Effects of arbitrary shear stress on unsteady free convection flow of Casson fluid past a vertical plate

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ABSTRACT

This article studies the unsteady free flow of a Casson fluid over an infinite vertical plate with constant wall temperature. The Problem is modelled by employing equations of continuity, momentum and energy. Exact solutions for the dimensionless velocity and temperature are established by the Laplace transform technique. The solutions that have been obtained, uncommon in the literature, satisfy all imposed initial and boundary conditions and can generate huge number of solutions for any motion problem with technical relevance of this type. For illustration, some special cases are considered. The velocity solutions are presented as a sum of convective and mechanical parts. Pertinent results are discussed and displayed graphically.

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Introduction

Heat transfer phenomenon in non-Newtonian fluids such as drilling muds, clay coatings and other suspensions, certain oils and greases, polymer melts, blood and many emulsions, is an important research area due to its relevance in the optimized processing of chocolate, toffee, and other foodstuffs. However, it is not as easy as in case of Newtonian fluids. Because there is not available a single constitutive relation same as for Newtonian fluids which describes all non-Newtonian fluids. Due to this difficulty several models or constitutive equations have been proposed [1–5]. Amongst them, some of the non-Newtonian fluid models are studied in a great length. But some of them are important but less investigated. Such as the rheological model of Casson fluid. This fluid model was originally introduced by Casson [6] to simulate industrial inks. However, it has also significant applications in polymer processing industries and biomechanics [7–9]. According to Swati and Mandal [10], the Casson fluid model is sometime more suitable compare to other viscoplastic models to fit for the rheological data and for many materials such as blood and chocolate. Mostly the Casson fluid problems are solved when velocity at the boundary is specified. Yet, no problem is reported on the

unsteady free convection flow of Casson fluid when instead of velocity, shear stress is specified at the boundary. The idea of arbitrary shear stress at the wall for the free convection flow of viscous fluid was introduced by Fetecau et al. [11]. More exactly, they studied the free convection flow near a vertical plate that applies arbitrary shear stress to the fluid with the additional effects of thermal radiation and porosity. Soon after, this idea is extended for other problems as we can see [12–15]. Moreover, some interesting numerical solutions for Casson fluids are investigated in details in [17–19]. In all these investigations, the exact solutions were obtained using the Laplace transform technique. However, it is worth pointing out that all these solutions correspond to the motion of problems for viscous fluids. For non-Newtonian fluids such solutions are scarce, more exactly for Casson fluids.

Based on this motivation, the main objective of this work is to study the unsteady flow of non-Newtonian Casson fluid over an infinite plate with isothermal temperature and arbitrary wall shear stress. Exact solutions are obtained using the Laplace transform method, plotted graphically and discussed.

Mathematical formulation

Let us consider the unsteady free convection flow of an incompressible viscous fluid over an infinite vertical plate. The physical configuration of the problem is shown in Fig. 1.The x-axis is taken
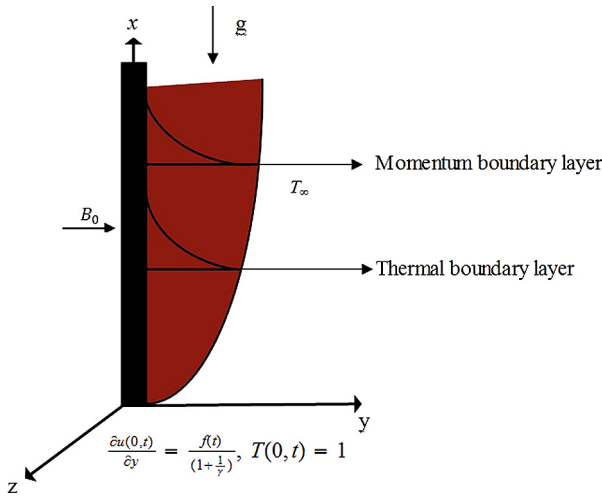

Fig. 1. Physical configuration of the problem.

along the vertical plate and the y-axis is taken normal to the plate. Initially, both the plate and fluid are at stationary condition with the constant temperature T_∞. After time $t = 0$, the plate applies a time dependent shear stress $f(t)$ to the fluid along the x-axis. Meanwhile, the temperature of the plate is raised to T_w. From [10], we know that that the rheological equation of state for the isotropic and incompressible flow of a Casson fluid is:

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y/\sqrt{2\pi}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + p_y/\sqrt{2\pi_c}\right)e_{ij}, \pi < \pi_c \end{cases} \tag{1}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i,j)-th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, p_y is the yield stress of the fluid. Under the usual Boussinesq's approximation and neglecting the viscous dissipation the continuity equation is identically satisfied. Thus the problem of unsteady free convection flow is governed by the following equations of momentum and energy

$$\frac{\partial u}{\partial t} = \nu\left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty); \quad y, t > 0, \tag{2}$$

$$\rho C_p \frac{\partial T}{\partial t} = k_1 \frac{\partial^2 T}{\partial y^2} \quad y, t > 0, \tag{3}$$

where u, T, ν, ρ, g, β_T, C_p, k_1 and $\gamma = \mu_B\sqrt{2\pi_c}/p_y$ are the velocity of the fluid in x-direction, its temperature, the kinematic viscosity, the constant density, the gravitational acceleration, the heat transfer coefficient, the heat capacity at constant pressure, the thermal conductivity of fluids and the non-Newtonian Casson parameter respectively.

The corresponding initial and boundary conditions for velocity and temperature are:

$$\begin{aligned} u(y,0) &= 0, \quad T(y,0) = T_\infty, \quad \forall y \geqslant 0, \\ T(0,t) &= T_w, \quad \forall t > 0, \\ u(\infty,t) &= 0, \quad T(\infty,t) = T_\infty. \end{aligned} \tag{4}$$

The arbitrary shear stress at the wall for Casson fluid is defined by [16]

$$\tau(0,t) = \left(\mu_B + p_y/\sqrt{2\pi_c}\right)\frac{\partial u(0,t)}{\partial y} = f(t), \tag{5}$$

after simplification reduces to

$$\left(1 + \frac{1}{\gamma}\right)\frac{\partial u(0,t)}{\partial y} = \frac{f(t)}{\mu_B}. \tag{6}$$

Introducing the non-dimensional variables

$$u^* = u\sqrt{\frac{t_0}{\nu}}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad y^* = \frac{y}{\sqrt{\nu t_0}},$$

$$t^* = \frac{t}{t_0}, \quad f^*(t^*) = \frac{t_0}{\mu_B}f(t), \tag{7}$$

into Eqs. (2) and (3), and initial and boundary conditions 4 and (6) ("*" notations are dropped for simplicity)

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} + GrT, \tag{8}$$

$$\text{Pr}\,\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2}, \tag{9}$$

$$\begin{aligned} u(y,0) &= 0, T(y,0) = 0, \quad \forall y \geqslant 0, \\ \frac{\partial u(0,t)}{\partial y} &= \frac{f(t)}{\left(1 + \frac{1}{\gamma}\right)}, \quad T(0,t) = 1, \\ T(\infty,t) &= 0, u(\infty,t) = 0, \end{aligned} \tag{10}$$

where

$$Gr = \frac{g\beta_T(T_w - T_\infty)\nu}{U_0^3}, \quad t_0 = \frac{\nu}{U_0^2}, \quad \text{Pr} = \frac{\mu_B C_p}{k_1}, \tag{11}$$

are the Grashof number, the characteristic time and the Prandtl number.

Exact solution

Applying Laplace transform to Eqs. (8) and (9), we obtained the following transformed ordinary differential equations

$$q\bar{u}(y,q) = \left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 \bar{u}(y,q)}{\partial y^2} + Gr\bar{T}(y,q), \tag{12}$$

$$\text{Pr}\,q\,\bar{T}(y,q) = \frac{\partial^2 \bar{T}(y,q)}{\partial y^2}, \tag{13}$$

with transformed boundary conditions

$$\bar{T}(0,q) = \frac{1}{q}, \quad \bar{T}(\infty,q) = 0,$$

$$\bar{u}(\infty,q) = 0, \quad \frac{\partial\bar{u}(0,q)}{\partial y} = \frac{F(q)}{\left(1 + \frac{1}{\gamma}\right)}. \tag{14}$$

Solution of Eq. (13) under boundary conditions (14) is obtained as

$$\bar{T}(y,q) = \frac{1}{q}e^{-y\sqrt{q\text{Pr}}}. \tag{15}$$

By taking the inverse Laplace transform of Eq. (15), we find

$$T(y,t) = \text{erf}\,c\left(\frac{y\sqrt{\text{Pr}}}{2\sqrt{t}}\right) \tag{16}$$

and

$$\left.\frac{\partial T(y,t)}{\partial y}\right|_{y=0} = -\frac{e^{-\frac{\text{Pr}\,y^2}{4t}}\sqrt{\text{Pr}}}{\sqrt{\pi}\sqrt{t}}, \tag{17}$$

is the corresponding heat transfer rate also known as Nusselt number.

The solution of Eq. (12) under boundary conditions (14) results

$$\bar{u}(y,q) = \frac{Gr\sqrt{Pr}}{\sqrt{\left(1+\frac{1}{\gamma}\right)}\left[Pr-\frac{1}{\left(1+\frac{1}{\gamma}\right)}\right]q^2}e^{-\frac{y}{\sqrt{\left(1+\frac{1}{\gamma}\right)}}\sqrt{q}} - \frac{Gr}{\left(1+\frac{1}{\gamma}\right)\left[Pr-\frac{1}{\left(1+\frac{1}{\gamma}\right)}\right]q^2}e^{-y\sqrt{Pr}\sqrt{q}}$$
$$-\frac{F(q)}{\sqrt{\left(1+\frac{1}{\gamma}\right)}\sqrt{q}}e^{-\frac{y}{\sqrt{\left(1+\frac{1}{\gamma}\right)}}\sqrt{q}}. \tag{18}$$

Applying the inverse Laplace transform to Eq. (18), we get

$$u(y,t) = u_c(y,t) + u_m(y,t), \tag{19}$$

where

$$u_c(y,t) = \frac{Gr\sqrt{Pr}}{\sqrt{\left(1+\frac{1}{\gamma}\right)}\left[Pr-\frac{1}{\left(1+\frac{1}{\gamma}\right)}\right]}$$
$$\cdot\left[\left(t+\frac{y^2}{2\left(1+\frac{1}{\gamma}\right)}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{\left(1+\frac{1}{\gamma}\right)t}}\right) - \frac{y\sqrt{t}}{\sqrt{\left(1+\frac{1}{\gamma}\right)\pi}}e^{-\frac{y^2}{4\left(1+\frac{1}{\gamma}\right)t}}\right]$$
$$-\frac{Gr}{\left(1+\frac{1}{\gamma}\right)\left[Pr-\frac{1}{\left(1+\frac{1}{\gamma}\right)}\right]}\left[\left(t+\frac{y^2Pr}{2}\right)\operatorname{erf}c\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}\right) - \frac{y\sqrt{Pr}\sqrt{t}}{\sqrt{\pi}}e^{-\frac{y^2Pr}{4t}}\right] \tag{20}$$

and

$$u_m(y,t) = -\frac{1}{\sqrt{\pi}\sqrt{\left(1+\frac{1}{\gamma}\right)}}\int_0^t f(t-s)\frac{e^{-\frac{y^2}{4\left(1+\frac{1}{\gamma}\right)s}}}{\sqrt{s}}ds, \tag{21}$$

correspond to the convective and mechanical parts of velocity.

It is noted from Eq. (16) that $T(y,t)$ is valid for all positive values of Pr while $u_c(y,t)$ is not valid for Pr = 1. Therefore, to get $u_c(y,t)$ when the Prandtl number is equal to one, we make Pr = 1 into Eq. (9), use a similar procedure as discussed above, and obtain

$$\bar{u}(y,q) = \frac{Gr}{\sqrt{\left(1+\frac{1}{\gamma}\right)}\left[1-\frac{1}{\left(1+\frac{1}{\gamma}\right)}\right]q^2}e^{-\frac{y}{\sqrt{\left(1+\frac{1}{\gamma}\right)}}\sqrt{q}} - \frac{Gr}{\left(1+\frac{1}{\gamma}\right)\left[1-\frac{1}{\left(1+\frac{1}{\gamma}\right)}\right]q^2}e^{-y\sqrt{q}}$$
$$-\frac{F(q)}{\sqrt{\left(1+\frac{1}{\gamma}\right)}\sqrt{q}}e^{-\frac{y}{\sqrt{\left(1+\frac{1}{\gamma}\right)}}\sqrt{q}}. \tag{22}$$

By taking the inverse Laplace transform, we find that

$$u_c(y,t) = \frac{Gr}{\sqrt{\left(1+\frac{1}{\gamma}\right)}\left[1-\frac{1}{\left(1+\frac{1}{\gamma}\right)}\right]}$$
$$\cdot\left[\left(t+\frac{y^2}{2\left(1+\frac{1}{\gamma}\right)}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{\left(1+\frac{1}{\gamma}\right)t}}\right) - \frac{y\sqrt{t}}{\sqrt{\left(1+\frac{1}{\gamma}\right)\pi}}e^{-\frac{y^2}{4\left(1+\frac{1}{\gamma}\right)t}}\right]$$
$$-\frac{Gr}{\left(1+\frac{1}{\gamma}\right)\left[1-\frac{1}{\left(1+\frac{1}{\gamma}\right)}\right]}\left[\left(t+\frac{y^2}{2}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}\right) - \frac{y\sqrt{t}}{\sqrt{\pi}}e^{-\frac{y^2}{4t}}\right] \tag{23}$$

Limiting cases

In this section we discuss few limiting cases of our general solutions.

Solution in the absence of Casson fluid parameter ($\gamma \to \infty$)

We substitute $\gamma \to \infty$ into Eq. (12) to get the corresponding solutions for viscous fluid, by adopting same procedure, obtained result for velocity is

$$u(y,t) = \frac{Gr\sqrt{Pr}}{[Pr-1]}\left[\left(t+\frac{y^2}{2}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}\right) - \frac{y\sqrt{t}}{\sqrt{\pi}}e^{-\frac{y^2}{4t}}\right]$$
$$-\frac{Gr}{[Pr-1]}\left[\left(t+\frac{y^2Pr}{2}\right)\operatorname{erf}c\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}\right) - \frac{y\sqrt{Pr}\sqrt{t}}{\sqrt{\pi}}e^{-\frac{y^2Pr}{4t}}\right]$$
$$-\frac{1}{\sqrt{\pi}}\int_0^t f(t-s)\frac{e^{-\frac{y^2}{4s}}}{\sqrt{s}}ds. \tag{24}$$

Solutions in the absence of free convection

Let us assume that the flow is caused only due to bounding plate and the corresponding buoyancy forces are zero equivalently it shows the absence of free convection due to the differences in temperature gradient i.e. the terms Gr is zero. This shows that the convective part of velocity is zero. Hence the flow is only governed by the mechanical part of velocity given by Eq. (21).

Solutions in the absence of mechanical effects

Let us assume that the infinite plate is in static position at every time i.e. the function $f(t)$ is zero for all values of t and the mechanical part is equivalently zero. In such a situation, the motion in the fluid is induced only due to the free convection which causes due to the buoyancy forces. Therefore, the velocity of the fluid is only represented by their convective part given by Eq. (20).

Special cases

As we noted that the solutions for velocity obtained in Section "Exact solution", are more general. Therefore, we want to discuss some special cases of the present solutions together with some limiting solutions in order to know in details about the physical aspects of the problem. Hence, we discuss the following important special cases.

Case-I: $f(t) = fH(t)$

In this first case we take the arbitrary function $f(t) = fH(t)$, where f is a dimensionless constant and $H(\cdot)$ denotes the unit step function. After time $t = 0$, the infinite vertical plate applies a constant shear stress to the fluid. The convective part of the velocity remains unchanged while the mechanical part takes the following form

$$u_m(y,t) = -\frac{f}{\sqrt{\left(1+\frac{1}{\gamma}\right)}\sqrt{\pi}}\int_0^t \frac{e^{-\frac{y^2}{4\sqrt{\left(1+\frac{1}{\gamma}\right)}s}}}{\sqrt{s}}ds. \tag{25}$$

Furthermore, in the absence of Casson fluid parameter, Eq. (25) reduce to

$$u_m(y,t) = -\frac{f}{\sqrt{\pi}}\int_0^t \frac{e^{-\frac{y^2}{4s}}}{\sqrt{s}}ds. \tag{26}$$

which is identical with [11]; Eq. (23).

Case-II: $f(t) = f \sin(\omega t)$

In the second case, we take the arbitrary function of the form $f(t) = f \sin(\omega t)$ in which the plate applies an oscillating shear stress to the fluid. Here ω denotes the dimensionless frequency of the shear stress. As previously, the convective part of velocity remains the same whereas the mechanical part takes the form

$$u_m(y,t) = -\frac{f}{\sqrt{\left(1 + \frac{1}{\gamma}\right)}\sqrt{\pi}} \int_0^t \frac{\sin(\omega t - \omega s)e^{-\frac{y^2}{4\sqrt{\left(1 + \frac{1}{\gamma}\right)s}}}}{\sqrt{s}} ds. \quad (27)$$

It can be further written as a sum of the steady-state and transient solutions

$$u_m(y,t) = u_{ms}(y,t) + u_{mt}(y,t), \quad (28)$$

where

$$u_{ms}(y,t) = -\frac{f}{\sqrt{\left(1 + \frac{1}{\gamma}\right)}\sqrt{\pi}} \int_0^t \frac{\sin(\omega t - \omega s)e^{-\frac{y^2}{4\sqrt{\left(1 + \frac{1}{\gamma}\right)s}}}}{\sqrt{s}} ds, \quad (29)$$

$$u_{mt}(y,t) = \frac{f}{\sqrt{\left(1 + \frac{1}{\gamma}\right)}\sqrt{\pi}} \int_t^\infty \frac{\sin(\omega t - \omega s)e^{-\frac{y^2}{4\sqrt{\left(1 + \frac{1}{\gamma}\right)s}}}}{\sqrt{s}} ds. \quad (30)$$

In addition when $\gamma \to \infty$, physically it corresponds to the absence of Casson fluid parameter, Eq. (29) results in

$$u_{ms}(y,t) = -\frac{f}{\sqrt{\pi}} \int_0^t \frac{\sin(\omega t - \omega s)e^{-\frac{y^2}{4s}}}{\sqrt{s}} ds, \quad (31)$$

equivalent to [12]; Eq. (38).

Results and discussion

Computations have been carried out by assigning values to the embedded parameters characterizing the fluid properties. The flow phenomenon is characterized by Grashof number Gr, dimensionless time t, Prandtl number Pr, shear stress f and Casson fluid parameter γ. The effects of these parameters on velocity and temperature profiles are shown in Figs. 2–8. The influence of thermal Grashof number Gr on velocity profiles is shown in Fig. 2. It is clear from this figure that in the absence of thermal effect ($Gr = 0$), when the effect of buoyant forces is negligible and the viscous forces are dominant, the velocity tends to steady-state faster than
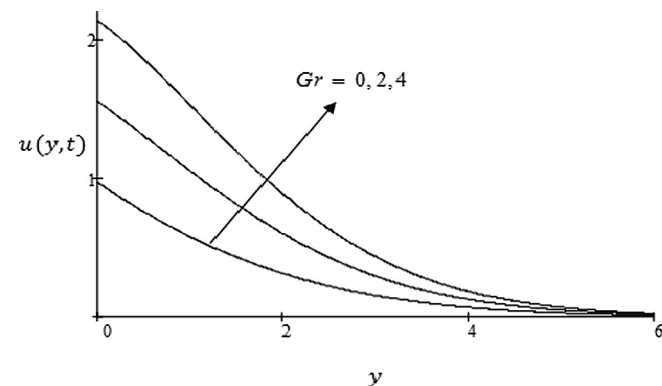
Fig. 2. Velocity profiles for for different values of Gr when the plate applies a constant shear stress $f = -2$.
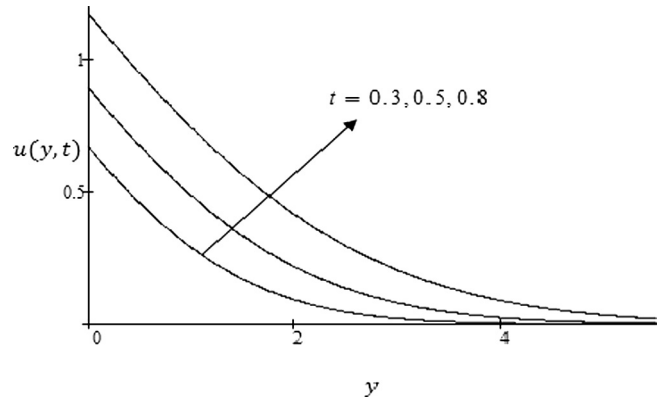
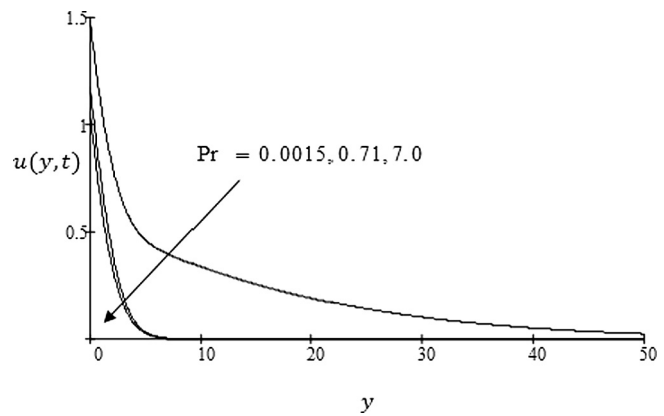Fig. 3. Velocity profiles for for different values of t when the plate applies a constant shear stress $f = -2$.

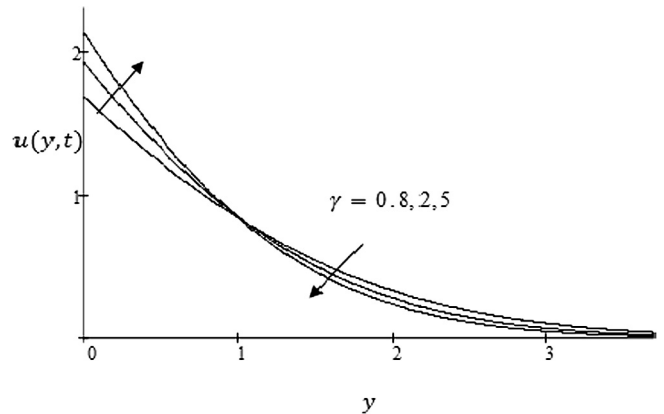Fig. 4. Velocity profiles different values of Pr when the plate applies a constant shear stress $f = -2$.

Fig. 5. Velocity profiles for different values of γ when the plate applies a constant shear stress $f = -2$.

for the values of $Gr > 0$. It can be observed that velocity increases for the increasing values of Gr. It is also true physically as the Grashof number Gr describes the ratio of buoyancy forces to viscous forces. Therefore, an increase in the values of Gr leads to increase in buoyancy forces, consequently velocity increases. On the other hand, it is clearly seen from Fig. 3 that velocity increases with increasing time. In Fig. 4 the velocity profiles for different values of Prandtl number Pr are shown. An increase in Pr leads to decrease in the velocity which suggests that low rate of thermal diffusion
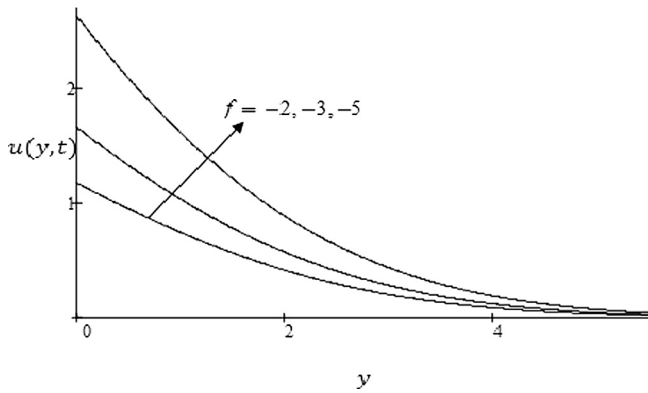
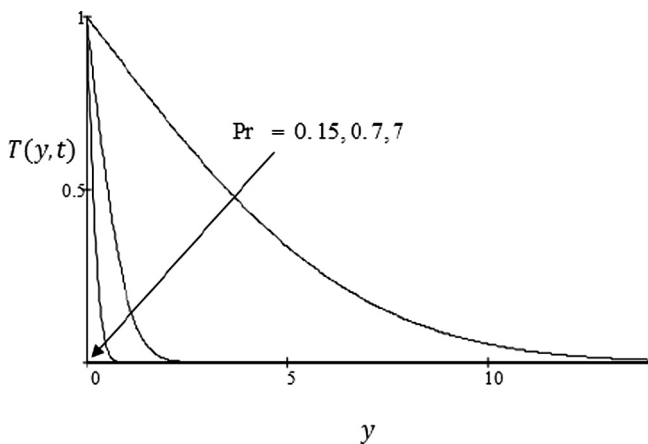**Fig. 6.** Velocity profiles for different values of constant shear stress f.
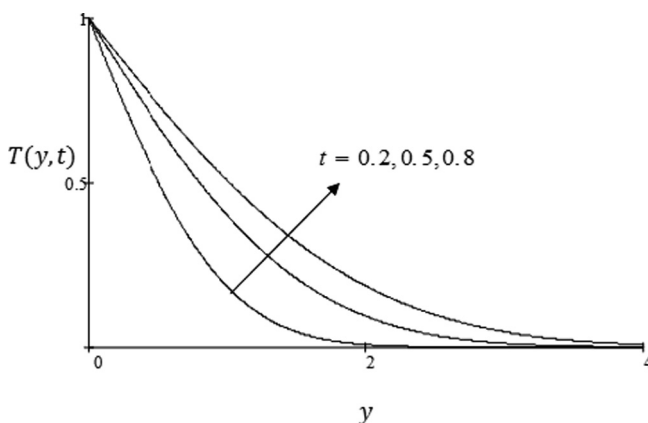

Fig. 7. Temperature profile for different values of Pr.

Fig. 8. Temperature profiles for different values of t.

Table 1
Variation of Nusselt number for different parameters.

t	Pr	Nu
1.2	0.015	0.0076
1.2	0.71	0.065
1.2	7	1.69
1.2	**0.71**	0.065
1.3	**0.71**	0.058
1.4	**0.71**	0.052

Table 2
Variation of velocity for different parameters.

f	Pr	t	Gr	u
−0.75	0.35	0.9	0.7	0.68
−0.45	0.35	0.9	0.7	0.57
−0.15	0.35	0.9	0.7	0.46
−0.25	**0.35**	0.9	0.7	0.49
−0.25	**3.50**	0.9	0.7	0.43
−0.25	**50**	0.9	0.7	0.39
−0.25	0.35	**0.2**	0.7	0.11
−0.25	0.35	**0.4**	0.7	0.24
−0.25	0.35	**0.6**	0.7	0.35
−0.25	0.35	0.9	**0.5**	0.42
−0.25	0.35	0.9	**0.8**	0.47
−0.25	0.35	0.9	**1**	0.51

Conclusions

Exact solutions for the problem of unsteady free convection flow of Casson fluid over an infinite plate are obtained under the conditions of arbitrary wall shear and constant wall temperature. These solutions are expressed in simplified forms in terms of exponential and complementary error functions. We observed that they satisfy all imposed initial and boundary conditions and reduce to some known solutions from the literature as special cases. The following main conclusions are drawn from this study (see Tables 1 and 2).

- In absence of thermal effects $(Gr = 0)$, the solutions purely explain the mechanical aspects of the fluid.
- The general solutions obtained here give several other solutions of important fluid motions as limiting cases.
- It is concluded from velocity profile that increasing Gr, t and f, increases fluid motion whereas fluid flow decays when Pr and γ are increased.
- Temperature increases with increasing t but decreases when Pr increases.

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leads to increase in the velocity boundary layer thickness. Fig. 5 illustrates the influence of Casson fluid parameter on velocity. It is observed that velocity decreases with increasing γ. The effects of the shear stress f induced by the bounding plate on the non-dimensional velocity profiles are shown in Fig. 6. The velocity of fluid is found to decrease with increasing f. The temperature variations against y for various values of Prandtl number Pr are displayed in Fig. 7. It is observed that increasing Pr results in the decrease of temperature distribution. It is because of the fact that fluid has a thinner thermal boundary layer with higher values of Pr. Fig. 8 shows that temperature increases with increasing time.

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