# Chapter 12
# Stackcelberg Game Inventory Model With Progressive Permissible Delay of Payment Scheme

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## ABSTRACT

*Supplier has many schemes to motivate retailer to buy more and of them one is a progressive permissible delay of payment. Instead of analyst from the retailer side alone, in this chapter, we develop the inventory model of supplier and retailer. In reality, some suppliers and retailers cannot have collaboration and they try to optimize their own decision so we develop a Stackelberg Game model. Two models are developed wherein the first model supplier acts as the leader and in the second model, the retailer acts a leader. Since the models are complex, a hybrid Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) is developed to solve the model. A numerical analysis and sensitivity analysis are conducted to get management insights of the model. The results show that a Stackelberg Game model for progressive permissible delay of payment is sensitive in varies values of the first and second delay interest rate if supplier acts as a leader. The retailer gets less inventory cost when he acts as a leader compared to when vendor acts a leader at high interest rate of the first and second delay period.*

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## INTRODUCTION

In order to motivate customers to buy more from supplier, supplier apply some scheme such as dynamic pricing, quantity discount gift on purchase and permissible delay of payments. Trade credit in the form of permissible delay of payment affects the conduct of business significantly (Jaggi *et al.*, 2013). In permissible delay of payment buyer does not have to pay the supplier immediately after receiving the goods but can delay the payment until the allowable time period. Buyer can get benefit from interest earn form goods that have been sold. Many research have been conducted by considering permissible delay of payment in inventory. Huang (2005) developed a buyer's inventory model by considering delay of payment and cash discount. Inventory model with collaboration between supplier and retailer by considering permissible delay of payment was developed by Jaber and Osman (2006). They concluded that coordination with permissible delay in payment is better than no-coordination system. Similar research of inventory model with permissible delay of payment under collaboration between single-vendor and single-buyer for deteriorating items was developed by Yang and Wee (2006). They found that permissible delay of payment is a win-win strategy when implemented under collaboration system. Liao (2007) developed deteriorating economic production quantity model by considering permissible delay in payments. Tsao and Sheen (2008) did not consider permissible delay of payment for deteriorating inventory model but they also include other schemes which are dynamic pricing and promotion. Instead only collaboration between supplier and retailer, Jaggi *et al.* (2008) developed an inventory model with two levels of credit policy. In their model supplier gives a fixed credit period to the retailer and retailer offer credit period to customers. Inventory model with permissible delay of payment by considering allowable shortage was introduced by Chung and Huang (2009). Huang *et al.* (2010) developed single-vendor single buyer integrated inventory model considering permissible delay of payments and order time reduction that can be attained through procedural changes, worker training and specialized equipment acquisition. Sarkar (2012) developed an EOQ model by considering stock dependent demand, production defective items and delay of payment scheme.

Soni and Shah (2008) develop an inventory model with stock dependent demand and progressive payment scheme. Progressive scheme is a variant of permissible delay of payment. In progressive payments scheme, there is more than one payment period. The supplier does not charge any interest if the buyer pays before the first payment period but interest will be charged after the first payment period and become higher for the next payment period. Similar payment scheme model using two-levels of credit policy was developed by Jaggi *et al.* (2012). This chapter tries to extend the work of Soni and Shah (2008) by considering not only retailers but supplier and retailer decisions simultaneously in just in time inventory model. Since supplier and the retailer try to optimize their own decision, the model is developed as a non-cooperative model. Single vendor-single buyer non-cooperative models with permissible delay in payments are developed using Stackelberg equilibrium (Chern *et al.*, 2013) and under Nash equilibrium (Chern *et al.*,2014). Teng *et al*. (2012) studied vendor-buyer inventory model with credit financing for both integrated and non-cooperative environment. They concluded that vendor should offer short permissible delay payment to reduce its cost. Li *et al.* (2014) introduced an inventory model with a transferable utility game under permissible delay of payment scheme. Supplier sells the same commodities and gives the retailers delay of payments.

Stackelberg Game can be described as follows: a leader of this game, for example a supplier, who knows the decision process of his buyers will react to maximize his own profit. The buyer, as a follower, answers the supplier's decision by setting new decision to improve his profit. On the other side, buyer can act as a leader and supplier acts of the follower depend on the bargaining power of the buyer and the supplier. Even though collaboration system is better than the competitive system, Stackelberg game theory has attracted many researchers' attention for practical reason. Esmaeili *et al.* (2008) developed Stackelberg game and cooperative game between several buyers and several sellers where marketing expenditure and unit price charged by the buyer influence the demand of the product being sold. Yu *et al.* (2009) discussed a Stackelberg Game theory in a Vendor Manage Inventory (VMI) system. They concluded that vendor can get benefit from his leadership in Stackelberg game model. In this chapter, Stackelberg game model between the supplier and the buyer is developed. Supplier produces an item continuously and then sends it to the buyer using just in time concept.

From our extensive literature research there are few papers discussing about inventory models with progressive delay of payment scheme and no research that consider two players in progressive delay of payment scheme using Stackelberg game approach. In this chapter, supplier will give two period permissible delay of payment to the buyer and the buyer decides replenishment period and delivery frequency at each replenishment time. The item will send to the buyer in discrete frequency and buyer can sell it directly to customer with constant demand rate. Buyer can pay the supplier at the first delay period, between first and second delay period and after the second delay period. In this chapter, we will show the effect of progressive delay in payment when supplier acts a leader and buyer acts as a leader. Since the model is complex, a hybrid of Genetic Algorithm (GA) and Simulate Annealing (SA) is used to solve the problem. This chapter is divided into five sections. The first section discusses the research gap and literature study. Inventory model development is shown in section two and the method to solve the model is described in section three. Section four shows a numerical example and sensitivity analysis to give management insights of the model and section five concluded the results.

## MATHEMATICAL MODEL

The entire of this chapter using assumptions, parameters and decision variables as follows:

### Assumptions

1. Demand rate is constant and deterministic.
2. Shortage is not allowed
3. Production rate is higher than demand rate
4. Delivery lead time is zero
5. Supplier does not charge any interest if buyer pay before delay period ($M_1$). Supplier charges buyer with interest $I_{c1}$ if buyer pay between first delay period ($M_1$) and second delay period ($M_2$) and interest ($I_{c2}$) if the buyer pay after second delay period ($M_2$), where ($I_{c2} > I_{c1}$).
6. Planning horizon is infinite.

### Parameters

$I$: Product quantity
$Q$: Order quantity
$q$: Delivery quantity

$K$: Delivery frequency
$w$: Delivery frequency during production up time
$P$: Supplier's production rate (unit/unit time)
$D$: Buyer's demand rate (unit/unit time)
$A$: Buyer's ordering cost
$A_v$: Supplier's setup cost
$C_t$: Transportation cost
$h$: Buyer's inventory cost
$h_o$: Buyer's opportunity cost
$h_v$: Supplier's inventory cost
$h_{vo}$: Supplier's opportunity cost
$IP$: Supplier average inventory
$I_{c1}$: Supplier's interest rate for buyer when the buyer pay between $M_1$ and $M_2$ Period
$I_{c2}$: Supplier's interest rate for buyer when the buyer pay after $M_2$ period
$I_e$: Bank's interest rate
$c$: product unit cost
$p_r$: product price
$TI_{ev}$: Total supplier's opportunity cost
$TI_{eb}$: Total buyer's interest profit
$TI_{c1}$: Total interest paid by the buyer if buyer pays between $M_1$ and $M_2$ period
$TI_{c2}$: Total interest paid by the buyer if buyer pays after $M_2$ period
$TBUC$: Buyer's total cost
$TVUC$: Supplier's total cost

## Decision Variables

$T$: Replenishment period
$K$: Delivery quantity during replenishment period
$M_1$: The first period of delay of payment
$M_2$: The second period of delay of payment

This chapter discusses an inventory model with progressive payment between single supplier and single buyer. Supplier has a specific production rate and delivers the buyer's order in a specific quantity ($q$) to the buyer in a discrete delivery frequency ($K$) as seen in Figure 1. Buyer receives items from the supplier in a specific $T/K$ period and then sells the items with a constant demand rate. Figure 2 show the inventory rate at the buyer's side. The buyer does not have to pay directly when the items are delivered. The supplier gives delay of payment to the buyer to give opportunity for buyer to buy in higher volume. Supplier has two deadline payment period which are $M_1$ and $M_2$. When the buyer pay before $M_1$ period, there is no interest charged by supplier to the buyer. If the buyer pays after $M_1$ and before $M_2$, supplier charges a certain interest rate to the buyer. If the buyer pays after $M_2$ period then the supplier charges the buyer with higher interest rate. The buyer can sell the items directly and can get cash payment from his customer. The buyer can save his income to banks and get extra interest.

*Figure 1. Supplier's inventory model*

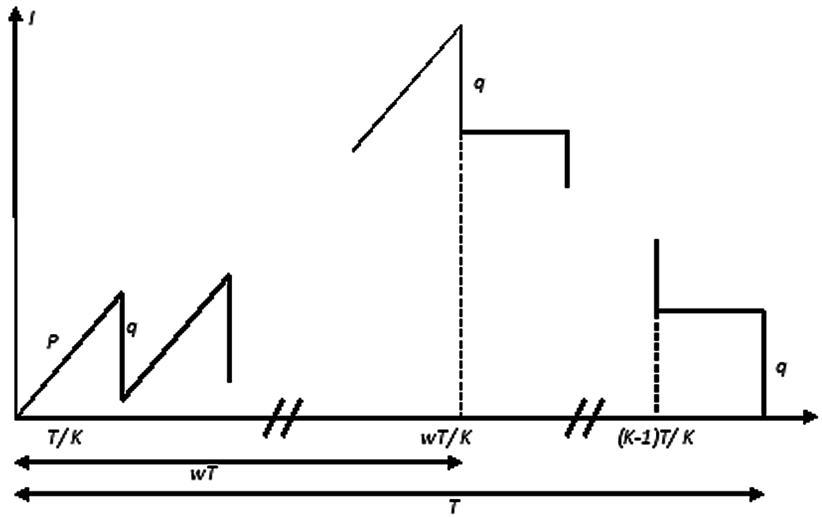
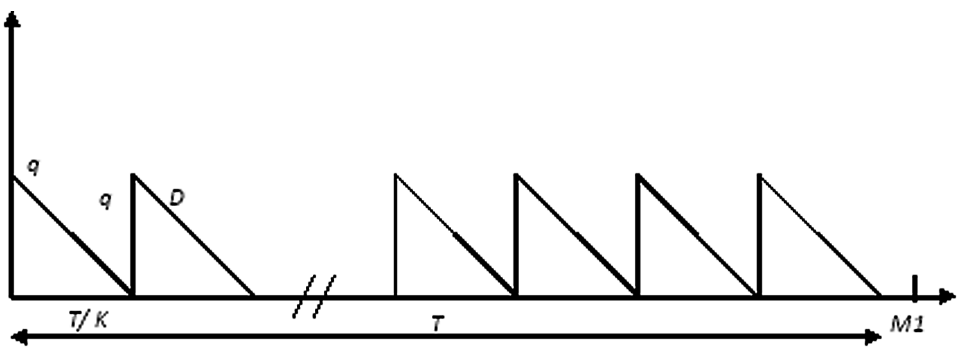



*Figure 2. Buyer inventory level*




There are three cases in this model. In the first model, the total order quantity ($Q$) in replenishment time ($T$) has shorter time than the first delay period ($M_1$). The second case where replenishment time in between the first delay period and second delay period and the last case where replenishment period is longer than the second delay period.

Using Figure 1, the supplier has production rate $P$ and has production period as long as ($wT/K$) period. Supplier delivers m unit/delivery every $T/K$ period for $K$ deliveries. Average supplier inventory can be modeled as:

$$IP = \frac{\frac{1}{2} \times q \times \left( \frac{\left(K-1\right)T}{K} - \frac{wT}{K} \right)}{\frac{T}{K}} = \frac{q\left(K-w-1\right)}{2} \tag{1}$$

and the buyer average inventory as seen in Figure 2, can be modeled as

$$IB = \frac{q}{2} \tag{2}$$

## Case 1

($T \leq M_1$)

In the first case replenishment time ($T$) is shorter than the first delay time ($M_1$). Buyer can has payment at last in the first delay time ($M_1$). When buyer pay at $M_1$, buyer can have opportunity cost as shown in Figure 3.

When vendor does not apply for delay of payment, supplier can receive payment directly after the products are sold to buyer. Since supplier applied delay of payment, supplier has opportunity cost as below.

$$TI_{ev} = \frac{h_{v0}PwT}{K}\left(M_1 - \frac{wT}{2K}\right) \tag{3}$$

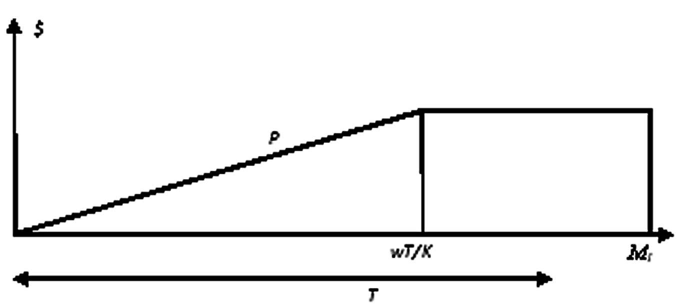
In the other side, buyer receives payment directly from his buyer and delay payment to supplier until the first delay period ($M_1$). With bank interest rate ($I_e$), buyer can get total interest gain as:

$$TI_{eb} = I_e DT\left(M_1 - \frac{T}{2}\right) \tag{4}$$

The total supplier cost consists of setup cost, inventory cost and opportunity lost cost, and one has:

$$TVUC_1 = \frac{A_v D}{mK} + \frac{h_v q\left(K - w + 1\right)}{2} + \frac{h_{v0}PwT}{K}\left(M_1 - \frac{wT}{2K}\right) \tag{5}$$

*Figure 3. Buyer opportunity cost for case 1*



The buyer cost consists of setup cost, transportation cost, inventory cost and buyer's opportunity gain as modeled below.

$$TBUC_1 = \frac{AD}{qK} + \frac{C_t D}{m} + \frac{hq}{2} - I_e DT\left(M_1 - \frac{T}{2}\right) \quad (6)$$

## Case 2

$(M_1 \leq T \leq M_2)$

In the case 2, there are two possibilities of buyer opportunity cost as shown in Figure 4. The first case occurs when the vendor production uptime longer than the first delay of payment period and the second case when the vendor production uptime is shorter than the first delay of payment period.

### Case 2.1.

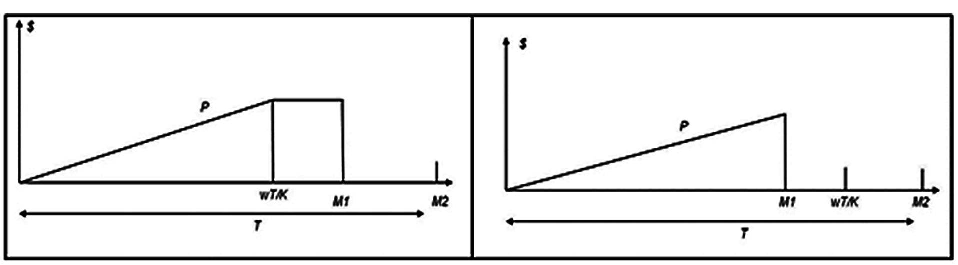
$p_r DM_1 + p_r I_e DM_1^2/2 \geq cDT$

In this case, the buyer can pay with full payment at $M_1$. For this case, there are two cases of the vendor opportunity cost. In the first cases, the vendor's production up time period is less than the first permissible time and in the second case, the vendor's production period is longer than the first permissible payment. For the first case, the vendor opportunity cost is same as equation (3). For the second case, the vendor opportunity cost is:

$$TI_{cv} = h_{v0}\left(\int_{t=0}^{M_1} Ptdt\right) = \frac{h_{v0}P\left(M_1\right)^2}{2} \quad (7)$$

Since the buyer pays all cost at $M_1$, so the vendor does not get interest earned. The buyer interest earned is equal to:

*Figure 4. Buyer opportunity cost for case 2*



$$TI_{eb} = I_e \left( \int_{t=0}^{M_1} Dtdt \right) = I_e \left( \frac{DM_1^{\ 2}}{2} \right) \tag{8}$$

Buyer does not have interest cost, since buyer pays all payment at $M_1$.

*Case 2.1.a.*

wT/K < $M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, interest cost, opportunity cost minus interest earned. The total inventory cost per unit can be modeled as:

$$TBUC_{21a} = \frac{AD}{qK} + \frac{C_t D}{Qq} + \frac{hq}{2} - I_e \left( \frac{DM_1^{\ 2}}{2} \right) \tag{9}$$

The vendor total cost consists of setup cost, physical inventory holding cost, interest cost minus interest earned. The vendor total cost is

$$TVUC_{21a} = \frac{A_v D}{qK} + \frac{h_v q(K - w + 1)}{2} + \frac{h_{v0} PwT}{K} \left( M_{1-} \frac{wT}{2K} \right) \tag{10}$$

*Case 2.1.b.*

wT/K > $M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, interest cost, opportunity cost minus interest earned. The total inventory cost is same as $TBUC_{21a}$. The vendor total cost consists of setup cost, physical inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{21b} = \frac{A_v D}{qK} + \frac{h_v q(K - w + 1)}{2} + \frac{h_{v0} P \left( M_1 \right)^2}{2} \tag{11}$$

## Case 2.2.

$p_r DM_1 + p_r I_e DM_1^2/2 < cDT$

In this case, the vendor opportunity cost is equal with the vendor opportunity cost in case 2.1. The vendor interest earned can be modeled as:

$$TI_{ev} = \frac{I_{c1}}{2p_r D}\left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)^2 \tag{12}$$

The buyer interest cost is equal to the vendor interest earned and the buyer interest earned is equal to case 2.1.

*Case 2.2.a.*

wT/K < $M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, interest cost, opportunity cost minus interest earned. The total inventory cost per unit can be modeled as:

$$TBUC_{22a} = \frac{AD}{qK} + \frac{C_t D}{Qq} + \frac{hq}{2} + \frac{I_{c1}}{2p_r D}\left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)^2 - I_e\left(\frac{DM_1^{\ 2}}{2}\right) \tag{13}$$

The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{22a} = \frac{A_v D}{qK} + \frac{h_v q(K-w+1)}{2} + \frac{h_{v0}PwT}{K}\left(M_{1-}\frac{wT}{2K}\right) - \frac{I_{c1}}{2p_r D}\left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)^2 \tag{14}$$

*Case 2.2.b.*

wT/K > $M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, interest cost, opportunity cost minus interest earned. The total inventory cost is same as $TBUC_{22a}$. The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{22b} = \frac{A_v D}{qK} + \frac{h_v q(K-w+1)}{2} + \frac{h_{v0}P\left(M_1\right)^2}{2} - \frac{I_{ci}}{2p_r D}\left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)^2 \tag{15}$$

## Case 3

$(M_2 \leq T)$

There are some possibilities when the replenishment time (*T*) is longer than the second delay of payment period ($M_2$). The buyer opportunity cost for case 3 has same trend as case 2 as shown in Figure 4.

## Case 3.1.

$$p_r DM_1 + p_r I_e DM_1^2 / 2 \geq cDT$$

In this case the vendor opportunity cost is equal with the vendor opportunity cost in case 2, the vendor interest earned is zero since all payment is paid at $M_1$, The buyer interest cost is zero since all payment is paid at $M_1$ and the buyer interest earned is equal to case 2.1. There are two possibilities where production period is shorter than the first delay of payment and the production period is longer than the first delay of payment.

### *Case 3.1.a.*

$$wT / K < M_1$$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, minus interest earned. The total inventory is same as $TBUC_{21a}$. The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost is same as $TBUC_{21a}$.

### *Case 3.1.b.*

$$wT/K > M_1$$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost minus interest earned. The total buyer inventory cost per unit is same as $TBUC_{21a}$. The vendor total cost consists of setup cost, inventory holding cost, and interest cost. The vendor total cost per unit time is same as $TVUC_{21a}$.

## Case 3.2.

$$p_r DM_1 + p_r I_e DM_1^2 / 2 < cDTbut \begin{bmatrix} p_r D\left(M_2 - M_1\right) \\ +p_r IeD\left(M_2 - M_1\right)^2 / 2 \end{bmatrix} \geq \left[cDTp_r DM_1 + p_r I_e DM_1^2 / 2\right]$$

The vendor opportunity cost is equal with the vendor opportunity cost in case 2. The vendor interest earned, the buyer interest cost and the buyer interest earned is equal to case 2.2.

*Case 3.2.a.*

$wT/K < M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, opportunity cost minus interest earned. The total inventory cost per unit is same as $TBUC_{22a}$.

The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost per unit time is same as $TVUC_{22a}$.

*Case 3.2.b.*

$wT / K > M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, opportunity cost minus interest earned. The total inventory cost per unit is same as $TBUC_{22a}$. The vendor total cost consists of setup cost, physical inventory holding cost, interest cost minus interest earned. The vendor total cost per unit time is same as $TVUC_{22b}$.

Case 3.3.

$$p_r DM_1 + p_r I_e DM_1^2 / 2 < cDT \text{and} \left[ p_r D\left(M_2 - M_1\right) + p_r IeD\left(M_2 - M_1\right)^2 / 2 \right] < \begin{bmatrix} cDT - p_r DM_1 \\ -p_r I_e DM_1^2 / 2 \end{bmatrix}$$

The vendor opportunity cost is equal with the vendor opportunity cost in case 2. The vendor interest earned is:

$$TIE_v = I_{c1}\left(\frac{a+b}{2}\right)\left(M_2 - M_1\right) + I_{c2}\left(\frac{b^2}{2p_r D}\right) \tag{16}$$

where

$$a = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)$$

and

$$b = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right) - \left(p_r D\left(M_2 - M_1\right) + p_r IeD\frac{(M_2 - M_1)^2}{2}\right)$$

The buyer interest cost is equal with the vendor interest earned and the buyer interest earned is equal with case 2.2.

*Case 3.3.a.*

$wT / K < M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory, opportunity cost minus interest earned. The total inventory cost per unit can be modeled as:

$$TBUC_{33a} = \frac{AD}{qK} + \frac{C_t D}{Qq} + \frac{hq}{2} + I_{c1}\left(\frac{a+b}{2}\right)\left(M_2 - M_1\right) + I_{c2}\left(\frac{b^2}{2p_r D}\right) - I_{eb}\left(\frac{DM_1^{\ 2}}{2}\right) \quad (17)$$

The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{33a} = \frac{A_v D}{qK} + \frac{h_v q(K-w+1)}{2} + \frac{h_{v0} PwT}{K}\left(M_{1-}\frac{wT}{2K}\right) - I_{c1}\left(\frac{a+b}{2}\right)\left(M_2 - M_1\right) + I_{c2}\left(\frac{b^2}{2p_r D}\right) \quad (18)$$

where

$$a = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)$$

and

$$b = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right) - \left(p_r D\left(M_2 - M_1\right) + p_r IeD\frac{(M_2 - M_1)^2}{2}\right)$$

*Case 3.3.b.*

$wT / K > M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, opportunity cost minus interest earned. The total inventory cost per unit is same as $TBUC_{33a}$. The vendor total cost consists of setup cost, physical inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{33b} = \frac{A_v D}{qK} + \frac{h_v q(K-w+1)}{2} + \frac{h_{v0} P\left(M_1\right)^2}{2} - I_{c1}\left(\frac{a+b}{2}\right)\left(M_2 - M_1\right) + I_{c2}\left(\frac{b^2}{2p_r D}\right) \quad (19)$$

where

$$a = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)$$

and

$$b = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right) - \left(p_r D\left(M_2 - M_1\right) + p_r IeD\frac{(M_2 - M_1)^2}{2}\right)$$

## THE STACKELBERG GAME SOLUTION

In the Stackelberg game model, the buyer decision variables are replenishment period (*T*) and delivery frequency (*K*). The vendor offers the first and second delay of the payment period ($M_1$ and $M_2$). Since the model is complex, hybrid of Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) is applied. The GA method is used to solve leader optimization problem and PSO is applied to solve the follower optimization problem. In the solution we use a simple GA and PSO method. The GA method algorithm as below:

### GA Algorithm

#### Chromosome

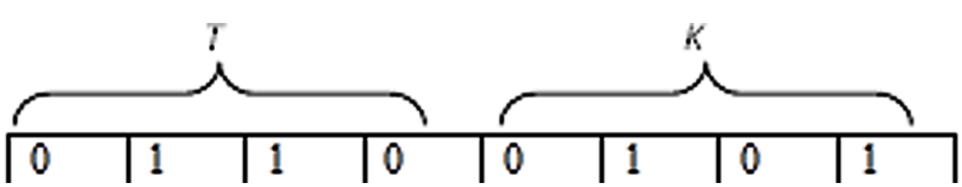
The chromosome (allele) is a binary The chromosome structure for 2 products is shown in Figure 5.

As example, the *T* in Figure 4 can be represented as:

$T = 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 / 15 = 5/15 = 0.333$

and

*Figure 5. Chromosome structure*



$K = 0x2^0+1x2^1+0x2^2+1x2^3 = 9$.

## Initial Population

The initial chromosome population is generated randomly and the population size is equal to 20.

## Evaluation of Fitness

A fitness function is calculated using the total vendor unit cost if vendor act as the leader and the total buyer unit cost if the buyer act as the leader.

## The Parent Selection

This model uses the roulette wheel method and a solution with a less fitness value has greater probability to be selected.

## Genetic Operators

Genetic operators are used to derive a better solution for each generation. Genetic operators consist of elitism, crossover and mutation. In this study, the population size is set at a constant through successive generation. In each generation, elitism is set and crossover and mutation are used to generate new children. Two point crossover function with crossover is used with probability is equal to 0.8. The mutation scheme is uniform with mutation probability is equal to 0.03.

## Stopping Criterion

The stopping criterion for Genetic Algorithm depends on the number of generations. If the number of generations is greater than the value previously determined, then stop. We used 100 number of generations.

## PSO Framework

Particle swarm optimization is a population-based computation technique where each particle moves according to its own best position and the best position of the other particle. It is like a flock of birds collectively foraging for food, where the food location is represented by the fitness function.

1. Initialize particle by setting number of particles (*pr*), number of iterations (α) and some initial parameters. Set $\overrightarrow{v_0} = 0$, personal best (pbest) $\overrightarrow{xl_{ps}} = \overrightarrow{x_{ps}}$ and iteration $i$=1.
2. For $i$=1,..., $p$ decode $\overrightarrow{x_{ps}}$ to a set of decision variable.
3. For $i$= 1..., $p$.calculate the performance measurement of $R_i$ as $Z_i$, where $Z_i$ is calculated using total vendor unit cost or total buyer unit cost depend on who acts as the follower. Update pbest by setting $\overrightarrow{xl_{ps}} = \overrightarrow{x_{ps}}$ if $Z_{x_{ps}} < Z_{xl_{ps}}$.

4. Update gbest by setting $\overrightarrow{xl_{ps}} = \overrightarrow{x_{ps}}$ if $Z_{xl_{ps}} < Z_{xg_s}$ $Z_{xl_{ps}} < Z_{xg_s}$.
5. Update the velocity and the position of each particle

$$\overrightarrow{v_{ps}}\left(i+1\right) = w\left(i\right)\times\overrightarrow{v_{ps}}\left(i\right) + u\left[0,1\right]\times c1\left(i\right)\times\left(\overrightarrow{xg_s} - \overrightarrow{x_{ps}}\left(i\right)\right) + u\left[0,1\right]\times c2\left(i\right)\times\left(\overrightarrow{xl_{ps}} - \overrightarrow{x_{ps}}\left(i\right)\right) \tag{20}$$

Update of the moment inertia using fitness distance ratio (FDR), and it can be can be shown as

$$w\left(i\right) = w\left(F\right) + \left(\frac{i-F}{1-F}\right)\left(w\left(1\right) - w\left(F\right)\right) \tag{21}$$

Calculate the new position using (20)

$$\overrightarrow{x_{ps}}\left(i+1\right) = \overrightarrow{x_{ps}}\left(i\right) + \overrightarrow{v_{ps}}\left(i+1\right) \tag{22}$$

6. If the generation meet the stopping criteria, stop. Otherwise add generation by one and return to step 2.
7. Set gbest of the last solution as the best solution for multi route inventory routing problem for deteriorating items.

## A NUMERICAL EXAMPLE

A numerical example is used to show how the model work and a sensitivity analysis is conducted to get management insights of the model. The numerical example use similar data as Goyal *et al.*(2007). The set of data that is used in this numerical example is $[A, A_v, I_e, c, p_r]$ = [200,150,4%,25,35,] and $[P, D, C_t, h_{vo}, h, h_v]$ = [4000,1000,100,110,4,4]. In the first case, vendor acts as a leader and buyer acts as the follower. The decision variables have lower bound and upper bound as shown in Table 1.

The algorithm is run ten times to get a solution that near to the global optimal solution. The best solution with different values of $I_{c1}$ and $I_{c2}$ when vendor acts as a leader is shown in Table 2. Table 2 shows that the buyer cost tends to increase when the second period interest increase. Buyer tries to reduce her cost by reducing replenishment period and delivery frequency. In the other side, vendor tries to reduce as small as possible the first and second delay of payment to get more profit and reduce her total cost. Buyer's and vendor's total cost tend to be stable in varies the interest of the first delay of the payment period. The buyer's and vendor's decisions are not significantly different in varies interest of the first delay of payment.

In the second sensitivity analysis, buyer acts as a leader and vendor acts as a follower. The lower bound and upper bound of each decision are set as shown in Table 3.

*Table 1. The decision variables bound*

| Decision Variables | Upper Bound | Lower Bound |
|---|---|---|
| $M_1$ | 1,2 year | 0 year |
| $M_2$ | 1,2 year | 0 year |
| $K$ | 20 times | 1 times |
| $T$ | 1,3 year | 0 year |

*Table 2. Sensitivity analysis with varies of $Ic_1$ and $I_{c2}$ (vendor as a leader)*

| $I_{c1}$ | $I_{c2}$ | $T^*$ | $K^*$ | $M_1^*$ | $M_2^*$ | *TVUC* | *TBUC* |
|---|---|---|---|---|---|---|---|
| 3,5% | 8% | 0,5976 | 3 | 0,0025 | 0,1045 | $1.355,20 | $1.427,00 |
| | 9% | 0,4762 | 2 | 0,0007 | 0,0007 | $1.324,20 | $1.497,60 |
| | 10% | 0,4617 | 2 | 0,0002 | 0,0002 | $1.294,30 | $1.518,10 |
| | 11% | 0,4589 | 2 | 0,0003 | 0,0009 | $1.268,20 | $1.536,50 |
| 4% | 8% | 0,5983 | 3 | 0,0010 | 0,1230 | $1.355,40 | $1.426,40 |
| | 9% | 0,4764 | 2 | 0,0014 | 0,0015 | $1.325,60 | $1.496,80 |
| | 10% | 0,467 | 2 | 0,0001 | 0,0002 | $1.294,20 | $1.518,10 |
| | 11% | 0,4591 | 2 | 0,0011 | 0,0014 | $1.269,30 | $1.535,70 |
| 4,5% | 8% | 0,5974 | 3 | 0,0010 | 0,1340 | $1.350,80 | $1.430,10 |
| | 9% | 0,4761 | 2 | 0,0002 | 0,0002 | $1.323,30 | $1.498,20 |
| | 10% | 0,4672 | 2 | 0,0002 | 0,0009 | $1.294,90 | $1.517,50 |
| | 11% | 0,4588 | 2 | 0,0003 | 0,0004 | $1.267,70 | $1.536,90 |
| 5% | 8% | 0,5975 | 3 | 0,0030 | 0,1530 | $1.351,50 | $1.431,20 |
| | 9% | 0,4764 | 2 | 0,0012 | 0,0012 | $1.325,00 | $1.497,10 |
| | 10% | 0,4671 | 2 | 0,0004 | 0,0004 | $1.294,60 | $1.517,80 |
| | 11% | 0,4588 | 2 | 0,0005 | 0,0005 | $1.267,80 | $1.536,80 |

The calculation results with varies of interest rate at the first delay of the payment period ($I_{c1}$) and the second delay of the payment period ($I_{c2}$) are shown in Table 4. Table 4 shows the buyer's and vendor's decision is not sensitive in different $I_{c1}$ and $I_{c2}$. Since the buyer is the leader, vendor tries to minimize the inventory cost by set the first delay of payment to be 0 but the second delay of payment period have to be set as long as possible. When the buyer acts as a leader, the progressive payment scheme becomes a single delay of payment scheme.

Figure 6 shows the vendor and buyer total cost for varies of percentage interest rate of the first and second delay of payment. When vendor acts as a leader, total vendor cost (TVUC VL) decreases as the percentage rate of a second delay of payment increase and as a consequence, the buyer total cost increase. The condition is different when the buyer acts as a leader. The vendor total cost and the buyer total cost are not significantly different in value of the percentage rate of the second delay of payment. This condition shows that the total cost is more sensitive when the vendor acts as leader than the buyer acts as a leader. The comparison of the vendor total cost in varies of $I_{c1}$ and $I_{c2}$ can be seen in Figure 7.

*Table 3. Lower bound and upper bound variables when buyer acts as a leader*

| Decision Variables | Upper Bound | Lower Bound |
|---|---|---|
| $K$ | 20 times | 1 times |
| $T$ | 1,3 year | 0 year |
| $M_1$ | 1,2 year | 0 year |
| $M_2$ | 1,2 year | 0 year |

*Table 4. Sensitivity analysis when buyer acts as a leader*

| $I_{c2}$ | $I_{c1}$ | $T^*$ | $K^*$ | $M_1^*$ | $M_2^*$ | *TBUC* | *TVUC* |
|---|---|---|---|---|---|---|---|
| 8% | 3,50% | 0,6956 | 3 | 0 | 1,2 | $ 1.333,74 | $ 1.571,60 |
| | 4% | 0,6638 | 3 | 0 | 1,2 | $ 1.353,10 | $ 1.506,80 |
| | 4,50% | 0,6464 | 3 | 0 | 1,2 | $ 1.372,30 | $ 1.464,70 |
| | 5% | 0,5995 | 3 | 0 | 1,2 | $ 1.394,10 | $ 1.388,70 |
| 9% | 3,50% | 0,6724 | 3 | 0 | 1,2 | $ 1.333,20 | $ 1.538,70 |
| | 4% | 0,6648 | 3 | 0 | 1,2 | $ 1.353,10 | $ 1.508,20 |
| | 4,50% | 0,6441 | 3 | 0 | 1,2 | $ 1.372,40 | $ 1.461,70 |
| | 5% | 0,5899 | 3 | 0 | 1,2 | $ 1.396,20 | $ 1.377,00 |
| 10% | 3,50% | 0,694 | 3 | 0 | 1,2 | $ 1.333,60 | $ 1.569,30 |
| | 4% | 0,6523 | 3 | 0 | 1,2 | $ 1.353,30 | $ 1.491,30 |
| | 4,50% | 0,643 | 3 | 0 | 1,2 | $ 1372,40 | $ 1460,30 |
| | 5% | 0,5997 | 3 | 0 | 1,2 | $ 1394,10 | $ 1388,90 |
| 11% | 3,50% | 0,7734 | 4 | 0 | 1,2 | $ 1.349,40 | $ 1.553,80 |
| | 4% | 0,665 | 3 | 0 | 1,2 | $ 1.353,20 | $ 1.508,50 |
| | 4,50% | 0,6472 | 3 | 0 | 1,2 | $ 1.372,30 | $ 1.465,70 |
| | 5% | 0,5962 | 3 | 0 | 1,2 | $ 1.394,80 | $ 1.384,70 |

Figure 7 shows that the vendor total cost is higher when vendor acts as a leader at small values of $I_{c1}$ and $I_{c2}$, but mostly the vendor total cost when vendor acts as a leader smaller than the buyer acts as a leader. Similar as previous research, vendor has bigger advantage when he acts as a leader than buyer acts as a leader. The difference of vendor total cost when vendor acts a leader and buyer acts a leader becomes wider as the percentage interest rate of the first delay of payment ($I_{c1}$) and percentage interest of the second delay of payment ($I_{c2}$) increase.

Figure 8 shows a comparison of buyer total cost when vendor acts as leader and buyer acts as a leader. In small first delay of payment interest rate ($I_{c1}$), the buyer total cost when buyer acts as a leader higher than vendor acts as a leader. The buyer total cost moves faster when vendor acts as leader and starts to become higher when $I_{c1} = 4\%$ and $I_{c2} = 10\%$ and 11%. The trends continue and the buyer total cost when vendor acts as a leader is higher than buyer acts as a leader in high first delay of payment interest rate ($I_{c1} = 5\%$). This situation shows that buyer has benefited even vendor acts as a leader when the vendor set small value of the first and second delay of payment interest rates.

*Figure 6. Sensitivity analysis in varies of $I_{c1}$ and $I_{c2}$*

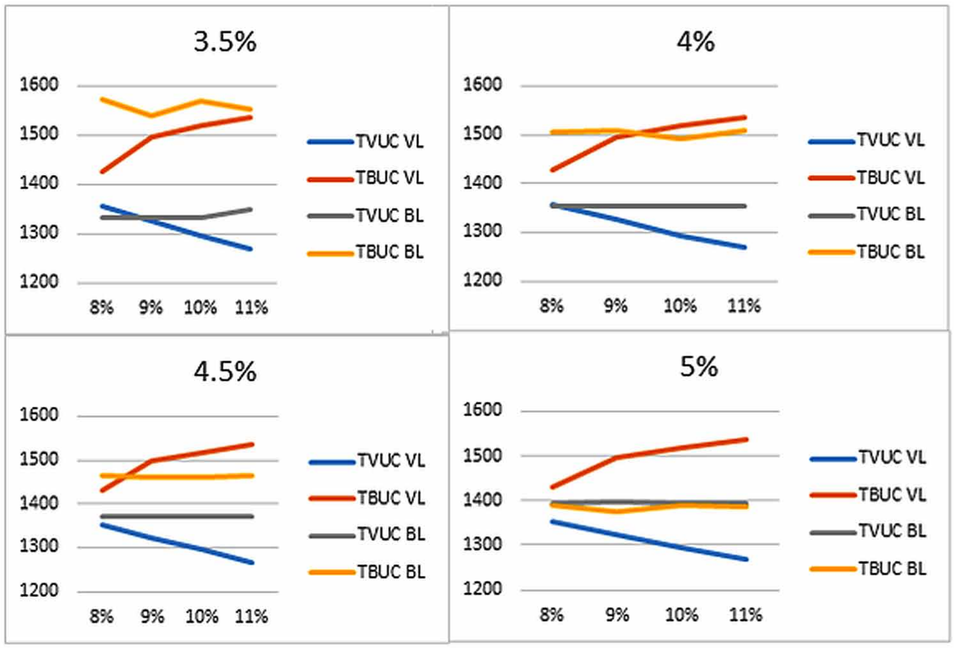



*Figure 7. Vendor total cost when vendor and buyer act as a leader*

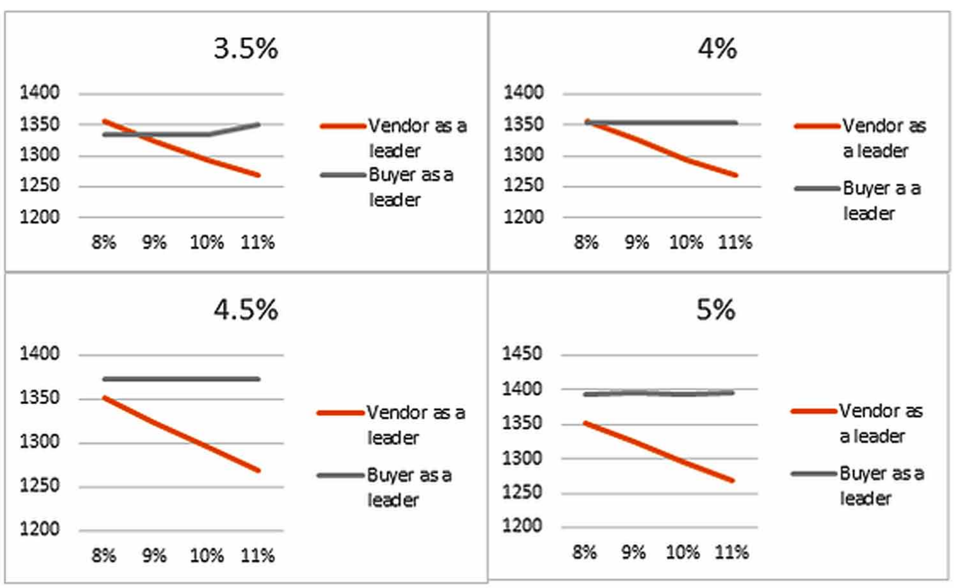
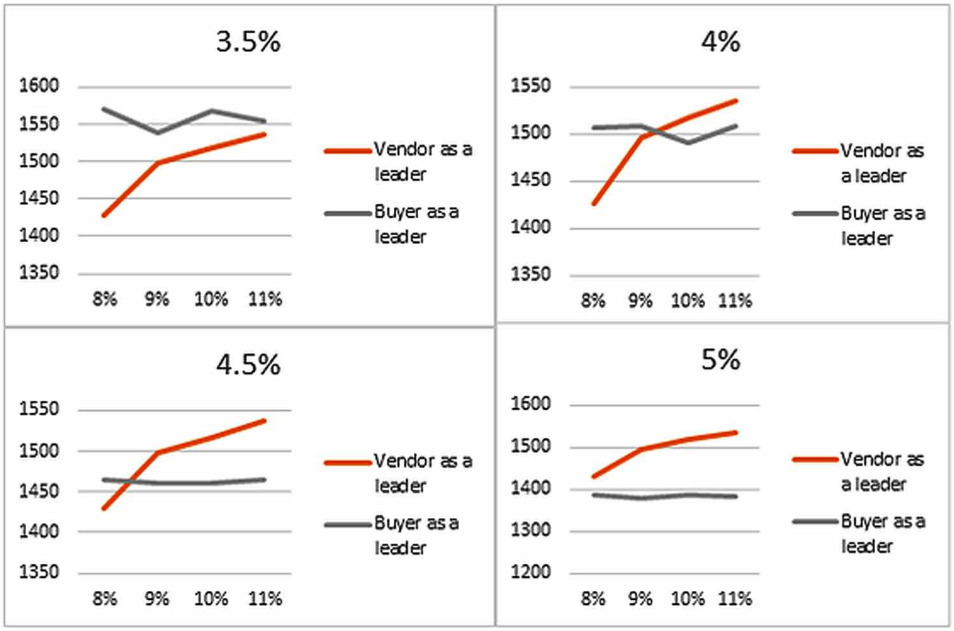


*Figure 8. Buyer total cost when vendor act as a leader and buyer act as a leader*




The sensitivity analysis shows that the progressive payment scheme works as a single delay of payment when buyer acts as a leader and the decision variables and total cost are more stable. On the other side, when the vendor acts as leader, the decision variables and the total cost are sensitive in varies values of the first and second delay of payment interest rate. The sensitivity analysis shows that the progressive payment scheme can be applied in the Stackelberg game model when vendor acts as the leader and vendor set small values of the first and second delay of payment interest rate.

## CONCLUSION

In this chapter, a Stackelberg game model for the progressive payment scheme is developed. The vendor offers to the buyer two delay of the payment period. When the buyer pays before the first delay of payment period, there is no interest charged by the vendor to the buyer. Interest is charged when a buyer pays after the first delay of the payment period and higher interest is charged when a buyer pays after the second delay of payments period. The model is solved using a hybrid of GA and PSO since the model is a nonlinear model. A numerical analysis and sensitivity analysis are conducted to show how the model works and gets some management insights. The sensitivity analysis shows that the progressive payment scheme is similar as a single payment scheme when the buyer acts as a leader. The vendor can get benefit when he acts as a leader, and the buyer can get benefit when he acts as a leader only when the first and second delay of payment interest rate offer by the vendor are high. The Stackelberg game model can work better when vendor acts as a leader and set small values of interest rates.

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