STRUCTURAL DYNAMIC UPDATING USING A GLOBAL OPTIMIZATION METHODOLOGY

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ABSTRACT

In this article, we present the application of a global optimization technique, in particular the *GlobalSearch* command from MatLab®, in the updating of structural dynamic models. For comparative purposes, we evaluate the efficiency of the global method relatively to the local search method previously used in the *Finite Element Model Updating* program. The *Finite Element Model Updating* programs are designed with the primary purpose of validating and optimizing structural numerical models. The first step for structural optimization process is to idealize the desired behavior of the dynamic model to develop, or collect experimental data of a physical model considered as the reference model. The process begins with the construction, on a finite element program, of a numerical model with initial physical parameters, preferably close to the reference model parameters. The numerical model is then submitted, through a *Finite Element Model Updating* program, to a successive parametric updating until improving its dynamic behavior described by their natural frequencies, mode shapes and damping properties, be similar to the dynamic behavior of the reference model. The *Sequential Quadratic Programming* algorithm was already used in the optimization of the *Finite Element Model Updating* program, and the obtained solutions showed that it can't achieve the global optimal value of the objective function. This kind of methods, used for nonlinear constrained optimization problems, have, generally, difficulties to achieve the global optimum, since they are local optimization methods.

Keywords: Updating, Finite Element Model, Global Optimization, Structural Dynamic

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1. INTRODUCTION

Optimization emerged with the ambition of solving problems in logistic, engineering, chemistry and biology areas. The last decades have allowed the appearance and evolution of computational tools able to solve optimization problems.

There are optimization problems with simple linear functions of few variables, until the most complex problems of non-linear functions with many variables and many optimal local solutions. Thus, emerge the need to investigate and develop methods for global optimization. An example of application of these optimization methods in the engineering field is in structural dynamics, to optimize structural models [1] [2] [3]. In these cases, the optimization methodologies resort to the methods of improving finite element models, usually denominated by *Finite Element Model Updating,* namely in the optimization of the geometrical and material parameters of the structure [4]. These improvements can be conducted under two types of approach:

- o first, in the updating of simplified numerical models, representative of detailed physical models which present high computation time. The simplified model is submitted to updating by a *Finite Element Model Updating* methodology until obtain dynamic behavior similar to the physical model, also denominated as reference model [5]. Thus, it is possible to obtain a light computationally model and representative of the physical model. It is important to refer that, in these cases, the main interest is in the correlation of dynamic behavior, independently of the parameters values optimized;

- o second, in the structural modification to optimization of the models. Detailed numerical models of physical models are built and submitted to optimization to improve the dynamic behavior and/or achieve a model with similar behavior but with geometrical and/or physical parameters more advantageous from the design point view [6].

The optimization methodologies help to fit on the control of updating process, nevertheless still constitute a developing task. The traditional optimization methods, based on local search, have difficulties in dealing with global optimization problems, because are easily trapped in local minimal and not evolve into other areas of the feasible region. There are two types of optimization methods: the direct methods and methods based on gradients. The direct methods just use information of the objective function and constraints. The gradient-based methods use the information of the objective function and constraints, and their first and/or second derivates [7] [8]. The *Finite Element Model Updating* program, developed by [9], incorporates the *Sequential Quadratic Programming* algorithm (SQP), as a local search method to find the optimal value, using the *fmincon* command from Matlab®. In this paper, with the purpose of improving the efficiency of the *Finite Element Model Updating* program, the local search methodology is replaced by a global search technique, using the *GlobalSearch* command available in Matlab®. In this global optimization method, a number of starting points is randomly generated and then uses a local solver to find the optimal value in the basins of attraction of the starting points. This methodology is performed in two phases: a local phase and a global phase. In the local phase, the sample of points, randomly obtained, is manipulated by the local search solver to find candidates for local minimum. In the global phase, the local minimum with best objective function value is used as an approximation to the optimal global [7].

The objective of this paper is to present the global optimization technique applied to the updating process of a structural model, and compare it with the local search method previously used in the *Finite Element Model Updating* program.

The organization of this paper is as follows: Section 2 presents the problem description; Section 3 describes the application of a new optimization process; Section 4 presents the models used in the optimization process; Section 5 shows the computational tests, the obtained results and their discussion. This paper is concluded in Section 6.

2. PROBLEM DESCRIPTION

The optimization process starts with the building of a numerical model in the finite element program ANSYS®, from which are obtained its dynamic characteristics. These dynamic characteristics are transferred to the optimizer, developed in Matlab®, where an optimization method is applied.

The optimization problem consists in the minimization of an objective function *f(x)* and the aim is to find the optimal value that represents the best improvement of the numerical model. The objective function *f(x),* as defined by [9] [10], is given by

$$f(\mathbf{x}) = -\frac{\sum_{i=0}^{N} MAC_{ii}(\mathbf{x})}{\sum_{i=0}^{N} MAC_{ii}(\mathbf{x}^0)} + \left(\frac{1}{N}\right)\frac{\sum_{j=1}^{N}\sum_{\substack{i=1\\i\neq j}}^{N} MAC_{ij}(\mathbf{x})}{\sum_{j=1}^{N}\sum_{\substack{i=1\\i\neq j}}^{N} MAC_{ij}(\mathbf{x}^0)} + \frac{\sum_{\substack{i=1\\j=1}}^{N}\left(\omega_i(\mathbf{x})-\omega_j\right)^2}{\sum_{\substack{i=1\\j=1}}^{N}\left(\omega_i(\mathbf{x}^0)-\omega_j\right)^2} \tag{1}$$

where **x** is the parameters vector to vary in the optimization of the numerical model and *N* is the number of mode shapes and frequencies involved. $\mathbf{x}^0$ is the parameters vector with the initial values. The MAC (Modal Assurance Criterion) [11] for a parameters vector **x** is given by *MAC*(**x**), affected by ASMAC (Alternated Search Modal Assurance Criterion) [9] [10] values. The frequencies *ω* for a parameters vector **x** is given by *ω*(**x**). The objective function is divided in 3 terms. The first term corresponds to the sum of *N* values of the diagonal MAC matrix, affected by ASMAC values, obtained from the correlation of the mode shapes between mode shapes of the reference model and mode shapes of the numerical model. To obtain a 100% correlation the first term should be 1. The second term is the sum of all values outside the diagonal MAC matrix, affected by values of ASMAC, obtained from the correlation of the mode shapes between mode shapes of the reference model and mode shapes of numerical model. The term should be zero. The third term is the sum of the differences between the reference frequency values *ω_i* and numerical frequency values *ω_j*. The number of frequencies is equal to the number of mode shapes. To obtain a 100% correlation between frequencies, these differences should be zero. At each iteration, the objective function value is divided by initial objective function value in order to normalize each term of the function.

Here, the optimization problem is a minimization of the objective function (1) subject to simple bounds in the decision variables **x** of the problem.

3. APPLICATION OF A NEW OPTIMIZATION PROCESS

The *Finite Element Model Updating* program, implemented in MatLab®, uses now the global solver *GlobalSearch*, to optimize and find the global optimal value of the objective function *f(x)*. This global solver works also with the local solver *fmincon*. Each solution of the local solver is an optimal local value of *f(x)*. The best optimal local value is the optimal global value of *f(x)*.

The optimization process uses the interaction between Matlab® and Ansys® to execute the numerical model and calculate the objective function value, and performs the following steps, as it is build in [9] [10]:

1.   Starts the ANSYS® program with a given numerical model input file;

2.   Reads the output file of the ANSYS program and processes it in order to build the objective function and constraints used for the optimization process;

3.   Stops the calculation process if an optimal value on the updating process has been achieved, or goes to the next iteration on the updating process;

4.   Obtains the new design parameters defined by the optimization algorithm;

5.   Modifies the ANSYS® finite element model input file with the new parameters;

6.   Starts a new analysis by going to step 1 with the new input file.

In order to compare the local and global solver performance in the optimization process, first, it is used the local solver *fmincon* and then the global solver *GlobalSearch*, from Matlab®.

3.1. Local solver

The local solver command used in the optimization process is the *fmincon*. Given an initial starting point, this solver can work with four algorithms type: *active-set*, *interior-point*, *Sequential Quadratic Programming* and *trust-region-reflective*, and performs the following steps [8]:

1. Runs the starting point defined by user;

2. Creates the local optimum solution vector.

3.2. Global solver

The global solver used in the optimization process, the *GlobalSearch* solver, uses a scatter search strategy in order to generate the trial points, and rejects all of those that are unlikely to achieve the best local minimum. The solver performs the following steps [8]:

1. Runs the local solver from initial point supplied by user, and saves the *f(x)* value for an initial estimate on the radius of a basin of attraction and for posterior use in the *score* function;

2. Generates trial points (potential starting points);

3. Runs the starting points defined in the option *NumStageOnePoints* and evaluates the score function of each one;

4. Runs the best starting point with the best score and removes the remaining points defined on stage one;

5. Examines the remaining trial points and runs the local solver if point satisfies the basin of attraction, the score and constraints;

6. Creates the global optimum solution vector.

4. MODELS DESCRIPTION

A numerical model will be optimized taking into account a reference model with reference values of mode shapes and natural frequencies.

4.1. Reference model

The reference model is a steel sheet with dimensions 200x300x10 mm^3, represented respectively by width (*w*), height (*h*) and thickness (*t*), as shown in **Figure 1**, and has mass of 4.708 kg. This model is built in ANSYS$^®$ with shell elements (SHELL63), and is submitted to modal analysis for extraction of mode shapes in 24 points and natural frequencies. The mechanical properties of the steel sheet are presented in **Table 1**.
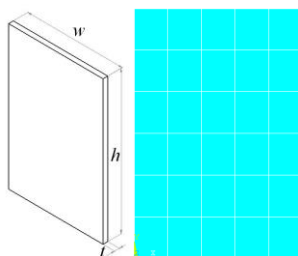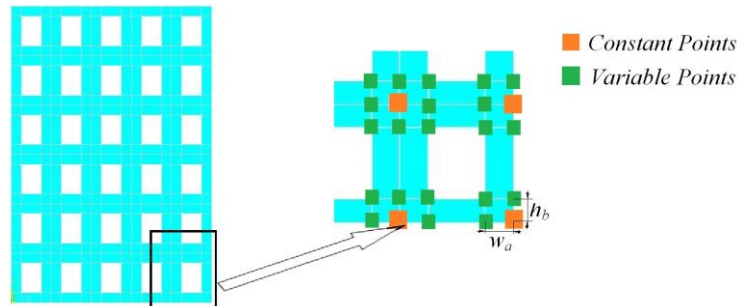

Figure 1 Reference model

Table 1 Mechanical properties of reference model

Property	Variable	Units	Value
Young's Module	E_x	Pa	2.1e^{11}
Young's Module	$E_y = E_z$	Pa	2.2e^{11}
Poisson's Ratio	$v_{xy} = v_{yz} = v_{zx}$	-	0.27
Density	ρ	kg/m^3	7847

4.2. Numerical model

The numerical model to be optimized has a set of 240 areas of variable geometry, as shown in **Figure 2**. The areas are created from points, some with variable coordinates, enabling the change of all areas of the model. The points with variable coordinates are function of the geometrical parameters: width (w_a) and height (h_b). The coordinates of the points chosen for reading the mode shapes are kept constant in order to coincide with the data of reference.

Figure 2 Initial numerical model

The numerical model built in ANSYS® with shell elements (SHELL63), has the mechanical properties presented in **Table 1**. The width (w), height (h) and thickness (t) dimensions are equal to the reference model, represented in **Figure 1**. The numerical model has initial mass of 3.296 kg, and will be subject to modifications of geometric parameters, such as thickness (t), width (w_a) and height (h_b), to be optimized in the *Finite Element Model Updating* program. The initial values of the parameters and their lower and upper bounds are indicated in **Table 2**.

Table 2 1 Parameters of the numerical model to be optimized

Property	Variable	Units	Initial Value	Lower bound	Upper bound
Thickness	t	mm	10	1	20
Width a	w_a	mm	10	10	19
Height b	h_b	mm	10	10	24

It is expected that the optimal value of width (w_a) and height (h_b) variables, have a clear tendency to converge to the upper bound, in order to fill the empty spaces of the steel sheet.

5. COMPUTATIONAL TESTS

In this section the numerical model is optimized and the obtained computational results are presented, first with the local solver *fmincon*, and then with the global solver *GlobalSearch*. The local solver *fmincon* is performed with the *active-set*, *interior-point*, *Sequential Quadratic Programming* and *trust-region-reflective* algorithms. The *GlobalSearch* solver is performed with 100 and 400 trial points, where the number of points analyzed in stage one is 100 and 400, respectively. So, the *GlobalSearch* applies the *fmincon* solver in two starting points, on supplied initial point x^0 and best starting point among the trial points of stage one.

5.1. Local solver results

In the local solver *fmincon* analysis, with results presented in **Table 3**, the search is only performed on the starting point x_0.

Table 3 Optimization results for *fmincon* solver

Output		active-set	interior-point	SQP	trust-region-reflective
Number of function evaluations		162	138	239	162
Optimization time [h]		~1.700	~1.500	~2.500	~1.700
x local [mm]	t	10.160	10.158	10.142	10.160
	w_a	14.582	14.677	15.318	14.582
	h_b	14.881	14.843	14.528	14.881
Optimal local $f(x)$ value		4.402	4.401	4.393	4.402

The best optimal value, 4.393, is achieved by *SQP* algorithm. With the other algorithms the optimal value of *f(x)* is very similar between them. The solver requires 239 function evaluations and 2.5 hours to achieve the best optimal global value of the objective function.

5.2. GlobalSearch solver results

In *GlobalSearch* solver analysis, there are, in general, improvements relatively to local solver solution. It is shown that these improvements are more evident when using 400 trial points. In the first experiment with 100 trial points, where the optimization results are presented in **Table 4**, the *SQP* algorithm obtains the best optimal value, 4.163. There is an improvement of 5.236% when compared with the optimal value obtained with the local search method (4.393), while with the other algorithms, only the *interior-point* achieves a slight improvement of 0.068% compared with the value obtained with the local solver (4.401). With the *active-set* and *trust-region-reflective* algorithms there is no evolution. The solver requires 532 function evaluations and 5.5 hours to achieve the best optimal global value of the objective function, needing more 120% of optimization time and function evaluations than with the local solver.

Table 4 Optimization results with *GlobalSearch* for 100 trial points

Output		active-set	interior-point	SQP	trust-region-reflective
Number of function evaluations		526	505	532	526
Optimization time [h]		~5.500	~5.200	~5.500	~5.500
x global [mm]	t	10.160	10.152	9.990	10.160
	w_a	14.582	14.878	19.000	14.582
	h_b	14.881	14.762	10.000	14.881
Optimal global *f(x)* value		4.402	4.398	4.163	4.402

The results obtained with 400 trial points, are presented in **Table 5**, where there is an evolution in the optimal global value achieved by *active-set*, *interior-point* and *trust-region-reflective* algorithms.

Table 5 Optimization results with *GlobalSearch* for 400 trial points

| Output		active-set	interior-point	SQP	trust-region-reflective
Number of function evaluations		1149	1224	1168	1215
Optimization time [h]		~12.500	~13.300	~12.800	~13.200
x global [mm]	t	10.145	10.135	9.990	10.138
	w_a	15.137	15.440	19.000	15.389
	h_b	14.645	14.517	10.000	14.511
Optimal global *f(x)* value		4.394	4.391	4.163	4.392

The *active-set* algorithm achieves an improvement of 0.182% compared with the value obtained with the local solver (4.402). The *interior-point* and *trust-region-reflective* algorithms achieve an improvement of 0.227% compared with the value obtained with the local solver (4.401 and 4.402, respectively). The optimal global value obtained with the *SQP* algorithm remains in 4.163. The solver requires 1168 function evaluations and 12.8 hours to achieve the best optimal global value of the objective function, needing more 412% of optimization time and function evaluations than with the local solver.

5.3. Discussion of Results

The discussion of results is presented for local and global solutions.

In general, the local solver *fmincon* converges to a good solution for the four algorithms, achieving a correlation of mode shapes and natural frequencies, between the two models, with good quality. The color graphs of **Figure 3** represent the MAC matrix and frequencies matrix, and quantifies the correlation among the two models. In MAC matrix the diagonal should be as dark as possible and bright outside of the diagonal, to represent a good correlation among mode shapes, and the frequencies matrix should be as bright as possible to represent a good correlation among the frequencies.

The value of the objective function for the first evaluation, for any used algorithms, is 23.042, because the initial point x^0 is the same for all cases. This value has the meaning of the geometric distance between the two models, imposed by initial variables of point x^0. This originates a weak correlation between, mainly, the natural frequency values of the two models, since the correlation between all mode shapes in diagonal MAC matrix is quite close to the unit, as shown in **Figure 3**.
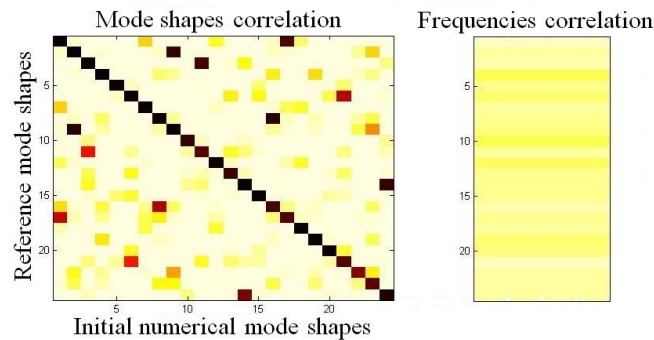

Figure 3 Initial correlation

After the optimization be finalized, the quality of the natural frequencies correlation shows a considerable improvement, and reveals a slight improvement in MAC matrix, as shown in **Figure 4a**. The *SQP* algorithm is the one that achieves the best optimal value of objective function, and consequently, the best correlation among the two models.
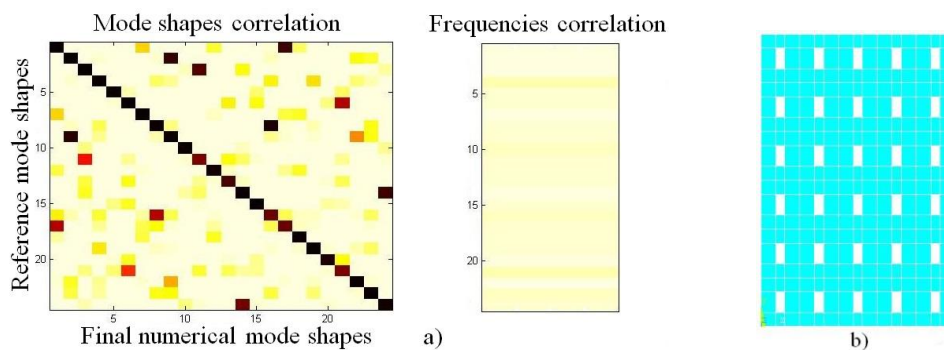
Figure 4 a) Best correlation for local optimization; **b)** Best final numerical model for local optimization

The final numerical model, presented in **Figure 4b**, suffers significant changes due to the width (w_a) and height (h_b) parameters converge to the upper bounds. As the thickness t parameter suffers a small change in relation to the initial value, the numerical model is now closer to the reference model, both geometrically and in terms of its dynamic behavior, but with a mass reduction of 8.5%.

The *GlobalSearch* solver, in general, is able to converge to a better solution than with the local solver *fmincon*. With 100 and 400 trial points, just the *SQP* algorithm is able to achieve the best optimal value of objective function. Consequently, the quality of the correlation between mode shapes and natural frequencies of the two models, presented in **Figure 5a**, reveals an improvement relatively to the local solver optimization.
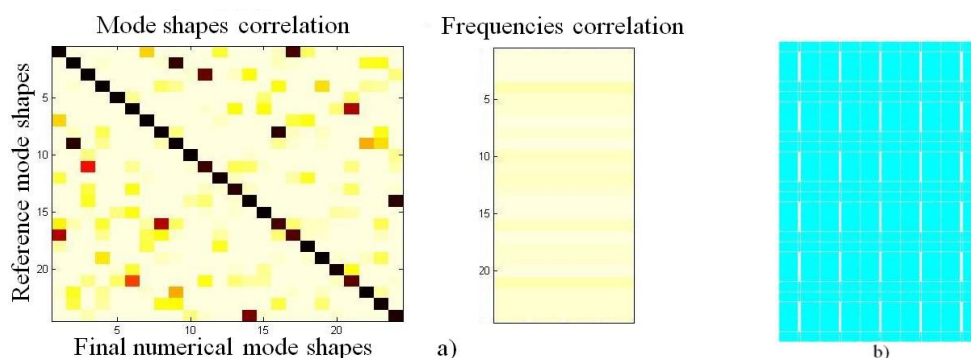
Figure 5 a) Best correlation for global optimization; **b)** Best final numerical model for global optimization

The final numerical model, presented in **Figure 5b**, is closer to the reference model because the width (w_a) parameter converges for the upper bound value. The height (h_b) parameter keeps the initial value, and the thickness (t) parameter suffers a small change, with regard to initial value, and converges to the lower bound. This parameter together with the other two, originates a final numerical model with very similar geometry and dynamic behaviour in relation to the reference model, but with the advantage of mass reduction by 3.1%.

6. CONCLUSIONS

The aim of this paper was to apply the global optimization techniques for the optimization of finite element models, and to establish a comparison with the local search. The global solver has the advantage of being able to work with a higher number of trial points, and therefore, is more efficient than the local solver. The global solver tested, *GlobalSearch*, achieves the best optimal value when working with the *SQP* algorithm.

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