Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 199 (2017) 3374–3379

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Operational modal analysis for the characterization of ancient water towers in Pompeii

F. Lorenzoni[a]*, M. R. Valluzzi[b], M. Salvalaggio[a], A. Minello[a], C. Modena[a]

[a] *University of Padova, Department of Civil, Environmental and Architectural Engineering, 35131 Padova, Italy*
[b] *University of Padova, Department of Cultural Heritage: Archaelogical and History of Art, Cinema and Music, 35139 Padova, Italy*

Abstract

In the framework of an investigation campaign carried out in June 2015 by the authors on four ancient water towers (10–20 BC) in the archaeological site of Pompeii, modal analysis and output-only identification techniques were employed to extract the dynamic properties in order to assess structural vulnerabilities and support numerical model updating.

The four investigated towers (selected among the fourteen present within the archaeological site) are free-standing structures at least 6 m tall, belonging to the Castellum Aquae, i.e. the ancient aqueducts system of the city. During the Roman Age, until the destruction of Pompeii due to the volcanic eruption in 79 AD, water towers provided fresh water to houses, palaces and villas. This particular type of structures are classified as no. 1, 2, 3 and 4 by archaeological literature: no. 1 and 4 are made of soft stone masonry (tuff, limestone), while no. 2 and 3 are composed by brickwork masonry.

The paper reports the outcomes of ambient vibration tests performed on four towers in terms of extracted modal parameters using various operational modal analysis techniques. Obtained data are then used to study numerically the soil-structure interaction problem and implement model updating procedures.

Keywords: operational modal analysis; archaeological structure; non destructive tests; model updating

---

* Corresponding author. Tel.: +39 049 827 5560; fax: +39 049 827 5613.
E-mail address: filippo.lorenzoni@dicea.unipd.it

10.1016/j.proeng.2017.09.446

1. Introduction

During the archaeological excavation of Pompeii numerous brick and stone masonry water towers were brought to light. Those elements were part of the ancient aqueduct system. The pilasters are located throughout the city and have different heights with the scope of providing the Pompeii water supply system with proper gravity flow of the water. The height and slenderness of some pilasters in combination with the importance of preservation of the cultural heritage in the area of Pompeii were the incentives for performing a vulnerability assessment of those structures.

The first step of the applied knowledge-based approach for the conservation of this type of structures was the execution of on-site ND investigations to (i) characterize the quality of materials and the state of conservation, (ii) detect and locate active damaging process and (iii) identify the dynamic behavior, necessary to calibrate and validate behavioral models.

The paper reports the results of ambient vibration tests performed by the authors on 4 water towers and the successive FE model updating. A convenient way to explore the instability mechanism of these free-standing elements is to rely on the inverted pendulum model, and a crucial step in the analysis is therefore to establish a reliable response for the tower foundations. The influence of soil-structure interaction was studied numerically, introducing elastic foundation and varying the linear properties of masonry elements to obtain an optimal matching between experimental and theoretical modal parameters.

2. Castellum Aquae hydraulic system

At Roman Age, water was carried to Pompeii by a public external aqueduct which ended at the entrance of *Porta Vesuvio,* in the main water tank of the town, the so called *Castellum Aquae.* Water was distributed to quarters through 14 secondary water towers, free-standing structures on whose tops lead tanks were placed. Most of the elements were probably built between 20-10 BC [1], whereas some else had been subjected to reconstruction after seismic event of AD 63.

The thirteen water towers remained nowadays usually have squared bases, while structural material is often soft stone or clay brick masonry [2][3].

Towers object of this paper are the ones numbered as no. 1, 2, 3 and 4 by archaeological literature. No. 1 is made of Nocera tuff masonry, no. 2 and no. 3 are made of clay brick masonry while no. 4 is composed of three layers (i.e. Sarno limestone, Nocera tuff and yellow tuff) (Figure 1).

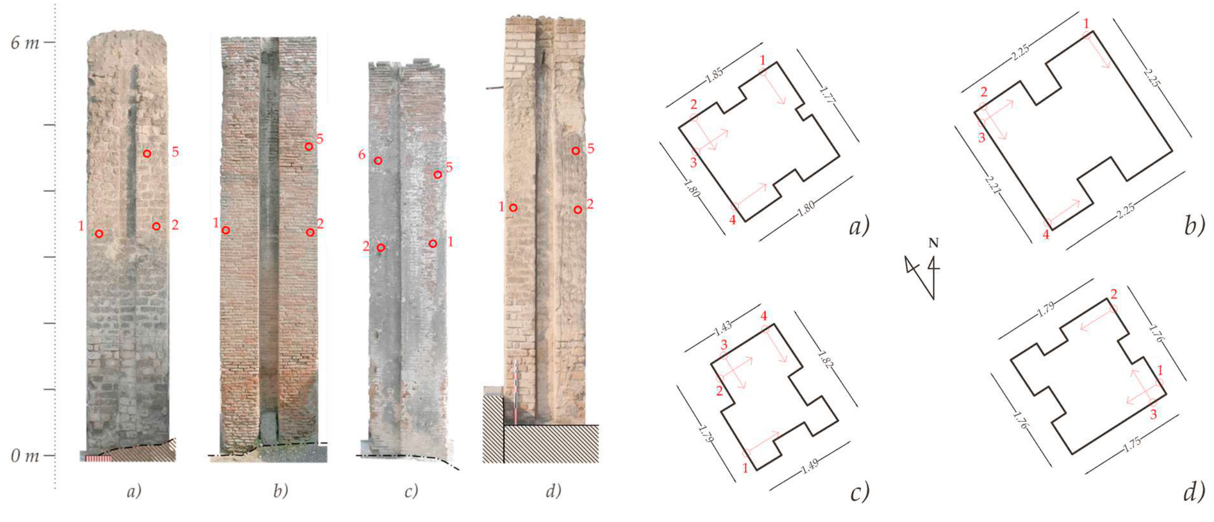


Figure 1. Elevations and bases of tower no. 1 (a), no. 2 (b), no. 3(c), no. 4 (d) [2][3].

3. Survey and on-site inspections

Water towers analyzed within the present study are at least 6 m tall and can be simplified from a structural point of view as on inverted pendulum model, i.e. a single degree of freedom (SDOF) system, fixed at base. The horizontal sections of the towers are generally squared and 1.2 m wide on average. The most irregular shape in plan is the tower no.3 one, which is also narrower than others. The geometric survey (plan and elevation) of the four towers are reported in Figure 1. Each element has vertical grooves at least on two sides along the entire height, used to place the pipes which carried water to and from the top tank.

A survey campaign was performed in March 2015, while experimental on site testing took place in June 2015. On this occasion, sonic tomographic tests were performed in order to identify the conservation status of the structures, detect internal voids and cores and assess qualitatively the masonry composition. With similar aims (i.e. evaluation of the inner composition of the towers) also ground penetrating radar (GPR) test were performed and the results provided by both techniques compared and validated. Moreover, dynamic identification tests were carried out on the four towers to characterize the dynamic behavior and extract modal parameters, later used to update numerical models.

4. Ambient vibration test and modal identification

Ambient vibrations tests (AVT) and output-only identification techniques were performed to extract modal parameters in terms of natural frequencies, damping ratios and mode shapes. Dynamic identification tests on the four water towers of *Castellum Aquae* are identified as follow: ID01 for tower no. 1, ID02 for tower no. 2, ID03 for tower no. 3, ID04 for tower no. 4.

Tests were carried out through the record of three acceleration time-histories in 6 points per each structure, according to the analysis of the expected mode shapes (Figure 1: red dot indicate the position of the accelerometers). This test layout allows identifying both bending and torsional mode shapes of the analyzed structures. High-sensitivity (10’000 mV/g) piezometric accelerometers are rigidly fastened to the masonry walls and connected by cable to the acquisition unit that converts the electric signals into accelerations and stored the recorded time histories.

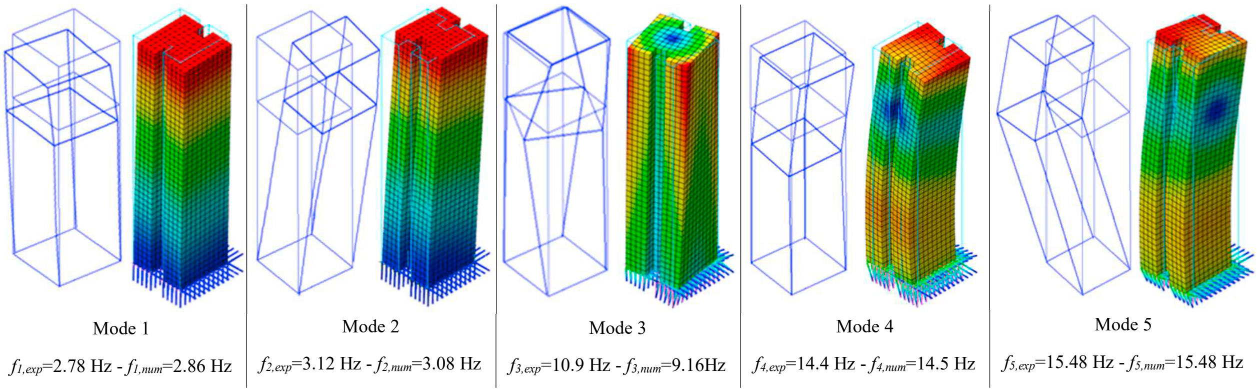
Ambient vibrations were acquired at a sampling frequency of 100 Hz. Each acquisition file is composed by 65’500 points and a recording duration of about 10 minutes.

Two non-parametric frequency domain techniques were implemented for system identification: FDD and EFDD [4]. Peaks in the frequency domain related to structural eigenfrequencies were selected and the corresponding modal parameters (damping ratios and eigenvectors) extracted.

Results of modal analysis on the 4 towers are summarized in Table 1.

Table 1. Modal parameters of water towers no. 1, 2, 3 and 4

	WT 01			WT02			WT03			WT04		
Mode	f [Hz]	ξ [%]	Mode shape	f [Hz]	ξ [%]	Mode shape	f [Hz]	ξ [%]	Mode shape	f [Hz]	ξ [%]	Mode shape
1	2.979	1.205	Bend. Y	2.783	1.201	Bending diagonal XY	2.686	1.656	Bending X	3.076	0	Bending X
2	3.174	1.09	Bend. X	3.125	1.285	Bending diagonal XY	3.320	1.227	Bending Y	3.125	1.127	Bending Y
3	16.89	0.9045	Torsional	10.94	0.5405	Torsional	12.23	0.9351	II Bend. X	14.01	0.6645	Torsional
4	17.09	0.3239	II Bend. Y	14.4	1.742	II Bending diagonal XY	15.48	0.667	Torsional	15.04	0.2774	II Bend. X
5	17.33	0.8979	II Bend. X	15.48	1.37	II Bending diagonal XY	16.31	1.412	II Bend. Y	15.19	0.809	II Bend. Y



Mode 1
$f_{1,exp}$=2.78 Hz - $f_{1,num}$=2.86 Hz

Mode 2
$f_{2,exp}$=3.12 Hz - $f_{2,num}$=3.08 Hz

Mode 3
$f_{3,exp}$=10.9 Hz - $f_{3,num}$=9.16Hz

Mode 4
$f_{4,exp}$=14.4 Hz - $f_{4,num}$=14.5 Hz

Mode 5
$f_{5,exp}$=15.48 Hz - $f_{5,num}$=15.48 Hz

Figure 2. Experimental and numerical mode shapes of water tower no. 2

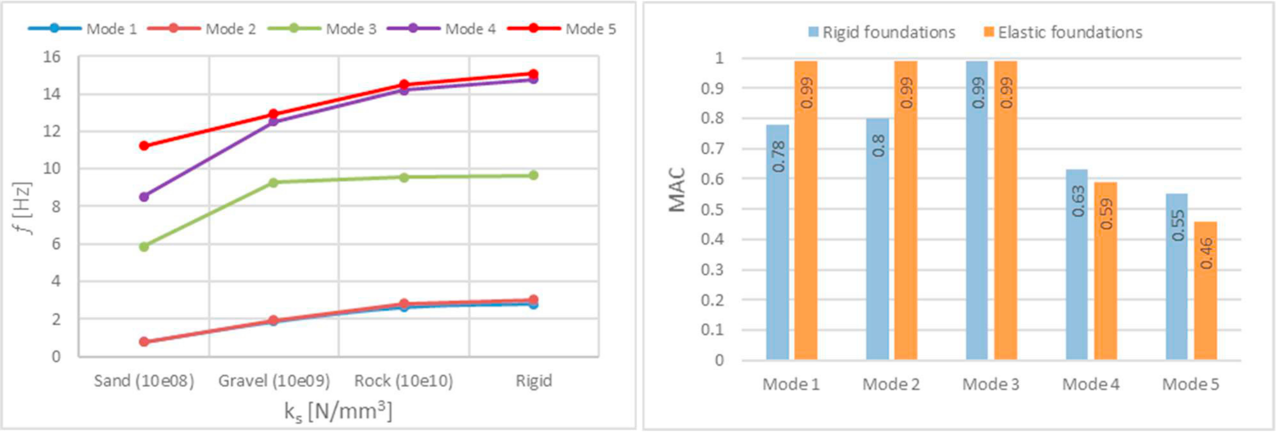
Water towers show generally the theoretical mode shapes for a SDOF structure, i.e. 2 first order bending modes along the principal horizontal directions (X, Y), 1 torsional mode and 2 second order bending modes. This is true for towers no. 1 and no. 4.

Tower no.2 show a slightly different dynamic behavior (e.g. bending mode shapes along the two mutually orthogonal diagonal directions instead of along the principal axes) probably due to an irregular connection at the base or soil-structure interaction effects (Figure 2). Tower no.3 presents inverted modes (no. 3 and no. 4) in terms of mode shapes most likely due to its non-symmetric shape.

5. Finite element modeling and structural identification

3D finite element models of the 4 towers have been successively created to assess the structural behavior. DIANA FEA 9.6 [5] environment was used to generate the four models, adopting 8-noded (HX24L) and 6-noded (TP18L) linear brick elements. All FE models are composed by a varying number of nodes (from 3700 to 8300) and elements (from 2500 to 6000). The average element size is about 15 cm for each side of the 3D elements.

Once numerical models were constructed the model updating process started, trying to reach a satisfying modal match between natural frequencies and mode shapes experimentally identified and numerically calculated.



a) b)

Figure 3. a) Sensitivity analysis of tower no.2: variations of the absolute value the numerical frequency in function of the type of soil, b) variation of the MAC index for tower no.2 with and without elastic foundations

The procedure for the model updating can be synthetized as follow:

1. Masonry was modeled as an homogeneous, linear elastic and isotropic material;
2. Mass densities ρ were selected according to the Italian code [6], i.e. 16 KN/m^3 for soft stone masonry, 18 KN/m^3 for clay brick masonry and kept fixed for all the analyses; Elastic modulus of masonry was selected as an updating parameter and its variation range defined according to the Italian code;
3. Firstly, rigid constraints were introduced at the base of the towers; then, elastic foundations were implemented according to the Winkler soil model formulation;
4. Sensitivity analysis was performed to select a proper range of variation of spring stiffness;
5. Calibration of FE models was achieved by an iterative procedure, changing the value of the updating parameters (i.e. Elastic modulus of masonry E and stiffness of foundation springs K), until a satisfactory agreement between experimental and numerical modal parameters was achieved [7].

The first attempt of model updating took into account rigid constraints at the base of the towers and the elastic modulus of masonry as unique updating parameter. The updating process lead to unreliable results of this mechanical property, especially for towers no. 1 and no. 2, as reported in Table 2 (see the column "lower bound of E" with rigid foundations). The updated elastic modulus of tower no.1 made of tuff stone is out of the range provided by the table of the Italian code and it is considerably lower than the value of the same parameter of tower no.4, composed by roughly the same type of masonry. Similar considerations can be made for the comparison between tower no.2 and no.3: both of them are composed by brick masonry even though the final values of the updated Young's modulus is totally different (450 MPa for tower no.2 and 1050 MPa for tower no.3).

Table 2. Initial and final values of the updating parameters with and without elastic foundations

Water tower	Structural elements	ρ [KN/m^3]	E [MPa] Lower bound (Rigid foundations)	K$_s$ [N/mm^3]	E [MPa] Upper bound (Elastic foundations)	E [MPa] Italian code range [6]
1	Tuff	16	530	$5{\cdot}10^8$	1000	900÷1260
2	Clay brick	18	450	$5{\cdot}10^8$	1200	1200÷1800
3	Clay brick	18	1050	10^9	1900	1200÷1800
4	Sarno limestone, yellow tuff, Nocera tuff	16	800	$5{\cdot}10^8$	1800	900÷1260

The assumption of rigid foundations inevitably leads to a decrement of the modulus of elasticity of masonry to match the experimental modal parameters. In order to reach an optimal calibration of the FE models elastic constraints at the foundation level were introduced, adopting a Winkler model for soil.

Three SP2TR spring elements were generated per each base node for all principal directions, simulating the soil as a bed of springs and assigning a value of the vertical stiffness. Horizontal stiffness of spring was initially considered as 1/10 of the vertical stiffness, in order to take into account the scarce connection with the soil and the difference between soil compressive and shear behavior [8]. Experimental data about soil characteristics are not available. Ranges of variation for this parameter were defined according to the values of masonry Young's moduli [6] and subgrade reaction moduli provided by literature [9]. Initially, sensitivity analysis on spring stiffness was performed for tower no. 2, in order to study the variation of the percentage error between numerical and experimental frequency (Figure 3). Young's modulus of masonry was kept constant and equal to E=450 Mpa (the value obtained from the calibration of the model with rigid constraints) and the stiffness of the springs varied. Thus, a range of variation for Winkler modulus was defined merging literature data and sensitivity analysis data, which goes from $K_{s,\,min}=5{\cdot}10^8$ N/mm^3 to $K_{s,\,max}=10^9$ N/mm^3. This range was then extended to all water towers.

The introduction of spring element at towers foundations provided a better calibration of the elastic characteristics of masonry. It is possible to state that:

- Tower no.1 and no.2 are probably characterized by a bad connection with the soil at the foundation level since the assumption of rigid constraints led to unreliable results;

- Tower no.3 and no.4 show a more rigid connection with the soil and the introduction of elastic foundations slightly overestimated the masonry Young's modulus.

Results of the final calibration are reported in Table 3: it shows for each tower the comparison between experimental and numerical modal parameters in terms of natural frequencies (average error ε) and mode shapes (MAC index). Comparison between experimental and numerical mode shapes of tower no. 2 are reported in (Figure 2).

Table 3. Calibration results of the FE model with elastic constraints: comparison between modal and numerical parameters

	WT01				WT02				WT03				WT04			
Mode	f_{exp}	f_{num}	ε [%]	MAC	f_{exp}	f_{num}	ε [%]	MAC	f_{exp}	f_{num}	ε [%]	MAC	f_{exp}	f_{num}	ε [%]	MAC
1	3.02	3.05	0.83	0.93	2.78	2.86	3.02	0.99	2.68	2.66	0.97	0.97	3.07	3.09	0.52	0.93
2	3.17	3.06	3.31	0.94	3.12	3.08	1.44	0.99	3.32	3.53	6.54	0.98	3.12	3.01	3.78	0.91
3	16.89	11.39	32	0.72	10.94	9.16	16	0.99	13.2	15.21	14.95	0.82	14.01	11.97	14.54	0.1
4	17.14	16.09	6.13	0.71	14.4	14.56	1.17	0.59	15.48	13.25	14.41	0.97	15.04	17.66	17.43	0.96
5	17.29	16.11	6.7	0.74	15.48	14.99	3.11	0.46	16.31	19.23	17.96	0.96	15.19	17.30	13.9	0.75

6. Conclusions

The combination of ambient vibration tests and numerical models (for model updating) proved to be an effective tool to study and assess the real structural behavior of four water towers in the archaeological site of Pompeii. From this study some important conclusions can be drawn:

- Modal parameters in terms of eigenfrequencies, damping factors and modes shapes were successfully extracted for all structures, implementing and compering various frequency-domain OMA techniques;
- Model updating procedures were applied considering the elastic modulus (E) of masonry and the stiffness of foundation springs (K) as updating parameters
- The results of the calibration provide interesting information on the soil-structure interaction for each free-standing element. It was possible to study and identify different boundary conditions and state of conservations for towers characterized by similar geometry and materials.
- The introduction of elastic foundations for this type of structures (SDOF systems fixed at the base) provide a better match between numerical and experimental modal parameters, meaning that both soil and masonry deformability contribute to their dynamic response.

References

[1] Ohlig C., De aquis Pompeiorum: das Castellum Aquae in Pompeii: Herkunft, Zuteilung, Verteilung des Wassers, Nimègue, 2001
[2] Dessales H., Carlit J., Charlier P., Coutelas A., De Sigoyer J., D'eau et de pierre. Recherches des laboratoires d'archéologie et de géologie de L'ecole normale supérieure sur le Réseau hydraulique du Pompéei et son bâti, coord. H. Dessales, 2008
[3] Dessales H., Le portage de l'eau. Fontaines et distribution hydraulique dans l'architecture domestique de l'Occident romain, Bibliotheque de l'Ecole française de Rome, Rome, 2013
[4] Brincker R., Andersen P., Jacobsen N.J., Automated Frequency Domain Decomposition for Operational Modal Analysis, Proceedings of the 25th SEM International Modal Analysis Conference, Orlando (USA), 2007
[5] TNO Diana, DIANA 9.6 Finite Element Analysis User Guide, TNO DIANA BV, Delft, 2014
[6] Cons. Sup. LL. PP., Circolare 2 febbraio 2009, n. 617, Istruzioni per l'applicazione delle « Norme Tecniche per le Costruzioni » di cui al D.M. 14/01/2008 G.U. del 26/02/2009 n. 47, supplemento ordinario n. 27/2009
[7] Lorenzoni F., Integrated methodologies based on strcutural health monitoring for the protection of culuìtural heritage buildings, PhD Thesis. Director : Prof. Modena C., University of Trento, University of Brescia, University of Padova, University of Trieste, University of Udine, IUAV University of Venezia
[8] Elyamani A., Integrated monitoring and structural analysis strategies for the study of large historical construction. Application to Mallorca cathedral. PhD Thesis. Director. Roca P., Universitat Politecnica de Catlunya, 2015
[9] Bowles J. E., Foundation analysis and design, McGraw-Hill, USA, 1996, pp. 121-127, 501-509