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Abstract

This paper explores theoretical implications of the efficient structure and quiet-life hypotheses on the basis of the generalized user-revenue model constructed by Homma (2009, 2012). From the perspective of the extended generalized-Lerner index (EGLI) on the cost frontier, the following two points are noteworthy: 1) it is not always possible to justify anti-monopoly and anti-concentration policies using support for the quiet-life hypothesis; and 2) new industrial organization policies are required if support for the efficient structure hypothesis is undesirable. Furthermore, where intertemporal regular linkage of single-period EGLIs on the cost frontier exists, the appropriate industrial organization policies must be determined based on a long-term perspective. If this linkage shows an upward trend caused mainly by an upwardly trending intertemporal regular linkage of single-period Herfindahl indices, then anti-monopoly and anti-concentration policies are justified from a long-term perspective. If the upward trend of the intertemporal regular linkage of single-period EGLIs on the cost frontier is, however, caused mainly by the intertemporal regular linkage of single-period dynamic cost efficiencies or single-period optimal planned financial goods, then other policies are desirable because in this case anti-monopoly and anti-concentration policies cause unnecessary distortion in the economy.

Keywords: Efficient structure hypothesis; Quiet-life hypothesis; Generalized user-revenue model; Extended generalized-Lerner index; Cost frontier; Dynamic cost efficiency; Intertemporal regular linkage
JEL classification: C61; D24; G21; L13

1 Introduction

On the basis of the generalized user-revenue model (hereafter the GURM) constructed by Homma (2009, 2012), we explore theoretical implications of the efficient structure hypothesis proposed by Demsetz (1973) and the quiet-life hypothesis first put forward by Berger and Hannan (1998). We develop mathematical formulations and subsequent interpretations covering the relative magnitude of both hypotheses, the relation between both hypotheses and the extended generalized Lerner index (hereafter the EGLI) on the cost frontier proposed by Homma (2009, 2012), and the relation between both hypotheses and the existence of intertemporal regular linkages of single-period dynamic cost efficiencies, single-period optimal planned financial goods, single-period Herfindahl indices, and single-period EGLIs on the cost frontier.

The first step in considering the theoretical implications of both hypotheses requires formulating them in mathematical terms; this is accomplished in Sections 2 and 3. Thus far, in the extant literature, formulations of these hypotheses have only been attempted in empirical contexts (e.g., Berger and Hannan 1998 and Homma et al. 2014). Consequently, they have been verifiable but lack theoretical depth because dynamic-uncertainty banking behavior has not been explicitly formulated under imperfect competition. This paper formulates both hypotheses on the basis of the GURM elaborated by Homma (2009, 2012). The GURM was developed from Hancock's (1985, 1987, 1991) user-cost model (hereafter UCM) of financial firms. Specifically, the GURM is a more general model that relaxes the following five implicit assumptions of the UCM. First, financial firms are risk neutral. Second, no strategic interdependence exists between financial firms. Third, no asymmetric information exists in the market for financial assets and liabilities. Fourth, no uncertainty exists in holding revenues and costs. Fifth, the utility function of financial firms does not depend on equity capital. Furthermore, in order to formulate both hypotheses, this paper develops the GURM in terms of relaxing the sixth implicit assumption of the UCM that no cost inefficiency exists in financial firms (i.e., financial firms are perfectly cost efficient).

Following the mathematical formulation of both hypotheses, Section 3 offers theoretical interpretations based on these elaborations. Demsetz's (1973) efficient structure hypothesis proposes that under the pressure of market competition, efficient firms prevail and grow, so that they become larger, capture greater market shares, and accrue higher profits. Under this hypothesis, a market becomes more efficient as a result of market concentration, thus anti-monopoly and anti-concentration policies cause unnecessary strain in the economy. Significantly, from the perspective of industrial organization, this hypothesis is a composite that suggests three stages of causal relations from firm efficiency to firm growth (i.e., the first stage), then to market structure (i.e., the second stage), and finally to market performance (i.e., the third stage).

Demsetz (1973) equated market structure to market share, whilst market performance was considered in terms of firms' profits. From the perspective of contemporary industrial organization, however, it is more desirable to regard market structure as the Herfindahl index that accounts for the distribution of a financial good, rather than using a simple market share proxy. In addition, market performance could be better captured by accounting for the degree of market competition (i.e., the Lerner index) rather than just considering individual firms' profits. Thus, there is scope for improving on how Demsetz's (1973) original ideas about the two stages of causal relations from firm growth to market structure (i.e., the second stage) and to market performance (i.e., the third stage) are operationalized. By contrast, there is no such need to reconsider Demsetz's (1973) approach in terms of the first stage causality from firm efficiency to firm growth. As noted by Homma et al. (2014), this first stage causality is the fundamental feature of the efficient structure hypothesis, so this paper also regards this causality as the efficient structure hypothesis. Specifically, by regarding firm efficiency as dynamic cost efficiency, and by considering firm growth as an increase in a financial good (e.g., a loan), this paper endeavors to theoretically interpret the efficient structure hypothesis on the basis of the mathematical formulations put forward in the first step. As will be seen, not only the original interpretation of Demsetz (1973) but also a more advanced interpretation of the efficient structure hypothesis is possible.

Moving on, according to Berger and Hannan (1998), the quiet-life hypothesis suggests that in a concentrated market, firms do not minimize costs for various reasons including insufficient managerial effort, lack of profit-maximizing behavior, wasteful expenditures to obtain and maintain monopoly power, and/or survival of inefficient managers. Consequently, increases in market concentration will decrease firm efficiency, thus justifying anti-monopoly and anti-concentration policies. Similar to Homma et al. (2014), by regarding the relationship between market concentration and firm efficiency as that between the Herfindahl index and dynamic cost efficiency, this paper seeks to theoretically interpret the quiet-life hypothesis on the basis of the mathematical formulations in the first step. Doing so suggests that not only the original interpretation of Berger and Hannan (1998) but also a more advanced interpretation of the quiet-life hypothesis is possible.

The third step involves theoretically clarifying the relative magnitude of the efficient structure hypothesis to the quiet-life hypothesis; this is approached in Section 4. Where support for both hypotheses decreases market performance (i.e., the degree of competition on the cost frontier) and if the quiet-life hypothesis is superior in magnitude to the efficient structure hypothesis, then anti-monopoly and anti-concentration policies are necessary. If the efficient structure hypothesis is, however, superior in magnitude to the quiet-life hypothesis, then new industrial organization policies which differ from existing anti-monopoly and anti-concentration policies, and under which efficiency improvements would increase the degree of competition on the cost frontier, are needed. Consequently, it is important to clarify which of these two hypotheses is superior, because the necessary industrial organization policy interventions depend on this.

The fourth step involves identifying and exploring the relation between both hypotheses and the EGLI on the cost frontier; this is covered in Section 5. According to Homma (2009, 2012), the EGLI is useful because it accounts for not only the effect of market structure and conduct but also the effect of financial firms' risk attitudes, the effect of fluctuation risk on short-run profits, and the effect of equity capital on the risk of burden from financial distress costs. This paper develops the EGLI in terms of explicitly accounting for dynamic cost efficiency, to clarify the theoretical relation between both hypotheses and the EGLI. Beyond theory, this development is desirable from a normative policy perspective because it facilitates evaluation by the standard of a frontier bank (i.e., the most cost-efficient bank). On the basis of this development, this paper theoretically clarifies under what assumptions either or both of the hypotheses increase or decrease the EGLI on the cost frontier and thus whether either or both of the hypotheses are desirable. Indeed, the results of the theoretical analysis conducted herein suggest that both desirable and undesirable cases exist, and the following two points are particularly noteworthy: (1) it is not always possible to use support for the quiet-life hypothesis to justify anti-monopoly and anti-concentration policies; and (2) new industrial organization policies are needed if support for the efficient structure hypothesis is undesirable. In terms of the first point, support for the quiet-life hypothesis can decrease the EGLI on the cost frontier (i.e., increase the degree of competition on the cost frontier), and hence where this occurs it cannot always be used to justify anti-monopoly and anti-concentration policies, even if an increase in market concentration decreases dynamic cost efficiency. Such policies are only justified where increased market concentration increases the EGLI on the cost frontier (i.e., decreases the degree of competition on the cost frontier). As such, the enactment and enforcement of anti-monopoly and anti-concentration policies requires careful consideration. Regarding the second point, so far, a theoretical foundation for suggesting that support for the efficient structure hypothesis is undesirable is lacking. However, at least theoretically, support for the efficient structure hypothesis can both decrease the EGLI on the cost frontier (i.e., increase the degree of competition on the cost frontier) and increase the EGLI on the cost frontier (i.e., decreases the degree of competition on the cost frontier). In terms of the latter, it is determined that support for the efficient structure hypothesis is undesirable. In this case, new industrial organization policies which differ from existing anti-monopoly and anti-concentration policies, and under which efficiency improvements would increase the degree of competition on the cost frontier, are advised.

The last step requires clarifying the relation between both hypotheses and the intertemporal regular linkages (i.e., cyclical linkages, monotonic trending linkages, and terminal up-and-down volatile linkages) of single-period dynamic cost efficiencies, single-period optimal planned financial goods, single-period Herfindahl indices, and single-period EGLIs on the cost frontier; this is considered in Section 6. These linkages serve to permit long-term forecasts and long-term dynamic analyses, so they are critical from the perspective of industrial organization.

2 Extending the GURM to Explicitly Account for Dynamic Cost Efficiency

To formulate both hypotheses, this section extends the GURM to explicitly account for dynamic cost efficiency. Specifically, a dynamic cost function is derived by employing the following two procedures, then dynamic cost efficiency is defined by using this dynamic cost function. First, a static transformation function of three vectors and one variable is defined, namely, a vector of real balances of financial goods, a vector of real resource inputs, a vector of exogenous (state) variables affecting the quality of financial goods, and an index of (exogenous) technical change. Moreover, using this defined function, a static cost function is derived from the vector of variable input prices in addition to two vectors and one variable other than the vector of real resource inputs in this defined function. Second, a dynamic transformation function is derived from the vector of Herfindahl indices in the previous period and static cost efficiency in the previous period in addition to three vectors and one variable in the static transformation function. The dynamic cost function of these vectors and efficiency in addition to three vectors and one variable in the static cost function is, furthermore, derived. Next, after deriving the dynamic cost function and defining dynamic cost efficiency, quasi-short-run profits are redefined using the derived dynamic cost function. The dynamic-uncertainty behavior of financial firms is then reformulated to explicitly account for the effects of the Herfindahl indices in the previous period and static cost efficiency in the previous period. Furthermore, on the basis of this formulation, stochastic Euler equations are derived, and generalized user-revenue prices (hereafter the GURPs) and EGLIs are redefined by transforming these equations.

The following preliminary assumptions are made. First, time is divided into discrete periods. Second, these periods are sufficiently short that variations in exogenous (state) variables within the period can be neglected. In other words, exogenous variables are constant within each period but can change discretely at the boundaries between periods. Third, the process of adjustment is essentially instantaneous, allowing stock adjustment problems to be ignored. These assumptions are made to facilitate empirical research, in a manner similar to that of Hancock (1985, 1987, 1991), Homma and Souma (2005), and Homma (2009, 2012), with the expectation that the GURM may provide a consistent basis for such research.

2.1 Dynamic Cost Efficiency and Dynamic Marginal Variable Costs

2.1.1 Static Efficient Production Technology (Static Transformation Function)

In order to derive and define the static cost function as a precursor to defining usual static cost efficiency, static efficient production technology is defined as follows.

Definition 1 (Static Efficient Production Technology) *The static efficient production technology of the i-th financial firm in period t is represented by the following static transformation function:*

$$\phi_i^S\left(\mathbf{q}_{i,t}, \mathbf{x}_{i,t}, \mathbf{z}_{i,t}^Q, \tau_{i,t}\right) = 0, (t \geq 0), \tag{2.1.1}$$

where $\mathbf{q}_{i,t} = (q_{i,1,t}, \cdots, q_{i,N_A+N_L,t})'$ *is a vector of real balances of financial goods, namely financial assets (i.e.,* $q_{i,1,t}, \cdots, q_{i,N_A,t}$*) and liabilities (i.e.,* $q_{i,N_A+1,t}, \cdots, q_{i,N_A+N_L,t}$*),* $\mathbf{x}_{i,t} = (x_{i,1,t}, \cdots, x_{i,M,t})'$ *is a vector of real resource in-*

puts, namely labor, materials, and physical capital, $\mathbf{z}_{i,t}^{Q} = \left(\mathbf{z}_{i,1,t}^{Q\prime}, \cdots, \mathbf{z}_{i,N_A+N_L,t}^{Q\prime}\right)'$ *is a vector of exogenous (state) variables affecting the quality of financial goods, namely, financial technological factors that affect financial goods and real resource inputs, and* $\tau_{i,t}$ *is an index of (exogenous) technical change.*

Similar to the conventional transformation function, this static transformation function has the following two properties. First, some elements of the real balance vector $\mathbf{q}_{i,t}$ may be outputs or inputs, but not all can be inputs, as the existence of outputs cannot otherwise be guaranteed. Second, the static transformation function ϕ_i^S satisfies appropriate regularity conditions. That is, ϕ_i^S is strictly convex in $(\mathbf{q}_{i,t}, \mathbf{x}_{i,t})$ and $\partial\phi_i^S/\partial q_{i,j,t} > 0$ if $q_{i,j,t}$ is an output, $\partial\phi_i^S/\partial q_{i,j,t} < 0$ if $q_{i,j,t}$ is an input, and $\partial\phi_i^S/\partial x_{i,j,t} < 0$, because $\mathbf{x}_{i,t}$ is an input vector.

2.1.2 Static Frontier Variable Cost Function

Next, to derive and define the static frontier cost function required for defining usual static cost efficiency, real resource inputs are assumed to be optimized within a single period, taking financial goods (i.e., outputs and fixed inputs) as given. Specifically, for a single period, it is assumed that the financial firm takes the vector of input prices $\mathbf{p}_{i,t} = (p_{i,1,t}, \cdots, p_{i,M,t})'$ as given and minimizes real resource variable costs $\sum_{j=1}^{M} p_{i,j,t} \cdot x_{i,j,t}$ with respect to the vector of real resource inputs $\mathbf{x}_{i,t}$ subject to the static transformation function ϕ_i^S given by Eq. (2.1.1). Under this assumption, the following static frontier variable cost function is derived and defined.

Definition 2 (Static Frontier Variable Cost Function) *The static frontier variable cost function of the i-th financial firm in period t, denoted by* $C_i^{SFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right)$*, is given by*

$$\begin{aligned} & C_i^{SFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right) \\ & \quad = \min_{\mathbf{x}_{i,t}} \left\{ \sum_{j=1}^{M} p_{i,j,t} \cdot x_{i,j,t} \middle| \phi_i^S\left(\mathbf{q}_{i,t}, \mathbf{x}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right) = 0 \right\}, (t \geq 0). \quad (2.1.2) \end{aligned}$$

From the first property of the static transformation function, some elements of the real balance vector $\mathbf{q}_{i,t}$ may be outputs or inputs, but not all can be inputs, so some elements of $\mathbf{q}_{i,t}$ in the static frontier variable cost function may be outputs or fixed inputs, but not all can be fixed inputs. In order to explicitly account for this property, let $\mathbf{q}^{O}_{i,t} = \left(q^{O}_{i,1,t}, \cdots, q^{O}_{i,N_O,t}\right)'$ denote the output vector of real balances of the i-th financial firm in period t, and let $\mathbf{q}^{F}_{i,t} = \left(q^{F}_{i,1,t}, \cdots, q^{F}_{i,N_F,t}\right)'$ be the fixed input vector. Both vectors include all elements of $\mathbf{q}_{i,t}$.[1] In this case, similar to the conventional variable cost function, because of the duality between transformation functions and variable cost functions, this static frontier variable cost function C^{SFV}_{i} also has the following properties: it is strictly increasing in $\mathbf{p}_{i,t}$ and $\mathbf{q}^{O}_{i,t}$, strictly decreasing in $\mathbf{q}^{F}_{i,t}$, homogeneous of degree one, and strictly concave in $\mathbf{p}_{i,t}$.

2.1.3 Static Actual Variable Cost Function

On the basis of the derived and defined static frontier variable cost function, the static actual variable cost function required to define usual static cost efficiency is defined as follows.

Definition 3 (Static Actual Variable Cost Function) *The static actual variable cost function of the i-th financial firm in period t, denoted by $C^{SAV}_{i}\left(\mathbf{a}^{SIE}_{i,t}, \mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}^{Q}_{i,t}, \tau_{i,t}\right)$, is given by*

$$\begin{aligned} C^{SAV}_{i}\left(\mathbf{a}^{SIE}_{i,t}, \mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}^{Q}_{i,t}, \tau_{i,t}\right) &= \sum_{j=1}^{M} p_{i,j,t} \cdot a^{SIE}_{i,j,t} \cdot \frac{\partial C^{SFV}_{i}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}^{Q}_{i,t}, \tau_{i,t}\right)}{\partial p_{i,j,t}} \\ &= \sum_{j=1}^{M} p_{i,j,t} \cdot a^{SIE}_{i,j,t} \cdot x^{SFD}_{i,j}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}^{Q}_{i,t}, \tau_{i,t}\right) \\ &\geq C^{SFV}_{i}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}^{Q}_{i,t}, \tau_{i,t}\right), (t \geq 0), \end{aligned} \tag{2.1.3.1}$$

where $\mathbf{a}^{SIE}_{i,t} = \left(a^{SIE}_{i,1,t}, \cdots, a^{SIE}_{i,M,t}\right)'$ is a vector of inefficiency coefficients of static factor demand functions denoted by $x^{SFD}_{i,j}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}^{Q}_{i,t}, \tau_{i,t}\right)$

[1] In this case, $\mathbf{q}_{t} = \left(\mathbf{q}^{O\prime}_{t}, \mathbf{q}^{F\prime}_{t}\right)'$ and $N_O + N_F = N_A + N_L$ are satisfied.

(= $\partial C_i^{SFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right)\Big/\partial p_{i,j,t},\ j = 1, ..., M$*). Some elements of this vector* $\mathbf{a}_{i,t}^{SIE}$ *may be less than, equal to, or greater than one, but not all can be less than one, as the static actual variable cost function is otherwise less than the static frontier variable cost function.*

From the duality between the static transformation function and the static frontier cost function, the following equations hold:

$$x_{i,j}^{SFD}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right) = \frac{\partial C_i^{SFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right)}{\partial p_{i,j,t}}, (j = 1, ..., M)\,. \tag{2.1.3.2}$$

From these equations, the j-th static factor demand function, $x_{i,j}^{SFD}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right)$, means the j-th optimal input for cost minimization, so the product of this static factor demand function and the inefficiency coefficient, $a_{i,j,t}^{SIE}\cdot x_{i,j}^{SFD}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right)$, is the j-th actual input that explicitly accounts for input inefficiency because the inefficiency coefficient $a_{i,j,t}^{SIE}$ does not necessarily equal one. The product of this actual input and the j-th factor price is the j-th actual input cost, so the sum of all actual input costs is the actual total cost (i.e., the static actual variable cost function) that is not less than the minimum total cost (i.e., the static frontier variable cost function). From the definition of the static actual variable cost function (Definition 3), this variable cost function also exhibits properties similar to the static frontier variable cost function. That is, the static actual variable cost function C_i^{SAV} is strictly increasing in $\mathbf{p}_{i,t}$ and $\mathbf{q}_{i,t}^{O}$, strictly decreasing in $\mathbf{q}_{i,t}^{F}$, homogeneous of degree one, and strictly concave in $\mathbf{p}_{i,t}$. Furthermore, if all the inefficiency coefficients of factor demand functions equal $a_{i,t}^{SIE}$ (i.e., $a_{i,t}^{SIE} = a_{i,j,t}^{SIE} \geq 1,\ j = 1, ..., M$), the following equation holds, so the inefficiency coefficient $a_{i,t}^{SIE}$ has a cost neutral property:

$$C_i^{SAV}\left(\mathbf{a}_{i,t}^{SIE}, \mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right) = a_{i,t}^{SIE}\cdot C_i^{SFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right). \tag{2.1.3.3}$$

2.1.4 Static Cost Efficiency

On the basis of the derived and defined static frontier and actual variable cost functions, static cost efficiency is defined as follows.

Definition 4 (Static Cost Efficiency) *The static cost efficiency of the i-th financial firm in period t, denoted by $EF_{i,t}^{S}$, is given by*

$$EF_{i,t}^{S} = \frac{C_i^{SFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right)}{C_i^{SAV}\left(\mathbf{a}_{i,t}^{SIE}, \mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right)}, (t \geq 0). \qquad (2.1.4)$$

From this definition and the definition of the static actual variable cost function (Definition 3), static cost efficiency $EF_{i,t}^{S}$ is not greater than one (i.e., $EF_{i,t}^{S} \leq 1$), and, if all the inefficiency coefficients of factor demand functions equal $a_{i,t}^{SIE}$ (i.e., $a_{i,t}^{SIE} = a_{i,j,t}^{SIE} \geq 1$, $j = 1, ..., M$), $EF_{i,t}^{S}$ is the inverse of the inefficiency coefficient $a_{i,t}^{SIE}$ (i.e., $EF_{i,t}^{S} = 1 \big/ a_{i,t}^{SIE}$).

2.1.5 Static Neutral Cost Efficiency

From the perspective of empirical feasibility, it is useful to account for a specification where all inefficiency coefficients of factor demand functions are equal, because most empirical models that estimate cost efficiency assume a cost neutral inefficiency coefficient. Consequently, also from the perspective of empirical analyses, consideration of this case is important for illuminating the theoretical foundation of many extant empirical models.

The following three points are assumed in addition to the cost neutral inefficiency coefficient. First, the static frontier variable cost function is identical for all financial firms. Second, the component other than the cost neutral inefficiency coefficient of the static frontier and actual variable cost functions is identical for all financial firms. Third, the cost neutral inefficiency coefficient is an exponential function of an individual function of an index of (exogenous) technical change. The reasons for this third assumption are that static cost efficiency, defined later, exhibits a time-variant property and many existing cost functions take a logarithmic form. Under these three assumptions, the static actual variable cost function can be specified as follows:

$$C_i^{SAV}\left(\mathbf{a}_{i,t}^{SIE},\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},\tau_{i,t}\right)=\exp\left\{a_i^S\left(\tau_{i,t}\right)\right\}\cdot C^{sf}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},\tau_{i,t}\right),(t\geq 0), \tag{2.1.5.1}$$

where the coefficient $\exp\left\{a_i^S\left(\tau_{i,t}\right)\right\}$ is the cost neutral inefficiency coefficient $a_{i,t}^{SIE}$ (i.e., $a_{i,t}^{SIE}=\exp\left\{a_i^S\left(\tau_{i,t}\right)\right\}$), and the function $C^{sf}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},\tau_{i,t}\right)$ is common to all financial firms. Similarly, the static frontier variable cost function can be specified as follows:

$$C^{SFV}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},\tau_{i,t}\right)=\exp\left\{\min_i a_i^S\left(\tau_{i,t}\right)\right\}\cdot C^{sf}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},\tau_{i,t}\right),(t\geq 0), \tag{2.1.5.2}$$

where the logarithm of the cost neutral inefficiency coefficient $\min_i a_i^S\left(\tau_{i,t}\right)$ is the minimum of the logarithms of the inefficiency coefficients $a_i^S\left(\tau_{i,t}\right)$ $(i=1,...,N_F)$ for all financial firms. This specification enables the static frontier variable cost function to always be no greater than the static actual variable cost function. On the basis of these specifications, static neutral cost efficiency can be specified as follows:

$$\begin{aligned} EF_{i,t}^{S} &= C^{SFV}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},\tau_{i,t}\right)\Big/C_i^{SAV}\left(\mathbf{a}_{i,t}^{SIE},\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},\tau_{i,t}\right) \\ &= \exp\left[\left\{\min_i a_i^S\left(\tau_{i,t}\right)\right\}-a_i^S\left(\tau_{i,t}\right)\right],(t\geq 0). \end{aligned} \tag{2.1.5.3}$$

Accordingly, static neutral cost efficiency is time variant and can be specified only by the cost neutral inefficiency coefficients. In practical terms, static neutral cost efficiency can be easily estimated by specifying these inefficiency coefficients as the time-variant coefficients of individual dummies of financial firms.

2.1.6 Dynamic Efficient Production Technology (Dynamic Transformation Function)

If we regard the economic behavior of financial firms as static within a single period, it is valid to also regard the efficient production technology as being static. However, for intertemporal dynamic behavior, it is desirable to also account for the possibility that the efficient production technology is also dynamic. To explicitly account for both the efficient structure and quiet-life hypotheses, the efficient production technology needs to be formulated to dynamically account for the effects of the Herfindahl indices in the previous period and static cost efficiency in the previous period. Accordingly, dynamic efficient production technology is defined as the following function of a vector of Herfindahl indices in the previous period and static cost efficiency in the previous period in addition to three vectors and one variable in the static transformation function.

Definition 5 (Dynamic Efficient Production Technology) *The dynamic efficient production technology of the i-th financial firm in period t is represented by the following dynamic transformation function:*

$$\phi_i^D\left(\mathbf{q}_{i,t},\mathbf{x}_{i,t},\mathbf{z}_{i,t}^Q,b_1\cdot\mathbf{HI}_{t-1},b_1\cdot EF_{i,t-1}^S,\tau_{i,t}\right)=0,(t\geq 0)\,, \tag{2.1.6}$$

where b_1 is a parameter used to distinguish between the initial period and the later period: $b_1=0$ for the initial period (i.e., $t=0$), and $b_1=1$ for the later period (i.e., $t\geq 1$). In addition, $\mathbf{HI}_{t-1}=(HI_{1,t-1},...,HI_{N_A+N_L,t-1})'$ is a vector of Herfindahl indices in the previous period, $EF_{i,t-1}^S$ is static cost efficiency in the previous period, and all others are as per the static transformation function.

From this definition, for the initial period, the dynamic transformation function equals the static transformation function, and, for the later period, they differ. Because static cost efficiency is included in the previous period as a variable, the dynamic transformation function in the current period is premised on the existence of the static transformation function in the previous period. Therefore, for all periods including the initial period, provided that the static transformation function exists, the dynamic transformation function can also exist. To the extent that Herfindahl indices in the previous period and static cost efficiency in the previous period affect the transformation function in the current period, the coexistence of both transformation functions continues; this provides the production-technological foundation for simultaneous support of both the efficient structure and quiet-life hypotheses. The properties of the dynamic transformation function with respect to the element of the vector of real balances of financial goods and the element of the vector of real resource inputs are similar to the static transformation function.

2.1.7 Dynamic Frontier Variable Cost Function

Next, the following dynamic frontier variable cost function is derived and defined as a precursor to defining dynamic cost efficiency.

Definition 6 (Dynamic Frontier Variable Cost Function) *The dynamic frontier variable cost function of the i-th financial firm in period t, denoted by* $C_i^{DFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)$*, is given by*

$$\begin{aligned} & C_i^{DFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right) \\ = & \min_{\mathbf{x}_{i,t}} \left\{ \sum\nolimits_{j=1}^{M} p_{i,j,t} \cdot x_{i,j,t} \middle| \, \phi_i^{D}\left(\mathbf{q}_{i,t}, \mathbf{x}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right) = 0 \right\}, \\ & (t \geq 0), \quad (2.1.7) \end{aligned}$$

where three vectors and a variable other than the vector of Herfindahl indices in the previous period and static cost efficiency in the previous period are similar to the static frontier variable cost function.

From this definition, similar to the relation between static and dynamic transformation functions, for the initial period, the dynamic frontier variable cost function equals the static frontier variable cost function, and, for the later period, they differ. The coexistence of both frontier variable cost functions due to the coexistence of both transformation functions (on which both frontier variable cost functions are based) yields the difference to the frontier criterion used for defining cost efficiency. To explicitly account for the possibility of simultaneous support for both the efficient structure and quiet-life hypotheses, static cost efficiency regarding the static frontier variable cost function and dynamic cost efficiency (defined later) regarding the dynamic frontier variable cost function are required to coexist. Properties of the dynamic frontier variable cost function with respect to the element of the vector of real balances of financial goods and the element of the vector of real resource input prices are also similar to the static frontier variable cost function.

2.1.8 Dynamic Actual Variable Cost Function

On the basis of the derived and defined dynamic frontier variable cost function, the dynamic actual variable cost function is defined as follows as a precursor to defining dynamic cost efficiency.

Definition 7 (Dynamic Actual Variable Cost Function) *The dynamic actual variable cost function of the i-th financial firm in period t, denoted by* $C_i^{DAV}\left(\mathbf{a}_{i,t}^{DIE}, \mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)$*, is given by*

$$\begin{aligned}
&C_i^{DAV}\left(\mathbf{a}_{i,t}^{DIE}, \mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right) \\
&= \sum\nolimits_{j=1}^{M} p_{i,j,t} \cdot a_{i,j,t}^{DIE} \cdot \frac{\partial C_i^{DFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)}{\partial p_{i,j,t}} \\
&= \sum\nolimits_{j=1}^{M} p_{i,j,t} \cdot a_{i,j,t}^{DIE} \cdot x_{i,j}^{DFD}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right) \\
&\geq C_i^{DFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right), (t \geq 0), \qquad (2.1.8.1)
\end{aligned}$$

where $\mathbf{a}_{i,t}^{DIE} = \left(a_{i,1,t}^{DIE}, \cdots, a_{i,M,t}^{DIE}\right)'$ *is the vector of inefficiency coefficients of dynamic factor demand functions denoted by*

$$\begin{aligned}
&x_{i,j}^{DFD}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right) \\
&(= \partial C_i^{DFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right) \Big/ \partial p_{i,j,t};\ j = 1, ..., M,).
\end{aligned}$$

Some elements of this vector $\mathbf{a}_{i,t}^{DIE}$ *may be less than, equal to, or greater than one, but not all can be less than one, as otherwise the dynamic actual variable cost function would be less than the dynamic frontier variable cost function.*

Similar to the static case, from the duality between the dynamic transformation function and the dynamic frontier cost function, the following equations hold:

$$x_{i,j}^{DFD}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},b_1\cdot\mathbf{HI}_{t-1},b_1\cdot EF_{i,t-1}^{S},\tau_{i,t}\right)$$
$$=\frac{\partial C_i^{DFV}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},b_1\cdot\mathbf{HI}_{t-1},b_1\cdot EF_{i,t-1}^{S},\tau_{i,t}\right)}{\partial p_{i,j,t}},(j=1,...,M). \tag{2.1.8.2}$$

From these equations, the j-th dynamic factor demand function, $x_{i,j}^{DFD}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},b_1\cdot\mathbf{HI}_{t-1},b_1\cdot EF_{i,t-1}^{S},\tau_{i,t}\right)$, means the j-th optimal input for cost minimization on the basis of dynamic efficient production technology, so the product of this dynamic factor demand function and the dynamic inefficiency coefficient, $a_{i,j,t}^{DIE}\cdot x_{i,j}^{DFD}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},b_1\cdot\mathbf{HI}_{t-1},b_1\cdot EF_{i,t-1}^{S},\tau_{i,t}\right)$, is the j-th actual dynamic input that explicitly accounts for input dynamic inefficiency because the dynamic inefficiency coefficient $a_{i,j,t}^{DIE}$ does not necessarily equal one. The product of this actual dynamic input and the j-th factor price is the j-th actual dynamic input cost, so the sum of all actual dynamic input costs is the actual dynamic total cost (i.e., the dynamic actual variable cost function) that is not less than the minimum dynamic total cost (i.e., the dynamic frontier variable cost function). From this definition of the dynamic actual variable cost function (Definition 7), this variable cost function also has properties similar to the dynamic frontier variable cost function. Furthermore, if all the dynamic inefficiency coefficients of dynamic factor demand functions equal $a_{i,t}^{DIE}$ (i.e., $a_{i,t}^{DIE}=a_{i,j,t}^{DIE}\geq 1,\ j=1,...,M$), similar to the cost neutral inefficiency coefficient $a_{i,t}^{SIE}$, the following equation holds so that the dynamic inefficiency coefficient $a_{i,t}^{DIE}$ also has a cost neutral property:

$$C_i^{DAV}\left(\mathbf{a}_{i,t}^{DIE},\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},b_1\cdot\mathbf{HI}_{t-1},b_1\cdot EF_{i,t-1}^{S},\tau_{i,t}\right)$$
$$=a_{i,t}^{DIE}\cdot C_i^{DFV}\left(\mathbf{p}_{i,t},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{Q},b_1\cdot\mathbf{HI}_{t-1},b_1\cdot EF_{i,t-1}^{S},\tau_{i,t}\right). \tag{2.1.8.3}$$

2.1.9 Dynamic Cost Efficiency

On the basis of the derived and defined dynamic frontier and actual variable cost functions, similar to the definition of static cost efficiency, dynamic cost efficiency is defined as follows.

Definition 8 (Dynamic Cost Efficiency) *The dynamic cost efficiency of the i-th financial firm in period t, denoted by $EF_{i,t}^{D}$, is given by*

$$EF_{i,t}^{D} = \frac{C_i^{DFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)}{C_i^{DAV}\left(\mathbf{a}_{i,t}^{DIE}, \mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)}, (t \geq 0)\,. \tag{2.1.9}$$

From this definition and the definition of the dynamic actual variable cost function (Definition 7), similar to the definition of static cost efficiency, dynamic cost efficiency $EF_{i,t}^{D}$ is also not greater than one (i.e., $EF_{i,t}^{D} \leq 1$), and, in the case that all the dynamic inefficiency coefficients of dynamic factor demand functions equal $a_{i,t}^{DIE}$ (i.e., $a_{i,t}^{DIE} = a_{i,j,t}^{DIE} \geq 1,\ j = 1, ..., M$), $EF_{i,t}^{D}$ is the inverse of the dynamic inefficiency coefficient $a_{i,t}^{DIE}$ (i.e., $EF_{i,t}^{D} = 1 \big/ a_{i,t}^{DIE}$).

2.1.10 Dynamic Neutral Cost Efficiency

For the dynamic case, the following assumption should be noted. For the later period, rather than the initial period, the cost neutral dynamic inefficiency coefficient is a function of not only the index of (exogenous) technical change but also the vector of Herfindahl indices in the previous period and static cost efficiency in the previous period. The purpose of this additional assumption is for dynamic cost efficiency to not only be time-variant but also depend on market structure in the previous period and cost efficiency in the previous period, and thereby to explicitly account for the possibility of simultaneous support for both the efficient structure and quiet-life hypotheses. Under these assumptions, the dynamic actual variable cost function can be specified as follows:

$$
\begin{aligned}
&C_i^{DAV}\left(\mathbf{a}_{i,t}^{DIE}, \mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right) \\
&\quad = \exp\left\{a_i^D\left(b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)\right\} \cdot C^{df}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right), \\
&\qquad\qquad (t \geq 0), \quad (2.1.10.1)
\end{aligned}
$$

where the coefficient $\exp\left\{a_i^D\left(b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)\right\}$ is the cost neutral dynamic inefficiency coefficient $a_{i,t}^{DIE}$ (i.e.,

$$
a_{i,t}^{DIE} = \exp\left\{a_i^D\left(b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)\right\}
$$

), and the function $C^{df}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right)$ is a common component for all financial firms. Similarly, the dynamic frontier variable cost function can be specified as follows:

$$
\begin{aligned}
&C^{DFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right) \\
&\quad = \exp\left\{\min_i a_i^D\left(b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)\right\} \cdot C^{df}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right), \\
&\qquad\qquad (t \geq 0), \quad (2.1.10.2)
\end{aligned}
$$

where the logarithm of the cost neutral dynamic inefficiency coefficient $\min_i a_i^D\left(b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)$ is the minimum of the logarithms of these dynamic inefficiency coefficients $a_i^D\left(b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)$ $(i = 1, ..., N_F)$ for all financial firms. This specification forestalls the dynamic frontier variable cost function from ever being greater than the dynamic actual variable cost function. On the basis of these specifications, dynamic neutral cost efficiency can be specified as follows:

$$
\begin{aligned}
EF_{i,t}^{D} &= \frac{C^{DFV}\left(\mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)}{C_i^{DAV}\left(\mathbf{a}_{i,t}^{DIE}, \mathbf{p}_{i,t}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{Q}, b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)} \\
&= \exp\left[\left\{\min_i a_i^D\left(b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)\right\}\right. \\
&\qquad \left. - a_i^D\left(b_1 \cdot \mathbf{HI}_{t-1}, b_1 \cdot EF_{i,t-1}^{S}, \tau_{i,t}\right)\right], (t \geq 0). \quad (2.1.10.3)
\end{aligned}
$$

From this specification, similar to static neutral cost efficiency, dynamic neutral cost efficiency can be specified only by the cost neutral dynamic inefficiency coefficients, is time-variant, and depends on market structure in the previous period and cost efficiency in the previous period. In practical estimations, dynamic neutral cost efficiency can be easily estimated by specifying these dynamic inefficiency coefficients as individual dummies of financial firms that are time variant and dependent on Herfindahl indices in the previous period and static cost efficiency in the previous period.

2.1.11 Dynamic Frontier and Actual Marginal Variable Costs

Because the relation between the marginal cost of the dynamic frontier variable cost function (hereafter the dynamic frontier marginal variable cost) and the marginal cost of the dynamic actual variable cost function (hereafter the dynamic actual marginal variable cost) is used in the mathematical formulations of both the efficient structure and quiet-life hypotheses considered later, this relation is clarified by the following proposition.

Proposition 1 *Dynamic frontier marginal variable cost (i.e., $\partial C_{i,t}^{DFV} \big/ \partial q_{i,j,t}$) is related to dynamic actual marginal variable cost (i.e., $\partial C_{i,t}^{DAV} \big/ \partial q_{i,j,t}$) as follows:*

$$\begin{aligned}\frac{\partial C_{i,t}^{DFV}}{\partial q_{i,j,t}} &= \left(EF_{i,t}^{D} + \frac{\partial EF_{i,t}^{D}}{\partial \ln q_{i,j,t}} \middle/ \frac{\partial \ln C_{i,t}^{DAV}}{\partial \ln q_{i,j,t}} \right) \cdot \frac{\partial C_{i,t}^{DAV}}{\partial q_{i,j,t}} \\ &= \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial C_{i,t}^{DAV}}{\partial q_{i,j,t}}; \; j = 1, ..., N_A + N_L,\end{aligned} \tag{2.1.11.1}$$

where $C_{i,t}^{DFV}$ is the dynamic frontier variable cost function, $C_{i,t}^{DAV}$ is the dynamic actual variable cost function, $q_{i,j,t}$ is the real balance of the j-th financial good, and $EF_{i,t}^{D}$ is dynamic cost efficiency.

Proof. From the definition of dynamic cost efficiency, the following equation holds:

$$EF_{i,t}^{D} = \frac{C_{i,t}^{DFV}}{C_{i,t}^{DAV}}.$$

Partially differentiating both sides of this equation with respect to the real balance of the j-th financial good $q_{i,j,t}$ leads to the following expression:

$$\frac{\partial EF_{i,t}^{D}}{\partial q_{i,j,t}} = \frac{1}{C_{i,t}^{DAV}} \cdot \left(\frac{\partial C_{i,t}^{DFV}}{\partial q_{i,j,t}} - EF_{i,t}^{D} \cdot \frac{\partial C_{i,t}^{DAV}}{\partial q_{i,j,t}} \right).$$

Transforming this equation with respect to dynamic frontier marginal variable cost $\partial C_{i,t}^{DFV} \big/ \partial q_{i,j,t}$ and rearranging yields

$$\begin{aligned} \frac{\partial C_{i,t}^{DFV}}{\partial q_{i,j,t}} &= C_{i,t}^{DAV} \cdot \frac{\partial EF_{i,t}^{D}}{\partial q_{i,j,t}} + EF_{i,t}^{D} \cdot \frac{\partial C_{i,t}^{DAV}}{\partial q_{i,j,t}} \\ &= \frac{C_{i,t}^{DAV}}{q_{i,j,t}} \cdot \frac{\partial EF_{i,t}^{D}}{\partial \ln q_{i,j,t}} + EF_{i,t}^{D} \cdot \frac{\partial C_{i,t}^{DAV}}{\partial q_{i,j,t}} \\ &= \left(EF_{i,t}^{D} + \frac{\partial EF_{i,t}^{D}}{\partial \ln q_{i,j,t}} \Bigg/ \frac{\partial \ln C_{i,t}^{DAV}}{\partial \ln q_{i,j,t}} \right) \cdot \frac{\partial C_{i,t}^{DAV}}{\partial q_{i,j,t}} \\ &= \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial C_{i,t}^{DAV}}{\partial q_{i,j,t}}. \end{aligned}$$

■

As noted, where the dynamic inefficiency coefficients are cost neutral, dynamic cost efficiency $EF_{i,t}^{D}$ is the inverse of the cost neutral dynamic inefficiency coefficient $a_{i,t}^{DIE}$ (i.e., $EF_{i,t}^{D} = 1 \big/ a_{i,t}^{DIE}$). Therefore, the following equation holds:

$$\left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} = C_{i,t}^{DAV} \cdot \frac{\partial EF_{i,t}^{D}}{\partial C_{i,t}^{DAV}} = C_{i,t}^{DAV} \cdot \frac{\partial \left(a_{i,t}^{DIE} \right)^{-1}}{\partial C_{i,t}^{DAV}} = 0.$$

Consequently, the following equation is obtained:

$$\frac{\partial C_{i,t}^{DFV}}{\partial q_{i,j,t}} = EF_{i,t}^{D} \cdot \frac{\partial C_{i,t}^{DAV}}{\partial q_{i,j,t}}. \tag{2.1.11.2}$$

Based on its definition, dynamic cost efficiency is not greater than one, so dynamic frontier marginal variable cost is not greater than dynamic actual marginal variable cost. Where the dynamic inefficiency coefficients are not cost neutral, if the inverse of the elasticity of the dynamic actual variable cost function with respect to dynamic cost efficiency is not greater than dynamic cost inefficiency (i.e., $\left(\partial \ln C_{i,t}^{DAV} \middle/ \partial EF_{i,t}^{D}\right)^{-1} \leq 1 - EF_{i,t}^{D}$), then the same relation between these two marginal variable costs holds. However, this relation cannot otherwise be established.

2.2 GURM Based on Dynamic Efficient Production Technology

In this subsection, the GURM is modified to explicitly account for dynamic cost efficiency based on the dynamic efficient production technology configured previously. Homma's (2009, 2012) quasi-short-run profits are redefined using the dynamic frontier and actual variable cost functions derived and defined in the previous subsection. Moreover, the dynamic-uncertainty behavior of financial firms configured by Homma (2009, 2012) is reformulated to explicitly account for the effects of Herfindahl indices in the previous period and static cost efficiency in the previous period. Furthermore, on the basis of this formulation, Homma's (2009, 2012) stochastic Euler equations are rederived, and the GURPs and EGLIs are redefined to explicitly reveal the difference between the frontier and the actual by transforming these equations.

2.2.1 Quasi-Short-Run Profits Using Dynamic Frontier and Actual Variable Cost Functions

In the context of dynamic efficient production technology, quasi-short-run profit defined by Homma (2009, 2012) is improved upon in the following three respects. First, the Herfindahl indices in the previous period and static cost efficiency in the previous period are added to the exogenous variables affecting quasi-short-run profit. Second, the stochastic endogenous holding-revenue and holding-cost rates defined by Homma (2009, 2012) are replaced by stochastic dynamic endogenous holding-revenue rates (hereafter SDEHRRs) and stochastic dynamic endogenous holding-cost rates (hereafter SDEHCRs), respectively. Third, the static frontier variable cost function is similarly replaced by a dynamic frontier variable cost function or a dynamic actual variable cost function. Quasi-short-run profits are defined as follows.

Definition 9 (Quasi-Short-Run Profit Based on Dynamic Frontier Cost) *The quasi-short-run profit based on the dynamic frontier cost of the i-th financial firm during period t, denoted by* $\pi_i^{QSF}\left(\mathbf{q}_{i,t-1}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{\pi}\right)$*, is defined as follows:*

$$\pi_i^{QSF}\left(\mathbf{q}_{i,t-1}, \mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{\pi}\right) = \sum_{j=1}^{N_A+N_L} b_j \cdot \left[\left\{1 + b_C \cdot h_{i,j}^R\left(Q_{j,t-1}, \mathbf{z}_{i,j,t-1}^{DH}\right) + \zeta_{i,j,t}\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}\right] - C_i^{DFV}\left(\mathbf{q}_{i,t}, \mathbf{z}_{i,t}^{C}\right), \ (t \geq 1), \quad (2.2.1.1)$$

$$\pi_i^{QSF}\left(\mathbf{q}_{i,0}, \mathbf{z}_{i,0}^{\pi}\right) = \sum_{j=1}^{N_A+N_L} b_j \cdot \left\{b_C \cdot h_{i,j}^R\left(Q_{j,0}, \mathbf{z}_{i,j,0}^{DH}\right) + \zeta_{i,j,0}\right\} \cdot p_{G,0} \cdot q_{i,j,0} - C_i^{DFV}\left(\mathbf{q}_{i,0}, \mathbf{z}_{i,0}^{C}\right), \quad (2.2.1.2)$$

where $\mathbf{z}_{i,t}^{\pi} = \left(\mathbf{z}_{i,t-1}^{DH\prime}, \boldsymbol{\zeta}_{i,t}^{\prime}, p_{G,t-1}, p_{G,t}, \mathbf{z}_{i,t}^{C\prime}\right)^{\prime}$ *(*$t \geq 0$*) are vectors of exogenous variables affecting quasi-short-run profit, and in the case of* $t = 0$, $\mathbf{z}_{i,0}^{\pi} = \left(\mathbf{z}_{i,0}^{DH\prime}, \boldsymbol{\zeta}_{i,0}^{\prime}, p_{G,0}, , \mathbf{z}_{i,0}^{C\prime}\right)^{\prime}$*. More specifically,* $\mathbf{z}_{i,t-1}^{DH} = \left(\mathbf{HI}_{t-2}^{\prime}, EF_{i,t-2}^{S}, \mathbf{z}_{i,t-1}^{H\prime}\right)^{\prime}$ *(*$t \geq 0$*) are vectors of exogenous variables affecting the certain or predictable components of SDEHRR and SDEHCR in the period* $t-1$ *(*≥ -1*), and in the case of* $t \leq 1$,

$$\mathbf{z}_{i,-1}^{DH} = \left(\mathbf{HI}_{-2}^{\prime}, EF_{i,-2}^{S}, \mathbf{z}_{i,-1}^{H\prime}\right)^{\prime} = \mathbf{z}_{i,0}^{DH} = \left(\mathbf{HI}_{-1}^{\prime}, EF_{i,-1}^{S}, \mathbf{z}_{i,0}^{H\prime}\right)^{\prime} = \mathbf{z}_{i,0}^{H}.$$

$\mathbf{z}_{i,t-1}^{H} = \left(\mathbf{z}_{i,1,t-1}^{H\prime}, \cdots, \mathbf{z}_{i,N_A+N_L,t-1}^{H\prime}\right)^{\prime}$ *(*$t \geq 0$*) are vectors of exogenous variables other than Herfindahl indices two periods prior and static cost efficiency two periods prior, and in the case of* $t = 0$, $\mathbf{z}_{i,-1}^{H} = \mathbf{z}_{i,0}^{H} = \left(\mathbf{z}_{i,1,0}^{H\prime}, \cdots, \mathbf{z}_{i,N_A+N_L,0}^{H\prime}\right)^{\prime}$.

$\boldsymbol{\zeta}_{i,t} = \left(\zeta_{i,1,t}, \cdots, \zeta_{i,N_A+N_L,t}\right)'$ ($t \geq 0$) are vectors of the uncertain or unpredictable components of SDEHRR and SDEHCR, and $p_{G,t}$ ($t \geq 0$) are general price indices. $\mathbf{z}^C_{i,t} = \left(\mathbf{p}'_{i,t}, \mathbf{z}^{Q\prime}_{i,t}, b_1 \cdot \mathbf{HI}'_{t-1}, b_1 \cdot EF^S_{i,t-1}, \tau_{i,t}\right)'$ ($t \geq 0$) are vectors of exogenous variables affecting the dynamic frontier variable cost function. b_j is a parameter distinguishing between financial assets and liabilities: $b_j = 1$ for financial assets (i.e., $j = 1, ..., N_A$), and $b_j = -1$ for liabilities (i.e., $j = N_A + 1, ..., N_A + N_L$). $b_C \cdot h^R_{i,j}\left(Q_{j,t-1}, \mathbf{z}^{DH}_{i,j,t-1}\right) + \zeta_{i,j,t}$ ($j = 1, ..., N_A + N_L$) are the SDEHRRs or the SDEHCRs of the j-th financial good of the i-th firm at the end of period $t-1$, and b_C is a parameter distinguishing cash from other financial assets. In other words, if $q_{i,j,t}$ represents cash (i.e., $j = 1$), then $b_C = 0$, whereas if the financial good is another type of financial asset (i.e., $j \neq 1$), then $b_C = 1$. $h^R_{i,j}\left(Q_{j,t-1}, \mathbf{z}^{DH}_{i,j,t-1}\right)$ is the certain or predictable component of the SDEHRR or the SDEHCR, and $Q_{j,t-1}$ is total j-th financial goods (i.e., financial assets or liabilities) in the market.

Definition 10 (Quasi-Short-Run Profit Based on Dynamic Actual Cost) *The quasi-short-run profit based on the dynamic actual cost of the i-th financial firm during period t, denoted by $\pi^{QSA}_i\left(\mathbf{q}_{i,t-1}, \mathbf{q}_{i,t}, \mathbf{z}^\pi_{i,t}\right)$, is defined by replacing the dynamic frontier variable cost function $C^{DFV}_i(\cdot,\cdot)$ in Definition 9 with the dynamic actual variable cost function $C^{DAV}_i(\cdot,\cdot)$ as follows:*

$$\begin{aligned}
&\pi^{QSA}_i\left(\mathbf{q}_{i,t-1}, \mathbf{q}_{i,t}, \mathbf{z}^\pi_{i,t}\right) \\
&= \sum_{j=1}^{N_A+N_L} b_j \cdot \left[\left\{1 + b_C \cdot h^R_{i,j}\left(Q_{j,t-1}, \mathbf{z}^{DH}_{i,j,t-1}\right) + \zeta_{i,j,t}\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}\right] \\
&\qquad - C^{DAV}_i\left(\mathbf{q}_{i,t}, \mathbf{z}^C_{i,t}\right), \ (t \geq 1), \qquad (2.2.1.3)
\end{aligned}$$

$$\begin{aligned}
&\pi^{QSA}_i\left(\mathbf{q}_{i,0}, \mathbf{z}^\pi_{i,0}\right) \\
&= \sum_{j=1}^{N_A+N_L} b_j \cdot \left\{b_C \cdot h^R_{i,j}\left(Q_{j,0}, \mathbf{z}^{DH}_{i,j,0}\right) + \zeta_{i,j,0}\right\} \cdot p_{G,0} \cdot q_{i,j,0} - C^{DAV}_i\left(\mathbf{q}_{i,0}, \mathbf{z}^C_{i,0}\right), \qquad (2.2.1.4)
\end{aligned}$$

where $\pi^{QSA}_i\left(\mathbf{q}_{i,t-1}, \mathbf{q}_{i,t}, \mathbf{z}^\pi_{i,t}\right)$ is not greater than $\pi^{QSF}_i\left(\mathbf{q}_{i,t-1}, \mathbf{q}_{i,t}, \mathbf{z}^\pi_{i,t}\right)$ (i.e.,

$\pi_i^{QSA}\left(\mathbf{q}_{i,t-1},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{\pi}\right) \leq \pi_i^{QSF}\left(\mathbf{q}_{i,t-1},\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{\pi}\right)$), *because* $C_i^{DAV}\left(\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{C}\right)$ *is not less than* $C_i^{DFV}\left(\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{C}\right)$ *(i.e.,* $C_i^{DAV}\left(\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{C}\right) \geq C_i^{DFV}\left(\mathbf{q}_{i,t},\mathbf{z}_{i,t}^{C}\right)$*).*

The SDEHRR (or the SDEHCR) in Definitions 9 and 10 is the revenue obtained (or cost required) from holdings per currency unit for a single time period. Thus, $\left\{b_C \cdot h_{i,j}^{R}\left(Q_{j,t-1},\mathbf{z}_{i,j,t-1}^{DH}\right)+\zeta_{i,j,t}\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1}$ is the holding revenue or cost, which is received or paid at the end of period $t-1$, and the net cash flow of the i-th firm produced by financial good j in period t is defined as

$$b_j \cdot \left[\left\{1+b_C \cdot h_{i,j}^{R}\left(Q_{j,t-1},\mathbf{z}_{i,j,t-1}^{DH}\right)+\zeta_{i,j,t}\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}\right].$$

For example, for an asset such as a loan (with the exception of cash), $b_j = 1$, in which case the second and third terms, $\left\{b_C \cdot h_{i,j}^{R}\left(Q_{j,t-1},\mathbf{z}_{i,j,t-1}^{DH}\right)+\zeta_{i,j,t}\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1}$, indicate holding revenues, and the first and fourth terms, $p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}$, represent the change in the nominal asset for the period. If loan repayments by the borrower exceed total new loans for the period, the revised balance indicates a positive change, and if repayments are lower than total new loans for the period, the value is negative. These terms thus express the net cash flow resulting from the acceptance of an asset. However, cash, which is an asset, generates no interest. As such, the holding revenue for cash is zero. Similarly, in the case of a liability such as a deposit, $b_j = -1$, the second and third terms, $-\left\{b_C \cdot h_{i,j}^{R}\left(Q_{j,t-1},\mathbf{z}_{i,j,t-1}^{DH}\right)+\zeta_{i,j,t}\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1}$, indicate holding costs, whereas the first and fourth terms, $-p_{G,t-1} \cdot q_{i,j,t-1} + p_{G,t} \cdot q_{i,j,t}$, represent nominal liability change. The change is therefore positive if new deposits exceed withdrawals and negative if new deposits are less than withdrawals. These terms thus indicate the net cash flow resulting from the issuance of a liability.

2.2.2 Dynamic-Uncertainty Behavior and Stochastic Euler Equations

To formulate the dynamic-uncertainty behavior of financial firms as a stochastic dynamic programming problem (hereafter SDP), similar to Homma

(2009, 2012), the following three key assumptions are made. First, the decision of the financial firm is made after uncertainty is resolved, such that, in each period, the financial firm chooses the state variable of the next period directly. Second, the financial firm chooses a plan that maximizes the expected value of the discounted intertemporal utility function of a stream of planned quasi-short-run profits and planned equity capital. Third, the intertemporal utility function is additively separable. The reason for the first key assumption is that the adjustment cost of stock variables is assumed to be zero and more reliable information on the decision leads to a rise in the value of the firm. In the second key assumption, the utility function is used to explicitly account for the effect of risk attitudes other than risk neutrality, and the utility function depends on planned equity capital to account for (although indirectly) the risk of the burden of financial distress costs from a banking theory perspective because an increase in equity capital reduces this risk. The third key assumption is conventional and widely held.

These key assumptions are based on the following three underlying assumptions. First, the state variables are classified as either endogenous or exogenous. The endogenous state variable vectors $\mathbf{q}_{i,t}$ $(t \geq 0)$ are vectors of real balances of financial goods, and the exogenous state variable vectors $\mathbf{z}_{i,t}$ $(t \geq 0)$ are those which affect quasi-short-run profits $\mathbf{z}^{\pi}_{i,t}$ $(t \geq 0)$ (i.e., $\mathbf{z}_{i,t} = \mathbf{z}^{\pi}_{i,t}$). Within these exogenous variables, the vectors of those exogenous variables that affect equity capital are defined as $\mathbf{z}^{e}_{i,t} = \left(p_{G,t}, \mathbf{z}^{C\prime}_{i,t}\right)'$ $(t \geq 0)$. Second, the exogenous state variable vectors $\mathbf{z}_{i,t}$ $(t \geq 0)$ are vectors of random variables, and the stochastic term $\{\mathbf{z}_{i,t}\}_{t \geq 0}$ follows a stationary Markov process. Let $(Z, \boldsymbol{B}_Z)$ be a measurable space, where Z is a set of $\mathbf{z}_{i,t}$, and $\boldsymbol{B}_Z$ is a σ-algebra of its subsets. In this case, the stochastic properties of the exogenous state variables can be expressed as a stationary transition function: $Q : Z \times \boldsymbol{B}_Z \to [0, 1]$.[2] The interpretation of this definition is that $Q\left(\mathbf{z}_{i,t}, A_{i,t+1}\right)$ is the probability that the state of the next period lies in the set $A_{i,t+1}$, given that the current state is $\mathbf{z}_{i,t}$. The product space of $(Z, \boldsymbol{B}_Z)$ is expressed as $\left(Z^t, \boldsymbol{B}^t_Z\right) = (Z \times \cdots \times Z, \boldsymbol{B}_Z \times \cdots \times \boldsymbol{B}_Z)$, and $\mathbf{z}_{i,0}$ $(\in Z)$ is

[2]For further details regarding the stationary transition function, see Stokey and Lucas (1989, p.212).

given. Third, the decision to be made in period t can depend on information that will be available at that time. This information can be expressed as a sequence of vectors of exogenous state variables. Let $\mathbf{z}_i^t = (\mathbf{z}_{i,1}, ..., \mathbf{z}_{i,t})$ $(\in Z^t)$ denote the partial history in periods 1 through t, and let $(Y, \boldsymbol{B}_Y)$ be a measurable space, where Y is a set of vectors of the endogenous state variables $\mathbf{q}_{i,t}$, and $\boldsymbol{B}_Y$ is a σ-algebra of its subsets. A plan $\mathbf{q}_i^p$ is then defined as the set of a value $\mathbf{q}_{i,0}^p$ $(\in Y)$ and a sequence of functions $\mathbf{q}_{i,t}^p : Z^t \to Y$ $(t \geq 1)$, where $\mathbf{q}_{i,t}^p(\mathbf{z}_i^t)$ is the value of $\mathbf{q}_{i,t+1}$ that will be chosen in period t if the partial history of the exogenous state variables in periods 1 through t is $\mathbf{z}_i^t$ (i.e., $\mathbf{q}_i^p = \left\{\mathbf{q}_{i,0}^p, \left\{\mathbf{q}_{i,t}^p(\mathbf{z}_i^t)\right\}_{t=1}^{\infty}\right\}$).

From the second underlying assumption, the following definition of probability measures is proposed.

Definition 11 (Probability Measure) *The probability measures on $\left(Z^t, \boldsymbol{B}_Z^t\right)$, $\mu^t(\mathbf{z}_{i,0}, \cdot) : \boldsymbol{B}_Z^t \to [0,1]$ $(t \geq 1)$, are defined as follows.*[3] *For any rectangle $A_i^t = A_{i,1} \times \cdots \times A_{i,t} \in \boldsymbol{B}_Z^t$,*

$$\mu^t\left(\mathbf{z}_{i,0}, A_i^t\right) = \int_{A_{i,1}} \cdots \int_{A_{i,t-1}} \int_{A_{i,t}} Q(\mathbf{z}_{i,t-1}, \mathbf{dz}_{i,t})\, Q(\mathbf{z}_{i,t-2}, \mathbf{dz}_{i,t-1}) \cdots Q(\mathbf{z}_{i,0}, \mathbf{dz}_{i,1}), \tag{2.2.2.1}$$

where the probability measure $\mu^t(\mathbf{z}_{i,0}, \cdot)$ satisfies the properties of measures, and $\mu^t(\mathbf{z}_{i,0}, Z^t) = 1$.

From this definition of probability measures and the above key and underlying assumptions, the SDP of the i-th financial firm is formulated as follows:

$$\begin{aligned}
&\max_{\mathbf{q}_i^p} u_i\left[\pi_i^{QSF}\left(\mathbf{q}_{i,0}, \mathbf{q}_{i,0}^p(\mathbf{z}_{i,0}), \mathbf{z}_{i,0}^{\pi}\right), q_{e,i}^p\left(\mathbf{q}_{i,0}^p(\mathbf{z}_{i,0}), \mathbf{z}_{i,0}^e\right)\right] \\
&\qquad + \lim_{T\to\infty} \sum_{t=1}^{T} \int_{Z^t} \beta_i^t \cdot u_i\left[\pi_i^{QSF}\left(\mathbf{q}_{i,t-1}^p\left(\mathbf{z}_i^{t-1}\right), \mathbf{q}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^{\pi}\right),\right. \\
&\qquad\qquad \left. q_{e,i}^p\left(\mathbf{q}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^e\right)\right] \mu^t\left(\mathbf{z}_{i,0}, \mathbf{dz}_i^t\right),
\end{aligned} \tag{2.2.2.2}$$

[3] For a comprehensive account of probability measures, see Stokey and Lucas (1989: pp. 220-225).

where $u_i(\cdot,\cdot)$ is the utility function, $\beta_i^t = \prod_{s=0}^{t-1} \beta_{i,s} = \prod_{s=0}^{t-1} \frac{1}{1+r_{i,s}^D}$ is the cumulative discount factor, and $r_{i,s}^D$ is the subjective rate of time preference.[4] $\pi_i^{QSF}\left(\mathbf{q}_{i,t-1}^p\left(\mathbf{z}_i^{t-1}\right), \mathbf{q}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^\pi\right)$ $(t \geq 1)$ and $\pi_i^{QSF}\left(\mathbf{q}_{i,0}, \mathbf{q}_{i,0}^p\left(\mathbf{z}_{i,0}\right), \mathbf{z}_{i,0}^\pi\right)$ are the planned quasi-short-run profit based on the dynamic frontier cost, which are as follows:

$$\begin{aligned}
&\pi_i^{QSF}\left(\mathbf{q}_{i,t-1}^p\left(\mathbf{z}_i^{t-1}\right), \mathbf{q}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^\pi\right) \\
&= \sum_{j=1}^{N_A+N_L} b_j \cdot \left[\left\{1 + b_C \cdot h_{i,j}^R\left(Q_{j,t-1}^p, \mathbf{z}_{i,j,t-1}^{DH}\right) + \zeta_{i,j,t}\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1}^p\left(\mathbf{z}_i^{t-1}\right)\right. \\
&\qquad \left. - p_{G,t} \cdot q_{i,j,t}^p\left(\mathbf{z}_i^t\right)\right] - C_i^{DFV}\left(\mathbf{q}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^C\right) \ (t \geq 1), \quad (2.2.2.3)
\end{aligned}$$

$$\begin{aligned}
&\pi_i^{QSF}\left(\mathbf{q}_{i,0}, \mathbf{q}_{i,0}^p\left(\mathbf{z}_{i,0}\right), \mathbf{z}_{i,0}^\pi\right) \\
&\quad = \sum_{j=1}^{N_A+N_L} b_j \cdot \left[\left\{1 + b_C \cdot h_{i,j}^R\left(Q_{j,0}, \mathbf{z}_{i,j,0}^{DH}\right) + \zeta_{i,j,0}\right\} \cdot p_{G,0} \cdot q_{i,j,0}\right. \\
&\qquad \left. - p_{G,0} \cdot q_{i,j,0}^p\left(\mathbf{z}_{i,0}\right)\right] - C_i^{DFV}\left(\mathbf{q}_{i,0}^p\left(\mathbf{z}_{i,0}\right), \mathbf{z}_{i,0}^C\right). \quad (2.2.2.4)
\end{aligned}$$

The functions in these planned quasi-short-run profits (Eqs. (2.2.2.3) and (2.2.2.4)) are defined as follows.

- $h_{i,j}^R\left(Q_{j,t-1}^p, \mathbf{z}_{i,j,t-1}^{DH}\right)$: Planned certain or predictable component of the SDEHRR or the SDEHCR. Using this component, the planned SDEHRR or the planned SDEHCR is defined as $b_C \cdot h_{i,j}^R\left(Q_{j,t-1}^p, \mathbf{z}_{i,j,t-1}^{DH}\right) + \zeta_{i,j,t}$; $j = 1, ..., N_A + N_L$, where $Q_{j,t-1}^p$ is the planned total j-th financial good in the market. Other vectors and variables (i.e., $\mathbf{z}_{i,j,t-1}^{DH}$ and $\zeta_{i,j,t}$) are as defined above.

- $C_i^{DFV}\left(\mathbf{q}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^C\right)$: Planned dynamic frontier variable cost function.

[4] For details regarding this optimization problem, see Stokey and Lucas (1989, pp.241-254).

In addition, $q_{e,i}^{p}\left(\mathbf{q}_{i,t}^{p}\left(\mathbf{z}_{i}^{t}\right),\mathbf{z}_{i,t}^{e}\right)$ $(t\geq 0)$ is the planned equity capital, given by

$$q_{e,i}^{p}\left(\mathbf{q}_{i,t}^{p}\left(\mathbf{z}_{i}^{t}\right),\mathbf{z}_{i,t}^{e}\right)=\sum_{j=1}^{N_A}p_{G,t}\cdot q_{i,j,t}^{p}\left(\mathbf{z}_{i}^{t}\right)+\sum_{j=1}^{M_F}p_{i,j,t}^{F}\cdot x_{F,i,j}^{p}\left(\mathbf{q}_{i,t}^{p}\left(\mathbf{z}_{i}^{t}\right),\mathbf{z}_{i,t}^{C}\right)$$
$$-\sum_{j=N_A+1}^{N_A+N_L}p_{G,t}\cdot q_{i,j,t}^{p}\left(\mathbf{z}_{i}^{t}\right),\ (t\geq 0),\quad (2.2.2.5)$$

where $p_{i,j,t}^{F}$ is the j-th real resource fixed factor price,[5] and $x_{F,i,j}^{p}\left(\mathbf{q}_{i,t}^{p}\left(\mathbf{z}_{i}^{t}\right),\mathbf{z}_{i,t}^{C}\right)$ is the conditional factor demand function for the j-th planned real resource fixed input.

The necessary conditions for the SDP in sequence form can be found by adopting a variational approach. Such conditions are represented by stochastic Euler equations, which for the above SDP (2.2.2.2) are expressed as

$$-\frac{\partial u_{i,t}^{F*}}{\partial\pi_{i,t}^{QSF*}}\cdot\left(b_j\cdot p_{G,t}+\frac{\partial C_{i,t}^{DFV*}}{\partial q_{i,j,t}^{p*}}\right)+b_j\cdot p_{G,t}\cdot\frac{\partial u_{i,t}^{F*}}{\partial q_{e,i,t}^{p*}}$$
$$+\beta_{i,t}\cdot b_j\cdot p_{G,t}\cdot\int_Z\left\{1+b_C\cdot\left(h_{i,j,t}^{R*}+\frac{\partial h_{i,j,t}^{R*}}{\partial\ln q_{i,j,t}^{p*}}\right)+\zeta_{i,j,t+1}\right\}$$
$$\cdot\frac{\partial u_{i,t+1}^{F*}}{\partial\pi_{i,t+1}^{QSF*}}Q\left(\mathbf{z}_{i,t},\mathbf{dz}_{i,t+1}\right)=0;\ j=1,...,N_A+N_L,\quad (2.2.2.6)$$

where $q_{i,j,t}^{p*}=q_{i,j,t}^{p*}\left(\mathbf{z}_{i}^{t}\right)$ $(j=1,\cdots,N_A+N_L)$ denote the optimal levels for financial goods. Furthermore, $\pi_{i,t}^{QSF*}=\pi_{i}^{QSF}\left(\mathbf{q}_{i,t-1}^{p*}\left(\mathbf{z}_{i}^{t-1}\right),\mathbf{q}_{i,t}^{p*}\left(\mathbf{z}_{i}^{t}\right),\mathbf{z}_{i,t}^{\pi}\right)$, $q_{e,i,t}^{p*}=q_{e,i}^{p}\left(\mathbf{q}_{i,t}^{p*}\left(\mathbf{z}_{i}^{t}\right),\mathbf{z}_{i,t}^{e}\right)$, $u_{i,t}^{F*}=u_i\left(\pi_{i,t}^{QSF*},q_{e,i,t}^{p*}\right)$, $C_{i,t}^{DFV*}=C_{i}^{DFV}\left(\mathbf{q}_{i,t}^{p*}\left(\mathbf{z}_{i}^{t}\right),\mathbf{z}_{i,t}^{C}\right)$, and $h_{i,j,t}^{R*}=h_{i,j}^{R}\left(Q_{j,t}^{p*},\mathbf{z}_{i,j,t}^{DH}\right)$ $(j=1,\cdots,N_A+N_L)$. For $\mathbf{dz}_{i,t+1}$, the following

[5] $p_{i,j,t}^{F}$ is an element of $\mathbf{p}_{i,t}$. $p_{i,j,t}^{F}$ is therefore an element of $\mathbf{z}_{i,t}^{C}$ because $\mathbf{p}_{i,t}$ is an element of $\mathbf{z}_{i,t}^{C}$.

equality holds:

$$\begin{aligned}
\mathbf{dz}_{i,t+1} &= \mathbf{dz}^{\pi}_{i,t+1} = \left(\mathbf{dz}^{DH\prime}_{i,t}, \mathbf{d\zeta}^{\prime}_{i,t+1}, dp_{G,t}, dp_{G,t+1}, \mathbf{dz}^{C\prime}_{i,t+1}\right)^{\prime} \\
&= \left(\mathbf{d\zeta}^{\prime}_{i,t+1}, dp_{G,t+1}, \mathbf{dp}^{\prime}_{i,t+1}, \mathbf{dz}^{Q\prime}_{i,t+1}, d\tau_{i,t+1}\right)^{\prime} \\
(\because\ & \mathbf{dz}^{DH}_{i,t} = \mathbf{0}^{Z},\ dp_{G,t} = 0,\ \mathbf{dHI}_{t} = \mathbf{0}^{HI}, \text{ and } dEF^{D}_{i,t} = 0).
\end{aligned}$$

If the utility function $u^{F*}_{i,t}$ is concave and continuously differentiable in $\mathbf{q}^{p*}_{i,t-1}$ and $\mathbf{q}^{p*}_{i,t}$ and is integrable,[6] and if each of the partial derivatives of $u^{F*}_{i,t}$ with respect to $\mathbf{q}^{p*}_{i,t-1}$ is absolutely integrable,[7] then the stochastic Euler equations (2.2.2.6) with the transversality conditions

$$\lim_{t\to\infty} \beta^{t}_{i} \cdot \int_{Z} \frac{\partial u^{F*}_{i,t+1}}{\partial \pi^{QSF*}_{i,t+1}} \cdot \frac{\partial \pi^{QSF*}_{i,t+1}}{\partial q^{p*}_{i,j,t}} \cdot q^{p*}_{i,j,t}\, Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) = 0;\ j = 1, ..., N_A + N_L \tag{2.2.2.7}$$

are sufficient conditions for an optimal plan $\mathbf{q}^{p*}_{i} = \left\{\mathbf{q}^{p*}_{i,0}, \left\{\mathbf{q}^{p*}_{i,t}\right\}^{\infty}_{t=1}\right\}$.

Equation (2.2.2.6) is the stochastic Euler equations in the case of no dynamic cost inefficiency and no dynamic price inefficiencies (i.e., no dynamic pricing errors). However, to derive not only the GURP on the cost frontier but also the GURP on the actual cost, these inefficiencies need to be explicitly considered. If these inefficiencies exist, Eq. (2.2.2.6) is corrected as follows:

$$\begin{aligned}
&-\frac{\partial u^{A*}_{i,t}}{\partial \pi^{QSA*}_{i,t}} \cdot \left(b_j \cdot p_{G,t} + \frac{\partial C^{DAV*}_{i,t}}{\partial q^{p*}_{i,j,t}}\right) + b_j \cdot p_{G,t} \cdot \frac{\partial u^{A*}_{i,t}}{\partial q^{p*}_{e,i,t}} \\
&\qquad + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot \int_{Z} \left\{1 + b_C \cdot \left(h^{R*}_{i,j,t} + \frac{\partial h^{R*}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}}\right) + \zeta_{i,j,t+1}\right\} \\
&\qquad\qquad \cdot \frac{\partial u^{A*}_{i,t+1}}{\partial \pi^{QSA*}_{i,t+1}} Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) = \varepsilon^{P}_{i,j,t};\ j = 1, ..., N_A + N_L,
\end{aligned} \tag{2.2.2.8}$$

where $\pi^{QSA*}_{i,t}$ $(= \pi^{QSA}_{i}\left(\mathbf{q}^{p*}_{i,t-1}\left(\mathbf{z}^{t-1}_{i}\right), \mathbf{q}^{p*}_{i,t}\left(\mathbf{z}^{t}_{i}\right), \mathbf{z}^{\pi}_{i,t}\right))$ is the maximum planned

[6] Integrability of $u^{F*}_{i,t}$ means that $\int_Z u^{F*}_{i,t}\, Q\left(\mathbf{z}_{i,t-1}, \mathbf{dz}_{i,t}\right) < \infty$.

[7] Absolute integrability of $\frac{\partial u^{F*}_{i,t}}{\partial q^{p*}_{i,j,t-1}}$ is defined as $\int_Z \left|\frac{\partial u^{F*}_{i,t}}{\partial q^{p*}_{i,j,t-1}}\right| Q\left(\mathbf{z}_{i,t-1}, \mathbf{dz}_{i,t}\right) < \infty$.

quasi-short-run profit based on dynamic actual cost, $u_{i,t}^{A*}$ $\left(= u_i\left(\pi_{i,t}^{QSA*}, q_{e,i,t}^{p*}\right)\right)$ is the maximum planned utility based on this quasi-short-run profit, $C_{i,t}^{DAV*}$ $\left(= C_i^{DAV}\left(\mathbf{q}_{i,t}^{p*}\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^C\right)\right)$ is the planned dynamic actual variable cost, and $\varepsilon_{i,j,t}^P$ $(j = 1, ..., N_A + N_L)$ are terms used to explicitly account for dynamic price inefficiencies (i.e., dynamic pricing errors). More specifically, if no dynamic price inefficiency exists, then $\varepsilon_{i,j,t}^P = 0$, whereas if any dynamic price inefficiency exists, then $\varepsilon_{i,j,t}^P \neq 0$. In the case of no dynamic cost inefficiency and no dynamic price inefficiencies, Eq. (2.2.2.8) equals Eq. (2.2.2.6).

2.2.3 Risk Corrections, GURP on the Cost Frontier, and GURP on the Actual Cost

Similar to Homma (2009, 2012), the GURP on the cost frontier and the GURP on the actual cost (see below) can be derived by transforming the stochastic Euler equations (Eqs. (2.2.2.6) and (2.2.2.8)). More specifically, first, similar to the treatment in the consumption-based capital asset pricing model (hereafter CCAPM), Eqs. (2.2.2.6) and (2.2.2.8) are transformed into an expression of risk correction. Next, these transformed equations are again transformed with respect to dynamic frontier marginal variable cost or dynamic actual marginal variable cost and rearranged. Finally, the right-hand sides of these retransformed equations are defined as the GURP on the cost frontier and the GURP on the actual cost, respectively. The form of the Eq. (2.2.2.6) expression of risk correction is provided by the following theorem.

Theorem 1 *Under the assumption that* $\partial u_{i,t}^{F*} \Big/ \partial \pi_{i,t}^{QSF*} \neq 0$ *and* $E\left[\zeta_{i,j,t+1} \middle| \mathbf{z}_{i,t}\right] = 0$, *Eq.(2.2.2.6) can be transformed into an expression of risk correction as fol-*

lows:

$$
\begin{aligned}
&- b_j \cdot p_{G,t} - MC_{i,j,t}^{DFV*} + b_j \cdot p_{G,t} \cdot MRS_{e,i,t}^{F\pi *} \\
&\qquad + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot \left\{ 1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*} \right) \right\} \cdot E \left[IMRS_{\pi,i,t+1}^{F*} \left| \mathbf{z}_{i,t} \right. \right] \\
&\qquad\quad + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot \frac{cov\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{F*} \left/ \partial \pi_{i,t+1}^{QSF*} \right. \left| \mathbf{z}_{i,t} \right. \right)}{\partial u_{i,t}^{F*} \left/ \partial \pi_{i,t}^{QSF*} \right.} = 0; \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad j = 1, ..., N_A + N_L, \quad (2.2.3.1)
\end{aligned}
$$

where $MC_{i,j,t}^{DFV*} = \partial C_{i,t}^{DFV*} \left/ \partial q_{i,j,t}^{p*} \right.$, $MRS_{e,i,t}^{F\pi *} = \left(\partial u_{i,t}^{F*} \left/ \partial q_{e,i,t}^{p*} \right. \right) \Big/ \left(\partial u_{i,t}^{F*} \left/ \partial \pi_{i,t}^{QSF*} \right. \right)$,[8] $\eta_{i,j,t}^{*} = \partial h_{i,j,t}^{R*} \left/ \partial \ln q_{i,j,t}^{p*} \right.$, $IMRS_{\pi,i,t+1}^{F*} = \left(\partial u_{i,t+1}^{F*} \left/ \partial \pi_{i,t+1}^{QSF*} \right. \right) \Big/ \left(\partial u_{i,t}^{F*} \left/ \partial \pi_{i,t}^{QSF*} \right. \right)$,[9] *and* $E \left[\cdot \left| \mathbf{z}_{i,t} \right. \right] = \int_Z \cdot Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1} \right)$.

[8] This term is the marginal rate of substitution (MRS) of quasi-short-run profit based on the dynamic frontier cost for equity capital. This MRS quantifies the rate at which the financial firm is just willing to substitute quasi-short-run profit for equity capital, or, in other words, it is a measure of the opportunity costs of equity capital.

[9] This term represents the intertemporal marginal rate of substitution (IMRS) with respect to quasi-short-run profit based on the dynamic frontier cost. This IMRS quantifies the rate at which the financial firm is just willing to substitute quasi-short-run profit in period t for profit in period t +1. If the financial firm is risk averse, the marginal utility of quasi-short-run profit is a decreasing function of quasi-short-run profit. The IMRS therefore declines if quasi-short-run profit increases from the current period to the next period and rises if profits fall.

Proof. Both sides of Eq. (2.2.2.6) are divided by $\partial u_{i,t}^{F*} \Big/ \partial \pi_{i,t}^{QSF*}$, provided $\partial u_{i,t}^{F*} \Big/ \partial \pi_{i,t}^{QSF*} \neq 0$, which gives

$$
\begin{aligned}
& - b_j \cdot p_{G,t} - \frac{\partial C_{i,t}^{DFV*}}{\partial q_{i,j,t}^{p*}} + b_j \cdot p_{G,t} \cdot \frac{\partial u_{i,t}^{F*} \Big/ \partial q_{e,i,t}^{p*}}{\partial u_{i,t}^{F*} \Big/ \partial \pi_{i,t}^{QSF*}} \\
& \quad + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot \int_Z \left\{ 1 + b_C \cdot \left(h_{i,j,t}^{R*} + \frac{\partial h_{i,j,t}^{R*}}{\partial \ln q_{i,j,t}^{p*}} \right) + \zeta_{i,j,t+1} \right\} \\
& \quad \cdot \frac{\partial u_{i,t+1}^{F*} \Big/ \partial \pi_{i,t+1}^{QSF*}}{\partial u_{i,t}^{F*} \Big/ \partial \pi_{i,t}^{QSF*}} Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) = 0;\ j = 1, ..., N_A + N_L. \quad \text{(T1.1)}
\end{aligned}
$$

To simplify the expressions, the notation of Theorem 1 is used. Eq. (T1.1) can then be rewritten as

$$
\begin{aligned}
& - b_j \cdot p_{G,t} - MC_{i,j,t}^{DFV*} + b_j \cdot p_{G,t} \cdot MRS_{e,i,t}^{F\pi*} \\
& + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot E\left[\left\{ 1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*} \right) + \zeta_{i,j,t+1} \right\} \cdot IMRS_{\pi,i,t+1}^{F*} \left| \mathbf{z}_{i,t} \right.\right] = 0; \\
& \qquad j = 1, ..., N_A + N_L. \quad \text{(T1.2)}
\end{aligned}
$$

To transform these equations into explicit expressions of risk correction, the expectation in the third term of the left-hand side of Eq. (T1.2) is transformed by the same method as employed in the CCAPM. Let $w_{i,j,t+1}^{*} = 1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*} \right) + \zeta_{i,j,t+1}$. The expectation in the third term is then expressed as $E\left[w_{i,j,t+1}^{*} \cdot IMRS_{\pi,i,t+1}^{F*} \right| \mathbf{z}_{i,t} \Big]$. As in the CCAPM, the covariance of $w_{i,j,t+1}^{*}$ with respect to $IMRS_{\pi,i,t+1}^{F*}$, $\operatorname{cov}\left(w_{i,j,t+1}^{*}, IMRS_{\pi,i,t+1}^{F*} \right| \mathbf{z}_{i,t} \Big)$, is the focus of attention. Using the property of covariance

$$
\begin{aligned}
\operatorname{cov}\left(w_{i,j,t+1}^{*}, IMRS_{\pi,i,t+1}^{F*} \middle| \mathbf{z}_{i,t} \right) &= E\left[w_{i,j,t+1}^{*} \cdot IMRS_{\pi,i,t+1}^{F*} \middle| \mathbf{z}_{i,t} \right] \\
&\quad - E\left[w_{i,j,t+1}^{*} \middle| \mathbf{z}_{i,t} \right] \cdot E\left[IMRS_{\pi,i,t+1}^{F*} \middle| \mathbf{z}_{i,t} \right],
\end{aligned}
$$

$E\left[\left. w^*_{i,j,t+1} \cdot IMRS^{F*}_{\pi,i,t+1} \right| \mathbf{z}_{i,t}\right]$ can be written as

$$E\left[\left. w^*_{i,j,t+1} \cdot IMRS^{F*}_{\pi,i,t+1} \right| \mathbf{z}_{i,t}\right] = E\left[\left. w^*_{i,j,t+1} \right| \mathbf{z}_{i,t}\right] \cdot E\left[\left. IMRS^{F*}_{\pi,i,t+1} \right| \mathbf{z}_{i,t}\right] + \mathrm{cov}\left(\left. w^*_{i,j,t+1}, IMRS^{F*}_{\pi,i,t+1} \right| \mathbf{z}_{i,t}\right). \quad \text{(T1.3)}$$

Substituting $w^*_{i,j,t+1} = 1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t}\right) + \zeta_{i,j,t+1}$ for $E\left[\left. w^*_{i,j,t+1} \right| \mathbf{z}_{i,t}\right]$, under the assumption that $E\left[\left. \zeta_{i,j,t+1} \right| \mathbf{z}_{i,t}\right] = 0$, leads to

$$E\left[\left. w^*_{i,j,t+1} \right| \mathbf{z}_{i,t}\right] = 1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t}\right). \quad \text{(T1.4)}$$

Substituting $w^*_{i,j,t+1} = 1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t}\right) + \zeta_{i,j,t+1}$ and $IMRS^{F*}_{\pi,i,t+1} = \left(\partial u^{F*}_{i,t+1} \middle/ \partial \pi^{QSF*}_{i,t+1}\right) \middle/ \left(\partial u^{F*}_{i,t} \middle/ \partial \pi^{QSF*}_{i,t}\right)$ for $\mathrm{cov}\left(\left. w^*_{i,j,t+1}, IMRS^{F*}_{\pi,i,t+1} \right| \mathbf{z}_{i,t}\right)$, the property of covariance gives the following:

$$\begin{aligned} & \mathrm{cov}\left(\left. w^*_{i,j,t+1}, IMRS^{F*}_{\pi,i,t+1} \right| \mathbf{z}_{i,t}\right) \\ = \; & \mathrm{cov}\left(\left. \zeta_{i,j,t+1}, IMRS^{F*}_{\pi,i,t+1} \right| \mathbf{z}_{i,t}\right) \\ = \; & \frac{\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u^{F*}_{i,t+1} \middle/ \partial \pi^{QSF*}_{i,t+1} \middle| \mathbf{z}_{i,t}\right)}{\partial u^{F*}_{i,t} \middle/ \partial \pi^{QSF*}_{i,t}}. \end{aligned} \quad \text{(T1.5)}$$

Substituting Eqs. (T1.4) and (T1.5) for Eq. (T1.3), the expectation in the third term of the left-hand side of Eq. (T1.2) can be transformed to explicitly express risk corrections, as follows:

$$\begin{aligned} & E\left[\left\{1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t}\right) + \zeta_{i,j,t+1}\right\} \cdot IMRS^{F*}_{\pi,i,t+1} \left| \mathbf{z}_{i,t}\right.\right] \\ & = \left\{1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t}\right)\right\} \cdot E\left[IMRS^{F*}_{\pi,i,t+1} \left| \mathbf{z}_{i,t}\right.\right] \\ & \quad + \frac{\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u^{F*}_{i,t+1} \middle/ \partial \pi^{QSF*}_{i,t+1} \middle| \mathbf{z}_{i,t}\right)}{\partial u^{F*}_{i,t} \middle/ \partial \pi^{QSF*}_{i,t}}. \end{aligned} \quad \text{(T1.6)}$$

Substituting Eq. (T1.6) into Eq. (T1.2) thus adds a risk-adjustment term, as given by Eq. (2.2.3.1). ■

Similarly, the form of the Eq. (2.2.2.8) expression of risk correction is provided by the following theorem.

Theorem 2 *Under the assumption that* $\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*} \neq 0$ *and* $E\left[\zeta_{i,j,t+1} \middle| \mathbf{z}_{i,t}\right] = 0$*, Eq. (2.2.2.8) can be transformed into an expression of risk correction as follows:*

$$\begin{aligned} &- b_j \cdot p_{G,t} - MC_{i,j,t}^{DAV*} + b_j \cdot p_{G,t} \cdot MRS_{e,i,t}^{A\pi*} \\ &\quad + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot \left\{1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*}\right)\right\} \cdot E\left[IMRS_{\pi,i,t+1}^{A*} \left| \mathbf{z}_{i,t}\right.\right] \\ &\quad + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot \frac{cov\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{A*} \Big/ \partial \pi_{i,t+1}^{QSA*} \middle| \mathbf{z}_{i,t}\right)}{\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*}} = PIE_{i,j,t}; \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad j = 1, ..., N_A + N_L, \quad (2.2.3.2) \end{aligned}$$

where $MC_{i,j,t}^{DAV*} = \partial C_{i,t}^{DAV*} / \partial q_{i,j,t}^{p*}$, $MRS_{e,i,t}^{A\pi*} = \left(\partial u_{i,t}^{A*} / \partial q_{e,i,t}^{p*}\right) \Big/ \left(\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*}\right)$,[10] $IMRS_{\pi,i,t+1}^{A*} = \left(\partial u_{i,t+1}^{A*} \Big/ \partial \pi_{i,t+1}^{QSA*}\right) \Big/ \left(\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*}\right)$,[11] *and* $PIE_{i,j,t} = \varepsilon_{i,j,t}^{P} \Big/ \left(\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*}\right)$*, which is the price inefficiency normalized by the marginal utility of quasi-short-run profit based on dynamic actual cost.*

Proof. The proof is similar to the proof of Theorem 1 with two exceptions, so we omit the derivation. First, $C_{i,t}^{DFV*}$, $u_{i,t}^{F*}$, $\pi_{i,t}^{QSF*}$, $u_{i,t+1}^{F*}$, and $\pi_{i,t+1}^{QSF*}$ in Eq. (2.2.3.1) are replaced by $C_{i,t}^{DAV*}$, $u_{i,t}^{A*}$, $\pi_{i,t}^{QSA*}$, $u_{i,t+1}^{A*}$, and $\pi_{i,t+1}^{QSA*}$, respectively. Second, $PIE_{i,j,t}$ is added to the right-hand side of Eq. (2.2.3.1). ■

As similarly described by Homma (2009, 2012), the fractions in the fifth terms on the left-hand sides of Eqs. (2.2.3.1) and (2.2.3.2),

$$\text{cov}\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{F*} \Big/ \partial \pi_{i,t+1}^{QSF*} \middle| \mathbf{z}_{i,t}\right) \Big/ \left(\partial u_{i,t}^{F*} \Big/ \partial \pi_{i,t}^{QSF*}\right) \text{ and}$$
$$\text{cov}\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{A*} \Big/ \partial \pi_{i,t+1}^{QSA*} \middle| \mathbf{z}_{i,t}\right) \Big/ \left(\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*}\right),$$

[10] The interpretation of this term is similar to $MRS_{e,i,t}^{F\pi*}$ in Eq. (2.2.3.1) with the exception of replacing $u_{i,t}^{F*}$ and $\pi_{i,t}^{QSF*}$ in $MRS_{e,i,t}^{F\pi*}$ with $u_{i,t}^{A*}$ and $\pi_{i,t}^{QSA*}$, respectively.

[11] The interpretation of this term is similar to $IMRS_{\pi,i,t+1}^{F*}$ in Eq. (2.2.3.1) with the exception of replacing $u_{i,t}^{F*}$, $\pi_{i,t}^{QSF*}$, $u_{i,t+1}^{F*}$, and $\pi_{i,t+1}^{QSF*}$ in $IMRS_{\pi,i,t+1}^{F*}$ with $u_{i,t}^{A*}$, $\pi_{i,t}^{QSA*}$, $u_{i,t+1}^{A*}$, and $\pi_{i,t+1}^{QSA*}$, respectively.

i.e., the ratio of the covariance of uncertain components of the SDEHRR and the SDEHCR with respect to the marginal utility of quasi-short-run profit based on the dynamic frontier cost in period $t+1$ to the same marginal utility in period t and the ratio of the covariance of the same uncertain components with respect to the marginal utility of quasi-short-run profit based on dynamic actual cost in period $t+1$ to the same marginal utility in period t, are risk-adjustment terms. If financial firms are risk averse, the marginal utility of quasi-short-run profit based on the dynamic frontier cost or dynamic actual cost is a decreasing function of profit. Therefore, $\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi_{i,t+1}^{QSF*} \,\middle|\, \mathbf{z}_{i,t}\right)$ and $\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi_{i,t+1}^{QSA*} \,\middle|\, \mathbf{z}_{i,t}\right)$ are positive if $\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{F*} \middle/ \partial \pi_{i,t+1}^{QSF*} \,\middle|\, \mathbf{z}_{i,t}\right)$ and $\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{A*} \middle/ \partial \pi_{i,t+1}^{QSA*} \,\middle|\, \mathbf{z}_{i,t}\right)$ are negative, respectively, and vice versa. In this case, the variance of quasi-short-run profit based on dynamic frontier cost or dynamic actual cost in the next period increases if a financial asset in the current period increases, whereas the same variance decreases if a liability in the current period increases, and vice versa. For example, if ξ $(0 < \xi < 1)$ of the j-th financial good in period t increases, then from Eq. (2.2.1.1) (or Eq. (2.2.1.3)), quasi-short-run profit based on the dynamic frontier cost (or the quasi-short-run profit based on dynamic actual cost) in the next period becomes

$$\pi_{i,t+1}^{QSF} + b_j \cdot \left\{1 + b_C \cdot h_{i,j}^R\left(Q_{j,t}, \mathbf{z}_{i,j,t}^{DH}\right) + \zeta_{i,j,t+1}\right\} \cdot p_{G,t} \cdot \xi$$
$$(\text{or } \pi_{i,t+1}^{QSA} + b_j \cdot \left\{1 + b_C \cdot h_{i,j}^R\left(Q_{j,t}, \mathbf{z}_{i,j,t}^{DH}\right) + \zeta_{i,j,t+1}\right\} \cdot p_{G,t} \cdot \xi).$$

In this case, its variance can be expressed as

$$\operatorname{var}\left(\pi_{i,t+1}^{QSF}+b_j\cdot\left\{1+b_C\cdot h_{i,j}^{R}\left(Q_{j,t},\mathbf{z}_{i,j,t}^{DH}\right)+\zeta_{i,j,t+1}\right\}\cdot p_{G,t}\cdot\xi\,\middle|\,\mathbf{z}_{i,t}\right)$$
$$=\operatorname{var}\left(\pi_{i,t+1}^{QSF}\,\middle|\,\mathbf{z}_{i,t}\right)+2\cdot b_j\cdot p_{G,t}\cdot\xi\cdot\operatorname{cov}\left(\zeta_{i,j,t+1},\pi_{i,t+1}^{QSF}\,\middle|\,\mathbf{z}_{i,t}\right)$$
$$+\left(b_j\cdot p_{G,t}\cdot\xi\right)^2\cdot\operatorname{var}\left(\zeta_{i,j,t+1}\,\middle|\,\mathbf{z}_{i,t}\right)$$
$$(\text{or }\operatorname{var}\left(\pi_{i,t+1}^{QSA}+b_j\cdot\left\{1+b_C\cdot h_{i,j}^{R}\left(Q_{j,t},\mathbf{z}_{i,j,t}^{DH}\right)+\zeta_{i,j,t+1}\right\}\cdot p_{G,t}\cdot\xi\,\middle|\,\mathbf{z}_{i,t}\right)$$
$$=\operatorname{var}\left(\pi_{i,t+1}^{QSA}\,\middle|\,\mathbf{z}_{i,t}\right)+2\cdot b_j\cdot p_{G,t}\cdot\xi\cdot\operatorname{cov}\left(\zeta_{i,j,t+1},\pi_{i,t+1}^{QSA}\,\middle|\,\mathbf{z}_{i,t}\right)$$
$$+\left(b_j\cdot p_{G,t}\cdot\xi\right)^2\cdot\operatorname{var}\left(\zeta_{i,j,t+1}\,\middle|\,\mathbf{z}_{i,t}\right)). \tag{2.2.3.3}$$

Thus, if ξ is sufficiently small, the third term on the right-hand side of this equation is much smaller than the second term. The sign of the second term, $\operatorname{cov}\left(\zeta_{i,j,t+1},\pi_{i,t+1}^{QSF}\,\middle|\,\mathbf{z}_{i,t}\right)$ (or $\operatorname{cov}\left(\zeta_{i,j,t+1},\pi_{i,t+1}^{QSA}\,\middle|\,\mathbf{z}_{i,t}\right)$), determines whether this variance is greater than $\operatorname{var}\left(\pi_{i,t+1}^{QSF}\,\middle|\,\mathbf{z}_{i,t}\right)$ (or $\operatorname{var}\left(\pi_{i,t+1}^{QSA}\,\middle|\,\mathbf{z}_{i,t}\right)$). Thus, if the j-th financial good is a financial asset (i.e., $b_j=1$), the variance is greater than $\operatorname{var}\left(\pi_{i,t+1}^{QSF}\,\middle|\,\mathbf{z}_{i,t}\right)$ (or $\operatorname{var}\left(\pi_{i,t+1}^{QSA}\,\middle|\,\mathbf{z}_{i,t}\right)$) if the sign of $\operatorname{cov}\left(\zeta_{i,j,t+1},\pi_{i,t+1}^{QSF}\,\middle|\,\mathbf{z}_{i,t}\right)$ (or $\operatorname{cov}\left(\zeta_{i,j,t+1},\pi_{i,t+1}^{QSA}\,\middle|\,\mathbf{z}_{i,t}\right)$) is positive. Similarly, if the j-th financial good is a liability (i.e., $b_j=-1$), this variance is greater than $\operatorname{var}\left(\pi_{i,t+1}^{QSF}\,\middle|\,\mathbf{z}_{i,t}\right)$ (or $\operatorname{var}\left(\pi_{i,t+1}^{QSA}\,\middle|\,\mathbf{z}_{i,t}\right)$) if the sign of $\operatorname{cov}\left(\zeta_{i,j,t+1},\pi_{i,t+1}^{QSF}\,\middle|\,\mathbf{z}_{i,t}\right)$ (or $\operatorname{cov}\left(\zeta_{i,j,t+1},\pi_{i,t+1}^{QSA}\,\middle|\,\mathbf{z}_{i,t}\right)$) is negative.

To derive and define the GURP on the cost frontier, the following corollary to Theorem 1 is established.

Corollary 1 (to Theorem 1) *Equation (2.2.3.1) can be expressed as follows:*

$$MC_{i,j,t}^{DFV*}=b_j\cdot p_{G,t}\cdot\left[\left(b_C\cdot h_{i,j,t}^{R*}-r_{i,t}^{FF*}\right)/\left(1+r_{i,t}^{FF*}\right)+b_C\cdot\eta_{i,j,t}^{*}/\left(1+r_{i,t}^{FF*}\right)\right.$$
$$\left.+MRS_{e,i,t}^{F\pi*}+\varpi_{i,j,t}^{F*}\right];\ j=1,...,N_A+N_L, \tag{2.2.3.4}$$

where $r_{i,t}^{FF*}$ $(=1/E\left[\beta_{i,t}\cdot IMRS_{\pi,i,t+1}^{F*}\left|\mathbf{z}_{i,t}\right.\right]-1)$ *is the reference rate on the cost frontier corresponding to the risk-free rate referred to in the CCAPM* *and* $\varpi_{i,j,t}^{F*}$ *(*$= \beta_{i,t} \cdot cov\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{F*} \left/ \partial \pi_{i,t+1}^{QSF*} \right| \mathbf{z}_{i,t}\right) \Big/ \left(\partial u_{i,t}^{F*} \left/ \partial \pi_{i,t}^{QSF*}\right.\right)$*) is the discounted risk-adjustment term on the cost frontier.*

Proof. Transforming Eq. (2.2.3.1) with respect to $MC_{i,j,t}^{DFV*}$ and rearranging then gives

$$MC_{i,j,t}^{DFV*} = b_j \cdot p_{G,t} \cdot \left[\left\{ b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*} \right) - \left(1 \left/ E\left[\beta_{i,t} \cdot IMRS_{\pi,i,t+1}^{F*} \left| \mathbf{z}_{i,t} \right.\right] \right. - 1 \right) \right\} \right.$$
$$\left. \cdot E\left[\beta_{i,t} \cdot IMRS_{\pi,i,t+1}^{F*} \left| \mathbf{z}_{i,t} \right.\right] + MRS_{e,i,t}^{F\pi*} + \beta_{i,t} \cdot \frac{\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{F*} \left/ \partial \pi_{i,t+1}^{QSF*} \right| \mathbf{z}_{i,t} \right)}{\partial u_{i,t}^{F*} \left/ \partial \pi_{i,t}^{QSF*} \right.} \right]$$
$$= b_j \cdot p_{G,t} \cdot \left[\left\{ b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*} \right) - r_{i,t}^{FF*} \right\} \left/ \left(1 + r_{i,t}^{FF*} \right) \right. + MRS_{e,i,t}^{F\pi*} + \varpi_{i,j,t}^{F*} \right]$$
$$= b_j \cdot p_{G,t} \cdot \left[\left(b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{FF*} \right) \left/ \left(1 + r_{i,t}^{FF*} \right) \right. + b_C \cdot \eta_{i,j,t}^{*} \left/ \left(1 + r_{i,t}^{FF*} \right) \right. + MRS_{e,i,t}^{F\pi*} + \varpi_{i,j,t}^{F*} \right];$$
$$j = 1, ..., N_A + N_L.$$

■

Similarly, to derive and define the GURP on the actual cost, the following corollary to Theorem 2 is formulated.

Corollary 2 (to Theorem 2) *Equation (2.2.3.2) can be expressed as follows:*

$$MC_{i,j,t}^{DAV*} + PIE_{i,j,t} = b_j \cdot p_{G,t} \cdot \left[\left(b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{FA*} \right) \left/ \left(1 + r_{i,t}^{FA*} \right) \right. \right.$$
$$\left. + b_C \cdot \eta_{i,j,t}^{*} \left/ \left(1 + r_{i,t}^{FA*} \right) \right. + MRS_{e,i,t}^{A\pi*} + \varpi_{i,j,t}^{A*} \right]; \; j = 1, ..., N_A + N_L, \quad (2.2.3.5)$$

where $r_{i,t}^{FA*}$ *(*$= 1 \left/ E\left[\beta_{i,t} \cdot IMRS_{\pi,i,t+1}^{A*} \left| \mathbf{z}_{i,t} \right.\right] \right. - 1$*) is the reference rate on the actual cost and* $\varpi_{i,j,t}^{A*}$ *(*$= \beta_{i,t} \cdot cov\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{A*} \left/ \partial \pi_{i,t+1}^{QSA*} \right| \mathbf{z}_{i,t}\right) \Big/ \left(\partial u_{i,t}^{A*} \left/ \partial \pi_{i,t}^{QSA*}\right.\right)$*) is the discounted risk-adjustment term on the actual cost.*

Proof. The proof is similar to the proof of Corollary 1 to Theorem 1 with two exceptions, so we omit the derivation. First, $MC_{i,j,t}^{DFV*}$, $r_{i,t}^{FF*}$, $MRS_{e,i,t}^{F\pi*}$, and $\varpi_{i,j,t}^{F*}$ in Eq. (2.2.3.4) are replaced by $MC_{i,j,t}^{DAV*}$, $r_{i,t}^{FA*}$, $MRS_{e,i,t}^{A\pi*}$, and $\varpi_{i,j,t}^{A*}$, respectively. Second, $PIE_{i,j,t}$ is added to the left-hand side of Eq. (2.2.3.4).

■

The right-hand sides of Eqs. (2.2.3.4) and (2.2.3.5) are then the prices of the j-th financial good because they are equivalent to $MC_{i,j,t}^{DFV*}$ and $MC_{i,j,t}^{DAV*}+PIE_{i,j,t}$, respectively. From the perspective of production theory, these corollaries are thus used as definitions for the GURP on the cost frontier and the GURP on the actual cost, respectively.

Definition 12 (Generalized User-Revenue Price on the Cost Frontier) *The generalized user-revenue price on the cost frontier of the j-th financial good of the i-th financial firm in period t, denoted by $p_{i,j,t}^{GURF}$, is defined as*

$$\begin{aligned} p_{i,j,t}^{GURF} &= b_j \cdot p_{G,t} \cdot \left[\left(b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{FF*}\right) / \left(1 + r_{i,t}^{FF*}\right) + b_C \cdot \eta_{i,j,t}^{*} / \left(1 + r_{i,t}^{FF*}\right)\right. \\ &\qquad \left. + MRS_{e,i,t}^{F\pi*} + \varpi_{i,j,t}^{F*}\right] \\ &= p_{i,j,t}^{SURF} + \eta_{i,j,t}^{BPF*} + MRS_{e,i,t}^{BPF\pi*} + \varpi_{i,j,t}^{BPF*};\ j = 1, ..., N_A + N_L, \quad (2.2.3.6) \end{aligned}$$

where $p_{i,j,t}^{SURF}$ $(= b_j \cdot p_{G,t} \cdot \left(b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{FF*}\right) / \left(1 + r_{i,t}^{FF*}\right))$ is the stochastic user-revenue price on the cost frontier similarly defined by Homma (2009, 2012), $\eta_{i,j,t}^{BPF*}$ $(= b_j \cdot p_{G,t} \cdot b_C \cdot \eta_{i,j,t}^{*} / \left(1 + r_{i,t}^{FF*}\right))$ expresses the market structure and conduct effect on the cost frontier, $MRS_{e,i,t}^{BPF\pi*}$ $(= b_j \cdot p_{G,t} \cdot MRS_{e,i,t}^{F\pi*})$ expresses the equity capital effect on the cost frontier, and $\varpi_{i,j,t}^{BPF*}$ $(= b_j \cdot p_{G,t} \cdot \varpi_{i,j,t}^{F*})$ expresses the risk-adjustment effect on the cost frontier.

Definition 13 (Generalized User-Revenue Price on the Actual Cost) *The generalized user-revenue price on the actual cost of the j-th financial good of the i-th financial firm in period t, denoted by $p_{i,j,t}^{GURA}$, is defined as*

$$\begin{aligned} p_{i,j,t}^{GURA} &= b_j \cdot p_{G,t} \cdot \left[\left(b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{FA*}\right) / \left(1 + r_{i,t}^{FA*}\right) + b_C \cdot \eta_{i,j,t}^{*} / \left(1 + r_{i,t}^{FA*}\right)\right. \\ &\qquad \left. + MRS_{e,i,t}^{A\pi*} + \varpi_{i,j,t}^{A*}\right] \\ &= p_{i,j,t}^{SURA} + \eta_{i,j,t}^{BPA*} + MRS_{e,i,t}^{BPA\pi*} + \varpi_{i,j,t}^{BPA*};\ j = 1, ..., N_A + N_L, \quad (2.2.3.7) \end{aligned}$$

where $p_{i,j,t}^{SURA}$ $(= b_j \cdot p_{G,t} \cdot \left(b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{FA*}\right) / \left(1 + r_{i,t}^{FA*}\right))$ is the stochastic user-revenue price on the actual cost similarly defined by Homma (2009, 2012), $\eta_{i,j,t}^{BPA*}$ $(= b_j \cdot p_{G,t} \cdot b_C \cdot \eta_{i,j,t}^{*} / \left(1 + r_{i,t}^{FA*}\right))$ expresses the market structure and conduct effect on the actual cost, $MRS_{e,i,t}^{BPA\pi*}$ $(= b_j \cdot p_{G,t} \cdot MRS_{e,i,t}^{A\pi*})$ expresses *the equity capital effect on the actual cost, and* $\varpi^{BPA*}_{i,j,t}$ $(= b_j \cdot p_{G,t} \cdot \varpi^{A*}_{i,j,t})$ *expresses the risk-adjustment effect on the actual cost.*

As similarly noted by Homma (2009, 2012), the four terms on the right-hand sides of Eqs. (2.2.3.6) and (2.2.3.7) represent the stochastic user-revenue price (hereafter SURP), market structure and conduct effects, equity capital effects, and risk-adjustment effects, respectively. Especially, $\eta^*_{i,j,t}$ in the second term of the right-hand side of Eqs. (2.2.3.6) and (2.2.3.7) reflects the effects of market structure of the j-th financial good and the strategic interdependence of financial firms, as expressed by

$$\begin{aligned}\eta^*_{i,j,t} &= \frac{\partial h^{R*}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}} = \frac{q^{p*}_{i,j,t}}{Q^{p*}_{j,t}} \cdot \frac{\partial h^{R*}_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} \cdot \left(1 + \sum\nolimits_{k \neq i}^{N_F} \frac{\partial q^{p*}_{k,j,t}}{\partial q^{p*}_{i,j,t}}\right) \\ &= s^*_{i,j,t} \cdot \eta^{Q*}_{i,j,t} \cdot \left(1 + CV^*_{i,j,t}\right); \; j = 1, ..., N_A + N_L, \qquad (2.2.3.8)\end{aligned}$$

where $s^*_{i,j,t}$ $(= q^{p*}_{i,j,t} / Q^{p*}_{j,t})$ is the ratio of the real balance of the j-th financial good of the i-th financial firm to the total balance in the market for the j-th financial good. The range of $s^*_{i,j,t}$ is $0 < s^*_{i,j,t} \leq 1$, and $s^*_{i,j,t} = 1$ if the i-th financial firm has a monopoly. In addition, $\eta^{Q*}_{i,j,t}$ $(= \partial h^{R*}_{i,j,t} / \partial \ln Q^{p*}_{j,t})$ is the elasticity of the certain or predictable components of the SDEHRR or the SDEHCR for the j-th financial good with respect to the total balance in the market, and represents the fractional change in the former due to a 1% increase in the latter. Furthermore, $CV^*_{i,j,t}$ $(= \sum\nolimits_{k \neq i}^{N_F} \partial q^{p*}_{k,j,t} / \partial q^{p*}_{i,j,t})$ is the conjectural derivative quantifying how the i-th financial firm regards the changes in the j-th financial good of other firms with respect to the change in the j-th financial good of the i-th financial firm in period t. If $s^*_{i,j,t} = 1$ and $CV^*_{i,j,t} = 0$, then the i-th financial firm has a monopoly in the j-th financial good market in period t. If $CV^*_{i,j,t} = 0$, then the i-th financial firm is a Cournot firm, i.e., the outputs of all other financial firms are not expected to change as the output of the i-th financial firm changes. If $CV^*_{i,j,t} = -1$, then the i-th financial firm is a competitive firm, i.e., $\eta^*_{i,j,t}$ is zero. Higher values of $CV^*_{i,j,t}$ correspond to larger absolute values of $\eta^*_{i,j,t}$, and thus represent less intense competition.[12]

From these definitions and the above two corollaries, the following two remarks immediately follow.

Remark 1 *From Corollary 1 to Theorem 1 and Definition 12,*

$$MC_{i,j,t}^{DFV*} = p_{i,j,t}^{GURF};\ j = 1, ..., N_A + N_L \tag{2.2.3.9}$$

holds, and thus the classification of financial goods into inputs and outputs based on the sign of each GURP on the cost frontier is consistent with the classification based on the sign of each partial derivative of the dynamic frontier variable cost function with respect to financial goods (i.e., the sign of each dynamic frontier marginal variable cost). The sign of the dynamic frontier marginal variable cost is the same as the sign of the GURP on the cost frontier, indicating that a financial good is an output if positive and a fixed input if negative.

Remark 2 *From Corollary 2 to Theorem 2 and Definition 13,*

$$MC_{i,j,t}^{DAV*} + PIE_{i,j,t} = p_{i,j,t}^{GURA};\ j = 1, ..., N_A + N_L \tag{2.2.3.10}$$

holds, and thus the classification of financial goods into inputs and outputs based on the sign of each GURP on the actual cost is not always consistent

[12] The concept of conjectural variation is popular in both theoretical and empirical studies of industrial organization. Theorists of industrial organization, however, regard it critically for the following reasons: 1) it represents ad hoc assumptions about the conduct of firms, 2) it lacks a game-theoretic foundation, and 3) it forces dynamics into an essentially static model with the strategy space and time horizon of the underlying game being only loosely defined (e.g., Fellner, 1949; Friedman, 1983, p. 110; Daughety, 1985; Makowski, 1987; Tirole, 1989, pp. 244–245). These shortcomings are often recognized as the cost that the modeler must pay for realism without sacrificing simplicity and tractability (i.e., parsimony). However, Dockner (1992), Cabral (1995), and Pfaffermayr (1999) have demonstrated that the concept of conjectural variation can be supported by a consistent theoretical foundation, if it is considered to be a reduced form of a dynamic game. Their findings can be used to justify a static conjectural variations analysis for both modeling dynamic interactions and estimating the degree of oligopoly power. From this viewpoint, we believe that the use of the conjectural derivative is rationalized by considering the derivative to be a reduced form of an (unmodeled) dynamic game.

with the classification based on the sign of each partial derivative of the dynamic actual variable cost function with respect to financial goods (i.e., the sign of each dynamic actual marginal variable cost). Both classifications are consistent in the following two limited cases: 1) the sign of dynamic actual marginal variable cost is the same as the sign of price inefficiency normalized by the marginal utility of quasi-short-run profits based on dynamic actual cost, and 2) if both signs are not equal, then the absolute value of dynamic actual marginal variable cost is greater than the absolute value of normalized price inefficiency.

From these remarks and Proposition 1, the following remark immediately follows.

Remark 3 *From Remarks 1 and 2 and Proposition 1, the GURP on the cost frontier is related to the GURP on the actual cost as follows:*

$$p_{i,j,t}^{GURF} = \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \left(p_{i,j,t}^{GURA} - PIE_{i,j,t} \right) \; ; \; j = 1, ..., N_A + N_L . \tag{2.2.3.11}$$

From this remark, similar to the relation between dynamic frontier marginal variable cost and dynamic actual marginal variable cost, if the inverse of the elasticity of the dynamic actual variable cost function with respect to dynamic cost efficiency is not greater than dynamic cost inefficiency (i.e., $\left(\partial \ln C_{i,t}^{DAV} \big/ \partial EF_{i,t}^{D} \right)^{-1} \leq 1 - EF_{i,t}^{D}$), then the GURP on the cost frontier is not greater than the GURP on the actual cost minus the normalized price inefficiency (i.e., $p_{i,j,t}^{GURA} - PIE_{i,j,t}$), and vice versa. In addition, if the sign of normalized price inefficiency is not negative (i.e., $PIE_{i,j,t} \geq 0$), then the GURP on the cost frontier is not greater than the GURP on the actual cost.

2.2.4 EGLIs on the Cost Frontier and the Actual Cost

Similar to Homma (2009, 2012), the EGLIs on the cost frontier and the actual cost can be derived using Eqs. (2.2.3.6) and (2.2.3.9), which represent the relationship between the GURP on the cost frontier and dynamic frontier marginal variable cost, and Eqs. (2.2.3.7) and (2.2.3.10), which represent the relationship between the GURP on the actual cost and the dynamic actual marginal variable cost, respectively. More specifically, dividing the discrepancy between the SURP on the cost frontier and the dynamic frontier marginal variable cost by the SURP on the cost frontier gives the EGLI on the cost frontier. Similarly, dividing the discrepancy between the SURP on the actual cost and the dynamic actual marginal variable cost by the SURP on the actual cost gives the EGLI on the actual cost. The SURP on the cost frontier is the price at which the market structure and conduct effect on the cost frontier, the equity capital effect on the cost frontier, and the risk-adjustment effect on the cost frontier are assumed to be zero, so the discrepancy between the SURP on the cost frontier and dynamic frontier marginal variable cost equals the product of negative one and the sum of these effects. Similarly, the SURP on the actual cost is the price at which the market structure and conduct effect on the actual cost, the equity capital effect on the actual cost, the risk-adjustment effect on the actual cost, and normalized price inefficiency are assumed to be zero, so the discrepancy between the SURP on actual cost and dynamic actual marginal variable cost equals the sum of the normalized price inefficiency and the product of negative one and the sum of these effects. Where there is no dynamic cost inefficiency and no dynamic price inefficiency, the EGLI on the actual cost equals the EGLI on the cost frontier. In this subsection, the case of positive SURPs on the cost frontier and the actual cost and the positive dynamic frontier and actual marginal variable costs is considered with respect to the relevant financial good as an output.

The discrepancy between the SURP on the cost frontier and the dynamic frontier marginal variable cost and the discrepancy between the SURP on the actual cost and the dynamic actual marginal variable cost are expressed in Remarks 4 and 5, respectively.

Remark 4 *From Eqs. (2.2.3.6) and (2.2.3.9), the discrepancy between the SURP on the cost frontier and the dynamic frontier marginal variable cost* *can be expressed as*

$$
\begin{aligned}
p_{i,j,t}^{SURF} - MC_{i,j,t}^{DFV*} &= -\left(\eta_{i,j,t}^{BPF*} + MRS_{e,i,t}^{BPF\pi*} + \varpi_{i,j,t}^{BPF*}\right) \\
&= -b_j \cdot p_{G,t} \cdot \left(\frac{b_C \cdot \eta_{i,j,t}^{*}}{1 + r_{i,t}^{FF*}} + MRS_{e,i,t}^{F\pi*} + \varpi_{i,j,t}^{F*}\right); \\
& \qquad j = 1, ..., N_A + N_L. \quad (2.2.4.1)
\end{aligned}
$$

Remark 5 *From Eqs. (2.2.3.7) and (2.2.3.10), the discrepancy between the SURP on the actual cost and the dynamic actual marginal variable cost can be expressed as*

$$
\begin{aligned}
p_{i,j,t}^{SURA} - MC_{i,j,t}^{DAV*} &= -\left(\eta_{i,j,t}^{BPA*} + MRS_{e,i,t}^{BPA\pi*} + \varpi_{i,j,t}^{BPA*}\right) + PIE_{i,j,t} \\
&= -b_j \cdot p_{G,t} \cdot \left(\frac{b_C \cdot \eta_{i,j,t}^{*}}{1 + r_{i,t}^{FF*}} + MRS_{e,i,t}^{A\pi*} + \varpi_{i,j,t}^{A*}\right) + PIE_{i,j,t}; \\
& \qquad j = 1, ..., N_A + N_L. \quad (2.2.4.2)
\end{aligned}
$$

The EGLIs on the cost frontier and the actual cost are defined by dividing both sides of Eqs. (2.2.4.1) and (2.2.4.2) by the SURPs on the cost frontier and the actual cost, respectively.

Definition 14 (Extended Generalized-Lerner Index on the Cost Frontier) *The extended generalized-Lerner index on the cost frontier of the j-th financial good of the i-th financial firm in period t, denoted by $EGLI_{i,j,t}^{F}$, is defined as*

$$
\begin{aligned}
EGLI_{i,j,t}^{F} &= \frac{p_{i,j,t}^{SURF} - MC_{i,j,t}^{DFV*}}{p_{i,j,t}^{SURF}} = -\frac{\eta_{i,j,t}^{BPF*} + MRS_{e,i,t}^{BPF\pi*} + \varpi_{i,j,t}^{BPF*}}{p_{i,j,t}^{SURF}} \\
&= -\frac{b_C \cdot \eta_{i,j,t}^{*} + \left(MRS_{e,i,t}^{F\pi*} + \varpi_{i,j,t}^{F*}\right) \cdot \left(1 + r_{i,t}^{FF*}\right)}{b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{FF*}}; \\
& \qquad j = 1, ..., N_A + N_L. \quad (2.2.4.3)
\end{aligned}
$$

Definition 15 (Extended Generalized-Lerner Index on the Actual Cost) *The extended generalized-Lerner index on the actual cost of the j-th financial* *good of the i-th financial firm in period t, denoted by $EGLI^{A}_{i,j,t}$, is defined as*

$$EGLI^{A}_{i,j,t} = \frac{p^{SURA}_{i,j,t} - MC^{DAV*}_{i,j,t}}{p^{SURA}_{i,j,t}} = \frac{PIE_{i,j,t} - \left(\eta^{BPA*}_{i,j,t} + MRS^{BPA\pi*}_{e,i,t} + \varpi^{BPA*}_{i,j,t}\right)}{p^{SURA}_{i,j,t}}$$

$$= \frac{PIE_{i,j,t} \cdot \left(1 + r^{FA*}_{i,t}\right) - b_j \cdot p_{G,t} \cdot \left\{b_C \cdot \eta^{*}_{i,j,t} + \left(MRS^{A\pi*}_{e,i,t} + \varpi^{A*}_{i,j,t}\right) \cdot \left(1 + r^{FA*}_{i,t}\right)\right\}}{b_j \cdot p_{G,t} \cdot \left(b_C \cdot h^{R*}_{i,j,t} - r^{FA*}_{i,t}\right)};$$

$$j = 1, ..., N_A + N_L. \quad (2.2.4.4)$$

As similarly noted by Homma (2009, 2012), under the assumption that the j-th financial good is an output (i.e., $p^{SURF}_{i,j,t}$, $MC^{DFV*}_{i,j,t}$, $p^{SURA}_{i,j,t}$, $MC^{DAV*}_{i,j,t} > 0$), the signs of $b_C \cdot h^{R*}_{i,j,t} - r^{FF*}_{i,t}$ and $b_C \cdot h^{R*}_{i,j,t} - r^{FA*}_{i,t}$ are positive if the j-th financial good is a financial asset other than cash, and negative if the j-th financial good is a liability. If the sign of $\eta^{*}_{i,j,t}$ is determined by the sign of the elasticity of the collected or paid interest rate of the SDEHRR or the SDEHCR with respect to the total balance in the market, then the sign of $\eta^{*}_{i,j,t}$ is negative if the j-th financial good is a financial asset and positive if the j-th financial good is a liability.[13] From Eqs. (2.2.3.1) and (2.2.3.2), the signs of $MRS^{F\pi*}_{e,i,t}$ and $MRS^{A\pi*}_{e,i,t}$ are positive, and from Eqs. (2.2.3.4) and (2.2.3.5), the signs of $\varpi^{F*}_{i,j,t}$ and $\varpi^{A*}_{i,j,t}$ can be either positive or negative. From the definitional identities of $\varpi^{F*}_{i,j,t}$ and $\varpi^{A*}_{i,j,t}$ in Eqs. (2.2.3.3), (2.2.3.4), and (2.2.3.5), if the j-th financial good is a financial asset and the risks (variances) of quasi-short-run profits based on dynamic frontier cost and dynamic actual cost increase due to an increase in the asset, then $(\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi^{QSF*}_{i,t+1} \middle| \mathbf{z}_{i,t}\right)$,

[13]If the j-th financial good is a financial asset (other than cash), then the elasticity of the certain or predictable components of the SDEHRR with respect to the total balance in the market (i.e., $\eta^{Q*}_{i,j,t}$; $j = 2, ..., N_A$) corresponds to the sum of the same elasticities of the collected interest rate, the uncollected interest rate, and the service charge rate, minus the same elasticity of the default rate. If the j-th financial good is a liability, then the elasticity of the certain or predictable components of the SDEHCR with respect to the total balance in the market (i.e., $\eta^{Q*}_{i,j,t}$; $j = N_A + 1, ..., N_A + N_L$) corresponds to the sum of the same elasticities of the paid interest rate, the unpaid interest rate, and the insurance premium rate, minus the same elasticity of the service charge rate. The sign of the elasticity of the certain or predictable component of the collected interest rate with respect to the total balance in the market is usually negative, and the sign of the same elasticity of the paid interest rate is usually positive. However, the sign of the other elasticities can be positive or negative.

$\text{cov}\left(\zeta_{i,j,t+1}, \pi^{QSA*}_{i,t+1}\middle| \mathbf{z}_{i,t}\right) > 0$), and if the financial firm is risk averse, the signs of $\varpi^{F*}_{i,j,t}$ and $\varpi^{A*}_{i,j,t}$ are negative, whereas if the risks (variances) of quasi-short-run profits based on dynamic frontier cost and dynamic actual cost decrease, then ($\text{cov}\left(\zeta_{i,j,t+1}, \pi^{QSF*}_{i,t+1}\middle| \mathbf{z}_{i,t}\right)$, $\text{cov}\left(\zeta_{i,j,t+1}, \pi^{QSA*}_{i,t+1}\middle| \mathbf{z}_{i,t}\right) < 0$), and if the financial firm is still risk averse, the signs of $\varpi^{F*}_{i,j,t}$ and $\varpi^{A*}_{i,j,t}$ are positive. On the other hand, if the j-th financial good is a liability and the risks (variances) of quasi-short-run profits based on dynamic frontier cost and dynamic actual cost increase due to an increase in the liability, then ($\text{cov}\left(\zeta_{i,j,t+1}, \pi^{QSF*}_{i,t+1}\middle| \mathbf{z}_{i,t}\right)$, $\text{cov}\left(\zeta_{i,j,t+1}, \pi^{QSA*}_{i,t+1}\middle| \mathbf{z}_{i,t}\right) < 0$), and if the financial firm is risk averse, the signs of $\varpi^{F*}_{i,j,t}$ and $\varpi^{A*}_{i,j,t}$ are positive, whereas if the risks (variances) of quasi-short-run profits based on dynamic frontier cost and dynamic actual cost decrease, then ($\text{cov}\left(\zeta_{i,j,t+1}, \pi^{QSF*}_{i,t+1}\middle| \mathbf{z}_{i,t}\right)$, $\text{cov}\left(\zeta_{i,j,t+1}, \pi^{QSA*}_{i,t+1}\middle| \mathbf{z}_{i,t}\right) > 0$), and if the financial firm is still risk averse, the signs of $\varpi^{F*}_{i,j,t}$ and $\varpi^{A*}_{i,j,t}$ are negative. From Eqs. (2.2.3.2), (2.2.3.5), and (2.2.3.10), the sign of $PIE_{i,j,t}$ can also be either positive or negative. Under the assumption that the j-th financial good is an output, if the sign of $PIE_{i,j,t}$ is positive, then ($MC^{DAV*}_{i,j,t} < p^{GURA}_{i,j,t}$), and the j-th financial good is short, whereas if the sign of $PIE_{i,j,t}$ is negative, then ($MC^{DAV*}_{i,j,t} > p^{GURA}_{i,j,t}$), and the j-th financial good is over.

From Definitions 14 and 15, we can appreciate that the factors that have an impact on the degree of competition are not those that affect market structure and conduct ($\eta^{*}_{i,j,t}$) from the perspective of conventional industrial organization. From a financial perspective, the risk-averse attitude of financial firms ($r^{FF*}_{i,t}$, $r^{FA*}_{i,t}$), the fluctuation risk of quasi-short-run profit ($\varpi^{F*}_{i,j,t}$, $\varpi^{A*}_{i,j,t}$), and the equity capital (which reflects the risk of the burden of financial distress costs) ($MRS^{F\pi*}_{e,i,t}$, $MRS^{A\pi*}_{e,i,t}$) also have an impact. Furthermore, from a productive efficiency perspective, the dynamic cost and price inefficiencies ($1 - EF^{D}_{i,t}$, $PIE_{i,j,t}$) also have an impact. Consequently, similar to Homma (2012, Propositions 1 and 2), the following two propositions can be derived.

Proposition 2 *If financial firms are risk averse, an increase in equity capital increases the EGLIs of financial assets other than cash on the cost frontier and the actual cost (decreases the degree of competition) and decreases the same EGLIs of liabilities (increases the degree of competition).*

Proof. The proof is similar to the proof of Proposition 1 in Homma (2012) with the exception of replacing $MRS^{\pi *}_{e,i,t}$, $u^{*}_{i,t}$, $\pi^{QS*}_{i,t}$, and $r^{F*}_{i,t}$ with $MRS^{F\pi *}_{e,i,t}$ (or $MRS^{A\pi *}_{e,i,t}$), $u^{F*}_{i,t}$ (or $u^{A*}_{i,t}$), $\pi^{QSF*}_{i,t}$ (or $\pi^{QSA*}_{i,t}$), and $r^{FF*}_{i,t}$ (or $r^{FA*}_{i,t}$), respectively, so we omit the derivation. ■

Proposition 3 *Under the assumption that the risks (variances) of quasi-short-run profits based on the dynamic frontier cost and the dynamic actual cost increase due to an increase in financial assets other than cash and liabilities, if the financial firm is risk averse, then the EGLIs on the cost frontier and the actual cost increase (the degree of competition decreases), whereas if it is assumed that the risks (variances) decrease, then the same EGLIs decrease (the degree of competition increases) if the financial firm is risk averse.*

Proof. The proof is similar to the proof of Proposition 2 in Homma (2012) with the exception of replacing $\varpi^{*}_{i,j,t}$ and $r^{F*}_{i,t}$ with $\varpi^{F*}_{i,j,t}$ (or $\varpi^{A*}_{i,j,t}$) and $r^{FF*}_{i,t}$ (or $r^{FA*}_{i,t}$), respectively, so we omit the derivation. ■

From Definitions 14 and 15, using the EGLIs on the cost frontier and the actual cost, the impact of dynamic cost and price inefficiencies on the EGLI, which was not considered in Homma (2009, 2012), can be defined.

Definition 16 (Impact of Inefficiencies on the EGLI (IIEE)) *The impact of the dynamic cost and price inefficiencies of the j-th financial good of the i-th financial firm in period t on the EGLI, denoted by $IIEE_{i,j,t}$, is defined as*

$$IIEE_{i,j,t} = EGLI^{F}_{i,j,t} - \frac{p^{SURA}_{i,j,t}}{p^{SURF}_{i,j,t}} \cdot EGLI^{A}_{i,j,t} = \frac{p^{SURF}_{i,j,t} - MC^{DFV*}_{i,j,t} - \left(p^{SURA}_{i,j,t} - MC^{DAV*}_{i,j,t}\right)}{p^{SURF}_{i,j,t}}$$
$$= \frac{\left(\eta^{BPA*}_{i,j,t} - \eta^{BPF*}_{i,j,t}\right) + \left(MRS^{BPA\pi *}_{e,i,t} - MRS^{BPF\pi *}_{e,i,t}\right) + \left(\varpi^{BPA*}_{i,j,t} - \varpi^{BPF*}_{i,j,t}\right) - PIE_{i,j,t}}{p^{SURF}_{i,j,t}};$$
$$j = 1, ..., N_A + N_L. \quad (2.2.4.5)$$

From Definition 16, the following proposition can be established.

Proposition 4 *If the financial firm is risk averse and the dynamic frontier variable cost function in period $t+1$ equals the dynamic actual variable* *cost function in period $t+1$ (i.e., $C^{DFV*}_{i,t+1} = C^{DAV*}_{i,t+1}$) (hereafter Assumption 1), and if the inverse of the elasticity of the dynamic actual variable cost function with respect to dynamic cost efficiency is not greater than the dynamic cost inefficiency (i.e., $\left(\partial \ln C^{DAV}_{i,t} \big/ \partial EF^{D}_{i,t}\right)^{-1} \leq 1 - EF^{D}_{i,t}$) (hereafter Assumption 2), and if the j-th financial good is a financial asset, or if the j-th financial good is a liability, the ratio of the subtraction of the certain or predictable components of the SDEHCR from the reference rate on the cost frontier to the subtraction of the certain or predictable components of the SDEHCR from the reference rate on the actual cost (hereafter RH) is not less than the ratio of the addition of one and the reference rate on the cost frontier to the addition of one and the reference rate on the actual cost (hereafter RR) (i.e., $RH_{i,j,t} = \left(r^{FF*}_{i,t} - h^{R*}_{i,j,t}\right) \big/ \left(r^{FA*}_{i,t} - h^{R*}_{i,j,t}\right) \geq RR_{i,j,t} = \left(1 + r^{FF*}_{i,t}\right) \big/ \left(1 + r^{FA*}_{i,t}\right)$), and if Assumption 2 holds, then the IIEE is not less than zero (i.e., $IIEE_{i,j,t} \geq 0$). Where dynamic cost and price inefficiencies exist, the degree of competition can therefore be overestimated. However, under Assumption 1, if the j-th financial good is a liability, the RH is less than the RR (i.e., $RH_{i,j,t} = \left(r^{FF*}_{i,t} - h^{R*}_{i,j,t}\right) \big/ \left(r^{FA*}_{i,t} - h^{R*}_{i,j,t}\right) < RR_{i,j,t} = \left(1 + r^{FF*}_{i,t}\right) \big/ \left(1 + r^{FA*}_{i,t}\right)$), and Assumption 2 holds, then the IIEE can be negative, zero, or positive.*

Proof. From Definitions 6 and 7, the dynamic actual variable cost function in period t is not less than the dynamic frontier variable cost function in period t (i.e., $C^{DAV*}_{i,t} \geq C^{DFV*}_{i,t}$), so quasi-short-run profit based on dynamic actual cost in period t is not greater than quasi-short-run profit based on dynamic frontier cost in period t from Definitions 9 and 10 (i.e., $\pi^{QSA*}_{i,t} \leq \pi^{QSF*}_{i,t}$). Furthermore, if the financial firm is risk averse, the marginal utility of the firm with respect to quasi-short-run profit is a decreasing function of quasi-short-run profit, so the marginal utility with respect to quasi-short-run profit based on dynamic actual cost in period t is not less than the marginal utility with respect to quasi-short-run profit based on dynamic frontier cost in period t (i.e., $\partial u^{A*}_{i,t} \Big/ \partial \pi^{QSA*}_{i,t} \geq \partial u^{F*}_{i,t} \Big/ \partial \pi^{QSF*}_{i,t}$). In this case, under the assumption that the dynamic frontier variable cost function in period $t+1$ equals the dynamic actual variable cost function in period $t+1$ (i.e., $C^{DFV*}_{i,t+1} = C^{DAV*}_{i,t+1}$), from the definitions of the reference rates on the cost frontier and the actual cost (i.e., $r_{i,t}^{FF*} = 1 \Big/ E\left[\beta_{i,t} \cdot \left(\partial u_{i,t+1}^{F*} \Big/ \partial \pi_{i,t+1}^{QSF*}\right) \Big/ \left(\partial u_{i,t}^{F*} \Big/ \partial \pi_{i,t}^{QSF*}\right) |\mathbf{z}_{i,t}\right] - 1$ and $r_{i,t}^{FA*} = 1 \Big/ E\left[\beta_{i,t} \cdot \left(\partial u_{i,t+1}^{A*} \Big/ \partial \pi_{i,t+1}^{QSA*}\right) \Big/ \left(\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*}\right) |\mathbf{z}_{i,t}\right] - 1$), the reference rate on the actual cost is not less than the reference rate on the cost frontier (i.e., $r_{i,t}^{FA*} \geq r_{i,t}^{FF*}$) because quasi-short-run profit based on dynamic actual cost in period $t+1$ equals quasi-short-run profit based on dynamic frontier cost in period $t+1$ (i.e., $\pi_{i,t+1}^{QSA*} = \pi_{i,t+1}^{QSF*}$) and thus the marginal utility with respect to quasi-short-run profit based on dynamic actual cost in period $t+1$ equals the marginal utility with respect to quasi-short-run profit based on dynamic frontier cost in period $t+1$ (i.e., $\partial u_{i,t+1}^{A*} \Big/ \partial \pi_{i,t+1}^{QSA*} = \partial u_{i,t+1}^{F*} \Big/ \partial \pi_{i,t+1}^{QSF*}$). In this case, from the definitions of the SURPs on the cost frontier and the actual cost (i.e., $p_{i,j,t}^{SURF} = b_j \cdot p_{G,t} \cdot \left(b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{FF*}\right) / \left(1 + r_{i,t}^{FF*}\right)$ and $p_{i,j,t}^{SURA} = b_j \cdot p_{G,t} \cdot \left(b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{FA*}\right) / \left(1 + r_{i,t}^{FA*}\right)$), if the j-th financial good is a financial asset (i.e., $b_j = 1$) or if the j-th financial good is a liability (i.e., $b_j = -1$) and the ratio of the subtraction of the certain or predictable components of the SDEHCR from the reference rate on the cost frontier to the subtraction of the certain or predictable components of the SDEHCR from the reference rate on the actual cost (RH) is not less than the ratio of the addition of one and the reference rate on the cost frontier to the addition of one and the reference rate on the actual cost (RR) (i.e., $RH_{i,j,t} = \left(r_{i,t}^{FF*} - h_{i,j,t}^{R*}\right) / \left(r_{i,t}^{FA*} - h_{i,j,t}^{R*}\right) \geq RR_{i,j,t} = \left(1 + r_{i,t}^{FF*}\right) / \left(1 + r_{i,t}^{FA*}\right)$), then the stochastic user-revenue price on the actual cost is not greater than the stochastic user-revenue price on the cost frontier (i.e., $p_{i,j,t}^{SURA} \leq p_{i,j,t}^{SURF}$), whereas if the j-th financial good is a liability (i.e., $b_j = -1$) and the RH is less than the RR (i.e., $RH_{i,j,t} = \left(r_{i,t}^{FF*} - h_{i,j,t}^{R*}\right) / \left(r_{i,t}^{FA*} - h_{i,j,t}^{R*}\right) < RR_{i,j,t} = \left(1 + r_{i,t}^{FF*}\right) / \left(1 + r_{i,t}^{FA*}\right)$), then the stochastic user-revenue price on the actual cost is greater than the stochastic user-revenue price on the cost frontier (i.e., $p_{i,j,t}^{SURA} > p_{i,j,t}^{SURF}$). Furthermore, from Proposition 1 (i.e., $MC_{i,j,t}^{DFV*} = \left\{EF_{i,t}^{D} + \left(\partial \ln C_{i,t}^{DAV} / \partial EF_{i,t}^{D}\right)^{-1}\right\} \cdot MC_{i,j,t}^{DAV*}$), if the inverse of the elasticity of the dynamic actual variable cost function with respect to dynamic cost efficiency is not greater than the dynamic cost inefficiency (i.e., $\left(\partial \ln C_{i,t}^{DAV} / \partial EF_{i,t}^{D}\right)^{-1} \leq 1 - EF_{i,t}^{D}$) (Assumption

2), then dynamic actual marginal variable cost is not less than dynamic frontier marginal variable cost (i.e., $MC^{DAV*}_{i,j,t} \geq MC^{DFV*}_{i,j,t}$). Consequently, under Assumption 1, if the j-th financial good is a financial asset and Assumption 2 holds, or if the j-th financial good is a liability, the RH is not less than the RR, and Assumption 2 holds, then the discrepancy between the SURP on the cost frontier and the dynamic frontier marginal variable cost is not less than the discrepancy between the SURP on the actual cost and the dynamic actual marginal variable cost (i.e., $p^{SURF}_{i,j,t} - MC^{DFV*}_{i,j,t} \geq p^{SURA}_{i,j,t} - MC^{DAV*}_{i,j,t}$), so the IIEE is not less than zero (i.e., $IIEE_{i,j,t} \geq 0$) because the sign of the SURP on the cost frontier is positive under the assumption that the j-th financial good is an output (i.e., $p^{SURF}_{i,j,t}$, $MC^{DFV*}_{i,j,t}$, $p^{SURA}_{i,j,t}$, $MC^{DAV*}_{i,j,t} > 0$). However, under Assumption 1, if the j-th financial good is a liability, the RH is less than the RR, and Assumption 2 holds, then the SURP on the actual cost is greater than the SURP on the cost frontier (i.e., $p^{SURA}_{i,j,t} > p^{SURF}_{i,j,t}$) and the dynamic actual marginal variable cost is not less than the dynamic frontier marginal variable cost (i.e., $MC^{DAV*}_{i,j,t} \geq MC^{DFV*}_{i,j,t}$), so the IIEE can be negative, zero, or positive. ■

Further, under the assumption that the j-th financial good is an output (i.e., $p^{SURF}_{i,j,t}$, $MC^{DFV*}_{i,j,t}$, $p^{SURA}_{i,j,t}$, $MC^{DAV*}_{i,j,t} > 0$), if the j-th financial good is a financial asset, then the sign of the elasticity of the certain or predictable components of the SDEHRR or the SDEHCR with respect to the j-th financial good is negative (i.e., $\eta^{*}_{i,j,t} < 0$), whereas if the j-th financial good is a liability, then the sign of this elasticity is positive (i.e., $\eta^{*}_{i,j,t} > 0$). From the definitions of market structure and conduct effects based on the cost frontier and the actual cost (i.e., $\eta^{BPF*}_{i,j,t} = b_j \cdot p_{G,t} \cdot b_C \cdot \eta^{*}_{i,j,t} \big/ \big(1 + r^{FF*}_{i,t}\big)$ and $\eta^{BPA*}_{i,j,t} = b_j \cdot p_{G,t} \cdot b_C \cdot \eta^{*}_{i,j,t} \big/ \big(1 + r^{FA*}_{i,t}\big)$), the signs of these effects are, therefore, negative (i.e., $\eta^{BPF*}_{i,j,t}$, $\eta^{BPA*}_{i,j,t} < 0$). Furthermore, from the proof of Proposition 4, under Assumption 1, the reference rate on the actual cost is not less than the reference rate on the cost frontier (i.e., $r^{FA*}_{i,t} \geq r^{FF*}_{i,t}$), so the market structure and conduct effect based on actual cost is not less than the market structure and conduct effect based on the cost frontier (i.e., $\eta^{BPA*}_{i,j,t} \geq \eta^{BPF*}_{i,j,t}$).

From the definition of the marginal rate of substitution of quasi-short-run profit based on the dynamic actual cost for equity capital (i.e., $MRS_{e,i,t}^{A\pi*} = \left(\partial u_{i,t}^{A*} \big/ \partial q_{e,i,t}^{p*}\right) \Big/ \left(\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*}\right)$), the following equation holds:

$$\frac{\partial MRS_{e,i,t}^{A\pi*}}{\partial \pi_{i,t}^{QSA*}} = \left(\frac{\partial u_{i,t}^{A*}}{\partial \pi_{i,t}^{QSA*}}\right)^{-1} \cdot \left(\frac{\partial^2 u_{i,t}^{A*}}{\partial \pi_{i,t}^{QSA*} \partial q_{e,i,t}^{p*}} - MRS_{e,i,t}^{A\pi*} \cdot \frac{\partial^2 u_{i,t}^{A*}}{\partial \pi_{i,t}^{QSA*2}}\right). \tag{2.2.4.6}$$

The signs of the marginal utility of the financial firm with respect to quasi-short-run profit based on dynamic actual cost and the marginal utility of the financial firm with respect to equity capital are positive (i.e., $\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*}$, $\partial u_{i,t}^{A*} \big/ \partial q_{e,i,t}^{p*} > 0$), so the sign of the marginal rate of substitution of quasi-short-run profit based on dynamic actual cost for equity capital is also positive (i.e., $MRS_{e,i,t}^{A\pi*} > 0$). If the financial firm is risk averse, the marginal utility of the financial firm with respect to quasi-short-run profit is a decreasing function of quasi-short-run profit, so the sign of the second-order partial derivative of the utility of the financial firm with respect to quasi-short-run profit based on dynamic actual cost is negative (i.e., $\partial^2 u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*2} < 0$). If the relationship between quasi-short-run profit and equity capital is, therefore, complementary (i.e., $\partial^2 u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*} \partial q_{e,i,t}^{p*} > 0$), or if this relationship is substitutive (i.e., $\partial^2 u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*} \partial q_{e,i,t}^{p*} < 0$) and the absolute value of the cross partial derivative of the utility of the financial firm with respect to quasi-short-run profit based on dynamic actual cost and equity capital is less than the product of the negative marginal rate of substitution of quasi-short-run profit based on dynamic actual cost for equity capital and the second-order partial derivative of the utility of the financial firm with respect to quasi-short-run profit based on dynamic actual cost (i.e., $\left|\partial^2 u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*} \partial q_{e,i,t}^{p*}\right| < -MRS_{e,i,t}^{A\pi*} \cdot \partial^2 u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*2}$), then the sign of the partial derivative of the marginal rate of substitution of quasi-short-run profit based on dynamic actual cost for equity capital with respect to quasi-short-run profit based on dynamic actual cost is positive (i.e., $\partial MRS_{e,i,t}^{A\pi*} \big/ \partial \pi_{i,t}^{QSA*} > 0$). Thus the marginal rate of substitution of quasi-short-run profit based on the dynamic frontier cost for equity capital is greater than the marginal rate of substitution of quasi-short-run profit based on the dynamic actual cost for equity capital (i.e., $MRS_{e,i,t}^{F\pi *} > MRS_{e,i,t}^{A\pi *}$) because quasi-short-run profit based on dynamic actual cost is not greater than quasi-short-run profit based on the dynamic frontier cost from Definitions 9 and 10 (i.e., $\pi_{i,t}^{QSA*} \leq \pi_{i,t}^{QSF*}$). Thus, from the definitions of equity capital effects based on the cost frontier and the actual cost (i.e., $MRS_{e,i,t}^{BPF\pi *} = b_j \cdot p_{G,t} \cdot MRS_{e,i,t}^{F\pi *}$ and $MRS_{e,i,t}^{BPA\pi *} = b_j \cdot p_{G,t} \cdot MRS_{e,i,t}^{A\pi *}$), if the j-th financial good is a financial asset (i.e., $b_j = 1$), then the equity capital effects based on the cost frontier are greater than the equity capital effects based on the actual cost (i.e., $MRS_{e,i,t}^{BPF\pi *} > MRS_{e,i,t}^{BPA\pi *}$), whereas if the j-th financial good is a liability (i.e., $b_j = -1$), then the equity capital effects based on the cost frontier are less than the equity capital effects based on the actual cost (i.e., $MRS_{e,i,t}^{BPF\pi *} < MRS_{e,i,t}^{BPA\pi *}$). However, if the relationship between quasi-short-run profit and equity capital is substitutive (i.e., $\partial^2 u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*} \partial q_{e,i,t}^{p*} < 0$) and the absolute value of the cross partial derivative of the utility of the financial firm with respect to quasi-short-run profit based on dynamic actual cost and equity capital is greater than the product of the negative marginal rate of substitution of quasi-short-run profit based on dynamic actual cost for equity capital and the second-order partial derivative of the utility of the financial firm with respect to quasi-short-run profit based on dynamic actual cost (i.e., $\left| \partial^2 u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*} \partial q_{e,i,t}^{p*} \right| > -MRS_{e,i,t}^{A\pi *} \cdot \partial^2 u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*2}$), the sign of the partial derivative of the marginal rate of substitution of quasi-short-run profit based on dynamic actual cost for equity capital with respect to quasi-short-run profit based on dynamic actual cost is negative (i.e., $\partial MRS_{e,i,t}^{A\pi *} / \partial \pi_{i,t}^{QSA*} < 0$). The marginal rate of substitution of quasi-short-run profit based on the dynamic frontier cost for equity capital is therefore less than the marginal rate of substitution of quasi-short-run profit based on the dynamic actual cost for equity capital (i.e., $MRS_{e,i,t}^{F\pi *} < MRS_{e,i,t}^{A\pi *}$). In this case, if the j-th financial good is a financial asset, then the equity capital effect based on the cost frontier is less than the equity capital effect based on actual cost (i.e., $MRS_{e,i,t}^{BPF\pi *} < MRS_{e,i,t}^{BPA\pi *}$), whereas if the j-th financial good is a liability, then the equity capital effect based on the cost frontier is greater than the equity capital effect based on actual cost (i.e., $MRS_{e,i,t}^{BPF\pi *} > MRS_{e,i,t}^{BPA\pi *}$).

From the proof of Proposition 4, under Assumption 1, the marginal utility of quasi-short-run profit based on dynamic actual cost in period t is not less than the marginal utility of quasi-short-run profit based on the dynamic frontier cost in period t (i.e., $\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*} \geq \partial u_{i,t}^{F*} \Big/ \partial \pi_{i,t}^{QSF*}$) and the marginal utility of quasi-short-run profit based on the dynamic actual cost in period $t{+}1$ equals the marginal utility of quasi-short-run profit based on the dynamic frontier cost in period $t+1$ (i.e., $\partial u_{i,t+1}^{A*} \Big/ \partial \pi_{i,t+1}^{QSA*} = \partial u_{i,t+1}^{F*} \Big/ \partial \pi_{i,t+1}^{QSF*}$). From the definition of the risk-adjustment effects based on the cost frontier and the actual cost (i.e., $\varpi_{i,j,t}^{BPF*} = b_j \cdot p_{G,t} \cdot \beta_{i,t} \cdot \mathrm{cov}\Big(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{F*} \Big/ \partial \pi_{i,t+1}^{QSF*} \Big| \, \mathbf{z}_{i,t} \Big) \Big/ \Big(\partial u_{i,t}^{F*} \Big/ \partial \pi_{i,t}^{QSF*} \Big)$ and $\varpi_{i,j,t}^{BPA*} = b_j \cdot p_{G,t} \cdot \beta_{i,t} \cdot \mathrm{cov}\Big(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{A*} \Big/ \partial \pi_{i,t+1}^{QSA*} \Big| \, \mathbf{z}_{i,t} \Big) \Big/ \Big(\partial u_{i,t}^{A*} \Big/ \partial \pi_{i,t}^{QSA*} \Big)$), the absolute value of the risk-adjustment effect based on the cost frontier is, therefore, not less than the absolute value of the risk-adjustment effect based on the actual cost (i.e., $\left|\varpi_{i,j,t}^{BPF*}\right| \geq \left|\varpi_{i,j,t}^{BPA*}\right|$) because the covariance of uncertain components of the SDEHRR and the SDEHCR with respect to the marginal utility of quasi-short-run profit based on the dynamic frontier cost in period $t+1$ equals the covariance of the same uncertain components with respect to the marginal utility of quasi-short-run profit based on dynamic actual cost in period $t{+}1$ (i.e., $\mathrm{cov}\Big(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{F*} \Big/ \partial \pi_{i,t+1}^{QSF*} \Big| \, \mathbf{z}_{i,t} \Big) = \mathrm{cov}\Big(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{A*} \Big/ \partial \pi_{i,t+1}^{QSA*} \Big| \, \mathbf{z}_{i,t} \Big)$). Consequently, if dynamic cost inefficiency exists, the impact of the risk-adjustment effect can be underestimated.

As noted above, under the assumption that the j-th financial good is an output, if the sign of $PIE_{i,j,t}$ is positive, then ($MC_{i,j,t}^{DAV*} < p_{i,j,t}^{GURA}$), the j-th financial good is short, whereas if the sign of $PIE_{i,j,t}$ is negative, then ($MC_{i,j,t}^{DAV*} > p_{i,j,t}^{GURA}$), the j-th financial good is over. In these cases, the sign of the IIEE is ambiguous because the market structure and conduct effect, the equity capital effect, and the risk-adjustment effect are also simultaneously affected.

3 Mathematical Formulations and Theoretical Interpretations of the Efficient Structure and Quiet-Life Hypotheses

This section formulates the efficient structure and quiet-life hypotheses on the basis of the extended GURM that accounts for dynamic cost efficiency. In terms of the former, three formulations are possible. The first is that the efficient structure hypothesis is expressed by the effect of the improvement in dynamic cost efficiency in the previous period on the planned optimal financial good in the current period, so it is a direct definition of the efficient structure hypothesis. The second formulation involves expressing the efficient structure hypothesis by the ratio of the following two sums, and provides the foundation for rigorous theoretical interpretations: the numerator is the sum of the net effect of the improvement in dynamic cost efficiency in the previous period and the effect of the same improvement. The former net effect is on the GURP on the cost frontier (i.e., the dynamic frontier marginal variable cost with respect to the planned optimal financial good) in the current period and on the dynamic actual marginal variable cost with respect to the planned optimal financial good in the current period. This net effect is normalized by the same dynamic actual marginal variable cost and accounts for the correction in dynamic marginal cost efficiency in the current period (as discussed below). The latter effect is on the elasticity of the dynamic actual variable cost in the current period with respect to dynamic cost efficiency in the current period. This effect is normalized by the square of the same elasticity. Similarly, the denominator is the sum of the net effect of an increase in the planned optimal financial good in the current period and the effect of the same increase in the planned optimal financial good. Similar to the numerator, the former net effect is on the same GURP and on the same dynamic actual marginal variable cost. This net effect is normalized by the same dynamic actual marginal variable cost and accounts for the correction in dynamic marginal cost efficiency in the current period. The latter effect is on the same elasticity of dynamic actual variable cost and is normalized by the square of the same elasticity. The third formulation is that the net effect in the numerator of the second formulation is expressed by the sum of the effects of the improvement in dynamic cost efficiency in the previous period on the efficiency difference of the GURP of the planned optimal financial good in the current period, the pricing error of the same financial good, and dynamic actual marginal variable cost with respect to the same financial good, which is corrected by dynamic marginal cost inefficiency in the current period. Similar to the numerator, the net effect in the denominator of the second formulation is expressed by the sum of the effects of an increase in the planned optimal financial good in the current period on the same factors as the numerator. This formulation is, therefore, used to extensively interpret the efficient structure hypothesis with these effects. Similarly, in terms of the quiet-life hypothesis, three formulations are also possible. The first is that the quiet-life hypothesis is expressed by the effect of an increase in the Herfindahl index in the previous period on dynamic cost efficiency in the current period, so it is a direct definition of the quiet-life hypothesis. The second formulation is that the quiet-life hypothesis is expressed by the following ratio, so it provides the foundation for rigorous theoretical interpretations: the numerator is the sum of the net effect of the same increase in the Herfindahl index and the effect of the same increase. Similar to the efficient structure hypothesis, the former net effect is on the same GURP and on the same dynamic actual marginal variable cost. This net effect is normalized by the same dynamic actual marginal variable cost and accounts for the same correction in dynamic marginal cost efficiency. The latter effect is on the same elasticity of dynamic actual variable cost, and is normalized by the same square of the same elasticity. The denominator is the product of the same dynamic actual marginal variable cost as per the efficient structure hypothesis and the same square of the same elasticity. The third formulation is that the same net effect in the second formulation is expressed by the sum of the effects of the same increase in the Herfindahl index on the same efficiency difference of the GURP as per the efficient structure hypothesis, the same pricing error, and the same corrected dynamic actual marginal variable cost, so it is the formulation that is used to extensively interpret the quiet-life hypothesis with these effects.

3.1 Mathematical Formulations and Theoretical Interpretations of the Efficient Structure Hypothesis

As already noted, the efficient structure hypothesis is a composite that suggests three stages of causal relations from firm efficiency to firm growth (i.e., the first stage), then to market structure (i.e., the second stage), and finally to market performance (i.e., the third stage). There is no scope for improving on Demsetz (1973) vis-à-vis the first stage causality from firm efficiency to firm growth. As noted by Homma et al. (2014), this first stage causality is the fundamental feature of the efficient structure hypothesis, so this paper also regards this causality as the efficient structure hypothesis. Specifically, by regarding firm efficiency as dynamic cost efficiency, and by considering firm growth as an increase in a financial good (e.g., a loan), this section endeavors to rigorously formulate and theoretically interpret the efficient structure hypothesis.

Definition 17 (Acceptance of the Efficient Structure Hypothesis) *If the planned optimal financial good (e.g., the planned optimal loan) in the current period increases because of improved dynamic cost efficiency in the previous period, then the efficient structure hypothesis is accepted. Specifically, if the sign of* $\partial q^{p*}_{i,j,t} / \partial EF^{D}_{i,t-1}$ *is positive (i.e.,* $\partial q^{p*}_{i,j,t} / \partial EF^{D}_{i,t-1} > 0$*), then the efficient structure hypothesis is accepted.*

From this definition, the following two propositions are derived.

Proposition 5 *$\partial q^{p*}_{i,j,t} / \partial EF^{D}_{i,t-1}$ is expressed as follows:*

$$\frac{\partial q^{p*}_{i,j,t}}{\partial EF^{D}_{i,t-1}} = \left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial EF^{D}_{i,t-1}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial EF^{D}_{i,t-1}}\right] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t-1} \partial EF^{D}_{i,t}}\right] \Bigg/ \left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial q^{p*}_{i,j,t}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial q^{p*}_{i,j,t}}\right] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial q^{p*}_{i,j,t} \partial EF^{D}_{i,t}}\right], \quad (3.1.1)$$

where $\partial p^{GURF}_{i,j,t} / \partial X$ $(X = EF^{D}_{i,t-1}$ or $q^{p*}_{i,j,t})$ is expressed as

$$\frac{\partial p^{GURF}_{i,j,t}}{\partial X} = \frac{\partial p^{SURF}_{i,j,t}}{\partial X} + \frac{\partial \eta^{BPF*}_{i,j,t}}{\partial X} + \frac{\partial MRS^{BPF\pi*}_{e,i,t}}{\partial X} + \frac{\partial \varpi^{BPF*}_{i,j,t}}{\partial X}. \quad (3.1.2)$$

Proof. Partial differentiation of both sides of Eq. (2.2.3.9) with respect to the j-th planned optimal financial good in the current period gives

$$\frac{\partial MC^{DFV*}_{i,j,t}}{\partial q^{p*}_{i,j,t}} = \frac{\partial p^{GURF}_{i,j,t}}{\partial q^{p*}_{i,j,t}}. \quad (P5.1)$$

Similarly, partial differentiation of both sides of Eq. (2.1.11.1) with respect to the j-th planned optimal financial good in the current period gives

$$\frac{\partial MC^{DFV*}_{i,j,t}}{\partial q^{p*}_{i,j,t}} = \left\{\frac{\partial EF^{D}_{i,t}}{\partial q^{p*}_{i,j,t}} - \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-2} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial q^{p*}_{i,j,t} \partial EF^{D}_{i,t}}\right\} \cdot MC^{DAV*}_{i,j,t} + \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial q^{p*}_{i,j,t}}. \quad (P5.2)$$

Substituting Eq. (P5.2) for the left-hand side of Eq. (P5.1) gives

$$\left\{ \frac{\partial EF_{i,t}^{D}}{\partial q_{i,j,t}^{p*}} - \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-2} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial q_{i,j,t}^{p*} \partial EF_{i,t}^{D}} \right\} \cdot MC_{i,j,t}^{DAV*} + \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial q_{i,j,t}^{p*}} = \frac{\partial p_{i,j,t}^{GURF}}{\partial q_{i,j,t}^{p*}}. \quad \text{(P5.3)}$$

Transforming Eq. (P5.3) with respect to $\partial EF_{i,t}^{D} / \partial q_{i,j,t}^{p*}$ and then rearranging gives

$$\frac{\partial EF_{i,t}^{D}}{\partial q_{i,j,t}^{p*}} = \left[\left[\frac{\partial p_{i,j,t}^{GURF}}{\partial q_{i,j,t}^{p*}} - \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial q_{i,j,t}^{p*}} \right] \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{2} + MC_{i,j,t}^{DAV*} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial q_{i,j,t}^{p*} \partial EF_{i,t}^{D}} \right] \Bigg/ \left\{ MC_{i,j,t}^{DAV*} \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{2} \right\}. \quad \text{(P5.4)}$$

From this equation, $\partial q_{i,j,t}^{p*} / \partial EF_{i,t}^{D}$ is expressed as follows:

$$\frac{\partial q_{i,j,t}^{p*}}{\partial EF_{i,t}^{D}} = \left\{ MC_{i,j,t}^{DAV*} \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{2} \right\} \Bigg/ \left[\left[\frac{\partial p_{i,j,t}^{GURF}}{\partial q_{i,j,t}^{p*}} - \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial q_{i,j,t}^{p*}} \right] \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{2} + MC_{i,j,t}^{DAV*} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial q_{i,j,t}^{p*} \partial EF_{i,t}^{D}} \right]. \quad \text{(P5.5)}$$

Similar to Eq. (P5.1), partial differentiation of both sides of Eq. (2.2.3.9) with respect to dynamic cost efficiency in the previous period gives

$$\frac{\partial MC_{i,j,t}^{DFV*}}{\partial EF_{i,t-1}^{D}} = \frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}}. \quad \text{(P5.6)}$$

Similar to Eq. (P5.2), partial differentiation of both sides of Eq. (2.1.11.1) with respect to dynamic cost efficiency in the previous period gives

$$\frac{\partial MC_{i,j,t}^{DFV*}}{\partial EF_{i,t-1}^{D}} = \left\{ \frac{\partial EF_{i,t}^{D}}{\partial EF_{i,t-1}^{D}} - \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-2} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial EF_{i,t-1}^{D} \partial EF_{i,t}^{D}} \right\} \cdot MC_{i,j,t}^{DAV*} + \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}}. \quad \text{(P5.7)}$$

Similar to Eq. (P5.3), substituting Eq. (P5.7) for the left-hand side of Eq. (P5.6) gives

$$\left\{ \frac{\partial EF_{i,t}^{D}}{\partial EF_{i,t-1}^{D}} - \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-2} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial EF_{i,t-1}^{D} \partial EF_{i,t}^{D}} \right\} \cdot MC_{i,j,t}^{DAV*} + \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} = \frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}}. \quad \text{(P5.8)}$$

Similar to Eq. (P5.4), transforming Eq. (P5.8) with respect to $\partial EF_{i,t}^{D} / \partial EF_{i,t-1}^{D}$ and then rearranging gives

$$\frac{\partial EF_{i,t}^{D}}{\partial EF_{i,t-1}^{D}} = \left[\left[\frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}} - \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \right] \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{2} + MC_{i,j,t}^{DAV*} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial EF_{i,t-1}^{D} \partial EF_{i,t}^{D}} \right] \Big/ \left\{ MC_{i,j,t}^{DAV*} \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{2} \right\}. \quad \text{(P5.9)}$$

From Eqs. (P5.5) and (P5.9), $\partial q^{p*}_{i,j,t}/\partial EF^{D}_{i,t-1}$ is expressed as follows:

$$\frac{\partial q^{p*}_{i,j,t}}{\partial EF^{D}_{i,t-1}}=\frac{\partial q^{p*}_{i,j,t}}{\partial EF^{D}_{i,t}}\cdot\frac{\partial EF^{D}_{i,t}}{\partial EF^{D}_{i,t-1}}$$

$$=\left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial EF^{D}_{i,t-1}}-\left\{EF^{D}_{i,t}+\left(\frac{\partial\ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\}\cdot\frac{\partial MC^{DAV*}_{i,j,t}}{\partial EF^{D}_{i,t-1}}\right]\cdot\left(\frac{\partial\ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2}\right.$$

$$\left.+MC^{DAV*}_{i,j,t}\cdot\frac{\partial^{2}\ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t-1}\partial EF^{D}_{i,t}}\right]\Bigg/\left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial q^{p*}_{i,j,t}}-\left\{EF^{D}_{i,t}+\left(\frac{\partial\ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\}\cdot\frac{\partial MC^{DAV*}_{i,j,t}}{\partial q^{p*}_{i,j,t}}\right]\right.$$

$$\left.\cdot\left(\frac{\partial\ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2}+MC^{DAV*}_{i,j,t}\cdot\frac{\partial^{2}\ln C^{DAV}_{i,t}}{\partial q^{p*}_{i,j,t}\partial EF^{D}_{i,t}}\right],$$

where, from Eq. (2.2.3.6), $\partial p^{GURF}_{i,j,t}/\partial X$ ($X=EF^{D}_{i,t-1}$ or $q^{p*}_{i,j,t}$) is expressed as

$$\frac{\partial p^{GURF}_{i,j,t}}{\partial X}=\frac{\partial p^{SURF}_{i,j,t}}{\partial X}+\frac{\partial\eta^{BPF*}_{i,j,t}}{\partial X}+\frac{\partial MRS^{BPF\pi*}_{e,i,t}}{\partial X}+\frac{\partial\varpi^{BPF*}_{i,j,t}}{\partial X}.$$

■

$\partial p^{GURF}_{i,j,t}/\partial EF^{D}_{i,t-1}$ in Eq. (3.1.1) is the effect of the improvement in dynamic cost efficiency in the previous period on the GURP of the j-th planned optimal financial good on the cost frontier (i.e., the dynamic frontier marginal variable cost with respect to the j-th planned optimal financial good) in the current period. $\left\{EF^{D}_{i,t}+\left(\partial\ln C^{DAV}_{i,t}/\partial EF^{D}_{i,t}\right)^{-1}\right\}$ $(=MC^{DFV*}_{i,j,t}/MC^{DAV*}_{i,j,t}=p^{GURF}_{i,j,t}/MC^{DAV*}_{i,j,t})$ in Eq. (3.1.1) is the dynamic marginal cost efficiency that can be interpreted as a coefficient quantifying the differential shapes of the dynamic frontier variable cost function and the dynamic actual variable cost function. If both shapes are perfectly equal (i.e., $MC^{DFV*}_{i,j,t}(=p^{GURF}_{i,j,t})=MC^{DAV*}_{i,j,t}$), then the following equation holds: $\left\{EF^{D}_{i,t}+\left(\partial\ln C^{DAV}_{i,t}/\partial EF^{D}_{i,t}\right)^{-1}\right\}=1$. However, for example, if the dynamic actual variable cost function is an increasing homothetic function of the dynamic frontier variable cost function (i.e., both shapes are not very different), then the following inequality holds: $MC^{DFV*}_{i,j,t}(=p^{GURF}_{i,j,t})\leq MC^{DAV*}_{i,j,t}$. Further, this inequality also holds: $\left\{EF^{D}_{i,t}+\left(\partial\ln C^{DAV}_{i,t}/\partial EF^{D}_{i,t}\right)^{-1}\right\}\leq 1$. In this case, dynamic marginal cost efficiency can be interpreted as a discount factor. In contrast, if both shapes are very different (for example, the dynamic frontier variable cost function has no area where the marginal cost decreases, whereas the dynamic actual variable cost function has an area where the marginal cost decreases), then this inequality can hold: $MC_{i,j,t}^{DFV*}(=p_{i,j,t}^{GURF})>MC_{i,j,t}^{DAV*}$. Further, this inequality would also hold: $\left\{EF_{i,t}^{D}+\left(\partial\ln C_{i,t}^{DAV}\big/\partial EF_{i,t}^{D}\right)^{-1}\right\}>1$. In this case, dynamic marginal cost efficiency can be interpreted as an extra factor. Consequently, $-\left\{EF_{i,t}^{D}+\left(\partial\ln C_{i,t}^{DAV}\big/\partial EF_{i,t}^{D}\right)^{-1}\right\}\cdot\partial MC_{i,j,t}^{DAV*}\big/\partial EF_{i,t-1}^{D}$ in Eq. (3.1.1) can be interpreted as the decreasing effect of the improvement in dynamic cost efficiency in the previous period on dynamic actual marginal variable cost with respect to the j-th planned optimal financial good in the current period (i.e., $-\partial MC_{i,j,t}^{DAV*}\big/\partial EF_{i,t-1}^{D}$), which is corrected by dynamic marginal cost efficiency in the current period (i.e., $\left\{EF_{i,t}^{D}+\left(\partial\ln C_{i,t}^{DAV}\big/\partial EF_{i,t}^{D}\right)^{-1}\right\}$). Considering the case that both shapes are perfectly equal (i.e., $\left\{EF_{i,t}^{D}+\left(\partial\ln C_{i,t}^{DAV}\big/\partial EF_{i,t}^{D}\right)^{-1}\right\}=1$) as a criterion for interpreting dynamic marginal cost efficiency, if the dynamic actual variable cost function is an increasing homothetic function of the dynamic frontier variable cost function (i.e., both shapes are not very different), then this decreasing effect is evaluated at a discount, whereas if this dynamic actual variable cost function is not an increasing homothetic function (i.e., both shapes are very different), then this decreasing effect is evaluated at an extra. Without this correction, the former case overestimates this decreasing effect, whereas the latter case underestimates it. Specifically, in order to compare these cases, it is assumed that, following an improvement in dynamic cost efficiency in the previous period, dynamic actual marginal variable costs with respect to the j-th planned optimal financial good in the current period are equal where dynamic marginal cost efficiencies in the current period are one and other than one. If dynamic marginal cost efficiency in the current period is less than one, then the decreasing effect (in terms of absolute value) is greater than in the case that this dynamic marginal cost efficiency is one (i.e., $-\partial MC_{i,j,t}^{DAV*}\big/\partial EF_{i,t-1}^{D}=-\partial MC_{i,j,t}^{DFV*}\big/\partial EF_{i,t-1}^{D}$), whereas if this dynamic marginal cost efficiency is greater than one, then the decreasing effect (in terms of absolute value) is less than in the case that this dynamic marginal cost efficiency is one. If taking this dynamic marginal cost efficiency to be one as a criterion, the need to correct this decreasing effect by multiplying by this dynamic marginal cost efficiency, therefore, arises. Consequently, $\partial p_{i,j,t}^{GURF} / \partial EF_{i,t-1}^{D} - \left\{ EF_{i,t}^{D} + \left(\partial \ln C_{i,t}^{DAV} / \partial EF_{i,t}^{D} \right)^{-1} \right\} \cdot \partial MC_{i,j,t}^{DAV*} / \partial EF_{i,t-1}^{D}$ in Eq. (3.1.1) can be concisely interpreted as the net effect of the improvement in dynamic cost efficiency in the previous period on the GURP of the j-th planned optimal financial good on the cost frontier in the current period and on the dynamic actual marginal variable cost with respect to the same planned optimal financial good in the current period, which accounts for the correction in dynamic marginal cost efficiency in the current period (hereafter "the net effect"). In addition, $\partial^2 \ln C_{i,t}^{DAV} / \partial EF_{i,t-1}^{D} \partial EF_{i,t}^{D}$ in Eq. (3.1.1) can be interpreted as the effect of the same improvement in dynamic cost efficiency on the elasticity of dynamic actual variable cost in the current period with respect to dynamic cost efficiency in the current period (hereafter "the effect on the elasticity"). The remainder of Eq. (3.1.1), $\left(\partial \ln C_{i,t}^{DAV} / \partial EF_{i,t}^{D} \right)^2$, and $MC_{i,j,t}^{DAV*}$ can be interpreted as coefficients connecting the net effect and the effect on the elasticity, which use the product of these coefficients as a common criterion (i.e., denominator). From the proof of Proposition 5, the net effect is based on $MC_{i,j,t}^{DAV*}$ and the effect on the elasticity is based on $\left(\partial \ln C_{i,t}^{DAV} / \partial EF_{i,t}^{D} \right)^2$, so the need to multiply these coefficients in order to connect these effects based on the product of these coefficients arises. Generally speaking, the numerator of Eq. (3.1.1) can be interpreted as the sum of the net effect based on $MC_{i,j,t}^{DAV*}$ and the effect on the elasticity, which is based on $\left(\partial \ln C_{i,t}^{DAV} / \partial EF_{i,t}^{D} \right)^2$. For the denominator of Eq. (3.1.1), the interpretation is similar to that of the numerator of Eq. (3.1.1) with the exception of replacing $EF_{i,t-1}^{D}$ with $q_{i,j,t}^{p*}$. The denominator is the sum of the net effect of an increase in the j-th planned optimal financial good in the current period and the effect of the same increase in the j-th planned optimal financial good. Similar to the numerator, the former net effect is on the same GURP and on the same dynamic actual marginal variable cost. This net effect is normalized by the same dynamic actual marginal variable cost and accounts for the correction in dynamic marginal cost efficiency in the current period. The latter effect is on the same elasticity of dynamic actual variable cost and is normalized by the square of the same elasticity. Consequently, if both the numerator and denominator are simultaneously positive or negative, then the efficient structure hypothesis is accepted.

Proposition 6 $\partial q_{i,j,t}^{p*} / \partial EF_{i,t-1}^{D}$ *is, furthermore, expressed as follows:*

$$\frac{\partial q_{i,j,t}^{p*}}{\partial EF_{i,t-1}^{D}} = \left[A_{i,j,t} \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^2 + MC_{i,j,t}^{DAV*} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial EF_{i,t-1}^{D} \partial EF_{i,t}^{D}} \right] \Bigg/ \left[B_{i,j,t} \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^2 + MC_{i,j,t}^{DAV*} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial q_{i,j,t}^{p*} \partial EF_{i,t}^{D}} \right], \quad (3.1.3)$$

where $A_{i,j,t}$ *and* $B_{i,j,t}$ *are respectively expressed as follows:*

$$\begin{aligned} A_{i,j,t} &= \left(\frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}} - \frac{\partial p_{i,j,t}^{GURA}}{\partial EF_{i,t-1}^{D}} \right) + \frac{\partial PIE_{i,j,t}}{\partial EF_{i,t-1}^{D}} \\ &\quad + \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \cdot \frac{MC_{i,j,t}^{DAV*} - MC_{i,j,t}^{DFV*}}{MC_{i,j,t}^{DAV*}}, \qquad (3.1.4) \\ B_{i,j,t} &= \left(\frac{\partial p_{i,j,t}^{GURF}}{\partial q_{i,j,t}^{p*}} - \frac{\partial p_{i,j,t}^{GURA}}{\partial q_{i,j,t}^{p*}} \right) + \frac{\partial PIE_{i,j,t}}{\partial q_{i,j,t}^{p*}} \\ &\quad + \frac{\partial MC_{i,j,t}^{DAV*}}{\partial q_{i,j,t}^{p*}} \cdot \frac{MC_{i,j,t}^{DAV*} - MC_{i,j,t}^{DFV*}}{MC_{i,j,t}^{DAV*}}. \qquad (3.1.5) \end{aligned}$$

Proof. From Proposition 5 (Eq. (3.1.1)), $A_{i,j,t}$ is initially expressed as follows:

$$A_{i,j,t} = \frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}} - \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}}. \qquad \text{(P6.1)}$$

Rearranging this equation then gives

$$
\begin{aligned}
A_{i,j,t} &= \frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}} - \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \\
&= \left(\frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}} - \frac{\partial p_{i,j,t}^{GURA}}{\partial EF_{i,t-1}^{D}} \right) + \left(\frac{\partial p_{i,j,t}^{GURA}}{\partial EF_{i,t-1}^{D}} - \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \right) \\
&\quad + \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \cdot \left[1 - \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \right]. \qquad \text{(P6.2)}
\end{aligned}
$$

From Remark 2 (Eq. (2.2.3.10)), the second term in this equation is expressed as follows:

$$
\frac{\partial p_{i,j,t}^{GURA}}{\partial EF_{i,t-1}^{D}} - \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} = \frac{\partial PIE_{i,j,t}}{\partial EF_{i,t-1}^{D}}. \qquad \text{(P6.3)}
$$

From Proposition 1 (Eq. (2.1.11.1)), the third term in the same equation is expressed as follows:

$$
\begin{aligned}
\frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \cdot \left[1 - \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \right] &= \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \cdot \left(1 - \frac{MC_{i,j,t}^{DFV*}}{MC_{i,j,t}^{DAV*}} \right) \\
&= \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \cdot \frac{MC_{i,j,t}^{DAV*} - MC_{i,j,t}^{DFV*}}{MC_{i,j,t}^{DAV*}}. \qquad \text{(P6.4)}
\end{aligned}
$$

Substituting Eqs. (P6.3) and (P6.4) into Eq. (P6.2) then gives

$$
A_{i,j,t} = \left(\frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}} - \frac{\partial p_{i,j,t}^{GURA}}{\partial EF_{i,t-1}^{D}} \right) + \frac{\partial PIE_{i,j,t}}{\partial EF_{i,t-1}^{D}} + \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \cdot \frac{MC_{i,j,t}^{DAV*} - MC_{i,j,t}^{DFV*}}{MC_{i,j,t}^{DAV*}}.
$$

The derivation of $B_{i,j,t}$ (Eq. (3.1.5)) is similar to the derivation of $A_{i,j,t}$ (Eq. (3.1.4)) with the exception of replacing $EF_{i,t-1}^{D}$ with $q_{i,j,t}^{p*}$, so we omit the derivation. ■

From Proposition 6, the net effect of the improvement in dynamic cost efficiency in the previous period on the GURP of the j-th planned optimal financial good on the cost frontier in the current period and on the dynamic actual marginal variable cost with respect to the same planned optimal financial good in the current period, which accounts for the correction in dynamic marginal cost efficiency in the current period, can be expressed as the sum of the effects of the improvement in dynamic cost efficiency in the previous period on the efficiency difference of the GURP of the j-th planned optimal financial good in the current period (i.e., $p_{i,j,t}^{GURF} - p_{i,j,t}^{GURA}$), the pricing error of the same financial good (i.e., $PIE_{i,j,t}$ $(= p_{i,j,t}^{GURA} - MC_{i,j,t}^{DAV*})$), and the dynamic actual marginal variable cost with respect to the same financial good (i.e., $MC_{i,j,t}^{DAV*}$), which accounts for the correction in dynamic marginal cost efficiency in the current period (i.e., $\left(MC_{i,j,t}^{DAV*} - MC_{i,j,t}^{DFV*}\right)/MC_{i,j,t}^{DAV*}$). Regarding the net effect of an increase in the j-th planned optimal financial good in the current period, the expression is similar to the net effect of the improvement in dynamic cost efficiency in the previous period with the exception of replacing $EF_{i,t-1}^{D}$ with $q_{i,j,t}^{p*}$.

3.2 Mathematical Formulations and Theoretical Interpretations of the Quiet-Life Hypothesis

As already noted, the quiet-life hypothesis concerns the relationship between market concentration and firm efficiency. Similar to Homma et al. (2014), by regarding this as the relationship between the Herfindahl index and dynamic cost efficiency, this section endeavors to rigorously formulate and theoretically interpret this hypothesis.

Definition 18 (Acceptance of the Quiet-Life Hypothesis) *If dynamic cost efficiency in the current period decreases because of an increase in the Herfindahl index in the previous period, then the quiet-life hypothesis is accepted. Specifically, if the sign of* $\partial EF_{i,t}^{D}/\partial HI_{j,t-1}$ *is negative (i.e.,* $\partial EF_{i,t}^{D}/\partial HI_{j,t-1} < 0$*), then the quiet-life hypothesis is accepted.*

Similar to Definition 17, from this definition, the following two propositions are derived.

Proposition 7 *$\partial EF^{D}_{i,t}/\partial HI_{j,t-1}$ is expressed as follows:*

$$\frac{\partial EF^{D}_{i,t}}{\partial HI_{j,t-1}} = \left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial HI_{j,t-1}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial HI_{j,t-1}}\right] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^2 \ln C^{DAV}_{i,t}}{\partial HI_{j,t-1}\partial EF^{D}_{i,t}}\right] \Big/ \left\{MC^{DAV*}_{i,j,t} \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2}\right\}, \quad (3.2.1)$$

where $\partial p^{GURF}_{i,j,t}/\partial HI_{j,t-1}$ is expressed as

$$\frac{\partial p^{GURF}_{i,j,t}}{\partial HI_{j,t-1}} = \frac{\partial p^{SURF}_{i,j,t}}{\partial HI_{j,t-1}} + \frac{\partial \eta^{BPF*}_{i,j,t}}{\partial HI_{j,t-1}} + \frac{\partial MRS^{BPF\pi*}_{e,i,t}}{\partial HI_{j,t-1}} + \frac{\partial \varpi^{BPF*}_{i,j,t}}{\partial HI_{j,t-1}}. \quad (3.2.2)$$

Proof. The proof of this proposition is similar to Eq. (P5.9) in the proof of Proposition 5 with the exception of replacing $EF^{D}_{i,t-1}$ with $HI_{j,t-1}$, so we omit the derivation. ■

The interpretation of Eq. (3.2.1) in Proposition 7 is similar to the numerator of Eq. (3.1.1) in Proposition 5 with the exception of replacing $EF^{D}_{i,t-1}$ with $HI_{j,t-1}$. Eq. (3.2.1) is the sum of the net effect of an increase in the Herfindahl index in the previous period and the effect of the same increase in the Herfindahl index. Similar to the numerator of Eq. (3.1.1) in Proposition 5, the former net effect is on the GURP of the j-th planned optimal financial good on the cost frontier in the current period and on the dynamic actual marginal variable cost with respect to the same planned optimal financial good in the current period. This net effect is normalized by the same dynamic actual marginal variable cost and accounts for the correction in dynamic marginal cost efficiency in the current period. The latter effect is on the elasticity of dynamic actual variable cost in the current period with respect to dynamic cost efficiency in the current period and is normalized by the square of the same elasticity. Under the assumption that the j-th financial good is an output (i.e., $p^{SURF}_{i,j,t}$, $MC^{DFV*}_{i,j,t} > 0$) and the sign of dynamic marginal cost efficiency in the current period is positive (i.e., $MC^{DFV*}_{i,j,t}/MC^{DAV*}_{i,j,t} > 0$), if the numerator of Eq. (3.2.1) in Proposition 7 is negative, then the quiet-life hypothesis is accepted.

Proposition 8 *$\partial EF^D_{i,t}/\partial HI_{j,t-1}$ is, furthermore, expressed as follows:*

$$\frac{\partial EF^D_{i,t}}{\partial HI_{j,t-1}} = \left[A_{i,j,t}\cdot\left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}}\right)^2 + MC^{DAV*}_{i,j,t}\cdot\frac{\partial^2 \ln C^{DAV}_{i,t}}{\partial HI_{j,t-1}\partial EF^D_{i,t}}\right] \Bigg/ \left\{MC^{DAV*}_{i,j,t}\cdot\left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}}\right)^2\right\}, \quad (3.2.3)$$

where $A_{i,j,t}$ is expressed as

$$A_{i,j,t} = \left(\frac{\partial p^{GURF}_{i,j,t}}{\partial HI_{j,t-1}} - \frac{\partial p^{GURA}_{i,j,t}}{\partial HI_{j,t-1}}\right) + \frac{\partial PIE_{i,j,t}}{\partial HI_{j,t-1}} + \frac{\partial MC^{DAV*}_{i,j,t}}{\partial HI_{j,t-1}}\cdot\frac{MC^{DAV*}_{i,j,t} - MC^{DFV*}_{i,j,t}}{MC^{DAV*}_{i,j,t}}. \quad (3.2.4)$$

Proof. The derivation of $A_{i,j,t}$ (Eq. (3.2.4)) is similar to the derivation of $A_{i,j,t}$ in Eq. (3.1.4) of Proposition 6 with the exception of replacing $EF^D_{i,t-1}$ with $HI_{j,t-1}$, so we omit the derivation. ■

The interpretation of $A_{i,j,t}$ (Eq. (3.2.4)) is similar to the interpretation of $A_{i,j,t}$ in Eq. (3.1.4) of Proposition 6 with the exception of replacing $EF^D_{i,t-1}$ with $HI_{j,t-1}$. The net effect of an increase in the Herfindahl index in the previous period on the GURP of the j-th planned optimal financial good on the cost frontier in the current period and on dynamic actual marginal variable cost with respect to the same planned optimal financial good in the current period, which accounts for the correction in dynamic marginal cost efficiency in the current period, can be expressed as the sum of the effects of an increase in the Herfindahl index in the previous period on the efficiency difference of the GURP of the j-th planned optimal financial good in the current period (i.e., $p^{GURF}_{i,j,t} - p^{GURA}_{i,j,t}$), the pricing error of the same financial good (i.e., $PIE_{i,j,t}$ $(= p^{GURA}_{i,j,t} - MC^{DAV*}_{i,j,t})$), and dynamic actual marginal variable cost with respect to the same financial good (i.e., $MC^{DAV*}_{i,j,t}$), which accounts for the correction in dynamic marginal cost efficiency in the current period (i.e., $\left(MC^{DAV*}_{i,j,t} - MC^{DFV*}_{i,j,t}\right)/MC^{DAV*}_{i,j,t}$).

4 Relative Magnitude of the Efficient Structure Hypothesis to the Quiet-Life Hypothesis

This section defines the relative magnitude of the efficient structure hypothesis to the quiet-life hypothesis and clarifies the condition whereby the former is superior (or inferior) in magnitude to the latter. As already noted, if a criterion for judging industrial organization policies is that support for both hypotheses should be associated with increased EGLI on the cost frontier, then anti-monopoly and anti-concentration policies are necessary if the quiet-life hypothesis is superior in magnitude to the efficient structure hypothesis. If the efficient structure hypothesis is, however, superior in magnitude to the quiet-life hypothesis, then new industrial organization policies which differ from existing anti-monopoly and anti-concentration policies, and in which an efficiency improvement would decrease the EGLI on the cost frontier, are needed. Consequently, from the perspective of industrial organization and anti-monopoly policies, it is important to clarify which of these hypotheses are superior, because this determines the recommended policy interventions.

Definition 19 (Relative Magnitude) *The relative magnitude of the efficient structure hypothesis to the quiet-life hypothesis, denoted by $RM_{i,j,t}$, is defined as follows:*

$$RM_{i,j,t} = \frac{\partial \ln q_{i,j,t}^{p*}}{\partial EF_{i,t-1}^{D}} \Big/ \frac{\partial EF_{i,t}^{D}}{\partial \ln HI_{j,t-1}}. \tag{4.1}$$

$RM_{i,j,t}$ is the ratio of the elasticity of the j-th planned optimal financial good in the current period with respect to dynamic cost efficiency in the previous period to the elasticity of dynamic cost efficiency in the current period with respect to the Herfindahl index in the previous period. From this definition and Propositions 5 and 7, the following proposition holds.

Proposition 9 *$RM_{i,j,t}$ is expressed as follows:*

$$
\begin{aligned}
RM_{i,j,t} = &\Bigg[\Bigg[\Bigg[\Bigg[\frac{\partial p^{GURF}_{i,j,t}}{\partial EF^{D}_{i,t-1}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial EF^{D}_{i,t-1}}\Bigg] \\
&\cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t-1}\partial EF^{D}_{i,t}}\Bigg] \cdot \left\{MC^{DAV*}_{i,j,t} \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2}\right\}\Bigg]\Bigg] \\
&\Big/ \Bigg[\Bigg[\Bigg[\frac{\partial p^{GURF}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}}\Bigg] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} \\
&+MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial \ln q^{p*}_{i,j,t}\partial EF^{D}_{i,t}}\Bigg] \cdot \Bigg[\Bigg[\frac{\partial p^{GURF}_{i,j,t}}{\partial \ln HI_{j,t-1}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \\
&\cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial \ln HI_{j,t-1}}\Bigg] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial \ln HI_{j,t-1}\partial EF^{D}_{i,t}}\Bigg]\Bigg]. \qquad (4.2)
\end{aligned}
$$

Proof. From Definition 19 and Proposition 5, the numerator of $RM_{i,j,t}$ is expressed as follows:

$$
\begin{aligned}
\frac{\partial \ln q^{p*}_{i,j,t}}{\partial EF^{D}_{i,t-1}} &= \frac{\partial q^{p*}_{i,j,t}}{\partial EF^{D}_{i,t-1}} \cdot \frac{1}{q^{p*}_{i,j,t}} \\
&= \Bigg[\Bigg[\frac{\partial p^{GURF}_{i,j,t}}{\partial EF^{D}_{i,t-1}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial EF^{D}_{i,t-1}}\Bigg] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} \\
&+MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t-1}\partial EF^{D}_{i,t}}\Bigg] \Big/ \Bigg[\Bigg[\frac{\partial p^{GURF}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \\
&\cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}}\Bigg] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial \ln q^{p*}_{i,j,t}\partial EF^{D}_{i,t}}\Bigg]. \qquad (P9.1)
\end{aligned}
$$

Similarly, from Definition 19 and Proposition 7, the denominator of $RM_{i,j,t}$ is expressed as follows:

$$\frac{\partial EF_{i,t}^{D}}{\partial \ln HI_{j,t-1}} = \frac{\partial EF_{i,t}^{D}}{\partial HI_{j,t-1}} \cdot HI_{j,t-1}$$
$$= \left[\left[\frac{\partial p_{i,j,t}^{GURF}}{\partial \ln HI_{j,t-1}} - \left\{EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}}\right)^{-1}\right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial \ln HI_{j,t-1}}\right] \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}}\right)^{2}\right.$$
$$\left. + MC_{i,j,t}^{DAV*} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial \ln HI_{j,t-1} \partial EF_{i,t}^{D}}\right] \Bigg/ \left\{MC_{i,j,t}^{DAV*} \cdot \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}}\right)^{2}\right\}. \quad \text{(P9.2)}$$

Substituting Eqs. (P9.1) and (P9.2) into Eq. (4.1) then yields Eq. (4.2). ■

From Proposition 9, the following proposition is then established.

Proposition 10 *Considering $CM_{i,j,t}$ as a criterion, if dynamic actual marginal variable cost with respect to the j-th planned optimal financial good in the current period is less than $CM_{i,j,t}$ (i.e., $MC_{i,j,t}^{DAV*} < CM_{i,j,t}$), then the efficient structure hypothesis is superior in magnitude to the quiet-life hypothesis, whereas if the same dynamic actual marginal variable cost is greater than the same criterion (i.e., $MC_{i,j,t}^{DAV*} > CM_{i,j,t}$), then the quiet-life hypothesis is superior in magnitude to the efficient structure hypothesis, where $CM_{i,j,t}$ is as* *follows:*

$$
\begin{aligned}
CM_{i,j,t} = -&\Bigg[\Bigg[\frac{\partial p^{GURF}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}}\Bigg] \\
&\cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial \ln q^{p*}_{i,j,t} \partial EF^{D}_{i,t}}\Bigg] \cdot \Bigg[\Bigg[\frac{\partial p^{GURF}_{i,j,t}}{\partial \ln HI_{j,t-1}} \\
&- \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial \ln HI_{j,t-1}}\Bigg] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} \\
&+ MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial \ln HI_{j,t-1} \partial EF^{D}_{i,t}}\Bigg] \Bigg/ \Bigg[\Bigg[\Bigg[\frac{\partial p^{GURF}_{i,j,t}}{\partial EF^{D}_{i,t-1}} - \left\{EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{-1}\right\} \\
&\cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial EF^{D}_{i,t-1}}\Bigg] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2} + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^{2} \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t-1} \partial EF^{D}_{i,t}}\Bigg] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}}\right)^{2}\Bigg].
\end{aligned}
\tag{4.3}
$$

Proof. From Proposition 9 (Eq. (4.2)), the following relations between inequalities hold:

$$
\begin{aligned}
& RM_{i,j,t} = \frac{\partial \ln q^{p*}_{i,j,t}}{\partial EF^D_{i,t-1}} \Big/ \frac{\partial EF^D_{i,t}}{\partial \ln HI_{j,t-1}} < (> , \ =) -1 \\
\iff & \left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial EF^D_{i,t-1}} - \left\{ EF^D_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^{-1} \right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial EF^D_{i,t-1}} \right] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^2 \right. \\
& \quad \left. + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^2 \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t-1} \partial EF^D_{i,t}} \right] \cdot \left\{ MC^{DAV*}_{i,j,t} \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^2 \right\} \\
& < (> , \ =) - \left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}} - \left\{ EF^D_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^{-1} \right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}} \right] \right. \\
& \quad \left. \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^2 + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^2 \ln C^{DAV}_{i,t}}{\partial \ln q^{p*}_{i,j,t} \partial EF^D_{i,t}} \right] \cdot \left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial \ln HI_{j,t-1}} \right. \right. \\
& \quad \left. - \left\{ EF^D_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^{-1} \right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial \ln HI_{j,t-1}} \right] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^2 \\
& \quad \left. + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^2 \ln C^{DAV}_{i,t}}{\partial \ln HI_{j,t-1} \partial EF^D_{i,t}} \right] \\
\iff & MC^{DAV*}_{i,j,t} < (> , \ =) - \left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}} - \left\{ EF^D_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^{-1} \right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}} \right] \right. \\
& \quad \left. \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^2 + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^2 \ln C^{DAV}_{i,t}}{\partial \ln q^{p*}_{i,j,t} \partial EF^D_{i,t}} \right] \cdot \left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial \ln HI_{j,t-1}} \right. \right. \\
& \quad \left. - \left\{ EF^D_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^{-1} \right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial \ln HI_{j,t-1}} \right] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^2 \\
& \left. + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^2 \ln C^{DAV}_{i,t}}{\partial \ln HI_{j,t-1} \partial EF^D_{i,t}} \right] \Big/ \left[\left[\left[\frac{\partial p^{GURF}_{i,j,t}}{\partial EF^D_{i,t-1}} - \left\{ EF^D_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^{-1} \right\} \right. \right. \right. \\
& \left. \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial EF^D_{i,t-1}} \right] \cdot \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^2 + MC^{DAV*}_{i,j,t} \cdot \frac{\partial^2 \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t-1} \partial EF^D_{i,t}} \right] \cdot \left. \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^D_{i,t}} \right)^2 \right] \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (= CM_{i,j,t}).
\end{aligned}
$$

■

5 Efficient Structure and Quiet-Life Hypotheses and the EGLI on the Cost Frontier

This section clarifies under what assumptions either or both the efficient structure and quiet life hypotheses increase or decrease the EGLI on the cost frontier and considers the implications thereof. Results suggest that both desirable and undesirable cases exist, and the following two points are particularly noteworthy: (1) it is not always possible to justify anti-monopoly and anti-concentration policies using support for the quiet-life hypothesis; and (2) new industrial organization policies are needed if support for the efficient structure hypothesis is undesirable. In terms of the first point, there is the case where support for the quiet-life hypotheses decreases the EGLI on the cost frontier (i.e., increases the degree of competition on the cost frontier), so support for this hypothesis cannot always be used to justify anti-monopoly and anti-concentration policies, even if an increase in market concentration decreases dynamic cost efficiency. Justification for such policies is restricted to the case where an increase in market concentration increases the EGLI on the cost frontier (i.e., decreases the degree of competition on the cost frontier). Thus the enactment and enforcement of anti-monopoly and anti-concentration policies requires careful consideration. In terms of the second point, thus far, a theoretical foundation suggesting that support for the efficient structure hypothesis is undesirable is not discerned. However, at least theoretically, there are cases where both support for the efficient structure hypothesis decreases the EGLI on the cost frontier (i.e., increases the degree of competition on the cost frontier) and increases the EGLI on the cost frontier (i.e., decreases the degree of competition on the cost frontier). In the latter case, it is judged that support for the efficient structure hypothesis is undesirable: new industrial organization policies would be needed which differ from existing anti-monopoly and anti-concentration policies and under which an efficiency improvement would increase the degree of competition on the cost frontier.

From Definition 14 and Proposition 5, regarding the relation between the efficient structure hypothesis and the EGLI on the cost frontier, the following two propositions can be derived for clarifying under what assumptions the efficient structure hypothesis increases or decreases the EGLI on the cost frontier.

Proposition 11 *The EGLI on the cost frontier decreases with dynamic cost efficiency in the previous period and the j-th optimal planned financial good in the current period (i.e., the degree of competition on the cost frontier increases with them, $\partial EGLI_{i,j,t}^{F} / \partial EF_{i,t-1}^{D} < 0$ and $\partial EGLI_{i,j,t}^{F} / \partial q_{i,j,t}^{p*} < 0$) if and only if the efficient structure hypothesis is accepted (i.e., dynamic efficiency improves, $\partial q_{i,j,t}^{p*} / \partial EF_{i,t-1}^{D} > 0$) under the following assumptions: (A1) The j-th financial good is an output (i.e., $p_{i,j,t}^{SURF} > 0$ and $MC_{i,j,t}^{DFV*} > 0$); and (A2) One of the following two pairs of inequalities holds:*

$$\begin{aligned} \partial p_{i,j,t}^{GURF} / \partial EF_{i,t-1}^{D} &> \max\left(ME_{i,j,t}, \left(MC_{i,j,t}^{DFV*} / p_{i,j,t}^{SURF}\right) \cdot \left(\partial p_{i,j,t}^{SURF} / \partial EF_{i,t-1}^{D}\right)\right) \\ \text{and } \partial p_{i,j,t}^{GURF} / \partial q_{i,j,t}^{p*} &> \max\left(MQ_{i,j,t}, \left(MC_{i,j,t}^{DFV*} / p_{i,j,t}^{SURF}\right) \cdot \left(\partial p_{i,j,t}^{SURF} / \partial q_{i,j,t}^{p*}\right)\right), \end{aligned}$$

or

$$\begin{aligned} \left(MC_{i,j,t}^{DFV*} / p_{i,j,t}^{SURF}\right) \cdot \left(\partial p_{i,j,t}^{SURF} / \partial EF_{i,t-1}^{D}\right) &< \partial p_{i,j,t}^{GURF} / \partial EF_{i,t-1}^{D} < ME_{i,j,t} \\ \text{and } \left(MC_{i,j,t}^{DFV*} / p_{i,j,t}^{SURF}\right) \cdot \left(\partial p_{i,j,t}^{SURF} / \partial q_{i,j,t}^{p*}\right) &< \partial p_{i,j,t}^{GURF} / \partial q_{i,j,t}^{p*} < MQ_{i,j,t}, \end{aligned}$$

where $ME_{i,j,t}$ and $MQ_{i,j,t}$ are respectively expressed as

$$\begin{aligned} ME_{i,j,t} &= \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial EF_{i,t-1}^{D}} \\ &\quad - MC_{i,j,t}^{DAV*} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial EF_{i,t-1}^{D} \partial EF_{i,t}^{D}} \Bigg/ \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{2}, \qquad (5.1) \\ MQ_{i,j,t} &= \left\{ EF_{i,t}^{D} + \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{-1} \right\} \cdot \frac{\partial MC_{i,j,t}^{DAV*}}{\partial q_{i,j,t}^{p*}} \\ &\quad - MC_{i,j,t}^{DAV*} \cdot \frac{\partial^2 \ln C_{i,t}^{DAV}}{\partial q_{i,j,t}^{p*} \partial EF_{i,t}^{D}} \Bigg/ \left(\frac{\partial \ln C_{i,t}^{DAV}}{\partial EF_{i,t}^{D}} \right)^{2}. \qquad (5.2) \end{aligned}$$

Proof. From Definition 14 (Eq. (2.2.4.3)), the following equation holds:

$$\frac{\partial EGLI_{i,j,t}^{F}}{\partial X} = \left(p_{i,j,t}^{SURF}\right)^{-1} \cdot \left(\frac{MC_{i,j,t}^{DFV*}}{p_{i,j,t}^{SURF}} \cdot \frac{\partial p_{i,j,t}^{SURF}}{\partial X} - \frac{\partial MC_{i,j,t}^{DFV*}}{\partial X} \right),$$
$$(X = EF_{i,t-1}^{D} \text{ or } q_{i,j,t}^{p*}). \quad \text{(P11.1)}$$

From this equation, under assumption (A1), the following relation is then revealed:

$$\frac{\partial EGLI_{i,j,t}^{F}}{\partial X} > (<) 0 \Longleftrightarrow \frac{MC_{i,j,t}^{DFV*}}{p_{i,j,t}^{SURF}} \cdot \frac{\partial p_{i,j,t}^{SURF}}{\partial X} > (<) \frac{\partial MC_{i,j,t}^{DFV*}}{\partial X} \left(= \frac{\partial p_{i,j,t}^{GURF}}{\partial X} \right),$$
$$(X = EF_{i,t-1}^{D} \text{ or } q_{i,j,t}^{p*}). \quad \text{(P11.2)}$$

In addition, from Proposition 5 (Eq. (3.1.1)), the following relation holds:

$$\frac{\partial q_{i,j,t}^{p*}}{\partial EF_{i,t-1}^{D}} > 0 \Longleftrightarrow \frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}} > ME_{i,j,t} \text{ and } \frac{\partial p_{i,j,t}^{GURF}}{\partial q_{i,j,t}^{p*}} > MQ_{i,j,t}, \text{ or}$$
$$\frac{\partial p_{i,j,t}^{GURF}}{\partial EF_{i,t-1}^{D}} < ME_{i,j,t} \text{ and } \frac{\partial p_{i,j,t}^{GURF}}{\partial q_{i,j,t}^{p*}} < MQ_{i,j,t}, \quad \text{(P11.3)}$$

where $ME_{i,j,t}$ and $MQ_{i,j,t}$ are respectively expressed as Eqs. (5.1) and (5.2).

From relations (P11.2) and (P11.3), under assumptions (A1) and (A2), the following relation is, therefore, established:

$$\frac{\partial q_{i,j,t}^{p*}}{\partial EF_{i,t-1}^{D}} > 0 \Longleftrightarrow \frac{\partial EGLI_{i,j,t}^{F}}{\partial EF_{i,t-1}^{D}} < 0 \,\text{and}\, \frac{\partial EGLI_{i,j,t}^{F}}{\partial q_{i,j,t}^{p*}} < 0.$$

■

Proposition 12 *The EGLI on the cost frontier increases with dynamic cost efficiency in the previous period and the j-th optimal planned financial good in the current period (i.e., the degree of competition on the cost frontier decreases with them,* $\partial EGLI_{i,j,t}^{F}/\partial EF_{i,t-1}^{D} > 0$ *and* $\partial EGLI_{i,j,t}^{F}/\partial q_{i,j,t}^{p*} > 0$*) if and only if the efficient structure hypothesis is accepted (i.e., dynamic efficiency improves,* $\partial q_{i,j,t}^{p*}/\partial EF_{i,t-1}^{D} > 0$*) under the following assumptions: (A3) assumption (A1) holds; and (A4) One of the following two pairs of inequalities holds:*

$$\begin{aligned} \left(MC_{i,j,t}^{DFV*}/\,p_{i,j,t}^{SURF}\right)\cdot\left(\partial p_{i,j,t}^{SURF}/\partial EF_{i,t-1}^{D}\right) &> \partial p_{i,j,t}^{GURF}/\partial EF_{i,t-1}^{D} > ME_{i,j,t} \\ \textit{and } \left(MC_{i,j,t}^{DFV*}/\,p_{i,j,t}^{SURF}\right)\cdot\left(\partial p_{i,j,t}^{SURF}/\partial q_{i,j,t}^{p*}\right) &> \partial p_{i,j,t}^{GURF}/\partial q_{i,j,t}^{p*} > MQ_{i,j,t}, \end{aligned}$$

or

$$\begin{aligned} \partial p_{i,j,t}^{GURF}/\partial EF_{i,t-1}^{D} &< \min\left(ME_{i,j,t}, \left(MC_{i,j,t}^{DFV*}/\,p_{i,j,t}^{SURF}\right)\cdot\left(\partial p_{i,j,t}^{SURF}/\partial EF_{i,t-1}^{D}\right)\right) \\ \textit{and } \partial p_{i,j,t}^{GURF}/\partial q_{i,j,t}^{p*} &< \min\left(MQ_{i,j,t}, \left(MC_{i,j,t}^{DFV*}/\,p_{i,j,t}^{SURF}\right)\cdot\left(\partial p_{i,j,t}^{SURF}/\partial q_{i,j,t}^{p*}\right)\right), \end{aligned}$$

where $ME_{i,j,t}$ *and* $MQ_{i,j,t}$ *are respectively expressed as Eqs. (5.1) and (5.2).*

Proof. The proof of this proposition is similar to the proof of Proposition 11, so we omit the derivation. ■

Consider the following. First, the effect of improved dynamic cost efficiency in the previous period on the GURP of the j-th planned optimal financial good on the cost frontier (i.e., the dynamic frontier marginal variable cost with respect to the j-th planned optimal financial good) in the current period (i.e., $\partial p_{i,j,t}^{GURF}/\partial EF_{i,t-1}^{D}$, hereafter EA) as a criterion for judging the two magnitudes of the subtraction of the effect of the same improvement in dynamic cost efficiency on the elasticity of dynamic actual variable cost in the current period with respect to dynamic cost efficiency in the current period, which is corrected by the ratio of dynamic actual marginal variable cost with respect to the j-th planned optimal financial good in the current period to the square of this elasticity, from the effect of the same improvement in dynamic cost efficiency on the same dynamic actual marginal variable cost, which is corrected by the dynamic marginal cost efficiency in the current period (i.e., $ME_{i,j,t}$, hereafter EB), and the effect of the same improvement in dynamic cost efficiency on the SURP of the j-th planned optimal financial good on the cost frontier in the current period, which is discounted by the ratio of dynamic frontier marginal variable cost with respect to the j-th planned optimal financial good in the current period to the same SURP on the cost frontier (i.e., $\left(MC_{i,j,t}^{DFV*}/\,p_{i,j,t}^{SURF}\right)\cdot\left(\partial p_{i,j,t}^{SURF}\,/\partial EF_{i,t-1}^{D}\right)$, hereafter EC). Second, consider the foregoing in terms of the effect of an increase in the j-th planned optimal financial good in the current period on the same GURP on the cost frontier (i.e., $\partial p_{i,j,t}^{GURF}\,/\partial q_{i,j,t}^{p*}$, hereafter QA) as a criterion for judging the two magnitudes of the subtraction of the corrected effect of the same increase in the j-th planned optimal financial good on the same elasticity of dynamic actual variable cost from the corrected effect of the same increase in the j-th planned optimal financial good on the same dynamic actual marginal variable cost (i.e., $MQ_{i,j,t}$, hereafter QB), and the discounted effect of the same increase in the j-th planned optimal financial good on the same SURP on the cost frontier (i.e., $\left(MC_{i,j,t}^{DFV*}/\,p_{i,j,t}^{SURF}\right)\cdot\left(\partial p_{i,j,t}^{SURF}\,/\partial q_{i,j,t}^{p*}\right)$, hereafter QC). Then from assumption (A2) in Proposition 11, the EB and EC are small from the perspective of the EA, and the QB and QC are also small from the perspective of the QA, or the EB is large whilst the EC is small from the perspective of the EA, and the QB is large whilst the QC is small from the perspective of the QA. Similarly, from assumption (A4) in Proposition 12, the EB is small and the EC is large from the perspective of the EA, and the QB is small and the QC is large from the perspective of the QA, or the EB and EC are large from the perspective of the EA, and the QB and QC are also large from the perspective of the QA.

Similar to Propositions 11 and 12, from Definition 14 and Proposition

7, regarding the relation between the quiet-life hypothesis and the EGLI on the cost frontier, the following two propositions can be derived to theoretically clarify under what assumptions the quiet-life hypothesis increases or decreases the EGLI on the cost frontier.

Proposition 13 *The EGLI on the cost frontier decreases with the Herfindahl index in the previous period (i.e., the degree of competition on the cost frontier increases with it,* $\partial EGLI^{F}_{i,j,t}/\partial HI_{j,t-1} < 0$*) if and only if the quiet-life hypothesis is accepted (i.e.,* $\partial EF^{D}_{i,t}/\partial HI_{j,t-1} < 0$*). Thus, the EGLI on the cost frontier increases with dynamic cost efficiency in the "current" period (i.e.,* $\partial EGLI^{F}_{i,j,t}/\partial EF^{D}_{i,t} > 0$*) under the following assumptions: (A5) The* j*-th financial good is an output (i.e.,* $p^{SURF}_{i,j,t} > 0$ *and* $MC^{DFV*}_{i,j,t} > 0$*) and the sign of* $MC^{DAV*}_{i,j,t}$ *is the same as the sign of* $MC^{DFV*}_{i,j,t}$ *(i.e.,* $MC^{DAV*}_{i,j,t} > 0$*); and (A6) The following inequality holds:*

$$\frac{MC^{DFV*}_{i,j,t}}{p^{SURF}_{i,j,t}} \cdot \frac{\partial p^{SURF}_{i,j,t}}{\partial HI_{j,t-1}} < \frac{\partial p^{GURF}_{i,j,t}}{\partial HI_{j,t-1}} < MH_{i,j,t},$$

where $MH_{i,j,t}$ *is expressed as*

$$\begin{aligned} MH_{i,j,t} &= \left\{ EF^{D}_{i,t} + \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}} \right)^{-1} \right\} \cdot \frac{\partial MC^{DAV*}_{i,j,t}}{\partial HI_{j,t-1}} \\ &\quad - MC^{DAV*}_{i,j,t} \cdot \frac{\partial^2 \ln C^{DAV}_{i,t}}{\partial HI_{j,t-1} \partial EF^{D}_{i,t}} \Bigg/ \left(\frac{\partial \ln C^{DAV}_{i,t}}{\partial EF^{D}_{i,t}} \right)^2 . \end{aligned} \quad (5.3)$$

Proof. From Eq. (P11.1), under assumption (A5), and replacing $EF^{D}_{i,t-1}$ or $q^{p*}_{i,j,t}$ with $HI_{j,t-1}$, the following relation is revealed:

$$\frac{\partial EGLI^{F}_{i,j,t}}{\partial HI_{j,t-1}} > (<) 0 \Longleftrightarrow \frac{MC^{DFV*}_{i,j,t}}{p^{SURF}_{i,j,t}} \cdot \frac{\partial p^{SURF}_{i,j,t}}{\partial HI_{j,t-1}} > (<) \frac{\partial MC^{DFV*}_{i,j,t}}{\partial HI_{j,t-1}} \left(= \frac{\partial p^{GURF}_{i,j,t}}{\partial HI_{j,t-1}} \right). \quad \text{(P13.1)}$$

In addition, from Proposition 7 (Eq. (3.2.1)), under assumption (A5), the following relation holds:

$$\frac{\partial EF_{i,t}^{D}}{\partial HI_{j,t-1}} < 0 \Longleftrightarrow \frac{\partial p_{i,j,t}^{GURF}}{\partial HI_{j,t-1}} < MH_{i,j,t}, \tag{P13.2}$$

where $MH_{i,j,t}$ is expressed as Eq. (5.3). Then from relations (P13.1) and (P13.2) under assumptions (A5) and (A6), the following relation is established:

$$\frac{\partial EF_{i,t}^{D}}{\partial HI_{j,t-1}} < 0 \Longleftrightarrow \frac{\partial EGLI_{i,j,t}^{F}}{\partial HI_{j,t-1}} < 0.$$

Consequently, from this relation, the following inequality holds:

$$\frac{\partial EGLI_{i,j,t}^{F}}{\partial EF_{i,t}^{D}} = \frac{\partial EGLI_{i,j,t}^{F}}{\partial HI_{j,t-1}} \cdot \left(\frac{\partial EF_{i,t}^{D}}{\partial HI_{j,t-1}} \right)^{-1} > 0.$$

■

Proposition 14 *The EGLI on the cost frontier increases with the Herfindahl index in the previous period (i.e., the degree of competition on the cost frontier decreases, $\partial EGLI_{i,j,t}^{F} / \partial HI_{j,t-1} > 0$) if and only if the quiet-life hypothesis is accepted (i.e., $\partial EF_{i,t}^{D} / \partial HI_{j,t-1} < 0$). The EGLI on the cost frontier decreases with dynamic cost efficiency in the "current" period (i.e., $\partial EGLI_{i,j,t}^{F} / \partial EF_{i,t}^{D} < 0$) under the following assumptions: (A7) Assumption (A5) holds; and (A8) The following inequality holds:*

$$\frac{\partial p_{i,j,t}^{GURF}}{\partial HI_{j,t-1}} < \min \left(MH_{i,j,t}, \frac{MC_{i,j,t}^{DFV*}}{p_{i,j,t}^{SURF}} \cdot \frac{\partial p_{i,j,t}^{SURF}}{\partial HI_{j,t-1}} \right),$$

where $MH_{i,j,t}$ is expressed as Eq. (5.3).

Proof. The proof of this proposition is similar to the proof of Proposition 13, so we omit the derivation. ■

Similar to Propositions 11 and 12, considering the effect of an increase in the Herfindahl index in the previous period on the GURP of the j-th planned optimal financial good on the cost frontier in the current period (i.e., $\partial p_{i,j,t}^{GURF} / \partial HI_{j,t-1}$, hereafter HA) as a criterion for judging the two magnitudes of the subtraction of the effect of the same increase in the Herfindahl index on the elasticity of dynamic actual variable cost in the current period with respect to dynamic cost efficiency in the current period, which is corrected by the ratio of dynamic actual marginal variable cost with respect to the j-th planned optimal financial good in the current period to the square of this elasticity, from the effect of the same increase in the Herfindahl index on the same dynamic actual marginal variable cost, which is corrected by dynamic marginal cost efficiency in the current period (i.e., $MH_{i,j,t}$, hereafter HB), and the effect of the same increase in the Herfindahl index on the SURP of the j-th planned optimal financial good on the cost frontier in the current period, which is discounted by the ratio of dynamic frontier marginal variable cost with respect to the j-th planned optimal financial good in the current period to the same SURP on the cost frontier (i.e., $\left(MC_{i,j,t}^{DFV*} / p_{i,j,t}^{SURF}\right) \cdot \left(\partial p_{i,j,t}^{SURF} / \partial HI_{j,t-1}\right)$, hereafter HC), assumption (A6) in Proposition 13 means that the HB is large and the HC is small from the perspective of the HA. Similarly, assumption (A8) in Proposition 14 means that the HB and HC are large from the perspective of the HA.

From the perspective of the EGLI on the cost frontier, in the case that the EC is small from the perspective of the EA, and the QC is also small from the perspective of the QA, then support for the efficient structure hypothesis is desirable, whereas if the EC is large from the perspective of the EA, and the QC is also large from the perspective of the QA, then support for this hypothesis is undesirable. In the former case, the ratio of the discrepancy between the SURP on the cost frontier and the dynamic frontier marginal variable cost to the same SURP decreases, so the EGLI on the cost frontier decreases (i.e., the degree of competition on the cost frontier increases), whereas, in the latter case, the ratio increases, so the EGLI on the cost frontier increases (i.e., the degree of competition on the cost frontier decreases). Regarding the quiet-life hypothesis, where HC is small from the perspective of HA, support for this hypothesis is desirable. In this case, the EGLI on the cost frontier decreases with the Herfindahl index (i.e., the degree of competition on the cost frontier increases with it), so anti-monopoly and anti-concentration policies are unnecessary, even if dynamic cost efficiency decreases with the Herfindahl index. Although it is for empirical studies to explore whether and when this case actually exists, at least theoretically, support for the quiet-life hypothesis need not become a justification for anti-monopoly and anti-concentration policies. Justification for such policies is restricted to where the EGLI on the cost frontier increases with the Herfindahl index (i.e., the degree of competition on the cost frontier decreases with it), so enactment and enforcement of such policies require careful consideration. Similarly, regarding the efficient structure hypotheses, where the EC is large from the perspective of the EA, and the QC is also large from the perspective of the QA, support for this hypothesis is undesirable, so policy interventions which decrease the EC and QC are necessary. Put differently, policies which do not substantially increase the SURP on the cost frontier, or which substantially decrease the discrepancy between the SURP on the cost frontier and the dynamic frontier marginal variable cost are required. In any case, new industrial organization policies which differ from existing anti-monopoly and anti-concentration policies, and under which an efficiency improvement would increase the degree of competition on the cost frontier, are required. This novel implication for existing industrial organization policies is revealed by providing the theoretical foundation for suggesting that support for the efficient structure hypothesis is undesirable.

6 Intertemporal Regular Linkages

This section theoretically clarifies the relations between the efficient structure and quiet-life hypotheses and the intertemporal regular linkages (i.e., cyclical linkages, monotonic trending linkages, and terminal up-and-down volatile linkages) of single-period dynamic cost efficiencies, single-period optimal planned financial goods, single-period Herfindahl indices, and single-period EGLIs on the cost frontier.

6.1 Intertemporal Regular Linkages of Single-Period Dynamic Cost Efficiencies

The intertemporal regular linkage (i.e., cyclical linkage, monotonic trending linkage, or terminal up-and-down volatile linkage) of single-period dynamic cost efficiencies is principally defined as the following relations between dynamic cost efficiencies in period $t-1$ and period $t-1+2T$ (i.e., $EF^D_{i,t-1}$ and $EF^D_{i,t-1+2T}$), where T is a natural number.

Definition 20 (Intertemporal Regular Linkage of Dynamic Cost Efficiencies) *The intertemporal regular linkage of single-period dynamic cost efficiencies exists if any one of the following linkages (i.e., cyclical linkage, monotonic trending linkage, and terminal up-and-down volatile linkage) exists mainly between the dynamic cost efficiencies in period $t-1$ and period $t-1+2T$ (i.e., $EF^D_{i,t-1}$ and $EF^D_{i,t-1+2T}$), where T is a natural number. (E1) (Cyclical Linkage) Dynamic cost efficiency in period $t-2+2T$ (i.e., $EF^D_{i,t-2+2T}$) is dependent on dynamic cost efficiency in period $t-3+2T$ (i.e., $EF^D_{i,t-3+2T}$), so $\partial EF^D_{i,t-2+2T}/\partial EF^D_{i,t-3+2T}$ is positive, negative, or zero. Dynamic cost efficiency in period $t-1+2T$ (i.e., $EF^D_{i,t-1+2T}$) is, moreover, dependent on dynamic cost efficiency in period $t-1$ (i.e., $EF^D_{i,t-1}$), so the sign of $\partial EF^D_{i,t-1+2T}/\partial EF^D_{i,t-1}$ is positive if T is an even number or negative if T is an odd number; (E2) (Monotonic Trending Linkage) $EF^D_{i,t-2+2T}$ is dependent on $EF^D_{i,t-3+2T}$, so $\partial EF^D_{i,t-2+2T}/\partial EF^D_{i,t-3+2T}$ is nonnegative (i.e., $\partial EF^D_{i,t-2+2T}/\partial EF^D_{i,t-3+2T} \geq 0$). $EF^D_{i,t-1+2T}$ is, moreover, dependent on $EF^D_{i,t-1}$, so the sign of $\partial EF^D_{i,t-1+2T}/\partial EF^D_{i,t-1}$ is positive (i.e., $\partial EF^D_{i,t-1+2T}/\partial EF^D_{i,t-1} > 0$); and (E3) (Terminal Up-and-Down Volatile Linkage) $EF^D_{i,t-2+2T}$ is dependent on $EF^D_{i,t-3+2T}$, so the sign of $\partial EF^D_{i,t-2+2T}/\partial EF^D_{i,t-3+2T}$ is negative (i.e., $\partial EF^D_{i,t-2+2T}/\partial EF^D_{i,t-3+2T} < 0$). $EF^D_{i,t-1+2T}$ is, moreover, dependent on $EF^D_{i,t-1}$, so the sign of $\partial EF^D_{i,t-1+2T}/\partial EF^D_{i,t-1}$ is positive (i.e., $\partial EF^D_{i,t-1+2T}/\partial EF^D_{i,t-1} \geq 0$).*

The relations between this linkage and the efficient structure and quiet-life hypotheses are derived from the following proposition.

Proposition 15 $\partial EF^{D}_{i,t-1+2T} / \partial EF^{D}_{i,t-1}$, *where* T *is a natural number, is expressed as follows:*

$$\frac{\partial EF^{D}_{i,t-1+2T}}{\partial EF^{D}_{i,t-1}} = \prod_{k=1}^{T} \left[\frac{\partial EF^{D}_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} \cdot \frac{dHI_{j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}} \cdot \frac{\partial q^{p*}_{i,j,t-2+2k}}{\partial EF^{D}_{i,t-3+2k}} \right]. \tag{6.1.1}$$

Proof. $\partial EF^{D}_{i,t+1} / \partial EF^{D}_{i,t-1}$ is expressed as follows:

$$\frac{\partial EF^{D}_{i,t+1}}{\partial EF^{D}_{i,t-1}} = \frac{\partial EF^{D}_{i,t+1}}{\partial HI_{j,t}} \cdot \frac{dHI_{j,t}}{dq^{p*}_{i,j,t}} \cdot \frac{\partial q^{p*}_{i,j,t}}{\partial EF^{D}_{i,t-1}}. \tag{P15.1}$$

Similarly, $\partial EF^{D}_{i,t+3} / \partial EF^{D}_{i,t-1}$ is expressed as follows:

$$\frac{\partial EF^{D}_{i,t+3}}{\partial EF^{D}_{i,t-1}} = \left[\frac{\partial EF^{D}_{i,t+3}}{\partial HI_{j,t+2}} \cdot \frac{dHI_{j,t+2}}{dq^{p*}_{i,j,t+2}} \cdot \frac{\partial q^{p*}_{i,j,t+2}}{\partial EF^{D}_{i,t+1}} \right] \cdot \left[\frac{\partial EF^{D}_{i,t+1}}{\partial HI_{j,t}} \cdot \frac{dHI_{j,t}}{dq^{p*}_{i,j,t}} \cdot \frac{\partial q^{p*}_{i,j,t}}{\partial EF^{D}_{i,t-1}} \right]. \tag{P15.2}$$

Consequently, from Eqs. (P15.1) and (P15.2), $\partial EF^{D}_{i,t-1+2T} / \partial EF^{D}_{i,t-1}$, where T is a natural number, is expressed as Eq. (6.1.1). ■

From Proposition 15 (Eq. (6.1.1)), the relations between the efficient structure and quiet-life hypotheses and the intertemporal regular linkages (i.e., cyclical linkage, monotonic trending linkage, and terminal up-and-down volatile linkage) of single-period dynamic cost efficiencies are shown as the following three propositions.

Proposition 16 *The cyclical linkage of single-period dynamic cost efficiencies occurs if one of two triplets of assumptions holds: (A0), (A1), and (A2); or (A0), (B1), and (B2).*

(A0) Dynamic cost efficiency in period $t-2+2T$ *(i.e.,* $EF^{D}_{i,t-2+2T}$*), where* T *is a natural number, is dependent on dynamic cost efficiency in period* $t-3+2T$ *(i.e.,* $EF^{D}_{i,t-3+2T}$*), so* $\partial EF^{D}_{i,t-2+2T} / \partial EF^{D}_{i,t-3+2T}$ *is positive, negative, or zero;*

(A1) The j*-th optimal planned financial goods in periods* $t-2+2k$ *(i.e.,* $q^{p*}_{i,j,t-2+2k}$*), where* $k = 1, \ldots, T$*, are large; that is, the following inequalities*

hold:

$$q^{p*}_{i,j,t-2+2k} > \frac{\sum_i \left(q^{p*}_{i,j,t-2+2k}\right)^2}{\sum_k q^{p*}_{k,j,t-2+2k}} \cdot \left(1 + \sum_{k \neq i} \frac{dq^{p*}_{k,j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}}\right) - \sum_{h \neq i} \left(q^{p*}_{h,j,t-2+2k} \cdot \frac{dq^{p*}_{h,j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}}\right), \textit{ (}k = 1, \ldots, T\textit{);} \quad (6.1.2)$$

(A2) Both the efficient structure hypothesis from period $t-3+2k$ to period $t-2+2k$ and the quiet-life hypothesis from period $t-2+2k$ to period $t-1+2k$ are supported or unsupported; that is, one of the two pairs of the following inequalities holds:

$$\frac{\partial q^{p*}_{i,j,t-2+2k}}{\partial EF^D_{i,t-3+2k}} > 0 \textit{ and } \frac{\partial EF^D_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} < 0, \textit{ or}$$
$$\frac{\partial q^{p*}_{i,j,t-2+2k}}{\partial EF^D_{i,t-3+2k}} < 0 \textit{ and } \frac{\partial EF^D_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} > 0, \textit{ (}k = 1, \ldots, T\textit{);} \quad (6.1.3)$$

(B1) $q^{p*}_{i,j,t-2+2k}$ $(k = 1, \ldots, T)$ are small; that is, the following inequalities hold:

$$q^{p*}_{i,j,t-2+2k} < \frac{\sum_i \left(q^{p*}_{i,j,t-2+2k}\right)^2}{\sum_k q^{p*}_{k,j,t-2+2k}} \cdot \left(1 + \sum_{k \neq i} \frac{dq^{p*}_{k,j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}}\right) - \sum_{h \neq i} \left(q^{p*}_{h,j,t-2+2k} \cdot \frac{dq^{p*}_{h,j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}}\right), \textit{ (}k = 1, \ldots, T\textit{);} \quad (6.1.4)$$

and (B2) Any one of the efficient structure hypothesis from period $t-3+2k$ to period $t-2+2k$ and the quiet-life hypothesis from period $t-2+2k$ to period $t-1+2k$ is supported or unsupported; that is, one of the two pairs of the following inequalities holds:

$$\frac{\partial q^{p*}_{i,j,t-2+2k}}{\partial EF^D_{i,t-3+2k}} > 0 \textit{ and } \frac{\partial EF^D_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} > 0, \textit{ or}$$
$$\frac{\partial q^{p*}_{i,j,t-2+2k}}{\partial EF^D_{i,t-3+2k}} < 0 \textit{ and } \frac{\partial EF^D_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} < 0, \textit{ (}k = 1, \ldots, T\textit{).} \quad (6.1.5)$$

Proof. Assumption (A0) is the same as the first part of the definition of the cyclical linkage of single-period dynamic cost efficiencies (i.e., (E1) in Definition 20). The remainder of this definition is met as follows. From the definition of the Herfindahl index, the following equations hold:

$$\frac{dHI_{j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}} = 2\cdot\left(\sum_k q^{p*}_{k,j,t-2+2k}\right)^{-3}\cdot\left[\left\{q^{p*}_{i,j,t-2+2k} + \sum_{h\neq i}\left(q^{p*}_{h,j,t-2+2k}\cdot\frac{dq^{p*}_{h,j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}}\right)\right\}\right.$$
$$\left.\cdot\left(\sum_k q^{p*}_{k,j,t-2+2k}\right) - \left\{\sum_i\left(q^{p*}_{i,j,t-2+2k}\right)^2\right\}\cdot\left(1+\sum_{k\neq i}\frac{dq^{p*}_{k,j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}}\right)\right],$$
$$(k = 1,\ldots,T). \quad \text{(P16.1)}$$

From these equations, the following relations are revealed:

$$\frac{dHI_{j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}} > (=,\ <)0$$
$$\Longleftrightarrow q^{p*}_{i,j,t-2+2k} > (=,\ <)\frac{\sum_i\left(q^{p*}_{i,j,t-2+2k}\right)^2}{\sum_k q^{p*}_{k,j,t-2+2k}}\cdot\left(1+\sum_{k\neq i}\frac{dq^{p*}_{k,j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}}\right)$$
$$-\sum_{h\neq i}\left(q^{p*}_{h,j,t-2+2k}\cdot\frac{dq^{p*}_{h,j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}}\right),\ (k = 1,\ldots,T). \quad \text{(P16.2)}$$

From these relations and assumptions (A1) and (B1), the signs of $dHI_{j,t-2+2k}$ $/dq^{p*}_{i,j,t-2+2k}$ $(k = 1,\ldots,T)$ in Eq. (P16.1) are positive and negative, respectively (i.e., $dHI_{j,t-2+2k}/dq^{p*}_{i,j,t-2+2k} > 0$ and $dHI_{j,t-2+2k}/dq^{p*}_{i,j,t-2+2k} < 0$, respectively, for $k = 1,\ldots,T$). In addition, from assumptions (A2) and (B2), the following inequalities for Eq. (6.1.1) hold:

$$\frac{\partial EF^D_{i,t-1+2k}}{\partial HI_{j,t-2+2k}}\cdot\frac{dHI_{j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}}\cdot\frac{\partial q^{p*}_{i,j,t-2+2k}}{\partial EF^D_{i,t-3+2k}} < 0,\ (k = 1,\ldots,T). \quad \text{(P16.3)}$$

From these inequalities and Eq. (6.1.1), the sign of $\partial EF^D_{i,t-1+2T}/\partial EF^D_{i,t-1}$ is positive if T is an even number or negative if T is an odd number. ■

Proposition 17 *The monotonic trending linkage of single-period dynamic cost efficiencies occurs if one of two triplets of the four assumptions of Propo-*

sition 16 and assumption (C0) holds: (C0), (A1), and (B2); or (C0), (B1), and (A2).

(C0) $EF^{D}_{i,t-2+2T}$ *is dependent on* $EF^{D}_{i,t-3+2T}$*, so* $\partial EF^{D}_{i,t-2+2T} / \partial EF^{D}_{i,t-3+2T}$ *is nonnegative (i.e.,* $\partial EF^{D}_{i,t-2+2T} / \partial EF^{D}_{i,t-3+2T} \geq 0$*).*

Proof. The proof of this proposition is similar to the proof of Proposition 16, so we omit the derivation. ■

Proposition 18 *The terminal up-and-down volatile linkage of single-period dynamic cost efficiencies occurs if one of two triplets of the four assumptions of Proposition 16 and assumption (D0) holds: (D0), (A1), and (B2); or (D0), (B1), and (A2).*

(D0) $EF^{D}_{i,t-2+2T}$ *is dependent on* $EF^{D}_{i,t-3+2T}$*, so the sign of* $\partial EF^{D}_{i,t-2+2T}$ $/\partial EF^{D}_{i,t-3+2T}$ *is negative (i.e.,* $\partial EF^{D}_{i,t-2+2T} / \partial EF^{D}_{i,t-3+2T} < 0$*).*

Proof. The proof of this proposition is similar to the proof of Proposition 16, so we omit the derivation. ■

6.2 Intertemporal Regular Linkages of Single-Period Optimal Planned Financial Goods

Similar to the intertemporal regular linkage (i.e., cyclical linkage, monotonic trending linkage, or terminal up-and-down volatile linkage) of single-period dynamic cost efficiencies, the same intertemporal regular linkage of single-period optimal planned financial goods is principally defined as the following relations between the j-th optimal planned financial goods in period t and period $t + 2T$ (i.e., $q^{p*}_{i,j,t}$ and $q^{p*}_{i,j,t+2T}$), where T is a natural number.

Definition 21 (Intertemporal Regular Linkage of Financial Goods) *The intertemporal regular linkage of single-period optimal planned financial goods exists if any one of the following linkages (i.e., cyclical linkage, monotonic trending linkage, and terminal up-and-down volatile linkage) exists mainly between the j-th optimal planned financial goods in period t and period* $t + 2T$ *(i.e.,* $q^{p*}_{i,j,t}$ *and* $q^{p*}_{i,j,t+2T}$*), where T is a natural number: (F1)*

(Cyclical Linkage) The j-th optimal planned financial good in period $t-1+2T$ (i.e., $q^{p*}_{i,j,t-1+2T}$) is dependent on the j-th optimal planned financial good in period $t-2+2T$ (i.e., $q^{p*}_{i,j,t-2+2T}$), so $\partial q^{p*}_{i,j,t-1+2T} / \partial q^{p*}_{i,j,t-2+2T}$ is positive, negative, or zero. The j-th optimal planned financial good in period $t+2T$ (i.e., $q^{p*}_{i,j,t+2T}$) is, moreover, dependent on the j-th optimal planned financial good in period t (i.e., $q^{p*}_{i,j,t}$), so the sign of $\partial q^{p*}_{i,j,t+2T} / \partial q^{p*}_{i,j,t}$ is positive if T is an even number, whereas this sign is negative if T is an odd number; (F2) (Monotonic Trending Linkage) $q^{p*}_{i,j,t-1+2T}$ is dependent on $q^{p*}_{i,j,t-2+2T}$, so $\partial q^{p*}_{i,j,t-1+2T} / \partial q^{p*}_{i,j,t-2+2T}$ is nonnegative (i.e., $\partial q^{p*}_{i,j,t-1+2T} / \partial q^{p*}_{i,j,t-2+2T} \geq 0$). $q^{p*}_{i,j,t+2T}$ is, moreover, dependent on $q^{p*}_{i,j,t}$, so the sign of $\partial q^{p*}_{i,j,t+2T} / \partial q^{p*}_{i,j,t}$ is positive (i.e., $\partial q^{p*}_{i,j,t+2T} / \partial q^{p*}_{i,j,t} > 0$); and (F3) (Terminal Up-and-Down Volatile Linkage) $q^{p*}_{i,j,t-1+2T}$ is dependent on $q^{p*}_{i,j,t-2+2T}$, so the sign of $\partial q^{p*}_{i,j,t-1+2T} / \partial q^{p*}_{i,j,t-2+2T}$ is negative (i.e., $\partial q^{p*}_{i,j,t-1+2T} / \partial q^{p*}_{i,j,t-2+2T} < 0$). $q^{p*}_{i,j,t+2T}$ is, moreover, dependent on $q^{p*}_{i,j,t}$, so the sign of $\partial q^{p*}_{i,j,t+2T} / \partial q^{p*}_{i,j,t}$ is positive (i.e., $\partial q^{p*}_{i,j,t+2T} / \partial q^{p*}_{i,j,t} > 0$).

Similar to Proposition 15, the relations between this linkage and the efficient structure and quiet-life hypotheses are derived from the following proposition.

Proposition 19 *$\partial q^{p*}_{i,j,t+2T} / \partial q^{p*}_{i,j,t}$, where T is a natural number, is expressed as follows:*

$$\frac{\partial q^{p*}_{i,j,t+2T}}{\partial q^{p*}_{i,j,t}} = \prod_{k=1}^{T} \left[\frac{\partial q^{p*}_{i,j,t+2k}}{\partial EF^{D}_{i,t-1+2k}} \cdot \frac{\partial EF^{D}_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} \cdot \frac{dHI_{j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}} \right]. \tag{6.2.1}$$

Proof. $\partial q^{p*}_{i,j,t+2} / \partial q^{p*}_{i,j,t}$ is expressed as follows:

$$\frac{\partial q^{p*}_{i,j,t+2}}{\partial q^{p*}_{i,j,t}} = \frac{\partial q^{p*}_{i,j,t+2}}{\partial EF^{D}_{i,t+1}} \cdot \frac{\partial EF^{D}_{i,t+1}}{\partial HI_{j,t}} \cdot \frac{dHI_{j,t}}{dq^{p*}_{i,j,t}}. \tag{P19.1}$$

Similarly, $\partial q^{p*}_{i,j,t+4} / \partial q^{p*}_{i,j,t}$ is expressed as follows:

$$\frac{\partial q^{p*}_{i,j,t+4}}{\partial q^{p*}_{i,j,t}} = \left[\frac{\partial q^{p*}_{i,j,t+4}}{\partial EF^{D}_{i,t+3}} \cdot \frac{\partial EF^{D}_{i,t+3}}{\partial HI_{j,t+2}} \cdot \frac{dHI_{j,t+2}}{dq^{p*}_{i,j,t+2}} \right] \cdot \left[\frac{\partial q^{p*}_{i,j,t+2}}{\partial EF^{D}_{i,t+1}} \cdot \frac{\partial EF^{D}_{i,t+1}}{\partial HI_{j,t}} \cdot \frac{dHI_{j,t}}{dq^{p*}_{i,j,t}} \right]. \tag{P19.2}$$

Consequently, from Eqs. (P19.1) and (P19.2), $\partial q^{p*}_{i,j,t+2T} / \partial q^{p*}_{i,j,t}$, where T is a natural number, is expressed as Eq. (6.2.1). ■

From Proposition 19 (Eq. (6.2.1)), the relations between the efficient structure and quiet-life hypotheses and the intertemporal regular linkages (i.e., cyclical linkage, monotonic trending linkage, and terminal up-and-down volatile linkage) of single-period optimal planned financial goods are distilled as the following three propositions.

Proposition 20 *The cyclical linkage of single-period optimal planned financial goods occurs if one of two triplets of the two assumptions of Proposition 16 and assumptions (E0), (C2), and (D2) holds: (E0), (A1), and (C2); or (E0), (B1), and (D2).*

(E0) The j-th optimal planned financial good in period $t-1+2T$ (i.e., $q^{p*}_{i,j,t-1+2T}$) is dependent on the j-th optimal planned financial good in period $t-2+2T$ (i.e., $q^{p*}_{i,j,t-2+2T}$), so $\partial q^{p*}_{i,j,t-1+2T} / \partial q^{p*}_{i,j,t-2+2T}$ is positive, negative, or zero;

(C2) Both the quiet-life hypothesis from period $t-2+2k$ to period $t-1+2k$ and the efficient structure hypothesis from period $t-1+2k$ to period $t+2k$ are supported or unsupported; that is, one of the following two pairs of inequalities holds:

$$\begin{aligned}
&\frac{\partial EF^{D}_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} < 0 \text{ and } \frac{\partial q^{p*}_{i,j,t+2k}}{\partial EF^{D}_{i,t-1+2k}} > 0, \text{ or} \\
&\frac{\partial EF^{D}_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} > 0 \text{ and } \frac{\partial q^{p*}_{i,j,t+2k}}{\partial EF^{D}_{i,t-1+2k}} < 0, \ (k = 1, \ldots, T);
\end{aligned} \tag{6.2.2}$$

and (D2) Any one of the quiet-life hypothesis from period $t-2+2k$ to period $t-1+2k$ and the efficient structure hypothesis from period $t-1+2k$ to period $t+2k$ is supported or unsupported; that is, one of the following two pairs of inequalities holds:

$$\begin{aligned}
&\frac{\partial EF^{D}_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} < 0 \text{ and } \frac{\partial q^{p*}_{i,j,t+2k}}{\partial EF^{D}_{i,t-1+2k}} < 0, \text{ or} \\
&\frac{\partial EF^{D}_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} > 0 \text{ and } \frac{\partial q^{p*}_{i,j,t+2k}}{\partial EF^{D}_{i,t-1+2k}} > 0, \ (k = 1, \ldots, T).
\end{aligned} \tag{6.2.3}$$

Proof. Assumption (E0) is the same as the first part of the definition of the cyclical linkage of single-period optimal planned financial goods (i.e., (F1) in Definition 21). The remainder of this definition is met as follows. From the proof of Proposition 16, assumptions (A1) and (B1) dictate that the signs of $dHI_{j,t-2+2k} \left/ dq^{p*}_{i,j,t-2+2k} \right.$ $(k = 1, \ldots, T)$ in Eq. (P16.1) are positive and negative, respectively (i.e., $dHI_{j,t-2+2k} \left/ dq^{p*}_{i,j,t-2+2k} \right. > 0$ and $dHI_{j,t-2+2k} \left/ dq^{p*}_{i,j,t-2+2k} \right. < 0$, respectively, for $k = 1, \ldots, T$). In addition, from assumptions (C2) and (D2), the following inequalities for Eq. (6.2.1) hold:

$$\frac{\partial q^{p*}_{i,j,t+2k}}{\partial EF^{D}_{i,t-1+2k}} \cdot \frac{\partial EF^{D}_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} \cdot \frac{dHI_{j,t-2+2k}}{dq^{p*}_{i,j,t-2+2k}} < 0, \ (k = 1, \ldots, T). \qquad \text{(P20.1)}$$

From these inequalities and Eq. (6.2.1), the sign of $\partial q^{p*}_{i,j,t+2T} \left/ \partial q^{p*}_{i,j,t} \right.$ is positive if T is an even number, and negative if T is an odd number. ■

Proposition 21 *The monotonic trending linkage of single-period optimal planned financial goods occurs if one of two triplets of the four assumptions of Propositions 16 and 20 and assumption (F0) holds: (F0), (A1), and (D2); or (F0), (B1), and (C2).*

(F0) $q^{p*}_{i,j,t-1+2T}$ *is dependent on* $q^{p*}_{i,j,t-2+2T}$*, so* $\partial q^{p*}_{i,j,t-1+2T} \left/ \partial q^{p*}_{i,j,t-2+2T} \right.$ *is nonnegative (i.e.,* $\partial q^{p*}_{i,j,t-1+2T} \left/ \partial q^{p*}_{i,j,t-2+2T} \right. \geq 0$*).*

Proof. The proof of this proposition is similar to the proof of Proposition 20, so we omit the derivation. ■

Proposition 22 *The terminal up-and-down volatile linkage of single-period optimal planned financial goods occurs if one of two triplets of the four assumptions of Propositions 16 and 20 and assumption G0 holds: (G0), (A1), and (D2); or (G0), (B1), and (C2).*

(G0) $q^{p*}_{i,j,t-1+2T}$ *is dependent on* $q^{p*}_{i,j,t-2+2T}$*, so the sign of* $\partial q^{p*}_{i,j,t-1+2T} \left/ \partial q^{p*}_{i,j,t-2+2T} \right.$ *is negative (i.e.,* $\partial q^{p*}_{i,j,t-1+2T} \left/ \partial q^{p*}_{i,j,t-2+2T} \right. < 0$*).*

Proof. The proof of this proposition is similar to the proof of Proposition 20, so we omit the derivation. ■

6.3 Intertemporal Regular Linkages of Single-Period Herfindahl Indices

Similar to the intertemporal regular linkages (i.e., cyclical linkages, monotonic trending linkages, and terminal up-and-down volatile linkages) of single-period dynamic cost efficiencies and single-period optimal planned financial goods, the same intertemporal regular linkage of single-period Herfindahl indices is mainly defined as the following relations between the Herfindahl indices in period t and period $t+2T$ (i.e., $HI_{j,t}$ and $HI_{j,t+2T}$), where T is a natural number.

Definition 22 (Intertemporal Regular Linkage of Herfindahl Indices) *The intertemporal regular linkage of single-period Herfindahl indices exists if any one of the following linkages (i.e., cyclical linkage, monotonic trending linkage, and terminal up-and-down volatile linkage) mainly exists between the Herfindahl indices in period t and period $t+2T$ (i.e., $HI_{j,t}$ and $HI_{j,t+2T}$), where T is a natural number. (H1) (Cyclical Linkage) The Herfindahl index in period $t-1+2T$ (i.e., $HI_{j,t-1+2T}$) is dependent on the Herfindahl index in period $t-2+2T$ (i.e., $HI_{j,t-2+2T}$), so $\partial HI_{j,t-1+2T} / \partial HI_{j,t-2+2T}$ is positive, negative, or zero. The Herfindahl index in period $t+2T$ (i.e., $HI_{j,t+2T}$) is, moreover, dependent on the Herfindahl index in period t (i.e., $HI_{j,t}$), so the sign of $\partial HI_{j,t+2T} / \partial HI_{j,t}$ is positive if T is an even number and negative if T is an odd number; (H2) (Monotonic Trending Linkage) $HI_{j,t-1+2T}$ is dependent on $HI_{j,t-2+2T}$, so $\partial HI_{j,t-1+2T} / \partial HI_{j,t-2+2T}$ is nonnegative (i.e., $\partial HI_{j,t-1+2T} / \partial HI_{j,t-2+2T} \geq 0$). $HI_{j,t+2T}$ is, moreover, dependent on $HI_{j,t}$, so the sign of $\partial HI_{j,t+2T} / \partial HI_{j,t}$ is positive (i.e., $\partial HI_{j,t+2T} / \partial HI_{j,t} > 0$); and (H3) (Terminal Up-and-Down Volatile Linkage) $HI_{j,t-1+2T}$ is dependent on $HI_{j,t-2+2T}$, so the sign of $\partial HI_{j,t-1+2T} / \partial HI_{j,t-2+2T}$ is negative (i.e., $\partial HI_{j,t-1+2T} / \partial HI_{j,t-2+2T} < 0$). $HI_{j,t+2T}$ is, moreover, dependent on $HI_{j,t}$, so the sign of $\partial HI_{j,t+2T} / \partial HI_{j,t}$ is positive (i.e., $\partial HI_{j,t+2T} / \partial HI_{j,t} > 0$).*

Similar to Propositions 15 and 19, the relations between this linkage and the efficient structure and quiet-life hypotheses are derived from the following proposition.

Proposition 23 $\partial HI_{j,t+2T}/\partial HI_{j,t}$, *where* T *is a natural number, is expressed as follows:*

$$\frac{\partial HI_{j,t+2T}}{\partial HI_{j,t}}=\prod_{k=1}^{T}\left[\frac{dHI_{j,t+2k}}{dq^{p*}_{i,j,t+2k}}\cdot\frac{\partial q^{p*}_{i,j,t+2k}}{\partial EF^{D}_{i,t-1+2k}}\cdot\frac{\partial EF^{D}_{i,t-1+2k}}{\partial HI_{j,t-2+2k}}\right]. \tag{6.3.1}$$

Proof. $\partial HI_{j,t+2}/\partial HI_{j,t}$ is expressed as follows:

$$\frac{\partial HI_{j,t+2}}{\partial HI_{j,t}}=\frac{dHI_{j,t+2}}{dq^{p*}_{i,j,t+2}}\cdot\frac{\partial q^{p*}_{i,j,t+2}}{\partial EF^{D}_{i,t+1}}\cdot\frac{\partial EF^{D}_{i,t+1}}{\partial HI_{j,t}}. \tag{P23.1}$$

Similarly, $\partial HI_{j,t+4}/\partial HI_{j,t}$ is expressed as follows:

$$\frac{\partial HI_{j,t+4}}{\partial HI_{j,t}}=\left[\frac{dHI_{j,t+4}}{dq^{p*}_{i,j,t+4}}\cdot\frac{\partial q^{p*}_{i,j,t+4}}{\partial EF^{D}_{i,t+3}}\cdot\frac{\partial EF^{D}_{i,t+3}}{\partial HI_{j,t+2}}\right]\cdot\left[\frac{dHI_{j,t+2}}{dq^{p*}_{i,j,t+2}}\cdot\frac{\partial q^{p*}_{i,j,t+2}}{\partial EF^{D}_{i,t+1}}\cdot\frac{\partial EF^{D}_{i,t+1}}{\partial HI_{j,t}}\right]. \tag{P23.2}$$

Consequently, from Eqs. (P23.1) and (P23.2), $\partial HI_{j,t+2T}/\partial HI_{j,t}$, where T is a natural number, is expressed as Eq. (6.3.1). ■

From Proposition 23 (Eq. (6.3.1)), the relations between the efficient structure and quiet-life hypotheses and the intertemporal regular linkages (i.e., cyclical linkage, monotonic trending linkage, and terminal up-and-down volatile linkage) of single-period Herfindahl indices are distilled as the following three propositions.

Proposition 24 *The cyclical linkage of single-period Herfindahl indices occurs if one of two triplets of the two assumptions of Proposition 20 and assumption (H0) holds: (H0), (E1), and (C2); or (H0), (F1), and (D2).*

(H0) The Herfindahl index in period $t-1+2T$ *(i.e.,* $HI_{j,t-1+2T}$*) is dependent on the Herfindahl index in period* $t-2+2T$ *(i.e.,* $HI_{j,t-2+2T}$*), so* $\partial HI_{j,t-1+2T}/\partial HI_{j,t-2+2T}$ *is positive, negative, or zero;*

(E1) The j*-th optimal planned financial goods in periods* $t+2k$*, where*

$k = 1, \ldots, T$, *(i.e.,* $q^{p*}_{i,j,t+2k}$*) are large, that is, the following inequalities hold:*

$$q^{p*}_{i,j,t+2k} > \frac{\sum_i \left(q^{p*}_{i,j,t+2k}\right)^2}{\sum_k q^{p*}_{k,j,t+2k}} \cdot \left(1 + \sum_{k \neq i} \frac{dq^{p*}_{k,j,t+2k}}{dq^{p*}_{i,j,t+2k}}\right) - \sum_{h \neq i} \left(q^{p*}_{h,j,t+2k} \cdot \frac{dq^{p*}_{h,j,t+2k}}{dq^{p*}_{i,j,t+2k}}\right), \ (k = 1, \ldots, T); \quad (6.3.2)$$

and (F1) $q^{p*}_{i,j,t+2k}$ *(*$k = 1, \ldots, T$*) are small, that is, the following inequalities hold:*

$$q^{p*}_{i,j,t+2k} < \frac{\sum_i \left(q^{p*}_{i,j,t+2k}\right)^2}{\sum_k q^{p*}_{k,j,t+2k}} \cdot \left(1 + \sum_{k \neq i} \frac{dq^{p*}_{k,j,t+2k}}{dq^{p*}_{i,j,t+2k}}\right) - \sum_{h \neq i} \left(q^{p*}_{h,j,t+2k} \cdot \frac{dq^{p*}_{h,j,t+2k}}{dq^{p*}_{i,j,t+2k}}\right), \ (k = 1, \ldots, T). \quad (6.3.3)$$

Proof. Assumption (H0) is the same as the first part of the definition of the cyclical linkage of single-period Herfindahl indices (i.e., (H1) in Definition 21). The remainder of this definition is met as follows. From relation (P16.2) of Proposition 16 and replacing periods $t - 2 + 2k$ ($k = 1, \ldots, T$) with $t + 2k$ ($k = 1, \ldots, T$), assumptions (E1) and (F1) mean that the signs of $dHI_{j,t+2k} / dq^{p*}_{i,j,t+2k}$ ($k = 1, \ldots, T$) in Eq. (P16.1) are positive and negative, respectively (i.e., $dHI_{j,t+2k} / dq^{p*}_{i,j,t+2k} > 0$ and $dHI_{j,t+2k} / dq^{p*}_{i,j,t+2k} < 0$, respectively, for $k = 1, \ldots, T$). In addition, from assumptions (C2) and (D2), the following inequalities for Eq. (6.3.1) hold:

$$\frac{dHI_{j,t+2k}}{dq^{p*}_{i,j,t+2k}} \cdot \frac{\partial q^{p*}_{i,j,t+2k}}{\partial EF^D_{i,t-1+2k}} \cdot \frac{\partial EF^D_{i,t-1+2k}}{\partial HI_{j,t-2+2k}} < 0, \ (k = 1, \ldots, T). \quad \text{(P24.1)}$$

From these inequalities and Eq. (6.3.1), the sign of $\partial HI_{j,t+2T} / \partial HI_{j,t}$ is positive if T is an even number and negative if T is an odd number. ■

Proposition 25 *The monotonic trending linkage of single-period Herfindahl indices occurs if one of two triplets of the four assumptions of Propositions 20 and 24 and assumption (I0) hold: (I0), (E1), and (D2); or (I0), (F1),* *and (C2).*

(I0) $HI_{j,t-1+2T}$ is dependent on $HI_{j,t-2+2T}$, so $\partial HI_{j,t-1+2T} / \partial HI_{j,t-2+2T}$ is nonnegative (i.e., $\partial HI_{j,t-1+2T} / \partial HI_{j,t-2+2T} \geq 0$).

Proof. The proof of this proposition is similar to the proof of Proposition 24, so we omit the derivation. ■

Proposition 26 *The terminal up-and-down volatile linkage of single-period Herfindahl indices occurs if one of two triplets of the four assumptions of Propositions 20 and 24 and assumption (J0) hold: (J0), (E1), and (D2); or (J0), (F1), and (C2).*

(J0) $HI_{j,t-1+2T}$ is dependent on $HI_{j,t-2+2T}$, so the sign of $\partial HI_{j,t-1+2T}$ $/ \partial HI_{j,t-2+2T}$ is negative (i.e., $\partial HI_{j,t-1+2T} / \partial HI_{j,t-2+2T} < 0$).

Proof. The proof of this proposition is similar to the proof of Proposition 24, so we omit the derivation. ■

6.4 Intertemporal Regular Linkages of Single-Period EGLIs on the Cost Frontier

Similar to intertemporal regular linkages in the previous subsections of this section, the same intertemporal regular linkage of single-period EGLIs on the cost frontier is mainly defined as the following relations in period t and period $t+2T$ (i.e., $EGLI^F_{i,j,t}$ and $EGLI^F_{i,j,t+2T}$), where T is a natural number.

Definition 23 (Intertemporal Regular Linkage of EGLIs on the Cost Frontier) *The intertemporal regular linkage of single-period EGLIs on the cost frontier exists if any one of the following linkages (i.e., cyclical linkage, monotonic trending linkage, and terminal up-and-down volatile linkage) mainly exists between the EGLIs on the cost frontier in period t and period $t+2T$ (i.e., $EGLI^F_{i,j,t}$ and $EGLI^F_{i,j,t+2T}$), where T is a natural number. (L1) (Cyclical Linkage) The EGLI on the cost frontier in period $t-1+2T$ (i.e., $EGLI^F_{i,j,t-1+2T}$) is dependent on the EGLI on the cost frontier in period $t-2+2T$ (i.e., $EGLI^F_{i,j,t-2+2T}$), so $\partial EGLI^F_{i,j,t-1+2T} / \partial EGLI^F_{i,j,t-2+2T}$ is positive, negative,* *or zero. The EGLI on the cost frontier in period* $t+2T$ *(i.e.,* $EGLI^F_{i,j,t+2T}$*) is, moreover, dependent on the EGLI on the cost frontier in period* t *(i.e.,* $EGLI^F_{i,j,t}$*), so the sign of* $\partial EGLI^F_{i,j,t+2T} / \partial EGLI^F_{i,j,t}$ *is positive (or negative) if* T *is an even number, whereas this sign is negative (or positive) if* T *is an odd number; (L2) (Monotonic Trending Linkage)* $EGLI^F_{i,j,t-1+2T}$ *is dependent on* $EGLI^F_{i,j,t-2+2T}$*, so* $\partial EGLI^F_{i,j,t-1+2T} / \partial EGLI^F_{i,j,t-2+2T}$ *is nonnegative (i.e.,* $\partial EGLI^F_{i,j,t-1+2T} / \partial EGLI^F_{i,j,t-2+2T} \geq 0$*).* $EGLI^F_{i,j,t+2T}$ *is, moreover, dependent on* $EGLI^F_{i,j,t}$*, so the sign of* $\partial EGLI^F_{i,j,t+2T} / \partial EGLI^F_{i,j,t}$ *is positive (i.e.,* $\partial EGLI^F_{i,j,t+2T} / \partial EGLI^F_{i,j,t} > 0$*); and (H3) (Terminal Up-and-Down Volatile Linkage)* $EGLI^F_{i,j,t-1+2T}$ *is dependent on* $EGLI^F_{i,j,t-2+2T}$*, so the sign of* $\partial EGLI^F_{i,j,t-1+2T} / \partial EGLI^F_{i,j,t-2+2T}$ *is negative (i.e.,* $\partial EGLI^F_{i,j,t-1+2T} / \partial EGLI^F_{i,j,t-2+2T} < 0$*).* $EGLI^F_{i,j,t+2T}$ *is, moreover, dependent on* $EGLI^F_{i,j,t}$*, so the sign of* $\partial EGLI^F_{i,j,t+2T} / \partial EGLI^F_{i,j,t}$ *is positive (i.e.,* $\partial EGLI^F_{i,j,t+2T} / \partial EGLI^F_{i,j,t} > 0$*). Otherwise* $\partial EGLI^F_{i,j,t-1+2T} / \partial EGLI^F_{i,j,t-2+2T} > 0$ *and* $\partial EGLI^F_{i,j,t+2T} / \partial EGLI^F_{i,j,t} < 0$*.*

Similar to Propositions 15, 19, and 23, the relations between this linkage and the efficient structure and quiet-life hypotheses are derived from the following proposition.

Proposition 27 $\partial EGLI^F_{i,j,t+2T} / \partial EGLI^F_{i,j,t}$*, where* T *is a natural number, is expressed as follows:*

$$\begin{aligned}
\frac{\partial EGLI^F_{i,j,t+2T}}{\partial EGLI^F_{i,j,t}} &= \frac{\partial EGLI^F_{i,j,t+2T}}{\partial EF^D_{i,j,t-1+2T}} \cdot \frac{\partial EF^D_{i,j,t-1+2T}}{\partial EF^D_{i,j,t-1}} \cdot \left(\frac{\partial EGLI^F_{i,j,t}}{\partial EF^D_{i,j,t-1}} \right)^{-1} \\
&= \frac{\partial EGLI^F_{i,j,t+2T}}{\partial q^{p*}_{i,j,t+2T}} \cdot \frac{\partial q^{p*}_{i,j,t+2T}}{\partial q^{p*}_{i,j,t}} \cdot \left(\frac{\partial EGLI^F_{i,j,t}}{\partial q^{p*}_{i,j,t}} \right)^{-1} \\
&= \frac{\partial EGLI^F_{i,j,t+2T}}{\partial HI_{j,t-1+2T}} \cdot \frac{\partial HI_{j,t-1+2T}}{\partial HI_{j,t-1}} \cdot \left(\frac{\partial EGLI^F_{i,j,t}}{\partial HI_{j,t-1}} \right)^{-1} . \quad (6.4)
\end{aligned}$$

Proof. $\partial EGLI_{i,j,t+2}^{F} / \partial EGLI_{i,j,t}^{F}$ is expressed as follows:

$$\begin{aligned}
\frac{\partial EGLI_{i,j,t+2}^{F}}{\partial EGLI_{i,j,t}^{F}} &= \frac{\partial EGLI_{i,j,t+2}^{F}}{\partial EF_{i,j,t+1}^{D}} \cdot \frac{\partial EF_{i,j,t+1}^{D}}{\partial EF_{i,j,t-1}^{D}} \cdot \left(\frac{\partial EGLI_{i,j,t}^{F}}{\partial EF_{i,j,t-1}^{D}} \right)^{-1} \\
&= \frac{\partial EGLI_{i,j,t+2}^{F}}{\partial q_{i,j,t+2}^{p*}} \cdot \frac{\partial q_{i,j,t+2}^{p*}}{\partial q_{i,j,t}^{p*}} \cdot \left(\frac{\partial EGLI_{i,j,t}^{F}}{\partial q_{i,j,t}^{p*}} \right)^{-1} \\
&= \frac{\partial EGLI_{i,j,t+2}^{F}}{\partial HI_{j,t+1}} \cdot \frac{\partial HI_{j,t+1}}{\partial HI_{j,t-1}} \cdot \left(\frac{\partial EGLI_{i,j,t}^{F}}{\partial HI_{j,t-1}} \right)^{-1} . \quad \text{(P27.1)}
\end{aligned}$$

Similarly, $\partial EGLI_{i,j,t+4}^{F} / \partial EGLI_{i,j,t}^{F}$ is expressed as follows:

$$\begin{aligned}
\frac{\partial EGLI_{i,j,t+4}^{F}}{\partial EGLI_{i,j,t}^{F}} &= \frac{\partial EGLI_{i,j,t+4}^{F}}{\partial EF_{i,j,t+3}^{D}} \cdot \frac{\partial EF_{i,j,t+3}^{D}}{\partial EF_{i,j,t-1}^{D}} \cdot \left(\frac{\partial EGLI_{i,j,t}^{F}}{\partial EF_{i,j,t-1}^{D}} \right)^{-1} \\
&= \frac{\partial EGLI_{i,j,t+4}^{F}}{\partial q_{i,j,t+4}^{p*}} \cdot \frac{\partial q_{i,j,t+4}^{p*}}{\partial q_{i,j,t}^{p*}} \cdot \left(\frac{\partial EGLI_{i,j,t}^{F}}{\partial q_{i,j,t}^{p*}} \right)^{-1} \\
&= \frac{\partial EGLI_{i,j,t+4}^{F}}{\partial HI_{j,t+3}} \cdot \frac{\partial HI_{j,t+3}}{\partial HI_{j,t-1}} \cdot \left(\frac{\partial EGLI_{i,j,t}^{F}}{\partial HI_{j,t-1}} \right)^{-1} . \quad \text{(P27.2)}
\end{aligned}$$

Consequently, from Eqs. (P27.1) and (P27.2), $\partial EGLI_{i,j,t+2T}^{F} / \partial EGLI_{i,j,t}^{F}$, where T is a natural number, is expressed as Eq. (6.4). ■

From Proposition 27 (Eq. (6.4)), the relations between the efficient structure and quiet-life hypotheses and the intertemporal regular linkages (i.e., cyclical linkage, monotonic trending linkage, and terminal up-and-down volatile linkage) of single-period EGLIs on the cost frontier are distilled as the following three propositions.

Proposition 28 *The cyclical linkage of single-period EGLIs on the cost frontier occurs if one of six pairs of the following assumptions holds: (SA1) and (SA2), (SB1) and (SB2), (SC1) and (SC2), (SD1) and (SA2), (SE1) and (SB2), or (SF1) and (SC2).*

(SA1) The signs of $\partial EGLI_{i,j,t}^{F} / \partial EF_{i,t-1}^{D}$ *and* $\partial EGLI_{i,j,t-2+2T}^{F} / \partial EF_{i,t-3+2T}^{D}$ *are the same as the signs of* $\partial EGLI_{i,j,t+2T}^{F} / \partial EF_{i,t-1+2T}^{D}$ *and* $\partial EGLI_{i,j,t-1+2T}^{F} / \partial EF_{i,t-2+2T}^{D}$*, respectively;*

(SA2) One of two triplets of the assumptions of Proposition 16 holds: (A0), (A1), and (A2); or (A0), (B1), and (B2);

(SB1) The signs of $\partial EGLI^F_{i,j,t} / \partial q^{p*}_{i,j,t}$ *and* $\partial EGLI^F_{i,j,t-2+2T} / \partial q^{p*}_{i,j,t-2+2T}$ *are the same as the signs of* $\partial EGLI^F_{i,j,t+2T} / \partial q^{p*}_{i,j,t+2T}$ *and* $\partial EGLI^F_{i,j,t-1+2T} / \partial q^{p*}_{i,j,t-1+2T}$*, respectively;*

(SB2) One of two triplets of the assumptions of Proposition 20 holds: (E0), (A1), and (C2); or (E0), (B1), and (D2);

(SC1) The signs of $\partial EGLI^F_{i,j,t} / \partial HI_{j,t-1}$ *and* $\partial EGLI^F_{i,j,t-2+2T} / \partial HI_{j,t-3+2T}$ *are the same as the signs of* $\partial EGLI^F_{i,j,t+2T} / \partial HI_{j,t-1+2T}$ *and* $\partial EGLI^F_{i,j,t-1+2T} / \partial HI_{j,t-2+2T}$*, respectively;*

(SC2) One of two triplets of the assumptions of Proposition 24 holds in $t-1$*: (H0), (E1), and (C2); or (H0), (F1), and (D2);*

(SD1) The signs of $\partial EGLI^F_{i,j,t} / \partial EF^D_{i,t-1}$ *and* $\partial EGLI^F_{i,j,t-2+2T} / \partial EF^D_{i,t-3+2T}$ *are different from the signs of* $\partial EGLI^F_{i,j,t+2T} / \partial EF^D_{i,t-1+2T}$ *and* $\partial EGLI^F_{i,j,t-1+2T} / \partial EF^D_{i,t-2+2T}$*, respectively;*

(SE1) The signs of $\partial EGLI^F_{i,j,t} / \partial q^{p*}_{i,j,t}$ *and* $\partial EGLI^F_{i,j,t-2+2T} / \partial q^{p*}_{i,j,t-2+2T}$ *are different from the signs of* $\partial EGLI^F_{i,j,t+2T} / \partial q^{p*}_{i,j,t+2T}$ *and* $\partial EGLI^F_{i,j,t-1+2T} / \partial q^{p*}_{i,j,t-1+2T}$*, respectively;*

(SF1) The signs of $\partial EGLI^F_{i,j,t} / \partial HI_{j,t-1}$ *and* $\partial EGLI^F_{i,j,t-2+2T} / \partial HI_{j,t-3+2T}$ *are different from the signs of* $\partial EGLI^F_{i,j,t+2T} / \partial HI_{j,t-1+2T}$ *and* $\partial EGLI^F_{i,j,t-1+2T} / \partial HI_{j,t-2+2T}$*, respectively.*

Proof. From Propositions 16, 20, and 24, assumptions (SA2), (SB2), and (SC2) mean that the cyclical linkages of single-period dynamic cost efficiencies, single-period optimal planned financial goods, and single-period Herfindahl indices, respectively, occur. From Definition L1 of Definition 23 and Proposition 27 (Eq. (6.4)), assumptions (SA1), (SB1), and (SC1) mean that the signs of these cyclical linkages are invariable, whereas assumptions (SD1), (SE1), and (SF1) mean that the signs of these cyclical linkages are inverse. From Definition L1 of Definition 23, the cyclical linkage of single-period EGLIs on the cost frontier occurs. ■

Proposition 29 *The monotonic trending linkage of single-period EGLIs on the cost frontier occurs if one of six pairs of the six assumptions of Proposition*

28 and assumptions (MA2), (MB2), and (MC2) holds: (SA1) and (MA2), (SB1) and (MB2), (SC1) and (MC2), (SD1) and (MA2), (SE1) and (MB2), or (SF1) and (MC2).

(MA2) One of two triplets of the assumptions of Propositions 16 and 17 holds: (C0), (A1), and (B2); or (C0), (B1), and (A2);

(MB2) One of two triplets of the assumptions of Propositions 16, 20, and 21 holds: (F0), (A1), and (D2); or (F0), (B1), and (C2);

(MC2) One of two triplets of the assumptions of Propositions 20, 24, and 25 holds in $t-1$*: (I0), (E1), and (D2); or (I0), (F1), and (C2).*

Proof. The proof of this proposition is similar to the proof of Proposition 28 with the exception of replacing the cyclical linkage and so forth with the monotonic trending linkage and so forth, so we omit the derivation. ■

Proposition 30 *The terminal up-and-down volatile linkage of single-period EGLIs on the cost frontier occurs if one of six pairs of the six assumptions of Proposition 28 and assumptions (TA2), (TB2), and (TC2) holds: (SA1) and (TA2), (SB1) and (TB2), (SC1) and (TC2), (SD1) and (TA2), (SE1) and (TB2), or (SF1) and (TC2).*

(TA2) One of two triplets of the assumptions of Propositions 16 and 18 holds: (D0), (A1), and (B2); or (D0), (B1), and (A2);

(TB2) One of two triplets of the assumptions of Propositions 16, 20, and 22 holds: (G0), (A1), and (D2); or (G0), (B1), and (C2);

(TC2) One of two triplets of the assumptions of Propositions 20, 24, and 26 holds in $t-1$*: (J0), (E1), and (D2); or (J0), (F1), and (C2).*

Proof. The proof of this proposition is similar to the proof of Proposition 28 with the exception of replacing the cyclical linkage and so forth with the terminal up-and-down volatile linkage and so forth, so we omit the derivation. ■

6.5 Policy Implications

According to the results in this section, where there is an intertemporal regular linkage of single-period EGLIs on the cost frontier, the EGLI can increase or decrease at least in the short term except for monotonic trending linkages. Therefore, over the short term, it is difficult to judge the need for industrial organization policies for promoting competition. However, from a long-term perspective, if the intertemporal regular linkage of single-period EGLIs on the cost frontier does not exhibit a downward trend, then industrial organization policies for promoting long-term competition are needed. If this linkage shows an upward trend caused mainly by an upward trend of the intertemporal regular linkage of single-period Herfindahl indices, then anti-monopoly and anti-concentration policies are justified from a long-term perspective. If the upward trend of the intertemporal regular linkage of single-period EGLIs on the cost frontier is, however, caused mainly by the intertemporal regular linkage of single-period dynamic cost efficiencies or single-period optimal planned financial goods, then other policies are desirable because, in this case, anti-monopoly and anti-concentration interventions cause unnecessary distortion in the economy. Specifically, if this upward trend is caused mainly by the downward (upward) trend of the intertemporal regular linkage of single-period dynamic cost efficiencies, then industrial organization policies for improving long-term dynamic cost efficiency (industrial organization policies in which a long-term improvement in dynamic cost efficiency increases long-term competition) are needed. Similarly, if the upward trend of the intertemporal regular linkage of single-period EGLIs on the cost frontier is mainly caused by the downward (upward) trend of the intertemporal regular linkage of single-period optimal planned financial goods, then industrial organization policies for stimulating long-term growth (industrial organization policies in which long-term growth increases long-term competition) are needed.

7 Conclusions

In this paper, on the basis of the GURM constructed by Homma (2009, 2012), we explored the efficient structure hypothesis proposed by Demsetz (1973) and the quiet-life hypothesis put forward by Berger and Hannan (1998). We clarified mathematical formulations and theoretical interpretations of both hypotheses, the relative magnitude of the efficient structure hypothesis to the quiet-life hypothesis, the relation between both hypotheses and the EGLI on the cost frontier proposed by Homma (2009, 2012), and the relation between both hypotheses and the existence of intertemporal regular linkages of single-period dynamic cost efficiencies, single-period optimal planned financial goods, single-period Herfindahl indices, and single-period EGLIs on the cost frontier. In the following, we summarize the major results and offer conclusions.

7.1 Formulations

On the efficient structure hypothesis, three formulations are possible. The first formulation is that the efficient structure hypothesis is expressed by the effect of improved dynamic cost efficiency in the previous period on the planned optimal financial good in the current period, so it is a direct definition of the efficient structure hypothesis. The second formulation is that the efficient structure hypothesis is expressed by the ratio of the following two sums, so it provides the foundation for rigorous theoretical interpretations: the numerator is the sum of the net effect of the improvement in dynamic cost efficiency in the previous period and the effect of the same improvement. The former net effect is on the GURP on the cost frontier (i.e., the dynamic frontier marginal variable cost with respect to the planned optimal financial good) in the current period and on dynamic actual marginal variable cost with respect to the planned optimal financial good in the current period. This net effect is normalized by the same dynamic actual marginal variable cost and accounts for the correction in dynamic marginal cost efficiency in the current period. The latter effect is on the elasticity of dynamic actual variable cost in the current period with respect to dynamic cost efficiency in the current period. This effect is normalized by the square of the same elasticity. Similarly, the denominator is the sum of the net effect of an increase in the planned optimal financial good in the current period and the effect of the same increase in the planned optimal financial good. Similar to the numerator, the former net effect is on the same GURP and the same dynamic actual marginal variable cost. This net effect is normalized by the same dynamic actual marginal variable cost and accounts for the correction in dynamic marginal cost efficiency in the current period. The latter effect is on the same elasticity of dynamic actual variable cost and is normalized by the square of the same elasticity. The third formulation is that the net effect in the numerator of the second formulation is expressed by the sum of the effects of the improvement in dynamic cost efficiency in the previous period on the efficiency difference of the GURP of the planned optimal financial good in the current period, the pricing error of the same financial good, and dynamic actual marginal variable cost with respect to the same financial good, which is corrected by dynamic marginal cost inefficiency in the current period, respectively. Similar to the numerator, the net effect in the denominator of the second formulation is expressed by the sum of the effects of an increase in the planned optimal financial good in the current period on the same factors as the numerator. This formulation is, therefore, used to thoroughly interpret the efficient structure hypothesis with these effects. Similarly, regarding the quiet-life hypothesis, three formulations are also possible. The first formulation is that the quiet-life hypothesis is expressed by the effect of an increase in the Herfindahl index in the previous period on dynamic cost efficiency in the current period, so it is a direct definition of the quiet-life hypothesis. The second formulation is that the quiet-life hypothesis is expressed by the following ratio, so it provides the foundation for rigorous theoretical interpretations: the numerator is the sum of the net effect of the same increase in the Herfindahl index and the effect of the same increase. Similar to the case of the efficient structure hypothesis, the former net effect is on the same GURP and the same dynamic actual marginal variable cost. This net effect is normalized by the same dynamic actual marginal variable cost and accounts for the same correction in dynamic marginal cost efficiency. The latter effect is on the same elasticity of dynamic actual variable cost, and is normalized by the same square of the same elasticity. The denominator is the product of the same dynamic actual marginal variable cost as per the efficient structure hypothesis and the same square of the same elasticity. The third formulation is that the same net effect in the second formulation is expressed by the sum of the effects of the same increase in the Herfindahl index on the same efficiency difference of the GURP as per the efficient structure hypothesis, the same pricing error, and the same corrected dynamic actual marginal variable cost, respectively, so it is the formulation that is used to thoroughly interpret the quiet-life hypothesis with these effects.

7.2 EGLI on the Cost Frontier

In terms of whether support for either or both of the hypotheses is desirable from the perspective of the EGLI on the cost frontier, the results of the theoretical analysis herein suggest that both desirable and undesirable cases exist, with the following two points being particularly noteworthy: 1) it is not always possible to invoke support for the quiet life hypothesis to justify anti-monopoly and anti-concentration policies; and 2) new industrial organization policies are needed where support for the efficient structure hypothesis is undesirable. Regarding the first point, support for the quiet-life hypothesis can decrease the EGLI on the cost frontier (i.e., increase the degree of competition on the cost frontier), so that support for this hypothesis does not always justify anti-monopoly and anti-concentration policies, even if an increase in market concentration decreases dynamic cost efficiency. Justification of such policies is restricted to the case that an increase in market concentration increases the EGLI on the cost frontier (i.e., decreases the degree of competition on the cost frontier), so enactment and enforcement of these policies requires careful consideration. In terms of the second point, so far, there is no theoretical foundation for suggesting that support for the efficient structure hypothesis is undesirable. At least theoretically, there are, however, both cases where support for the efficient structure hypothesis decreases the EGLI on the cost frontier (i.e., increases the degree of competition on the cost frontier) and increases the EGLI on the cost frontier (i.e., decreases the degree of competition on the cost frontier). Regarding the latter, it is judged that support for the efficient structure hypothesis is undesirable. In this case, new industrial organization policies which differ from existing anti-monopoly and anti-concentration policies, and under which efficiency improvements increase the degree of competition on the cost frontier, are required.

7.3 Trends in Intertemporal Regular Linkages

Where intertemporal regular linkage of single-period EGLIs exists on the cost frontier, the need for industrial organization policies must be judged from a long-term perspective. As discussed, policy implications differ depending on the direction and cause of this linkage, and as such, careful consideration is required to determine when and why anti-monopoly/anti-concentration policies and policies designed to increase long-term competition via improving dynamic cost efficiency or long-term growth are needed.

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