Customers as Investment Objects – a New Perspective on Marketing

Christoph Heitz[1], Stephan Bütikofer[1], Marcel Dettling[1]

[1] Zurich University of Applied Sciences, Institute of Data Analysis and Process Design, Winterthur, Switzerland

Abstract

Customers have always considered as one of the most important assets of a firm. The concept of Customer Equity (Rust, Zeithaml, Lemon, 2000) has given this idea a conceptual and methodological foundation. Marketing can then be seen as the attempt to maximize Customer Equity by corresponding activities. For doing this, we focus on the Customer Lifetime Value (CLV) and ways for its maximization.

While the CLV has become a well-studied concept in marketing with a lot of literature coverage, much less work has been done in developing methods how to increase CLV. As an active increase in CLV always requires some activities, it is coupled with costs, and a cost-benefit analysis has to be made.

We develop a new model for such a cost-benefit-analysis, by considering a customer as an investment object: A customer treatment incurs costs today, while, on the other hand, it generates returns in the future, over the lifecycle of the customer. Since a customer can receive different forms of treatment, there are in fact different investment options available. When trying to increase the CLV, these options have to be compared to each other.

By formalizing this approach in a quantitative model, we create a framework for describing the cost-benefit profile of an individual customer. With this framework, optimum treatments can be identified, and the optimum height of investment into a single customer can be determined.

1   Introduction

The notion of customer lifetime value (CLV) of customers has become a popular research topic during the last decade (see Reinartz and Kumar, 2003, Rust et al 2000). It has been shown to be an important and useful concept in marketing, especially in micro marketing and direct marketing. The CLV is usually defined as the estimated present value of an individual customer's future cash flows, including future revenues as well as future costs (Jain and Singh, 2002). Conceptually, the notion of CLV is defined on an individual customer level. However, integrating all customers of a firm, a natural extension is to define the customer equity as the sum of the future profits of all customers (Blattberg and Deighton 1996, Rust et al. 2004, Gupta, Lehmann, Stuart 2005, Villanueva and Hanssens 2007, Berger et al. 2006). Customer Equity is usually considered as a good proxy for the value of a firm (Gupta, Lehmann and Stuart 2005). For calculating the CLV, different models have been proposed. An excellent overview on CLV

models in a wider context of customer-base analysis is given by Fader and Hardie (2009).

The concepts of CLV and CE have been used for solving relevant management questions on, e.g., allocation of marketing spending, understanding the connection between market spending and financial performance, and optimizing customer relationship management (Rust et al, 2000, Gupta and Lehmann, 2003, Rust et al., 2004, Kumar, Lemon and Parasuramin, 2006, Villanueva and Hanssens, 2007). Especially in relationship marketing, CLV has become central (Berger and Nasr, 1998, Malthouse and Blattberg, 2004).

The present paper builds on this body of knowledge by applying the concept of CLV and CE for optimizing the customer treatment on an invidual customer level. For this, we propose a new model of a customer as a set of investment options. We assume that each customer treatment is associated with costs which can be seen as investments in customers, as each marketing treatment is done to influence customers in order to increase the future profit coming from the customers. Thus, there is an expected return for this investment. In a CLV perspective, this return consists of an increase of the CLV, as the CLV covers all future profits of a customer. We can thus interpret a marketing strategy as an investment strategy whose ultimate goal is to increase the CE of the firm. So, each treatment has both costs and benefit, and we can apply a cost-benefit-analysis of the different investment strategies, which may be used for optimally investing in customers.

In this paper, we derive a quantitative model for implementing this idea, and we show how to apply this model in general and with some example cases.

The paper is structured as follows: In Section 2, we formulate the general model of a customer as a set of investment options. In Section 3, we derive a method of optimally investing in a set of customers, by using classical economic arguments from investment theory. Section 4 is a short conclusion.

2 Customer treatment: costs and benefit

2.1 General model

In this section, we describe our model for modeling customer treatments in a CLV framework, and we show how a cost-benefit analysis of a specific treatment can be done. We assume that a customer i is described by a CLV model, giving her customer lifetime value as a function of parameters which include behavioral ones. For example, the CLV may be calculated by the model of Rust, Zeithaml, Lemon (2000):

$$CLV = \sum_{t=1}^{\infty} m \cdot \frac{r^{t-1}}{\left(1+d\right)^{r-1}} = m \frac{r}{1+d-r} \qquad (1)$$

where m is the annual margin for the customer, d is an annual discount factor ($0<d<1$), and r is the retention rate ($0<r<1$). In this case, m and r are customer individual parameters, describing the behavior of a customer, and typically differing for different customers, whereas d is a global parameter which is identical for each customer.

Many different CLV models have been proposed in the literature (see e.g. Fader and Hardie (2009)). Our approach is not based on a specific CLV model but can be used for any CLV model.

The second ingredient of our model is the formalization of marketing actions and customer treatments. We assume that we consider a specific treatment period, for example the next year, where different marketing actions can be applied to a customer. For example, the firm might plan to send a specific marketing offer each 3 month during the next year. A specific customer may get the whole set of the four offers, but the marketing department might also decide to send him only two out of these four offers.

In general, we assume that a treatment T is composed of a set of different marketing actions or campaigns. We assume that N different campaigns T_j, j=1,…,N are run, and that each customer receives a subset of these campaigns. In practice, when the marketing department plans to launch N different campaigns during the next planning period, not all 2^N possible combinations of treatments are typically taken into account. In most cases, some of these treatments are excluded, for example because a specific treatment would result in too many customer contacts, or in customer contacts in a too short period. In addition, there may be rules on customer level which exclude treatments, for example when a customer has declared that she does not want to be contacted by phone.

So, with N planned marketing actions, the number of feasible treatments M is smaller than 2^N, and, in many cases, M is customer dependent.

Any treatment T should influence the behavior of the customer, e.g. trigger some additional revenues, or increase the customer loyalty. In a CLV context, this should be reflected in the CLV of the corresponding customer. So, assume that customer i is exposed to treatment j, we expect that the customer lifetime value CLV_i is changed, ideally it is increased.

An important property of a treatment j is thus the change of CLV_i. We denote this change with ΔCLV_{ij}:

$$\Delta CLV_{ij} = CLV_i(\text{with treatment j}) - CLV_i(\text{w/o treatment}) \qquad (2)$$

The reference case is that customer i does not get any treatment at all, and the effect of the treatment is the change of the CLV under this treatment. Note that the quantity ΔCLV_{ij} can be considered as the net benefit of the treatment. This benefit includes more than the direct reaction of the customer to the marketing actions, for example a one-time additional sales. Since the CLV includes the total future behavior of the customer, any changes in the future behavior that are created by a marketing action is integrated as well. As an example, a marketing action that increases the loyalty of a customer without changing the sales also leads to an increase of the CLV.

Every treatment also incurs some costs. This is the second element for modeling a treatment. We denote the cost of treatment j by C_{ij}, and we allow the costs to depend on the specific customer i. Note that C_{ij} are not the costs for the campaign but rather the costs per customer of a campaign. Summing up these costs over all customers that are subjected to the campaign yields the total campaign costs.

So, a treatment j which is applied to customer i, is modelled by the two quantities cost and benefit: (C_{ij}, B_{ij}), where we identify B_{ij} with the change of CLV: $B= \Delta CLV_{ij}$. For a given customer, the set of all possible treatments can be visualized in a twodimensional cost-benefit space as shown in Fig. 1. In Fig.1, a set of 15 different treatments is shown. The treatment 1 has costs C_1 and a benefit B_1, wheres the treatment 2 is described by (C_2, B_2).
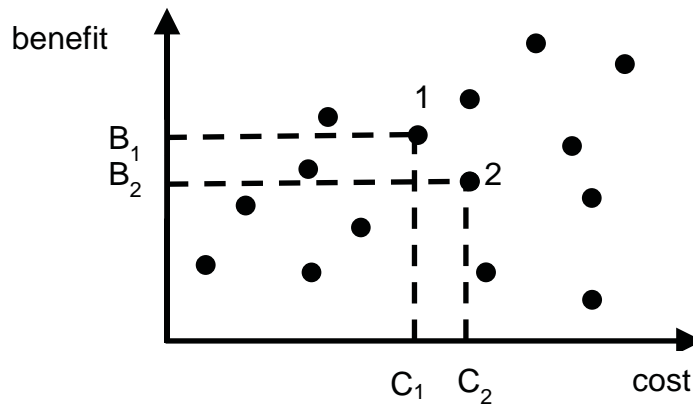
Fig. 1: A set of M possible marketing treatments for a specific customer can be modelled as a set of M points in a 2-dimensional cost-benefit space. The cost dimension denotes the costs of the treatment, and the benefit is the change of the CLV of the considered customer.

Note that a visualization such as in Fig. 1 is customer specific as each customer may react differently to a given treatment. In general, different customers thus have different such pictures, even if the set of treatments is identical.

2.2 Example

As an example, we consider a service provider which serves customers continuously. The customers pay for the service with a monthly subscription fee. Examples for such providers are telecom providers or video rentals such as Netflix. In addition, the service provider sells products like a retailer which are related to the service. For example, a video rental might sell video DVDs, in addition to the rental service.

The company plans to run four marketing campaigns in the next 12 months, one each three months. The campaigns are for selling products, but are expected to increase the customer loyalty in general as well.

We assume that it is January, and the first campaign will run this month. The second campaign is planned for April, the third for July, and the last one for October. The campaigns have different costs of 10, 15, 4, and 18, respectively. These costs are cost per customer, and are identical for each customer.

Each customer can be exposed to any combinations of these campaigns. For example, a customer may be selected for the first and the third campaign. This treatment would have costs of C=10+4 = 14. There are 2^4=16 possible treatments, including the reference treatment of excluding the customer of all campaigns. We code a treatment with a sequence of four binary numbers (a_1, a_2, a_3, a_4), where a_k is either 0 (customer will not receive campaign k), or 1 (customer will receive campaign k). For example, the treatment T=(1,0,0,1) means that the customer is selected for the first and the fourth campaign, but is not subjected to campaigns 2 and 3.

The CLV model is assumed to have the form of Eq. (2), with an annual discount factor of 0.1 (10% discount rate).

We consider a customer with an annual margin of m=120, and an annual retention rate of r=0.7. This means that the probability that this customer is still a customer one year from now is 70%. We can transform this to a monthly retention rate of $\tilde{r} = 0.9707$, since 0.9707^{12}=0.7.

The CLV without any campaigns would be

$$CLV_i\left(T = (0,0,0,0)\right) = 10 + \frac{0.9707}{1+\tilde{d}} \cdot 10 + \frac{0.9707^2}{(1+\tilde{d})^2} \cdot 10 + \frac{0.9707^3}{(1+\tilde{d})^3} \cdot 10 + ...$$
$$= 10 \cdot \frac{0.0087}{1+0.0087-0.9707} \tag{3}$$

where $\tilde{d}$ is the monthly discount factor ($\tilde{d} = 0.0087$ in our example), and each term of the sum denotes the profit of a month in the future.

As for the effect of the marketing campaigns, we assume the following:

› The probability that the customer reacts to the campaign is 30%, in which case an additional one-time margin of 20 is obtained in form of an additional sale.

› If the customer reacts to the campaign, her annual retention rate is increased to r=0.9 (or $\tilde{r} = 0.9913$) for the following 5 months, then it drops back to 0.7 as before. This means that it is very unlikely to loose this customer in the five months following a sale.

So, there are different effects of the treatment of a customer:

› A treatment increases the expected margin. The more treatments, the more additional margin is created. This increases the CLV of the customer.

› A treatment increases the loyalty over a certain period of time, if the customer reacts. Thus it increases the lifetime of the customer, and accordingly her CLV.

For example, consider a treatment T=(1,0,0,0) where the customer only receives the first marketing campaign. The CLV then can be calculated as

$$CLV_i\big(\mathrm{T}=(1,0,0,0)\big)=0.7\cdot CLV_i\big(T=(0,0,0,0)\big)$$

$$+0.3\cdot\bigg(20+10+\frac{0.9913}{(1+\tilde{d})}\cdot 10+\frac{0.9913^2}{(1+\tilde{d})^2}\cdot 10$$

$$+\frac{0.9913^3}{(1+\tilde{d})^3}\cdot 10+\frac{0.9913^4}{(1+\tilde{d})^4}\cdot 10$$

$$+\frac{0.9913^4\cdot 0.9707}{(1+\tilde{d})^5}\cdot 10+\frac{0.9913^4\cdot 0.9707^2}{(1+\tilde{d})^6}\cdot 10$$

$$+\frac{0.9913^4\cdot 0.9707^3}{(1+\tilde{d})^7}\cdot 10+\frac{0.9913^4\cdot 0.9707^4}{(1+\tilde{d})^8}\cdot 10+.....\bigg)$$

The CLV under treatment T=(1,0,0,0) is identical to the reference CLV with 70% probability (in the case where the customer does not react to the campaign). With 30% probability, there is an additional sales contribution of 20, and the CLV of the subscription revenues is changed by changing the churn rates for the four months after the campaign.

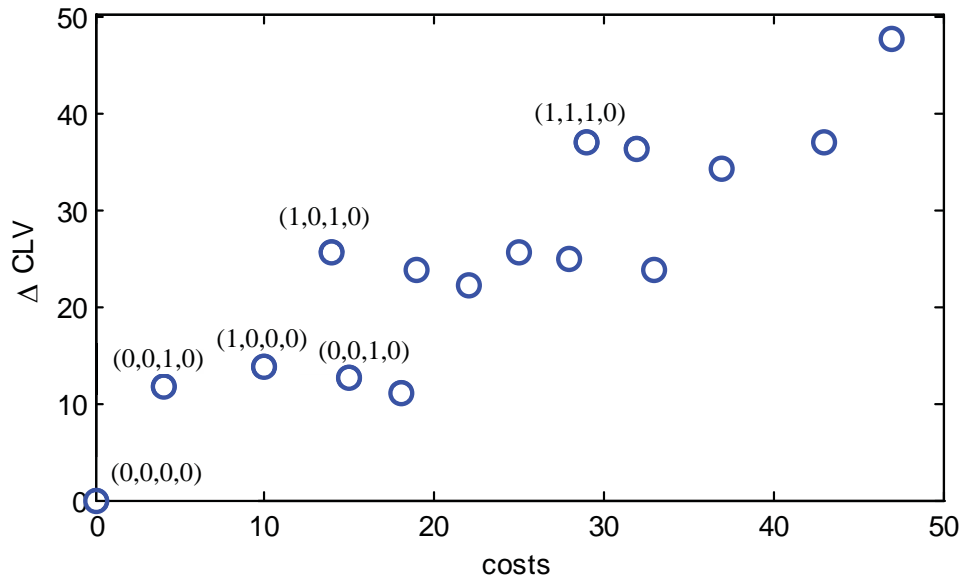Other treatments can be calculated similarly.

Fig. 2: Plot of the 16 possible treatments of a customer in the cost-benefit space. Each dot denotes a specific treatment, i.e. a selection some of the four planned campaigns. The treatments are indicated for some selected treatments.

In Fig. 2, the change ΔCLV vs. the costs of each of the 16 possible treatments are shown. The treatment (0,0,0,0) has no costs, and ΔCLV=0. The cheapest treatment consists of a single campaign in month 7, i.e. the treatment (0,0,1,0). This incurs costs of 4. Since the acceptance probability is 30%, the expected additional sales is 0.3·20 = 6,

which is discounted with the factor $0.9913^7=0.9404$, since the sales is in month 7. As can be seen in Fig. 2, this increase of $0.94*6=5.64$ does not account for the full increase of the CLV – an additional increase in CLV is obtained by the change of the retention rate in case of acceptance. Taking both effects together, an increase of 11.8 is achieved.

The next cheapest option is T=(1,0,0,0), which generates costs of 10. The change in CLV is a bit higher since the sales is earlier, thus less discounted. Additionally, the change in retention rate is earlier, thus leading to higher expected submission revenues than for treatment (0,0,1,0). Treatment (0,1,0,0) has costs of 15, and a ΔCLV which is in between the ones of (1,0,0,0) and (0,0,1,0).

The treatment (1,0,1,0) generates costs of 10+4=14, and ΔCLV=25.7. Since the acceptance probability of the two campaigns has been chosen to be independent, each campaign adds an expected additional sales of 5.64. The rest of the CLV increase is generated by the increased customer loyalty, and is a nonlinear function of the number of campaigns.

So, each possible treatment can be assessed with respect to its cost and its benefit, a visualization of which gives a complete overview of the different options that are available for the customer. The task is now to choose one of these options (see next section).

Remarks:

› The example demonstrates that non-trivial effects such as the superposition of different campaigns, with a mix of additional sales and change of the retention rate can be modelled. However, it should be noted that the approach also works for even more complex dynamics. An example for more complex dynamics has been shown in Heitz et al. (2011). There, a non-Markovian state-space dynamics with complex dynamical rules has been studied. The only requirement is that a CLV calculation can be performed, but the CLV model to do this calculation can be chosen freely, and according to the specific case.

› Restrictions such as choosing the acceptance probabilities as constant for all campaigns and independent from each other, can be released. The calculation of the CLV becomes more complicated, but the general approach is still feasible.

› Furthermore, both the costs and the change in CLV can be made customer individual, as done in Heitz at al (2011). In this case, a plot such as Fig. 2 becomes a description of the choices for a specific customer. We will come back to this in the next section.

3 Optimum treatments

3.1 The Pareto frontier

In the last section, we have shown that a specific treatment of a customer can be viewed as an investment option with associated costs and benefits, and can be visualized in a

two-dimensional cost-benefit space. In this section, we will analyze the taks of finding an optimal treatment for the customer.

When analyzing Fig. 1 or Fig. 2, it is immediately clear that there are some treatments that make no sense, since they are more expansive than other treatments, but yield a lower benefit. So, when faced with the decision problem of which treatment to choose, we should sort out all treatments that do not lie on the Pareto frontier (for an introduction in multi-criteria optimization and Pareto optimality, see e.g. Censor (1977) or Da Cunha and Polak (1967)). In Fig. 3, the Pareto frontier of the treatments of Fig. 1 is shown. Each treatment on the Pareto frontier is Pareto-optimal: There is no other treatment in the whole set of treatments that generates more added CLV with lower costs.
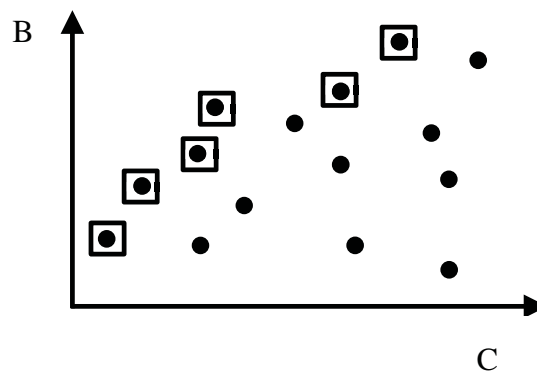

Fig. 3: The Pareto frontier of the treatment options of Fig. 1 consists of all the Pareto-optimal points (indicated by squares).

Choosing a treatment on the Pareto frontier makes sure that, for the given invested money (costs of the treatment), there is no other treatment that generates more value. In this sense, the treatments of the Pareto front are all optimal.

However, there are still different options available. The treatments of the Pareto frontier have the property that, for increasing costs, they yield increasing benefit. So, the more we invest in a customer (in the form of marketing campaigns), the more we get in return (in the form of additional CLV). Thus, the Pareto frontier can be interpreted as a characteristic of the customer as an investment option: We have a choice of how much we would like to invest in a customer, and each investment has a specific return.

Given a specific customer, we could now determine the optimum treatment by maximizing the difference between investment and ΔCLV:

$$\Delta CLV - \text{costs} = \max. \tag{4}$$

This would lead to a maximum return on investment, expressed in the chosen metrics of the CLV. When done over all customers, this would, in turn, result in an optimal marketing budget. However, in practical applications it is often not possible to apply this

simple method since there are restrictions on the total budget for marketing, or there are operational restrictions such as the number of available agents for outbound calls, or channel restrictions, or the like.

In the next subsection, we therefore propose a more general way of optimizing the marketing activities over a given customer base, by introducing a budget constraint.

3.2 Marketing optimization

In this section, we investigate the question of finding the optimum treatments for a given set of customers under a specified budget constraint.

Let's assume that we consider a set of M customers i=1,…,M. We denote the costs of a specific treatment j of customer i with C_{ij}, and the resulting benefit (in terms of ΔCLV) by B_{ij}. The budget constraint thus reads

$$\sum_{i=1}^{M} C_{ij} \leq B \tag{5}$$

with a given total budget B. The task is to determine the optimum treatments T_i for each customer such that the sum of the benefits is maximal:

$$\sum_{i=1}^{M} B_{i,j(i)} = \max \tag{6}$$

where j(i) denotes the treatment of customer i.

In order to address this problem, we decompose the investment into a customer in a sequence of partial investments: We start with the leftmost treatment (i.e. the treatment with minimum costs), and increase the investment stepwise by adding more budget.

In Figure 4 (a), we start with treatment A, with is the minimum investment that may be chosen for this customer. Note that, in most cases, treatment A is the origin (C,B)=(0,0), if there is the option to have the NULL treatment for this customer. However, this may not be the case, as marketing rules might force at least a minimum treatment. The second partial investment might, in principle, lead from A to B, to C, or to any other treatment on the Pareto frontier. However, in order to get the maximum return for this second investment, one should choose the partial investment that generates the maximum added CLV per invested dollar. This requirement leads to the selection of the partial investment leading to policy B. The third partial investment leads to policy D, after which it is policy F that is chosen (see Figure 4 (b)). Note that this procedure omits some of the treatments of the Pareto frontier. We call the remaining treatments, i.e. the treatments A,B,D,F, the optimum-investment frontier.
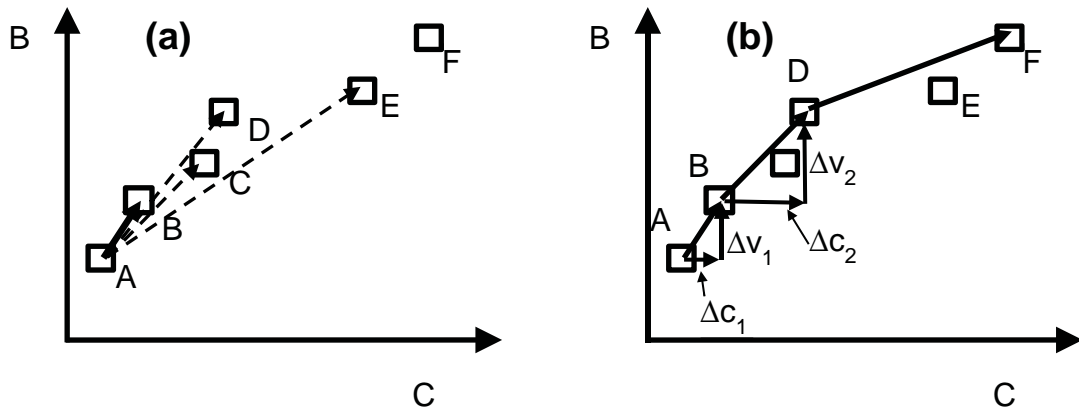
Fig. 4: The investment in one customer can be decomposed in a sequence of partial investments, starting with the leftmost treatment and increasing the investment by sequentially increasing the budget for the customer, taking into account only treatments on the Pareto frontier. (a) After the initial investment for treatment A, the second investment step may lead to any other treatment on the Pareto frontier. However, treatment B yields the maximal added value per invested dollar. (b) Sequence of optimum partial investments with decreasing returns.

In Fig. 5, the optimum investment frontier of the customer of the example in Section 3 is shown (cmp. Fig. 2)
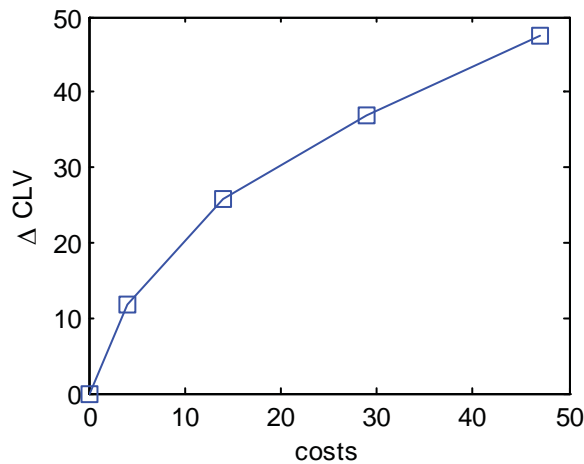

Fig. 5:The optimum investment frontier of the customer of the example in Section 3.

We call the ratio of added value and partial investment the Marginal Cost Effectiveness MCE (cmp., e.g., Uddin et al, 2013).

$$MCE_k = \frac{\Delta B_k}{\Delta C_k}$$

(7)

where ΔB_k is the additional investment in step k, and ΔC_k is the additional benefit obtained in step k.

It can be shown easily that, for the case of a large number M of customers, the optimum investment strategy is the following:

(1) Allocate the minimum investment level m_i to each customer i. Reduce the available budget by $\sum_i m_i$

(2) For each customer: calculate the MCE for the partial investment leading to the next investment level on the optimal-investment frontier.

(3) Find the treatment with the highest MCE. Say this is customer j.

(4) If the available budget is greater than the necessary amount for the partial investment of customer j, then
   a. Increase the investment in customer j by this partial investment
   b. Reduce the available budget by this amount
   c. Go to Step 2.
   d. If the remaining budget is smaller than the necessary amount for the partial investment of customer j, take the customer with the next smaller MCE and go to start of step 4.
   If there is no partial investment that can be done with the available budget, then stop.

For the case of a continuous optimum-investment frontier, this optimum solution is equivalent to the Equimarginal Principle, also known as Gossen's Law (Gossen 1983). The Equimarginal Principle states that the optimum allocation is characterized by the fact the derivative $\frac{dB}{dC}$ is equal for each customer, and is a well-known principle in economics.

Note that the problem of optimal allocation is an investment problem, but it cannot be solved by optimizing each customer individually. The central issue is not to find an individually defined optimal investment level for each customer. In contrast, the optimal investment levels are found by comparing different partial investment options (for the different customers) and choosing the best one. The basic property which is used for deriving the above defined procedure is the concavity of the optimal-investment frontier, i.e. the fact that returns are decreasing with increasing investment.

As a result, we end up with specific treatments for each single customer, which in turn defines the optimum treatment policy for the considered customer base.

As an example for demonstrating this procedure, we re-analyze the example of Section 2, assuming that we have three different customers:

› Customer 1 has exactly the properties as stated in Section 2.
› Customer 2 has a higher margin of 20/month, instead of 10/month. Thus, the CLV of customer 2 is higher.

› Customer 3 has again a margin of 10/month like customer 1, but his acceptance proability is only 20% instead of 30%. So, his CLV is identical to customer 1, but the change of CLV in response to a campaign is smaller.

In Fig. 5, the optimal-investment frontiers are shown for these three customers. As can be seen, the investments in customer 2 generally create a higher additional benefit than for customer 1 or customer 3.
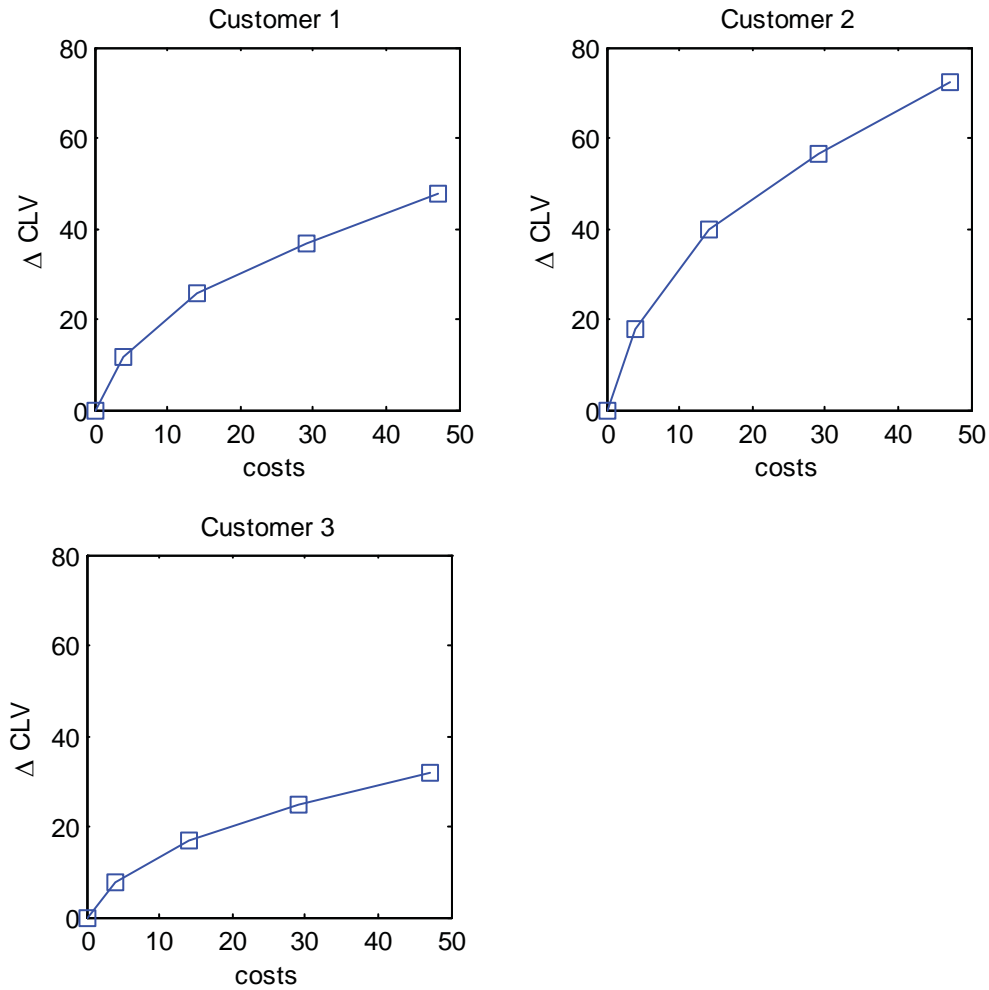

Fig. 6: The optimal-investment frontier of the three customers

In Table 1, the numerical data for the three customers is shown. For each investment step, the resulting value of benefit B and the MCE of the corresponding investment step is shown.

costs	Δcosts	customer 1		customer 2		customer 3	
		B	MCE	B	MCE	B	MCE
0	0	0	0	0	0	0	0
4	4	11.7876	2.946899	17.93285	4.483213	7.858398	1.964599
14	10	25.68096	1.389336	39.77203	2.183918	17.07541	0.921702

29	15	36.87612	0.746344	56.36941	1.106492	24.87689	0.520099
47	18	47.52607	0.591664	72.17364	0.878013	32.05854	0.39898

Table 1: Numerical data for the partial investments for the individual customers

As can seen in Table 1, when allocating a given marketing budget stepwise to the pool of customers, the first partial investment is to be made for customer 2: investing 4 for getting an increase of CLV of $\Delta B=17.93$, corresponding to MCE=4.48. The second investment is the first partial investment for customer 1 (MCE=2.95), leading to additional investment of 4, and additional CLV of 11.78. The third investment is the second partial investment for customer 2, with an additional investment of 10, and additional benefit of 39.77-17.93 = 21.8. This investment has an MCE of 2.18. The fourth investment is now on customer 3, with an investment of 4, and a return in form of additional CLV of 7.86.

We can now proceed with this procedure until the total budget is used.

For example, if a total budget of 50 is given, then we end up with the following solution: We should invest 29 in customer 2, 14 in customer 1, and 4 in customer 3. In terms of treatments, this corresponds to treatment (1,1,1,0) for customer 2, (1,0,1,0) for customer 1, and (0,0,1,0) for customer 3. The total investment is 29+14+4 = 47. The next possible investment would lead to a total budget which is larger than 50.

Note that the solution of the investment problem not only leads to an optimal investment over the complete set of customers, but also to the optimal treatments for each individual customer.

4 Conclusion

We have presented a method for optimally investing in customers in the form of marketing treatments, such that over a set of customers, the investment yields to maximal total return in form of increased customer equity. The basis for this method is a model that describes a customer as a set of investment options, each option corresponding to a specific treatment. Each investment option is described by a cost-benefit pair, the cost meaning the treatment costs, and the benefit meaning the increase of CLV of this customer under the chosen treatment. We have seen that the set of investment options can be reduced to a subset of the optimal-investment frontier, which selects the Pareto-optimal treatments that fulfill the addition requirement that each partial investment has an optimum return on investment (MCE).

By decomposing the total investment in the customers into small partial investments, we could derive a budget allocation heuristics which is optimal in the case of a large number of customers, making sure that the total investment yields a maximum increase of the customer equity.

The model can be applied to arbitrary CLV models, and is thus generally applicable to a large number of marketing problems.

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