# A Characterization of Verifiability and Observability in Contracts

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Reference to the notions of verifiability and observability is widespread in contract theory. This paper is a contribution towards a formalization, and related characterizations, of these two notions. In particular we first define them, through knowledge operators, and then provide characterization results in terms of the relevant state spaces. Since, when referring to a contract, observability typically pertains to parties while verifiability to the court, we define them differently. A main finding of the paper is that for proper contract verifiability to obtain the court must imagine true states and have information processing abilities as good as the parties.

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## 1. Introduction

The notions of observability and verifiability are fundamental in contract theory [Holmström, 1982; Hermalin and Katz, 1991; Hart, 1995; Bolton and Dewatripont, 2005]. Indeed they refer to how parties to a contract, and the court that may have to enforce it, would process the available information originated by the actual *state of the world* (*state* henceforth). Even though both concepts refer to the ability to *correctly understand* the state by the relevant agents, typically, in the literature, the term observability is associated to the parties while verifiability to the court. Despite their widespread use and conceptual importance so far, the epistemics behind the two notions, that is how parties and the court should process the relevant information, does not appear to have been investigated and characterized in a rigorous manner. This seems to be an important missing point in the theory for both because a formal characterization should convey the ideas more clearly and, moreover, because it would help reasoning on the needed conditions for observability and verifiability to obtain. The issue is even more important in view of the fact that, at least in principle, alternative definitions may be contemplated.

A first meaningful point which is often left unmentioned, that may potentially distinguish the contract parties from the court, is that the latter would typically not only be asked to draw correct inferences on what really occurred, the true state, but also to provide a (possibly) *consistent* (logical and with respect to empirical evidence) argument motivating her final verdict. Indeed, for example, Hermalin and Katz [1991] adopt a notion of verifiability which does not seem to require the judge to justify the verdict. For this reason, their notion of verifiability appears to be closer to our notion of observability. More specifically, we assume that while observability may exempt parties from providing a correct justification of their beliefs, this would not be the case for the court. In essence, the ability to verify a contract seems to require a court with a degree of information processing skills at least as good as the parties'. We shall see that this distinction lies at the heart of our definitions and characterizations.

As for courts clearly, in reality, they may have a meaningful role in resource allocation [Tirole, 1999; Djankov *et al.*, 2003], to the extent that factors like possible lack of competence, corruption, time constraints, difficulties in gathering relevant information, etc. may bias the verdict. However, in this paper, we shall only investigate a plausible characterization of courts who draw justifiable correct conclusions.

To do so, we only focus on the role of agents as "information processors" (IP), however without discussing what their (possibly bounded) abilities are due to. Moreover, we will not touch upon strategic issues related to the formulation of optimal contracts, as in Bernheim and Whinston [1998].

The framework we use has been widely adopted in economic theory to address a number of foundational issues related to agents' epistemics, such as the formation of consensus [Aumann, 1976] and the investigation of sufficient conditions for Nash Equilibria in normal form games [Aumann and Brandenburger, 1995].

We shall also discuss how our proposed formalization of verifiability could be linked to the notion of *awareness*. In particular, we shall refer to the seminal version formalized and discussed by Modica and Rustichini [1994, 1999] and further investigated by Dekel *et al.* [1998]. As pointed out by Heifetz *et al.* [2008] the extent to which later models of unawareness, based on epistemic logic, are connected to Modica and Rustichini's and Dekel *et al.*'s frameworks is not immediate. It follows that this is also true for our setting. Epistemic logic approaches typically constructed states building on elementary formulae and knowledge, modal, operators as primitives; therefore, these models have high expressive power. In this paper instead, states of the world are the primitives; in exchange for a lower descriptive ability we believe this approach gains in simplicity with no major loss in effectiveness to convey the main findings.

The work has the following structure. Section 2 is devoted to the fundamentals of the model. More specifically, we formalize the information processing abilities of the agents to a contract, the notion of a contract, and discuss the interpretation of the relevant state spaces that will appear in the analysis. In Sec. 3, we introduce observability and verifiability of a single event, of contracts and briefly discuss the relation between verifiability and awareness as defined in Modica and Rustichini [1994]. Section 4 presents the main characterization results, in terms of the relevant state spaces, for a given contract. In particular, we shall see that according to our definition a contract is correctly observed when the relevant parties imagine only states that can truly occur, but not necessarily all of them. This implies that correct contract observability can still obtain when some of the states that can indeed obtain, are not thought as possible by the parties. Verifiability instead requires the court to imagine states which coincide with those that can truly occur. In this sense, correct verifiability asks for a deeper understanding of the states that can possibly take place. Section 5 concludes the paper.

## 2. Agents, Contracts and Relevant Spaces

In this section, we model the parties to a contract and the court as IP; what follows introduces the basics of our framework.

Suppose $\Omega$ is the set of states (state space henceforth), with $\omega \in \Omega$ being the generic state, and let $P(\omega) \subseteq \Omega$ be the possibility set associated to $\omega$. A possibility correspondence is a mapping $P : \Omega \rightarrow 2^{\Omega} \backslash \{\varnothing\}$, where $2^{\Omega}$ is the power set of $\Omega$. Events are subsets of $\Omega$.

The possibility correspondence represents the set of states that IP thinks as possible when $\omega$ obtains. The sets $P(\omega)$ model IP's cognitive abilities at the *true* state $\omega$. The collection of $P(\omega)$, normally referred to as IP's *information structure* [Geanakoplos, 1989], is then taken to formalize her overall information processing skills. Information structures allow for IP's bounded rationality in interpreting the world. However, when this collection is a partition of $\Omega$ such that $\omega \in P(\omega)$, the individual is typically considered to be a rational, consistent, IP. Perfect information processing obtains when $P(\omega) = \{\omega\}$.[a]

Not surprisingly, since the court that we intend to model is a *proper* IP, we shall ask her cognitive abilities to satisfy some of the epistemic properties underlying partitions.

[a]The reason why rationality is formalized by partitions can be better appreciated upon introducing the knowledge operator. A knowledge operator is a mapping $K : 2^{\Omega} \rightarrow 2^{\Omega}$ which, for all $A \subseteq \Omega$, is defined as $KA = \{\omega \in \Omega | P(\omega) \subseteq A\}$. Knowledge could be incorrect, in the very common sense that personal beliefs may not correspond to *reality*. A partition of $\Omega$ obtains when for all $A \subseteq \Omega$, $KA \subseteq A$ and $\neg KA \subseteq K\neg KA$, where$\neg$ stands for event complementation; i.e. $\neg A = \Omega - A$. In this case it also follows that $KA \subseteq KKA$. In axiomatic approaches, the first property is called the axiom of *truth*, the second property axiom of *negative introspection* while the third property axiom of *positive introspection*. Following the suggestion of one of the referees, in the Appendix we further elaborate on why two remarkable properties of the knowledge operator, $K\Omega = \Omega$ and $K(A \cap B) = K(A) \cap K(B)$ are omitted. Indeed, they hold for any information structure and not only for partitions.

For an illustration consider the following simple example. Two parties, $a$ and $b$, signed a contract specifying that "if tomorrow it rains at day time, $a$ pays 1€ to $b$ while if it snows $b$ pays 1€ to $a$". Suppose that indeed it does rain so that $a$ should pay $b$, but $a$ refuses to. To resolve this case we have in mind a court that would be able to gather, and correctly process, the information needed to draw appropriate conclusions. Imagine that the only information the court collects is "there were many people in the streets, there was some sun light, almost everybody was wet, streets were wet, there were thunders". The court should then rightly infer that it rained, and so that $a$ must pay $b$, without using the fact that "there was some sun light" as the crucial piece of evidence motivating the verdict. We would moreover want the court to be able to draw correct conclusions, even when a contingency not specified in the contract would take place. Indeed, if it was sunny, rather than rainy or snowy, we think that appropriate verification should again request the right inference and possibly correct justification, so that no money would be transacted.

As far as observability is concerned, we understand its fundamental intuition as given by the *self-evidence* of contract contingencies. Formally, event $A$ is self-evident for IP if $A \subseteq KA$, namely whenever it occurs IP believes in it.

When referred to contracts however, the sense in which we interpret observability does not seem to require self-evidence over contingencies outside the contract. In the above example, if the weather is humid, but neither rainy nor snowy, this may not be self-evident to parties $a$ and $b$. Nonetheless, it would not prevent parties' consensus on contract contingencies when rain or snow were to obtain.

Alternatively, we see the decision concerning dismissal of a possible case mainly in the hands of the court. Consider again the above example; party $b$ (say), in case of no rain but with very high humidity, may go to the court and make the case that "since it was very humid it rained" and so $a$ should pay. The fundamental point here is not so much which contract contingency has obtained but, prior to that, whether the contract should at all be implemented. We see this as a necessary component of verifiability but not of observability. In view of the above considerations, the main results of the paper will not be surprising.

### 2.1. *Two relevant spaces*

There are $(N+1)$ agents: $N$ parties to a contract and the court. To save on notation, $(N+1)$ will indicate both the set and the number of agents. The $(N+1)$th agent is the court. In the model, the relevant timing would be made of three periods: (i) before the contract is written (ii) after the contract is written and before the state realizes (iii) after the state realizes. For our purposes however (i) and (ii) could be merged together and the model reduced to two periods: before and after the state realizes.

**Definition 1 (True (Objective) State Space).** The true state space $T$, with generic element $t \in T$, is the set of states that can truly (objectively) occur.

That is, the set $T$ represents all *objectively* possible states of world and it is independent of timing. As we shall see, the idea of correct information processing is built on $T$.

The following definition instead introduces the relevant space of the parties, before the state realizes.

**Definition 2 (Contract Space).** The contract space $C$, with generic element $c \in C$, is the set of states identified by the parties in the contract.

Some comments are in order. The interpretation of $C$ is analogous to that of $T$ but with the following important difference: unlike states in $T$ those in $C$ reflect the parties' subjective views of the world *before* the state realizes and might not *effectively* (*truly*) occur. To illustrate the difference that we have in mind consider, as an example, the following agreement between individuals $a$ and $b$, stipulated (say) on 31 July 2017: "If on 1 August 2017 the noon temperature in Rome is below $-50°$C, then $a$ gives 100€ to $b$ otherwise $b$ gives 100€ to $a$". Even though it is perfectly conceivable that in Rome it could be so cold in the middle of the summer, it did not happen and the assumed could not occur (given the actual state in our solar system). In the agreement, individual $b$ is the one who is more likely to think the event as truly possible and be worried about it. In this sense, her subjective view would be wrong.

Moreover, in a contract, parties do not associate clauses to single states of the world defined in all possible details. They rather base their agreement on some aspects of a state, namely on sets of states with common relevant characteristics. For example, two parties may stipulate an agreement founded on whether or not *tomorrow it will rain* in a certain location. The events *tomorrow rains* or *tomorrow does not rain* in the specified location are defined by the set of states that include those two contingencies in their definition.

### 2.2. *Agents' information processing ability*

Each agent $i = 1, \ldots, N+1$ is endowed with a possibility correspondence, $P^i : T \to 2^C \backslash \varnothing$: that is, also the state space conceived by each agent after the state realizes it is a subset of $C$. Moreover, for all events $A \subseteq C$, from $P^i$ we derive the knowledge operator as

$$K^i A = \{t \in T \mid P^i(t) \subseteq A\}.$$

Finally, we assume the following.

**Assumption.** For all $i = 1, \ldots, N+1, C = \bigcup_{t \in T} P^i(t)$ is the state space conceived after the state realizes by both the parties and the court.

Therefore, $C = \bigcup_{t \in T} P^i(t)$ amounts to assuming that parties maintain the same view of the world before and after the state occurs. We can now introduce the general state space, containing the subjective and objective state spaces.

**Definition 3 (General State Space).** $\Omega = T \cup C$ is the general state space, with generic element $\omega \in \Omega$.

Notice that $K^iA \subseteq T$. If $\sim K^iA$ is the set of true states in which $A$ is not known then, unlike the standard definition of set complementation, in this case $\sim K^iA = T - K^iA \subseteq T$ and not $\sim K^iA = \Omega - K^iA$, as it would be for nonepistemic events, since $T$ and $\Omega$ may not coincide. Henceforth, in the paper, this will hold for all events including knowledge operators.

### 2.3. *Contracts*

**Definition 4 (Contract).** A contract is a partition $\Pi$ of $C$, with $\Pi(c)$ being the partition element containing state $c \in C$. Then $\Pi(c)$ is a contract contingency and, moreover, $\Pi(C)$ the set of all contracts (partitions) defined on $C$.

In other words, since a contract could be seen as a collection of contingencies, each with an associated clause, for the purpose of the paper it could simply be thought of as a family of disjointed and exhaustive subsets of $C$, each formalizing a contract contingency (event). Defining a contract as a partition bears analogies with the recent Ahan and Ergin [2010] approach to modeling partition contingent preferences under uncertainty, where each partition represents a description of the state space.

The following example, capturing an agency setting, illustrates Definition 4.

**Example 1.** Consider a Principal–Agent model (P–A), where A works for P in a production activity, and his effort $e$ could be either low $(l)$ or high $(h)$, with $l < h$. As a joint result of A's effort and of a random component, the outcome of the production activity $x$ could take two values, $x \in \{0, 1\}$. Therefore, assuming that the wage $w(e, x)$ paid by P to A will only be conditioned on $e$ and $x$, and that whether or not the wage is paid by P can always be verified by the court, the relevant contract space is defined by four states $C = \{c_1 = (l, 0); c_2 = (l, 1); c_3 = (h, 0); c_4 = (h, 1)\}$. Hence, the reward scheme will characterize the contract as a partition of $C$. For instance, suppose $w(l, 0) = 0 = w(h, 0)$ and $w(l, 1) = 0,5 = w(h, 1)$, that is a payment positively related to the outcome and independent of the effort level. Then, the associated contract $\Pi$ can be represented by the following partition of the space $C : \Pi(c_1) = \{c_1, c_3\} = \Pi(c_3)$ and $\Pi(c_2) = \{c_2, c_4\} = \Pi(c_4)$. Instead, if the payment scheme is for instance $w(l, 0) = 0 = w(l, 1)$ and $w(h, 0) = 0,5 = w(h, 1)$, that is based only on the effort, then the contract partition would be $\Pi(c_1) = \{c_1, c_2\} = \Pi(c_2)$ and $\Pi(c_3) = \{c_3, c_4\} = \Pi(c_4)$.

## 3. Verifiability and Observability of Contracts

We are now ready to introduce the two main notions of the paper.

### 3.1. *Verifiability and observability of events and contracts*

In the following definitions, the knowledge operator $K$ refers to an IP with subjective state space $C$; the true state space will still be $T$ so that $\Omega = T \cup C$. We start introducing the notion of event verifiability, which we shall extend to a contract. The same will be done for observability. As for verifiability, the definition aims to formalize the intuition that the court must draw a correct conclusion and, furthermore, provide a consistent explanation of her verdict.

**Definition 5 (Event Verifiability).** Event $A \subseteq \Omega$ is locally verifiable at state $t \in T$ if there could be an IP such that $t \in VA = (A \cap KA \cap KKA) \cup (\neg A \cap \sim KA \cap K \sim KA) \subseteq T$, where $V : 2^{\Omega} \to 2^{T}$ is the verifiability operator. Event $A$ is verifiable if $VA = T$. Finally, the nonverifiability operator is defined as $\sim VA = T - VA$, with $\sim VA \subseteq T$.

Few comments are in order. First note that the definition captures a weak, minimal, notion of verifiability. More explicitly, it establishes that an event/contract is verifiable if there could be a court with the appropriate information processing skills. Below, in Theorem 1, we shall see that with such a weak notion, contract verifiability depends only on the relation between $C$ and $T$. However, the definition does not say if such a court will be effectively "available" in a specific situation. A stronger connotation, which takes into account court availability, seems to be the one most commonly used in the economic literature. Consider again a P–A setting with moral hazard, where A's action is hidden to P. Then a contract conditioning A's wage to his effort level could be verifiable according to our notion while typically, in the literature, nonverifiable. This is because our definition captures the idea that, in principle, there could be a monitoring device (however expensive) allowing the court to verify the effort. Instead, in the literature, the notion of hidden action typically means that such device, for whatever reason, is unavailable, even if $C$ and $T$ satisfy Theorem 1 below. Later, analogous considerations would also hold for observability.

We can extend the notion of event verifiability to contract verifiability.

**Definition 5a (Contract Verifiability).** Contract $\Pi$ is verifiable if there could be an IP such that all contingencies $\Pi(c)$ are verifiable.

The above definitions formalize verifiability according to the idea previously anticipated. In particular, IP verifies event $A$ when she always correctly believes whether or not $A$ occurred, and *properly* motivates her conviction (belief). In Definition 5, correct beliefs are formalized by the operators $KA$ and $\sim KA$. Indeed, when $A$ does not obtain, we do not impose in the definition that IP knows that $\neg A$ occurred, but only the milder requirement that IP should not believe that $A$ occurred. However, as we shall see later, contract verifiability implies $K\neg\Pi(c) = \sim K\Pi(c)$ for all contract contingencies $\Pi(c)$.

Therefore, when a contract is verifiable, condition $K\neg\Pi(c) = \sim K\Pi(c)$ is derived rather than assumed. Accordingly, with $K \sim KA$ we want to capture the idea of *proper* motivation: that is, when $A$ does not occur IP knows why he does not believe in $A$. Again, as we shall see, contract verifiability will imply $KK\neg\Pi(c) = K \sim K\Pi(c)$ for all contract contingencies. To summarize, our definition entails that with contract verifiability IP will "*know whether contract contingency* $\Pi(c)$" occurred, but the same may not hold for all possible events in $\Omega$.

Note that from Definition 5 it immediately follows that for all verifiable events $A$ the axiom of truth $KA \subseteq A$, of positive introspection $KA \subseteq KKA$ and of negative introspection $\sim KA \subseteq K{\sim}KA$ all hold.

To conclude, the relatively mild request on epistemic abilities embodied in Definition 5a will enable the court to correctly decide whether or not a case based on a contract should be dismissed.

The next definition still refers to an IP with subjective space $C$. It formalizes event and contract observability according to the notion of self-evidence when a contract contingency occurs. However, when it does not occur the definition simply requires that IP should not believe it did occur.

**Definition 6 (Event and Contract Observability).** Event $A \subseteq \Omega$ is observable if there could be an IP such that $A \subseteq KA$ and $\neg A \subseteq \sim KA$. Contract $\Pi$ is observable if there could be an IP such that all contingencies $\Pi(c)$ are observable.

The following continuation of Example 1 illustrates the difference between observability and verifiability.

**Example 1a.** Consider again the P–A model of Example 1, where contract $\Pi$ signed by the parties is defined by $\Pi(c_1) = \{c_1, c_3\} = \Pi(c_3)$ and $\Pi(c_2) = \{c_2, c_4\} = \Pi(c_4)$. Further, suppose that A can also undertake a medium $(m)$ effort, and that $T = \{t_1, \ldots, t_6\}$, where $c_i = t_i$ with $i = 1, \ldots, 4, t_5 = (m, 0), t_6 = (m, 1)$. If $P^i(t) = \{t\}$ for $t \in \{t_1, t_2, t_3, t_4\}$ and $P^i(t) = \{t_1, t_2, t_3, t_4\}$ for $t \in \{t_5, t_6\}$, with $i = 1, \ldots, (N{+}1)$, then the contract could be observed by the parties but non verified by the court. Indeed, $V(c_1) = V\Pi(c_2) = V\Pi(c_3) = V\Pi(c_4) = \{t_1, t_2, t_3, t_4\} \subset T$. However, at states $t_5$ and $t_6$, when the agent effort is *medium*, the court has a limited understanding of the objectively possible states failing to explain why, even though she considers both of them as possible, she neither feels sure of low $(l)$ nor of high $(h)$ effort.

As previously anticipated, we model observability of $A$ in terms of self-evidence of the relevant event and ask for IP not to believe in $A$ when it does not occur.

Definition 6 can be considered rather demanding in the sense of excluding that event nonobservability might occur because IP misinterprets the state space. Indeed, it only allows nonobservability due to incorrect detection of a true state. This is an important implication and, not surprisingly, we shall see that the two previous definitions capture the intuitive idea that when observability and verifiability of a contract hold, both the parties and the court only imagine truly possible contingencies. They must be *proper* IP with, again, the important distinction that observability does not require "motivating" beliefs at all true states.

Although we consider Definition 6 to be the appropriate formalization of our intuition, it is worth mentioning that a weaker notion of observability, one which at true states would entail correct information processing of the kind appearing in Definition 6, but which would also allow for an IP not necessarily imagining true states, i.e., $C - T \neq \varnothing$, could be conceived as follows.

**Definition 7 (Weak Event Observability).** Event $A \subseteq \Omega$ is weakly observable if there could be an IP such that $A \cap T \subseteq KA$ and $\neg A \cap T \subseteq {\sim}KA$.

The following example may help clarifying the difference between the two definitions.

**Example 2.** Let $T = \{\alpha, \beta\}$, $P(\alpha) = \{\alpha\}$, $P(\beta) = \{\gamma\}$ so that $C = \{\alpha, \gamma\}$ and $\Omega = \{\alpha, \beta, \gamma\}$. Take $A = \{\alpha\}$ and so ${\sim}KA = \{\beta\}$; hence, $A \subseteq KA$ but $\neg A = \{\beta, \gamma\} \nsubseteq {\sim}KA$ implying $A$, according to Definition 6, to be unobserved. However, $A \cap T = A \subseteq KA$ and $\neg A \cap T = \{\beta\} = {\sim}KA$ so that $A$ is weakly observed. When $A$ is not the case IP thinks $\{\gamma\}$ (an impossible contingency) to be possible. Although at $\{\beta\}$ she has a fully incorrect view of the world, IP does not however take the wrong decision of believing in $A$. In this weaker sense, she still has enough understanding of $A$.

### 3.2. *Relation between verifiability and awareness*

The understanding of an event, according to our notion of verifiability, evokes in a natural way some idea of *awareness* of that event. For this reason, we now discuss in more detail the relation between verifiability and awareness, as defined by Modica and Rustichini [1994]. However, before doing so it is worth stating the following interesting property of the verifiability operator $V$.

**Proposition 1.** (i) *If* $KT = \varnothing$, *then for all* $A \subseteq \Omega$ *is* $VA = \varnothing$. (ii) *If for all* $A \subseteq \Omega$ *is* $VA = \varnothing$, *then* $KKT = \varnothing$.

The above proposition identifies minimal (intuitive) conditions for verifiability. In particular, necessity says that at least one state $t$ must exist where all contingencies believed by IP could truly obtain.

As mentioned above, the idea of verifiability is intuitively linked to the notion of awareness, a topic which attracted much attention in recent years [Dekel *et al.*, 1998; Heifetz *et al.*, 2008; Li, 2009]. Though casted in different frameworks, a main common view shared by these contributions is that meaningful notions of unawareness call for some "nonstandard" state space modeling. By allowing for the possibility that subjective and objective state spaces may differ, this work is indeed along this same line. For the purpose of this paper, we shall confine ourselves to a simple comparison between our nonverifiability operator ${\sim}VA$ and [Modica and Rustichini, 1994] formalization of unawareness. Modica–Rustichini define the unawareness operator $UA \subseteq T$ as

$$\begin{aligned} UA &= \sim KA \cap \sim K \sim KA \\ &= (A \cap \sim KA \cap \sim K \sim KA) \cup (\neg A \cap \sim KA \cap \sim K \sim KA) \end{aligned}$$

namely, ignorance of one's ignorance, to capture the epistemic state of an individual who does not even have $A$ in her mind. This is explicit in the right term of the definition, where it is clear that unawareness is independent of whether or not $A$ is obtaining; i.e., a state of unawareness is unrelated to empirical evidence.

Hence, in our more general framework, if at state $t$ the individual is unaware of $A$, namely $t \in UA$ then either $t \in (A \cap \sim KA \cap \sim K \sim KA)$ or $t \in (\neg A \cap \sim KA \cap \sim K \sim KA)$, and it is easy to see that in both cases $t \in \sim VA$ and so $UA \subseteq \sim VA$. Therefore, the notion of nonverifiability is less strict than unawareness; alternatively, if IP verifies event $A$ then she is aware of it but not necessarily *vice versa*.

## 4. The Main Characterization Results

In this section, we formulate the main results of the paper, the proofs of which are in Appendix A. The following theorem provides a full characterization of verifiable contracts.[b]

**Theorem 1.** *Contract* $\Pi \in \Pi(C)$ *is verifiable if and only if* $C = T$.

Therefore, in this case the only possibility to verify a contract is to specify only those contingencies that can truly obtain. Any kind of misspecification might either prevent the court from drawing correct inferences or their appropriate motivations. Hence, verifiability of a contract requires a judge to have a good understanding of the states that can possibly occur. Indeed contracts, such as the ones in the following example, could not be verifiable.

**Example 2a.** Suppose $T = \{\alpha, \beta\}$ and $C = \{\alpha, \beta, \gamma\}$; moreover, let $P(\alpha) = \{\alpha, \gamma\}$, $P(\beta) = \{\beta\}$, for the parties and the court, and consider contract $\Pi = \{\{\alpha, \gamma\}, \{\beta\}\}$. It is easy to see that $\Pi$ is not verified by the above court and, more in general, is not verifiable.

The example is interesting since the same setting, without state $\gamma$, would make the contract verifiable. Namely, the mere introduction of what we may call a "phantom" state prevents verifiability. This is so because, for event $A = \{\alpha, \gamma\}$, it is $KA = \{\alpha\}$ but $KKA = \varnothing$. That is, though the court at state $\alpha$ draws the correct inference, namely that $A$ is the case, there is no proper understanding of the true reasons why it occurred since state $\gamma$, which is impossible, is considered as possible.

An analogous characterization can also be presented for observability.

**Theorem 2.** *Contract* $\Pi \in \Pi(C)$ *is observable if and only if* $C \subseteq T$.

[b]One of the referees suggested to rephrase Theorems 1 and 2 in a more extended way, which is done in the Appendix.

Theorems 1 and 2 establish that, given $T$ some contracts may be observable/verifiable while others may not. Indeed, it depends on the relation between $C$ and $T$: contracts for which $C - T \neq \varnothing$ can neither be observable nor verifiable, as they would contemplate states that could not occur.[c]

Taking Theorems 1 and 2 together, we immediately obtain the following further characterization, linking verifiability and observability, the simple proof of which is omitted.

**Corollary 1.** *If contract $\Pi \in \Pi(C)$ is verifiable then it is observable.*

In the economic literature observable, but nonverifiable, contracts are among the explanations for incomplete contracts [Hart and Moore, 1999; Tirole, 1999], that is contracts where parties do not include all welfare relevant contingencies. In the model, this means that parties could anticipate that no court would exist to verify some, or all, contract contingencies. This would require agents' epistemics to be part (at least informally) of the definition of a state of the world. With nonpartitional structures this may not necessarily ask for parties to know the entire possibility correspondence but, more simply, that courts cannot verify contract contingencies.

It may be interesting to further characterize a contract based on $C$, which would be observable by the court but not verifiable. However, before doing so consider the following example.

**Example 3.** Suppose $T = \{\alpha, \beta, \gamma\}$, $C = \{\alpha, \beta\}$; $P(t) = \{t\}$ for $t \in \{\alpha, \beta\}$ and $P(\gamma) = C$. Finally, let $\Pi = \{\{\alpha\}, \{\beta\}\}$. It is easy to see that $\Pi$ is observable but not verifiable by a court. At $t = \gamma$ the above court may be said to have a limited understanding of the objectively possible contingencies failing to explain why, even though it considers both of them as possible, it neither feels sure of $\alpha$ nor of $\beta$.

Example 3 suggests how we could proceed to provide *what's missing* for verifiability. In that case, when the true state is $t \notin \Pi(c)$, the court not only is unable to take the correct decision but, more in general, *a decision*. She has only a partial (though correct) view of the true world and when an unforeseen state obtains she does not recognize it. It is this ability to *decide*, that would be meaningful to introduce, and through which we now formalize the gap between observability and verifiability.

**Definition 8 (Decidable Contract).** Contract $\Pi \in \Pi(C)$ is decidable if there exists a court such that $\bigcup_{c \in C} K\Pi(c) = T$.

That is, a contract is decidable if there is a court that always thinks one of the contract contingencies to have obtained. Having introduced decidability we can now state the following result.

[c] It is worth noticing again that what really qualifies contract observability is $\Pi(c) \subseteq K\Pi(c)$, for all contract contingencies. Indeed, the request $\neg\Pi(c) \subseteq\sim K\Pi(c)$ is introduced to prevent parties from drawing the wrong conclusion that a contract contingency took place, when it did not.

**Theorem 3.** *Contract* $\Pi \in \Pi(c)$ *is verifiable if and only if it is observable and decidable.*

The above result formalizes the idea that contract verifiability requires higher information processing skills than observability. This same point could also be made from an alternative perspective. In the following theorem, the easy proof of which is omitted, we formulate the result.

**Theorem 4.** *Contract* $\Pi \in \Pi(C)$ *is verifiable if and only if* $K\Pi(c) = \Pi(c)$ *and* $K\neg\Pi(c) = \neg\Pi(c)$ *for all contract contingencies* $\Pi(c)$*. Contract* $\Pi \in \Pi(C)$ *is observable if and only if* $K\Pi(c) = \Pi(c)$ *and* $\sim K\Pi(c) = \neg\Pi(c)$ *for all contract contingencies* $\Pi(c)$.

Although neither observability nor verifiability require the axiom of truth to be valid, for all possible events $A \subseteq \Omega$, they both require it to hold for all contract contingencies $\Pi(c)$

However, while verifiability implies the axiom validity also for their complementary events $\neg\Pi(c)$, this is not so for observability, which could be seen as an alternative way to characterize the difference between the two notions.

## 5. Conclusions

The paper provides a formal definition and characterization of the notions of verifiability and observability, two fundamental and widespread concepts in contract theory. Typically, in the literature, both of them appear to refer to correct detection of the events (contingencies) specified in the contract. However, in the work, we conceived this correct detection to be different in the two cases. Indeed, a sense in which observability seems to be intended suggests self-evidence of contract contingencies. However, when no contract contingency occurs no request, except for not drawing the wrong conclusion that a contract event has occurred, is apparently requested to parties' information processing skills. Alternatively, when a true state does not belong to the contract space, its correct detection would not necessarily be asked to parties. Consequently, parties may not believe that any contract event obtained without necessarily understanding that this is due to none of them having occurred.

This, evidently, cannot be so when verifiability is considered. In case of a controversy among parties, the court should not only formulate a correct verdict but also *properly* motivate it. Therefore, if a state not included in the contract is the true one, the court would have to properly explain to parties that the case has to be dismissed because no contract contingency applies. The main finding of the paper says that for a contract to be verified the court's state space must coincide with the true state space. This agrees with our broad intuition, that no correct detection of contract events and no proper motivation could be formulated unless the court imagines states that are always truly possible.

## Appendix

**Proof of Proposition 1.** If $KT = \varnothing$, then $P(t) \nsubseteq T$, for all $t \in T$. This means that $P(t)$ can neither be included in $KA$ nor in $\sim KA$ so that $KKA = \varnothing = K{\sim}KA$ which implies $VA = \varnothing$ for all $A \subseteq \Omega$.

If instead, for all $A \subseteq \Omega$ is $VA = \varnothing$, then $VT = \varnothing$ and so $T \cap KT \cap KKT = \varnothing$. Hence, if $KT = \varnothing$, then $KKT = \varnothing$ while if $KT \neq \varnothing$, then $KKT = \varnothing$, since if $KKT \neq \varnothing$, then $KT \cap KKT \neq \varnothing$.

**Proof of Theorem 1.** Since in what follows we only refer to the court, to save on notation superscript $(N+1)$ will be omitted from the relevant epistemic operators.

Assume $\Pi \in \Pi(C)$ to be verifiable, namely that for all $\Pi(c)$ in the partition contract $\Pi$, there is a court such that $V\Pi(c) = T$. We first show that $C \subseteq T$ and then that $T - C = \varnothing$. Indeed, since

$$\begin{aligned} V\Pi(c) &= (\Pi(c) \cap K\Pi(c) \cap KK\Pi(c)) \cup (\neg\Pi(c) \cap {\sim}K\Pi(c) \cap K{\sim}K\Pi(c)) \\ &= K\Pi(c) \cup {\sim}K\Pi(c) = T \end{aligned}$$

and noticing that $V\Pi(c)$ is the union of two disjoint subsets of $T$, for all $t \in T$ it must be either that

$$t \in \Pi(c) \cap K\Pi(c) \cap KK\Pi(c) = K\Pi(c) \qquad (*)$$

or

$$t \in \neg\Pi(c) \cap {\sim}K\Pi(c) \cap K \sim K\Pi(c) = {\sim}K\Pi(c) \qquad (**)$$

However, if (*) is true, then $P(t) \subseteq K\Pi(c) \subseteq T$ while if (**) is true, it is $P(t) \subseteq {\sim}K\Pi(c) \subseteq T$ and since by assumption $C = \bigcup_{t \in T} P(t)$ it follows that $C \subseteq T = \Omega$.

Suppose now that $T - C \neq \varnothing$. From (*) it follows that $K\Pi(c) \subseteq \Pi(c) \subseteq T$ while from (**) that ${\sim}K\Pi(c) \subseteq \neg\Pi(c) \subseteq T$, which taken together imply $K\Pi(c) = \Pi(c)$ and ${\sim}K\Pi(c) = \neg\Pi(c)$. Then, for all $t \in T - C$ and all $c \in C$ it must be $P(t) \subseteq C$ and $P(t) \nsubseteq \Pi(c)$, because otherwise verifiability would be violated. If $\Pi(c) = C$, a contradiction follows immediately. Alternatively, there must exist states $c$ and $c'$ such that $\Pi(c) \neq \Pi(c')$ with $\Pi(c) \cap P(t) \neq \varnothing$ and $\Pi(c') \cap P(t) \neq \varnothing$. But this would imply $P(t) \nsubseteq {\sim}K\Pi(c)$, $P(t) \nsubseteq {\sim}K\Pi(c')$, and so that $t \notin K \sim K\Pi(c)$ and $t \notin K \sim K\Pi(c')$ which means that at such $t \in T - C$ neither $\Pi(c)$ nor $\Pi(c')$ are verifiable. Hence, the contract is nonverifiable, contradicting the initial assumption.

Assume now $C = T = \Omega$, consider contract $\Pi$ and take $P(t) = \Pi(c)$, for all $t \in \Pi(c)$ and $c \in C$. Hence, for all $\Pi(c)$ we have $K\Pi(c) = \Pi(c)$, $KK\Pi(c) = K\Pi(c), {\sim}K\Pi(c) = \neg\Pi(c)$ and $K{\sim}K\Pi(c) = \neg\Pi(c)$ so that $V\Pi(c) = T$.

**Rephrasing of Theorem 1.** Suppose $(T, P)$ characterises the fundamentals of the model, from the perspective of an IP, so that he can be identified with $(T, P)$.

Consider any $C$ and any $\Pi \in \Pi(C)$. Then

(1) For any $T$ such that $C = T$ there exists some $P$ such that $\Pi$ is verifiable by $(T, P)$.
(2) If $\Pi$ is verifiable by $(T, P)$ then $C = T$.

In other words: for any $T$, any $C$ and any $\Pi \in \Pi(C)$ there exists some $P$ such that $\Pi$ is verifiable by $(T, P)$ if and only if $C = T$.

**Proof of Theorem 2.** If $\Pi$ is observable then for all $c \in C$ and $i = 1, \ldots, N$ it is $\Pi(c) \subseteq K^i\Pi(c) \subseteq T$ and $\neg\Pi(c) \subseteq \sim K^i\Pi(c) \subseteq T$, entailing $C \subseteq T$.

Assume instead $C \subseteq T$; then parties whose possibility correspondences, for all $t \in \Pi(c)$ and $c \in C$, satisfy $P^i(t) = \Pi(c)$ and, for all $t \in T - C$, satisfy $P^i(t) = C$ would make the conclusion hold true.

**Rephrasing of Theorem 2.** As in the rephrasing of Theorem 1, consider any $C$ and any $\Pi \in \Pi(C)$. Then

(1) For any $T$ such that $C \subseteq T$ there exists some $P$ such that $\Pi$ is observable by $(T, P)$.
(2) If $\Pi$ is observable by $(T, P)$ then $C \subseteq T$.

In other words: for any $T$, any $C$ and any $\Pi \in \Pi(C)$ there exists some $P$ such that $\Pi$ is observable by $(T, P)$ if and only if $C \subseteq T$.

**Proof of Theorem 3.** Suppose $\Pi \in \Pi(c)$ is verifiable; then, by Theorem 1, $C = T$ and for all $c \in C$ a court such that $K\Pi(c) = \Pi(c)$ would make $\Pi$ observable and decidable. Instead, if $\Pi \in \Pi(C)$ is observable then $\Pi(c) \subseteq K\Pi(c)$ and $\neg\Pi(c) \subseteq \sim K\Pi(c)$; hence, $K\Pi(c) = \Pi(c)$ and $\sim K\Pi(c) = \neg\Pi(c)$. Moreover, decidability implies $\bigcup_{c\in C} K\Pi(c) = \bigcup_{c\in C} \Pi(c) = T$ and verifiability follows.

*Proof that* $K\Omega = \Omega$ *and* $K(A \cap B) = K(A) \cap K(B)$.

The two properties derive directly from the definition of the *knowledge operator*, and hold for any information structure, including but not exclusively, partitions.

Indeed, as for the former, since $K : 2^\Omega \to 2^\Omega$ it follows that $P(\omega) \subseteq \Omega$ for all $\omega \in \Omega$; therefore, $K\Omega = \{\omega \in \Omega \mid P(\omega) \subseteq \Omega\} = \Omega$.

As for the latter, suppose

(i) $\omega \in K(A \cap B)$. Then $P(\omega) \subseteq A \cap B$; hence, $P(\omega) \subseteq A$ and $P(\omega) \subseteq B$ and so $\omega \in K(A) \cap K(B)$.
(ii) $\omega \in K(A) \cap K(B)$. Then $P(\omega) \subseteq A$ and $P(\omega) \subseteq B$; hence, $P(\omega) \subseteq A \cap B$ and so $\omega \in K(A \cap B)$.

So the two properties do not hold for partitions only but, more in general, for any possibility correspondence.

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## References

Ahan, D. and Ergin, H. [2010] Framing contingencies, *Econometrica* **78**, 655–695.

Aumann, R. [1976] Agreeing to disagree, *Ann. Stat.* **4**, 1236–1239.

Aumann, R. and Brandenburger, A. [1995] Epistemic conditions for nash equilibrium, *Econometrica* **63**, 1161–1180.

Bernheim, D. and Whinston, M. [1998] Incomplete contracts and strategic ambiguity, *Am. Econ. Rev.* **88**, 902–932.

Bolton, P. and Dewatripont, M. [2005] *Contract Theory* (MIT Press, Massachusetts).

Dekel, E., Lipman, B. and Rustichini, A. [1998] Standard state space preclude unawareness, *Econometrica* **66**, 159–173.

Djankov, S., La Porta, R., Lopez-De-Silanes, F. and Schleifer, A. [2003] Courts, *Q. J. Econ.* **118**, 453–517.

Geanakoplos, J. [1989] Game theory without partitions and applications to speculation and consensus, Cowles Foundation Discussion Paper No. 914, Yale University, USA.

Hart, O. [1995] *Firms, Contracts and Financial Structure*, Clarendon Lectures in Economics (Oxford University Press, Oxford-UK).

Hart, O. and Moore, J. [1999] Foundations of incomplete contracts, *Rev. Econ. Stud.* **66**, 15–138.

Heifetz, A., Meier, M. and Schipper, B. [2008] A canonical model for interactive unawareness, *Games Econ. Behav.* **62**, 304–324.

Hermalin, B. and Katz, M. [1991] Moral hazard and verifiability: The effects of renegotiation in agency, *Econometrica* **59**, 1735–1753.

Holmström, B. [1982] Moral hazard in teams, *Bell J. Econ.* **13**, 324–340.

Li, J. [2009] Information structures with unawareness, *J. Econ. Theory* **144**, 977–993.

Modica, S. and Rustichini, A. [1994] Awareness and partitional information structures, *Theory Decis* **37**, 107–124.

Modica, S. and Rustichini, A. [1999] Unawareness and partitional information structures, *Games Econ. Behav.* **27**, 265–298.

Tirole, J. [1999] Incomplete contracts: Where do we stand? *Econometrica* **67**, 741–781.