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Water holdup estimation from pressure drop measurements in oil-water two-phase flows by means of the two-fluid model

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Water holdup estimation from pressure drop measurements in oil-water two-phase flows by means of the two-fluid model

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Abstract. The Two-Fluid Model (TFM) has been applied to determine water holdup from pressure drop measurements for core-annular flows in horizontal pipes. The fluids are Milpar 220 oil (ρ_o=890 kg/m^3, μ_o=0.832 Pa·s at 20 °C) and tap water (μ_w=1.026x10^{-3} Pa·s at 20 °C). The investigated volume flow rates range from 2 to 6 m^3/h, for water, and from 1 to 3.5 m^3/h, for oil, respectively. The results are in very good agreement with available experimental data from the literature and a simple correlation between water holdup and water input fraction has been benchmarked to the overall data set. Eventually, the TFM endowed with the holdup correlation has been adopted to predict the pressure drop with quite satisfactory results: 98% of data fall within a percentage error of ±10%, 99% of the data fall within ±15%, and all the data are predicted within ±20%. On the other hand, the mean absolute relative error for the pressure drop reduction factor is 5.5%.

1. Introduction

The flow of oil-water mixtures in pipes is widely studied, mainly due to the importance of long-distance transportation of oil products. Growing interest is devoted, in particular, to heavy-oils because of the progressive depletion of on-shore fields and light-oil reserves. However, the increased pressure drop resulting from the higher viscosity raises both extraction and transportation energy costs, which can represent up to one third of the overall operational expense [1]. A technique to reduce the pressure drop is water injection aimed at creating the so-called core annular flow (CAF), a flow regime characterised by the presence of an oil core enveloped in a water annulus wetting the pipe wall, so that the apparent viscosity of the mixture is considerably reduced. Fundamental literature on this subject, focusing on heavy-oil flows in horizontal tubes, may be summarized as follows. Clark and Shapiro [2] got the earliest patent of an injector able to reduce the pressure drop up to 30%. Charles et al. [3] considered three oils and concluded that flow patterns were largely independent of the oil viscosity. They also found that the pressure gradient was reduced to a minimum by the addition of water in case of laminar flow of the oil core. In addition, the maximum pressure gradient reduction (10 times) was achieved with the most viscous oil. Charles and Redberger [4] proposed a numerical procedure to solve the Navier-Stokes equations for the general case, in which the liquids are stratified as a result of the different densities of oil and water. The calculated pressure gradient reduction resulted considerably lower than experimental values. The authors concluded that wave motion and mixing at the oil-water interface might act as mechanisms of further reduction. Ooms et al. [5] developed a model based on the hydrodynamic lubrication theory that highlighted the importance of interface phenomena for CAF. They showed that the ripples on the interface, moving with respect to the pipe wall, can generate pressure variations in the annular layer. These result in a force acting normal to the core, which can counterbalance the buoyancy effect. Oliemans et al. [6] added the effect of turbulence in the water film surrounding the oil core, improving the predicting ability, provided that actual wave amplitudes and wavelengths observed during experimental tests are used as input data. On the other hand, Brauner [7] developed an analytical model based on the integral forms of the momentum equation for each phase (Two-fluid Model – TFM) regardless of the flow regime. The wall and the interfacial shear stresses are expressed in terms of the phase actual average velocities and the corresponding friction factors, hence empirical closure equations are needed. For the case of horizontal laminar core, explicit solutions for the holdup and the pressure gradient reduction factor are presented. Ullman and Brauner [8] have more recently provided further refinement of closure equations. Grassi et al. [9] assessed the effectiveness of such an approach in the prediction of pressure drop for two-phase liquid-liquid flows with high-viscosity ratio in horizontal and slightly inclined pipes. Differently, Arney et al. [10] reported on holdup and pressure drop measurements for waxy crude oil and No. 6 fuel oil (ASTM Classification). The comparison with previous sources [3, 11-13] led to a correlation formula for the water holdup as a function of the input fraction. Accordingly, they defined a modified Reynolds number to extend the single-phase friction factor definition to two-phase flows. In this paper, the TFM is used to relate the holdup and the pressure drop so that the former can be determined from the usual measurement of the latter. This is convenient as, generally, it is simpler to measure pressure drop rather than holdup, and in many practical situations, it is impossible to directly measure holdup. The simple scheme of correlation provided by Arney et al. [10] is then adopted to provide a correlation between water input fraction and holdup to be used in the TFM instead of a correlation for the interfacial shear stress. The results are then discussed and compared with the other available approaches.

2. Theoretical background

The Two-Fluid Model (TFM) for horizontal pipes, assuming fully developed flow, writes

$$\begin{cases} -A_o \dfrac{dp}{dx} - \tau_i S_i = 0 \\ -A_w \dfrac{dp}{dx} - \tau_w S_w + \tau_i S_i = 0 \end{cases} \tag{1}$$

where "*o*" denotes the core phase (oil) and "*w*" the phase in contact with the wall (water), *S* the wetted perimeter and A the cross-sectional area of the single phase, as indicated in Fig. 1. Eliminating the interfacial shear stress, and denoting the overall cross-sectional area $A = A_o + A_w$, it follows

$$-A\frac{dp}{dx} - \tau_w S_w = 0 \tag{2}$$

The wall shear stress can be replaced making use of the friction factor as

$$\tau_w = f_w \frac{\rho_w U_w^2}{2} \tag{3}$$

where the friction factor depends on the water Reynolds number. Introducing the water holdup as the ratio of the superficial velocity J_w to the actual one U_w

$$H_w = \frac{J_w}{U_w} \tag{4}$$

and the hydraulic diameter for the water phase (adjoining the pipe wall, see Fig. 1)

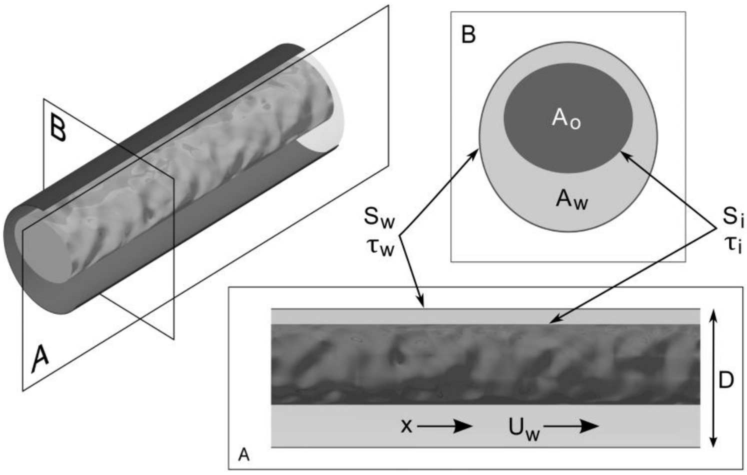

Figure 1. Sketch of an oil-water annular flow.

$$D_w = \frac{4A_w}{S_w} = H_w D \tag{5}$$

the Reynolds number is then

$$\mathrm{Re}_w = \frac{D_w U_w}{\nu_w} = \frac{DJ_w}{\nu_w} \tag{6}$$

so that

$$f_w = C_w \, \mathrm{Re}_w^{-n_w} \tag{7}$$

For the laminar flow regime $C_w = 16$ and $n_w = 1$, whereas for developed turbulent flows, the Blasius formulation is often used; accordingly, $C_w = 0.079$ and $n_w = 0.25$ for Re < 50000. $C_w = 0.046$ and $n_w = 0.2$ for Re > 50000. Replacing (4) to (7) in (2), the water holdup as a function of the superficial velocity and the measured pressure drop per unit length results

$$H_w = \frac{C_w \left(\frac{DJ_w}{\nu_w} \right)^{-n_w} \rho_w J_w^2}{\left(-\frac{dp}{dx} \right) \frac{D}{2}} \tag{8}$$

3. Experimental setup and data acquisition

The liquid-liquid flow facility is shown in Fig. 2. Oil (Milpar 220, ρ_o=890 kg/m^3, μ_o=0.832 Pa·s at 20 °C) and water (tap water, μ_w=1.026×10^{-3} Pa ·s at 20 °C) are pumped from their respective storage tanks. A magnetic flow meter (uncertainty ±0.5 % of the reading) and a calibrated metering pump (uncertainty ±2 % of the set point) are used for measuring water and oil flow rates, respectively. The range of investigated volume flow rates is 2 to 6 m^3/h for water and 1 to 3.5 m^3/h, for oil, respectively. The two fluids are pumped into the test section after going through a coaxial mixer, where oil flows parallel to the pipe axis while water is injected through an annulus into the oil stream. This procedure ensures the onset of stable annular flows provided that the superficial velocities of the phases lie in the range of existence of this flow regime, as reported in flow pattern maps available in [14]. The test section consists of a 12 m long circular pipe made of Plexiglas® that can be composed by segments with different diameters to realize sudden variations in the cross-sectional area. Pressure gradients along the pipe are measured by connecting 15 pressure taps (500 mm apart from each other) to differential piezoresistive pressure transducers (Kulite IPTE, 0-350 mbar, uncertainty ±2% full scale, and SETRA 0-1 PSI, uncertainty ±1.5% full scale). The temperature dependence of oil viscosity has been measured in the range 20 °C ≤ T ≤ 40 °C where it is well described as $\mu_o = 3.444 \cdot \exp(-0.071 \cdot T)$. Flow temperature during the experiments is detected by a K-type thermocouple (uncertainty 0.2 °C) located 50 mm before the first pressure tap to avoid flow disturbance. All sensor signals are collected by means of National Instruments acquisition boards and processed by *ad hoc* software. Operating conditions are defined by varying the superficial velocities of the phases within the range of existence of annular flow. In particular, water is supplied starting from the maximum value of the superficial velocity $J_{w,max}$. Then, oil is added at the selected superficial velocity J_o. At each run, J_w is decreased until its minimum value is reached. The value of J_o is then changed and the sequence is repeated.

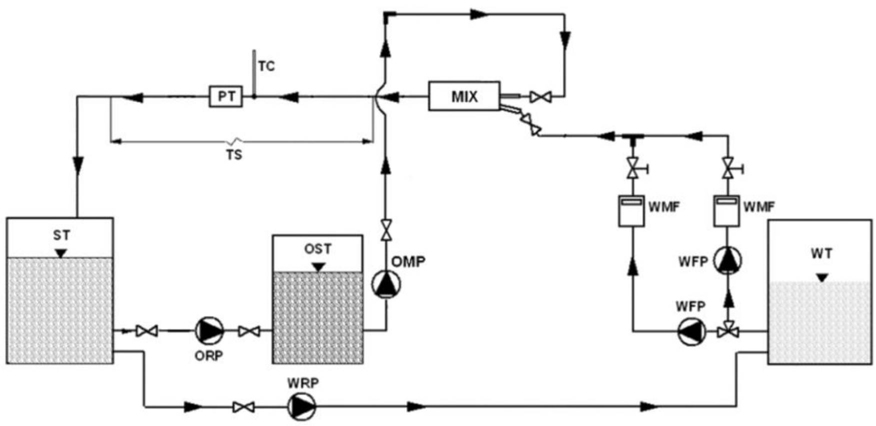

Figure 2. Schematic representation of the oil-water loop. MIX phase inlet mixer, OMP oil metering pump, ORP oil recovering pump, OST oil supply tank (0.5 m^3), PT pressure transducer, ST phase collector/separator tank (1.0 m^3), TC thermocouple (K type), TS test section, WFP water feeding pump, WMF water magnetic flow meter, WRP water recovering pump, WT water supply tank (5 m^3).

4. Results and discussion

4.1. Holdup estimation

Different experimental campaigns were run to analyse test section configurations including pipes with uniform diameter or upstream/downstream of sudden variations in the cross-sectional area. In all the cases, the Reynolds number of water, Eq. (6), resulted within about 17000 and 102000 (turbulent range) and the qualitative behavior of the pressure drop data showed quite similar characteristics. Thus, in the following, reference is made to pipes with D = 30 mm, for which data can be compared in all the tested configurations. Starting with the straight tube, the pressure gradient is reported as a function of the water input fraction, ε_w, for the different oil superficial velocities, J_o, in Fig. 3. As expected, the pressure gradient increases with J_o, at constant ε_w. On the other hand, the data points at constant J_o show that the pressure gradient increases with the water content. This confirms that for core-annular flows the pressure drop lowers by reducing the water content and hence thinning the water annulus [4]. Incidentally, parabolic fitting seems to reproduce very well the behavior with a regression coefficient always higher than 0.99. Moreover, it has been observed that interface instability may determine a sudden transition to stratified-wavy flow, if the mixture velocity, J_{mix}, lowers below a critical value [14]. This would cause an abrupt increase in the pressure drop. It is customary to define the pressure reduction factor, R, as the inverse of the ratio between the two-phase pressure drop and the pressure drop of the oil-only flow with the same superficial velocity. Figure 4 shows the pressure reduction factor as a function of the water input fraction, ε_w, for the different oil superficial velocities, J_o. The monotonic behavior shows that all the operating conditions correspond to stable core-annular flow regimes. A straight line fits properly the data and the slope seems to lower by increasing J_o. It is worth noting that the same pressure reduction factor is achieved at a reduced water input fraction as the oil superficial velocity increases. Quite similar characteristics have been found for the flow upstream and downstream of an abrupt change in the cross-sectional area. Figure 5 compares the pressure gradient as a function of the water input fraction, ε_w, for the different oil superficial velocities, J_o, for all the configurations. Evidently, the plot reproduces the behavior of Fig. 3 with a slightly higher dispersion of data points. This seems to arise from the perturbation caused by the sudden contraction: though the flow regime remains annular, it has been observed that the oil-water interface becomes more irregular and shows a tendency to form small drops [15,16].

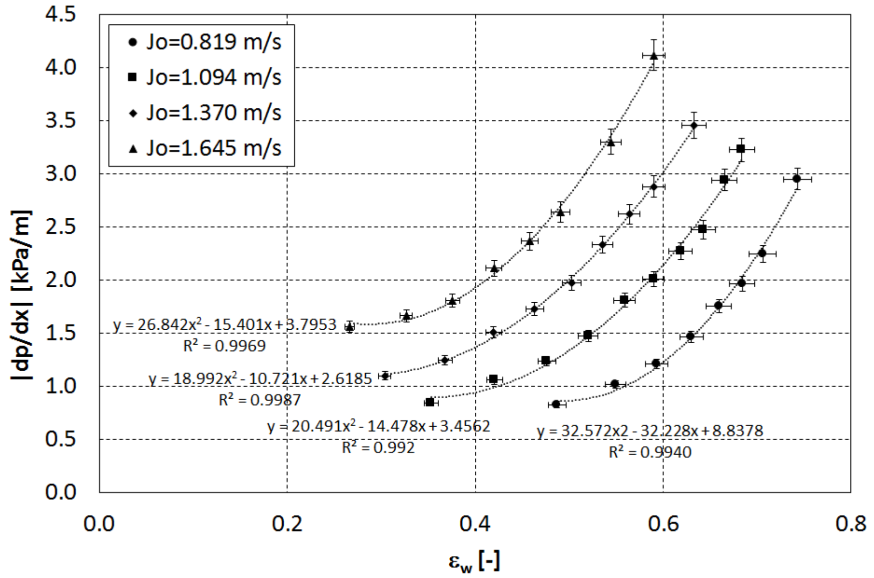

Figure 3. Pressure gradient versus water input fraction at constant oil superficial velocity (straight tube, D = 30 mm).

The water holdup is calculated from the measured pressure gradient according to Eq. (8). It is reported in Fig. 6(a) together with the experimental data collected by means of the quick closing valves technique for pipes of 30 mm and 40 mm i.d., respectively [16]. Substantial agreement is found between the measured and the calculated values. According to Arney et al. [10], the water holdup as a function of the water input fraction can be expressed by a parabolic fitting

$$H_w = \varepsilon_w \left[1 + C(1 - \varepsilon_w)\right] \tag{9}$$

Least square fitting gives $C = 0.356$ with regression parameter $R^2 = 0.98$ for the calculated holdup, $C = 0.358$ with regression parameter $R^2 = 0.95$ for the measured holdup, and $C = 0.357$ with regression parameter $R^2 = 0.95$ for both calculated and measured holdup. Hence, a unique value of 0.36 is assumed without significant differences (solid line in Fig. 6(a)). Mean Percentage Error (MPE) and Mean Absolute Percentage Error (MAPE) are defined as:

$$MPE = \frac{100\%}{N} \sum_{i=1}^{N} \frac{a_i - f_i}{a_i} \qquad MAPE = \frac{100\%}{N} \sum_{i=1}^{N} \left| \frac{a_i - f_i}{a_i} \right| \tag{10),(11}$$

where a_i is the actual value of the quantity being forecast, f_i is the forecast, and N is the population of the sample. Accordingly, Eq. (9) predicts the holdup data with *MPE* = 0.1% and *MAPE* = 2.9%.

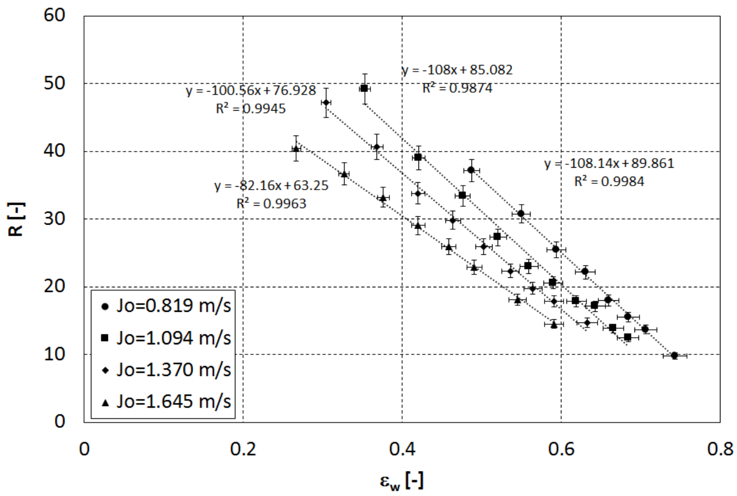

Figure 4. Pressure reduction factor versus water input fraction at constant oil superficial velocity (straight tube, D = 30 mm).

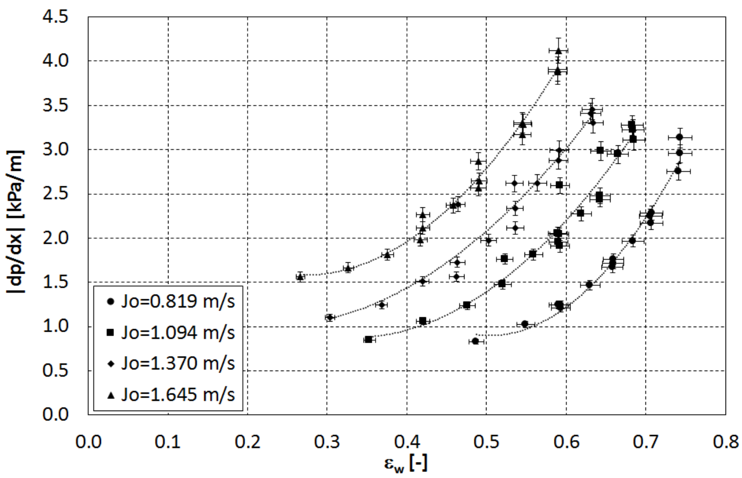

Figure 5. Pressure gradient versus water input fraction at constant oil superficial velocity (all data, D = 30 mm).

These findings are in very good agreement with Arney et al. [10] that deals with holdup measurements for core-annular flows of waxy crude oil and No. 6 fuel oil, leading to C = 0.35 (see also Table 1). Figure 6(b) reports the comparison of Eq. (9) and all the available experimental data from [3,11-13]. *MPE* is 0.2% and *MAPE* is 3.8%. *MAPE* values indicate a very satisfactory accuracy of the prediction, whereas the very small *MPE* indicates that the forecast by Eq. (9) is not significantly biased. On the other hand, Fig. 6(c) reports the comparison between Eq. (9) and the other models available in the literature for core-annular flow of liquid-liquid mixtures, listed in Table 1 (Arney et al. correlation is not reported since, being in the same form as Eq. (9), with C = 0.35 instead of 0.36, it cannot be distinguished). The model by Oliemans [11] always underestimates the water holdup with *MPE* = –13% and *MAPE* = 16%. Ullmann and Brauner [8] provided an analytical solution of the Two-Fluid Model introducing as a closure equation a suitable correlation for the interfacial shear stress. According to Grassi et al. [9] the parameter c_i^0, reported in Table 1, has been set to 1.17. This approach, which generalizes a former result from Brauner [7], results slightly underestimating with *MPE* = –0.5% and *MAPE* = 8.4%.

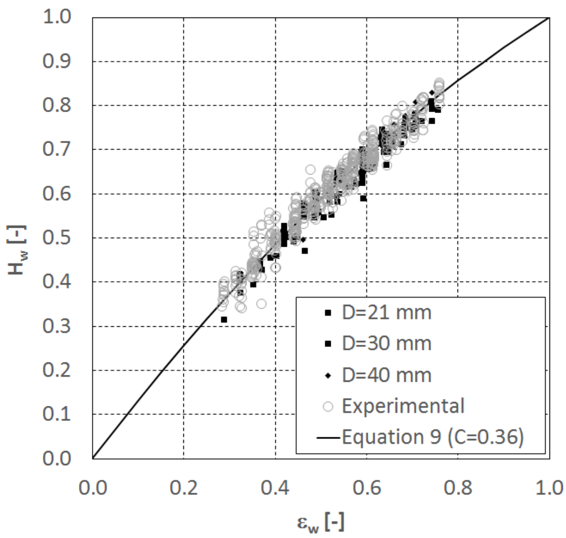

Figure 6(a). Water holdup versus water input fraction. Comparison between quick-closing valves data and TFM prediction.

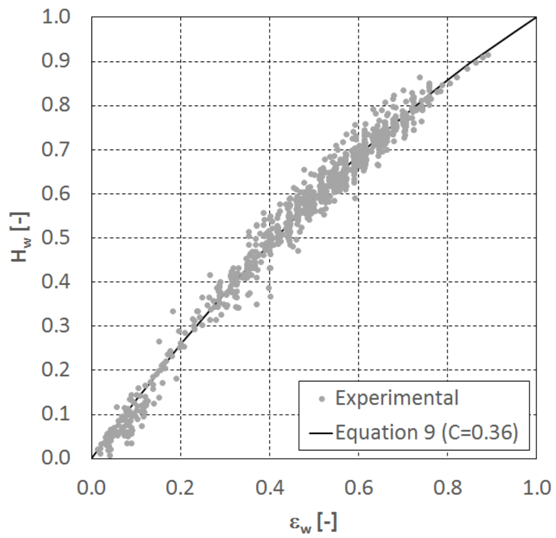

Figure 6(b). Water holdup versus water input fraction. Comparison between the proposed correlation and all the available data from the literature.

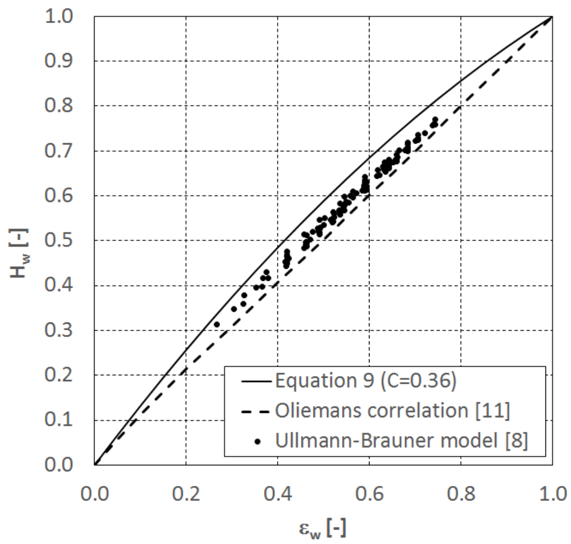

Figure 6(c). Water holdup versus water input fraction. Comparison between the proposed correlation and available models from the literature.

4.2. Pressure gradient estimation

The semi-empirical expression of the water holdup, Eq. (9), can be used in the Two-Fluid Model to predict the pressure drop, simply rearranging Eq. (8):

$$-\frac{dp}{dx} = 2C_w \left(\frac{DJ_w}{\nu_w}\right)^{-n_w} \frac{\rho_w J_w^2}{DH_w} \tag{12}$$

As shown in the parity plot of Fig. 7, the result is satisfactory. In particular, the model is able to predict 98% of the data within a percentage error of ±10%, 99% of the data fall within ±15%, and all the data are predicted within ±20%. Globally, *MPE* = 0.5% and *MAPE* = 4.4%. Similarly, for the pressure reduction factor 96% of the data are predicted within a percentage error of ±10%, 99% of the data fall within ±20%, and all the data are predicted within ±25%. Globally, *MPE* = 1.6% and *MAPE* = 5.5%. Figure 8 shows the comparison with the pressure reduction factor according to the Ullmann-Brauner model [8]. With the latter, *MPE* = –18.0% and *MAPE* = 18.1% hence the pressure reduction factor is generally underestimated. Nevertheless, such a model allows acceptable results in a broad range of operating conditions.

Table 1. Holdup models for oil-water flows from the literature.

Ref.	Model	Additional information
Arney [10]	$H_w = \varepsilon_w\left[1+0.35(1-\varepsilon_w)\right]$	-
Oliemans [11]	$H_w = \varepsilon_w\left[1+0.2(1-\varepsilon_w)^5\right]$	-
Ullmann and Brauner [8]	$H_w = \dfrac{c_i^0/2-\chi^2\phi/F_i+\dfrac{c_i^0}{2}\left[1+4\chi^2\left(\dfrac{\phi}{c_i^0}\right)^2\Big/F_i\right]^{1/2}}{c_i^0+\phi-\chi^2\phi/F_i}$ $c_i^0 = 1.15 \div 1.2$ $F_i = 1$	$\phi = J_o/J_w$ $\chi^2 = \dfrac{0.046}{16}\left(\dfrac{\mu_w}{\mu_o}\right)^{0.2}\left(\dfrac{\rho_w}{\rho_o}\right)^{0.8}\dfrac{1}{\phi^{1.8}}\mathrm{Re}_o^{0.8}$ (laminar oil core – turbulent water annulus) $\mathrm{Re}_o = \rho_o J_o D/\mu_o$

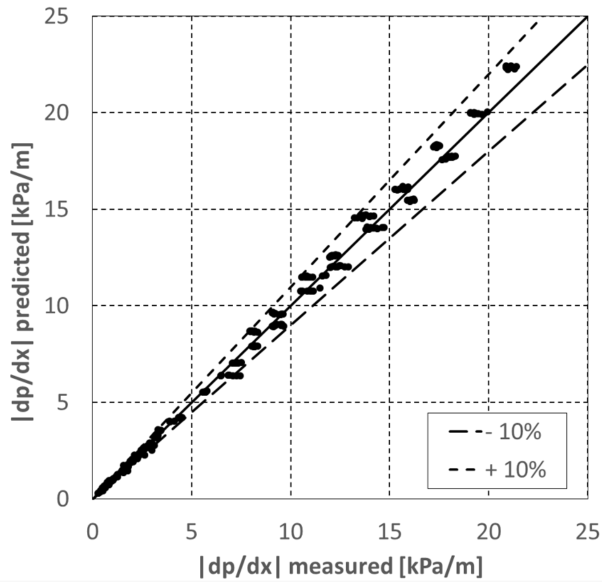

Figure 7. Predicted vs. measured pressure gradient.

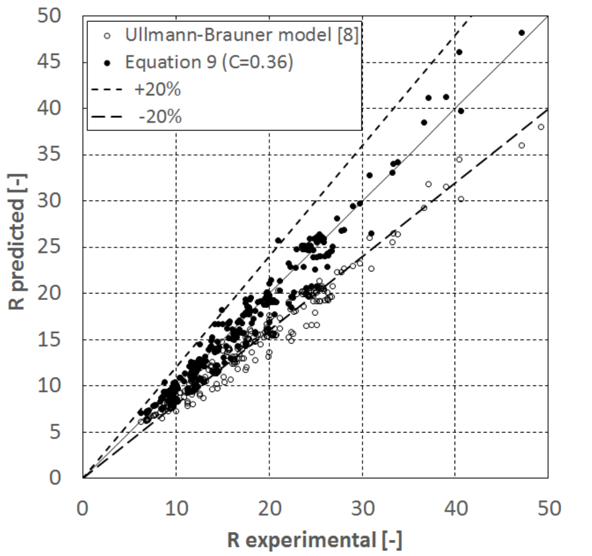

Figure 8. Comparison between predictions of the pressure drop reduction factor.

5. Conclusions

The water holdup has been estimated from the measurements of the pressure drop by means of the Two-Fluid Model. This is convenient as in general it is simpler to measure pressure drop than holdup. Moreover, holdup estimation is thus possible also when holdup cannot be directly measured. Hence, a semi-empirical formulation of the holdup as a function of the input water fraction has been derived, which generalizes on a broader data set an approach presented in the literature [10]. The expression for the holdup is then used in the Two-Fluid Model to predict the pressure drop and, in particular, the pressure reduction factor. This approach leads to better results compared with the use of a correlation for the interfacial shear stress in the Two-Fluid Model.

6. Nomenclature

A	cross-sectional area (m^2)
C	constant (-)
D	pipe diameter (m)
f	Fanning friction factor (-)
H	holdup (-)
J	superficial velocity (m/s)
MPE	mean percentage error (%)
MAPE	mean absolute percentage error (%)
p	pressure (Pa)
Re	Reynolds number (-)
S	wetted perimeter (m)
TFM	Two-Fluid Model
T	temperature (°C)
U	average velocity (m/s)

Greek Symbols

ε	volume ratio (-)
μ	dynamic viscosity (kg/m-s)
ρ	density (kg/m^3)
τ	shear stress (N/m^2)

Subscripts and Superscripts

n	power-law exponent
o	oil
w	water

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