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The Inverse Relation Between the Size and the Number of Parts

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Abstract

This study analyzes children's understanding of the inverse relationship between size and number of parts when fractions and division situations are involved. A survey by questionnaire was conducted with 42 Portuguese fourth-graders trying to address two questions: 1) How do children understand the inverse relation between size and number of parts in partitive and quotitive division situations? And 2) How do children understand the inverse relation when fractions are involved in part-whole and quotient interpretations? Results suggest that these distinct situations have different impacts on children's understanding of the inverse relations between the size and the number of parts.

Key words: Inverse relation, fractions, division.

Framework

This study investigates the understanding of the inverse relationship between size and number of parts in division situations and when fractions are presented to children using the part-whole and quotient interpretations. To explore this understanding, children's performance is analysed as well as their written justifications when solving the tasks.

Understanding the inverse relation between quantities

Considering the mathematical contexts approaching the inverse relationship between quantities, fractions and division situations are emphasized. Literature presents several studies focused on the students' understanding of the inverse relationship between quantities. Some are focused on the concept of division (Correa, Nunes & Bryant, 1998; Mamede & Silva, 2012), others focused on the concept of fraction (Behr, Wachsmuth, Post & Lesh, 1984; Kornilaki & Nunes, 2005; Mamede, Nunes & Bryan, 2005; Mamede & Cardoso, 2010).

In order to understand the aspects involved in division situations, it is important to distinguish the difference between partitive division and quotitive division. In partitive division, the quantity is divided between the number of recipients, and the part received by each recipient is the unknown part (e.g., John has 8 sweets to be shared between 4 children. How many sweets each child will receive?). In quotitive division, a quantity is divided and what each recipient will receive is already known; what is left to know is the number of recipients (e.g., Mary has 6 sweets and will give 2 sweets to each child. How many children will receive sweets?). Exploring the inverse relation between divisor and quotient in division situations, it makes sense to consider two kinds of tasks, when one of the dimensions is held constant (dividend or divisor) (Correa, Nunes & Bryant, 1998). For the first situation, the divisor can be held constant and the dividend can be changed. In this case, children must understand that the bigger the whole, the bigger the parts, if the number of parts is held constant. For the second situation, the dividend is held constant and the divisor is changed. The divisor corresponds to the number of recipients or the size of the quote. In either case, the inverse relationship is applied - the bigger the number of parts, the smaller the size of the part, or vice-versa.

Kornilaki and Nunes (2005) argue that children understand more easily partitive division than quotitive division, because they use term-by-term correspondence as the procedure to solve this type of division, once it is more simple thinking about the inverse relationship than building each quote.

More recently, Mamede & Silva (2012) investigated children's understanding of partitive division with discrete quantities with 30 children aged 4 and 5. In individual interviews, children were asked to make judgments in tasks with inverse relationship between divisor and quotient when the dividend is the same. The tasks involved division of 12 and 24 discrete quantities by 2, 3 and 4 recipients. Results showed that children aged 4 and 5 have some idea about the division, are able to estimate the quotient when the divisor changes and the dividend is constant, and are able to justify their answers.

The inverse relation between quantities is essential to understand the concept of fraction. Research has been giving evidence that children struggle with the concept of rational number (Behr, Wachsmuth, Post & Lesh, 1984; Mamede & Cardoso, 2010). Studies focused on different interpretations of rational number suggest that these interpretations affect differently children's understanding of fractions. Some authors argue that the quotient interpretation favours the understanding of the inverse relationship between numerator and denominator of the fraction (Mamede, *et al.*, 2005). Nunes *et al.* (2004) suggest that this understanding is facilitated in quotient interpretation because numerator and denominator are variables of different natures. Nevertheless, fractions are traditionally introduced to children using the part-whole interpretation of fractions. In quotient interpretation, $\frac{a}{b}$ can represent the relation between the number of recipients and items to be shared (e.g., $\frac{2}{3}$ can represent 2 chocolates to be shared fairly by 3 children), but it also can represent the quantity of an item received by each recipient (e.g., $\frac{2}{3}$ corresponds to the quantity of chocolate received by each child). In part-whole interpretation, $\frac{a}{b}$ represents the relation between the number of equal parts into which the whole is divided and the number of these parts to be taken (e.g., $\frac{2}{3}$ of a chocolate bar means that this was divided into 3 equal parts and 2 of them were considered).

Mamede, *et al.* (2005) investigated whether the quotient and part-whole interpretation of fraction influence the children's performance in problem solving tasks. Eighty children, aged between 6 and 7-years-old, who have not had formal instruction on fractions, but some of them were already familiar with the words "half" and "fourths" in social contexts, participated in the study. The authors analysed how children understand fractions when using part-whole and quotient interpretations, in tasks related to reasoning of fractions - equivalence and ordering of fractions, and labeling of fractions. Results indicated that children performed better in quotient interpretation than in part-whole regarding ordering and equivalence of fractions; children performed similarly when solving labeling tasks presented in quotient and in part-whole interpretations. Children's success levels in ordering and equivalence of fractions in quotient interpretation suggests that they have some informal knowledge about the logic of fractions, developed in their daily life, without school instruction. These results emphasize the idea that different interpretations of fractions create distinct opportunities for children to understand the inverse relation between quantities.

As children possess an informal knowledge that allows them to understand the inverse relation between quantities in division situations and understand the logical invariants (ordering and equivalence) of fractions it particular situations, it is important to explore children's ideas about these issues and how these aspects are related. For that it becomes relevant to analyse children's performance but also their justifications, either oral or written, when solving problems.

An insight on children's reasoning through justifications

Primary school children must be able to communicate their ideas and to interpret and understand others ideas, organizing and clarifying their mathematical thinking. When children are encouraged to talk, write, read and listen they learn to communicate mathematically. Thus, in mathematics classes they should be challenged to discuss ideas, processes and solutions. Through oral discussion in class, children have the opportunity to compare their strategies to solve problems and identify the arguments produced by their colleagues. Through written texts and explanations, children have the opportunity to clarify and elaborate in greater depth their strategies and their arguments, developing their recognition of the importance of rigor in the use of mathematical language.

The National Council of Teachers of Mathematics (2000) points out that teachers should encourage students' thinking through the tasks they provide and the issues they raise in the mathematics classroom. The question "why?", as well as requests for explanations, should be presented regularly and consistently after students' presentation of solutions. To listen carefully their ideas and to ask them to give justifications and explanations for their solutions using written communication are powerful ways of stimulating mathematics communication.

Portuguese official curricular guidance (see DGIDC, 2007) point out that the promotion of written communication, in particular concerning the explanations and reports related to the tasks students develop should be part of the classroom practice. Written communication allow children to reflect on the developed work as it demands a review of their procedures, think about how to organize their reasoning, and how to present it in a clear way (Fonseca, 2000). To be able to create and understand mathematics, it is important for children to have the opportunity to write in their own words and using their own symbols (Fonseca, 2000). Children should be encouraged to communicate by means of graphs, tables, diagrams and drawings, but also to present arguments and explain their mathematical ideas. Thus, children's justifications can be seen as a way to have an insight about their reasoning, becoming a powerful tool when exploring children's mathematical ideas.

This study aims to analyse how the inverse relation between size and number of parts in division situations is related to the concept of fraction in quotient and part-whole interpretations. Two questions were addressed: (1) How do children understand the inverse relation between size and number of parts in partitive and quotitive division situations? (2) How do children understand the inverse relation when fractions are involved in part-whole and quotient interpretations?

Methods

Participants

To assess the children's understanding of the inverse relation between quantities in division and fraction situations, a survey by questionnaire was carried out with 42 fourth-graders of 9- to 10-year-olds (mean age 9 years, 6 months), from a public school in Braga, Portugal.

The survey took place in January and in the same day for all students. By this time of the academic year, all the children already contacted either with division or fractions. According to the Portuguese curriculum, children are formally introduced to division and to fractions in the 3rd grade.

According to the teachers' information, these children were all introduced to fractions using the part-whole interpretation of fractions; the quotient interpretation of fractions was an unexplored type of problems in their practices, and in some cases, unknown by the teachers in spite of being referred in the official guidances.

Tasks

The questionnaire included 22 tasks: 6 division problems (3 partitive division problems and 3 quotitive division problems); 16 problems with fractions (8 problems with part-whole interpretation (4 problems of ordering; 4 problems of equivalence); 8 problems with quotient interpretation (4 problems of ordering; 4 problems of equivalence)).

The division problems involved only whole numbers, all of them less than *16*. All fractions involved in the tasks were less than 1 and were the same for the problems presented to the children using the quotient and part-whole interpretations of fractions.

The tasks used were adapted from the studies of xxx (2005) and Spinillo and Lautert (2011).

Tables 1 and 2 show examples of problems presented for each type of division and fraction situation, respectively.

Table 1
Examples of problems presented in division situations

Division	Problem
Partitive	Mary and Louise have the same quantity of sweets. Mary will distribute her sweets by 3 children and Louise will distribute hers by 4 children. Will the children at Mary's group receive more sweets than, less sweets than, or the same quantity of sweets as the children at Louise's group? Explain your answer.
Quotitive	John and Paul bought the same quantity de marbles. John will put 3 marbles in each bag and Paul will put 6 marbles in each bag. Will John need more bags than, less bags than, or the same quantity of bags as Paul? Explain your answer.

Table 2
Examples of problems presented with fractions.

Fraction	Equivalence	Ordering
Part-whole	Marco and Lara have each a pizza with the same size. Marco divided his pizza into 5 equal parts and ate one part. Lara divided her pizza into 10 equal parts and ate 2 parts. Did Marco eat more pizza than, less pizza than, or the same quantity of pizza as Lara? Explain why.	Ana and Rita have each a chocolate bar with the same size. Ana ate $\frac{1}{2}$ of her chocolate bar and Rita ate $\frac{1}{3}$ of her chocolate bar. Did Ana eat more chocolate than, less chocolate than, or the same quantity of chocolate as Rita? Explain why.
Quotient	Children share two same-sized cakes. Two girls share one cake fairly; three boys share the other cake fairly. Does each girl eat more cake than, less cake than, or the same quantity of cake as each boy? Explain why.	Two girls will share a chocolate bar and each one will eat $\frac{1}{2}$ of the chocolate. Three boys will share a chocolate bar and each one will eat $\frac{1}{3}$ of the chocolate. Does each girl eat more chocolate than, less chocolate than, or the same quantity of chocolate as each boy? Explain why.

Procedures

The questionnaire was solved individually and lasted for 40 minutes, being implemented in one session in the classroom. The questionnaire was implemented in the presence of the class teacher, but without any type of teachers' interference.

Each child received a booklet with one problem per sheet to be solved. In each problem, multiple-choice questions were present, and the judgment for relative value of the quotients by using relations "more than/ less than/ same quantity as" was favoured.

The questions were presented to the class and read by the researcher using PowerPoint. Each child solved each problem individually and had to indicate the right answer on the booklet. Then, they were asked to justify their answer when asked to "Explain why".

Results

Results of the children's performances when solving the proposed tasks were analysed, by assigning 1 to each right answer and 0 to each wrong answer. Table 3 presents the proportion of means for the right answers and standard deviation according to the type of problem.

Table 3
Mean and (standard deviation) of the proportion of correct responses.

Quotient		Part-whole		Division	
Ordering	Equivalence	Ordering	Equivalence	Partitive	Quotitive
.75 (.29)	.58 (.29)	.49 (.37)	.35 (.33)	.38 (.36)	.48 (.37)

Results suggest that, in quotient fractions problems children seem to have a better understanding of the inverse relation between quantities. They also suggest that quotient seems to be easy for children, even not having been explored in mathematics classes before, as these children only were introduced to fractions relying on the part-whole interpretation of fractions. Surprisingly, they also suggest that quotitive division seems to be easier than partitive division for children.

The analysis by type of problem proposed allows a better understanding of the children's performance on the tasks presented. Figures 1 and 2 present, respectively, the distribution of the ordering and equivalence fractions problems correctly solved in quotient interpretation.

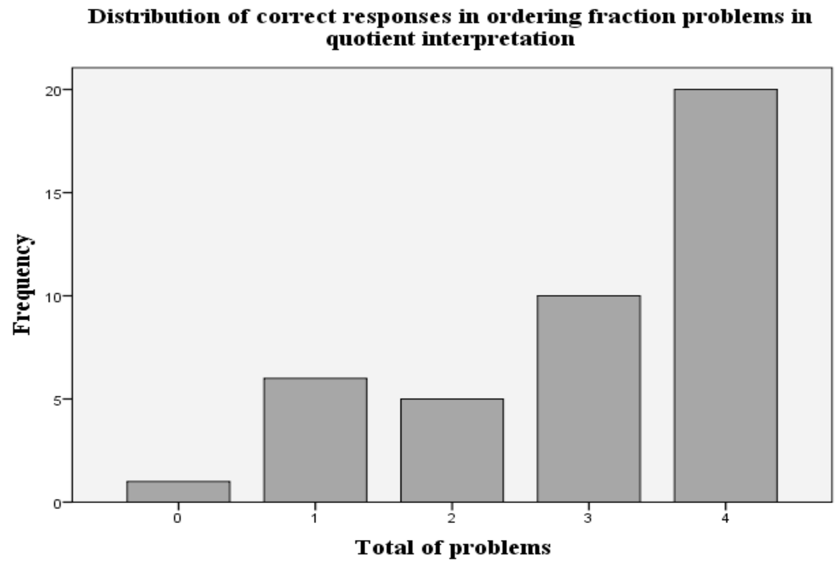


Figure 1. Correct responses in ordering fraction problems in quotient interpretation.

Ordering problems seem to be more accessible to understand the inverse relation between numerator and denominator. About 47.6% of the children answered correctly to all ordering problems and 14.3% answered correctly to all fraction equivalence problems presented to them in quotient interpretation of fractions; and 35.7% solved correctly 3 of the 4 problems of this type.

A T-test indicates that, in quotient interpretation, children's performance solving ordering problems (Prop. Mean = .75; S.E. =.04) was significantly better than their performance solving equivalence problems (Prop. Mean = .58; S. E. = .04), ($t(41)= 3.15$, $p<.05$).

The fraction problems presented in part-whole interpretation seem to be more difficult for children to understand the inverse relations between numerator and denominator. Figures 3 and 4 present the number of correct responses given by children when solving ordering and equivalence fractions problems presented in part-whole interpretation, respectively.

In ordering problems presented in part-whole interpretation, only 19% of the children answered correctly all problems and about 24% answered correctly to 3 of the 4 presented problems. In equivalence problems, about 5% of the children answered correctly to all problems and 21.4% answered correctly to 3 of the 4 problems.

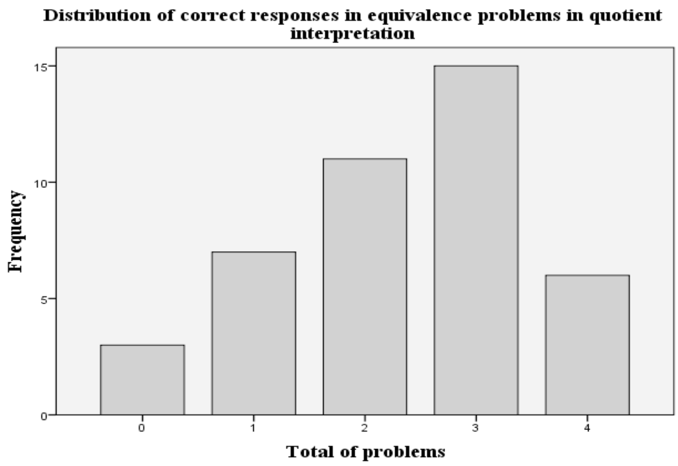


Figure 2. Correct responses in equivalence fraction problems in quotient interpretation.

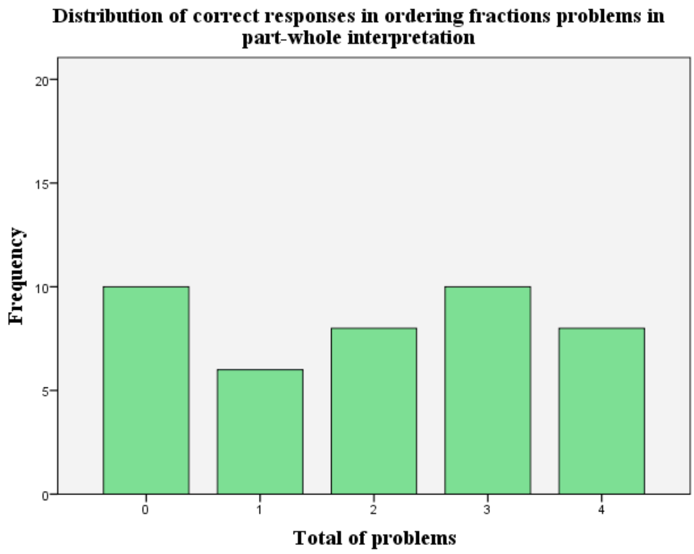


Figure 3. Correct responses in ordering fraction problems, part-whole interpretation.

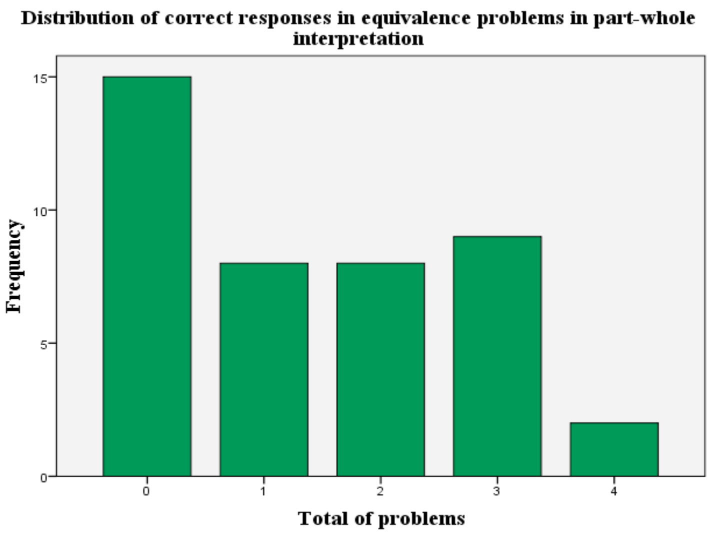


Figure 4. Correct responses in equivalence fraction problems, part-whole interpretation.

A T-test indicates that, in part-whole interpretation, children's performance solving ordering problems (Prop. Mean = .49; S.E. =.06) was significantly better than their performance solving equivalence problems (Prop. Mean = .35; S. E. = .05), ($t_{(41)}$= 2.71, p<.05).

Concerning the division, children seem to struggle with partitive division situation problems on the inverse relation between quantities. Figures 5 and 6 present, respectively, the number of correct answers in partitive and quotitive division problems. In partitive division problems, about 17% of children answered correctly to all problems; 14.3% answered 2 of the 3 problems correctly, and 35.7% answered correctly to only 1 problem. In quotitive division problems, 26.2% of the children answered correctly to all problems; 16.7% correctly answered to 2 of the 3 problems; and 33.3% gave a correct response to only 1 problem.

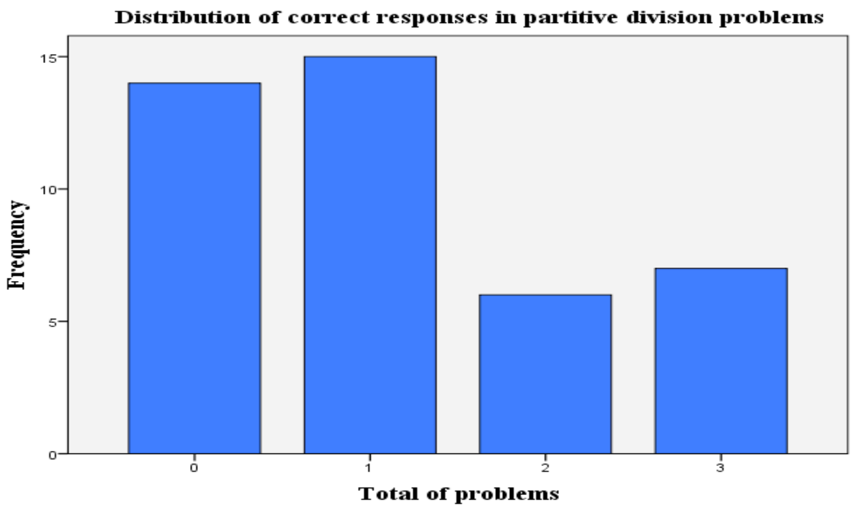


Figure 5. Correct responses in partitive division problems.

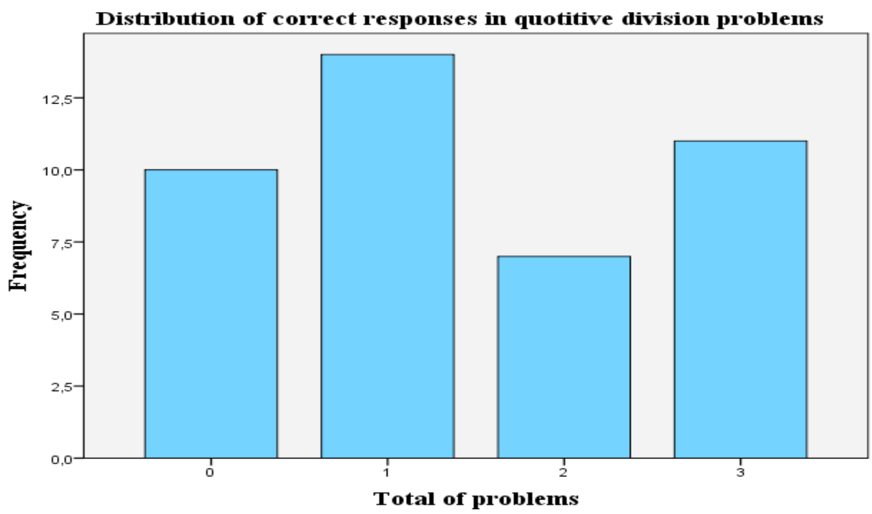


Figure 6. Correct responses in quotitive division problems.

A T-test indicates that children's performance solving partitive division situations (Prop. Mean = .38; S.E. =.06) and quotitive division situations (Prop. Mean = .48; S. E. = .06) were not significantly different, ($t_{(41)}$= 1.87, n.s.).

These results indicate that the exploration of fractions in quotient and part-whole interpretation and the partitive and quotitive divisions contribute differently for the understanding of inverse relationship between quantities.

As quotient and part-whole interpretations comprise different meanings for the values of the numerator and the denominator, it becomes relevant to explore if there is a relationship between children's understanding of quantities represented by fractions, in each type of problems presented in these interpretations. For that a correlational analysis was carried out on children's performance on problems of fractions presented in quotient and part-whole interpretations, to identify associations between the type of problems (ordering and equivalence of fractions) presented to the children. Table 4 resumes the correlations identified.

Table 4
Correlations of correct responses by type of problem presented to the children.

	Quotient interpretation		Part-whole interpretation	
	Ordering	Equivalence	Ordering	Equivalence
Quotient Ordering	1			
Quotient Equivalence	.306*	1		
Part-whole Ordering	.406**	.102	1	
Part-whole Equivalence	.282	.005	.509**	1

*.p<.05; **.p<.001; Coeficiente de correlação de Pearson.

There is an association between ordering and equivalence of fractions that are presented in quotient interpretation; ordering problems are related in both interpretations; and there is a stronger association among ordering and equivalence problems presented in part-whole interpretation, suggesting that children's understanding of these problems in part-whole in quite similar.

The written justifications of the children's answers were analysed to reach a better insight of their reasoning and their ideas about the inverse relations between quantities. While systematizing the explanations given by the children, 4 categories of their justifications were distinguished: 1) *inverse relationship* – it attends to the inverse relation between the quantities involved in the problem, producing a valid justification (e.g.,"[…] because he divided his pizza into 2 equal parts and she divided hers into 4 equal parts and hers become smaller."); 2) *proportional reasoning* – it comprises an establishment of a proportional relation between the quantities of the problem, producing a valid argument (e.g., "They eat the same because there are 2 girls for 1 chocolate bar and the boys are the double of girls and they have the double of chocolate bars."); 3) *direct relationship* – it sets a direct relation between the quantities (e.g., "He eats more because he has more cake, thus he eats more cake."); and 4) *inconclusive/ invalid* – it comprises all inconclusive, inappropriate, or blank explanations.

Table 5 summarizes the percentages of each type of argument used by the children according to the type of problem presented to them.

Table 5.
Percentage of types of justifications presented by the children when solving the problems.

	Quotient (%)		Part-whole (%)		Division (%)	
	Ordering	Equiv.	Ordering	Equiv.	Partitive	Quotitive
Inverse relationship	88.0	53.6	33.3	28.6	26.2	42.8
Proportional reasoning	2.4	19.0	4.8	11.9	0.0	4.8
Direct relationship	7.2	21.4	45.2	51.2	57.1	38.1
Inconclusive	2.4	6.0	16.7	8.3	16.7	14.3

When solving the ordering problems in quotient interpretation of fractions, the children presented more than 90% of valid arguments based in the inverse relationship between the quantities or on proportional reasoning. This was also observed by more than 70% of the children's arguments when solving the equivalence problems in quotient interpretation. This result was a surprise as these children were not used to explore the quotient interpretation of fractions in their mathematics classes, as referred by their teachers. In quotitive division problems, almost 48% of the children justifications were also considered valid arguments, based mostly on the inverse relation between the quantities involved in the problem.

Thus, the success levels regarding the children's performances for the problems presented were not obtained randomly, since they seem to be followed by explanations supported by valid arguments. Figure 7 illustrates an example of valid justifications presented when solving an equivalence fraction problem, in quotient interpretation. In the problem, a group of three girls are going to share fairly a cake, and there is nothing left; and the group of six boys are going to share fairly among them 2 cakes, and there is nothing left. The cakes all equal. The child was asked to answer who would eat more, each boy or each girl, or would they eat the same amount of cake. Then, they were required to give a written justification of their answer.

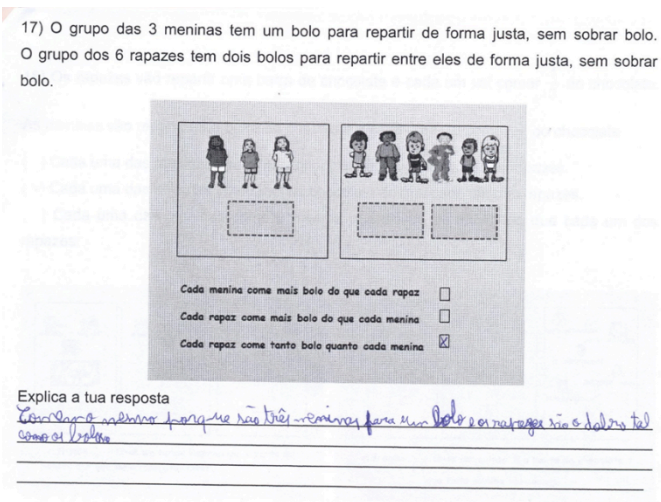


Figure 7. A child resolution when comparing $\frac{1}{3}$ and $\frac{2}{6}$ in quotient interpretation.

The child justification refers that "They eat the same because there are three girls for one cake and the boys are in double and the cakes too." suggesting some kind of proportional reasoning supporting the given answer.

Figure 8 illustrates an example of a valid justification presented by a child when solving an equivalence of fractions problem, in part-whole interpretations, comparing the fractions $\frac{1}{4}$ and $\frac{2}{8}$. In this problem it was explained that Marco and Rita have each a chocolate bar. The bars are equal. Marco divided his into four equal pieces and ate one piece; Rita divided hers into eight equal parts and ate 2 pieces. Then the child was asked if Marco ate more, less or the same amount of chocolate of Rita's.

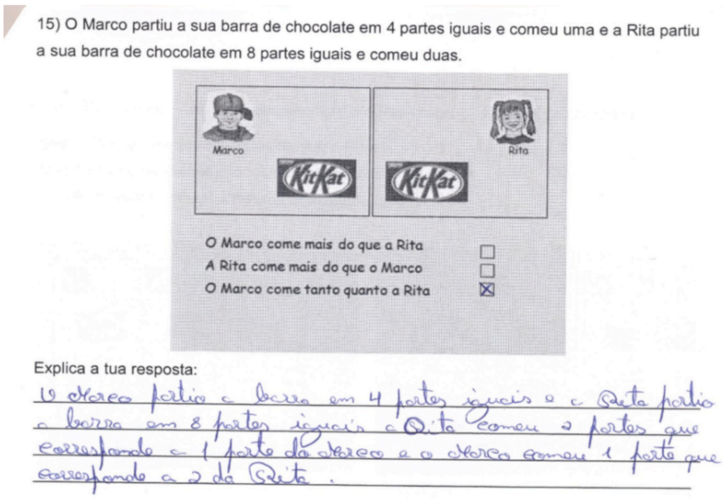


Figure 8. A child resolution comparing $\frac{1}{4}$ and $\frac{2}{8}$ in part-whole interpretation.

In the justification, the child explains that "Marco divides his bar into 4 equal parts and Rita divides hers into 8 equal parts. Rita eats 2 parts that correspond to 1 of Marco's; and Marco eats 1 part that corresponds to 2 of Rita's.", indicating an inverse quantities reasoning sustained in the correspondence established among the number of pieces and their sizes.

Figures 9 and 10 illustrate, respectively, examples of children's answers to the partitive and quotitive division problems, presenting valid justifications.

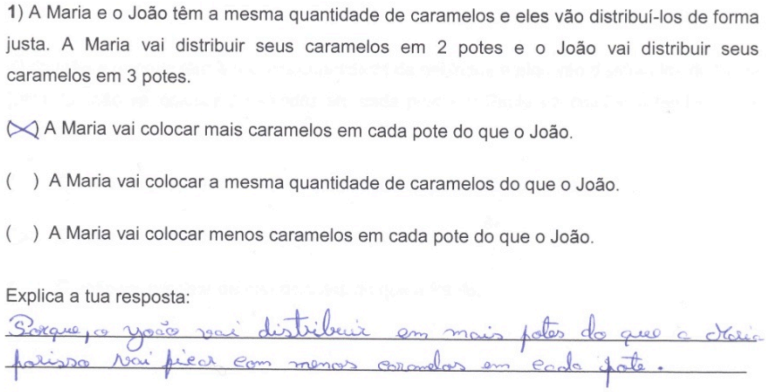
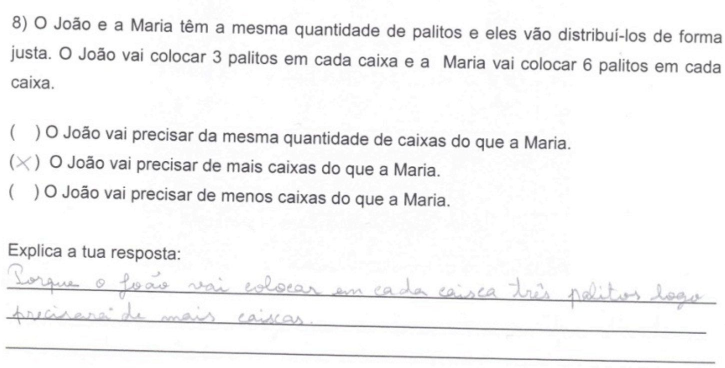


Figure 9. A justification presented in partitive division problem.

In the problem of Figure 9 the children were told that Maria and John have the same amount of candies and they are going to share them fairly. Maria is going to share hers among 2 bags and John is going to share his candies among 3 bags. Is Maria putting more candies in each bag than John, the same amount or fewer candies? The children were also asked to explain their answer. This child solved the problem correctly and justified arguing that "Because, John is going to share among more bags than Mary thus is going to have fewer candies in each bag.". This type of explanation reveals an understanding of the inverse relation between the number of recipients and the size of the shares.

Figure 10 presents a child resolution to a quotitive division problem in which the children were told that John and Maria have the same amount of wood sticks, and they are going to share them fairly. John is going to put 3 wood sticks in each of his boxes and Maria is going to put 6 wood sticks in each of hers. The children were asked if they think that John is going to need the same amount of boxes as Maria, more or fewer boxes. Then, they were challenged to explain their answer.



8) O João e a Maria têm a mesma quantidade de palitos e eles vão distribuí-los de forma justa. O João vai colocar 3 palitos em cada caixa e a Maria vai colocar 6 palitos em cada caixa.

( ) O João vai precisar da mesma quantidade de caixas do que a Maria.
(×) O João vai precisar de mais caixas do que a Maria.
( ) O João vai precisar de menos caixas do que a Maria.

Explica a tua resposta:
Porque o João vai colocar em cada caisca três palitos logo precisará de mais caiscas.

Figure 10. A justification presented in quotitive division problem.

In this resolution, this child presented a correct response and explained that "Because, John is going to put three wood sticks thus he will need more boxes" giving evidence that the reasoning established was supported by the inverse relation between the number of recipients and the size of the shares, understanding this inverse relationship between size and number of parts.

Final remarks

This study suggests that children understand better the inverse relation between quantities in quotitive division situations than in partitive division ones. It is an interesting result, because it was not reported in previous studies (see Correa, Nunes & Bryant, 1998; Kornilaki & Nunes, 2005). Possibly this is due to the existence of an unknown quantity involved in the problem. Kornilaki and Nunes (2005) suggest that 5- to 7-year-olds children have some ideas on the inverse relationship between divisor and quotient in partitive division tasks, when asked to judge the relative size of the shared sets. The results of the present study suggest that 8- to 10-year-olds children also understand the inverse relationship between quantities, but quotitive division seems to be easier for them. This idea is supported by children's written justifications which indicate that a correct reasoning was established when solving the tasks. Surprisingly, children understand better the inverse relation between quantities when this relation is associated with fractions than when is associated to division situations.

To understand the inverse relation between quantities when fractions are involved in quotient interpretation is easier than when the part-whole interpretation is involved. Again, children's explanations support the idea of their understanding of this relation. Consistent with previous studies (see xxxx, 2005), quotient interpretation still reveals to be important for the children's understanding of inverse relation between quantities, regardless the children's age differences. This study involved children that did not explore the quotient interpretation of fractions in their mathematics classes. Nevertheless, these children were able to rely on their informal knowledge and establish a correct reasoning to solve either ordering or equivalence fractions problems.

More research is needed regarding these issues in order to know more about children's understanding of the inverse relation between size and number of parts when fractions and division situations are involved. This is an important relation in the children's development of number sense, as it is essential for the learning of rational numbers.

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