1.0521	0.9326	0.9381	0.9896	1.0366
31	1.2086	1.0726	1.0746	1.0724	0.9903	1.0420	0.8758	0.9307
Average	1.0001	1.0076	1.0143	1.0141	1.0120	1.0870	1.0207	1.0563
Std. Dev.	0.0703	0.0634	0.0451	0.0816	0.1265	0.3717	0.1160	0.1157
Max.	1.2086	1.0838	1.1404	1.2575	1.4359	3.0239	1.3596	1.3560
Min.	0.8110	0.7840	0.9223	0.7514	0.7280	0.7815	0.6961	0.7644

Table S.62: GPROD decomposition along path H: Input orientation, comparison period viewpoint, specific components.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	1.0515	1.0042	1.0614	1.0416	1.2503	1.1055	1.0535	1.0276
2	1.0438	1.0468	1.0325	1.0048	0.9612	0.8799	0.9380	1.3449
3	0.9915	1.0593	1.0530	0.9929	0.9907	0.9379	0.9111	1.2193
4	1.0260	1.1059	1.0137	1.0658	0.8734	0.8406	0.9937	0.9077
5	1.0157	1.0919	1.0108	1.0443	0.9839	0.9972	1.0043	0.9959
6	1.0055	1.0092	1.0183	1.0262	1.0837	NaN	1.1517	1.0374
7	0.9978	1.0299	1.0590	1.0287	1.4150	0.9811	0.9233	1.1248
8	1.0071	1.0095	1.0614	1.0039	1.0221	0.9679	0.9809	0.9924
9	0.9906	1.0327	1.0453	1.0265	0.9754	0.9833	0.9998	0.9963
10	1.0315	1.0116	1.0697	1.0541	0.9934	1.2707	1.0464	1.1128
11	0.9814	1.0137	1.1404	1.0274	0.9198	1.0838	0.9849	0.9911
12	0.9748	0.9977	0.9556	1.0311	0.7331	0.9745	1.0495	0.9788
13	0.9841	1.0414	1.0180	1.0269	1.1427	1.0360	0.6961	0.7644
14	0.9794	0.9312	0.9656	1.1079	0.9474	1.0123	1.1681	1.2403
15	1.0092	1.0206	1.0513	1.0707	0.9572	0.9726	0.9324	1.1710
16	0.8110	1.0517	1.0216	1.0156	0.9138	1.2786	1.3596	0.9802
17	1.0528	1.0217	0.9920	1.0545	0.9387	1.0581	0.9946	1.1132
18	1.0303	0.9228	1.0290	0.9850	0.9725	0.9697	1.0749	1.0375
19	1.0416	1.0487	0.9223	1.0876	0.9719	1.0278	1.0959	1.1743
20	0.9743	0.9753	0.9680	0.7586	1.1232	0.7829	1.2081	1.0608
21	1.0821	1.0626	1.0601	0.9476	NaN	0.9482	0.9547	1.0979
22	1.0058	1.0402	1.0263	1.0470	1.0394	0.9749	0.9805	0.9613
23	0.9815	0.9929	0.9978	0.9796	1.0768	1.0322	1.0483	1.1161
24	1.0263	1.0264	1.0258	1.0399	0.9471	1.0227	0.9998	0.9998
25	0.9941	1.0528	1.0387	1.0325	1.0208	1.0019	0.9617	0.9613
26	1.1189	1.1143	1.0070	1.2726	1.0713	2.7774	1.1068	1.0228
27	1.0073	1.0730	0.9481	0.9709	0.8397	1.1264	1.0126	0.9346
28	0.9763	0.7818	0.9772	0.8687	1.0782	1.0450	0.9651	1.0435
29	0.8289	0.9436	0.9688	0.9633	1.1426	1.0200	0.9784	1.0745
30	0.9484	1.0267	1.0170	1.0521	0.8857	0.9381	0.9902	1.0366
31	1.2086	1.0726	1.0746	1.0724	0.9903	1.0420	0.8758	0.9307
Average	1.0057	1.0198	1.0203	1.0226	1.0087	1.0696	1.0142	1.0468
Std. Dev.	0.0689	0.0619	0.0441	0.0794	0.1252	0.3323	0.1122	0.1099
Max.	1.2086	1.1143	1.1404	1.2726	1.4150	2.7774	1.3596	1.3449
Min.	0.8110	0.7818	0.9223	0.7586	0.7331	0.7829	0.6961	0.7644

Table S.63: GPROD decomposition along paths EFGH: Input orientation, ‘mean’ period viewpoint

Bank	GPROD(07,06)	GPROD(08,07)	GPROD(09,08)	GPROD(10,09)	EC(07,06)	EC(08,07)	EC(09,08)	EC(10,09)	TC(07,06)	TC(08,07)	TC(09,08)	TC(10,09)
1	1.1904	1.0982	1.1279	0.9426	1.0000	1.0000	1.0000	1.0000	1.1385	1.1381	1.1426	0.9474
2	1.0512	0.8952	0.9616	1.3185	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
3	1.0705	0.9897	0.9875	1.4911	1.1387	0.9873	1.0827	1.2124	0.9582	1.0125	0.9756	1.0985
4	1.0299	1.0030	0.9859	1.0974	1.0267	1.0571	0.9553	1.1672	1.1061	1.0508	1.0323	0.9899
5	1.0104	1.0958	0.9946	1.0377	1.0000	1.0000	1.0000	1.0000	1.0525	1.1345	1.0426	1.1363
6	1.0556	1.0056	0.9892	1.0058	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	NaN
7	1.0423	1.0998	0.9942	1.0845	1.0000	1.0000	1.0000	1.0000	0.8776	1.1933	NaN	NaN
8	1.0516	1.1225	1.0105	1.0437	1.0730	1.1437	0.9494	1.0349	0.9513	1.1193	NaN	1.0001
9	0.9866	1.0138	1.0274	1.0158	1.0452	1.0064	1.0157	0.9897	0.9635	0.9817	0.9881	1.0053
10	0.9723	0.9668	1.0384	1.0125	1.0000	1.0000	1.0000	1.0000	1.2919	0.9348	NaN	1.0190
11	0.8849	1.1320	0.9191	1.0950	1.0261	1.0107	0.8668	1.0766	0.9562	1.0081	0.9616	1.0007
12	1.0468	1.0571	1.0291	1.0669	1.4838	1.1371	0.9537	0.9296	1.0918	1.0215	1.0971	1.1864
13	1.1927	0.8692	0.5862	0.8495	1.0638	0.8253	0.8335	1.1185	0.9937	0.9763	0.9902	0.9869
14	0.9217	0.9254	1.1736	1.6410	0.9899	0.9400	0.9994	1.0753	1.0113	1.0446	1.0880	1.9133
15	1.0203	0.9136	0.8560	1.5026	1.1043	0.9403	0.9098	1.2038	0.9591	0.9856	0.9621	1.1062
16	0.8732	0.9666	0.9327	0.9549	1.1294	0.6381	0.5833	1.0457	1.0662	1.1394	1.1305	0.9189
17	0.9919	1.0507	0.9250	1.2740	1.0465	0.9539	0.9599	1.1158	0.9613	1.0172	0.9777	1.0050
18	0.9208	0.9524	0.9939	1.2282	0.8557	1.0253	0.8517	1.2369	1.1011	1.0443	1.0668	1.0679
19	0.9222	0.8241	1.0437	1.2630	0.8984	0.7245	0.9722	1.0785	1.0206	1.0749	1.0619	0.9722
20	0.9015	0.8029	1.2209	1.1137	0.8257	0.9943	1.0095	1.2760	1.0104	1.0559	1.0501	1.0685
21	1.0041	1.1167	1.0224	0.9995	1.0000	1.0000	1.0000	1.0000	NaN	NaN	NaN	0.9917
22	0.9839	0.9093	0.9780	1.0614	0.9265	0.8814	0.9634	1.0742	1.0133	1.0398	1.0310	0.9890
23	0.9546	0.9373	0.9448	0.9257	0.8671	0.8356	0.8394	0.8762	1.0539	1.1006	1.0832	0.9583
24	1.0135	1.0336	1.0157	1.0172	1.0902	0.9708	1.0235	0.9801	0.9546	1.0115	0.9700	1.0006
25	1.0574	1.0360	1.0435	1.1071	1.0288	0.9344	1.0442	1.1069	1.0161	1.0515	0.9938	1.0296
26	0.9637	0.7713	0.9271	1.0913	1.0000	1.0000	1.0000	1.0000	1.6359	0.4571	0.9428	1.1479
27	1.1137	0.9242	1.0315	1.0464	0.9963	0.6174	0.9289	1.2474	1.2716	1.2482	1.1415	0.9418
28	1.0841	0.9559	0.9592	1.1226	0.8299	0.9853	0.9950	1.3134	1.2659	1.1789	1.0176	0.9452
29	0.9395	1.0903	0.8020	1.1318	1.0717	1.0665	0.9995	1.0005	0.9311	1.0881	0.8630	1.1013
30	0.8786	1.1294	1.0046	1.1222	0.9254	1.2266	0.8615	1.0368	1.1189	0.9095	1.1804	1.0447
31	1.3263	1.0815	0.8985	1.0538	1.0964	0.9260	0.9652	1.0643	0.9898	1.0340	0.9938	1.0048
Average	1.0147	0.9926	0.9814	1.1199	1.0174	0.9622	0.9537	1.0729	1.0629	1.0376	1.0314	1.0563
Std. Dev.	0.0977	0.0985	0.1084	0.1716	0.1177	0.1270	0.0905	0.1037	0.1500	0.1350	0.0726	0.1776
Max.	1.3263	1.1320	1.2209	1.6410	1.4838	1.2266	1.0827	1.3134	1.6359	1.2482	1.1804	1.9133
Min.	0.8732	0.7713	0.5862	0.8495	0.8257	0.6174	0.5833	0.8762	0.8776	0.4571	0.8630	0.9189

Table S.64: GPROD decomposition along paths EFGH: Input orientation, 'mean' period viewpoint.

Bank	IME(07,06)	IME(08,07)	IME(09,08)	IME(10,09)	SEC(07,06)	SEC(08,07)	SEC(09,08)	SEC(10,09)
1	1.0205	0.9986	1.0196	0.9869	1.0246	0.9663	0.9682	1.0082
2					NaN	NaN	NaN	NaN
3	0.9911	1.0599	1.0235	0.9992	0.9899	0.9341	0.9135	1.1205
4	1.0253	1.0886	1.0151	1.0437	0.8845	0.8294	0.9849	0.9101
5	0.9889	0.9859	0.9921	0.9935	0.9708	0.9797	0.9616	0.9191
6	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
7	0.9834	1.0107	NaN	NaN	1.2077	0.9119	NaN	NaN
8	1.0073	1.0078	NaN	1.0035	1.0228	0.8701	NaN	NaN
9	0.9902	1.0314	1.0379	1.0243	0.9894	0.9950	0.9862	0.9968
10	0.9497	0.9937	NaN	0.9517	0.7925	1.0408	NaN	1.0441
11	0.9832	1.0137	1.1188	1.0258	0.9173	1.0960	0.9856	0.9908
12	0.9663	0.9791	0.9506	1.0075	0.6687	0.9294	1.0346	0.9601
13	0.9820	1.0438	1.0162	1.0248	1.1489	1.0336	0.6990	0.7510
14	0.9801	0.9323	0.9559	1.1153	0.9394	1.0109	1.1291	0.7152
15	1.0098	1.0206	1.0431	1.0629	0.9540	0.9659	0.9375	1.0616
16	0.8178	1.0593	1.0153	1.0147	0.8867	1.2551	1.3931	0.9794
17	1.0561	1.0213	0.9933	1.0505	0.9336	1.0603	0.9924	1.0815
18	1.0231	0.9229	1.0359	0.9423	0.9552	0.9639	1.0560	0.9867
19	1.0423	1.0528	0.9307	1.0797	0.9650	1.0051	1.0862	1.1156
20	0.9750	0.9755	0.9378	0.7426	1.1083	0.7839	1.2281	1.1000
21	NaN	NaN	NaN	0.9432	NaN	NaN	NaN	1.0685
22	1.0060	1.0429	1.0239	1.0416	1.0419	0.9514	0.9616	0.9592
23	0.9815	0.9914	0.9982	0.9816	1.0642	1.0280	1.0409	1.1231
24	1.0271	1.0278	1.0213	1.0373	0.9481	1.0242	1.0017	0.9999
25	0.9937	1.0538	1.0493	1.0102	1.0180	1.0006	0.9583	0.9615
26	1.0173	0.9821	1.0013	1.1215	0.5791	1.7181	0.9821	0.8477
27	1.0073	1.0797	0.9804	0.9473	0.8727	1.1109	0.9923	0.9403
28	0.9750	0.8147	0.9736	0.8644	1.0583	1.0102	0.9731	1.0462
29	0.8257	0.9134	0.9675	0.9561	1.1401	1.0286	0.9610	1.0743
30	0.8736	1.0260	0.9899	1.0097	0.9714	0.9867	0.9980	1.0260
31	1.2272	1.0782	1.0652	1.0679	0.9960	1.0476	0.8794	0.9227
Average	0.9902	1.0074	1.0063	1.0018	0.9660	1.0192	1.0042	0.9893
Std. Dev.	0.0715	0.0576	0.0412	0.0736	0.1299	0.1603	0.1203	0.1002
Max.	1.2272	1.0886	1.1188	1.1215	1.2077	1.7181	1.3931	1.1231
Min.	0.8178	0.8147	0.9307	0.7426	0.5791	0.7839	0.6990	0.7152