Inventory Management Based on Simulation of Ordering Process

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Abstract

Inventory management is one of the key operational functions of a company which, in the context of modern Supply Chain Management schemes, plays an important role both for the company itself and the coordination with the SC partners. Especially the aspect of service level and stockout probability has become critical.

Classical methods of inventory management are based on simple analytical formulae which, however, only treat special cases. In this contribution we present a tool for the optimization of inventory management which is based on a simulation of the ordering processes. The full stochastic properties of the ordering process are incorporated which allows an accurate determination of performance measures like service level. With this tool, the determination of cost-minimal inventory parameters (reorder level, reorder quantity) for given stockout probability is easily possible.

1. INTRODUCTION

Inventory management is one of the key operational functions of companies and has a strong influence on the behavior of the company within a Supply Chain (SC) or Supply Network (SN). Regarding the flow of goods within a SC, inventory problems lead to unforeseen interruptions which, in the complex dynamical environment of SCs, tend to have a large impact on the performance of the whole chain.

From the perspective of Supply Chain Management (SCM), the main performance measure is the service level which is defined as the percentage of orders which can be fulfilled in time– a critical measure for modern SC processes like, for example, Just in Time (JIT) production.

In this context, customers may be external customers or internal customers. For example, a production process that needs a specific sort of screws is an internal customer of the screw inventory. A lack of screws leads to a stop of the production process which, in turn, may cause further problems within the factory: need of change of the scheduled production plan, production delays for other products, and so on. Thus, the impact of inventories to internal processes may cause severe distortions in the SC behavior of the whole company.

An inventory is a buffer which enables to de-couple the refill mechanism (buying items) with the usage process (use items for serving customers). The benefit of inventories is twofold: First, it allows buying items in large quantities which reduces the sourcing costs (lower fix costs per item, better price) and leads to a direct economic impact. However, in many cases, the second benefit is much more important: An inventory is a means for reducing uncertainty effects and increasing the supply reliability. With an inventory, variation in the delivery of goods can be absorbed. Thus, inventories are necessary in order to damp the effect of unforeseen (stochastic) variations in the supply chain.

For SCM, this leads to different tradeoff situations: For modern JIT concepts, the service level has to be very high. This requires large inventories. On the other side, one of the main aims of JIT is to *reduce* inventories. Apparently, these two aims are contradictory. They only can be met if the uncertainty, i.e. the unforeseen variability, is drastically reduced. Concepts like Six Sigma (reduction of uncertainty by total quality management) help to realize this.

Two main characteristic quantities describe the economic performance of an inventory: The inventory costs per time, and the service level. As seen above, both quantities have to be regarded. The general aim of inventory management is to produce maximum service level at minimum costs.

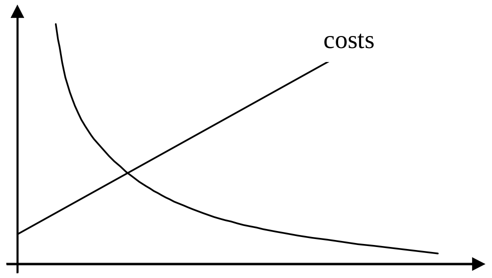

Figure 1. Schematic relation of inventory level and costs/stockouts.

Both costs and service level are a result of the inventory management which, roughly speaking consists in a rule which defines when to reorder how much, based on information about the actual inventory, the costs structure of the inventory, and forecasts. In many cases, the reordering rule is based on the present inventory level: Each time the inventory level falls below a certain reorder point, an order is placed. The value of the order may be constant, leading to an (r,Q)-policy [1], or may be chosen according the present inventory level (order-up-to policy, (s,S) policy [1]).

In this case, the task for an inventory manager is to choose the optimum values for the reorder point and the reorder quantity. There are classical analytical methods to do that which are widely used. However, as will be shown below, these methods have severe drawbacks and often lead to suboptimum solutions. The problems arise in two aspects: First, the usually used assumptions on the stochastic structure of the customer demand often are not adequate and lead to wrong results. Second, the cost structure of the analytical models is very simple and, often, does not coincide with the real situation. Due to these reasons, inventories often are managed suboptimal. Either costs are larger than needed, or the service level is lower than necessary.

In the present paper, we describe a tool which helps inventory managers to choose an optimum inventory strategy with realistic customer demand and realistic cost structures. The tool is based on simulation of the reordering and inventory processes. Thus, simplifications as used in analytical formulae are not necessary. The two key performance indicators “costs” and “service level” can be calculated for each setting of reorder point and reorder quantity thus helping to find an optimum inventory strategy.

2. LIMITATIONS OF CLASSICAL ANALYTICAL INVENTORY MODELS

In this section, the usually used analytical models are shortly reviewed and their limitations are indicated.

In order to determine an optimum inventory policy, the future demand and its stochastic variability has to be characterized. The usually used stochastic models are based on fixed periods (weeks, months, reordering lead time). They assume a normal distribution of the demand in each period, sometimes including correlations [1]. Often, however, the assumption of normal distribution is not fulfilled. In practice, often a skew distribution is observed which has higher probabilities of large demands as predicted by the normal distribution. Using a normal distribution assumption leads to a systematic under-estimation of stockouts.

The distribution can differ from a Gaussian one from other reasons, too. For example, if there is a single large customer with unusually high order quantities, the demand process is quite different from a Gaussian model.

In reality, the demand process consists of a stream of orders which are characterized by a time point and an order quantity. When regarding the cumulative demand over a fixed period, under certain conditions a normal distribution is obtained motivating the Gaussian models. However, often there are large deviations which require describing the ordering process in an order-by-order fashion [2].

For calculating an optimum inventory strategy, the cost structure of the inventory has to be known. Different cost components contribute to the total costs, i.e. storage costs, ordering costs (fixed costs), transport costs, quantity dependent prices (discounts for large quantities).

While storage costs and fixed ordering costs are included in most inventory models, the other costs components are often neglected because they cannot be treated easily in an analytical mathematical framework. However, they may have a large impact on the costs and, thus, on the optimum inventory policy.

As seen in the above mentioned examples, the classical inventory models have severe limitations due to the simplifications which are needed in order to treat the inventory problem analytically. Consequently, the "optimum" inventory policies obtained with these models tend to be suboptimum.

In this situation, simulation methods often can help substantially. Simulation allows a large freedom in the specification of the processes and is not restricted to simplifying assumptions. Thus, the real situation of the inventory can be modeled with a much higher accuracy which leads to a higher accuracy of the models.

3. SIMULATION BASED INVENTORY MANAGEMENT TOOL

A simulation based inventory management tool has been developed in a joint project with the Institute of Data Analysis and Process Design, Winterthur, and Siemens Business Services. It consists of a graphical user interface which allows to specify customer demand, lead times and the cost structure of the inventory, and a simulation kernel which evaluates the performance of a given inventory policy. The results of the calculations are given in figures and graphical displays which helps the user to easily identify optimum policies.

The GUI was developed in Visual Basic where the process simulation and the mathematical calculations for the performance measurement were developed in MATLAB. GUI and simulation were integrated into a single executable which can be run under Windows as a standalone application. No additional software or licenses are necessary.

The demand process is modeled as a stream of single orders with different order quantities as a Compound Poisson Process [3]. The orders arrive as a Poisson process with a rate λ (mean number of orders per time unit). This model can be deduced from theoretical arguments and has been validated with real data [2]. The order quantity is assumed to be an iid process with a lognormal distribution. This assumption is reasonable if there are many customers of different size [2]. The mean of this distribution corresponds to the user defined quantity "mean order quantity". The standard deviation has been set to 50% of the mean. The lead times are assumed to be constant.

As inventory policy, a (r,Q)-policy is used which is characterized by the following rule: Each time the inventory level falls below the reorder level r, the fixed quantity Q is ordered. A policy is thus characterized by the reorder level r, and the reorder quantity Q. The task of the inventory manager is to find the optimum parameters with respect to costs and service level.

Different costs structures can be defined. The following cost components are integrated into the tool: Storage costs, fixed costs per order, interest rates for bound capital, transport costs depending on the ordered quantity, price in dependence of the ordered quantity.

There are two different modes of simulation: The policy evaluation mode and the inventory policy optimization mode.

In the policy evaluation mode, for a given policy (defined by reorder level and reorder quantity), the inventory processes are simulated (see **Figure 2**). The resulting mean inventory level and the stockout probability are calculated. Additionally, the different cost components as well as the total inventory costs are calculated.

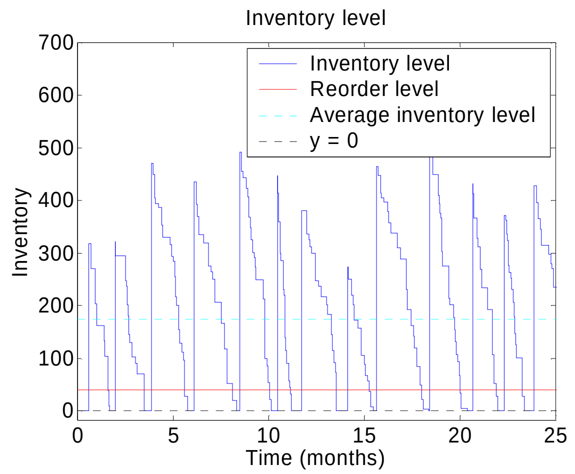

Figure 2. The inventory level as a result of a single run.

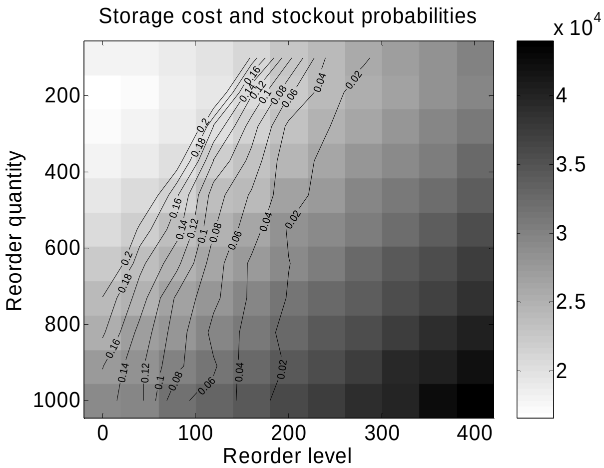

Figure 3. Result of the optimization run. Different inventory policies can be compared directly in terms of costs and stockout probability.

In the optimization mode, many different policies are simulated and the results of all simulations are shown graphically. Thus, the user may compare different policies and find the optimum one.

In **Figure 3**, the result of such a simulation run is displayed. The two axes correspond to reorder level and reorder quantity. Thus, a specific position in the picture corresponds to a specific reorder level and reorder quantity. The mean cost of a policy is indicated by colors. The stockout probability is indicated by isolines defining lines of constant stockout probability.

This plot may be used in different ways:

- If a desired service level is given, one can follow the isoline with this service level and determine the cost-minimum policy which creates the desired service level.
- If a certain maximum cost is given, one can determine the policy which maximizes the service level in the given cost range.

A graphical user interface allows entering the required parameters through input masks. The required parameters are process parameters (e.g. average reorder quantity, average number of orderings per month, average lead time), cost parameters (storage, personal, interests, …) and some parameters for the simulation runs.

4. EXAMPLES

4.1 INFLUENCE OF COSTS STRUCTURE TO OPTIMUM INVENTORY POLICY

The following example shows the effect of costs structure to optimum inventory policy. **Figure 4** , left side is the result of simulation runs with a constant price of the goods which are ordered. It can be seen that the inventory costs are smooth within the parameter space and that costs increase as we approach the downright corner of the figure (i.e. high reorder level and large reorder quantity). In contrast to this, the result of simulation runs where block-pricing is implemented in the model is shown in **Figure 4** (right side). In this simulation, the item costs depend on the ordered quantity:

Reorder quantity	<100	100-500	>500
Price per item	8	6	4

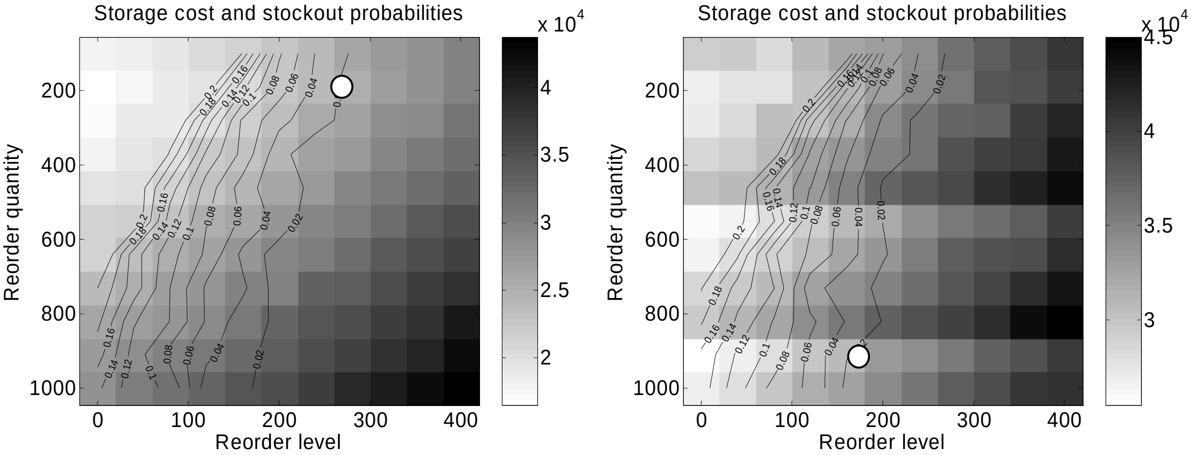

Figure 4 . Reorder prices with constant reorder prices (left) and quantity dependent reorder prices (right). The cost optimum inventory policy for stockout probability 0.02 is indicated by a white circle.

The surface of the inventory costs is no longer smooth and the optimum inventory policy is different from **Figure 4** , left side. Note that the stockout probability lines are not affected by the price structure except by some fluctuations due to the stochastic character of the simulation.

4.2 INFLUENCE OF STOCHASTIC PROPERTIES TO OPTIMUM INVENTORY POLICY

The stochastic properties of the ordering process may also have a noticeable influence on the optimum inventory policy. In the example below we use two different distributions for the times between two orders (interarrival times). In **Figure 5** it is shown that the distributions of the interarrival times have a strong influence on the service levels. On the left side of **Figure 5**, the simulation result for exponentially distributed interarrival times with a mean of 5 orderings per month is shown. On the right side, the results of a run with Weibull distributed interarrival times with the same mean demand is indicated. It can be seen clearly that the stockout probabilities differ which is caused by the different stochastic properties of the demand processes.

4. DISCUSSION

The presented approach of simulation based inventory management is based on a given structure for customer demand, lead time and cost structure which, in the present form, covers many cases which are not treated by the simple classical analytical models. It must be stressed, however, that the tool easily could be extended for including even more situations, e.g.

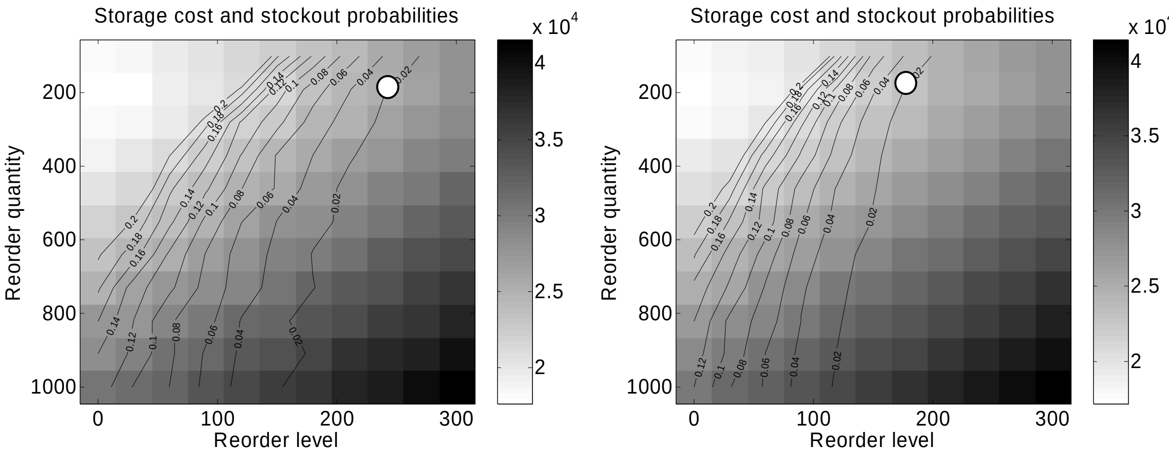

Figure 5. Left: Exponentially distributed interarrival times for the orders. Right: Weibull distributed interarrival times. The cost optimum inventory policy for stockout probability 0.02 is indicated by a white circle.

- Customer demand could be generated not only by a stochastic model (Compound Poisson Process) but also based on available data. Bootstrap methods could be used for generating new data sets from existing historical data. This would result in a non-parametric demand model which would be maximally adapted to the concrete situation.
- Other inventory policies, e.g. (s,S)-policies, could be integrated.
- Different or more complicated cost structures could be used
- Additional restrictions like, e.g., short lifetime of the stored products, could be added.
- Predictable deviations from the constant demand assumption (e.g. demand increase, seasonalities, …) can be added.

This list shows some extensions but is by no means exhaustive. The implementation of additional requirements or structures is simple because the tool is based on simulation instead of analytical models.

4. CONCLUSION

In this contribution, a recently developed inventory management tool based on simulation of inventory processes has been presented. The relevant information on customer demand, lead times and cost components can be specified by the user. For a given inventory policy, the performance in terms of cost and service level / stockout probability can be evaluated by simulation.

The tool can be used as a means of inventory optimization. The optimum inventory policy can be found easily by means of a multidimensional graphical representation of the inventory performance parameters costs and service level.

The main advantage of this tool in comparison with classical analytical inventory models is its flexibility which allows the correct description of nearly any possible situation.

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