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Multiobjective evolutionary algorithm based on vector angle neighborhood

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ABSTRACT

Selection is a major driving force behind evolution and is a key feature of multiobjective evolutionary algorithms. Selection aims at promoting the survival and reproduction of individuals that are most fitted to a given environment. In the presence of multiple objectives, major challenges faced by this operator come from the need to address both the population convergence and diversity, which are conflicting to a certain extent. This paper proposes a new selection scheme for evolutionary multiobjective optimization. Its distinctive feature is a similarity measure for estimating the population diversity, which is based on the angle between the objective vectors. The smaller the angle, the more similar individuals. The concept of similarity is exploited during the mating by defining the neighborhood and the replacement by determining the most crowded region where the worst individual is identified. The latter is performed on the basis of a convergence measure that plays a major role in guiding the population towards the Pareto optimal front. The proposed algorithm is intended to exploit strengths of decomposition-based approaches in promoting diversity among the population while reducing the user's burden of specifying weight vectors before the search. The proposed approach is validated by computational experiments with state-of-the-art algorithms on problems with different characteristics. The obtained results indicate a highly competitive performance of the proposed approach. Significant advantages are revealed when dealing with problems posing substantial difficulties in keeping diversity, including many-objective problems. The relevance of the suggested similarity and convergence measures are shown. The validity of the approach is also demonstrated on engineering problems.

1. Introduction

Evolutionary algorithms proved effective when solving multiobjective optimization problems (MOPs) in different application domains [1]. Similarly to single-objective counterparts, multiobjective evolutionary algorithms (MOEAs) process a population of solutions in a probabilistic manner. This allows to perform the global search with little knowledge about the objectives and to approximate the Pareto set efficiently in a single simulation run. In doing so, MOEAs rely on three major mechanisms such as mating (parent) selection, variation (reproduction) and environmental selection (replacement). In turn, these concepts draw inspiration from natural evolution. Variation aims at exploring the search space by producing new candidate solutions. This process makes use of stochastic operators applied to one or more parent solutions. Overwhelmingly, existing MOEAs simply rely on variation operators initially designed for single-objective optimization. Variation can be adopted without any modifications and plugged into a framework being enable to perform selection in the presence of multiple objectives. This makes selection a key feature of MOEAs, with its effectiveness playing a crucial role in their performance.

In nature, selection is responsible for adaptation of species to their environment. It gives an extra survival and reproduction probability to the most fitted individuals. In MOEAs, selection operators attempt to mimic this process. This is conducted on the basis of some fitness measure designed to reflect how suited individuals are in the context of the environment defined by the problem being solved. Depending on the fitness assignment and selection, most existing MOEAs can be classified into three major category.

Dominance-based MOEAs rely on the concept of the Pareto dominance to direct the search. Selection is motivated by the idea that nondominated individuals are preferred over dominated ones. It typically incorporates some mechanism to promote diversity. A representative MOEA belonging to this category was proposed in [2]. It combines convergence and diversity measures into a scalar fitness value. The former is based on the number of individuals the dominator of a given solution dominates, whereas the latter employs a nearest neighbor technique. Another popular approach ranks the population into different non-domination levels, thereby highlighting nondominated individuals [3]. A diversity preserving mechanism is applied when the last accepted level cannot be completely accommodated.

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Dominance-based MOEAs are often used with variation relying on genetic algorithm concepts. Though, in [4], it was demonstrated that a dominance-based selection is also effective when extending immune clonal algorithm to solving MOPs. The major advantage of dominance-based selection is that it naturally reflects the concepts of optimality in multiobjective optimization. Although such selection often works well for two and three objectives, its performance severely deteriorates in high-dimensional objective spaces [5]. This is caused by the fact that almost all individuals in the population become nondominated. Thus, a selection pressure is significantly decreased. In such circumstances, many diversity preserving mechanisms favor solutions that are far away from the Pareto front. And dominance-based MOEAs can perform even worse than random search algorithms. This issue can be addressed by modifying the dominance relation. In [6], a method for widening the area dominated by a given solution was suggested. In [7], the concept of dominance was applied to the grid in the objective spaces. Defining the grid requires setting the number of devisions for each objective, with an inappropriate value leading to a poor performance. On the other hand, improving a diversity preserving mechanism can increase the scalability of MOEAs [8–11]. Also, the dominance relation can provide a high selection pressure, thereby producing a harmful impact on the search due to the loss of diversity [12]. To improve the population diversity, one can consider the diversity of genetic information as an objective in a nondominated sorting [13]. Overall, the search in the decision space is an important aspect of multiobjective optimization that heavily influences both the population convergence and diversity. This was particularly explored in [14] by suggesting a promising framework based on decision variable analysis that divides and process the decision variables in accordance to their role in the given MOP.

Decomposition-based approaches attempt to decompose an original MOP into a number of subproblems and solve them in parallel. Different possibilities are available for this. On one hand, decomposition can be based on the aggregation of multiple objectives by means of scalarization that involves traditional mathematical techniques [15]. This way, a scalar fitness value is assigned to each population member reflecting its quality. Convergence is provided by minimizing a corresponding scalarizing function, whereas diversity is ensured by a well-distributed weight vectors. MOEA/D [16,12] is a popular framework relying on this principle. When producing offspring, it explores the neighborhood relation defined on the closeness of weight vectors. Replacement is performed favoring better values of the scalarizaing function. Recently, MOEA/D has been extensively investigated, leading to numerous modifications of its framework. The impact of different scalarizion schemes was studied in [17,18], with the results suggesting that a proper choice of scalarizing function is important for the performance of MOEA/D. Also, it was shown that a better exploration of the search space can be achieved by performing replacement in the neighborhood of the subproblem that the best matches offspring [19]. Another important issue in MOEA/D is an efficient allocation of computational resources between different subproblems [20]. For this purpose, the concept of successful solutions, those entering an external archive, was introduced in [21]. For each subproblem, a number of successful solutions is used to calculate a selection probability, thereby allocating computational recourses to most promising subproblems. It was demonstrated that MOEAs relying on scalarization can better balance convergence and diversity [12]. A highly competitive performance for problems with a large number of objectives was also shown [22]. The major advantage of such MOEAs is efficiency, as a little computational effort is needed to compute a scalarizing function value.

On the other hand, directional vectors can be used for defining directions of search. This way, population members are evolved being associated with corresponding directions. In [23], direction vectors uniformly distributed on a hypersphere are utilized to divide the population and assign fitness among subpopulations based on convergence along these vectors. In [24], a diversity preservation mechanism was modified to extend a nondominated sorting genetic algorithm to many-objective optimization. Another MOEA exploiting the dominance and decomposition-based strategies was suggested in [25]. An approach that performs selection considering the distance to the reference direction first and the distance to the reference point second was proposed in [26]. The promising performance exhibited by such algorithms comes with the cost of increasing the human's burden in the form of providing a proper set of vectors before the search. Generating such set may be not an easy task, especially for high-dimensional spaces. Although there are some strategies allowing to automatize this process [27], an arbitrary number of weights cannot be obtained and the population size must be adjusted to the resultant number. Alternatively, polar coordinates can be used to decompose the objective space into a set of grids as suggested in [28], where population members are evolved while maintaining them associated with corresponding grids. A significant drawback of weight vector-based algorithms has been pointed out in [29], showing these algorithms are largely overspecialized for popular test suites such as DTLZ and WFG. This is due to the consistency between the shape of the Pareto front and the shape of the distribution of the weight vectors, which allows doing well on these problems but not in a general case.

The working principle of indicator-based approaches is based on optimizing quality indicators that are often utilized for the performance assessment of MOEAs. Although there have been developed various types of such indicators, the epsilon and hypervolume are the most frequently used ones within MOEAs. A general framework for incorporating quality indicators was proposed in [30]. This approach can use an arbitrary indicator to compare a pair of candidate solutions instead of entire approximation sets. A scheme similar to summing up the indicator values for each population member with respect to the rest of population is used to assign a scalar fitness value reflecting its quality with respect to the convergence and diversity. Indicator-based selection is often used to refine the Pareto dominance relation [31]. In [32], it is shown that maintaining two archives separately, one for diversity and another for convergence with indicator-based selection, can be beneficial. Indicator-based MOEAs are successful in dealing with many-objective problems [33]. The difficulty in their application arises from a high computational cost. As shown in [34], the computation time of the hypervolume grows exponentially with the number of objectives, significantly limiting its applicability. This problem can be mitigated by approximation. For this purpose, a method based on Monte Carlo simulation was proposed in [35], where the hypervolume is approximated by computing a number of dominated point in a sample. This requires to accept some trade-off between accuracy and complexity. Alternatively, a method based on scalarizing functions was suggested in [36], though this necessitates a large number of weight vectors. Another developments make use of computationally less expensive quality indicators [37,38].

In spite of recent advances and numerous existing frameworks for solving MOPs, it is theoretically impossible to have an optimization approach that works the best for all the problems [39]. The only way one approach can outperform another if it implements some characteristics that are particularly suitable for dealing with a problem at hand. This fact stresses the importance of innovative approaches and motivates the research in the field of evolutionary multiobjective optimization. The present study seeks to advance the state-of-the-art by proposing a multiobjective evolutionary algorithm based on vector angle neighborhood (MOEA/VAN). The main novelty of the proposed framework is a selection scheme. In some sense, MOEA/VAN can be viewed as a decomposition-based approach. As opposed to existing MOEAs relying on decomposition, the proposed approach does not use any kind of weight of directional vectors. This alleviates the user's burden and the issue of consistency between the shapes of the Pareto front and the distribution of weight vectors. An important feature of MOEA/VAN is a similarity measure, which is defined on the basis of the angle between population members in the objective space. The smaller the angle, the more similar individuals. This concept is exploited during mating and environmental selections by determining the neighborhood and the most crowded region in the objective space, respectively. Diversity is ensured by forming a pool for replacement from most similar individuals, whereas convergence is provided by removing an individual with the worst value of a convergence measure. Different types of such measure are considered. Also, an external archive with nondominated solutions is maintained, with truncation procedure being developed based on proposed mechanisms for ensuring convergence and diversity. The proposed framework offers different possibilities for further extension and improvement. This is illustrated by a simple adaptation mechanism employed when estimating individuals similarity.

The first attempt to develop the MOEA/VAN framework was presented in [40]. The present study conducts more thorough investigation along this direction. When compared with its predecessor, the herein proposed approach incorporates an external archive with an appropriate truncation procedure based on the developed concepts. A new convergence measure is suggested to overcome some limitations identified in the previous work. Also, an adaptation strategy is devised to properly deal with different Pareto front geometries. Further contribution of this study is a set of test problems with known characteristics that can be useful for investigating the ability of MOEAs to balance convergence and diversity.

In the view of the present work, it is worthy to mention several studies available in the literature that employ angle information during the search. In [41], unit vectors generated in advance are used during the evolution to decompose the population into subpopulations according to the closeness of individuals to these vectors. For this purpose, the angle between the corresponding vectors in the objective space is utilized. The selection procedure employs a nondominated sorting to trim subpopulations if the number of available slots is exceeded. Similar decomposition approach is used in [42] for many-objective optimization. In this approach, subpopulations are also defined by associating individuals to closest reference directions using angles between reference and solution vectors. The difference lies in the selection where for each population member a fitness value is calculated using an angle-penalized distance measure. This measure takes into consideration the individual's convergence that is penalized by the angle between its objective values and the reference vector. In [43], constraints based on angles between individuals and corresponding weight vectors are introduced into MOEA/D. A new precedence relation is established for updating subproblems. When comparing two individuals, the one satisfying constraint is preferred. If both are infeasible in terms of angle-based constraint, the one having a smaller constraint violation value is favored. Otherwise, the scalarizing function values are used for decision as usual. Another variant of MOEA/D using angle information was suggested in [44]. To improve the balance between convergence and diversity, this approach employs decomposition-based sorting for controlling convergence and angle-based selection for maintaining diversity. The former defines different solution sets based on scalarizing function values. The latter selects solutions from the set, which cannot be completely accommodated into the new population, based on angles so that the minimum angle is maximized.

The common feature of the above approaches is that vectors defining directions of search are generated in advance and used for associating population members with certain directions, depending on how small is the angle between the direction and solution vectors. Although the herein proposed approach uses angle information, the important difference with existing approaches resides in the fact that no direction vectors are generated before and used during the search. To certain extent, this reduces a human labor. The proposed MOEA/VAN is the first algorithm demonstrating that the search can be effectively performed only exploiting the angle information between population members without associating them with predefined directions.

The remainder of this paper is organized as follows. Section 2 presents some concepts necessary for the development of the present work. Section 3 describes the design of the proposed framework. Section 4 reports and discusses the results of experimental study with state-of-the-art algorithms. This also includes the methodology employed for experimental validation. Section 5 concludes the study and outlines some possible future work.

2. Concepts

This study considers a multiobjective optimization problem (MOP) of the form

$$\begin{array}{ll}\text{minimize} & (f_1(\boldsymbol{x}),\ldots,f_m(\boldsymbol{x}))\\ \text{subject to} & \boldsymbol{x}\in\Omega\end{array}\tag{1}$$

where $\boldsymbol{f}=(f_1,\ldots,f_m)$ is the objective vector defined in the objective space $\Theta\subseteq\mathbb{R}^m$, $\boldsymbol{x}=(x_i,\ldots,x_n)$ is the decision vector, $\Omega\subseteq\mathbb{R}^n$ is the feasible decision space such that

$$\Omega=\{\boldsymbol{x}\in\mathbb{R}^n:\ \boldsymbol{l}\le\boldsymbol{x}\le\boldsymbol{u}\wedge\boldsymbol{c}(\boldsymbol{x})\le\boldsymbol{0}\}$$

and $\boldsymbol{c}=(c_1,\ldots,c_k)$ is the vector of inequality constraints, $\boldsymbol{l}=(l_1,\ldots,l_n)$ and $\boldsymbol{u}=(u_1,\ldots,u_n)$ are the lower and upper bounds of the decision vector, respectively.

There is no natural ordering in the objective space when multiple objectives are simultaneously considered. Instead, the objective space is partially ordered. In such a scenario, solutions are often compared on the basis of the Pareto dominance relation. This is also used to define the concepts of optimality in multiobjective optimization.

For two solutions $\boldsymbol{x}$ and $\boldsymbol{y}$ from Ω, a solution $\boldsymbol{x}$ is said to dominate a solution $\boldsymbol{y}$ (denoted by $\boldsymbol{x}\prec\boldsymbol{y}$) if

$$\begin{array}{ll}\forall\ i\in\{1,\ldots,m\}: & f_i(\boldsymbol{x})\le f_i(\boldsymbol{y})\wedge\\ \exists\ j\in\{1,\ldots,m\}: & f_j(\boldsymbol{x})<f_j(\boldsymbol{y}).\end{array}\tag{2}$$

There is a relaxed form of the Pareto dominance relation called ϵ-dominance. This has been widely used for the comparison of different multiobjective algorithms.

Given two solutions $\boldsymbol{x}$ and $\boldsymbol{y}$ from Ω, a solution $\boldsymbol{a}$ is said to ϵ-dominate a solution $\boldsymbol{y}$ (denoted by $\boldsymbol{x}\preceq_\epsilon\boldsymbol{y}$) if for a given ϵ

$$\forall\ i\in\{1,\ldots,m\}: f_i(\boldsymbol{x})-\epsilon\le f_i(\boldsymbol{y}).\tag{3}$$

A solution $\boldsymbol{x}^*\in\Omega$ is Pareto optimal if and only if

$$\nexists\boldsymbol{y}\in\Omega:\ \boldsymbol{y}\prec\boldsymbol{x}^*.\tag{4}$$

The presence of multiple conflicting objective typically gives rise to a set of optimal solutions. This set is generally known as the Pareto optimal set (or Pareto set for short). For a MOP (1), the Pareto set is defined as

$$\mathcal{PS}=\{\boldsymbol{x}^*\in\Omega\wedge\nexists\boldsymbol{y}\in\Omega:\ \boldsymbol{y}\prec\boldsymbol{x}^*\}.\tag{5}$$

For a MOP (1) and the Pareto set $\mathcal{PS}$, the Pareto optimal front (or Pareto front for short) is defined as

$$\mathcal{PF}=\{\boldsymbol{f}(\boldsymbol{x}^*)\in\Theta:\ \boldsymbol{x}^*\in\mathcal{PS}\}.\tag{6}$$

There are some special points often used in the context of multiobjective optimization. These points define the range of the Pareto front and are widely used in the decision making process.

An objective vector minimizing each of the objective functions is called an ideal objective vector $\boldsymbol{z}^{\text{ideal}}\in\mathbb{R}^m$. The elements of the ideal objective vector are the lower bounds of all objectives

$$\forall\ i\in\{1,\ldots,m\}:\ z_i^{\text{ideal}}=\inf_{x\in\mathcal{PS}}f_i(\boldsymbol{x}).\tag{7}$$

On the contrary, the nadir objective vector $\boldsymbol{z}^{\text{nadir}}$ represents the upper bound of each objective in the Pareto front. The elements of the nadir objective vector correspond to the corner points of the Pareto front

$$\forall\ i \in \{1, \ldots, m\}:\ z_i^{\text{nadir}} = \sup_{x \in \mathcal{PS}} f_i(\boldsymbol{x}). \tag{8}$$

Multiobjective optimization is concerned with solving a MOP (1) and aims at obtaining the Pareto set. Though, practically it is usually not possible to generate the entire Pareto set due to a large or even infinite number of points. Therefore, this is usually addressed by approximating the Pareto set that means obtaining a set of solutions that are as close as possible to the true Pareto set and as diverse as possible.

3. Proposed framework

Algorithm 1. MOEA/VAN.

1: input μ, δ_m, δ_r, T_m
2: // initialization
3: $A \leftarrow \{\}$
4: initialize and evaluate P
5: // evolution
6: **repeat**
7: // mating selection
8: randomly select i-th population member
9: generate $u \sim \mathbb{U}(0, 1)$
10: $P_m \leftarrow \begin{cases} T_m \text{ most similar individuals to } i & \text{if } u < \delta_m \\ \{1, \ldots, \mu\} & \text{otherwise} \end{cases}$
11: select parents from P_m
12: // variation
13: generate offspring y
14: repair y, if necessary
15: evaluate y
16: // environmental selection
17: find x so that $\nexists a \in P$: $s(a, y) < s(x, y)$
18: **if** $CV(y) \le CV(x) \wedge \exists\ i \in \{1, \ldots, m\}$: $f_i(y) < f_i(x)$ **then**
19: $P \leftarrow P \cup y$
20: identify replacement pool P_r
21: find two most similar individuals
22: remove one with worse convergence measure
23: **end if**
24: **until** the stopping criterion is met
25: output A

Herein proposed multiobjective evolutionary algorithm based on vector angle neighborhood (MOEA/VAN) embraces three major steps - namely selection, variation and replacement - reflecting a general framework of evolutionary algorithms. MOEA/VAN is outlined in Algorithm 1.

During the search, MOEA/VAN maintains the population P consisted of μ individuals and an external archive storing nondominated solutions that represents an approximation to the Pareto set. The population is randomly generated in the initialization phase, also a problem specific mechanism can be employed for this purpose. The population is evolved in the evolutionary process consisted of selection, variation and replacement. MOEA/VAN is a steady-state algorithm that means a single offspring is produced in each generation. For variation, any evolutionary operator can be adopted, depending on the problem at hand and genetic representation. Since the way in which selection is performed is a main novelty, in what follows respective aspects of MOEA/VAN are discussed in more detail.

3.1. Mating selection

The aim of mating selection (lines 7–11 in Algorithm 1) is to pick up a set of promising population members to undergo reproduction. By means of selection, useful genetic characteristics are expected to propagate, as individuals better fitted to the environment are more likely to produce promising offspring. In MOEA/VAN, there are two major steps constituting this process. The first is to identify a mating pool, a subset of the population from which parents are selected. At each evolutionary step, the mating pool is determined by the neighborhood of a randomly selected population member with probability δ_m. With probability $(1 - \delta_m)$, the entire population constitutes the mating pool. Next, a certain number of parents necessary for producing offspring are picked up from the mating pool. Different strategies can be adopted for this purpose, albeit a uniform selection is used in this study. The exploitation of neighborhood is motivated by the hope that similar individuals will produce promising offspring. This can be particularly relevant when handling problems with strong dependencies between the variables and high dimensional spaces.

3.2. Environmental selection

Environmental selection (lines 16–23 in Algorithm 1) aims at forming the population of the next generation by selecting the most promising individuals. Population members that succeed to survive during this process must ensure a good performance of the population with respect to convergence and diversity. Once an offspring is generated, it enters the population if the condition shown in line 18 (Algorithm 1) is fulfilled. This condition ensures offspring being far away from the current population are immediately rejected. Thus, offspring is accepted if its constraint violation value CV is not greater and it has a better value of at least one objective when compared with the most similar population member. $CV(\boldsymbol{x})$ is estimated as

$$CV(\boldsymbol{x}) = \sum_{j=1}^{k} \max(c_j(\boldsymbol{x}), 0). \tag{9}$$

Once the population is enlarged by accepting offspring, environmental selection is performed by forming replacement pool and removing worst performing individual from it. Replacement pool contains population members that will compete for survival. There are different possibilities for the creation of this pool in the MOEA/VAN framework. For instance, a nondominated sorting procedure can be used to identify the last nondominated front. This approach would be in line with dominance-based algorithms and can be used to foster the population convergence. Though, this study adopts a simple strategy in which replacement pool is formed by the entire population. This is particularly intended to promote the population diversity. Also, a combination of both strategies is possible.

In the replacement pool, the worst performing individual must be identified and removed. The factors influencing the survival of individuals are their dissimilarity and the performance in terms of convergence. First, two most similar individuals are identified. Then, an individual having a larger constraint violation value is removed. In the case when both individuals are feasible or their constraint violation values are equal, the decision is made based on a convergence measure. An individual having a worse value of the convergence measure is removed from the population.

3.3. Archive maintenance

Algorithm 2. ArchiveMaintenance.

input y
if $\nexists a \in A$: $a \prec y$ **then**
for $a \in A$ **do**
if $y \prec a$ **then**
$A \leftarrow A \backslash a$
end if

7: **end for**
8: $A \leftarrow A \cup y$
9: **end if**
10: **if** $|A| > \mu$ **then**
11: find two most similar individuals
12: remove one with worse convergence measure
13: **end if**

Each time a newly generated individual y is evaluated, it attempts to enter into an external archive A. The outline of the archive maintenance procedure is given in Algorithm 2. The candidate individual is compared with each member of the archive and is rejected if it is dominated by at least one archive member. In the case, there are archive members that are dominated by the candidate, those are removed from the archive, whereas the candidate is added into the archive. If the candidate and the archive members are mutually nondominated, then the candidate is accepted into the archive. If the maximum capacity of the archive is reached after the inclusion of individual, an archive truncation procedure is invoked to reduce its size. This procedure aims at preserving most promising individuals while identifying and removing the worst performing one. Truncation relies on the developed similarity and convergence measures, which play an essential role in the archive maintenance. A simple but effective scheme proposed in this study embraces two steps (lines 11–12 in Algorithm 2). First, a pair of individuals with the smallest similarity measure are identified. Next, an individual performing worse with respect to the convergence measure is removed.

3.4. Similarity and convergence measures

A thorough observation of the proposed algorithm reveals two important features responsible for its functionality. These are similarity and convergence measures. The former is intended to address the issue of diversity, whereas the latter is accountable for providing a selection pressure necessary for convergence.

To avoid a bias caused by differently scaled objectives, objective values used for calculating both measures are normalized. The normalization is performed as

$$\tilde{f}_i = \frac{f_i - f_i^{\min}}{f_i^{\max} - f_i^{\min}} \tag{10}$$

where $f_i^{\min}$ and $f_i^{\max}$ are the minimum and maximum values of the i-th objective in the current population. Eq. (10) ensures all the objective values are in the same range, namely $\tilde{\boldsymbol{f}} \in [0, 1]^m$.

MOEA/VAN makes use of the angle between population members in the objective space to measure how similar they are. An angle-based similarity is a popular technique to measure similarity between two vectors in the fields of information retrieval and data mining. This is in contrast to evolutionary multiobjective optimization where the Euclidean distance is the most common approach, which can be inappropriate especially in high dimensions. The similarity measure proposed in this study is calculated as

$$s(a, b) = 1 - \frac{\langle \tilde{\boldsymbol{f}}(a), \tilde{\boldsymbol{f}}(b) \rangle}{\| \tilde{\boldsymbol{f}}(a) \| \| \tilde{\boldsymbol{f}}(b) \|} \tag{11}$$

where $\langle \cdot,\cdot \rangle$ denotes the dot product and $\| \cdot \|$ denotes the Euclidean norm. The smaller the value of $s(a, b)$, the more similar the individuals a and b.

This study investigates two different measures for estimating the convergence of individuals in the population. The first is based on the notion that the more objective space is dominated by a given individual the better. For calculating this measure, the objective space must be bounded by some reference point. The portion of the objective space that is dominated by a given solution is also known as the hypervolume. It can be estimated by the product of differences between the coordinates of the reference point and the objective values of the given individual as

$$V(i) = \prod_{j=1}^{m} (r_j - \tilde{f}_j(i)). \tag{12}$$

where r_j is the j-th component of a reference point, with r_j=2 $\forall\, j = 1,\ldots,m$ being considered in this work to ensure the presence of extreme points, and $\tilde{f}_j$ is the j-th component of the normalized objective vector of individual i. Individuals having lager values of V are preferred.

Another convergence measure exploits the principle that individuals having smaller deviations from an ideal objective vector are preferred. This deviation is measured by the p-norm as

$$L_p(i) = \left(\sum_{j=1}^{m} |\tilde{f}_j(i)|^p \right)^{\frac{1}{p}} \tag{13}$$

where $p \geq 1$. The lower the value of $L_p(i)$, the better convergence. One possible disadvantage of this measure is that the extreme solutions of the Pareto front approximation with convex geometry can be lost. This is because solutions located closer to the knee point will have smaller values of this measure that those closer to the extremes.

The ideal of the developed selection is illustrated in Fig. 1, where for two objectives the population composed of six solutions needs to be reduced. Based on the angle-based similarity measure, two most similar solutions a and b are identified, with the angle between them denoted by θ. The convergence measures for solutions are given by the volumes $V(a)$ and $V(b)$ dominated by a and b, respectively, and bounded by the reference point r. Since $V(a) > V(b)$, solution a is retained in and solution b is removed from the population.

4. Experimental validation

The proposed MOEA/VAN is validated through computational experiments with representative state-of-the-art algorithms. This section presents the experimental study and discusses the obtained results. The experiments are divided into three parts according to characteristics of considered problems. These include a set of suggested test problems, state-of-the-art test suite and engineering problems.

4.1. Performance comparison methodology

Due to a stochastic nature, 30 independent runs of each tested MOEA are performed on each problem. The outcome of each algorithm is defined by a set of nondominated solutions, designated as an approximation set [45]. Quality indicators are used to quantitatively

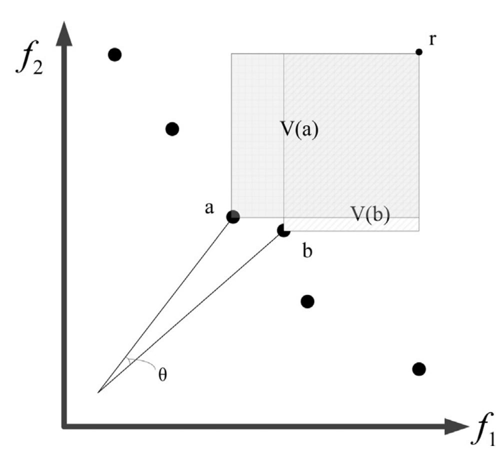

Fig. 1. Illustration of selection mechanism in biobjective space.

Table 1
Definition of test instances.

	α_1	α_2	β	γ
MOP1	x_1	$1-\sqrt{x_1}\sin(\pi^2 x_1)$	$-x_1\log x_1-(1-x_1)\log(1-x_1)$	$\parallel \boldsymbol{x} \parallel \sum_{j=2}^{n}\lvert x_1-x_j\rvert$
MOP2	x_1	$1-x_1^2$	$1-\cos(2\pi x_1)$	$\parallel \boldsymbol{x} \parallel \sum_{j=2}^{n}\lvert\sin(x_1)-x_j\rvert$
MOP3	x_1	$1-x_1$	$x_1(1-x_1)$	$\parallel \boldsymbol{x} \parallel \sum_{j=2}^{n}\lvert x_1^2-x_j\rvert$
MOP4	x_1	$1-\sqrt{x_1}$	$e^{x_1}\sin(\pi x_1)$	$\parallel \boldsymbol{x} \parallel \sum_{j=2}^{n}\lvert\cos(x_1)-x_j\rvert$
MOP5	x_1	$1-x_1\cos(8\pi x_1)$	$\begin{cases} x_1 & \text{if } x_1 \le 0.5 \\ 1-x_1 & \text{otherwise} \end{cases}$	$\parallel \boldsymbol{x} \parallel \sum_{j=2}^{n}\lvert x_1-x_j\rvert$
MOP6	x_1	$1-x_1e^{x_1-x_1^2}$	$\sin(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_1)$	$\parallel \boldsymbol{x} \parallel \sum_{j=2}^{n}\lvert e^{-x_1}-x_j\rvert$

assess the outcomes. Following suggestions in [46], the Pareto compliant quality indicators are employed in this study. Specifically, the epsilon [45] and hypervolume [47] indicators are used. The former assesses the convergence, whereas the latter can measure both the convergence and diversity of an approximation set.

For problems with known Pareto fronts, a reference set is constructed by generating 10^4 points along the true Pareto front. Otherwise, the reference set is created by combining all the nondominated solutions obtained in the experiments. Before calculating the indicator values, all the objective values are normalized using the minimum and maximum objective values in the reference set. When computing the hypervolume, solutions that do not dominate the nadir point are discarded. The hypervolume is calculated with respect to the reference point ***1.1*** and the normalized hypervolume values are presented.

A statistical analysis of the results is performed to get statistically sound conclusions. As the assumptions required for the application of parametric tests are usually not met, a nonparametric statistical procedure is adopted for the analysis. In particular, a pairwise comparison of the algorithms on a specific problem is performed by applying the Wilcoxon rank sum test [48]. The null hypothesis is that two algorithms perform equally, with the difference in the results being purely due to chance. The alternative hypothesis states the observed difference is nonrandom. The hypothesis testing is performed at a significance level of $\alpha = 0.05$.

The performance score [35] is used to access an overall performance of an algorithm. This approach attempts to rank different algorithms on the basis of statistical tests. For a specific problem and a set of algorithms $Alg_1,\ldots,Alg_l$, let $\delta_{i,j}$ be 1, if the algorithm Alg_j is statistically better that Alg_i. Otherwise $\delta_{i,j}$ is 0. An algorithm is considered to statistically outperform another algorithm if it yields a better median value of the quality indicator and there is a significant difference between the distributions of quality indicator values produced by the two algorithms. For each algorithm Alg_i, the performance score is determined as

$$P(Alg_i) = \sum_{\substack{j=1 \\ i\neq j}}^{l} \delta_{i,j}. \tag{14}$$

This value reveals how many other algorithms are better than the corresponding algorithm on the specific problem. The smaller the index, the better the algorithm. In particular, the value of zero means that no other algorithm produces significantly better approximation sets in terms of a given quality indicator.

4.2. Experimental setup

The main concern of the present study is the selection in MOEAs. Owing to this, each set of experiments is conducted adopting the algorithms with the same variation operators and parameter settings allowing fair comparison of the algorithms.

Experiments involve MOEAs with differential evolution and genetic algorithm reproduction operators. In particular, a differential evolution operator is adopted with the scale factor of F=0.5 and the crossover probability of CR=1. The SBX crossover is used with the distribution index of $\eta_c = 20$ and applied the probability of p_c=1. The polynomial mutation has the distribution index of $\eta_m = 20$ and the application probability of $p_m = 1/n$ (where n is the number of decision variables).

In all experiments, the population size is fixed to $\mu = 100$. The algorithms are run for $500 \times \mu$ and $300 \times \mu$ function evaluations in experiments reported in Sections 4.3 and 4.4–4.5, respectively. MOEA/VAN uses T_m=20 and $\delta_m = 0.8$, the default similarity measure is as shown in (12). The remaining parameter setting for the other algorithms are as in the original papers.

4.3. Proposed problems

A set of challenging two-objective problems is constructed. Following suggestions in [49], each problem is composed of different functions. The problems developed in this study conform to the following format

$$\begin{aligned} f_1(\boldsymbol{x}) &= \alpha_1(1+\beta\gamma) \\ f_2(\boldsymbol{x}) &= \alpha_2(1+\beta\gamma). \end{aligned} \tag{15}$$

The formulation of each problem requires three different functions: $\boldsymbol{\alpha}$, β and γ. Each function plays a particular role in the difficulties posed by a resultant MOP for MOEAs. The function $\boldsymbol{\alpha}$ accounts for the shape of the Pareto front, the function β introduces the bias into the search space, whereas γ is the distance function whose minimization results in a Pareto optimal solution.

Building a test instance requires the definition of each function. Table 1 lists the definitions of six test problems suggested in this study. For all MOPs, the decision space is $\boldsymbol{x} \in [0,1]^n$. The problems are scalable to any number of the decision variables. This study uses n=10. A major difference of the proposed test problems with the existing ones - for instance [12,50,51] - lies in the definition of the function β. For the problems in Table 1, β is a concave function with a maximum lying close to the middle of its domain. Particularly in MOP1 and MOP5, β corresponds to the binary entropy and triangular functions, respectively. In both cases, the maximum of β is achieved when x_1=0.5, when monotonically decreasing approaching extremes of its domain. This feature is intended to introduce difficulties in obtaining intermediate Pareto optimal solutions, as it would be easy to converge to the corner points of the Pareto front for $\beta = 0$ while leading to a loss of diversity in the other regions of the search space.

The performance of MOEA/VAN on these problems is tested against GDE3 [52], IBEA [30], MOEA/D [12], Two_Arch2 [32] and MOEA/DVA [14]. These algorithms serve as an important reference for performance evaluation because they encompass the major strategies for the fitness assignment and selection in MOEAs. In particular, the first three MOEAs are well-established state-of-the-art algorithms that represent dominance-, indicator- and decomposition-based types of selection. Whereas Two_Arch2 can be viewed as an improvement of indicator-based selection where two separate archives for diversity and

Table 2
Results for proposed problems. The values refer to median and interquartile range of quality indicators. Best performance is highlighted with gray background. The symbol † indicates a statistical difference between the respective and best performing algorithm.

		MOEA/VAN	GDE3	IBEA	Two_Arch2	MOEA/D	MOEA/DVA
MOP1	eps	1.15e-02 (2.2e-03)	8.94e-01 (0.0e+00)†	8.94e-01 (0.0e+00)†	8.94e-01 (4.8e-01)†	8.94e-01 (0.0e+00)†	2.62e-02 (1.3e-02)†
	hv	7.03e-01 (2.3e-03)	1.10e-01 (0.0e+00)†	1.10e-01 (0.0e+00)†	1.10e-01 (2.4e-01)†	1.10e-01 (0.0e+00)†	6.91e-01 (6.4e-03)†
MOP2	eps	3.88e-02 (4.5e-03)	3.82e-01 (1.1e-01)†	3.68e-01 (8.1e-04)†	3.67e-01 (8.4e-04)†	2.70e-01 (1.5e-02)†	7.93e-02 (2.1e-02)†
	hv	5.12e-01 (4.2e-03)	2.10e-01 (1.6e-01)†	2.35e-01 (1.2e-03)†	2.37e-01 (1.6e-03)†	3.75e-01 (1.9e-02)†	4.89e-01 (6.7e-03)†
MOP3	eps	1.81e-02 (2.4e-03)†	5.00e-01 (0.0e+00)†	5.00e-01 (0.0e+00)†	4.97e-01 (7.6e-04)†	5.00e-01 (0.0e+00)†	1.38e-02 (8.9e-04)
	hv	6.92e-01 (1.4e-03)†	2.10e-01 (0.0e+00)†	2.10e-01 (0.0e+00)†	2.17e-01 (1.6e-03)†	2.10e-01 (0.0e+00)†	6.96e-01 (1.3e-03)
MOP4	eps	3.62e-02 (6.1e-03)	6.18e-01 (0.0e+00)†	4.48e-01 (8.6e-03)†	4.29e-01 (1.3e-02)†	3.18e-01 (3.4e-01)†	3.99e-02 (1.6e-02)
	hv	8.46e-01 (4.1e-03)	1.10e-01 (0.0e+00)†	4.28e-01 (1.1e-02)†	4.57e-01 (2.3e-02)†	6.43e-01 (4.7e-01)†	8.47e-01 (1.5e-02)
MOP5	eps	1.12e-02 (2.6e-03)	2.50e-01 (0.0e+00)†	5.00e-01 (0.0e+00)†	4.95e-01 (1.6e-03)†	5.00e-01 (0.0e+00)†	1.31e-02 (5.3e-03)†
	hv	6.06e-01 (1.7e-03)	4.60e-01 (0.0e+00)†	2.10e-01 (0.0e+00)†	2.19e-01 (2.5e-03)†	2.10e-01 (0.0e+00)†	6.05e-01 (2.7e-03)†
MOP6	eps	2.14e-02 (4.9e-03)†	5.61e-01 (0.0e+00)†	5.47e-01 (2.8e-03)†	5.48e-01 (1.8e-03)†	5.61e-01 (0.0e+00)†	1.35e-02 (6.8e-04)
	hv	7.85e-01 (1.5e-03)†	1.10e-01 (0.0e+00)†	2.35e-01 (3.6e-03)†	2.31e-01 (2.3e-03)†	1.10e-01 (0.0e+00)†	7.88e-01 (1.4e-03)

convergence are maintained. MOEA/DVA is an extension of MOEA/D that includes an elaborate mechanism for analyzing and processing decision variables to improve the algorithm's search ability. MOEA/DVA differs from the other MOEAs used in this study in the sense that its main focus is on the exploration of the decision space.

Table 2 presents the results obtained for problems in Table 1. These results clearly indicate that MOEA/VAN completely outperforms the other algorithms, except for MOEA/DVA. Both MOEA/VAN and MOEA/DVA produce a comparable performance. The former yields better results for three MOPs, whereas the latter works better for two MOPs. The results for MOP4 are dependent on the quality indicator used, though no statistical difference was detected between the algorithms on this problem. An important observation is that MOEA/VAN exhibits a superior performance to other MOEAs that mainly differ by the selection mechanism. This is due to the ability of its selection scheme to maintain diversity among the population, which is ensured by performing a survival contest between two most similar individuals in the replacement pool. Since this pool consists of the entire population, the diversity is emphasized. On the contrary, a dominance-based selection considers convergence first and diversity second. A poor diversity maintenance capability is also exhibited by scalarizing- and indicator-based strategies, as a better fitness is assigned to better converged but poorly distributed individuals. A competitive performance of MOEA/DVA is explained by an elaborate mechanism for exploration of the decision space, which consists of decomposing the decision variables into position an distance related variables and learning the linkage between them. Such decomposition allows to evolve separately different types of variables, thereby reducing the complexity of MOP. Though, this can be ineffective for problems with no clear separation between the variables and a large number of mixed variables [14]. Conversely, MOEA/VAN uses a simple DE operator for exploring the decision space. Thus, the obtained results somewhat stress the importance of the selection in multi-objective search and its influence on the overall performance of MOEAs.

The difference in the performance of algorithms can be further understood by considering Fig. 2. The presented plots depict approximation sets of the run closest to the median value of the hypervolume indicator obtained by four best performing algorithms. It is clear that Two_Arch2 and MOEA/D fail to approximate the entire Pareto front for at least one MOP. Solutions are crowded close to the corners of the Pareto front. A slightly better distribution is observed for MOP2 and MOP4, though it is still unsatisfactory. On the other hand, MOEA/VAN provides adequate Pareto front approximations for all the problems. Quite similar performance is produced by MOEA/DVA. Though, it is a conceptually different algorithm as discussed early. A promising performance of MOEA/VAN and MOEA/DVA is due to its selection scheme promoting diversity in the objective spaces and an elaborate mechanism for exploring the decision space, respectively.

A distinctive feature of MOEA/VAN is a similarity measure. It plays an important role in mating and environmental selections during the course of evolution. To evaluate its impact on the algorithm's search ability, two versions of MOEA/VAN are investigated. The one uses the angle-based similarity measure as described in Section 3. The other relies on the Euclidean distance. Table 3 shows results in terms of quality indicators obtained by MOEA/VAN with two different similarity measures. From these results, it is evident that the variant using the angles between the vectors in the objective space works much better. Actually, it was observed that the population does not approach the Pareto front when using Euclidean distance within the above definition of MOEA/VAN framework. This can be also confirmed from large values of the epsilon indicator and small hypervolume values.

Thus, the results obtained on the six proposed problem indicate the competitiveness of the proposed framework and the relevance of using angles between the objective vectors of population members as similarity measure to guide the search and keep the population diversity.

4.4. WFG problems

The competitiveness of MOEA/VAN on state-of-the-art benchmarks is evaluated adopting the WFG test suite [51]. These problems possess some of the pertinent problem characteristics, including nonseparable, multimodal, deceptive problems and with different geometry of the Pareto front. All problems are scalable to the number of objectives and variables. The vector associated with a simple underlying problem that defines the fitness space is derived, via a series of transition vectors, from a vector of working parameters. Unlike other test suites available in the literature, this allows the user to control the number of parameters responsible for convergence and diversity, i.e. the number of position- and distance-related parameters. The characteristics of the WFG problems are summarized in Table 4.

In the experiments, the problems were tested with the number of objectives $m = 5, 7, 10$ and the number of decision variables $n = k + l$, where l=20 is the number of distance parameters and $k = m - 1$ is the number of position parameters. As experiments involve many-objective problems, a set of recent MOEAs proved effective in dealing with high-dimensional objective spaces is selected for comparison. These are HypE [35], an indicator-based MOEAs using Monte Carlo sampling for a hypervolume-based fitness assignment, MOEA/D [16], decomposition-based approach with a fitness assignment based on scalarization, MOEA/DD [25], an algorithm that combines dominance- and decom-

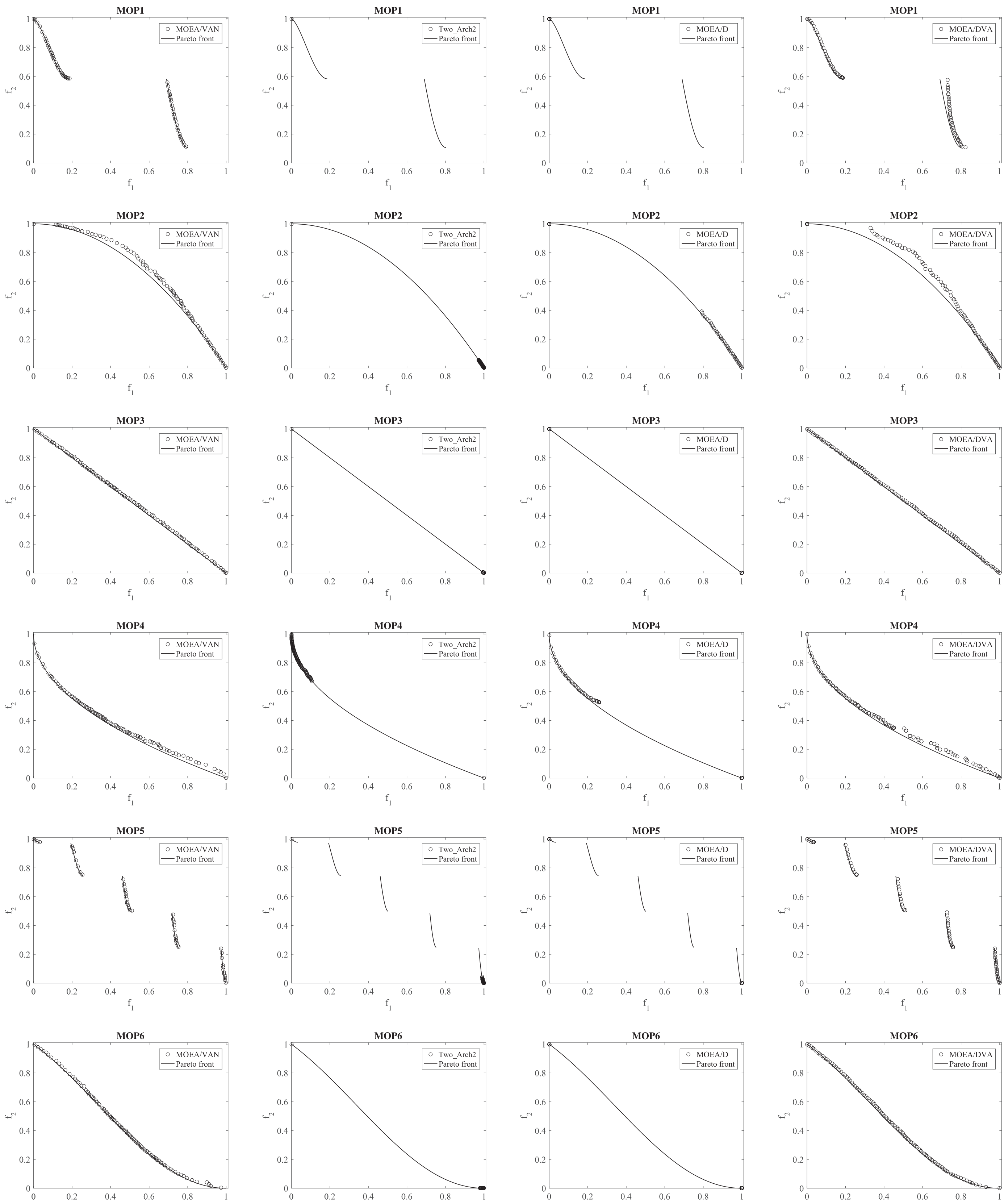

Fig. 2. Approximation sets obtained by different MOEAs for proposed problems. Plots refer to the median run in terms of the hypervolume.

position-based strategies specifically for many-objective optimization, and NSGA-III [24] that is an extension of a nondominated sorting genetic algorithm to handling many-objective problems.

The results in terms of the quality indicators obtained by the five algorithms are presented in Table 5. These results indicate a superior performance of MOEA/VAN to HypE, MOEA/D and MOEA/DD.

Table 3
Results for proposed problems obtained by MOEA/VAN with different similarity measures. The values refer to median and interquartile range of quality indicators. Best performance is highlighted with gray background. The symbol † indicates a statistical difference between the respective and best performing algorithm.

		angle	Euclidean distance
MOP1	eps	1.27e-02 (4.9e-03)	8.94e-01 (0.0e+00)†
	hv	7.03e-01 (3.0e-03)	1.10e-01 (0.0e+00)†
MOP2	eps	3.85e-02 (8.0e-03)	9.99e-01 (0.0e+00)†
	hv	5.10e-01 (5.8e-03)	1.10e-01 (0.0e+00)†
MOP3	eps	1.63e-02 (2.5e-03)	5.00e-01 (0.0e+00)†
	hv	6.94e-01 (1.5e-03)	2.10e-01 (0.0e+00)†
MOP4	eps	3.64e-02 (7.2e-03)	9.99e-01 (1.3e-15)†
	hv	8.46e-01 (4.7e-03)	0.00e+00 (0.0e+00)†
MOP5	eps	9.85e-03 (3.4e-03)	5.00e-01 (0.0e+00)†
	hv	6.06e-01 (1.1e-03)	2.10e-01 (0.0e+00)†
MOP6	eps	2.10e-02 (3.7e-03)	5.61e-01 (0.0e+00)†
	hv	7.87e-01 (1.1e-03)	1.10e-01 (0.0e+00)†

Table 4
Characteristics of the WFG problems.

Problem	Separability	Modality	Bias	Geometry
WFG1	separable	uni	polynomial, flat	convex, mixed
WFG2	non-separable	$f_{1:m-1}$ uni, f_m multi	no bias	convex, disconnected
WFG3	non-separable	uni	no bias	linear, degenerate
WFG4	separable	multi	no bias	concave
WFG5	separable	deceptive	no bias	concave
WFG6	non-separable	uni	no bias	concave
WFG7	separable	uni	parameter dependent	concave
WFG8	non-separable	uni	parameter dependent	concave
WFG9	non-separable	multi, deceptive	parameter dependent	concave

NSGA-III works better with respect to the epsilon indicator on several 7-objective problems and WFG3 with 5 and 10 objectives. Though, MOEA/VAN gives constantly better results with regard to the hypervolume on all the considered instances, stressing its overall competitiveness.

The above observations can be further confirmed when looking at Fig. 3. The plots clearly show the solutions obtained by MOEA/VAN are better distributed in the objective space. A different scale of the objectives can be regarded as a major reason causing a poor performance of decomposition-based MOEAs. It can be seen that the solutions obtained by MOEA/D and MOEA/DD are mostly located near the corner point with the lowest objective values. The promising results obtained by MOEA/VAN can be attributed to the ability of it selection scheme to keep the population diversity and provide a necessary selection pressure in high-dimensional objective spaces. The former is ensured by selecting for survival tournament, at each evolutionary step, the most similar individuals on the basis of the introduced similarity measure. Whereas convergence is provided by favoring the survival of an individual that is best performing with respect to the suggested convergence measure.

In the light of the ongoing discussion, it can be easily understood that the convergence measure is an important feature of MOEA/VAN. It bears the primary responsibility for providing a required selection pressure. This issue is particularly relevant in high-dimensional objective spaces. The comparison of the similarity measures, defined in (12) and (13), is graphically shown in Fig. 4. In the case of minimizing the distance to the ideal point, three types of distance measures are considered, such as Manhattan, Euclidean and Chebyshev distances denoted as L_1, L_2, L_∞, respectively. It can be seen that the best performance on the majority of problems is achieved by the convergence measure based on the dominated volume (V). Though, there are instances where L_1 works better. In particular, these are the problems WFG1,2 and 3, regarding the hypervolume. The deterioration on the other problems can be due to the concave geometry of the Pareto fronts. The results with respect to the epsilon indicator indicate quite similar performance for all the metrics on WFG3, which has linear Pareto front geometry. Whereas, trends similar with the results in terms of the hypervolume are observed. Thus, the obtained results suggest that MOEA/VAN is able to produce a competitive performance on problems with high-dimensional objective spaces and making use of the volume-based convergence measure can be advantageous for the search.

4.5. Engineering problems

The comparative analysis of MOEAs on artificially constructed test problems offers certain advantages, as the properties and the Pareto sets of these problems are typically known. Though, such problems often do include difficulties being encountered in real-world applications. Due to this fact three engineering problems from the literature are addressed to demonstrate a practical relevance and validity of MOEA/VAN.

The first is car side impact problem [24]. This problem aims to minimize the weight of a car, the pubic force experienced by a passenger, and the average velocity of the V-Pillar responsible for bearing the impact load. This involves constraints limiting values of abdomen load, pubic force, velocity of V-Pillar, rib deflection, etc. Thickness parameters of critical parts are the design variables. The second is the design of I-beam [53]. The problem involves the minimization of the total cross-sectional area and the deflection at the midspan under applied external loads. The geometric parameters of the I-beam are the four design variables. The problem is subject to stress and geometric constraints. The third is the design of welded beam [54]. In this problem, a beam must carry a certain load and needs to be welded on another beam. Thickness, length of weld, width of the beam, and thickness of the beam are the design variables. The cost of beam fabrication and the deflection at the end are to be minimized. The constraints are imposed on the shear stress, bending stress in the beam and buckling load on the bar.

As MOEA/VAN can be viewed as a decomposition-based algorithm, MOEA/D is selected as a reference algorithm for comparison. Also, there are several similarities in the frameworks of MOEA/D and MOEA/VAN. Whereas the major distinction is that MOEA/VAN does not use weight vectors during the search, with the population being process solely based on interactions between individuals in the objective space. MOEA/D is investigated with Chebyshev (CHB), penalty-based boundary intersection (PBI) and weighted sum (WSUM) scalarization schemes. For constraint handling, a strategy suggested in [55] is adopted, as it proved effective on a set of challenging constrained problems.

The obtained results in terms of the quality indicators are presented in Table 6. These results suggest that MOEA/VAN performs better than MOEA/D variants, except for the car side impact problem. On this problem, there is no statistical difference between MOEA/VAN and MOEA/D with Chebyshev method, with both algorithms alternately giving best median values of the two quality indicators. The Pareto front approximations obtained by MOEA/VAN are depicted in Fig. 5. The plots show that MOEA/VAN can provide adequate approximation sets for these problems, also the obtained results are consistent with those available in the literature.

Table 5
Results for WFG problems. The values refer to median and interquartile range of quality indicators. Best performance is highlighted with gray background. The symbol † indicates a statistical difference between the respective and best performing algorithm.

		MOEA/VAN	HypE	MOEA/D	MOEA/DD	NSGA-III
5-objectives						
WFG1	eps	4.33e-01 (9.5e-03)	6.81e-01 (2.9e-02)†	5.41e-01 (1.6e-02)†	5.65e-01 (1.4e-02)†	5.80e-01 (2.8e-02)†
	hv	5.44e-01 (1.1e-02)	3.22e-01 (3.0e-02)†	4.72e-01 (2.2e-02)†	4.43e-01 (1.1e-02)†	4.19e-01 (1.7e-02)†
WFG2	eps	8.95e-02 (1.6e-02)	1.76e-01 (3.8e-02)†	1.43e-01 (2.5e-02)†	1.02e-01 (9.0e-03)†	1.11e-01 (2.7e-02)†
	hv	1.53e+00 (1.8e-02)	1.46e+00 (3.6e-02)†	1.26e+00 (4.9e-02)†	1.48e+00 (2.6e-02)†	1.52e+00 (2.0e-02)
WFG3	eps	3.00e-01 (1.0e-02)	3.62e-01 (6.5e-02)†	3.20e-01 (1.0e-02)†	3.12e-01 (7.1e-03)†	2.98e-01 (1.3e-02)
	hv	9.67e-01 (3.9e-02)	8.74e-01 (7.1e-02)†	7.09e-01 (5.2e-02)†	7.99e-01 (2.8e-02)†	9.53e-01 (2.4e-02)†
WFG4	eps	2.18e-01 (1.9e-02)	3.90e-01 (4.7e-02)†	5.81e-01 (5.6e-02)†	2.75e-01 (2.0e-02)†	2.47e-01 (2.5e-02)†
	hv	1.14e+00 (1.0e-02)	8.43e-01 (6.3e-02)†	8.54e-01 (3.7e-02)†	1.01e+00 (1.6e-02)†	1.00e+00 (1.7e-02)†
WFG5	eps	2.21e-01 (1.6e-02)	3.53e-01 (2.3e-02)†	5.92e-01 (4.1e-02)†	2.79e-01 (1.2e-02)†	2.45e-01 (2.5e-02)†
	hv	1.13e+00 (8.0e-03)	8.75e-01 (6.1e-02)†	8.63e-01 (3.2e-02)†	9.71e-01 (1.3e-02)†	1.01e+00 (2.8e-02)†
WFG6	eps	2.30e-01 (1.4e-02)	4.05e-01 (6.8e-02)†	6.45e-01 (5.8e-02)†	2.79e-01 (2.7e-02)†	2.82e-01 (4.0e-02)†
	hv	1.14e+00 (1.9e-02)	7.66e-01 (1.4e-01)†	6.71e-01 (9.9e-02)†	9.65e-01 (2.8e-02)†	9.38e-01 (3.1e-02)†
WFG7	eps	2.20e-01 (1.2e-02)	4.19e-01 (4.5e-02)†	6.69e-01 (5.0e-02)†	2.86e-01 (1.6e-02)†	2.73e-01 (1.9e-02)†
	hv	1.22e+00 (8.0e-03)	8.37e-01 (1.1e-01)†	7.23e-01 (1.3e-01)†	9.86e-01 (1.9e-02)†	8.61e-01 (4.7e-02)†
WFG8	eps	2.43e-01 (1.2e-02)	6.40e-01 (6.4e-02)†	6.44e-01 (5.3e-02)†	2.90e-01 (1.3e-02)†	3.06e-01 (5.4e-02)†
	hv	1.06e+00 (1.1e-02)	6.88e-01 (5.4e-02)†	5.72e-01 (1.5e-01)†	8.78e-01 (1.8e-02)†	8.58e-01 (2.4e-02)†
WFG9	eps	2.12e-01 (1.7e-02)	3.78e-01 (5.2e-02)†	6.22e-01 (6.1e-02)†	3.05e-01 (2.6e-02)†	2.52e-01 (2.0e-02)†
	hv	1.15e+00 (1.0e-02)	7.73e-01 (1.1e-01)†	6.98e-01 (1.2e-01)†	8.30e-01 (5.2e-02)†	9.56e-01 (3.6e-02)†
7-objectives						
WFG1	eps	4.45e-01 (6.1e-03)	6.47e-01 (2.9e-02)†	5.19e-01 (2.0e-02)†	5.44e-01 (1.3e-02)†	6.00e-01 (1.7e-02)†
	hv	2.49e-01 (8.1e-03)	1.47e-01 (1.1e-02)†	2.29e-01 (1.7e-02)†	2.10e-01 (7.1e-03)†	1.78e-01 (7.8e-03)†
WFG2	eps	5.98e-02 (1.2e-02)	1.47e-01 (3.9e-02)†	1.38e-01 (4.0e-02)†	7.28e-02 (6.9e-03)†	8.10e-02 (2.2e-02)†
	hv	9.53e-01 (1.3e-02)	8.81e-01 (2.7e-02)†	7.32e-01 (7.5e-02)†	8.79e-01 (2.4e-02)†	9.48e-01 (1.5e-02)
WFG3	eps	2.83e-01 (1.7e-02)	3.29e-01 (4.4e-02)†	3.88e-01 (4.1e-02)†	3.07e-01 (1.9e-02)†	2.99e-01 (2.8e-02)†
	hv	5.56e-01 (2.8e-02)	4.70e-01 (4.0e-02)†	2.41e-01 (2.1e-02)†	3.76e-01 (2.0e-02)†	4.20e-01 (4.3e-02)†
WFG4	eps	2.82e-01 (2.3e-02)	5.46e-01 (1.1e-01)†	6.96e-01 (7.6e-02)†	5.52e-01 (4.0e-02)†	2.76e-01 (1.3e-02)
	hv	6.81e-01 (1.3e-02)	2.87e-01 (5.6e-02)†	2.30e-01 (2.0e-02)†	4.14e-01 (2.7e-02)†	6.12e-01 (1.7e-02)†
WFG5	eps	2.94e-01 (2.1e-02)†	4.89e-01 (7.9e-02)†	7.03e-01 (5.4e-02)†	5.39e-01 (1.5e-01)†	2.79e-01 (1.7e-02)
	hv	6.68e-01 (5.8e-03)	3.13e-01 (8.9e-02)†	2.51e-01 (1.9e-02)†	3.93e-01 (1.8e-02)†	6.12e-01 (1.4e-02)†
WFG6	eps	3.25e-01 (2.8e-02)†	5.57e-01 (1.3e-01)†	7.70e-01 (5.8e-02)†	5.62e-01 (3.7e-02)†	2.88e-01 (2.1e-02)
	hv	6.55e-01 (1.9e-02)	2.10e-01 (5.6e-02)†	1.35e-01 (4.0e-02)†	3.78e-01 (2.8e-02)†	5.96e-01 (1.7e-02)†
WFG7	eps	3.22e-01 (3.2e-02)†	5.45e-01 (1.5e-01)†	7.07e-01 (4.0e-02)†	5.59e-01 (3.5e-02)†	2.78e-01 (1.9e-02)
	hv	7.03e-01 (2.4e-02)	2.68e-01 (6.1e-02)†	2.03e-01 (4.0e-02)†	4.18e-01 (1.8e-02)†	6.31e-01 (2.5e-02)†
WFG8	eps	3.22e-01 (2.2e-02)	6.96e-01 (1.3e-01)†	8.82e-01 (5.5e-02)†	4.23e-01 (2.4e-02)†	3.23e-01 (2.2e-02)
	hv	5.92e-01 (1.8e-02)	1.82e-01 (3.1e-02)†	1.65e-02 (2.7e-02)†	2.78e-01 (2.2e-02)†	5.10e-01 (2.2e-02)†
WFG9	eps	2.84e-01 (2.8e-02)	5.81e-01 (1.3e-01)†	7.63e-01 (7.7e-02)†	4.59e-01 (3.7e-02)†	2.80e-01 (1.7e-02)
	hv	6.78e-01 (8.7e-03)	2.39e-01 (4.3e-02)†	1.70e-01 (1.1e-01)†	2.74e-01 (3.4e-02)†	5.52e-01 (3.3e-02)†
10-objectives						
WFG1	eps	4.42e-01 (6.3e-03)	6.38e-01 (2.3e-02)†	4.99e-01 (2.3e-02)†	5.06e-01 (1.4e-02)†	5.44e-01 (2.1e-02)†
	hv	2.16e-01 (5.2e-03)	1.31e-01 (8.9e-03)†	2.14e-01 (1.9e-02)	2.04e-01 (1.1e-02)†	1.76e-01 (7.6e-03)†
WFG2	eps	4.02e-02 (7.2e-03)	1.13e-01 (3.5e-02)†	1.29e-01 (2.0e-02)†	6.96e-02 (1.5e-02)†	5.57e-02 (8.7e-03)†
	hv	9.60e-01 (8.5e-03)	8.77e-01 (4.5e-02)†	7.27e-01 (5.3e-02)†	8.67e-01 (4.0e-02)†	9.31e-01 (1.7e-02)†
WFG3	eps	2.89e-01 (2.4e-02)†	3.49e-01 (5.8e-02)†	5.34e-01 (7.2e-02)†	3.04e-01 (2.7e-02)†	2.55e-01 (1.8e-02)
	hv	5.58e-01 (3.4e-02)	4.30e-01 (4.4e-02)†	1.39e-01 (1.5e-02)†	3.61e-01 (1.5e-02)†	4.74e-01 (3.0e-02)†
WFG4	eps	3.30e-01 (2.8e-02)	6.96e-01 (2.0e-01)†	9.08e-01 (5.8e-02)†	6.19e-01 (1.0e-01)†	4.46e-01 (1.4e-01)†
	hv	7.74e-01 (2.0e-02)	2.16e-01 (6.4e-02)†	6.37e-02 (2.5e-02)†	2.27e-01 (2.5e-02)†	5.30e-01 (3.0e-02)†
WFG5	eps	3.29e-01 (1.7e-02)	5.64e-01 (1.1e-01)†	8.36e-01 (9.1e-02)†	6.45e-01 (1.0e-01)†	4.16e-01 (1.1e-01)†
	hv	7.66e-01 (6.5e-03)	2.45e-01 (6.8e-02)†	8.62e-02 (3.4e-02)†	2.12e-01 (2.8e-02)†	5.36e-01 (2.4e-02)†
WFG6	eps	3.44e-01 (3.3e-02)	6.56e-01 (9.3e-02)†	9.63e-01 (3.0e-02)†	6.69e-01 (9.1e-02)†	5.60e-01 (7.0e-02)†
	hv	7.84e-01 (1.7e-02)	1.66e-01 (7.3e-02)†	2.35e-02 (2.0e-02)†	2.03e-01 (3.5e-02)†	4.79e-01 (4.3e-02)†
WFG7	eps	3.49e-01 (2.3e-02)	6.01e-01 (9.0e-02)†	9.39e-01 (1.6e-02)†	5.79e-01 (1.1e-01)†	3.64e-01 (6.2e-02)†
	hv	8.45e-01 (1.3e-02)	2.13e-01 (8.6e-02)†	4.70e-02 (1.3e-02)†	2.53e-01 (3.1e-02)†	5.48e-01 (3.1e-02)†
WFG8	eps	3.43e-01 (1.7e-02)	7.16e-01 (9.1e-02)†	1.14e+00 (2.5e-01)†	6.78e-01 (1.1e-01)†	5.45e-01 (1.0e-01)†
	hv	6.86e-01 (1.6e-02)	1.64e-01 (7.4e-02)†	0.00e+00 (1.8e-03)†	8.63e-02 (4.1e-02)†	3.99e-01 (7.0e-02)†
WFG9	eps	3.29e-01 (2.1e-02)	6.09e-01 (1.2e-01)†	9.56e-01 (6.4e-02)†	7.29e-01 (8.7e-02)†	4.14e-01 (1.2e-01)†
	hv	7.45e-01 (1.9e-02)	2.28e-01 (9.4e-02)†	1.69e-02 (2.1e-02)†	1.30e-01 (3.8e-02)†	5.00e-01 (4.0e-02)†

So far the MOEA/VAN framework was discussed by addressed similarity of individuals estimated with respect to the ideal point. Although this strategy works sufficiently well for some problems, it turns out that certain difficulties may be encountered when dealing with Pareto front geometries having high degrees of convexity, as shown in Fig. 5 for two-objective problems. In such circumstances, there may exist a bias in the distribution of solutions along the Pareto front. Also, this can adversely influence the performance of MOEA/VAN. In order to overcome this difficulty, the nadir point can be used when estimating the similarity of population members. This does not necessitate major modifications in the algorithm. After performing normalization, all components of the objective vector are subtracted from 1. The similarity measure for two individuals is calculated as shown in Eq. (11). The decision on when to use either the ideal or nadir point as a reference point is made based on the location of the current population in the objective space. If the distance from the closest individual to the nadir point is smaller from the closest individual to the ideal point, the ideal point is used as a reference point. Otherwise, the nadir point is utilized. This way, the population is expected to adapt to the geometry of the Pareto front.

The results obtained by two versions of MOEA/VAN are presented in Table 7. The one without adaptation always uses the ideal point

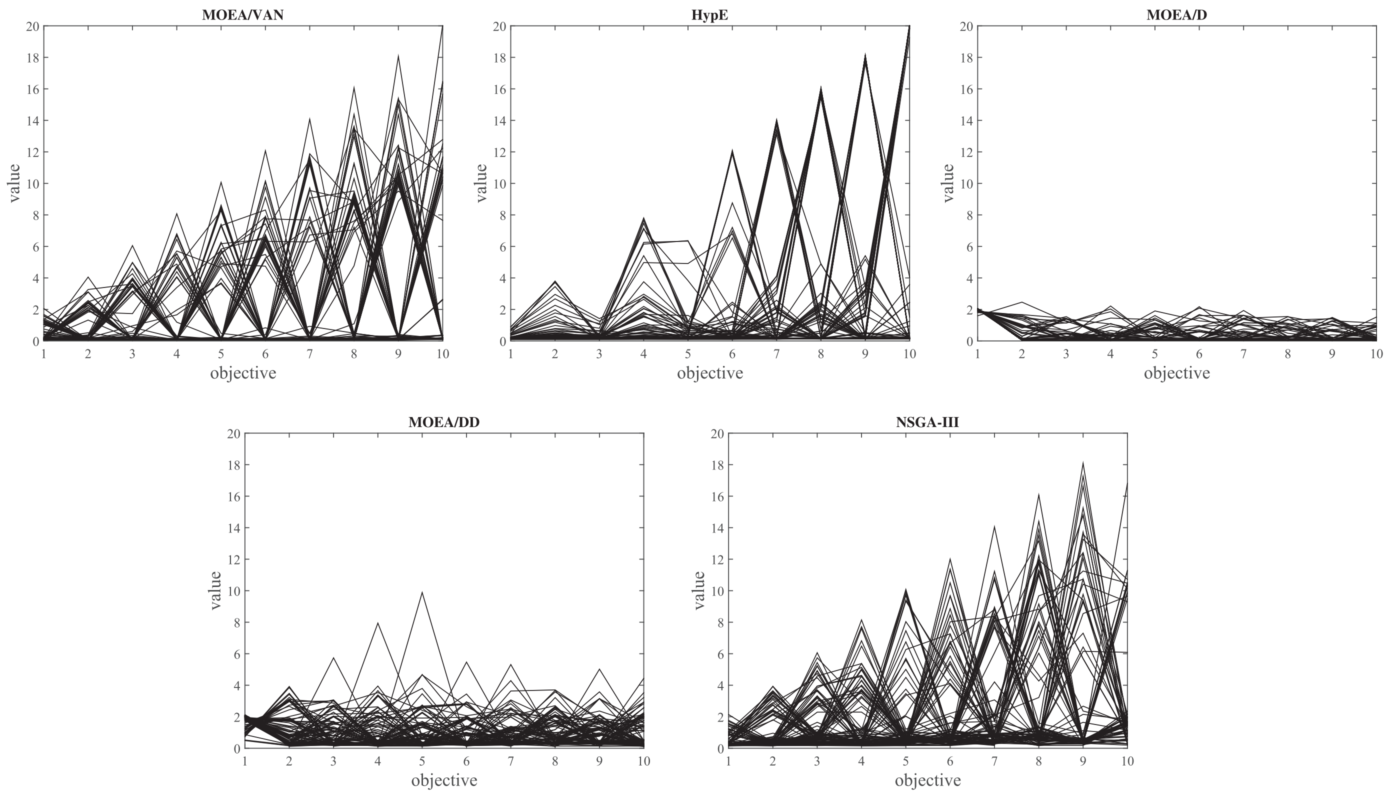

Fig. 3. Results for 10-objective WFG5 problem shown by parallel coordinates.

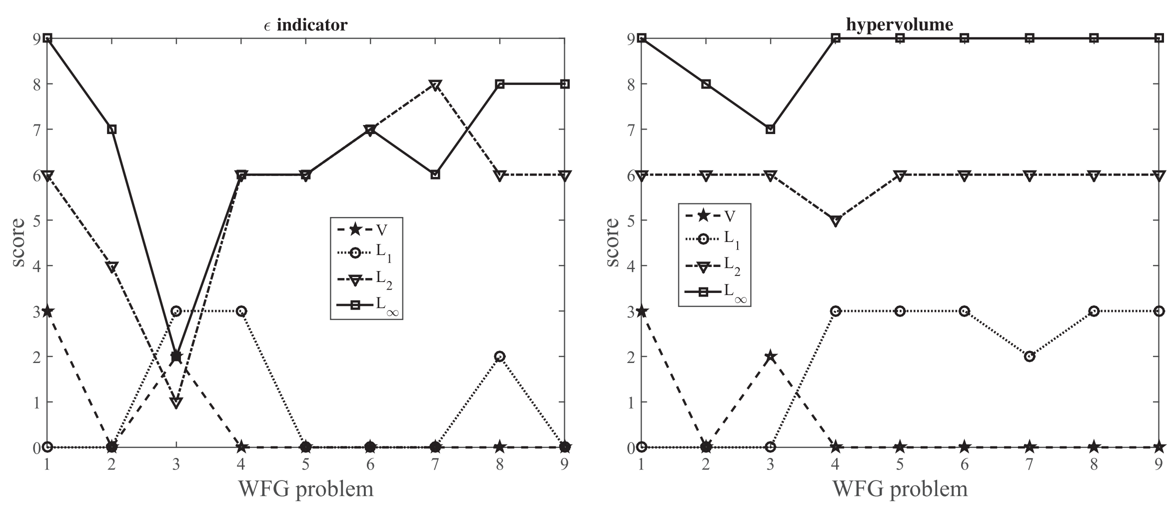

Fig. 4. Performance score for MOEA/VAN with different convergence measures.

Table 6
Results for engineering problems. The values refer to median and interquartile range of quality indicators. Best performance is highlighted with gray background. The symbol † indicates a statistical difference between the respective and best performing algorithm.

		MOEA/VAN	MOEA/D-CHB	MOEA/D-PBI	MOEA/D-WSUM
CarSideImpact	eps	3.37e-01 (9.1e-02)	3.41e-01 (8.8e-02)	9.99e-01 (2.9e-03)†	6.07e-01 (0.0e+00)†
	hv	6.98e-01 (2.5e-02)	7.01e-01 (2.9e-02)	1.22e-01 (3.5e-03)†	4.57e-01 (2.2e-02)†
Ibeam	eps	3.03e-02 (9.0e-03)	9.79e-02 (4.0e-03)†	1.41e-01 (1.4e-02)†	1.02e-01 (2.0e-03)†
	hv	1.15e+00 (7.1e-03)	1.10e+00 (4.2e-03)†	1.05e+00 (1.4e-02)†	1.09e+00 (2.1e-03)†
WeldedBeam	eps	2.44e-02 (2.8e-02)	7.42e-02 (1.5e-02)†	7.76e-02 (1.5e-02)†	7.12e-02 (1.6e-02)†
	hv	1.18e+00 (3.4e-02)	1.13e+00 (1.6e-02)†	1.12e+00 (1.5e-02)†	1.13e+00 (1.7e-02)†

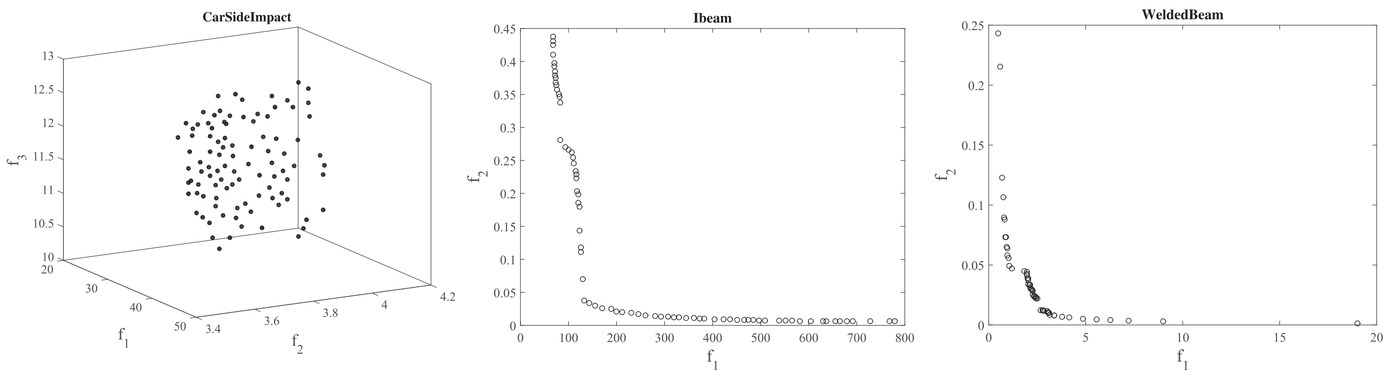

Fig. 5. Pareto front approximations for engineering problems.

Table 7
Results for engineering problems obtained by MOEA/VAN variants. The values refer to median and interquartile range of quality indicators. Best performance is highlighted with gray background.

		without adaptation	with adaptation
CarSideImpact	eps	3.37e-01 (9.1e-02)	1.91e-01 (5.1e-02)
	hv	6.98e-01 (2.5e-02)	7.43e-01 (1.7e-02)
Ibeam	eps	3.03e-02 (9.0e-03)	4.23e-02 (8.9e-03)
	hv	1.15e+00 (7.1e-03)	1.16e+00 (4.3e-03)
WeldedBeam	eps	2.44e-02 (2.8e-02)	2.15e-02 (4.1e-03)
	hv	1.18e+00 (3.4e-02)	1.19e+00 (5.7e-03)

when calculating the similarity measure. The other with adaptation choses the reference point based on the above explained strategy. The results indicate that adaptively selecting either the ideal or nadir objective vector can provide better results. Also, MOEA/VAN with adaptation becomes able to outperform all variants of MOEA/D presented in Table 6. Thus, the obtained results suggest that the proposed MOEA/VAN is able to effectively deal with constrained and real-world problems, as well as the adaptation of the selection strategy can further improve the algorithm performance.

5. Conclusions

Selection is an important feature in the design of MOEAs. Effective mechanisms to address both convergence and diversity must be embedded to ensure its proper functionality. Though, this might be uneasy task. This paper presented a new selection scheme for evolutionary multiobjective optimization. A distinct feature of the proposed approach is the way in which the population diversity is ensured. A key role plays a similarity measure that is based on angles between population members in the objective space. Different mechanisms for providing convergence are considered. By the working principle, the proposed MOEA/VAN is close to the category of decomposition-based MOEAs. Its major distinction is that no weight or directional vectors are required, as the objective space is implicitly decomposed by the interactions of individuals in the population.

The competitiveness of the proposed approach was evaluated by computational experiments with state-of-the-art algorithms on problems with different characteristics. The set of biobjective problems was suggested with the aim to assess the ability of different selection strategies to balance the population convergence and diversity on problems with a significant bias in the search space. The obtained results reveal a highly competitive performance of MOEA/VAN. Its major strength is the ability to keep the population diversity. It is also observed that the proposed selection provides a sufficient selection pressure and is able to effectively direct the search in high-dimensional objective spaces. The results confirm the ability of MOEA/VAN to deal with constrained real-world problems. When comparing with other decomposition-based MOEAs, an attractive feature of the proposed selection is that it does not require a set of weight vectors, thereby reducing user's burden.

As future work, it would be interesting to investigate the MOEA/VAN framework combined with other evolutionary search strategies. For instance, an intrinsic capability to maintain a diverse set of solutions may be useful for estimation of distribution algorithms, whose extensions to multiobjective optimization are quite limited. Also, the performance of MOEA/VAN can be further improved by developing a more elaborate mating selection procedure that does not only exploit the neighborhood relations but also favors fitter individuals. Further, the adaptation of control parameters is another promising research direction.

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