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QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA E STATISTICA

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Advanced or postponed wage payments: Sraffa validates Marx

n.756 – Giugno 2017

Abstract

In his theoretical model of production prices Marx follows the classical economists in treating wages as being paid in advance. Sraffa, instead, tends to treat them as being paid post factum. However, when Marx tackles the problem under less abstract scrutiny, he abandons the classical approach and declares that, as a matter of fact, wages are postponed. We prove that, if the period of postponed wage payment differs from the length of the production process, the correct prices are better approximated by an equation with the full post-payment of wages than by one with full pre-payment. Under perfect competition and postponed wage payments, Sraffa's approach to price determination is the correct one, and validates Marx's non-classical vision, whatever the period of wage payment.

Keywords: Value theory, Marx, Sraffa
JEL Codes: B310, B510

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We thank Neri Salvadori and Alberto Valli for their comments on an earlier draft of this article. All disclaimers apply.

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ADVANCED OR POSTPONED WAGE PAYMENTS: SRAFFA VALIDATES MARX

When dealing with a capitalist economy, Sraffa (1960, 9-11) determines production prices with a formula corresponding to the following equation:

$$\boldsymbol{p_1} = (1 + r)\boldsymbol{p_1}\boldsymbol{A} + w\boldsymbol{l}, \tag{1}$$

where $\boldsymbol{p_1}$ is a vector of prices, $\boldsymbol{A}$ an indecomposable matrix of technical coefficients, $\boldsymbol{l}$ a vector of labour coefficients, and the scalars r and w represent the profit and wage rates. In equation (1) the wage is treated as being paid *post factum*. Since it is not paid in advance, it is not capitalized.

Marx, instead, defines the profit rate $r = S/(C + V)$, where $S = \boldsymbol{p_2}(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{q} - w\boldsymbol{l}\boldsymbol{q}$, $C = \boldsymbol{p_2}\boldsymbol{A}\boldsymbol{q}$, $V = w\boldsymbol{l}\boldsymbol{q}$, and $\boldsymbol{q}$ is a vector of activity levels. This implies that wages are paid in advance and the price equation is:

$$\boldsymbol{p_2} = (1 + r)\boldsymbol{p_2}\boldsymbol{A} + (1 + r)w\boldsymbol{l}. \tag{2}$$

On the other hand, he declares unequivocally that in a capitalist economy 'the labourer is not paid until after he has expended his labour power' (1867-93, I, 567). As he observes, despite the common view that 'the capitalist, *using the jargon of political economy*, advances the capital laid down in wages […,] as a matter of fact the reverse takes place. It is the labourer who advances his labour to the capitalist' (1867-93, II, 219). He also thinks that the practice of taking C and V as a base for calculation of the profit rate is the result of a distorted point of view: 'the whole thing amounts to a capitalist *quid pro quo*, and the advance which the labourer gives to the capitalist in labour is turned into an advance of money given to the labourer by the capitalist' (ibid). Finally, he admits that, by adopting the formula $r = S/(C + V)$, he proceeds 'according to the usual way of reckoning' (1867-93, I, 227) and thus complies with 'the jargon of political economy', i.e. that of the classical economists. Sraffa (1960, 10), instead, makes it clear that he 'assume[s] that the wage is paid *post factum* […] thus abandoning the classical economists' idea of a wage "advanced" from capital'.[1]

[1] When he considers 'wages as consisting of the necessary subsistence of the workers', Sraffa treats them 'on the same footing as the fuel for the engines or the feed for the cattle' (1960, 9). However, when he comes to 'consider the

What are the reasons for the *quid pro quo*? One might be that equation (2) represents the usual practice of price fixing followed by firms: normal prices are determined by applying a gross markup to variable costs $(C + V)$. Then equation (2) holds independently of whether wages are advanced or postponed, but simply because it corresponds to the procedure by which firms fix prices. Price fixing by the firms, however, implies that markets are not fully competitive. If perfect competition is assumed – as done by Marx, following Smith and Ricardo – market prices are not fixed by firms, but are determined by the forces of demand and supply. If the market process is stable, and wages are postponed, market prices must gravitate around the production prices represented by equation (1), not equation (2).

Another reason for the *quid pro quo* could be that in many sectors (e.g. agriculture) the length of the production process (one year) is longer than the length of the sub-period for wage payment (a week or a month). Therefore, even if paid at the end of the week or the month, wages are advanced by capitalists during the production process and thence must be capitalized at the end of the year. This observation, however, does not justify equation (2).

In fact, suppose the annual wage, w, is post-paid in t sub-period instalments during the production process, the length of the wage payment sub-period being $1/t$ of the length of the production process. The sub-period wage is w/t. The annual factor of profit is $1 + r = (1 + i)^t$, where i is the sub-period rate of interest. As shown by Steedman (1977, 103-4), prices are determined as:

$$\boldsymbol{p_3} = (1 + r)\boldsymbol{p_3 A} + [1 + (1 + i) + \cdots (1 + i)^{t-1}]\frac{w}{t}\boldsymbol{l}.$$

Since $[1 + (1 + i) + \cdots (1 + i)^{t-1}] = \frac{(1+i)^t - 1}{i} = \frac{r}{i}$,

$$\boldsymbol{p_3} = (1 + r)\boldsymbol{p_3 A} + \frac{r}{it}w\boldsymbol{l}. \tag{3}$$

which is equal to (1) when t=1. Now, a real economy involves production processes of different lengths. Some of them are longer than the wage payment period, while others are shorter. In abstract theory, this difficulty is overcome by assuming that all production processes, as well as the wage payment period, have the same length. Then, the question is whether (1) or (2) is more plausible in this idealized economy.[2] The answer is: the most plausible is the one that better approximates equation (3).

Steedman (1977, 105) proves that (1) gives a good approximation for low profit rates. The degree of approximation weakens when r rises. Steedman's result can be generalized. We prove that equation (1) always yields a better approximation than

division of the surplus between capitalists and workers' (ib.) and, more generally, when he treats wages as a variable 'share of the annual product' (ib., 10), he assumes they are postponed.

[2] Sraffa (1960, 10) does not raise this question. He just assumes that all the production processes and the wage payment period last one year.

(2). Meanwhile, let us show in figure 1 the behaviour of r/it and $1+r$ for t=12 and i∈[0.001, 0.1]. It is evident that $(1+r)-r/it > r/it-1$, which is the condition under which the full wage post-payment equation yields a better approximation than the full wage pre-payment equation.

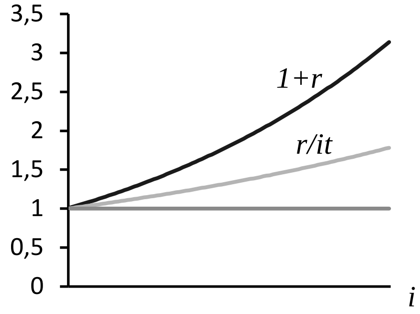

Figure 1

More generally, we prove in the appendix that

$$x_t(i) := \left[(1+r) - \frac{r}{it}\right] - \left(\frac{r}{it} - 1\right) = (1+i)^t - 2\frac{(1+i)^t - 1}{it} + 1 > 0, \ \ \forall t > 0, \ \forall i > 0.$$

The behaviour of $x_t(i)$, for $t = 12$ and $t = 52$, is shown in figure 2.

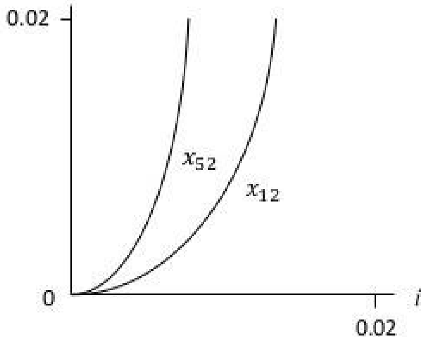

Figure 2

Appendix

PROPOSITION: for every positive i and every positive t,
$(1+i)^t - 2\frac{(1+i)^t-1}{it} + 1 > 0.$
PROOF: the relation $(1+i)^t - 2\frac{(1+i)^t-1}{it} + 1 > 0$ can be rewritten as

$$\left(1-\frac{2}{it}\right)(1+i)^t+\left(1+\frac{2}{it}\right)>0, \tag{a1}$$
which is trivially true if $it \geq 2$.
If $0 < it < 2$, the quantity $\left(1-\frac{2}{it}\right)$ is negative, thus (a1) is equivalent to
$$(1+i)^t < -\frac{1+\frac{2}{it}}{1-\frac{2}{it}},$$
or
$$(1+i)^t < \frac{2+it}{2-it}. \tag{a2}$$
In lemma 3 we prove (a2) for every positive i and $0 < t < \frac{2}{i}$.
Lemma 3 uses the relation
$$\left(1-\frac{it}{2}\right)^2(1+i)^t < 1,$$
which is proved in lemma 2.
This uses the relation
$$\frac{i^2}{4}\ln(1+i)\,t^2 - i\left(\ln(1+i)-\frac{i}{2}\right)t + \ln(1+i) - i < 0,$$
which is proved in lemma 1.

LEMMA 1: for every positive i, if $0 < t < \frac{2}{i}$,
$$\frac{i^2}{4}\ln(1+i)\,t^2 - i\left(\ln(1+i)-\frac{i}{2}\right)t + \ln(1+i) - i < 0. \tag{a3}$$
PROOF: consider the parabola with equation:
$$y_i(t) = \frac{i^2}{4}\ln(1+i)\,t^2 - i\left(\ln(1+i)-\frac{i}{2}\right)t + \ln(1+i) - i,$$
which is convex since $\frac{i^2}{4}\ln(1+i) > 0$. It is
$$y_i(0) = \ln(1+i) - i < 0$$
or
$$0 < \ln(1+i) < i.$$
Moreover
$$y_i\left(\frac{2}{i}\right) = \frac{i^2}{4}\ln(1+i)\frac{4}{i^2} - i\left(\ln(1+i)-\frac{i}{2}\right)\frac{2}{i} + \ln(1+i) - i$$
$$= \ln(1+i) - 2\ln(1+i) + i + \ln(1+i) - i = 0.$$
(a3) follows from non-positivity at the extremes and convexity of the parabola. ■

LEMMA 2: for every positive i, if $0 < t < \frac{2}{i}$,
$$\left(1-\frac{it}{2}\right)^2(1+i)^t < 1. \tag{a4}$$
PROOF: put $g_i(t) = \left(1-\frac{it}{2}\right)^2(1+i)^t$, and take the derivative:
$$g_i'(t) = 2\left(1-\frac{it}{2}\right)\left(-\frac{i}{2}\right)(1+i)^t + \left(1-\frac{it}{2}\right)^2(1+i)^t\ln(1+i)$$

$$= \left(\frac{i^2 t}{2} - i\right)(1+i)^t + \left(1 - it + \frac{i^2 t^2}{4}\right)(1+i)^t \ln(1+i)$$
$$= \left(\frac{i^2}{4}\ln(1+i)\, t^2 - i\left(\ln(1+i) - \frac{i}{2}\right) t + \ln(1+i) - i\right)(1+i)^t.$$

By (a3) $g_i'(t) < 0$ for every t between 0 and $\frac{2}{i}$. (a4) follows from this and from $\lim_{t\to 0^+} g_i(t) = 1$. ■

LEMMA 3: for every positive i, if $0 < t < \frac{2}{i}$, (a2) is true.

PROOF: by (a4) and relation $0 < \ln(1+i) < i$,

$$(1+i)^t \ln(1+i) < \frac{1}{\left(1-\frac{it}{2}\right)^2}\, i,$$

or

$$(1+i)^t \ln(1+i) < \frac{4i}{(2-it)^2}\ .$$

From the isotonic property of integrals we know that, if $0 < t < \frac{2}{i}$,

$$\int_0^t (1+i)^u \ln(1+i)\, du < \int_0^t \frac{4i}{(2-iu)^2}\, du.$$

This is equivalent to

$$\int_0^t d\left((1+i)^u\right) < \int_0^t d\left(\frac{4}{2-iu}\right),$$

which, after integration, yields

$$(1+i)^t - 1 < \frac{4}{2-it} - 2,$$

equivalent to (a2). ■

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