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Symmetry in portuguese fishing communities: students critical sense while solving symmetry tasks

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Abstract: This article analyzes the knowledge and the critical thinking level of students from two schools of different cultural contexts, regarding the mathematical topic of symmetries. The primary goal is to find and compare the critical thinking of students from a fishing community with students of a school of an urban type, when faced with tasks on symmetries involving artefacts / geometric motifs related to fishing activity. With a theoretical foundation based on Ethnomathematics, it gives prominence to the cultural and professional context students are from, so tasks were built taking into account the fishing context. Students revealed difficulties in symmetries identification and particularly in the representation of figures with symmetry. It also found that in tasks completion, students of the fishing context school proved to have a more accurate critical sense than students from the urban type school.

Résumée : Cet article analyse les connaissances et le niveau de la pensée critique des élèves de deux écoles de différents contextes culturels, concernant le sujet mathématique de symétries. L'objectif principal est de trouver et de comparer la pensée critique des élèves d'une communauté de pêche avec des étudiants d'une école de type urbain, lorsqu'ils sont confrontés à des tâches sur les symétries, comprenant des artefacts / motifs géométriques liés à l'activité de pêche. Avec une base théorique basée sur Ethnomathématique, il met en évidence le contexte culturel et professionnel d'ou les étudiants viennent, alors les tâches ont été construites en tenant compte du contexte de la pêche. Les étudiants ont révélé des difficultés dans l'identification des symétries et en particulier dans la représentation des figures avec symétrie. On a également constaté que dans l'achèvement des tâches, les élèves de l'école de contexte de pêche ont montré d'avoir un sens critique plus précis que les élèves de l'école de type urbain.

Objectives and main idea

From the most basic act of drawing a line segment in carpentry, to the more complex geometric reasoning in tracing a large bridge, there is geometry knowledge being applied. Research in the context of professionals with low educational level, demonstrate a strong presence of mathematics and the use of geometry in their everyday professional activities (Sousa, 2006; Lucena, 2002; 2005). In this work we will take into account geometric motifs used in two fishing communities routines, with special attention to symmetries. The focus on symmetry relates to the fact that there are investigations on this subject in professional groups (Sousa, 2006; Vieira, Palhares and Sarmento, 2008), but also because curriculum documents recommend work with isometries and symmetries in elementary schools. Still, students reveal many difficulties when faced with situations involving symmetries. Being this work grounded on ethnomathematics, one of the central concerns (besides investigating fishing context situations revealing the application of symmetry), is to assess students critical sense in performing tasks on symmetries, in two communities, one belonging to a fishing community the other not.

In this research work, we highlight the symmetries of reflection, rotation and translation. These are the ones identified, as a result of previous fieldwork, as used in daily life of fishing communities.

Ethnomathematics

Ethnomathematics is the theoretical foundation that frames this work both from a cultural point of view and from mathematics education. Etymologically, the word Ethnomathematics can be understood as the art or technique (techne = tica) to explain or to understand reality (matheme) within a specific cultural context (ethno) (D'Ambrosio, 2012). Ethnomathematics, rather than an association to ethnic groups (D'Ambrosio, 1998), is the research of mathematical practices and conceptions of a social group, including also an educational work (Oliveira, 2004; Miller, 2004) which develops in order to identify and decode the group's knowledge and draw comparisons between knowledge of everyday life and academic knowledge (Knijnik, 2008).

It is multicultural societies like ours that educators should reflect on which culture should be considered in the classroom. The dominant? Of the minority? Or maybe we can create a new culture, made up of all the cultures of all citizens living in a given territory. Knowledge and culture are the two foundations of mathematics. So we can assume that in a given area where there are many cultures, we are not facing the Ethnomathematics of these cultures, but in the presence of several (ethno) mathematics (Sousa, 2006) developed over years by cultures in that territory. From the epistemological relativism and demarcation of cultures as interpretation systems of the world, maybe we can talk about ethnomathematics or multimathematics (Oliveras, 2006).

In this paper we give special emphasis to the Ethnomathematics within the context of fishing cultures and also within classroom contexts. We look for a didactic transposition, so that Ethnomathematics, seen as the math arising from social groups (D'Ambrosio, 2006), help in this contextualization process, but also in humanizing mathematics (Palhares, 2012). Thus, the position assumed is taken from D'Ambrosio (1993; 1998; 2002); Olive (2004); Monteiro (2004); and Knijnik (2008), who evoke an investigative nature on the mathematics present in social minorities and an educational character, aiming to combine the mathematical knowledge of everyday life with the formal / school mathematical knowledge. The investigative nature of this work concerns the collection of everyday (ethno) mathematical aspect within fishing communities. The educational refers to the context and implementation of these (ethno) mathematics in the classroom, after analysis and cultural and curricular consideration.

Methodology

Qualitative research is our methodological framework because it is based on a holistic view of the particular setting to be researched without isolating it from its natural context (Amado, 2013).

One of the specific methods used was the multiple-cases study in ethnographic context. With it was possible to observe in detail individuals in each of the specific contexts (Lessard-Herbert, Goyette & bouti, 1994).
We also used participant observation, unstructured interviews (with the help of audio and video recordings) and document analysis, mainly documents and small teaching experiences in the classroom context.

It is therefore intended, in accordance with the theme "symmetry", to answer the question:

- What level of critical sense do students of the Caxinas fishing community reveal in comparison with the most urban contexts of other students?

We prepared a set of tasks, taking into account the curricular documents, as well as age and cultural background of the students. Its focus is on symmetry, while the context addresses the fishing everyday. We tried to draw up various tasks on this content, asking students different levels of knowledge regarding the symmetry.

The proposed tasks are contextualized tasks arising from the fieldwork carried out in fishing communities of Câmara de Lobos (Madeira) and Caxinas (Vila do Conde).

The cultural context involves boat construction, objects used in daily fishing and tiles used in the homes of fishermen. The mathematical content is symmetries.

Tasks that were used, adopting the knowledge that fishermen use in their work, were validated with regard to the suitability to the specific context of these fishing communities, but also validated by a panel of mathematics education experts.

Each task was applied in the classroom, in two separate phases. Initially, the tasks were applied before the approach to the subject related to symmetry. After teaching symmetry, the same tasks were reapplied to the same students. The tasks were applied in 2 groups of 5 and 6 years of schooling (10-12 years). The tasks were applied in two schools with different cultural and professional contexts: one belongs to the geographical area of a fishing community, and the other, is embedded in an urban school. The tasks were applied with 45 students of fishing context school, and 39 students from another school.

To apply this set of tasks in the classroom, three 90 minutes periods were dedicated in each class. Students individually solved each task to try to understand the concepts / knowledge that each student had about the symmetries and the critical sense of the students in relation to their own resolutions tasks. Their arguments and strategies were recorded on paper and on video.

Symmetries and critical sense

School of the fishing community of Caxinas

Critical sense about the rigour in the construction of artifacts

In some artifacts of this task (Figures 1 and 2), the overwhelming majority of students feel they have symmetry, but they question the fact that the objects were not built with mathematical rigor necessary to have symmetry. In other words, students admit that the craftsman who built each artifact intended to build with the symmetry and therefore consider themselves to have symmetry, but mathematically think they should be more "perfect". From the close observation of the represented figures (Figure 1 and Figure 2) and the symmetry axis represented by the students, the objects in all the rigor do not have symmetry of reflection due to imperfections in artisanal construction. Students turn out to be critical of this and despite considering the objects have symmetry (because the craftsman have that intention at the time of manufacture), make the repair that the final product should have more harmony. In this situation the combination of knowledge of everyday life and the formal school knowledge is evident, but also the critical sense with regard to the construction of the artifacts and the presence of symmetry in them.

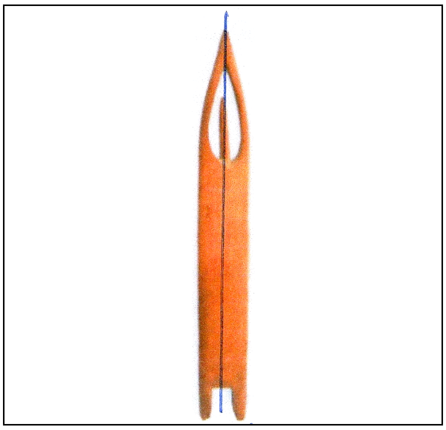

Figure 1. fishing nets sewing needle

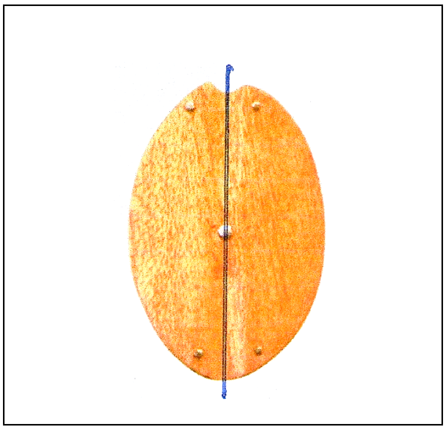

Figure 2. Davit

Critical sense about errors

In this task it is intended that students complete in Figure 3 in accordance with the represented part, obtaining a helix with rotational symmetry. It is found that the students reveal it hard to correctly complete the picture.

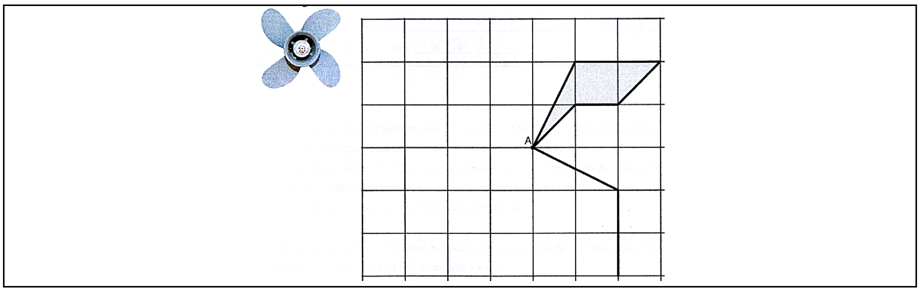

Figure 3. Propeller for students to complete

Taking as its starting point the already represented blade (1st quadrant), the overwhelming majority of students can correctly complete the representation of the 4th quadrant blade, but few can represent all blades correctly as in the representation of one of the students (Figure 4).

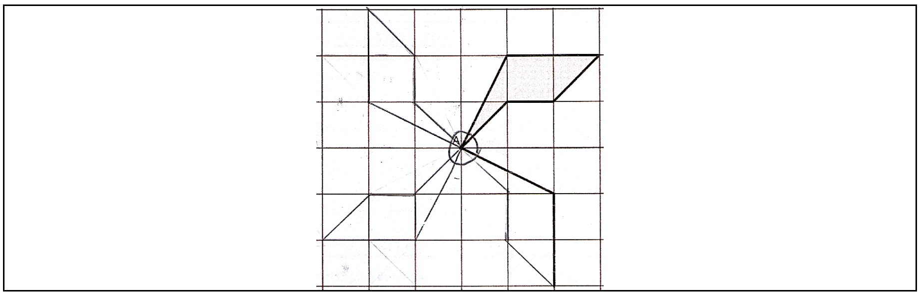

Figure 4. Correct representation of the propeller after several attempts

Although the student has managed to correctly represent the propeller, there were several unsuccessful attempts as seen in the transcript of the field work (Figure 5). One can also verify that the student recognizes that something is not correct, identifies the part of the picture that is not correct, but does not know why it is inaccurate. His keen critical sense in relation to his representations is salient, erasing immediately each blade that he considers not to be visually in harmony with the rest.

Student - Teacher, I have a doubt here. The propeller leaves, is it like this? (Pointing to the propeller blade already represented in the figure).
Student - Ahhh ... wait. (Deletes the part he feels it is wrong and represents again).
Student - I know. (Correctly representing the blade of the 4th quadrant).
Student - This (the blade of the 3rd quadrant) must be reversed.
Researcher - Reversed how?
Student - This part must be here.

Figure 5. Dialogue on propeller representation

The greatest difficulties arise in the 2nd and 3rd quarters. A considerable part of the students complete the picture according to Figure 6.

In this case the student begins to represent the blade in 2nd quadrant by reflecting the blade of the first quadrant. He watched for a few seconds the representation stating, "The blade is in reverse." The student makes several attempts, but fails to correctly represent the propeller. The final product, is given in Figure 6, however, the student said that the representation is not correct, that is, despite failing to represent the propeller with correction, the student is aware that its representation is not correct and is able to identify the parts of the figure to be changed. There is in this situation, strong evidence that the student is critical of his work because he recognizes that something is not right in the representation. The student delivers the task saying, "Teacher, this is not right." This is just one example of many students who, although can not properly represent the propeller, reveal quite critical sense and assume that the representation is not correct.

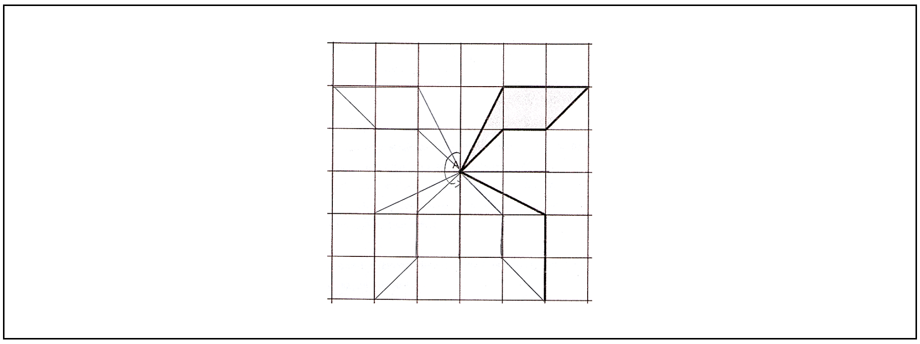

Figure 6. Incorrect propeller representation

School of Calendário

Critical sense about the rigour in the construction of artifacts

In the school of Calendário a large part of the students indicated that the artifacts shown in Figure 7 have only the vertical axis of reflection symmetry. However a considerable number of students said that the artifact to the right (Davit) has vertical and horizontal axis of reflection symmetries; others indicate 4 symmetry axes (Figure 9) and still others believe that the objects do not have symmetry (Figure 8). Only two students point out that both artifacts were not built with due rigor so that mathematically can be considered to have symmetry. The remaining relate nothing regarding this aspect. They do not reveal a critical keen sense as happens in the fishing environment school. In the city context in which the school is located, students externalize little sensitivity to analyze everyday aspects, at least from fishing everyday. They have difficulties in mobilizing fishing everyday knowledge for math classes and connect this knowledge with mathematics learning on the classroom. In the fishing context of the tasks these students are less critical than students from the fishing community of Caxinas. Although many of the students from both schools recognize that artifacts have symmetry, students of the school of Calendário tend to be more mathematically formal (not more able) which may affect the critical sense in relation to the fishing everyday situations.

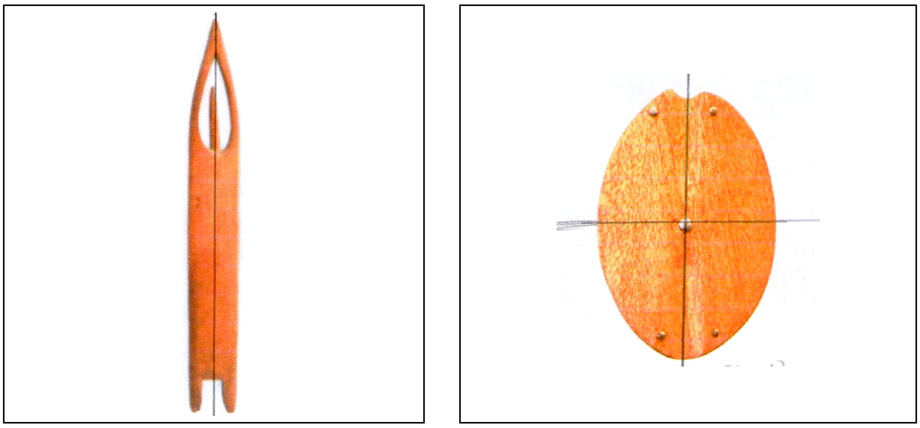

Figure 7. The student believes that the artifacts have one or two axes of symmetry.

Figure 8. Students consider the artifacts have no symmetry

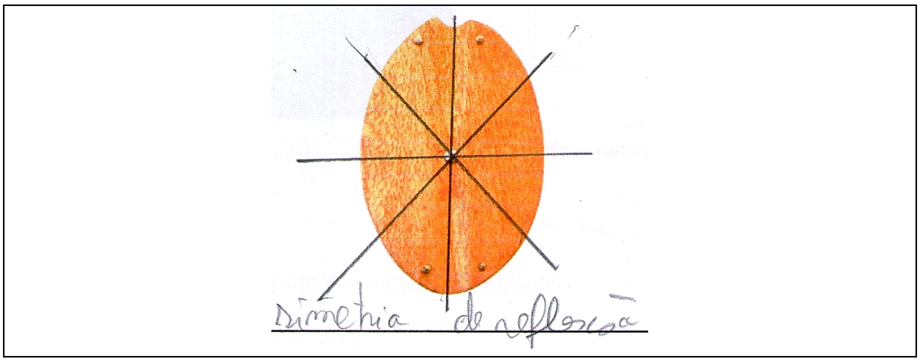

Figure 9. Students consider that the artifact has 4 axes of symmetry

4.1.1. Critical sense about errors

At the school of Calendário most students also failed to properly represent the propeller. Most students completed the propeller as shown in Figure 10. The students completed the helix so that it had vertical axis of reflection symmetry, not realizing that the symmetry involved was rotational symmetry.

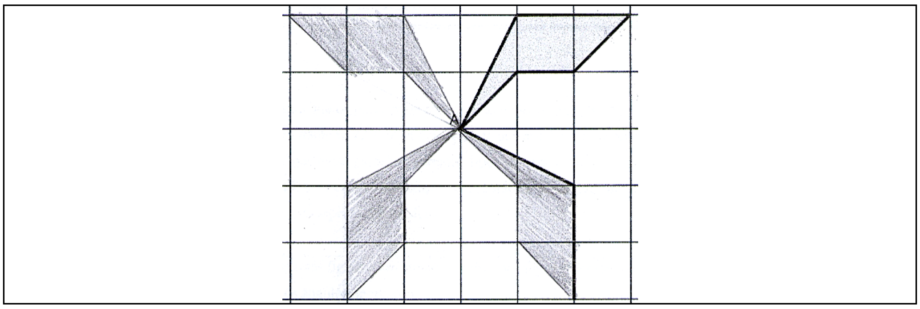

Figure 10. Incorrect representation of the propeller (with vertical axis of reflection symmetry)

In the following cases, the propellers presented do not have any symmetry, although students use isometries in their reproductions. In Figure 11 the student properly represent the blades of 3rd and 4th quadrants using rotation, but in the blade of the 2nd quadrant uses reflection. In Figure 12 and Figure 13 students use rotation on all the blades, but represent incorrectly the blades of 2nd and 3rd quarters. In Figure 14 the student tries to represent the helix so it has vertical axis of reflection symmetry, however, the blade of 4th quadrant is not correct. In addition to this failure it is not correct to build the propeller so that it has symmetry of reflection, as in reality it has only rotational symmetry.

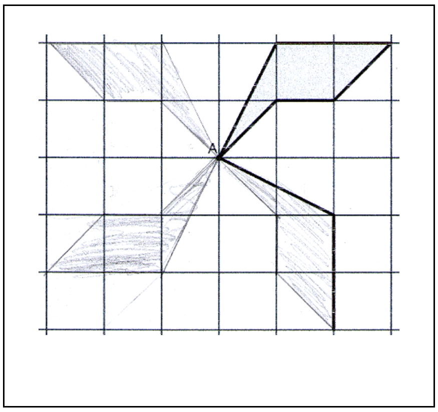

Figure 1. Incorrect representation of the helix (no symmetry)

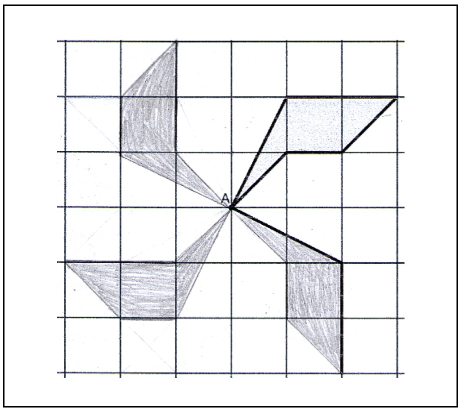

Figure 2. Incorrect representation of the helix (no symmetry)

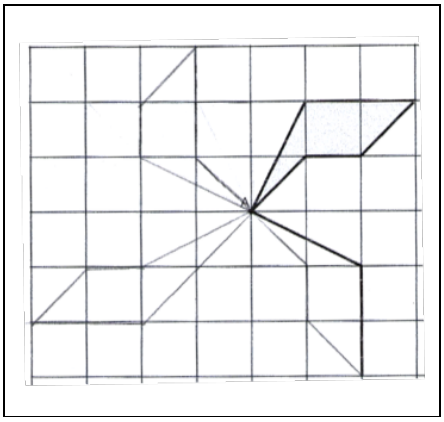

Figure 3. Incorrect representation of the helix (no symmetry)

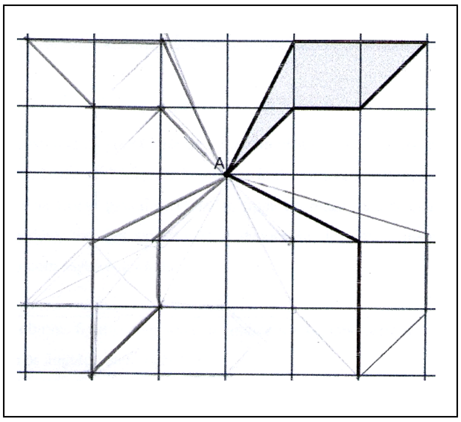

Figure 4. Incorrect representation of the helix (no symmetry)

Most students make mistakes of this kind in their representations, which reveals something unusual. More curious still is that students consider that their representations are correct as shown in the testimony of this one student (Figure 15).

Student - Teacher, I have done.
Researcher - Do you think it is right?
Student - Yes, it's right. It was a little difficult and I had to delete a few times, but I could do it right.

Figure 15. Student opinion on the representation of the figure 10

In this task there was a huge disparity in the critical sense of students. At Caxinas school students sometimes fail in their representations, but are aware that they are wrong and even warn the teacher that something is not right in their representations. In the school of Calendário students find it difficult to complete the propeller, fail in their representation, however they consider their representations correct and do not realize they are wrong. There is a number of situations in this school where the propellers represented not even have symmetry, yet the students feel they have symmetry. These facts reveal that the level of critical sense of students in these two schools are very different realities.

Conclusion

The implementation of the tasks in two schools of different cultural contexts and at different times (before and after teaching of symmetries), allow us to assess the level of critical sense that students have with regard to mistakes and to results in their resolutions. Given that the tasks were built within fishing contexts, the discrepancies in levels of critical thinking among students from both schools are well visible. Students from the fishing community of Caxinas reveal a more refined critical than students of the other school. Students of the fishing community turn out to be more able to detect and censor errors both in resolutions and in obtained results. There has also been no significant changes regarding the performance and the level of judgment when tasks are applied before or after teaching of symmetries. Thus, one can also conclude that students immersion in school/formal mathematics had no influence on their critical thinking ability in the schools involved.

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