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Thermoeconomic diagnosis and entropy generation paradox

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Abstract:

In the entropy generation paradox, the entropy generation number, as a function of heat exchanger effectiveness, counter-intuitively approaches zero in two limits symmetrically from a single maximum. In thermoeconomic diagnosis, namely in the characteristic curve method, the exergy destruction is proposed as the dependent variable, along with a set of independent variables, to locate the actual cause of malfunction. This relies on the assumption that in case of an operation anomaly its exergy destruction rate strictly increases. We examine the behaviour of the diagnosis method with regards to the entropy generation paradox, as a decreased heat exchanger effectiveness (as in the case of an operation anomaly in the component) can counter-intuitively result in decreased exergy destruction rate of the component. Therefore, along with an improper selection of independent variables, the heat exchanger can be deduced to be working more effectively from the resulting indicator, when it is actually degraded. From an extensive analysis of the diagnosis method an alternative dependent variable was proposed in the form of exergy destruction rate normalised with the exergy fuel rate, which strictly increases in case of an operation anomaly in a component. The normalised exergy destruction rate as the dependent variable therefore resolves the relation of the characteristic curve method with the entropy generation paradox.

Keywords:

Thermoeconmic diagnosis, Entropy generation paradox, Characteristic curve method.

1. Introduction

Thermoeconomic diagnosis is a field of research that studies thermal energy systems, in which the actual performance may vary from the expected performance because of malfunctions. The thermoeconomic diagnosis methods are primarily aimed at locating and identifying components with operation anomalies, causing malfunctions, and evaluate their effects on other components and the energy system. The operation anomalies cause a decreased efficiency of the system and therefore require a higher amount of resources to obtain the same power output.

Malfunctions can be classified into external, intrinsic and induced malfunctions. The external malfunctions occur because of modifications in the conditions outside the system boundary. The intrinsic malfunctions are the actual causes of malfunctions and along with the external malfunctions, make the induced malfunctions arise in the components that are not affected by operation anomalies. The identification between the intrinsic and induced malfunctions is a challenging task, as the effects of an operation anomaly in one component spreads through the whole system.

An extensive review of the thermoeconomic diagnosis methods, to identify the causes of malfunctions, has been covered in [1] and four of them have been compared on the TADUES test case in [2], which is a steady state model of a combined cycle power plant for both on- and off-design conditions. One of the thermoeconomic diagnosis methods is the characteristic curve method and its primary aim is to identify the component with operation anomalies [3]. It relies on the notion that in case of an operation anomaly, the complex interaction of the thermodynamic variables that describe the component, the characteristic curves are altered. This can be achieved by characterising the behaviour of the component, using the exergy destruction rate as the dependent variables and the independent thermodynamic variables. The malfunctions are then located by using the characteristic curves and comparing the actual performance and the expected performances. In addition to the application on a combined cycle power plant in [3] the method is considered for use on a transcritical/subcritical booster refrigeration system [4], as well as on an energy system for ship propulsion in [5].

The relation of the entropy generation paradox to thermoeconomic diagnosis methods has so far not been discussed in the literature, even if the entropy generation paradox itself is discussed extensively already. As discussed in [6] the entropy generation paradox results from the analysis of the entropy generation number, which is defined as the ratio of the entropy generation to the minimum heat capacity rate, with regards to changes in heat exchanger effectiveness. Counter-intuitively, the entropy generation number, as a function of heat exchanger effectiveness, symmetrically approaches zero in two limits from a single maximum for a balanced counter-flow heat exchanger. The paradox is even more apparent when observing that the entropy generation number increases when the heat exchanger effectiveness increases from zero to the maximum. In [7] an alternative entropy generation number is proposed, which resolves the entropy generation paradox by normalising the entropy generation with the ambient temperature and the heat transfer rate.

As exergy destruction rate is used as the dependent variable in the characteristic curve method, it was of interest in this study to examine its behaviour with regards to the entropy generation paradox. To further examine the behaviour, two different sets of the independent variables were examined. Two alternative dependent variables and resulting indicators were suggested with the goal of resolving the respective problem. Two components considered as already existing and operating components in an energy system were considered as test cases. First, a counter-flow heat exchanger was considered to illustrate the behaviour of the characteristic curve method with regards to the entropy generation paradox. Secondly, a gas turbine expander was considered to further examine that the alternative dependent variable is applicable for other components as well.

2. Methods

2.1. Thermodynamic model

To fulfil the objective of the study, thermodynamic models of the heat exchanger and the gas turbine under consideration were developed. The thermodynamic models were implemented in EES [8]. According to the objective of the study, both models were relatively simple and act to provide the thermodynamic states of the operating conditions needed for the thermoeconomic diagnosis. The operating conditions resemble the design point and off-design conditions near to the design point. Moreover, the mathematical formulation of the models also serves the purpose to determine the degrees of freedom of the components. Both components were considered adiabatic and under steady state operation, with negligible kinetic and potential energy effects. Air, modelled with ideal gas behaviour, was considered as the working fluid.

2.1.1 Heat exchanger

Figure 1 shows the process flow diagram of the heat exchanger under consideration.

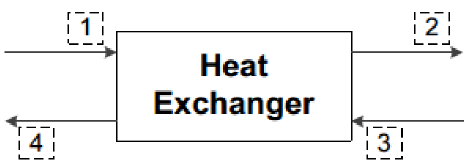

Fig. 1. Process flow diagram of the heat exchanger.

In addition to mass balance, energy balance and state equations, the heat exchanger was described with a heat transfer relation according to Equations 1-8:

$$\dot{m}_1 = \dot{m}_2, \qquad (1)$$

$$\dot{m}_3 = \dot{m}_4, \qquad (2)$$

$$\dot{m}_1 c_{p,1} T_1 - \dot{m}_2 c_{p,2} T_2 = \dot{m}_4 c_{p,4} T_4 - \dot{m}_3 c_{p,3} T_3, \qquad (3)$$

$$c_{p,1} = f(T_1), \quad (4)$$

$$c_{p,2} = f(T_2), \quad (5)$$

$$c_{p,3} = f(T_3), \quad (6)$$

$$c_{p,4} = f(T_4), \quad (7)$$

$$\dot{m}_1 c_{p,1} T_1 - \dot{m}_2 c_{p,2} T_2 = UA \frac{(T_1 - T_3) - (T_2 - T_4)}{\ln\left(\left((T_1 - T_3)/(T_2 - T_4)\right)\right)}. \qquad (8)$$

As the off-design conditions considered were close to the design point, the overall heat transfer coefficient was assumed to be fixed. Therefore, the UA value was assumed to be fixed at the design point value. The heat exchanger was assumed to have no pressure drops and thus no momentum equations were not included.

Tables 1 shows the thermodynamic parameters of the heat exchanger. In addition, two UA values were considered for the heat exchanger, UA = 7.5 kW/K and UA = 20 kW/K and these apply for two reference operating conditions (by two different existing components). The UA values represent relatively low and high heat exchanger effectiveness on the entropy generation paradox curve. These values were chosen to illustrate the behaviour of the characteristic curve method with regards to the paradox, as described in more detail later.

Table 1. Thermodynamic parameters for the heat exchanger.

Parameter	*Parameter value*
UA (rel. low)	7.5 kW/k
UA (rel. high)	20 kW/k
$\dot{m}_1$	10 kg/s
$\dot{m}_2$	10 kg/s
T_1	100 °C
T_3	25 °C

2.1.1 Gas Turbine

Figure 2 shows the process flow diagram of the gas turbine expander under consideration.

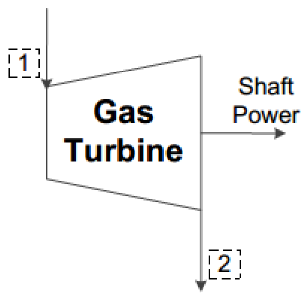

Fig. 2. Process flow diagram of the gas turbine expander.

In addition to mass balance, energy balance and state equations, the gas turbine expander was described by a second law relation in terms of an isentropic efficiency according to Equations 9-15:

$\dot{m}_1 = \dot{m}_2$ (9),

$\dot{W} = \dot{m}_1(h_2 - h_1)$, (10)

$h_1 = f(T_1)$, (11)

$s_1 = f(P_1, T_1)$, (12)

$h_{2,s} = f(P_2, s_1)$, (13)

$s_2 = f(P_2, h_2)$, (14)

$\eta_{is} = \frac{h_2 - h_1}{h_{2,s} - h_1}$ (15).

Moreover, a turbine constant relation was assumed to describe the relation among flow capacity, pressure ratio and inlet temperature according to Equation 16:

$C_T = \frac{\dot{m}_1\sqrt{T_1}}{\sqrt{P_1^2 - P_1^2}}$, (16)

For the considered off-design conditions, an isentropic efficiency and a turbine constant relation results in reasonable values for the thermodynamic variables, as discussed in [9].

Tables 2 shows the thermodynamic parameters of the heat exchanger.

Table 2. Thermodynamic parameters for the gas turbine expander.

Parameter	*Parameter value*
η_{is}	0.8
C_T	50 kg/s $\sqrt{K}$/bar
P_1	8 bar
P_2	1 bar
T_1	1200 °C

2.2. Characteristic curve method

The characteristic curve method relies on the information given by the characteristic curves of a component. The curves consist of a set of expressions that describe the performance or a thermodynamic quantity that characterises the component behaviour, the dependent variable. The fundamental idea of the method is that a component operates differently with respect to its characteristic curves depending on whether the malfunction in the component is intrinsic or induced. In case of an intrinsic malfunction the characteristic curve of a component will be altered and the real operating condition will be on a different characteristic curve compared to the reference operating condition [3]. A component that only has induced malfunctions continues to operate on the same characteristic curve but on different operating point, corresponding to the real operating point, because of the spreading of malfunctions.

Similar to other thermoeconomic diagnosis methods, the identification between the intrinsic and induced malfunctions is accomplished with a comparison between the reference and real operating conditions. In the characteristic curve method, this can be done by linearly approximating the characteristic curve with its derivatives, from the real operating condition back to the reference operating condition. Exergy destruction rate and the thermodynamic variables that describe the components are used as the dependent and independent variables, respectively, as recommended in [3]. The exergy destruction rate is suggested, as it describes the mass and energy flows in a single quantity and is defined in a unique way, and furthermore it is assumed that an operation anomaly alters the exergy destruction rate in a negative way and therefore results in a strictly positive indicator.

The sets of independent thermodynamic variables for the heat exchanger and the gas turbine expander are determined from the degrees of freedom of the respective components. The heat exchanger has thirteen equations and eight variables. One of these variables, the UA value, is fixed be the design of the specific choice of heat exchanger. Therefore, the heat exchanger has four degrees of freedom. There are also eight equations and thirteen variables that describe the gas turbine expander. Similarly, as a specific choice of a gas turbine expander was to be characterised in the thermoeconomic diagnosis method, its isentropic efficiency and turbine constant were assumed to be fixed by the design of the gas turbine. Therefore, the gas turbine expander has three degrees of freedom.

Table 3 shows the independent variables for the heat exchanger and the gas turbine expander. Two sets of independent variables were examined to understand their effects on the relation of the characteristic curve method with the entropy generation paradox. In addition to the mass flow, these choices represent the variations in the temperature selection, where inlet and outlet temperatures were examined.

Table 3. Sets of independent variables for the heat exchanger and gas turbine.

Component	*Independent variables* τ_k
Heat exchanger (inlet temp)	$\dot{m}_1, T_1, \dot{m}_3, T_3$
Heat exchanger (outlet temp)	$\dot{m}_1, T_2, \dot{m}_3, T_4$
Gas turbine	P_1, T_1, P_2

To develop the characteristic curves of each component, additional operating conditions were selected to approximate the derivatives of the exergy destruction rate with respect to each independent variable $\partial \dot{E}_D^{i,ref} / \partial \tau_k^i$ in the vicinity of the reference operating condition. These operating conditions are equal in number to the degrees of freedom of the component and were obtained by varying the independent thermodynamic variables near the reference state. Tables 4 and 5 show the additional operating conditions for the heat exchanger and the gas turbine expander, respectively.

Table 4. Additional operating conditions for the heat exchanger.

Independent variables τ_k	*Ref.*	*Add. #1*	*Add. #2*	*Add. #3*	*Add. #4*
$\dot{m}_1$ [kg/s]	10	9.50	10.50	10.25	9.75
$\dot{m}_2$ [kg/s]	10	10.25	9.50	10.75	9.75
T_1 [K]	100	100.0	97.5	92.50	105.00
T_2 [K]	25	27.50	22.50	23.00	27.00

Table 5. Additional operating conditions for the gas turbine.

Independent variables τ_k	*Ref.*	*Add. #1*	*Add. #2*	*Add. #3*
P_1 [bar]	8	8.100	8.050	7.950
P_2 [bar]	1	1.005	0.995	0.998
T_1 [K]	1200	1150.00	1205.00	1155.00

The characteristic curve derivatives for the heat exchanger, e.g., for the set of independent variables in terms of the inlet temperatures, were calculated by solving the following systems of equations for the unknowns:

$$\begin{bmatrix} \Delta^{op1}\dot{m}_1 & \Delta^{op1}T_1 & \Delta^{op1}\dot{m}_3 & \Delta^{op1}T_3 \\ \Delta^{op2}\dot{m}_1 & \Delta^{op2}T_1 & \Delta^{op2}\dot{m}_3 & \Delta^{op2}T_3 \\ \Delta^{op3}\dot{m}_1 & \Delta^{op3}T_1 & \Delta^{op3}\dot{m}_3 & \Delta^{op3}T_3 \\ \Delta^{op4}\dot{m}_1 & \Delta^{op4}T_1 & \Delta^{op4}\dot{m}_3 & \Delta^{op4}T_3 \end{bmatrix} \begin{bmatrix} \partial \dot{E}_D / \partial \dot{m}_1 \\ \partial \dot{E}_D / \partial T_1 \\ \partial \dot{E}_D / \partial \dot{m}_3 \\ \partial \dot{E}_D / \partial T_3 \end{bmatrix} = \begin{bmatrix} \Delta^{op1}\dot{E}_D \\ \Delta^{op2}\dot{E}_D \\ \Delta^{op3}\dot{E}_D \\ \Delta^{op4}\dot{E}_D \end{bmatrix}. \quad (17)$$

The characteristic curve derivatives for the gas turbine expander were calculated similarly, by solving the following systems of equations for the unknowns:

$$\begin{bmatrix}\Delta^{op1}P_1 & \Delta^{op1}T_1 & \Delta^{op1}P_2\\ \Delta^{op2}P_1 & \Delta^{op2}T_1 & \Delta^{op2}P_2\\ \Delta^{op3}P_1 & \Delta^{op3}T_1 & \Delta^{op3}P_2\end{bmatrix}\begin{bmatrix}\partial\dot{E}_D/\partial\dot{P}_1\\ \partial\dot{E}_D/\partial T_1\\ \partial\dot{E}_D/\partial P_2\end{bmatrix}=\begin{bmatrix}\Delta^{op1}\dot{E}_D\\ \Delta^{op2}\dot{E}_D\\ \Delta^{op3}\dot{E}_D\end{bmatrix}.(18)$$

The inlet conditions (the set of independent variables in terms of the inlet temperatures) at the reference and real operating conditions were kept fixed at the same values as in Tables 1 and 2. The comparison between the reference and the expected reference operating condition, yields in an indicator that uses the thermodynamic variables at the reference and real operating conditions and the characteristic curve derivatives. Because of the choice of dependent and independent variables, the indicator is expressed for the ith component as

$$I^i = \Delta\dot{E}_D^i - \Delta\dot{E}_D^{i,calc} = \Delta\dot{E}_D^i - \sum_k \left[\frac{\partial\dot{E}_D^{i,ref}}{\partial\tau_k^i}\right]_{\partial\tau_k^{i,ref}} \Delta\tau_k^i,(19)$$

where the $\Delta\dot{E}_D^i$ denotes the difference between exergy destruction rate at the real operating condition and the exergy destruction rate at the reference operating condition. Further, $\Delta\dot{E}_D^{i,calc}$ and $\Delta\tau_k^i$ denote the expected variation in exergy destruction rate and the variation in thermodynamic variables between the real and reference operating conditions, respectively. As the exergy rates differ in magnitude among components, a relative indicator can be used and is considered as a more meaningful measure [3]. For the *ith* component the relative indicator is:

$$I_{rel}^i = \frac{I^i}{\Delta\dot{E}_D^{i,calc}} \quad (20).$$

2.3. Normalised exergy destruction rate as the dependent variable

The entropy generation paradox can be avoided by using an alternative entropy generation number, which is normalised by the heat transfer rate of the heat exchanger and the ambient temperature, as discussed in [7]. It is thus assured that the alternative entropy generation number is increased with decreased heat exchanger effectiveness at all ranges of effectiveness. This line of thought can be applied to the characteristic curve method by using exergy destruction rate, which is normalised by heat transfer rate of the heat exchanger, as the dependent variable. The resulting dependent variable for the *ith* component (heat exchanger) is:

$$N_Q^i = \frac{\dot{S}_{gen}^i T_0}{\dot{Q}} = \frac{\dot{E}_D^i}{\dot{Q}_i}, \qquad (21)$$

The normalised exergy destruction rate suggested above only applies to heat exchangers, as the normalisation is done by using its heat transfer rate. To make the approach more general and to apply it to other components than heat exchangers, the exergy destruction rate can be normalised by the exergy fuel rate, i.e., by assuming that the exergy fuel rate is directly proportional to the heat transfer rate. The resulting dependent variable for the *ith* component is:

$$N_{E,F}^i = \frac{\dot{E}_D^i}{\dot{E}_F^i}, \qquad (22)$$

where the exergy destruction and fuel rates are defined for the heat exchanger and the gas turbine expander according to the conventions in [10].

The independent thermodynamic variables can still be recommended, as any choice of dependent variables can be expressed as a function of the independent thermodynamic variables [3]. Therefore, the same sets independent variables, given in Table 3, were with the new dependent variable. To develop the characteristic curves for each component in terms of the normalised exergy destruction rate, the same line of thought was used to generate the curves as in terms of the exergy destruction rate.

The resulting indicator, when using the normalised exergy destruction rate using exergy fuel rate (or similarly for heat transfer rate) is used as the dependent variable and the independent thermodynamic variables are used as the independent variables, for the *ith* component is:

$$I^i = \Delta N_{E,F}^i - \Delta N_{E,F}^{i,calc} = \Delta N_{E,F}^i - \sum_k \left[\frac{\partial N_{E,F}^{i,ref}}{\partial \tau_k^i}\right]_{\partial \tau_k^{i,ref}} \Delta \tau_k^i. \quad (23)$$

Because of the choice of the dependent variable, the indicator was normalised and thus not expressed in terms of exergy flow. A relative indicator in this case therefore gives a different purpose compared to the relative indicator given by Equation [20].

Inspired by [11] a relative indicator can however be used, which describes the ratio of the indicator to the dependent variable at the reference operating condition. For the *ith* component, the relative indicator for exergy destruction rate as the dependant variable is:

$$I_{rel}^i = \frac{I^i}{\dot{E}_D^{i,ref}}, \quad (24)$$

This relative indicator can be equally applied for all three dependent variables, i.e., for the one using exergy destruction rate and both the ones using normalised exergy destruction rate. For the *ith* component, the relative indicator for the normalised exergy destruction rate, using exergy fuel rate, as the dependent variable is:

$$I_{rel}^i = \frac{I^i}{N_{E,F}^{i,ref}}. \quad (25)$$

3. Results

3.1. Heat exchanger

Figure 3 shows the exergy destruction rate and the normalised exergy destruction rate as a function of UA value of a heat exchanger for the given inlet conditions. The exergy destruction rate as a function of the UA value has a similar trend to the entropy generation number as a function of heat exchanger effectiveness, as described in [7] where the latter is denoted as the entropy generation paradox curve. When the UA value is decreased from a relatively high value, the exergy destruction rate increases (initially from zero, represented by an infinitely large heat exchanger). The exergy destruction rate decreases after approximately UA = 10 kW/K and approaches zero (represented by a vanished heat exchanger) for a relatively low UA value. However, the normalised exergy destruction rate steadily increases as the UA value is decreased from a relatively high value to a relatively low value. Figure 3 also shows that the entropy generation paradox can be equivalently expressed in terms of a UA value instead of heat exchanger effectiveness. In addition to that, it shows the analogy between the alternative entropy generation number and the normalised exergy destruction rate.

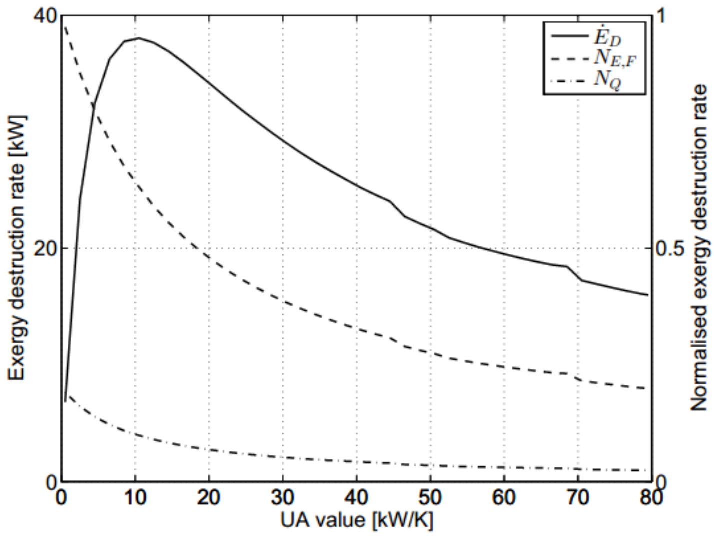

Fig. 3. Exergy destruction rate and normalised exergy destruction rate of the heat exchanger as a function of UA value.

As mentioned previously, it is of interest to examine the characteristic curve method with two reference operating conditions for the heat exchanger, represented by relatively high and low UA values. As seen in Figure 4 these UA values, i.e., UA = 20 kW/K and UA = 7.5 kW/K, represent the two different sides of the entropy generation paradox curve. The approximated characteristic curve derivatives for the heat exchanger are shown in Tables 6 and 7, for both reference operating conditions and sets of independent variables. The derivatives using the exergy destruction rate as the dependent variable are two orders of magnitude larger compared to the derivatives using the normalised exergy destruction rate as the dependent variable.

Table 6. Characteristic curve derivatives of the heat exchanger using the set of independent variables in terms of inlet temperatures.

	UA value	
	20 kW/K	7.5 kW/K
Derivative		Derivative value
$\partial \dot{E}_D/\partial \dot{m}_1$ [kW/kg/s]	1.675	0.834
$\partial \dot{E}_D/\partial \dot{T}_1$ [kW/K]	0.784	0.854
$\partial \dot{E}_D/\partial \dot{m}_3$ [kW/kg/s]	2.018	1.648
$\partial \dot{E}_D/\partial \dot{T}_3$ [kW/K]	-0.975	-1.138
$\partial N_D/\partial \dot{m}_1$ $\times 10^2$ [-/kg/s]	0.445	0.550
$\partial N_D/\partial \dot{T}_1$ $\times 10^2$ [-/K]	-0.024	-0.015
$\partial N_D/\partial \dot{m}_3$ $\times 10^2$ [-/kg/s]	2.163	1.832
$\partial N_D/\partial \dot{T}_3$ $\times 10^2$ [-/K]	-0.960	-1.222

Table 7. Characteristic curve derivatives of the heat exchanger using the set of independent variables in terms of outlet temperatures.

	UA value	
	20 kW/K	7.5 kW/K
Derivative		Derivative value
$\partial \dot{E}_D/\partial \dot{m}_1$ [kW/kg/s]	5.081	-0.048
$\partial \dot{E}_D/\partial \dot{T}_2$ [kW/K]	-5.641	3.472
$\partial \dot{E}_D/\partial \dot{m}_3$ [kW/kg/s]	4.284	-2.424
$\partial \dot{E}_D/\partial \dot{T}_4$ [kW/K]	4.025	-2.815
$\partial N_D/\partial \dot{m}_1$ $\times 10^2$ [-/kg/s]	6.252	6.517
$\partial N_D/\partial \dot{T}_2$ $\times 10^2$ [-/K]	-0.023	0.301
$\partial N_D/\partial \dot{m}_3$ $\times 10^2$ [-/kg/s]	3.057	2.024
$\partial N_D/\partial \dot{T}_4$ $\times 10^2$ [-/K]	-0.089	-0.057

Figure 4 shows the indicator, given by Equation 19, as a function of UA value, for both the reference operating conditions of the heat exchanger and using the set of independent variables in terms of inlet temperatures. The indicators have the same trend as the entropy generation paradox curve and as in Figure 4. When the characterisation is done at the right side of the entropy generation paradox curve, the indicator increases at first when the UA value is decreased below the reference value, but after UA ≈ 10 kW/K the indicator decreases and eventually falls below zero. However, when the characterisation is done at the left side of the entropy generation paradox curve, the indicator has a negative value, when the UA value is decreased below the reference value.

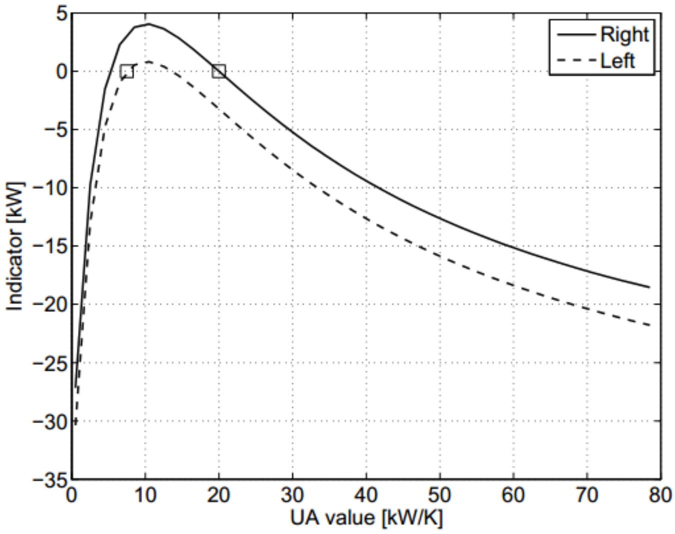

Fig. 4. Indicator for the heat exchanger as function of UA value and using the set of independent variable in terms of inlet temperatures.

Figure 5 shows the indicator, given by Equation 19, as a function of UA value, for both the reference operating conditions of the heat exchanger and using the set of independent variables in terms of outlet temperatures. When the characterisation is done at the right side of the entropy generation paradox curve, the indicator steadily increases from zero when the UA value is decreased below the reference value and vice-versa, decreases below zero when the UA value is increased above the reference value. However, when the characterisation is done at the left side of the entropy generation paradox curve, the indicator has the counter-intuitive behaviour, i.e., the indicator decreases when the UA value is decreased below the reference value and an increased UA value results in an increased indicator. Sometimes the indicators using exergy destruction rate as the dependent variable are thus unable to identify operation anomalies and the heat exchanger is deduced to be working more effectively.

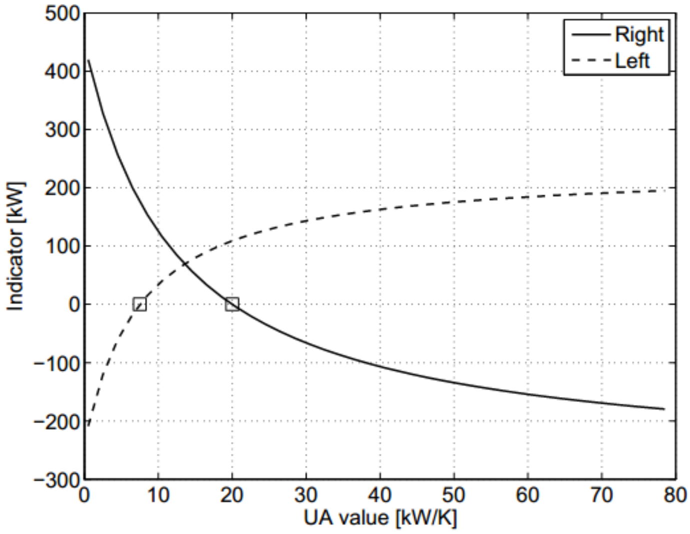

Fig. 5. Indicator for the heat exchanger as function of UA value and using the set of independent variable in terms of outlet temperatures.

Figure 6 shows the normalised indicator, using exergy fuel rate as given in Equation 23, as a function of the UA value, for the two reference operating conditions of the heat exchanger and using both of the two sets of independent variables. The normalised indicators are strictly increased when the UA value is decreased below the reference values, and vice-versa, is strictly decreased when the UA value is increased. This is observed for both reference operating conditions and sets of independent variables.

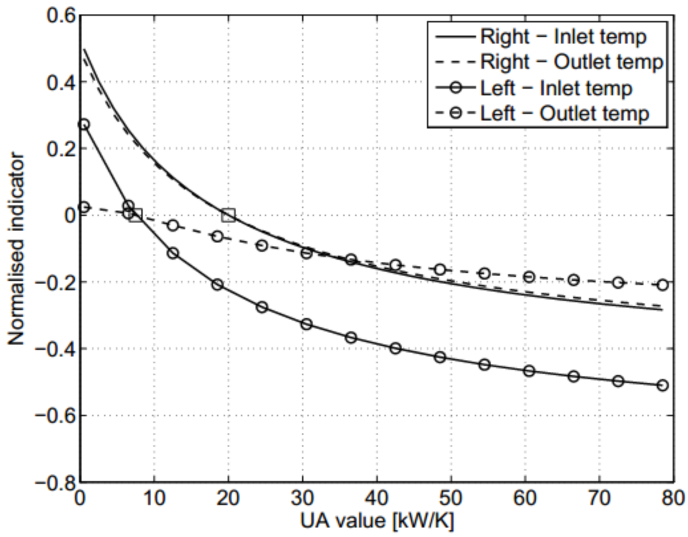

Fig. 6. Normalised indicator for the heat exchanger as a function of UA value for both sets of independent variables.

3.1. Gas turbine

Figure 7 shows the exergy destruction rate and the normalised exergy destruction rate, using exergy fuel rate as given in Equation 23, as a function of isentropic efficiency for the gas turbine expander at the given inlet and ambient conditions. Both the exergy destruction rate and the normalised exergy destruction rate increase steadily with decreased isentropic efficiency.

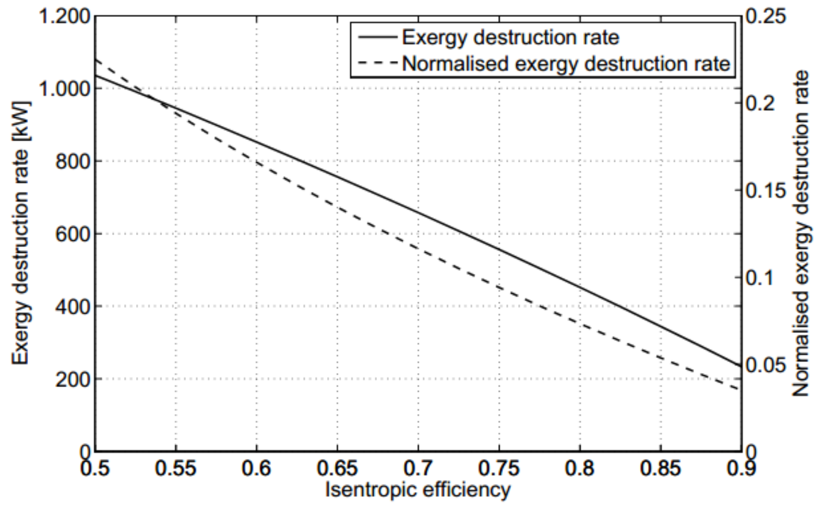

Fig. 7. Exergy destruction rate and normalised exergy destruction rate of the gas turbine expander as a function isentropic efficiency.

Table 8 shows the approximated characteristic curve derivatives for the gas turbine expander at the reference operating condition. The derivatives, using the exergy destruction rate as the dependent variable, are four orders of magnitude larger compared to the derivatives using the normalised exergy destruction rate as the dependent variable.

Table 8. *Characteristic curve derivatives of the gas turbine.*

Derivative	Derivative value
$\partial \dot{E}_D/\partial P_1$ [kW/kPa]	91.614
$\partial \dot{E}_D/\partial T_1$ [kW/K]	-0.169
$\partial \dot{E}_D/\partial P_2$ [kW/kPa]	-264.149
$\partial N_D/\partial P_1 \times 10^4$ [-/kPa]	0.195
$\partial N_D/\partial T_1 \times 10^4$ [-/K]	-0.005
$\partial N_D/\partial P_2 \times 10^4$ [-/kPa]	-1.518

Figure 8 shows the indicator as given by Equation 19 and normalised indicator as given by Equation 23, as a function of the isentropic efficiency of the gas turbine expander. The indicator and normalised indicator both steadily increase from zero when the isentropic efficiency is decreased below the reference operating condition and vice-versa, steadily decrease when the isentropic efficiency is increased.

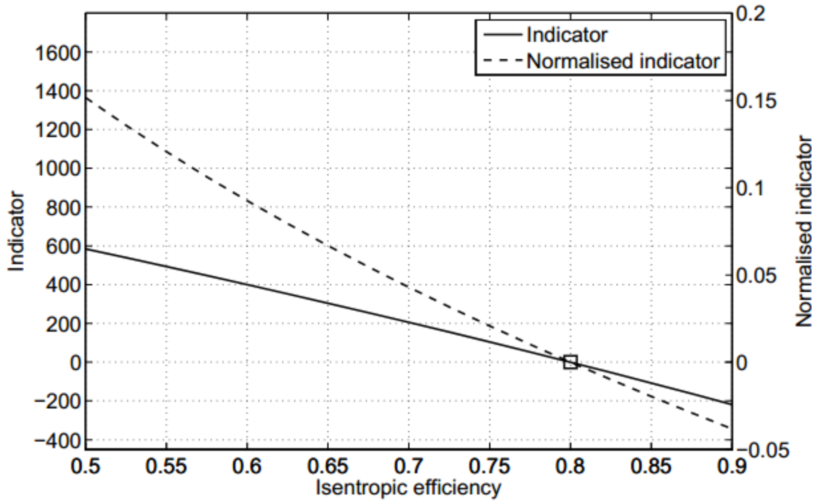

Fig. 8. Indicator and normalised indicator for the gas turbine expander as a function isentropic efficiency.

4. Discussion

When the characteristic curves are approximated at both sides of the entropy generation paradox curve and using the set of independent variables in terms of the inlet temperatures, the indicators can become smaller than zero when the UA value is decreased. The component can thus be deduced to be working more effectively, when it is actually degraded. This behaviour can be explained from the definition of the indicator, given by Equation 19, that consists of two terms. The first term compares the exergy destruction rates at the real and reference operating conditions. This term can become negative because of the entropy generation paradox, where the exergy destruction rate does not steadily increase when the UA value is decreased from a relatively high UA value. The second term compares the exergy destruction rates because of the different operating point at the real and reference operating conditions. Due to the entropy generation paradox, this term does not compensate for the first term. In principle the second term vanishes if the independent variables have the same values at the reference and real operating conditions. When the characteristic curves are approximated at the left side of the entropy generation paradox curve and using the set of independent variables in terms of the outlet temperatures, the indicator becomes negative when the UA value is decreased. This can be explained by the counter-intuitive behaviour of the component in terms of exergy destruction rate, where a decreased UA value results in increased exergy destruction rate.

A possible weakness in the definition of the relative indicator (given by Equation 20) is illustrated in the test cases where the independent variables at the real and reference operating conditions have the same value. In these cases, the relative indicator, which is a relative measure of the two terms of the indicator, results in unity values as the second term vanishes. This can give misleading information, while the alternative relative indicator (given by Equation 24) can avoid this problem.

The indicator shows that the choice of the independent thermodynamic variables in relation to the entropy generation paradox is significant. Using the set of independent variables in terms of the outlet temperatures, rather than the inlet temperatures, improves the usability of the exergy destruction rate as the dependent variable. However, this is not the case when the derivatives are approximated for the reference operating condition at the left side of the entropy generation paradox curve. In analogy with the alternative entropy generation number [7], the first term in the definition of the normalised indicator always becomes positive for degradations, both when operating at right and left side of the entropy generation paradox curve. Thus, the choice of the independent variables does not affect the normalised indicator as much.

Although the entropy generation paradox in some instances imposes restrictions on the use of exergy destruction rate as the dependent variable, the characteristic curve method is still applicable. The principle of the characteristic curves as the foundation of the method and its strict mathematical formulation in terms of independent variables is still valid, as discussed in [1]. Therefore, its indicator can be defined in a unique way, but by using a different dependent variable, e.g., the normalised exergy destruction rate.

Other thermoeconomic diagnosis methods were not considered in the relation to the entropy generation paradox. However, the diagnosis methods based on specific exergetic consumption, e.g., as suggested in [12-14] presumably avoid the problems associated with entropy generation paradox. This is because the dependent variable in the method is defined as the inverse of the exegetic efficiency, which is directly related to the normalised exergy destruction rate and is thus not based on exergy flow. Alternatively, the normalised exergy destruction rate could be used as the dependent variable in the previous mentioned method.

It can be reasoned that the problems associated with the entropy generation paradox and using the exergy destruction rate as the dependent variable may seldom be expected in practice. Heat exchangers may not necessarily degrade from a relatively high UA value to a relatively low UA value without it being noticed in the meantime. Moreover, heat exchangers are rarely so poorly designed that they operate with such low UA values as considered in one of the reference operating conditions.

5. Conclusion

The relation of the characteristic curve method with the entropy generation paradox has been analysed. Naturally, as the entropy generation paradox describes the entropy generation number as a function of heat exchanger effectiveness, the focus has been on heat exchangers. Two different reference operating conditions have been examined for the heat exchanger with two different sets of independent variables. To further extend the analysis, a gas turbine expander has also been considered.

It is shown that an operation anomaly in a heat exchanger does not always alter the exergy destruction rate in a negative way because of the entropy generation paradox. A negative indicator can thus be observed where the component is deduced to be working more effectively, when it is actually degraded. The choice of the independent variables also becomes an important factor. Using the set of independent variables in terms of the inlet temperatures restricts the application of the characteristic curve method, as exergy destruction rate does not strictly increase when the UA value is decreased from a relatively high UA value. Using the set of independent variables in terms of the outlet temperatures, the application of the method is valid at the right side of the entropy generation paradox curve. At the left side of the entropy generation paradox curve, negative indicators were observed when the UA value was decreased as the exergy destruction rate counter-intuitively decreased.

To resolve the relation of the characteristic curve method with the entropy generation paradox, normalised exergy destruction rate was proposed as an alternative dependent variable. Opposed to exergy destruction rate, the normalised exergy destruction rate strictly increased in case of an operation anomaly, e.g. with decreased UA value of a heat exchanger. The resulting indicator improved the usability of the characteristic curve method as it ensures positive indicators in cases of operation anomalies but at the same time ensures the strength of the method in the form of its strict mathematical formulation. Moreover, the alternative dependent variable allows for more flexibility in the choice of the set of independent variables.

Nomenclature

A Area, m^2

c_p Specific heat capacity, $\mathrm{kW/kg\ K}$

C_T Turbine constant, kg/s $\sqrt{\mathrm{bar}}$

$\dot{E}$ Exergy rate, kW

h Specific enthalpy, kJ/kg

I Indicator, kW

N Normalised exergy destruction rate

P Pressure, bar

s Specific entropy, $\mathrm{kJ/kg}$ K

T Temperature, K

U Overall heat transfer coefficient, $\mathrm{kW/m^2}$ K

$\dot{W}$ Work rate, kW

Greek symbols

Δ Deviation from reference operating condition

∂ Partial derivative

η_{is} Isentropic efficiency

τ Independent thermodynamic variable

Subscripts and superscripts

calc Calculated

D Destruction

F Fuel

i Component index

k Variable index

op Additional operating condition

rat Ratio

real Real operation condition

ref Reference operating condition

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