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Frequency control modelling - basics

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Resume (mask. 2000 char.):
The purpose of this report is to provide an introduction on how the system balance in an island system can be maintained by controlling the frequency. The power balance differential equation, which is fundamental in understanding the effect on the system frequency of the unbalance between generation and consumption, is addressed. Basic topics on the main components of a generating unit, such generators, prime movers and governors are presented. A simple dynamic model for an island power system, containing realistic dynamic representations of generators, loads, prime movers, governors, is described specifically for the assessment of the performance of frequency droop control loop, i.e. primary control.

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Content

Introduction

An electric power system is by definition a system where power is produced, transmitted and consumed in real time, namely contrarily to storage systems, power is intermittent and produced as demand calls for it. This means that a balance between production of electricity and consumption of electricity must always be maintained in a power system. A change in this balance alters the system frequency and if this violates a strictly predefined frequency range, it can threaten the stability and thus the security of the power system. Steady state frequency of 50Hz is thus an indication that the generation and the consumption are in balance.

As illustrated in Figure 1, on one side of the balance there are the large power plants, which produce electrical power. Notice that besides the traditional ones, namely the thermal and hydro plants, nowadays there are also more and more renewable generators like wind turbines and solar cells. On the other side of the balance there are the consumers, i.e. houses, hospitals, offices and electric vehicles. The consumers are also called loads or demand.

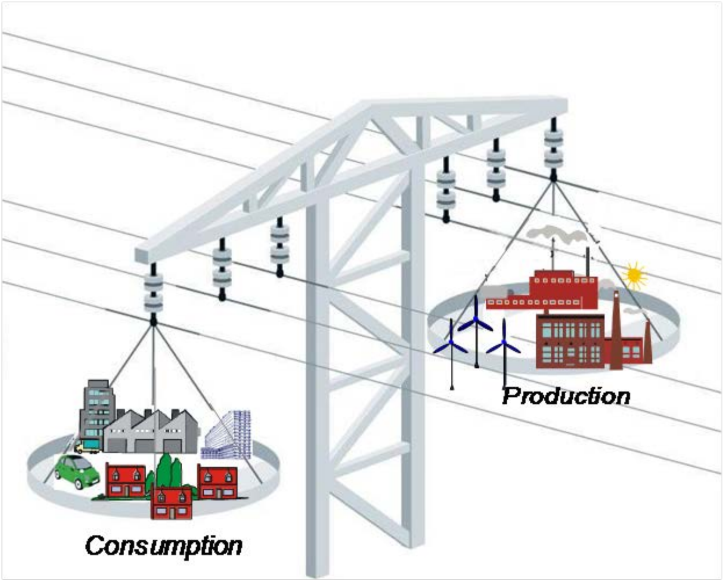

Figure 1: Power balance between production and consumption.

The maintenance of the power balance implies that in case of too much electricity production (supply), the frequency will increase, while in case of too much electricity consumption the frequency will decrease.

System balance is in general the responsibility of transmission system operators (TSOs), who utilize some predefined large power plants existing in the system to maintain the frequency constant at its nominal value. This balance is typically ensured through functions such as i.e. activation of additional generation or disconnection of loads, this last action being referred to as load shedding.

The goal of the present report is to provide an introduction on how the system balance in an island system can be maintained by controlling the frequency.

1. General issues on power systems

In this chapter a general introduction to power systems and their frequency control is given. It is not the intention to include the fundamentals of electrical engineering related with power systems. However some basic terminologies will be shortly addressed from time to time, when necessary, for the sake of completeness. The equation of motion, which is fundamental in understanding the effect of an instantaneous imbalance between load and generation on the system frequency, is described.

1.1 Power system frequency

Nowadays power systems are mostly based on alternating current (AC) applications [1]. An alternating current occurs typically when the voltage source (i.e. potential energy source) alternates around zero with nil average value over a period T. Figure 2 shows an example on how an alternating signal oscillates between positive and negative value.

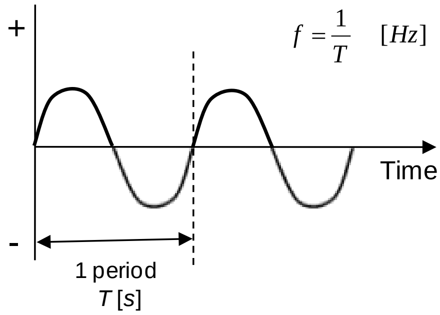

Figure 2: Alternating signal.

The time in seconds it takes the signal to complete one cycle is called the period (revolution), denoted typically by T. As indicated in Figure 2, the frequency *f* is the term used to describe the number of cycles of an alternative signal in a second[1].

The synchronous electrical angular speed ω_{e0} is defined as:

$$\omega_{e0} = 2 \cdot \pi \cdot f_0 \quad [rad/s] \quad (1)$$

where f_0 denotes the synchronous frequency (i.e. 50 Hz or 60Hz).

1.2 Synchronous power systems

A synchronous region is a regional power system that operates at a synchronized frequency and it is tied together during normal system operations[2].

All the grids in Europe are synchronized at 50Hz[3], while those in USA operate at 60Hz.

Figure 3 illustrates the five synchronous regions existing in the ENTSO-E association for European Network of Transmission System Operators for Electricity [2]:

- Continental Europe[4] synchronous grid
- Ireland's synchronous grid

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1 The number of cycles per second is also called hertz (Hz).

2 Notice that small local deviations can still appear due e.g. oscillations of the synchronous machines

3 In Europe, the period (revolution) of al alternating signal with a synchronous frequency is 20ms.

4 Includes the members of UCTE (Union for the Coordination of Transmission of Electricity) system, where Western Denmark is included.

- United Kingdom's synchronous grid
- Nordic synchronous grid[5]
- Baltic synchronous grid

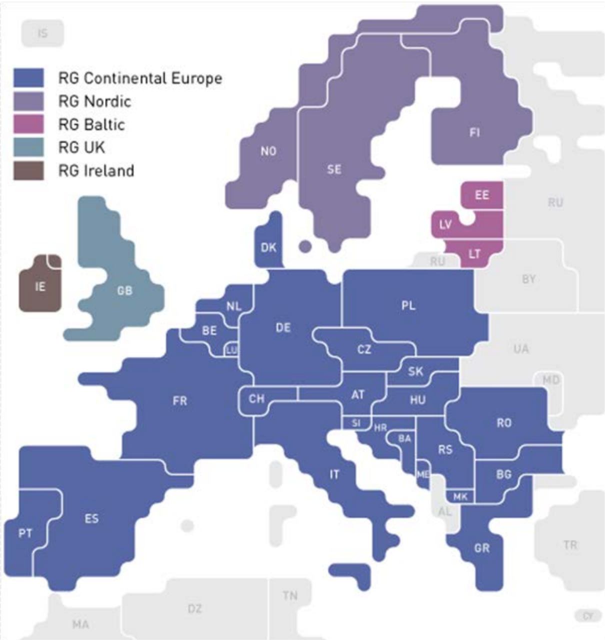


Figure 3: European network of transmission operators for electricity [2]

The main task of ENTSO-E is to maintain the frequencies in all five synchronous power systems balanced around the standard nominal value of 50Hz ensuring a safe European power supply.

Electrical power is typically produced by central power plants connected to the system. They convert mechanical energy into electrical energy by using synchronous generators. These generators are normally driven by prime movers such as steam turbines, gas turbines or hydro turbines.

Frequency in the power system is directly related to the rotating speed of the synchronous generators connected to the system. The frequency is synchronous all over the system, but small deviations can happen locally owing to the oscillations of the synchronous machines. Figure 4 illustrates a mechanical equivalent for the power system, namely a tandem configuration: the synchronous generators are running similar to a tandem, sharing the same load through the chain [3]. Load division between power plants is dependent on their speed-droop characteristics [1].

5 Nordic region consists of Finland, Sweden, Norway and Eastern Denmark – members of TSO association NORDEL.

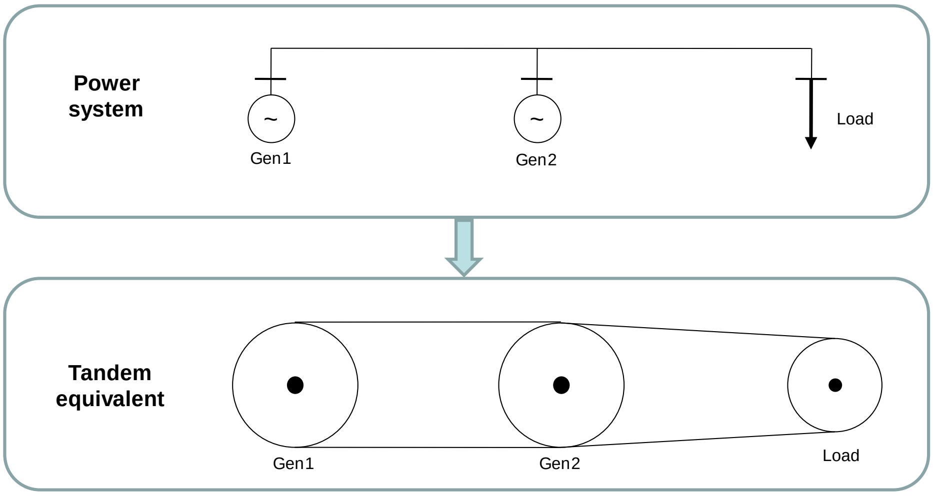


Figure 4: Mechanical equivalent of the power system operation.

1.3 Why frequency control in power systems?

Frequency in a power system is typically maintained in a narrow band. Nevertheless changes in frequency may occur if production or consumption exceeds its counterpart.

Figure 5 shows the legal operational states in the Nordic system. They are typically indicated by colours similar to those used in traffic lights, namely green for normal operation, yellow for alert operation and red for fault operation (disturbed or emergency operation). The colours are thus indirectly reflecting the security state of the power system.

The frequency in the Nordic power system is maintained during normal operation at the synchronous frequency 50Hz within operational limits of ±0.1Hz from the nominal value. If the frequency starts to deviate out of this range because of faults in the system, several consecutive warnings are activated in the power system. Notice that the system may still be in normal operation condition even in case of small power plant faults, as long as the frequency is maintained within the limits. However a large negative deviation can be a threat for the security of the system, because it might lead to potential blackout. A blackout means that the whole power system is going down due to problems in the generation of the electricity and as result the loads in the system are disconnected or are taken out of operation. On the other hand, a large positive deviation in frequency, due to generation exceeding the consumption, is more easily tackled and less harmful to the security of the entire power system.

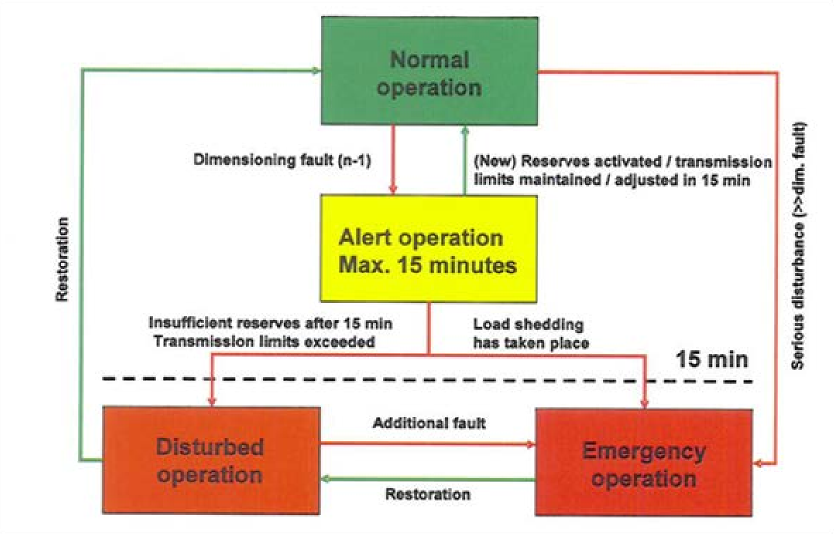


Figure 5: Operational states in the Nordic system [4].

One particular failure in the system, typically called as (n-1) dimensioning fault[6] situation, may lead the system into an alert operation (yellow area). Notice in Figure 5 that in this operational state, the power system should be restored in less than 15 minutes; otherwise the system enters disturbed operation state. In this situation, many systems can be protected from frequency collapse by importing large amounts of power from neighbouring systems to counter balance for the lost generation[7]. Beside power plants, a frequency drop in a power system is also registered by the loads. Another way to solve an emergency situation and to prevent a frequency collapse is therefore load shedding, where the least important loads are shed/ disconnected first. A load shedding can be performed manually or automatically as a part of security protection system. In an emergency situation, it is preferred to shed loads in the system to avoid a complete blackout, where all power plants in the system would disconnect. In the extreme situation of a blackout it may take very long time to recover the frequency. Another way to manage an emergency situation is to split up the network and isolate the part of the grid, which has caused emergency situation.

Figure 6 indicates that in an emergency state, power actions on the HVDC (high voltage DC) links are taken in the Nordic system.

6 A dimensioning fault is defined in the grid codes as the situation when the biggest component in the system falls.

7 Denmark can for example import power generation from Germany or Norway in case fault operation.

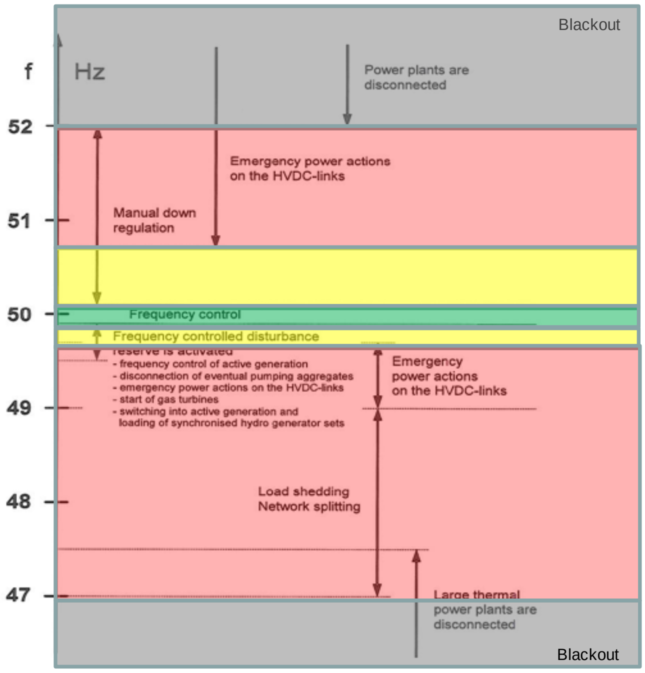


Figure 6: Frequency controlled actions in the Nordic system [4].

A sudden connection of a large load or disconnection of a generating unit by the protection equipment may result in a significant distortion of the power balance in the power system. This imbalance, which is initially covered by the release of the kinetic energy of rotating masses connected to the system, has as result a drastic drop in the frequency - as indicated in Figure 7. The rate at which the frequency drops, depends on the total angular momentum (rolling inertia) in the system, calculated as the sum of the angular momenta of all generators and spinning loads connected to the system.

The attention in the frequency control studies existing in the literature is typically drawn to the under-frequency events, namely to the events that lead to frequency drops. Frequency drops are also in focus in the present report.

The process of stabilization the frequency to a steady state (constant) value is typically performed through the primary control function, while the process of restoring the frequency back to the steady state corresponding to the nominal frequency value (i.e. 50Hz in the European system) is known as secondary control.

Primary control is provided automatically (within typically 30 seconds after a frequency drop) by fast generating units through a frequency droop (usually proportional) control loop. In the case of a frequency drop, these generating units start immediately to produce more power to avoid a further decrease of the frequency in the system. As illustrated in Figure 7, the task of primary control is to bring the frequency back to short term acceptable frequency values. During primary control, the frequency is thus stabilized temporarily to a lower level than the initial one, the system being still in an alert condition (i.e. yellow area).

Secondary control, also called Load Frequency Control, is a supplementary control loop, much slower than the primary control, which re-distributes load among the various generating units in order to restore the frequency back to its nominal value. In secondary control, the power setpoints of the generators are adjusted in order to compensate for the remaining frequency error after the primary control function. In the Danish power system, the secondary response must be delivered within 15 minutes. Secondary control is made either via automatic generation control systems (i.e. Continental Europe synchronous grid) where available, or via manual intervention by the system operators (i.e. in Nordic System).

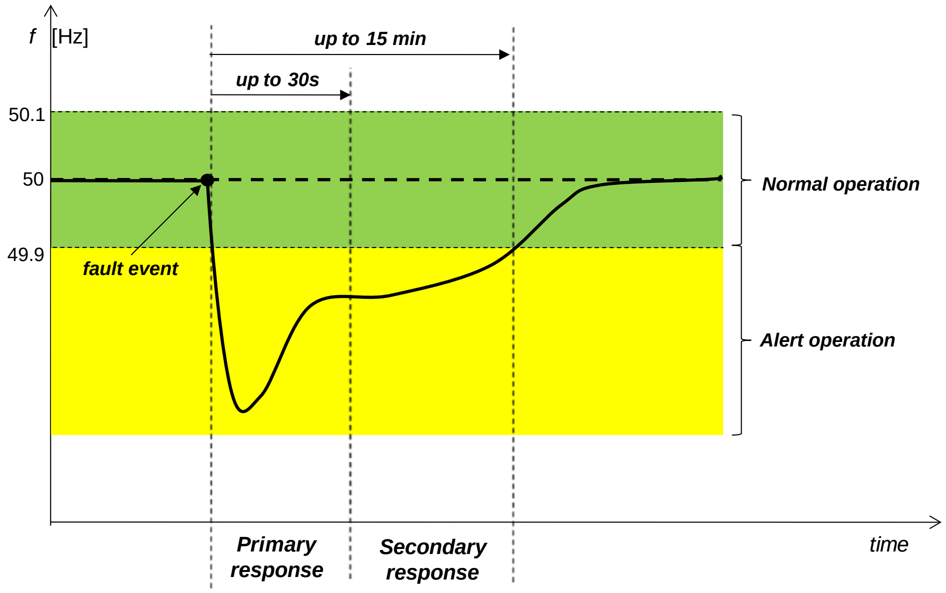


Figure 7: Frequency control in the Danish power system – for a fault event e.g. when a generating unit trips (disconnects).

In this report, the mechanisms for primary frequency control in island power systems are addressed and modelled. Primary control is implemented on a purely local level (on power plant level), i.e. without any coordination between different power plants.

1.4 Equation of motion (EoM)

Before presenting the equation of motion of the power system, it is useful to visualise the analogies between quantities and relationships associated with rotational motion with those associated with linear motion, since the latter is more familiar and easy to understand. Table 1 summarizes these analogies.

Table 1: Analogies between linear and rotational motion. Source: [1]

Linear Motion			Rotational Motion		
Quantity	Symbol/ Equation	SI unit	Quantity	Symbol/ Equation	SI unit
Length	L	*m*	**Rotational angle**	θ	*rad*
Mass	m	*kg*	**Moment of inertia**	$I = \int r^2 dm$	$kg \cdot m^2$
Velocity	v	*m/s*	**Angular velocity**	$\omega = d\theta / dt$	*rad/s*
Linear Momentum	$p = m \cdot v$	*kg m/s*	**Angular momentum**	$M = I \cdot \omega$	$kg \cdot m^2 / s$
Acceleration	a	m / s^2	**Angular acceleration**	$\alpha = d\omega / dt$	rad / s^2
Force	*F=m ·a*	N $[kg \cdot m / s^2]$	**Torque**	$T = I \cdot \alpha$	$N \cdot m$ $[kg \cdot m^2 / s^2]$
Power	***P*** *=F·v*	*W*	**Power**	$P = T \cdot \omega$	*W*

Power plants (generating units) transform other sources of energy in the process of producing electrical energy. For example hydraulic, heat, solar, wind sources of energy can be used in the production of electrical energy, as illustrated in Figure 8. Notice that the generator is driven by a prime mover, which can be for example a steam, gas or hydro turbine.

From the viewpoint of frequency control, the power system can be thought as only one large power plant supplying one load. Two opposite torques act on the large rotating mass, namely a mechanical torque T_{mec} and an electrical torque T_{elec}, as shown in Figure 8. In case of an imbalance between these torques, the rotating mass will experience an angular acceleration or deceleration $d\omega/dt$ according to Newton's Second Law for rotating systems:

$$T_{mech} - T_{elec} = I \frac{d\omega}{dt} \qquad \left[kg \cdot m^2 / s^2\right] \qquad (2)$$

where I is the moment of inertia of the rotational mass. Notice that the inertia *I* has a stabilization effect, i.e. in case of an imbalance in torques, the frequency change is smaller for a system with high inertia compared to a system with low inertia, meaning that a high inertia system is more stable.

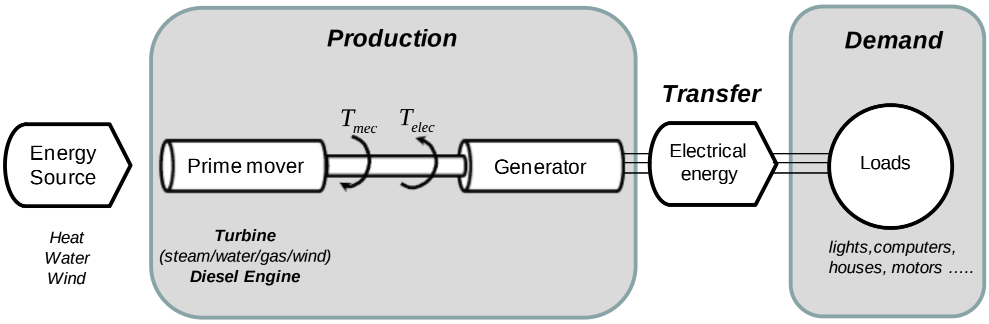


Figure 8: Simplified electric power system with a single prime mover, generator and load.

The previous equation is also known as equation of motion (EoM). The equation of motion includes generally rotating components of the power system i.e. generating units and loads. Notice that a sudden increase in the generation, namely in the mechanical torque T_{mec}, implies an increase in the rotational speed ω and thus in the frequency f in the system. Vice-versa, an increase in the consumption, namely in the electrical torque T_{elec}, implies a decrease in the frequency. It is worth noticing that the larger the inertia of the rotating mass is, the smaller the speed rate-of-change following a torque imbalance is.
The equation of motion can also be expressed in power terms by using the proportional relationship between power and torque:

$$P = T\,\omega \qquad \left[\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^3\right] \qquad (3)$$

By applying this relationship, the equation of motion in power terms (power balance equation) can be expressed as:

$$P_{mech} - P_{elec} = I\omega\frac{d\omega}{dt} = M\frac{d\omega}{dt} \qquad (4)$$

where P_{mech} is the mechanical power, P_{elec} is the electrical power and M is the angular momentum of a rotating system, defined as:

$$M = I\,\omega \qquad \left[\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}\right] \qquad (5)$$

2. Generating units components

The scope of this chapter is simply to provide an overview over the components of a generating unit, addressing shortly the role and the inputs/outputs of each component in a generating unit. Basic topics on generators, prime movers and governors are presented. No models are presented in this chapter, since this is the topic of a subsequent chapter.

The principle of the balance between production and consumption in a power system can be best presented by considering an isolated generating unit supplying a local load, as illustrated in Figure 9. The electrical energy is produced by a generator driven by a prime mover, which is usually a turbine or a diesel engine. The turbine is equipped with a governor, which controls the speed of the generating unit according to a predefined power-frequency characteristic.

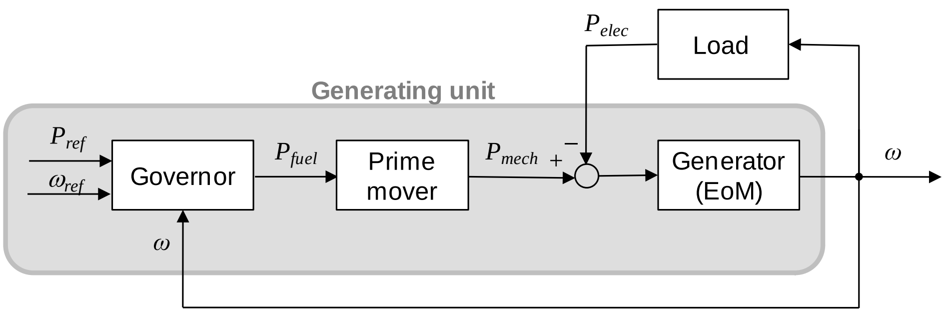


Figure 9: Generating unit supplying isolated load.

2.1 Generator

As illustrated in Figure 10, generators have typically coils of wire mounted in a certain pattern in a stationary housing, called stator, where the voltage is produced due to the magnetic field provided by the spinning rotor.

The rotor, which is the rotating part of the generator, is responsible for the inducing magnetic field of the generator. A rotor can have a permanent magnet or an electromagnet. In the case of an electromagnet, a magnetic field is produced around the rotor windings due to the current flowing through them, when they are connected to a stationary voltage source[8]. By its rotation, the rotor's magnetic field, that passes the stator windings, induces voltages at the stator's terminals. A magnetic field is further created around stator's windings (induced field), if current flows in the stator windings, i.e. the generator is loaded.

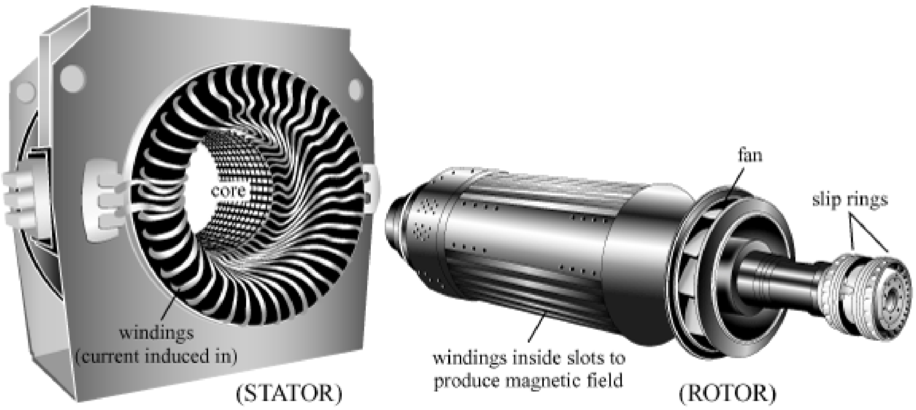
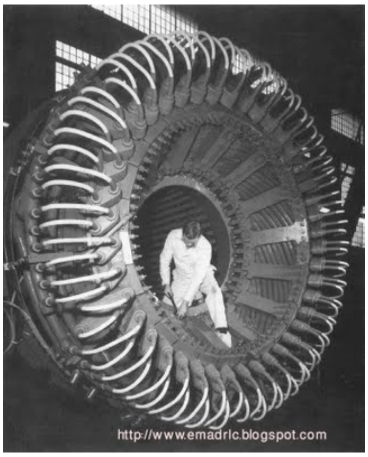


Figure 10: Generator's stator and rotor.

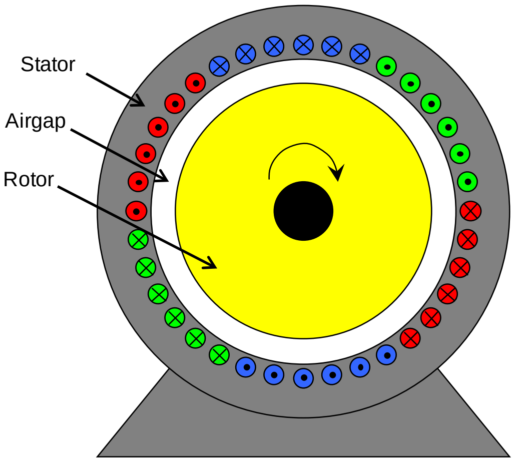
When the magnetic field of the stator is following the magnetic field of the rotor at the same speed, the generator is called synchronous. Such generator is synchronized to the power system, namely its shaft speed is the same as the frequency of the voltage in the power system. If the magnetic fields are not following each other synchronously the generator is called asynchronous.

The synchronous generator is the workhorse for the generation of electricity in the power system. Synchronous generators with power rating of several hundred MW are very common. Figure 11a) shows as an example the size of a 660MW synchronous generator. Figure 11b) illustrates a front side visual perspective of the generator, namely the inner rotor and the outer stator with the three-phase windings, separated by an airgap.

8 The rotor windings are connected to a stationary voltage source via electrical contacts (slip rings).



a)



b)

Figure 11: a) Size example of a 660MW AC generator.

Source: http://emadric.blogspot.com b) Generator cross section sketch.

For synchronous generators in power plants, the current which generates the rotor's magnetic field is normally supplied by an exciter (DC - Direct Current voltage source).

2.1.1 Electricity generation - basic principles

It is not the task of this section to provide a detailed description of the electrical system terminologies. However for the sake of a good understanding of how electricity in the power system is produced, some basic concepts are quickly addressed.

There are actually two physical laws that describe how the electric power system works:

- *Faraday's Law* - says that by placing a coil of wire (conductor) next to a moving/rotating magnet, an AC (Alternating Current) voltage is induced in that coil. This principle is graphically illustrated in Figure 12 both for single-phase and three-phase AC voltage generation. Notice how by placing one coil of wire next to a moving magnetic field (i.e. magnet), a voltage is induced in the coil. Each time the North pole and the South pole passes the coil, with the layout adopted in the figure, the induced voltage in the coil has a positive peak (denoted by N) and a negative peak (denoted by S), respectively. Figure 12 shows also that when three coils spaced physically with 120 degrees apart in a 360 degree circle are placed in the presence of a moving magnetic field, a so called three-phase AC voltage is induced.
- *Ampere's law* - says that a current flowing through a wire creates a magnetic field around the wire. This law describes how a magnetic field can be created, when the coil of the rotor is connected to a voltage source (e.g. battery, exciter). The induced magnetic field in the rotor's coil has of course a north and a south pole.

The simple combination of these two laws depicts the phenomena behind how electric generators produce electricity. Generators, for example, use a moving magnetic field next to a coil to produce voltage, while the magnetic field is created by applying a voltage (e.g. battery) to a coil of wires.

A three-phase AC generator has basically three single-phase windings mounted on the stator. The windings are physically spaced so that the angle between the magnetic fields induced in each winding is 120 degrees – as illustrated in Figure 12.

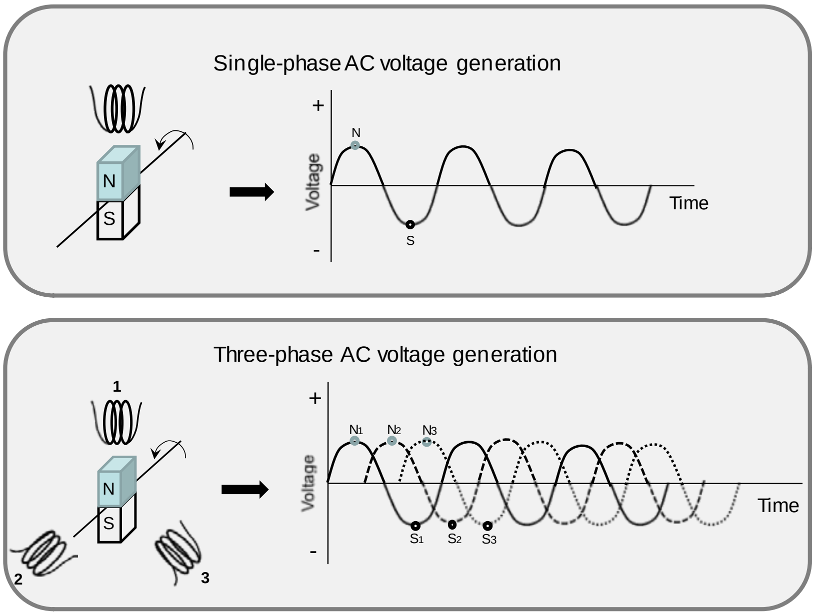


Figure 12: Single-phase and three-phase voltage generation.

2.1.2 Rotor poles

Many synchronous generators in the power system have more than one pole-pair (i.e. more than two poles) in the rotor. They may have four, six, eight or more poles, depending on the desired speed of rotation. Figure 13 illustrates the effect of increasing the number of poles in the generator rotor, namely higher number of poles implies slower rotational speed.

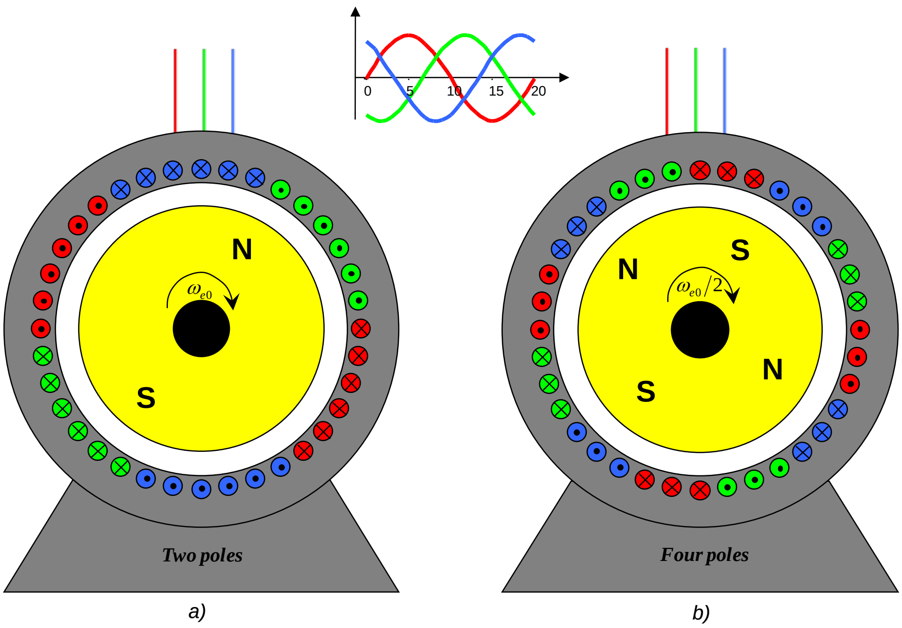


Figure 13: Three phase generators with different rotor poles:
a) Generator with two poles (one pole-pair $N_{pp}=1$)
b) Generator with four poles (two pole-pairs $N_{pp}=2$)

In a synchronous generator the generated frequency is determined by the rotational speed and the number of poles in the generator. Synchronous generators, even with differing number of poles, can thus operate in parallel at the same synchronous frequency, due to their different rotational speeds.

The synchronous rotor speed ω_{gen0} of a generator with two poles ($N_{pp} = 1$) is equal to the synchronous electrical angular speed ω_{e0}, namely it corresponds to the synchronous frequency of the AC (Alternating Current) voltage on the terminals. This means that the rotor speed of a synchronous generator with one pole pair can be provided by a measurement of the frequency of the voltage:

$$N_{pp} = 1 \quad \rightarrow \quad \omega_{gen0} = \omega_{e0} = 2 \cdot \pi \cdot f_0 \qquad [rad/s] \qquad (6)$$

Figure 13 shows that for a generator with two pole-pairs, the windings in the coil for each phase of the stator are rearranged according to the increased number of poles in the rotor. In this case, one cycle of voltage corresponds to only half rotation compared to that for a generation with one pole-pair, namely:

$$N_{pp} = 2 \quad \rightarrow \quad \omega_{gen0} = \frac{\omega_{e0}}{2} = \frac{2 \cdot \pi \cdot f_0}{2} \qquad [rad/s] \qquad (7)$$

Increasing the number of magnetic poles on the rotor enables thus the generator's rotor speed to be slower and still maintain the same electrical output frequency f_0.

For a certain number of pole-pairs N_{pp}, the rotor speed of the synchronous generator ω_{gen0} can be expressed as follows:

$$N_{pp} \quad \rightarrow \quad \omega_{gen0} = \frac{\omega_{e0}}{N_{pp}} = \frac{2 \cdot \pi \cdot f_0}{N_{pp}} \qquad [rad/s] \qquad (8)$$

Its expression in rotations per minute [rpm] is:

$$n_{gen0} = 60 \cdot \frac{f_0}{N_{pp}} \qquad [rpm] \qquad (9)$$

The electrical power P_{gen} produced by a synchronous generator is depending on its rotational speed ω_{gen0} and torque T_{gen} as follows:

$$P_{gen} = T_{gen}\, \omega_{gen0} \qquad [W] \qquad (10)$$

Notice that slower rotational speed implies higher torque to get the same electrical power, i.e. if a 2-poles and a 4-poles synchronous generator provide same electrical power, then the torque of the 4-poles generator is two times bigger the torque of the 2-poles generator.

2.2 Prime mover

As illustrated in Figure 9, the production of electrical energy by mechanical means always requires a prime mover to drive a generator.

A prime mover is mainly a turbine or a diesel engine, used to convert an energy source (i.e. steam, hydro, wind) into mechanical rotational movement of the rotor of a generator. A prime mover has typically a throttle valve, which position (open/close) is adjusted continuously by a control device called governor.

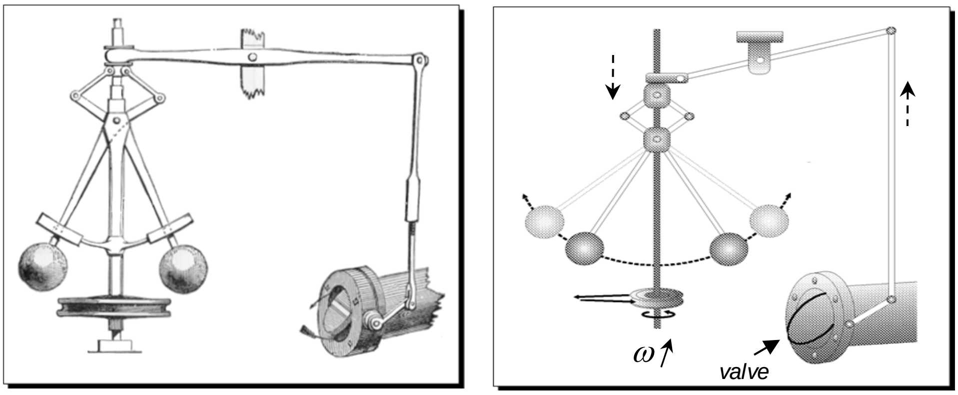
The input of the prime mover is a power flow related with the position of the throttle valve, while its output is the mechanical power, which drives the generator.

A simple model for a prime mover is presented in Section 5.3.

2.3 Governor

Each prime mover is equipped with a governor that senses the speed changes of the turbine and controls it to a predefined setpoint value. Governor is thus a mechanism, which controls the unit speed by adjusting (increasing or decreasing) the power flow into the prime mover to change the mechanical power output to compensate for load changes and thus to maintain frequency (speed) at a desired value.

For many years governors were of mechanical-hydraulic type, which was based on the widely used Watt flyballs centrifugal mechanism, shown in Figure 14, as the speed responsive device. Depending on turbine speed, the flyballs actuate a mechanical-hydraulic system to open or close the throttle valve of the prime mover and adjust (i.e. increase or decrease) the energy input to the prime mover. In this way, the governor manages to maintain the frequency (speed) at a desired value. As illustrated in Figure 14b), an increase in the speed ω causes the flyballs to move away from the rotating shaft, which forces the throttle valve to close a little. This action implies that the power flow through the prime mover is reduced and the frequency (speed) is controlled back to its setpoint.



a) b)

Figure 14: James Watt flyballs governor: a) in repose b) in action

Nowadays, electronic governors are more frequently applied. They measure the frequency and actuate hydraulic devices to control gate or throttle position without the use of flyballs [5].

There are two types of governors:

- The isochronous governor[9], which is the simplest type governor, controls and maintains the frequency regardless of the generator loading. It adjusts the prime mover's valve to bring the frequency back to its setpoint value. This governor mode is also referred to as frequency control operation mode. Such governor works satisfactorily when only one generator in the system has to respond to changes in load. This type of governor is problematic to use when several generators, connected to run

[9] Isochronous means constant speed.

in parallel, have to share the load in the system, because they can work against each other, trying to control the frequency to their own settings [6].

- The speed-droop governor controls the frequency taking the generator loading into account. It provides the primary control function, described in Section 1.3. It is utilized when two or more generating units connected to the power system have to share a load change.

3. Generating unit characteristics

The scope of this chapter is to explain the basic characteristics of a generating unit, like droop, power setpoint, nominal power, frequency bias and how multiple generating units can share load changes in the system.

3.1.1 Droop characteristic

The droop characteristic of a generating unit is a steady-state power-speed characteristic, as illustrated in Figure 15. It reflects an idealized relation between the system frequency and the power, which can be delivered by a generating unit into the system during its operation. Notice that the power of the unit slides back and forth along the droop characteristic in the attempt to stabilize supply and demand regardless of the frequency. According to the droop curve a decrease in frequency yields to a higher power production and vice-versa.

By definition:

- The power setpoint P_{ref} of a unit is the power, which is delivered by the unit at the nominal frequency f_n.
- The nominal power P_n of the unit is the maximum power, which can be continuously produced by the unit, and as illustrated in Figure 15, it corresponds to full load condition.

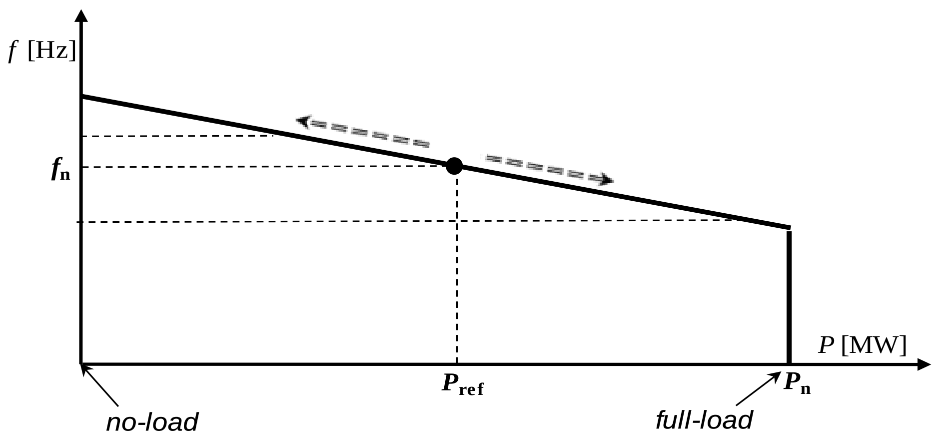


Figure 15: Steady-state droop characteristic.

Besides providing a graphical definition of droop, Figure 16 also illustrates the effect of changing the droop of a generating unit. The droop R of a unit is defined as the percentage of the frequency change required for a governor to move a unit from no-load to full-load or vice-versa[10]. Notice that by definition the droop is a non-negative value.

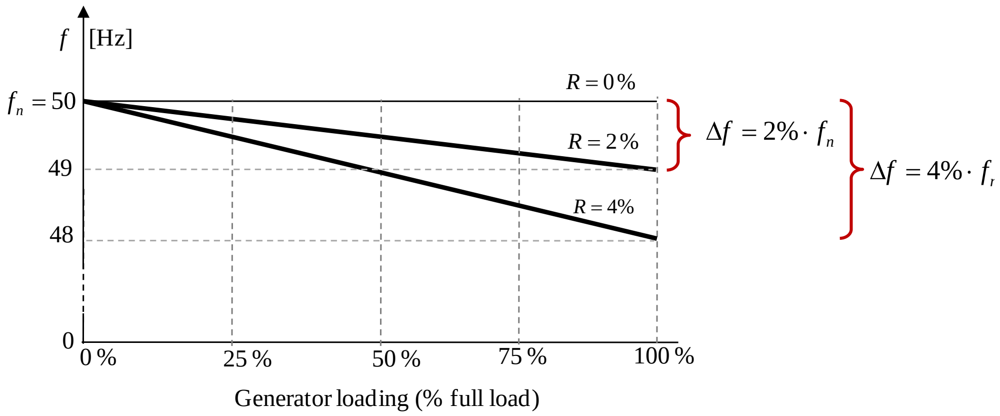


Figure 16: Graphical signification of droop.

Figure 17 indicates graphically different characteristics related to a generating unit: the droop *R*, the nominal power P_n, the frequency bias *B*, the change in power ΔP from power setpoint and the frequency deviation Δf from the initial steady state. By writing the proportional relations sketched in Figure 17, the mathematical expression of the droop can easily be obtained as follows:

$$\frac{\Delta f}{R \cdot f_n} = -\frac{\Delta P}{P_n} \quad \Rightarrow \quad R = -\frac{\Delta f / f_n}{\Delta P / P_n} \qquad [\%] \tag{11}$$

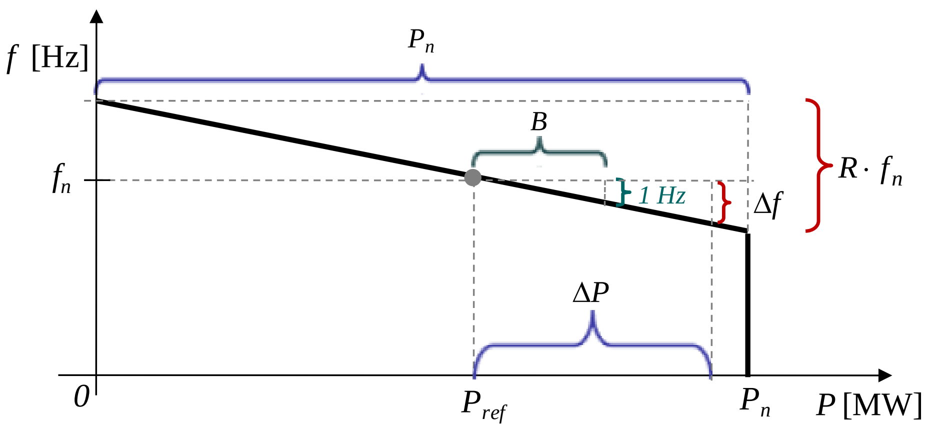


Figure 17: Droop R and frequency bias B graphically illustration.

This is the mathematical expression of the power balance between generation and demand, when the system is in steady state.

10 For example, a 4% droop means that a 4% frequency deviation causes 100% change in the power output of the generating unit.

If the generating units in a power system do not have the same droops, an equivalent droop R_{PS} for the whole power system can be calculated according to the following expression:

$$\frac{1}{R_{PS}} = \frac{\sum_{i=1}^{N_{units}} \frac{P_{n,i}}{R_i}}{\sum_{i=1}^{N_{units}} P_{n,i}} \qquad (12)$$

where R_i and $P_{n,i}$ denote the droop and the nominal power of unit i, respectively. N_{units} is the number of generating units in the system. Notice that if the individual droops are equal, the total droop of the system is equal with one individual droop.

3.2 Frequency bias

Another characteristic of a unit, illustrated graphically in Figure 17, is the frequency bias B.

Frequency bias reflects the power ability (reserve) of the unit in [MW] to compensate for a frequency deviation of 1Hz, as following[11]:

$$B = -\frac{\Delta P}{\Delta f} \qquad [\text{MW/Hz}] \qquad (13)$$

Notice that the frequency bias is per definition a positive value and can further be expressed as:

$$B = \frac{P_n}{R \cdot f_n} \qquad [\text{MW/Hz}] \qquad (14)$$

This formula can also be used to calculate the bias of a whole power system B_{PS}, when it is assumed that all units existing in a power system are sharing the same nominal frequency f_n and droop R:

$$B_{PS} = \frac{P_{n,PS}}{R \cdot f_n} \qquad [\text{MW/Hz}] \qquad (15)$$

where $P_{n,PS}$ is the nominal power of the whole power system. Otherwise the total bias of the power system B_{PS} has to be calculated by simply adding the biases of the individuals generating units B_i, as follows:

$$B_{PS} = \sum_{i=1}^{N_{units}} B_i \qquad [\text{MW/Hz}] \qquad (16)$$

Notice that the larger the number of generating units running in a power system is, the larger the frequency bias may be. The droop characteristic of a system with a very large number of units tends to be almost horizontal. This means that frequency deviation in a system with a

11 For example, if the bias of a unit is B=100 [MW/Hz], it means that for a decreasing in frequency of 1Hz, the unit is able to increase its power production with 100MW.

very large number of units due to a change in power may be very small even for a large power change.

Once the frequency bias B_{PS} of a power system is determined, the steady state deviation in frequency from the initial steady state produced by a changing in power ΔP can be calculated as follows:

$$\Delta f = -\frac{\Delta P}{B_{PS}} \qquad [Hz] \qquad (17)$$

It is worth noticing that frequency deviation Δf refers to the new steady state (i.e. new constant value) the system frequency has reached after a transient change in power ΔP, as depicted in Figure 19. During the first seconds of the transient change ΔP, the frequency deviation typically gets higher values.

3.3 Result of changing the droop or the power setpoint of a unit

Figure 18 illustrates the implication of changing the droop or the power setpoint of a unit.

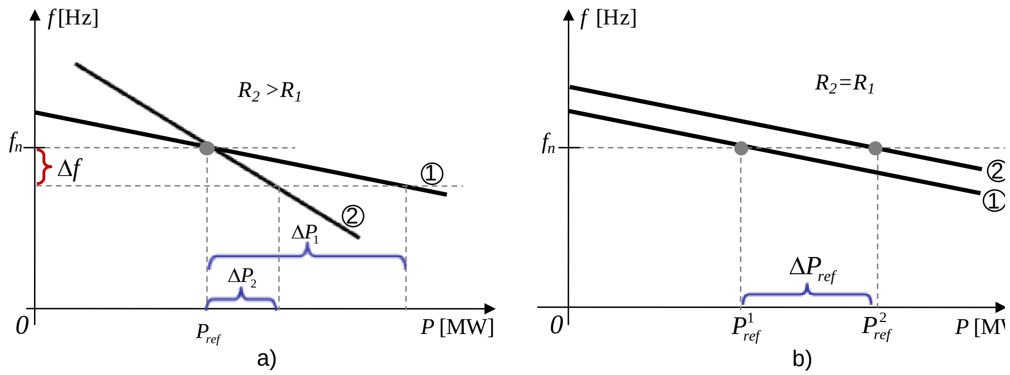


Figure 18: Implication of changing the droop or the power setpoint of a unit.

Notice in Figure 18a), that the higher the droop R, the less responsive the generating unit is to changes Δf in frequency, namely the change in production ΔP of a unit with a droop characteristic with a higher droop R_2 is smaller than that for a unit with a droop characteristic with a smaller droop R_1, i.e. $\Delta P_2 < \Delta P_1$. The change in production ΔP of a unit refers here to the new steady state power the unit should reach in order to compensate for the frequency change in the system.

Figure 18b) illustrates that a change in the power setpoint P_{ref} of a unit implies that the unit runs on a parallel droop characteristic (i.e. $R_1=R_2$).

It has already been mentioned in the abstract that the attention of this report, as the frame of the present course, is directed specially towards primary control action of generating units, namely the frequency droop control loop.

Generating units providing primary control in the power system are characterised by a fixed power setpoint P_{ref} , namely they are producing power only according to one droop characteristic. This means that in primary control $\Delta P_{ref}=0$.

In Section 1.3, it has been illustrated that during primary control, following a change in load, the frequency in the system does not return to its initial steady state. In order to force the frequency to return to the initial steady state, the power setpoint and thus the droop characteristic of the unit must be shifted (as for example illustrated in Figure 18b), from position *1* to *2*. It is said that a generating unit having this ability participates in the secondary control of the system. In secondary control a unit has to produce more at the same frequency. This means that the power setpoint of the unit has to be changed, i.e. $\Delta P_{ref} \neq 0$, and that the droop characteristic of the unit has to be moved upward, as shown in Figure 18b. Notice that no change in power setpoints can force a unit to exceed its nominal power. As a conclusion, the power output of a generating unit at a given frequency can be modified by adjusting its power setpoint P_{ref}, which has as result the shifting of the droop characteristic.

Remark that both terminologies described in this section, i.e. droop R and power setpoint P_{ref}, are essential for describing precisely the operation of a generating unit. It is thus not sufficient to have given the droop value only for a generating unit, but also its power setpoint value P_{ref}, i.e. to know how much the unit has to produce at the nominal frequency.

3.4 Unit response to changes in frequency in an infinite power system

If a unit is connected to a large interconnected system, it can be modelled, as it is connected to an infinite power system. In such system the outputs of the unit are locked to the system values and they cannot be changed by any action on the unit. Hence the infinite power system dictates the frequency, while the unit only controls its power (not also the frequency).

Figure 19 illustrates the time response of a generating unit, when the frequency decreases suddenly in the infinite power system due to e.g. an increase in load. The turbine governor opens the main control valves to increase the flow of working fluid through the valve to the prime mover (turbine) and so it increases the turbine mechanical power output. The unit increases thus its production, as result of the decrease in the frequency. Notice that the system reaches another steady state lower than the nominal one, after the power output of the generating unit has been increased with ΔP.

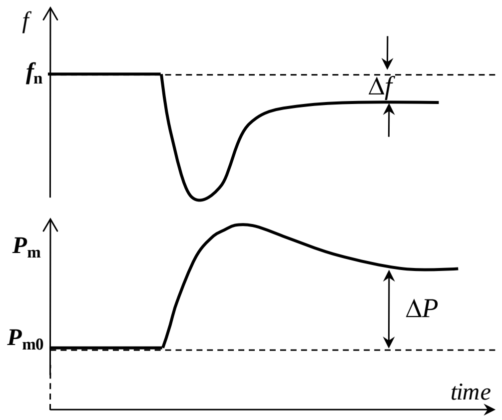


Figure 19: Time response of a unit when the system frequency drops.

Figure 20 illustrates how the generator output slides back and forth along the droop characteristic in the attempt to stabilise supply and demand, when changes in frequency occur in the system, i.e. generator production is reduced when the frequency in the system is increasing and vice-versa.

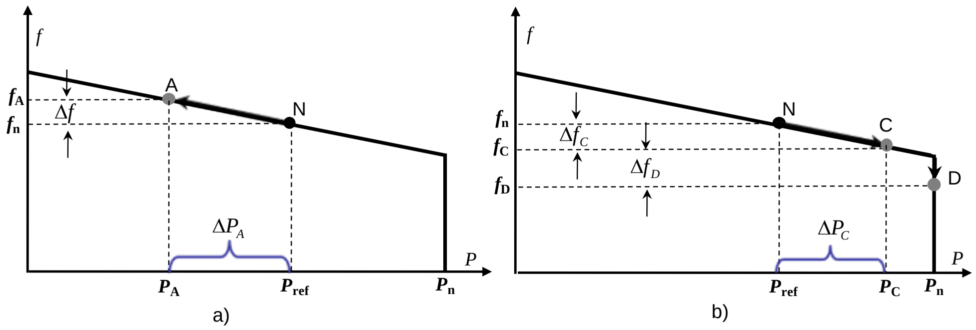


Figure 20: Unit response to frequency changing:
a) increase in frequency b) decrease in frequency

Notice in Figure 20b), that if the frequency decreases even lower than the frequency value corresponding to full load operation, the generating unit comes in a limited power production situation (as it is for example the operation case D). In point D the unit is operating at its upper power limit and has therefore no ability to further compensate for deviations in frequency. The frequency bias of the generating unit in this situation is equal to zero. Notice thus that the frequency bias of the whole system is dependent on the number of units, which are not operating at full load.

3.5 Load sharing in island power system

All generating units connected to a power system operate at the same frequency. They share and supply any load change in the power system according to their droop characteristic. Figure 21 illustrates a load share example between two units for a sudden increase in load.

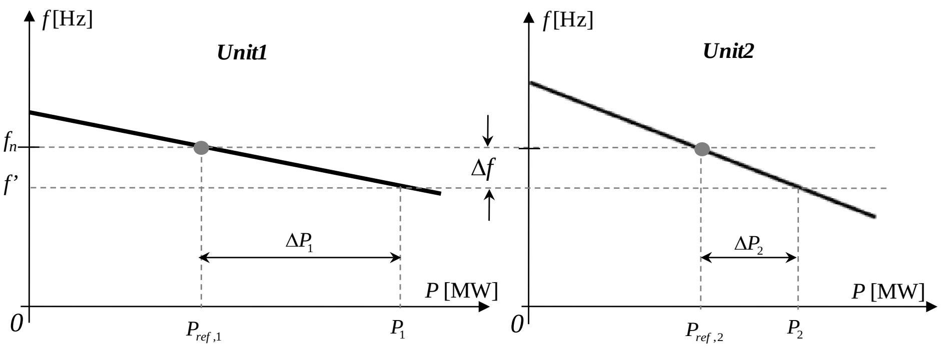


Figure 21: Load sharing example between two parallel generating units.

The two generating units are initially at nominal frequency f_n, with power outputs corresponding to their power setpoints $P_{ref,1}$ and $P_{ref,2}$, respectively. Notice that an increase in load causes both units to slow down (as the frequency decreases) and to increase their power output according to their droop characteristic. A new steady state operating frequency f' is established by both units corresponding to a new balance between generation and demand.

The change in generated power of the two units is then:

$$\begin{aligned} \Delta P_1 &= P_1 - P_{ref,1} = -B_1 \cdot \Delta f \\ \Delta P_2 &= P_2 - P_{ref,2} = -B_2 \cdot \Delta f \end{aligned} \qquad (18)$$

Notice that the change in the generated power of the two units is, as expected, proportional with their frequency bias, namely their ability to react and compensate for frequency deviations:

$$\frac{\Delta P_1}{\Delta P_2} = \frac{B_1}{B_2} = \frac{P_{n,1}}{R_1} \cdot \frac{R_2}{P_{n,2}} \quad (19)$$

This expression shows that the change in the generated power of two units with equal nominal powers ($P_{n,1}$=$P_{n,2}$) is inverse proportional to their droops. This implies also that when the generators have equal droops, the generators share the total load in proportion to their nominal powers:

$$\frac{\Delta P_1}{\Delta P_2} = \frac{P_{n1}}{P_{n2}} \quad (20)$$

In a power system with N_{units} generating units:

- The overall change in the total generated power in the system ΔP_{genPS} caused by a deviation in frequency Δf, can be calculated as the sum of changes in power in all units, as follows:

$$\Delta P_{genPS} = \sum_{i=1}^{N_{units}} \Delta P_i = \sum_{i=1}^{N_{units}} (P_i - P_{ref,i}) = \sum_{i=1}^{N_{units}} P_i - \sum_{i=1}^{N_{units}} P_{ref,i} = -\Delta f \cdot \sum_{i=1}^{N_{units}} B_i \quad (21)$$

- The total generated power P_{genPS} can be expressed as follows:

$$P_{genPS} = \sum_{i=1}^{N_{units}} P_i = \sum_{i=1}^{N_{units}} (P_{ref,i} + \Delta P_i) = \sum_{i=1}^{N_{units}} P_{ref,i} + \sum_{i=1}^{N_{units}} \Delta P_i = \sum_{i=1}^{N_{units}} P_{ref,i} - \Delta f \cdot \sum_{i=1}^{N_{units}} B_i \quad (22)$$

The system frequency reaches steady-state at a value that corresponds to the situation when power production (sum of power for on-line units MW) is equal to the system load (demand MW):

$$P_{genPS} = P_{loadPS} \quad (23)$$

4. Modeling representations

In this chapter different modeling representations, typically used in the electric power system literature, are described. Per-units and delta models are presented and exemplified. Some basics of Laplace transform are reviewed and an overview of its use in dynamic system analysis is given.

4.1 Per-unit representation

The per-unit system, mostly used in the electric power systems literature, is a scaling procedure, which normalizes the value of a variable with a corresponding base value.

Essentially, the idea of per-unit system is to express the value of a physical variable as percentage of its corresponding base value [1].

The per-unit value x_{pu} of a certain quantity x can be calculated as follows:

$$x_{pu} = \frac{x}{x_{base}} \tag{24}$$

where x_{base} is the base value and x is the value in physical units. The base value is a normalization factor. For a principal variable the base value is typically chosen so that its per-unit value is equal to one under rated conditions. In other words this means that in the per-unit system, 1pu corresponds to 100%.

The base value has always the same unit as the physical value and therefore the per-unit value is dimensionless. For the variables where the base values are chosen independently (i.e. power, frequency), the per-unit value x_{pu} is typically in the interval $0 \leq x_{pu} \leq 1$ (e.g. for power varying from 0 to 100 %) or close to 1 (e.g. for frequency or rotational speed). For other variables like inertia and angular momentum, where the base values are calculated based on interrelated quantities, per-unit values can be much bigger than 1. Furthermore some other variables are typically maintained in the physical units, e.g. phase angle (rad).

Table 2 provides an overview on the typical base value and on the calculation of per-unit value for different common quantities like: power, frequency, speed, torque, angular momentum, moment of inertia and damping. Notice that some base values in Table 2 are chosen independently (e.g. power and frequency), while other are defined directly based on relationships between interrelated quantities (e.g. torque, angular momentum, moment of inertia) [12].

As shown in Table 2, the base value for the torque is calculated in the similar way as the base value for the angular momentum:

$$\begin{aligned} T_{base} &= \frac{P_{base}}{\omega_{base}} \\ M_{base} &= \frac{P_{base}}{s_{base} \cdot \omega_{base}} = \frac{P_{base}}{\omega_{base}} \quad \text{as} \quad s_{base} = 1 \end{aligned} \tag{25}$$

eventhough their units are different, namely [kg m^2/s^2] for torque and [kg m^2/s] for angular momentum. The reason for this is that the unit for the base value of Laplace frequency operator s is [s^{-1}].

[12] A selection of two independent base values is typically enough to determine the base values of the others.

Table 2: Base and per-unit values.

Variable name	Base value	Per-unit value
Power *P* [W=kg m²/s³]	P_{base} = nominal power	$P_{pu} = \frac{P}{P_{base}}$
Frequency *f* [Hz]	$f_{base} = f_0 = \begin{cases} \text{50Hz in Europe} \\ \text{60Hz in USA} \end{cases}$	$f_{pu} = \frac{f}{f_{base}}$
Speed *ω* [rad/s]	$\omega_{base} = 2 \cdot \pi \cdot f_{base}$	$\omega_{pu} = \frac{\omega}{\omega_{base}}$
Torque *T* [kg m²/s²]	$T_{base} = \frac{P_{base}}{\omega_{base}}$	$T_{pu} = \frac{T}{T_{base}}$
Time *t* [s]	$t_{base} = 1$	$t_{pu} = \frac{t}{t_{base}} = t$
Laplace frequency (operator) *s* [s⁻¹]	$s_{base} = 1$	$s_{pu} = \frac{s}{s_{base}} = s$
Angular momentum *M* [kg m²/s]	$M_{base} = \frac{P_{base}}{\omega_{base} \cdot s_{base}}$	$M_{pu} = \frac{M}{M_{base}}$
Moment of inertia *I* [kg m²]	$I_{base} = \frac{M_{base}}{\omega_{base}} = \frac{P_{base}}{(\omega_{base})^2 \cdot s_{base}}$	$I_{pu} = \frac{I}{I_{base}}$
Inertia constant *H* [s]	$H_{base} = 1$	$H_{pu} = \frac{H}{H_{base}} = H$
Damping **D** [kg m²/s²]	$D_{base} = \frac{P_{base}}{\omega_{base}} = T_{base}$	$D_{pu} = \frac{D}{D_{base}}$

Table 3 contains basic formulas for power and angular momentum, used in physical, base and per-unit calculations.

Table 3: Examples of physical, base and per-unit relations.

Physical relations	Base relations	Per-unit relations
$P = T \cdot \omega$	$P_{base} = T_{base} \cdot \omega_{base}$	$P_{pu} = T_{pu} \cdot \omega_{pu}$
$M = I \cdot \omega$	$M_{base} = I_{base} \cdot \omega_{base}$	$M_{pu} = I_{pu} \cdot \omega_{pu}$

Notice that each generation unit in a power system may have its own individual base corresponding to its own technology. The summation of the power generated by all the units in the system is typically performed using a common base[13]. A power system base is usually used as common base. By definition, in this report, the power system base is considered to be the power capacity of the whole system.

As mentioned previosly, a per-unit value can be in general calculated referred to an individual (local) base or to a global base. The calculation of a per-unit value for a quantity (like power, torque or inertia) in a new base $x_{pu}^{new\_base}$, based on its per-unit value in an old base $x_{pu}^{old\_base}$, can be calculated according to:

$$x_{pu}^{new\_base} = \frac{x}{x_{new\_base}} = x_{pu}^{old\_base} \cdot \frac{P_{old\_base}}{P_{new\_base}} \qquad (26)$$

where P_{old_base} and P_{new_base} are the old and the new considered power bases, respectively.

In spite of its abstractness, per-unit system offers some advantages like:

- Computational simplicity for both manual and automatic calculations, by eliminating units and expressing quantities as dimensionless ratios.
- Quick evaluation and understanding of the system condition, i.e. the per-unit or percentage value of a certain variable contains more information than its corresponding physical value. For example, if a power plant unit is set to produce 0.8pu, it means that the power plant unit has to produce 80% of its nominal/base value. The per-unit value may also convey a message whether the value is or is not an acceptable value.
- Easy comparison between power plants with different ratings or speeds of generators with different number of pole-pairs.

In the following, some examples are given on how to calculate per-unit values for different quantities.

4.1.1 Calculation of power in per-unit

The power in per-unit is calculated as:

$$P_{pu} = \frac{P}{P_{base}} \qquad (27)$$

where P is the power expressed in physical units (W), while P_{base} is the power base which is typically defined equal to the nominal power (i.e. maximum possible continuous power). This means that the power per-unit value is equal to one at rated conditions ($P_{pu}=1$ when $P=P_n$).

As mentioned previosly, a per-unit value can be in general calculated referred to an individual (local) base, or to a global (common) base, as for example the power system base.

The power base $P_{base,i}$ of an individual unit i is typically defined as: $P_{base,i} = P_{n,i}$, namely equal with the nominal power $P_{n,i}$ of the individual unit i, while the power base for the whole power system P_{basePS} is typically defined as the total sum of the individual nominal powers of all units in the system, i.e. the system installed capacity:

$$P_{basePS} = \sum_{i=1}^{N_{units}} P_{n,i} = \sum_{i=1}^{N_{units}} P_{base,i} \qquad (28)$$

[13] Individual machines can be typically treated using their own base system. However, the quantities of all components in the system are scaled to a common new base when the whole power system has to be analysed.

4.1.2 Calculation of speed in per-unit

The electrical speed in per-unit is calculated as:

$$\omega_{pu} = \frac{\omega}{\omega_{base}} \tag{29}$$

where ω is the speed expressed in physical units, while ω_{base} is the speed base, which is typically defined thus that the per-unit value is equal to one under normal conditions, namely $\omega_{pu}=1$, when the speed is equal to the synchronous electrical angular speed $\omega=\omega_{e0}=2\cdot\pi\cdot f_0$.

The speed base for the whole power system is typically defined as illustrated in Table 2:

$$\omega_{base,PS} = 2\cdot\pi\cdot f_0 \tag{30}$$

Notice that speed in per-units is equal with frequency in per-units:

$$\omega_{pu} = \frac{\omega}{\omega_{base}} = \frac{2\cdot\pi\cdot f}{2\cdot\pi\cdot f_0} = f_{pu} \tag{31}$$

In order to calculate the base and the per-unit value of a generator speed, it is necessary to use the expression used in Section 2.1.2 for the generator speed, which expresses the generator speed depending on the electrical angular speed ω_{e0} and on the number of pole-pairs N_{PP}. The base and the per-unit value of a generator speed can be thus calculated as follows, respectively:

$$\omega_{gen,base} = \frac{\omega_{base,PS}}{N_{PP}} = \frac{2\cdot\pi\cdot f_0}{N_{PP}} \tag{32}$$

$$\omega_{gen,pu} = \frac{\omega_{gen}}{\omega_{gen,base}}$$

4.1.3 Calculation of torque in per-unit

The torque in per-units is calculated as:

$$T_{pu} = \frac{T}{T_{base}} \tag{33}$$

where T is the torque expressed in physical units, while T_{base} is the torque base defined based on relationship between the interrelated quantities power and speed:

$$T_{base} = \frac{P_{base}}{\omega_{base}} \tag{34}$$

The torque in per-units can be then calculated as:

$$T_{pu} = \frac{T}{T_{base}} = \frac{P}{\omega}\cdot\frac{\omega_{base}}{P_{base}} = \frac{P_{pu}}{\omega_{pu}} \tag{35}$$

The torque base of a unit i using the individual unit base or the power system base, respectively, is calculated as:

$$T_{base,i} = \frac{P_{base,i}}{\omega_{base,i}}$$

$$T_{basePS,i} = \frac{P_{basePS}}{\omega_{base,i}} \quad (36)$$

Similarly, the generator torque base and per-unit value can be expressed as:

$$T_{gen,base} = \frac{P_{gen,base}}{\omega_{gen,base}} = \frac{P_n}{\omega_{base,PS}} \cdot N_{pp} \quad (37)$$

$$T_{gen,pu} = \frac{T_{gen}}{T_{gen,base}}$$

where P_n is the nominal power of the generator.

4.1.4 Calculation of moment of inertia in per-unit

Inertia is a very important parameter for the frequency control in the power system. The larger system inertia is, the smaller the frequency rate-of-change following a power imbalance is.

The moment of inertia in per-units is calculated as:

$$I_{pu} = \frac{I}{I_{base}} \quad (38)$$

where I is the inertia expressed in physical units, while I_{base} is the inertia base defined as:

$$I_{base} = \frac{M_{base}}{\omega_{base}} = \frac{T_{base}}{s_{base} \cdot \omega_{base}} = \frac{T_{base}}{\omega_{base}} = \frac{P_{base}}{(\omega_{base})^2} \quad (39)$$

where $s_{base}=1$ [s^{-1}], as shown in Table 2.

The inertia base of a unit i using the individual unit base or the power system base, respectively, is calculated as:

$$I_{base,i} = \frac{P_{base,i}}{(\omega_{base,i})^2} \quad (40)$$

$$I_{basePS,i} = \frac{P_{basePS}}{(\omega_{base,i})^2}$$

Similarly, the generator inertia base and per-unit value can be expressed as:

$$I_{gen,base} = \frac{M_{gen,base}}{\omega_{gen,base}} = \frac{P_{gen,base}}{(\omega_{gen,base})^2} = \frac{P_n}{(\omega_{base,PS})^2} \cdot (N_{pp})^2$$

$$I_{gen,pu} = \frac{I_{gen}}{I_{gen,base}}$$

where P_n is the nominal power of the generator.

4.1.5 Calculation of angular momentum in per-unit

The angular momentum in per-units is calculated as:

$$M_{pu} = \frac{M}{M_{base}} \qquad (41)$$

where M is the angular momentum expressed in physical units, while M_{base} is the base of the angular momentum. The expression can be further extended as follows:

$$M_{pu} = \frac{M}{M_{base}} = \frac{I \cdot \omega}{I_{base} \cdot \omega_{base}} = I_{pu} \cdot \omega_{pu} \qquad (42)$$

Notice that the angular momentum is variable with speed, while inertia is a constant variable. Assuming now small variations in speed (to make possible the linearization of differential equation of *EoM*), as it is the case for synchronous generators in conventional power plants, it means that:

$$\omega \approx \omega_{e0} = const \quad \rightarrow \quad \omega_{pu} \approx 1 \quad \rightarrow \quad M_{pu} \approx I_{pu} \qquad (43)$$

The per-unit expression of the angular momentum can be approximated to be equal with the per-unit value of the inertia.

4.1.6 Calculation of inertia constant in per-unit

Inertia constant H is defined as the ratio of stored kinetic energy at rated (nominal) speed to the rated power:

$$H = \frac{\frac{1}{2} I \omega_n^2}{P_n} \quad [\sec] \qquad (44)$$

Notice that the inertia constant is expressed in seconds. An inertia constant of 4 seconds means that the energy stored in the rotating part could supply the nominal load during 4 seconds.

The inertia constant in per-units is calculated as:

$$H_{pu} = \frac{H}{H_{base}} = H \qquad (45)$$

where H is the inertia constant expressed in physical units, while H_{base} is the inertia base and it is equal to 1. The expression of the inertia constant in per unit can be further expressed as:

$$H_{pu} = H = \frac{1}{2} \cdot \frac{I \omega_n^2}{P_n} = \frac{1}{2} \cdot \frac{I}{I_{base}} = \frac{1}{2} \cdot I_{pu} = \frac{1}{2} \cdot M_{pu} \qquad (46)$$

The angular momentum in per-unit M_{pu} can be then expressed as:

$$M_{pu} = 2 \cdot H_{pu} \qquad (47)$$

4.2 Delta representation

Delta representation is another modelling approach often applied together with per-unit representation in electrical systems literature. It is typically used to describe deviations of quantities from initial steady-state values [6].

The idea of delta representation is that any variable *x* can be expressed at any moment *t* as the sum of its initial value denoted by $x_0=x(0)$ and a small deviation $\Delta x(t)$ from its initial value, as follows:

$$x(t) = x_o + \Delta x(t) \qquad (48)$$

with the condition of course that in the steady state initial condition the deviation is zero, i.e. $\Delta x(0) = 0$.

The delta model can then be expressed as:

$$\Delta x(t) = x(t) - x_o \quad and \quad \Delta x(0) = 0 \qquad (49)$$

Notice that delta models are very convenient to use, as their states do not need to be initialised, since they start automatically in zero.

In the following a small example is given on how to calculate the delta model for the equation of motion (EoM), described in Section 1.4.

4.2.1 Equation of motion – delta representation example

Consider the equation of motion (*EoM*) expressed in power, as described in Section 1.4:

$$P_{mech} - P_{elec} = I\omega \frac{d\omega}{dt} = M \frac{d\omega}{dt} \qquad (50)$$

Suppose that the variables of *EoM* can be expressed as the sum of their initial value (i.e. P_{mech0} for P_{mech}) and a small variation (i.e. P_{mech} for ΔP_{mech}), as follows:

$$\begin{aligned} P_{mech} &= P_{mech0} + \Delta P_{mech} \\ P_{elec} &= P_{elec0} + \Delta P_{elec} \\ \omega &= \omega_0 + \Delta\omega \end{aligned} \qquad (51)$$

By applying the superposition principle, the equation can be then rewritten like:

$$\Delta P_{mech} - \Delta P_{elec} + (P_{mech0} - P_{elec0}) = I(\omega_0 + \Delta\omega)\frac{d}{dt}(\omega_0 + \Delta\omega) \qquad (52)$$

Notice that the steady state quantities can be factored out, since:

$$\begin{aligned} P_{mech0} &= P_{elec0} \\ \frac{d\omega_0}{dt} &= 0 \end{aligned} \qquad (53)$$

and further assuming small deviations in speed, the terms involving products of $\Delta\omega$ with $\frac{d}{dt}(\Delta\omega)$ can be neglected. The power balance equation can then be approximated as:

$$\Delta P_{mech} - \Delta P_{elec} \approx I\omega_0 \frac{d}{dt}\Delta\omega = M_0 \frac{d}{dt}\Delta\omega \qquad (54)$$

This delta representation of *EoM* is going to be used in Section 5.1 as the basic formulation of a generator model.

4.3 Dynamic modelling in frequency domain (Laplace transform)

Understanding dynamic systems in Laplace and time domains is extremely important in the study of process dynamics and control. Dynamic system modelling in frequency domain, using Laplace transform, is often used in the analysis of continuous time dynamical systems. It is not the scope here to describe Laplace transform in details, as it can be found in relevant literature. The chapter focuses on the application of Laplace transform in analysis of process dynamics. The transfer function of a dynamic system is also addressed.

4.3.1 Linear time domain modelling

Any dynamic linear system can be completely characterized in time domain by its impulse response. This means that for any input function *x(t)* of the system, the output function *y(t)* of the system can be calculated in terms of the input *x(t)* and the impulse response *h(t)*.

The impulse response *h(t)* of a dynamic system is defined as the output of the system, when it has as input an impulse $\delta(t)$. The impulse $\delta(t)$ can be modeled as a Dirac delta function for continuous time, as follows:

$$\delta(t) = \begin{cases} 0 & for \quad t \neq 0 \\ \infty & for \quad t = 0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t) dt = 1 \qquad (55)$$

The Dirac delta function $\delta(t)$ is roughly speaking, a pulse of unbounded amplitude and zero duration.

The output of the system *y(t)* is then determined by the input *x(t)* according to the convolution defined as follows:

$$y(t) = y(t_0) + \int_0^t h(t-\tau) x(\tau) d\tau \qquad (56)$$

Notice thus that output *y(t)* of a linear system in time domain requires the convolution of the input *x(t)* with the impulse response function *h(t).* The convolution technique, described in more details in [7], is based on a decomposition of the input signal *x(t)* into impulses, while the output *y(t)* is expressed as a sum of the responses resulting from the individual impulses.

The calculation of the output of the system in time domain is quite difficult and it requires the use of integrals. The calculation is much easier to perform in Laplace domain, where the convolution turns into a simple multiplication of two algebraic functions.

4.3.2 Formal definition and characteristics of Laplace transform

The Laplace transform of a function *f(t)* is represented by a continuous sum of exponential functions of the form e^{-st}, where the operator 's' is defined as a complex frequency[14], namely:

$$F(s) = L\{f(t)\} = \int_0^{\infty} f(t)\, e^{-st}\, dt \qquad (57)$$

[14] The Laplace operator s, already mentioned in Table 2, is a complex variable. It is defined as $s \equiv \sigma + j\omega$. For the analyse of the frequency response in steady state, $s \equiv j\omega$ is used.

where L{.} is used to denote Laplace transformation. The function *f(t)* is a function of time, *s* is the Laplace operator, and *F(s)* is the transformed function. Notice that the Laplace transform converts functions with a time dependent variable into functions with a complex dependent variable, such as frequency (known as Laplace frequency), represented by operator '*s*'.

Table 4 contains the most fundamental properties of Laplace Transforms. More comprehensive lists of Laplace transforms may be found in numerous control literatures e.g. [7].

Table 4: Some Laplace properties and transforms.

Time domain	Laplace domain
$f(t)$	$F(s)$
$x(t)+y(t)$	$X(s)+Y(s)$
$k\ f(t)$	$k\ F(s)$
$df(t)/dt$	$s\ F(s)-f(0)$
$d^n f(t)/dt^n$	$s^n F(s)-s^{n-1}f(0)-s^{n-2}f^1(0)-\cdots-f^{n-1}(0)$
$\int_0^t f(t)dt$	$F(s)/s$
$1\ (step)$	$1/s$
t	$1/s^2$
e^{-at}	$1/(s+a)$
$f(t-a)$	$e^{-as}F(s)$

Notice that operation such as differentiation and integration in time domain can also be replaced by simple algebraic operations in Laplace domain.

Laplace transform is used extensively in electrical engineering. Laplace transform is for example a strong technique to solve linear differential equations, as it overcomes some of the complexities encountered in the time domain solution of differential equation. Laplace transform is used to transform time domain relationships to a set of equations expressed in terms of the Laplace operator 's'. Thereafter, the solution of an original problem can be found by simple algebraic manipulations in the Laplace domain rather than the time domain. To return to the time domain from the Laplace domain, the inverse Laplace Transform L^{-1} is used [7].

Example of solving differential equation using Laplace transform

Consider the linear differential equation to be solved:

$$\ddot{f}(t)+8\dot{f}(t)+15f(t)=1$$

with initial conditions $f(0)=\dot{f}(0)=0$

Applying Laplace transform:

$$L\{\ddot{f}(t)+8\dot{f}(t)+15f(t)\}=1 \;\;\Rightarrow\;\; s^2L\{f(t)\}+8sL\{f(t)\}+15L\{f(t)\}=1$$

as $L\{f(t)\} = F(s)$ *then the equation can be written as:*

$$s^2F(s) + 8sF(s) + 15F(s) = 1$$

This can be further written and decomposed as:

$$F(s) = \frac{1}{s^2 + 8s + 15} = \frac{0.5}{s+3} - \frac{0.5}{s+5}$$

Applying the inverse Laplace transformation, the solution of the differential equation can be then found as follows:

$$f(t) = 0.5e^{-3t} - 0.5e^{-5t}$$

Notice that the condition f(0)=0 is also satisfied.

4.3.3 Transfer function of a linear system

In system analysis, the Laplace transform is seen as a transformation from the time domain, in which inputs and outputs are functions of time, to the frequency domain, where the same inputs and outputs are functions of operator ‘s’ (complex frequency). Laplace transform provides thus an alternative functional description that simplifies the process of analysing the behaviour of the system.

Figure 22 sketches graphically how the Laplace transform can be used to transfer a time domain modeling into frequency domain modeling and back again.

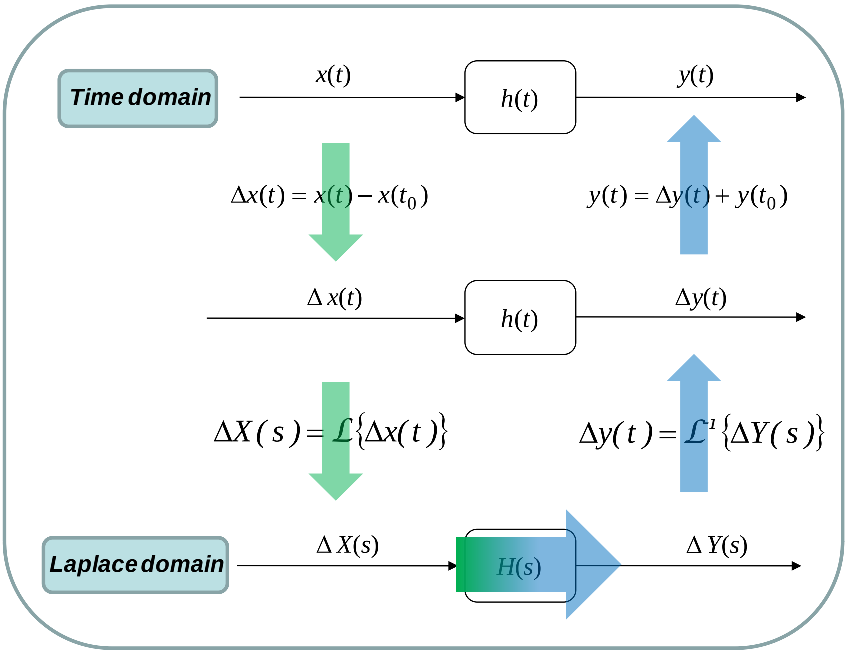


Figure 22: Application of Laplace transform.

Notice that in order to make the transformation from time domain in frequency domain, it is preferable to work with delta values, i.e. *Δx(t)*, because in this way the initial value of the variable *x* is automatically included in the analysis. The dynamic of variable *x* is the same as the dynamic of *Δx*, if of course initial condition is zero, i.e. *x(0)=0*.

As illustrated in Figure 22, the corresponding variables in frequency domain are:

- Input Laplace transform $\Delta X(s)$
- Output Laplace transform $\Delta Y(s) = H(s) \cdot \Delta X(s)$
- Transfer function *H(s),* which fully describes the system dynamic through the relation between the input $\Delta X(s)$ and the output $\Delta Y(s)$, as follows:

$$H(s) = \frac{\Delta Y(s)}{\Delta X(s)} \tag{58}$$

The transfer function *H(s)* is thus the Laplace transform of the impulse response function *h(t)*. It is a linear function and ratio of polynomials in '*s*'. The Laplace transform of a system's output can thus be determined by the multiplication of the transfer function with the input function the Laplace domain. The output function in time domain may be then calculated by applying the inverse Laplace transform.

As illustrated in Figure 22, the transfer function shows the flow of signal through a system, from the input to the output, describing thus the dynamics in operational sense.

The general procedure to find the transfer function from input to output of a system modeled by a linear differential equation is to apply the Laplace transform on both sides of the differential equation and to solve for the ratio of the output Laplace over the input Laplace.

Transfer functions play a central role in the analysis of a dynamic system, because it is usually easier to analyze systems using transfer functions instead of impulse response functions.

Example of first order differential system

Consider a linear system characterized by the following differential equation:

$$x(t) = a\,\frac{dy(t)}{dt} + b\;y(t)$$

where the input signal is $x(t) = \begin{cases} 0 & for\ t < 0 \\ 1 & for\ t \geq 0 \end{cases}$

The output y(t) of the system to the input x(t) is determined by applying Laplace transformation. Based on formulas presented in Table 4, the differential equation can be expressed in Laplace domain as follows:

$$\Delta X(s) = a\;s\;\Delta Y(s) + b\;\Delta Y(s)$$

The transfer function of the system is then given by:

$$H(s) = \frac{\Delta Y(s)}{\Delta X(s)} = \frac{1}{a\;s + b}$$

By using further the notation k=1/b and τ*=a/b, the transfer function can also be expressed as:*

$$H(s) = \frac{k}{\tau\;s + 1}$$

k is the final value, while τ *denotes typically the time constant. Based on Table 4, the output y(t) of the system in time domain can be then expressed and illustrated as in Figure 23.*

4.3.4 Block diagram of a system

$$y(t) = k\left(1-\exp\left(-\frac{t}{\tau}\right)\right) \Rightarrow$$

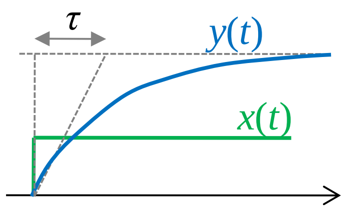


Figure 23: Output of a first order system at a step input.

Using block diagram is another way to represent dynamic systems pictorially. Each block represents a transfer function, while the signal flow between the blocks is illustrated by the block connections.

Dynamic systems can thus be easily visualized through block diagrams. Figure 24 illustrates examples for three basic arrangements of transfer functions and their respective equivalent transfer functions. Notice that transfer functions can consist of combinations of other transfer functions.

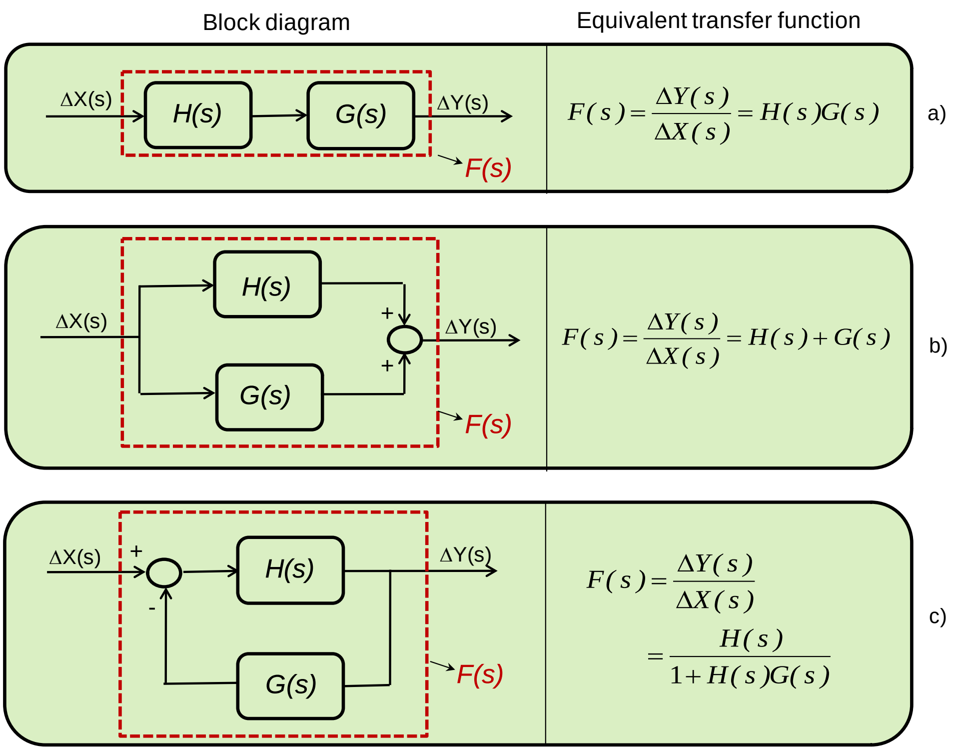


Figure 24: Three basic arrangements of transfer functions:

a) in series b) in parallel c) in feedback form

4.3.5 Initial and final value theorems

Initial and final value theorems, illustrated in Figure 25, permits one to calculate the output of a dynamic system approached at time *t=0*, i.e. *y(0)*, and after very long time *t*, i.e. *y(∞)*, respectively, for a certain input *x(t)*, directly from the transfer function *H(s)*, without the need of applying inverse Laplace transform.

Initial Value Theorem	Final Value Theorem
$y(0) = \lim_{t \to 0} y(t) = \lim_{s \to \infty} s\,\Delta Y(s)$	$y(\infty) = \lim_{t \to \infty} y(t) = \lim_{s \to 0} s\,\Delta Y(s)$

Figure 25: Initial and final theorems.

Example for initial and final value theorem of first order differential system

Consider a dynamic system with the following transfer function:

$$H(s) = \frac{1}{2s+1}$$

If the input ΔX(s) into the system is a step function, the output of the system ΔY(s) can be then calculated as:

$$\Delta Y(s) = \frac{1}{2s+1} \cdot \Delta X(s) = \frac{1}{2s+1} \cdot \frac{1}{s}$$

Applying now the initial and final value theorem the value of the output y(t) in time domain in the initial and final moment is as follows:

$$y(0) = s\Delta Y(s)\Big|_{s=\infty} = \frac{1}{(2s+1)}\Big|_{s=\infty} = 0$$

$$y(\infty) = s\Delta Y(s)\Big|_{s=0} = \frac{1}{(2s+1)}\Big|_{s=0} = 1$$

This result means that by applying a step signal as input to this dynamic system, the output will start from zero and it will converge to one after a while.

5. Dynamic models in frequency control loop

In this chapter, a basic dynamic frequency model for island power systems is introduced. It is based on simple models for the equation of motion, prime mover, governor and load. The frequency dependent loads, which have a stabilizing effect on the frequency, are also modelled.

Having so far described the fundamentals of the frequency control, it is now possible to model the frequency control loop. Such a model can be used to simulate the response of an island power system to a power imbalance caused, for example, by the tripping of a large generating unit.

Figure 26 sketches the block diagram of the frequency control loop. Models for the following components are considered:

- Equation of motion[15]
- Load

15 Equation of motion (*EoM*) includes in general both generators and rotating load. However, in an island power system, it can be assumed that all the inertia is given by the generators.

- Prime mover
- Governor

Notice that the model illustrated in Figure 26 is in per-unit and delta representation. Therefore all the models described from now on are going to be in per-unit and delta representation.

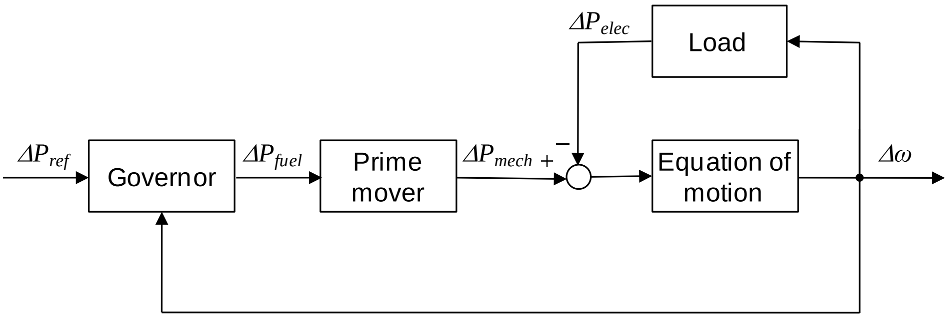


Figure 26: Frequency control loop model (delta and per-unit).

5.1 Generator model (equation of motion EoM)

As already mentioned, the equation of motion (*EoM*) is expressed as follows [1]:

$$T_{mech} - T_{elec} = I \frac{d\omega}{dt} \qquad (59)$$

Any change in the load of the system is instantaneously reflected in the electrical torque T_{elec}. This generates an imbalance between the mechanical torque T_{mech} and the electrical torque T_{elec}, which results further in speed/frequency changes according to the dynamics of the equation of motion. As shown in Section 1.4, the equation of motion can be expressed in terms of mechanical and electrical power, as follows:

$$P_{mech} - P_{elec} = M \frac{d\omega}{dt} \qquad (60)$$

As presented in Section 4.2.1, assuming very small variations in frequency, i.e. frequency almost constant (ω=ω_0), the generator model can be simple expressed as:

$$\Delta P_{mech} - \Delta P_{elec} = M \frac{d}{dt} \Delta\omega \qquad (61)$$

This can be further rewritten in the Laplace domain as:

$$\Delta P_{mech} - \Delta P_{elec} = M \ s \ \Delta\omega \qquad (62)$$

where '*s*' is the Laplace operator. The equation of motion can be further expressed as:

$$\Delta\omega = \frac{1}{Ms}(\Delta P_{mech} - \Delta P_{elec}) \qquad (63)$$

This expression is graphically sketched in Figure 27.

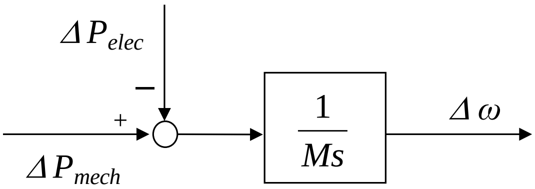


Figure 27: Generator model (delta and per-unit).

where ΔP_{mech} is the mechanical power from the prime mover, while ΔP_{elec} is the electrical power consumed by the loads in the system.

5.2 Load model

In general, the loads in a power system consist of different types of devices. They can be:

- frequency-dependent loads, as i.e. motors, fans, pumps
- frequency-independent loads, as i.e. lights, heaters, computers.

The total load in a power system can therefore be modelled as follows:

$$\Delta P_{elec} = \Delta P_L + D \cdot \Delta\omega \tag{64}$$

where:

- ΔP_L represents the frequency-independent load
- $D \cdot \Delta\omega$ represents the frequency-dependent load
- D is known as the damping constant and in a per-unit model it is expressed as a percent change in load for one percent change in frequency[16].

It is worth noticing that the frequency dependent loads, as i.e. motors in the power system, have a beneficial influence on the power system [1]. They have a stabilizing effect on the power system during a frequency drop[17].

The block diagram for a dynamic model of a load is presented in Figure 28.

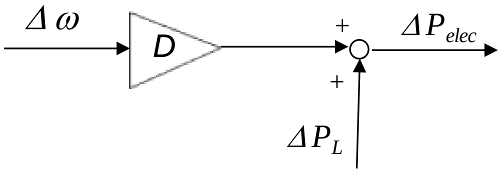


Figure 28: Load model (delta and per-unit).

By combining now the generator model (*EoM*), shown in Figure 27, and the load model, presented in Figure 28, the following expression of the system frequency can be obtained:

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[16] For example a value of D=2, means that a 1% change in frequency implies a 2% change in load.

[17] A frequency drop means that the frequency dependent load also decreases. This load reaction adjusts the momentary unbalance between production and consumption and transforms it into a new balanced situation, where the frequency is stabilised at a new value.

$$\Delta\omega = \frac{1}{Ms + D}(\Delta P_{mec} - \Delta P_L) \qquad (65)$$

This represents the integrated *EoM* and load model. Notice that, in the absence of the governor to control the speed $\Delta\omega$, the system response $\Delta\omega$ to any change in load ΔP_L is dependent only on the total angular momentum *M* and the damping constant *D* existing in the system.

The integrated *EoM* and load model is shown in the cascade form in Figure 29a. This form can be further rearranged in the equivalent transfer function form shown in Figure 29b.

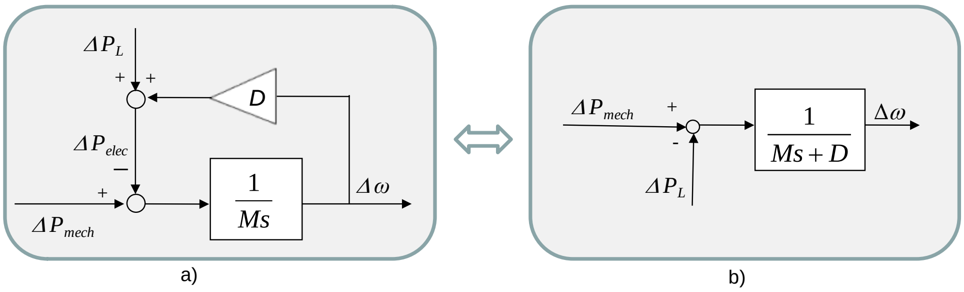


Figure 29: Integrated EoM & load model (delta and per-unit).

Typically more than one generator and load are connected to a power system. The collective performance of all generators in the system is in general analysed by assuming that they all operate synchronously at the same constant frequency. Using this assumption, all generators can be then represented by an equivalent lumped generator driven by the sum of all individual prime mover mechanical outputs [6], as illustrated in Figure 30.

The equivalent angular momentum M_{sys} of the system is then equal to the sum of the angular momenta of all generators (rolling inertia) and spinning loads connected to the system.

It is worth noticing that, besides frequency dependent loads[18], in a power system there are different devices, which through their control system contribute actively and largely to the total damping in the power system D_{sys}.

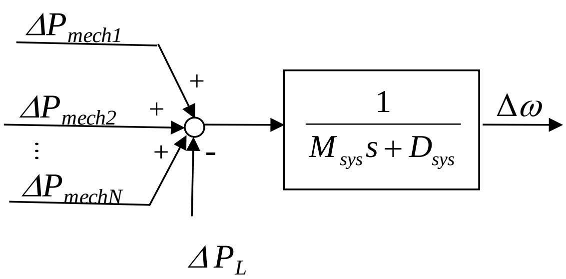


Figure 30: System equivalent with lumped angular momentum and lumped damping constant (delta and per-unit).

[18] Frequency dependent loads are not controllable, namely their damping contribution is passive.

Example of system parameter calculation

A power system consists of two generating units and each of them has a nominal capacity of 100MW. The system has a total load of 105 MW at 50Hz. Each generating unit has an inertia constant[19] of 1 p.u. calculated in 100 MW base. For every 1% change in frequency the corresponding variation of the load is 1.2%. If there is a sudden drop in load by 5 MW:

(a) Build the block diagram of the system expressed on 200 MW base

(b) Calculate the corresponding frequency variation

The power system calculations in per-unit are typically done using the total system power base, which is defined as the total nominal capacity of all generating units installed in the system. In this exercise the total system power base is therefore 200MW.

The calculation of the per-unit value for the inertia constant on the 200MW base can be done based on its per-unit value on the 100MW base, using the formula indicated in equation 26:

$$H_{unit}^{200MW\ base} = H_{unit}^{100MW\ base} \times \frac{100}{200} = 1 \times \frac{100}{200} = 0.5$$

The per-unit value on 200MW base of the inertia constant for the power system consisting of the two generating units can be then calculated as follows:

$$H = 2 \times H_{unit}^{200MW\ base} = 2 \times 0.5 = 1$$

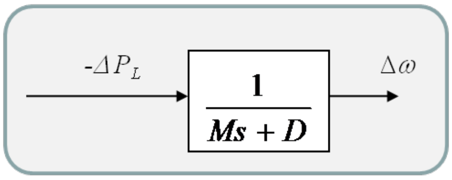
The angular momentum in per-unit on 200MW base can be then expressed as:

$$M = 2 \times H = 2$$

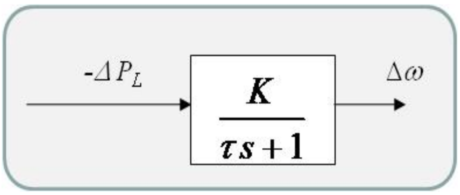
For the load after the change (105-5=100 MW) on 200 MW base,

$$D = 1.2 \times \frac{100}{200} = 0.6$$

The block diagram would be:



And expressed in standard transfer function form (see also section 4.3.3):



The parameters (gain and time constant) can be calculated according to the following:

[19] Definition in equation 44, page 28.

$$k = \frac{1}{D} = \frac{1}{0.6} = 1.67$$
$$\tau = \frac{M}{D} = \frac{2}{0.6} = 3.33$$

The load deviation is:

$$\Delta P_L = -5MW = -\frac{5}{200} = -0.025\,pu$$

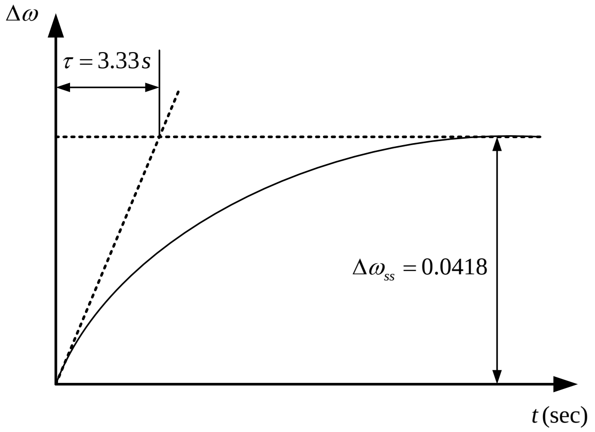
For the reduction in the load, the Laplace transformation of the change is (see also section 4.3):

$$\Delta P_L(s) = -\frac{0.025}{s}$$

Therefore,

$$\Delta\omega(s) = -\left(\frac{-0.025}{s}\right)\left(\frac{k}{\tau s + 1}\right) \overset{\Im^{-1}}{\Rightarrow}$$
$$\Delta\omega(t) = -0.025ke^{-\frac{t}{\tau}} + 0.025k$$
$$= -0.025 \times 1.67e^{-\frac{t}{3.33}} + 0.025 \times 1.67$$
$$= 0.0418e^{-0.3t} + 0.0418$$

The pu deviation in the speed is qualitatively shown here:



The time constant is 3.33 s and the steady-state speed deviation is

$$\Delta\omega_{ss} = -\frac{\Delta P_L}{D} = 0.0418\,pu$$
$$= 0.0418 \times 50 = 2.09\,Hz$$

5.3 Prime mover model

As mentioned in Section 2.2, a prime mover is a turbine, which converts steam energy, produced by burning some kind of fuel, into mechanical energy. Each turbine consists typically of a number of stages, where the steam can be reheated. The fuel is supplied through the governor valve, which controls the frequency by adjusting the fuel infeed flow.

Figure 29 illustrates the simplest dynamic model of a prime mover, corresponding to a non-reheat turbine (i.e. one turbine stage). It is a first order model with a time constant T_{CH}, known also as the charging time constant.

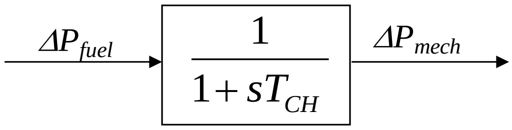


Figure 31: Prime mover model (delta and per-unit).

The model of a prime mover relates thus the power generated by burning fuel to the output mechanical power:

$$\Delta P_{mech} = \frac{1}{1+sT_{CH}} \cdot \Delta P_{fuel} \qquad (66)$$

5.4 Governor model

Figure 32 depicts the droop characteristic models for the two types of governors explained in Section 2.3, i.e. isochronous governor and speed droop governor.

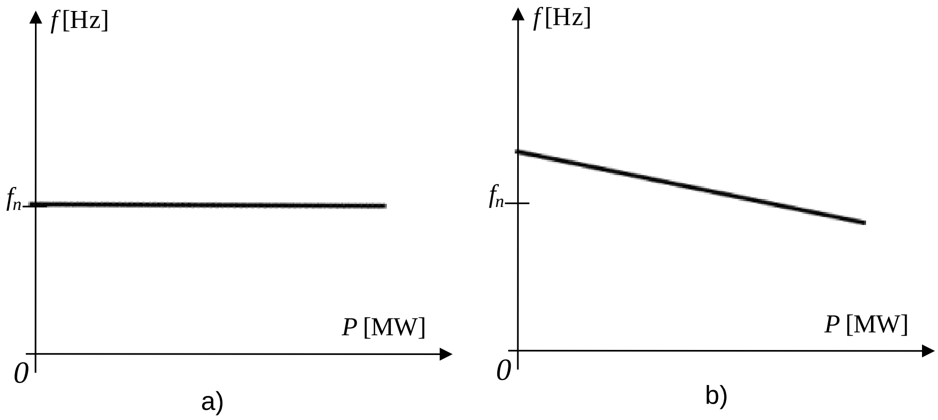


Figure 32: Droop characteristic of: a) isochronous governor; b) speed droop governor.

5.4.1 Isochronous governor model

As shown in Figure 32a), an isochronous governor keeps the frequency constant independent of the generator loading (zero droop).

The model of an isochronous governor is shown in Figure 33:

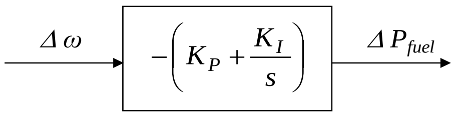


Figure 33: Model for an isochronous governor (delta and per-unit).

where K_p and K_I are the proportional and integral parameters of the controller, respectively. Notice that the isochronous generator has only as input signal the deviation of the speed $\Delta\omega$, between the measured speed ω and the reference synchronous electrical speed ω_{e0}. This speed deviation is amplified and integrated to produce the control signal ΔP_{fuel}, which actuates the throttle valve of the prime mover. Notice in Figure 34a) that the steady state operation for an isochronous governor is defined by the condition $\Delta\omega=0$.

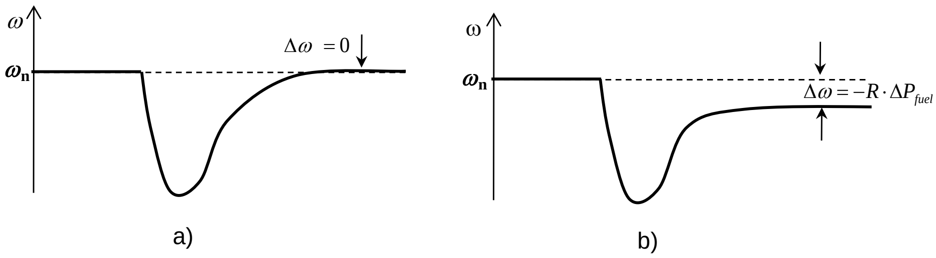


Figure 34: Frequency response of a generating unit equipped with:
a) Isochronous governor b) Speed droop governor

5.4.2 Speed droop governor model

As illustrated in Figure 32b), a speed droop governor has a characteristic with a negative slope. It reacts to load variations by changing its speed, namely it controls the relation between power and frequency. This means that a higher (lower) power output is established when the frequency drops (rises).

The model of a speed droop governor is shown in Figure 35.

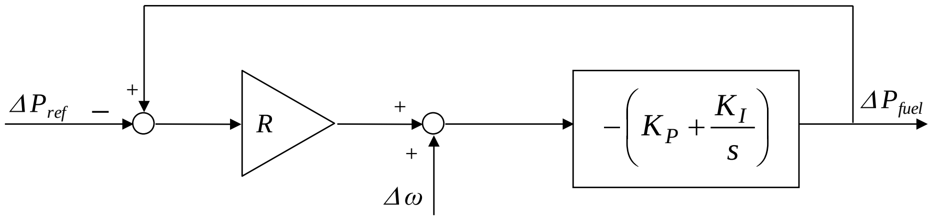


Figure 35: Model for a speed droop governor (delta and per-unit).

where R represents the droop of the characteristic. Notice that the speed droop governor has two input signals, namely deviation of the speed $\Delta\omega$ and the deviation of the power setpoint ΔP_{ref}.

The steady state operation for a speed droop governor is defined by the condition:

$$\Delta P_{ref} = 0 \quad \Rightarrow \quad \Delta\omega + R \cdot \Delta P_{fuel} = 0 \quad \Rightarrow \quad \Delta\omega = -R \cdot \Delta P_{fuel} \qquad (67)$$

If frequency drops due to e.g. an increased load, the speed droop governor will yield to an increased ΔP_{fuel} which will be order to the prime mover.

Speed droop governors are utilized when multiple generators have to share changes in load in the power system. For this sharing to be equal the speed droop governors need to have same droop characteristic.

5.5 Frequency control loop model

Figure 36 illustrates a simplified model of the frequency control loop in an island power system. It consists of models for speed droop governor, prime mover and integrated equation of motion and load model.

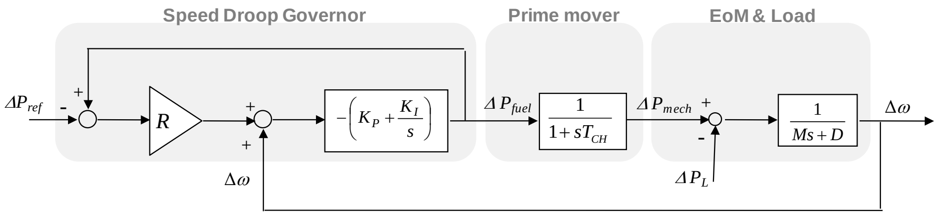


Figure 36: Frequency control loop model (delta and per-unit).

This model can be used to analyze the response of the frequency control loop to deviations in speed (frequency) caused by i.e. a sudden disconnection of a large generating unit from the system or by connecting a large load into the system.

6. Frequency control with wind power

In this chapter, the frequency control loop of an island power system, presented in the previous chapter, is extended with a simple model of a wind turbine.

Figure 37 shows an overview block diagram of an island power system including speed droop governor, prime mover, and equation of motion, load and wind turbine model. All generating units in the system are lumped together as a unique generator (equation of motion), as explained in Chapter 5.

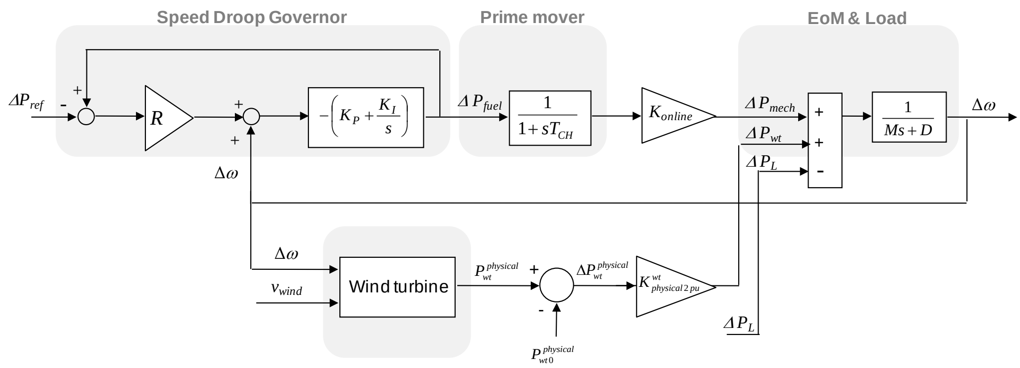


Figure 37: Overview of frequency control loop model including wind turbine model (delta and per-unit).

Notice that, the output of the wind turbine model is added to the mechanical power produced by the others prime movers in the system. This is because wind turbines are seen as negative loads, i.e. they produce power and reduce thus the load (demand) in the system, independently of the system conditions and of the actions of governors and prime movers of the conventional power stations.

The additional amplification blocks K_{online} and $K_{physical2pu}^{wt}$ are defined as follows:

$$K_{online} = \frac{\sum_{i=1}^{N_{units}} k_i \cdot P_{n,i}}{\sum_{i=1}^{N_{units}} P_{n,i}}$$ (68)

$$K^{wt}_{physical2pu} = \frac{N_{wtr}}{\sum_{i=1}^{N_{units}} P_{n,i}}$$

where k_i is 1 if unit *i* is online and 0 if unit *i* is offline and N_{units} is the number of units. The gain K_{online} is containing information about how much production capacity is in the system, e.g. how many units are online or not. Notice that K_{online} is equal to one, when all the units in the system are online. It adjusts properly the mechanical power in per-unit (power system base) produced by the aggregated prime mover model for all units in the system. $K^{wt}_{physical2pu}$ is used to transform first the wind turbine delta power (physical!) in the delta power of a wind farm consisting of N_{wtr} wind turbines and second from physical to per-units system (power system base). Remark that the power system base is by definition equal with the total power capacity, namely the sum of nominal power of all units installed in the system no matter whether they are online or not. $P^{physical}_{wt0}$ is the wind turbine power in the initial steady state condition.

Figure 38 sketches a more detailed block diagram of the wind turbine modeled. Description of the aerodynamic model is found in details in [8]. ω^{pu}_{gen0} is the generator speed in the initial steady state condition.

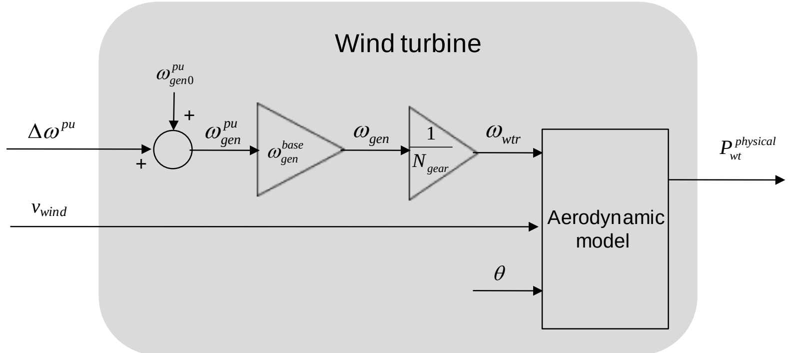


Figure 38: Wind turbine model.

References

[1] Kundur P. [1994], Power system stability and control, McGraw-Hill.

[2] https://www.entsoe.eu/the-association/

[3] Søder L. [2002], Explaining Power system operation to nonengineers, Power Engineering Review, IEEE 22(4), 25-27.

[4] Nordic grid code - Entsoe https://www.entsoe.eu/fileadmin/user_upload/_library/publications/nordic/planning/070115_entsoe_nordic_NordicGridCode.pdf

[5] Miller R., Malinowski J. H. [1995], Power system operation, McGraw-Hill.

[6] Wood A.J, Wollenberg B.F. [1996], Power generation, Operation and Control, John Wiley and Sons.

[7] Ogata K. [2010], Modern control engineering, Prentice-Hall International.

[8] Hansen A.D. [2010], Introduction to wind power models for frequency control studies.

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