# **Remapping algorithms: application to trimming operations in sheet metal forming**

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**Abstract**. Most of sheet metal forming processes comprise intermediate trimming operations to remove superfluous material. These operations are required for subsequent forming operations. On the other hand, the springback is strongly influenced by the trimming operations that change the part stiffness and the stress field. From the numerical point of view, this involves the geometrical trimming of the finite element mesh and subsequent remapping of the state variables. This study presents a remapping method based on Dual Kriging interpolation, specifically developed for hexahedral finite elements, which has been implemented in DD3TRIM in-house code. Its performance is compared with the one of the Incremental Volumetric Remapping method, using the split-ring test to highlight their advantages and limitations. The numerical simulation of the forming processes is performed with DD3IMP finite element solver.

#### 1. Introduction

Typically, the finite element simulation of multi-stage sheet metal forming processes requires the modification of the blank geometry, which is the consequence of trimming operations. Therefore, some remapping procedure is required to transfer the state variables from the old mesh to the new one, including the ones evaluated at the integration points, within each finite element [1]. The number of state variables depends on the constitutive model adopted, but usually includes the current flow stress, the stress field, the plastic strain, etc.

The remapping methods can be divided into four groups [2]. The first group refers to pointwise interpolation and extrapolation methods, where the variables are transferred from the old mesh to the new one using a function that interpolates/extrapolates the variables. The second group refers to the area/volume weighted averaging methods, which use the area/volume intersection between the old and the new mesh to define weighting factors, expressing the contribution of each finite element from the old mesh to the new one. The third group refers to mortar element methods, which are general techniques for projecting data at interfaces between non-conforming subdomains [3]. From a mathematical point of view, this method comprises the minimization of a weighted residual, where the weight functions are usually chosen from the space spanned by the basis functions of the mortar side. The last group refers to specialized methods, which are designed for specific applications [4].

This study presents and compares two distinct remapping methods for finite element meshes composed by linear hexahedral elements, namely the Incremental Volumetric Remapping (IVR) [5] and

the Dual Kriging (DK) [6]. Both methods were implemented in the in-house code DD3TRIM, which was specifically developed to perform trimming and remapping operations involved in the multi-stage sheet metal forming processes. In order to assess the performance of each remapping method, the springback of an aluminum alloy is quantified, using the split-ring test example.

## 2. Remapping algorithms

The accuracy and the computational performance are key points of any remapping algorithm. In fact, the remapping operation introduces errors in the new mesh, due to the approximation of the state variables. In order to try to control and reduce the error arising from the remapping operations, several authors point out some desirable characteristics. The method should be self-consistent, guarantee the locality and avoid spurious local extreme values. Accordingly, the IVR and the DK methods are described in detail, where the old and the new finite element meshes are denoted by donor and target mesh, respectively [2,7].

## 2.1. Incremental volumetric remapping

The IVR method is based on the concept that the value of each state variable, at the region corresponding to a Gauss volume (an eight part of the standard brick element), is equal to the state variable of the correspondent Gauss point. Therefore, the variable value assigned to a given Gauss point of the target mesh element is evaluated based on a weighted average of the values of the donor mesh [5]. Hence, the value of the state variable  $\alpha$  in a Gauss point *i* of the target mesh is given by:

$$\alpha_i = \sum_{j=1}^{ng} w_{ij} \alpha_j, \quad \text{with} \quad w_{ij} = \frac{1}{\nu_i^{\text{total}}} \sum_{k=1}^{nl^3} \nu_{ij}^k, \tag{1}$$

where ng denotes the number of donor Gauss volumes that contributes for the target Gauss volume i.  $w_{ij}$  is the fraction of the target Gauss volume i, contained within the donor Gauss volume j, and  $\alpha_j$  is the value of the state variable in the donor Gauss volume j. The target Gauss volume is partitioned in nlequal subdivisions in each direction (see Figure 1 (c)), in order to approximate the fraction of the target Gauss volume contained in each donor Gauss volume (see Figure 1 (d)). The weight function  $w_{ij}$  is the fraction between the summation of the elementary volumes  $v_{ij}^k$  of the target Gauss volume i, contained in the donor Gauss volume j, and the total volume of the target Gauss volume i, denoted by  $v_i^{\text{total}}$ .

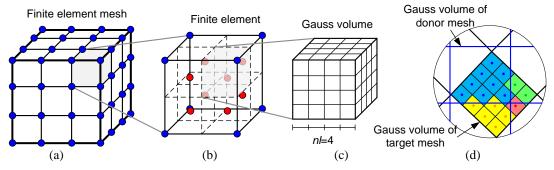


Figure 1. Schematic representation of the IVR method: (a) finite element mesh; (b) subdivision of each element into 8 Gauss volumes; (c) subdivision of the target Gauss volume; (d) intersection between target and donor Gauss volumes.

#### 2.2. Dual-Kriging method

The Dual Kriging method is applied in the present study to remap the state variables between different finite element meshes. This method provides an explicit parametric interpolation, allowing to evaluate the value of the state variables in any point (Gauss points of the target mesh). In order to reduce the computational effort, only the donor Gauss points neighboring to the target Gauss point under analysis are selected and used [6]. The general form of the DK model is decomposed into the sum of two terms:

$$\alpha(\mathbf{x}) = d(\mathbf{x}) + f(\mathbf{x}) = (d_1 + d_2 x + d_3 y + d_4 z) + \sum_{i=1}^n b_i K(h_i),$$
(2)

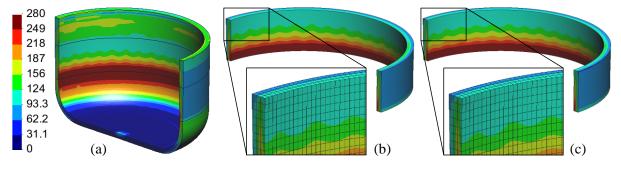
where  $d(\mathbf{x})$  denotes the global trend function (polynomial) and  $f(\mathbf{x})$  represents the fluctuation or deviation function, which captures the local deviations from the trend function. The fluctuation depends on the *n* donor Gauss points used to calculate the state variables in the target Gauss point. Indeed, it is constructed using *n* parameters  $b_i$ , which are weighted by the generalized covariance function  $K(h_i)$ associated with each donor Gauss point. The accuracy of the DK method is affected by the covariance function selected. In the present study, it is simply the Euclidean distance between the target Gauss point and the donor Gauss point. The *n* parameters  $b_i$  and the four parameters  $d_i$  (the last term  $d_{4Z}$  disappears for 2D applications) are determined by solving a system of n+4 linear equations, which has to be solved only once for each target Gauss point. Then, the interpolated value of all state variables is computed.

For each target Gauss point, the set of donor Gauss points located in its region of influence must be selected. Since the sheet metal forming simulation is the main application of the proposed remapping method, the selection method is carried out by layers through the thickness, i.e. only the donor Gauss points located in the same layer of the target Gauss point are considered. Accordingly, the DK method is applied in a 2D framework, where each donor Gauss point is projected into a plane defined by the finite element of the donor mesh containing the target Gauss point. Considering a structured discretization, nine is the maximum number of donor Gauss points selected (based on the mesh connectivity).

## 3. Trimming example

The split-ring test was used to measure the effect of the remapping method on the springback prediction. The cylindrical cup forming (fully draw) was carried using the aluminum alloy AA5754-O [8]. The opening diameter of the die is 35.25 mm and the punch diameter is 33 mm, while the shoulder radius of both the die and the punch is 5 mm. The circular aluminum blank presents 60 mm of diameter and 1 mm of thickness. The residual stress state is evaluated by measuring the opening of a ring (7 mm high) cut from the sidewall of the formed cylindrical cup (8 mm from the bottom).

The finite element simulation of the forming operation was performed with the in-house static implicit finite element code DD3IMP [9], while the ring cut and splitting was performed with the in-house code DD3TRIM [10]. The plastic behavior of the sheet is described by the isotropic work hardening (voce law) and isotropic yield criterion (von Mises). Only half model is simulated, allowing to perform the discretization of the blank with 30.237 solid finite elements (3 layers through the thickness). The von Mises stress distribution in the cylindrical cup is presented in Figure 2 (a), showing the position of the ring cutting. The von Mises stress distribution in the ring, obtained with the IVR and DK remapping methods is presented in Figure 2 (b) and (c), respectively. Both methods provide identical results for the state variables, in agreement with the variable distribution before the cutting and remapping operations (see Figure 2).



**Figure 2.** von Mises stress distribution in the: (a) formed cylindrical cup; (b) ring before splitting using the IVR method; (c) ring before splitting using the DK method.

The springback of the ring after cutting and splitting operations is strongly affected by the stress state in the ring, which is dictated by the remapping algorithm adopted. Since both remapping methods (IVR and DK) provide identical stress states in the ring (see Figure 2), the obtained ring opening after springback is similar for both methods (approximately 4.3 mm). The ring gap measurements were performed along the straight line connecting the two ends of the split ring. Regarding the computational performance of each remapping procedure, the computational cost is about 300 seconds using the IVR method, while the DK method requires less than 2 seconds. The calculations were carried out on a computer equipped with an Intel Core i7–4770K Quad-Core processor (3.5 GHz).

## 4. Conclusions

This study presents the application of the Dual Kriging (DK) interpolation method as remapping scheme, which is compared with the Incremental Volumetric Remapping (IVR) method, both in terms of accuracy and computational performance. The split-ring test is the example considered, which involves the cut of a ring from the sidewall of a cylindrical cup and consequent splitting operation, for springback evaluation. Both remapping methods provide results with a good level of accuracy, specifically the stress field, which presents a strong impact in the springback prediction. Thus, the predicted ring opening is identical in both remapping methods. On the other hand, the computational cost of the DK method is significantly lower (about 100 times) than the IVR method.

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