▸ FRANCO PARLAMENTO, FLAVIO PREVIALE, *The Cut Elimination and Non-lengthening Property for Gentzen's Sequent Calculus for First Order Logic with Equality*.
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Leibniz's indiscernibility principle, in the framework of second order logic, leads to a sequent calculus for first order logic with equality, that satisfies the cut elimination theorem, but cut free derivations may not satisfy the subformula property. We note that instead, the path described by von Plato in his historical reconstruction of Gentzen's discovery of the sequent calculus in [2], leads to a calculus that is fully satisfactory. In addition to the reflexivity axiom $\Rightarrow t = t$, it has the following two left introduction rules for $=$:

$$\frac{\Gamma \Rightarrow \Delta, F\{v/r\}}{\Gamma, r = s \Rightarrow \Delta, F\{v/s\}} =_1 \qquad \frac{\Gamma \Rightarrow \Delta, F\{v/s\}}{\Gamma, r = s \Rightarrow \Delta, F\{v/r\}} =_2$$

Other satisfactory calculi can be obtained by taking into account the following other rules:

$$\frac{\Gamma, F\{v/r\} \Rightarrow \Delta}{\Gamma, F\{v/s\}, r = s \Rightarrow \Delta} =_1^l \qquad \frac{\Gamma, F\{v/s\} \Rightarrow \Delta}{\Gamma, F\{v/r\}, r = s \Rightarrow \Delta} =_2^l$$

We give a very simple proof that cut elimination holds for a calculus obtained by adding to the reflexivity axiom some of the above four rules if and only if it contains (at least) $=_1$ and $=_2$ or $=_1$ and $=_1^l$ or $=_2$ and $=_2^l$. The admissibility results that are used, can be refined and extended in order to show that if (and only if) all the above four rules are added, then every derivation can be trasformed into one that is cut-free and satisfies the nonlenthening property of [1].

[1] A.V. Lifschitz, *Specialization of the form of deduction in the predicate calculus with equality and function symbols*, ***The Calculi of Symbolic Logic I*** (V.P. Orevkov, editor), Proceedings of the Steklov Institute of Mathematics 98, 1971, pp. 1–23.

[2] J. von Plato,, *Gentzen's Proof Systems: Byproducts in a Work of Genius*, ***The Bulletin of Symbolic Logic*** , vol. 18 (2012), no. 3, pp. 317–367