Original Papers

Polish Psychological Bulletin
2008, vol. 39 (1), 42-46
DOI - 10.2478/v10059-008-0006-7

Jakub Barbasz *
Zbigniew Stettner **
Michał Wierzchoń **
Krzysztof T. Piotrowski **,***
Anna Barbasz ****

How to estimate the randomness in random sequence generation tasks?

The aim of the paper was to discuss the accuracy of the multiple indexes used for random sequences generation results calculation. In the first part of the paper the models explaining deviations from randomness were presented. The key role of the structural limitations interpretation was suggested. Secondly, the multiple indexes of the deviation from randomness used in random sequence generation task studies were presented. The authors concluded that too many indexes are used in the studies of deviation from randomness. In order to avoid such problems two indexes were proposed: entropy and correlation function. The last part of the paper presents the preliminary version of the mathematical of random sequences generation in which the limited capacity of the short-term memory assumption was introduce.

Keywords: *Random sequences generation task, entropy, correlation function*

Introduction

Random sequences generation [RSG] is a popular task in experimental psychology. Using this task multiple studies on quite remote problems where examined. First of all, RSG was used to examine whether humans could produce random sequences and whether they can judge randomness of the sequences (see: Falk & Konold, 1997; Lopes & Oden, 1987; Vandierendonck, 2000). RSG is also used as a test of executive functions (dual task paradigm, see: Miyake, Witzki & Emerson, 2001; Friedman & Miyake, 2004). The task is also used in neuropsychological research (frontal lobe damages, see: Brugger, Monsh, Salmon & Butters, 1996; Brown, Soliveri & Jahanshahi, 1998).

Random generation of numbers, letters, or time intervals seems to be a very difficult task. Multiple studies suggested that people cannot react randomly. Sequences produced by humans deviate from randomness in numerous ways: the distribution of the possible options is usually unequal (Rapoport & Budescu, 1997; Budescu, 1987; Kareev, 1992; Falk & Konold, 1997), participants tend to avoid immediate repetitions of the same reaction (Brugger, 1997) and usually some type of counting is observed (Baddeley, Emslie, Kolodny & Duncan, 1998). The conclusion that humans cannot generate random sequences seems to be well-founded. Therefore the question occurs, why people failed to produce random sequences?

Why the deviations from randomness are observed?

At least a few concurrent accounts of cognitive processes involved in RSG were proposed, most of them being very limited. Wagenaar (1972) summed up the RSG studies done till early seventies. Researchers cited in this metaanalysis explained deviation from randomness with respect either to deficiency of understanding the concept of randomness or to the limitations of short term memory.

Deficiency of understanding the concept of randomness was investigated in Lopes and Oden (1987) studies. In one of their experiments they have proved that randomness was improved when appropriate feedback information is provided. Nevertheless, in another study they have observed that even professional statisticians cannot produce random

* Institute of Catalysis and Surface Chemistry, Polish Academy of Sicence, email: ncbarbas@cyf-kr.edu.pl
** Institute of Psychology, Jagiellonian University
*** Center for Educational Studies, Jagiellonian University
**** Department of Analytical Biochemistry, Faculty of Biotechnology Jagiellonian University

sequences. These results suggest that even understanding the concept of randomness does not guarantee randomness of produced sequences.

The second approach, relating deviation from randomness to the limitations of short-term memory, was investigated by Kareev (1992). This author has claimed that people understand the concept of randomness quite well – when only short fragments of sequences generated by participants in the RSG experiment were analyzed, the basic criteria of randomness were fulfilled. Therefore, he has argued, deviation from randomness in longer sequences occurred mainly due to short-term memory limitations. Similar conclusions can be found in Rapoport and Budescu (1997). They expleined the deviation from randomness in RSG task as a result of structural limitations (working memory capacity) as well as a result of using specific heuristics (local representativness - tendency to balance frequency of different possibilities in short fragments of sequences).

In a more recent metaanalysis proposed by Brugger (1997) deviation from randomness has been explained in terms of control processes. This view is currently quite popular. Vandierendonck (2000), for example, proposed the model of control processes involved in the random time interval generation task. One of the processes is responsible for generating responses and the other process is responsible for monitoring the results (involving a decision whether the string is random enough or not). In the case of a negative decision the second control process modifies the outcomes of the first process. Similar model was proposed by Towse and Valentine (1997) that included two sets of processes: those responsible for generating reactions (or rather candidates for reaction) and those involved in assessing the randomness.

The role of control processes in RSG was investigated also in the neuropsychological studies. Brugger et al. (1996) have observed that Alzheimer patients exhibit a strong tendency for counting in RSG task. This tendency was also positively correlated with a degree of dementia, as well as with results of other neuropsychological tests assessing executive functions. Very interesting results were also obtained by Brown, Soliveri & Jahanshahi (1998). They have reported that patients with Parkinson disease and a control group did not differ at general indexes of randomness in RSG task, although the two groups differed in terms of strategies used to generate the sequences. The evolution of Baddeley explanations of deviations from randomness provides a good example of the progress in the field. In one of his first studies on randomness Baddeley (1966) has argued that capacity limited mechanism must be involved in RSG task because randomness depends on pace of generating numbers. In his study randomness was decreasing when participants were asked to speed up. However, he has not specified the exact nature of these limitations. In his more recent works Baddeley (1996; Baddeley et al., 1998) stresses the role of control processes in random generation suggesting that switching between different retrieval strategies is the most important factor behind (but also e.g. monitoring).

To sum up, the deviations from randomness could be explained as a result of: (a) a poor understanding of a concept of randomness or a lack of experience in random sequence production; (b) structural limitations (e.g. short-term memory capacity limits); or (c) limitations of control processes.

In this paper we focus on the structural limitation explanation. We believe that in order to understand mechanisms of generation of randomness by humans it is necessary to explain the structural limitations, which affects information processing. We think that other explanations are secondary to this basic kind of limits. In order to explain control processes operating on the structures which are involved in the generation of random sequences, the basic processes and a role of these structures (e.g. working memory) should be explained.

How the randomness could be measured?

To assess the randomness of sequences generated by humans, many different indexes were suggested. Each index emphasizes only one part of the phenomenon and measures its slightly different aspects (e.g. distribution of elements in the sequence, dependencies between contiguous reactions, counting tendencies, etc.), but they could be classified into one of a few groups (Miyake et. al., 2000; Friedman & Miyake, 2004; Towse & Neil, 1998). The first group identifies concerns equality of distribution of different possibilities. Second factor contains indexes concerning relationships between consecutive responses. The third factor concerns repetitions of the same options in different distances. Towse and Neil (1998) have described main indexes used in random generation tasks. We present them in short bellow.

In the first group we can mention R (redundancy, Towse & Neil, 1998) – an index describing the distribution of possible responses. This measure is based on the information theory assumption (Shannon, 1948) - a sequence with equal distribution of elements yields maximum amount of information. If the distribution of the possibilities is not equal, the redundancy in the material could be observed, so the predictions about the likelihood of appearance of certain items are increased. The second index in this group is Random Number Generation (RNG; Evans, 1978), describing a distribution of pairs of elements. Similar index is RNG2, describing a distribution of pairs of reactions separated by one other reaction.

The examples of indexes from the second group are: number of ascending pairs (ASC) reflecting a tendency for counting, number of descending pairs (DESC) measuring backward counting tendency and TPI (Turning Point Index; Towse & Neil, 1998), which allows to estimate the regularity of the sequence by counting *turning points*. These points can be observed when the order of the sequence switches from increasing into decreasing, or the other way around (e.g. the sequence "1, 3, 5, 2, 1" has one turning point - 5, while at the sequence "1, 3, 7, 5, 2, 4" two of them could be observed - 7 and 2). It is possible to determine the number of such points in a real random sequence of a given length and TPI is calculated by dividing number of observed turning points by the number of expected turning points. Another index in this group is RUNS (Towse & Neil, 1998), defined as a variance of length of ascending subsequences.

Indexes in the third group measure biases for repeating the same reaction over different distances, as exemplified by the Phi index, described by Towse & Neil (1998).

In the most of the previous studies only selected indexes were used and therefore only selected features of randomness were analyzed. We think that this practice is inappropriate for at least a few reasons. First of all, it is difficult to compare the results across various RSG experiments when different measures are used. Secondly and perhaps more importantly, results of studies that claim the relationships between ability to generate randomness and level of other cognitive variables may be confusing. These variables may be connected only with a specific aspect of randomness generation ability. On the other hand, it is always possible that the connection between two variables really exists, but it simply cannot be found by using only selected measures. Finally, using many measures in one study at the same time may be confusing as well, providing results which are difficult to analyze, understand and interpret. Therefore we think there is a need of simple measures of randomness that would cover most aspects of the phenomena measured by the indexes described above. Therefore, we would like to propose two mathematical indexes that meet these criteria.

The first index is entropy proposed by the mathematical theory of communication (Shannon, 1948), defined by equation:

$$H(x) = \sum_{i=1}^{n} p(i)\log_2\left(\frac{1}{p(i)}\right) = -\sum_{i=1}^{n} p(i)\log_2 p(i) \quad (1)$$

where: *H(x)* is entropy, *p(i)* is probability of i^{th} events, *n* is number of possible events. For example, maximal entropy for a sequence of 10 different events is 3.32193 bits/character.

The redundancy index described above (Towse & Neil, 1998) is resolved to enropy. Redundancy is calculated as a difference between entropy of ideal chaotic sequence and entropy of analyzed sequence. The entropy index was used in a few of RSG studies (Vandierendonck, 2000b).

The adequacy of the entopy index is well documented, but we can find sequences in which events are correlated and its entropy equals maximal value. Let us consider

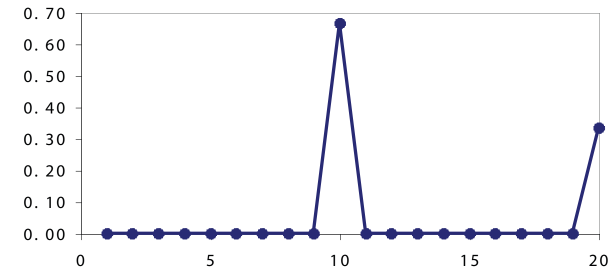

Figure 1. Plot of ***Cf*** for example sequence S_E.

sequence S_E as an example.

$$S_E=(1,2,3,4,5,6,7,8,9,10,1,2,3,4,5,6,7,8,9,10,1,2,3,4,5,6,7,8,9,10) \quad (2)$$

The entropy of the S_E sequence equals 3.32193… bits/character (maximal value), but with no doubt the events in the sequence are correlated. The example suggests that entropy index is not always sufficient for randomess estimation. Thus, to characterize the sequence better, one more index should be used. We propose to use the correlation function (Cf) as the additional measure of the deviations from randomness.

The ***Cf*** index inform us about distance of interactions between two elements in a sequence. Using ***Cf*** it is possible to measure how distant items could interact each other. We believed that the index could be interpratated as an information about structural limitations of the information processing. The index gives information about distant relations which could be interpretated as an information about limitation of the working memory capacity. From this perspective other indexes describe above give us an information about a strategy that people use to generate random sequences. The Fig. 1 present the plot of Cf for the sequence S_E (2).

Equlibrum defined ***Cf***, is given by:

$$Cf(i) = \frac{X(i)}{n} \quad (3)$$

where ***X(i)*** is a number of pairs of an identical events separated by *i* position at the sequence with the *n* number of elements. A ***Cf*** can be calculated for $i = 1...(n-1)$, but usable range in most of the RSG experiments, where participants produce more or less one-houndred elements sequences, is <1,20>. From statistical point of view, to use higer *i* we should ask participants to produce much longer sequences. Maximal value of $X(i)$ (for sequence of all identical events) can be calculated from equation:

$$X_{max}(i) = n - i \quad (4)$$

Where *n* stands for length of the sequence and *i* represents the position of the element in the sequence.

By substituting Eq.(4) into Eq.(3) we could calculate the Cf_{max} (maximal value of ***Cf)***

$$Cf_{max}(i) = \frac{n-i}{n} \quad (5)$$

Table 1
The values of Cf for two different sequences and the sequence with the maximal value of Cf.

S_1=(1,1,2,1,0,2,3,0,2,3)	S_2=(1,4,2,1,0,5,1,2,7,6)	S_{max}=(1,1,1,1,1,1,1,1,1,1)
Cf(1)=0.1	*Cf(1)*=0	$Cf_{max}(1)=0.9$
Cf(2)=0.1	*Cf(2)*=0	$Cf_{max}(2)=0.8$
Cf(3)=0.5	*Cf(3)*=0.2	$Cf_{max}(3)=0.7$
Cf(4)=0	*Cf(4)*=0	$Cf_{max}(4)=0.6$
Cf(5)=0	*Cf(5)*=0.1	$Cf_{max}(5)=0.5$
Cf(6)=0.1	*Cf(6)*=0.1	$Cf_{max}(6)=0.4$
Cf(7)=0	*Cf(7)*=0	$Cf_{max}(7)=0.3$
Cf(8)=0	*Cf(8)*=0	$Cf_{max}(8)=0.2$
Cf(9)=0	*Cf(9)*=0	$Cf_{max}(9)=0.1$
$\int_1^9 Cf_1(i) = 0.8$	$\int_1^9 Cf_2(i) = 0.4$	$\int_1^9 Cf_{max}(i) = 4.5$

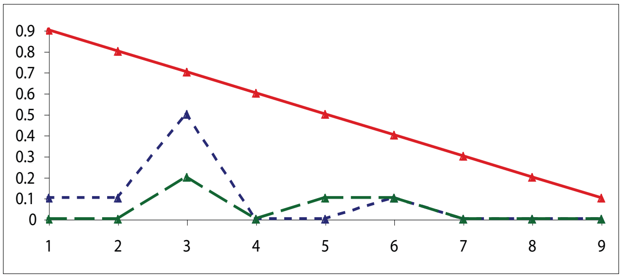

Figure 2. Plot of ***Cf*** for sequences: dot line – S_1, dash line – S_2, continuous line – S_{max}.

To asses the power of connections between the elements of the sequence the integral of Cfmax can be calculated. Equilibrium is defined by:

$$\int_i^{n-1} Cf_{max}(i) = \int_i^{n-1} \frac{n-i}{n} = \frac{1}{2}(n-1) \quad (6)$$

If not all elements of the set are correlated in 100 percent:

$$\int_i^{n-1} Cf(i) < \int_i^{n-1} Cf_{max}(i) \quad (7)$$

Let us compare two sequences with different ***Cf*** to the sequence with the maximal corelations as an illustration of the equlibrum (7) (see table 1).

This data shows that S_1 is correlated more strongly then S_2.

$$\frac{\int_1^9 Cf_1(i)}{\int_1^9 Cf_{max}(i)} = \frac{0.8}{4.5} = 0.1(7) > \frac{\int_1^9 Cf_2(i)}{\int_1^9 Cf_{max}(i)} = \frac{0.4}{4.5} = 0.0(8) \quad (8)$$

Figure 2 presents the plots of Cf for the sequences.

In our opinion, the S_E and ***Cf*** indexes gives us enough information about randomness of the sequences and could substitute the indexes described previously. The indexes could be also used as a measure of the human randomness generation data as well as to fit the results of experiments with data obtained by using computational model of the random sequences generation which we would like to propose below.

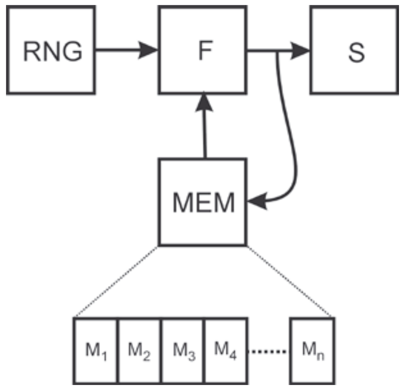

Figure 3. Schematic diagram of mathematical model of random sequence generation. RNG – random number generator, F – correction filter, S – output sequence, MEM – short-term memory, $M_1 \ldots M_n$ – cells of memory.

The mathematical model of the random sequence generation

Our model assumes the limited capacity of the short-term memory. This assumption was proposed by Miller (1956), and since that time is present at most of the short-term memory (Atkinson & Schiffrin, 1968) and working memory models (Baddeley & Hitch, 1974). Simmilarly to Miller (1956), we assume that the short-term memory capacity is constant in time.

When the first value is generated by the random number generator, it is located in the first cell of memory (M_1). In the second step, the next value is generated and it is put to M_1 while the value from M_1 is moved to M_2 (generally value from M_i is moved to $M_{(i+1)}$). Importantly, the probability of generating of a particular value changes, when the value has been already allocated to any of the memory cells. For different memory cells probability of value generation is independent, but the changes are summed up. It is implemented by correction filter. It compares generated value with all memory cells and passes it with certain probability depending on contents of short-term memory cells.

Model is fitted to experimental data by selecting changes of probability for all memory cells. The fit of the model is good when error between indexes for sequence generated by it and human is smaller then assumed before (admissible error of model).

General disscusion

We have sugessted that ethropy and correlation function could be used as measures of randomness of sequences generated by humans, as well as to assess the performance of the random sequences generation task using computational models of memory. We believed, that these indexes can describe the most important deviations from randomness, so using any of the other indexes desribed in the paper for assessing the random sequence generation task performance would note be necessery. We have also suggested that the proposed indexes could be used to build the mathematical model of generation the pseudo-random sequences by humans. In the second part of the paper, such a model was proposed. We believed that models assuming the structural limitations of the short-term memory could generate similar results as humans. In a future publications the results of model fitting will be presented.

The capacity model of random sequence generation can be used as a simple mathematical model of human short-term memory. The proposed methodology - the mathematical computational models with structural limitations of the memory - is the new way of modeling the results of random sequence generation task.

Typicaly, the computational models utilises algorithms of human performance, whereas our mathematical model reproduce the human data without the suggestion, that such rules simulates human information processing. Most of the computational models were critisised for the suggestion, that they were build analogicaly to the human information processing system. Our approach suggest to simulate the human data using most elementary computational model.

References

Atkinson, R. C. & Shiffrin, R. M. (1968). Human memory: A proposed system and its control processes. In: K. W. Spence & J. T. Spence (Eds.) *The psychology of learning and motivation: Advances in research and theory, 2*, New York: Academic Press.

Baddeley, A. D. (1966). The capacity for generating information by randomization. *Quarterly Journal of Experimental Psychology, 18*, 119-120.

Baddeley, A. D. (1996). Exploring the Central Executive. *The Quarterly Journal of Experimental Psychology, 49A(1),* 5-28.

Baddeley, A. D., Emslie, H., Kolodny, J. & Duncan, J. (1998). Random Generation and the Executive Control of Working Memory. *The Quarterly Journal of Experimental Psychology, 51A(4)*, 819-852.

Baddeley, A. & Hitch, G. (1974). Working memory In: G. A. Bower (Ed.), *The psychology of learning and motivation, 8,* New York: Academic Press, 47-89.

Brown, R. G., Soliveri, P. & Jahanshahi, M. (1998). Executive processes in Parkinson's disease – random number generation and response suppression. *Neuropsychologia, 36(12),* 1355-1362.

Brugger, P. (1997). Variables that influence the generation of random sequences: An update. *Perceptual and Motor Skill, 84*, 627-661.

Brugger, P., Landis, T. & Regard, M. (1992). The brain as a random generator: The relevance of subjective randomization for neuropsychology. *Journal of Clinical Experimental Neuropsychology, 14(84).*

Brugger, P., Monsch, A. U., Salmon, D. P. & Butters, N. (1996). Random number generation in dementia of the Alzheimer type: A test of frontal executive functions. *Neuropsychologia, 34(2)*, 97-103.

Budescu, D. V. (1987). A Markov model for generation of random binary sequences. *Journal of Experimental Psychology: Human Perception & Performance 13*, 25-39.

Evans, F. J. (1978). Monitoring attention deployment by random number generation: An index to measure subjective randomness. *Bulletin of the Psychonomic Society, 12,* 35-38.

Falk, R. & Konold, C. (1997). Making Sense of Randomness. Implicit Encoding as Basis for Judgement. *Psychological Review. 104 (2)*, 301-308.

Kahneman, D. & Tversky, A. (1972). Subjective probability: A judgment of representativness. *Cognitive Psychology, 3*, 430-454.

Kareev, Y. (1992). Not That Bad After All. Generation of Random Sequences. *Journal of Experimental Psychology: Human Perception & Performance 18 (4)*, 1189-1194.

Miller, G. A. (1956). The Magical Number Seven, Plus or Minus Two: Some Limits on our Capacity for Processing Information. *Psychological Review, 63(2)*, 81-97.

Miyake, A., Witzki, A. H. & Emerson, M. J. (2001). Field dependence-independence from a working memory perspective: a dual task investigation of the Hidden Figure Test. *Memory, 9(4/5/6),* 445-457.

Lopes, L. L. & Oden, G. C. (1987). Distinguishing between random and nonrandom events. *Journal of Experimental Psychology: Learning, Memory and Cognition, 13(3)*, 392-400.

Pollux, P. M. J., Wester, A. & De Haan, E. H. (1995). Random generation deficit in alcoholic Korsakoff patients. *Neuropsychologia, 33(1), 125-129.*

Neuringer, A. (1986). Can People Behave „Randomly?": The Role of Feedback. *Journal of Experimental Psychology: General, 115(1),* 62-75.

Rapoport, A. & Budescu, D. V. (1997). Randomization in Individual Choice Behavior. *Psychological Review, 104(3)*, 603-617.

Shannon, C. (1948). A Mathematical Theory of Communication, Reprinted with corrections from *The Bell System Technical Journal,* Vol. 27, pp. 379–423, 623–656

Towse, J. N. & Neil, D. (1998). Analyzing human random generation behavior: A review of methods used and a computer program for describing performance. *Behavior Research Methods, Instruments & Computers. 30(4)*, 583-591.

Towse, J. N. & Valentine, J. D. (1997). Random generation of numbers: A search for underlying processes. *European Journal of Cognitive Psychology. 9(4)*, 381-400.

Vandierendonck, A. (2000). Is judgment of random time intervals biased and capacity-limited? *Psychological Research, 63*, 199-209.

Vandierendonck, A. (2000b). Analyzing human random time generation behavior: A methodology and computer programs. *Behavior Research Methods, Instruments, & Computers, 32,* 555-565.

Wagenaar, W. A. (1970). Subjective randomness and the capacity to generate information. *Acta Psychologica, 3*, 233-242.

Weiss, R. L. (1965). Variables that influence random generation: An alternative hypothesis. *Perceptual and Motor Skills, 62,* 163-168.

Wiegersma S. (1982). Can repetition avoidance in randomization be explained by randomness concepts? *Psychological Research, 44,* 189-198.