On the *resting abyss* of a two-layered ocean

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Summary. — In the framework of the theory of geostrophic contours, a sufficient condition is pointed out in order that the lower layer of a two-layered ocean be motionless.

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1. – Introduction

The complexity of the vertical structure of the horizontal flow in geophysical fluid dynamics has induced investigators to resort to a hierarchy of schematizations of the fluid in motion, in order to circumvent the mathematical complexity of the problem at hand, and to make any progress possible. In making an analysis of basin scale ocean circulation, we note the coexistence of two different kinds of simple models since the very beginning of modern physical oceanography [1-3]: those with homogeneous density in a single layer, and those with two layers with different density values. The second case represents the simplest baroclinic extension of the first one and it is sometimes further simplified, assuming the lower layer to be at rest, so that the ocean circulation is dynamically described as the motion of a single fluid layer above a resting abyss. A typical case is reported in [4]. However, as far as the choice of a moving or a resting layer depends on the subjective judgement of the investigator, the resulting model can be satisfactory at most for its given purpose, but certainly not deductive enough. This difficulty has only been overcome [5-7] in the early years of the past decade, through the theory of the geostrophic contours, which deals with the mechanism of motion transmission from the upper to the lower layer with the *possible* formation of a pool of recirculating fluid in the lower layer. On the basis of this theory, we deduce a sufficient condition for the lower layer to be at rest. While the motion in the pool region (if any) is completely determined by resorting to a weak frictional coupling between the layers, our criterion purely depends on the leading vorticity equations in the limit of low dissipation. On grounds of uniformity, we keep our notation to be the same as in Pedlosky [8], who has extensively discussed the two-layer model in his recent monograph.

2. – The criterion

The dimensional vorticity equations controlling the basin dynamics of a two-layered ocean model are

$$F_1 J(\psi_1, \psi_2) + \beta \frac{\partial \psi_1}{\partial x} = \frac{f_0}{H_1} w_{\mathrm{E}}, \qquad \text{upper layer}, \tag{1}$$

$$F_2 J(\psi_2, \psi_1) + \beta \frac{\partial \psi_2}{\partial x} = 0, \qquad \text{lower layer}, \tag{2}$$

where H_i, $i = 1, 2$ is the layer thickness, $F_i = f_0^2/\gamma H_i$ is the Froude number and $\gamma = g(\Delta\varrho/\varrho_0)$ is the reduced gravity. The Ekman pumping vertical velocity w_{E} plays the role of forcing, since it is established by the external wind stress curl field.

Multiplying eq. (1) by H_1, eq. (2) by H_2 and adding the results, we obtain

$$\frac{\partial}{\partial x}(H_1 \psi_1 + H_2 \psi_2) = \frac{f_0}{\beta} w_{\mathrm{E}}. \tag{3}$$

Once the barotropic transport streamfunction $\psi_{\mathrm{B}} = (H_1 \psi_1 + H_2 \psi_2)/H$ is introduced ($H = H_1 + H_2$ is the total fluid depth), we have from eq. (3)

$$\beta \frac{\partial \psi_{\mathrm{B}}}{\partial x} = \frac{f_0}{H} w_{\mathrm{E}}$$

and hence

$$\psi_{\mathrm{B}} = -\frac{f_0}{\beta H} \int_x^{x_{\mathrm{e}}} w_{\mathrm{E}}(x', y)\, \mathrm{d}x'. \tag{4}$$

In eq. (4) x_{e} is the longitude of the eastern boundary, where the Sverdrup transport streamfunction ψ_{B} vanishes. Differentiating (4) with respect to y results in, as a function of ψ_1 and ψ_2,

$$\frac{\partial}{\partial y}(H_1 \psi_1 + H_2 \psi_2) = -\frac{f_0}{\beta} \frac{\partial}{\partial y} \int_x^{x_{\mathrm{e}}} w_{\mathrm{E}}(x', y)\, \mathrm{d}x'. \tag{5}$$

Consider now the lower layer, where eq. (2) holds. In order to decouple this equation from eq. (1), we express $\partial\psi_1/\partial x$ and $\partial\psi_1/\partial y$ as a function of ψ_2 and w_{E} by using eqs. (3) and (5) in eq. (2). The result is

$$J\left(\psi_2, \beta y - \widehat{F} \frac{f_0}{\beta H} \int_x^{x_{\mathrm{e}}} w_{\mathrm{E}}(x', y)\, \mathrm{d}x'\right) = 0, \tag{6}$$

where $\widehat{F} = f_0^2 H/\gamma H_1 H_2$. Equation (6) means that a functional dependence between the two arguments of the Jacobian operator holds and therefore we can write

(7) $$\psi_2(x, y) = \Psi_2\left(\beta y - \widehat{F}\frac{f_0}{\beta H}\int_x^{x_e} w_E(x', y)\,dx'\right).$$

About the dependence of Ψ_2 on its argument, we observe that the eastern boundary is a streamline of ψ_2. Without loss of generality we can assume $\psi_2(x_e, y) = 0$. This boundary condition implies, via eq. (7), that $\Psi_2(\beta y) = 0$ *for every* y *in the latitudinal strip of the basin*, say for $0 \leqslant y \leqslant L$. As βy is here the argument of Ψ_2, putting $\beta y = \xi$, we are able to state the explicit functional dependence of Ψ_2 on ξ within the interval $[0, \beta L]$:

(8) $$\Psi_2(\xi) = 0 \qquad \forall \xi \in [0, \beta L].$$

In particular, if, for every longitude x of the fluid domain

$$\beta y - \widehat{F}\frac{f_0}{\beta H}\int_x^{x_e} w_E(x', y)\,dx' \in [0, \beta L],$$

that is to say

(9) $$0 \leqslant \beta y - \widehat{F}\frac{f_0}{\beta H}\int_x^{x_e} w_E(x', y)\,dx' \leqslant \beta L,$$

then $\psi_2(x, y) = 0$ (for $x < x_e$ as well) and the lower layer is at rest. We clarify this point. If $(\bar{x}, \bar{y})$ is such that

$$\beta\bar{y} - \widehat{F}\frac{f_0}{\beta H}\int_{\bar{x}}^{x_e} w_E(x', \bar{y})\,dx' = \beta y,$$

then

$$\psi_2(\bar{x}, \bar{y}) = \Psi_2\left(\beta\bar{y} - \widehat{F}\frac{f_0}{\beta H}\int_{\bar{x}}^{x_e} w_E(x', \bar{y})\,dx'\right) = \Psi_2(\beta y) = 0,$$

so ψ_2 takes the same vanishing value along the line connecting $(\bar{x}, \bar{y})$ to (x_e, y) which therefore belongs to a *blocked* geostrophic contour. Inequality (9) states a sufficient condition in order that the sole upper layer be in motion. On the contrary, a necessary condition for the formation of a pool region in motion in the lower layer is that there exists a point x_r such that

(10) $$\beta y - \widehat{F}\frac{f_0}{\beta H}\int_{x_r}^{x_e} w_E(x', y)\,dx' > \beta L.$$

Obviously, if inequality (9) is fulfilled, then $\psi_1 \equiv \psi_B$. We stress once again that the dependence of Ψ_2 on ξ for $\xi > \beta L$, outside the interval $[0, \beta L]$, cannot be obtained readily since, to determine in this case the amplitude of ψ_2, eqs. (1) and (2) must be supplemented by higher-order viscosity terms (see ref. [8] for details).

3. – Concluding remarks

We apply the criterion above to an idealized subtropical gyre whose northernmost latitude $y = L$ satisfies, by definition, the equation $w_{\mathrm{E}}(L) = 0$, assuming moreover, for simplicity, a longitude-independent Ekman pumping. If the gyre extends in longitude from x_{w} to x_{e}, by using the truncated expansion $w_{\mathrm{E}} \approx [\partial w_{\mathrm{e}} / \partial y]_{y = L}(y - L)$ into inequality (9), the criterion itself takes the form

$$\frac{\beta^2 H}{\widehat{F} f_0 [\partial w_{\mathrm{E}} / \partial y]_{y = L}} \geqslant x_{\mathrm{e}} - x_{\mathrm{w}} . \tag{11}$$

Inequality (11) is equivalent to the statement that the zonal Sverdrup transport $u_{\mathrm{B}} = - \partial \psi_{\mathrm{B}} / \partial y$ evaluated from (4) is *not* large enough to arrest, at some longitude and at the latitude L where u_{B} is strongest, the baroclinic propagating Rossby wave on the interface, whose speed is $\beta / \widehat{F}$. In fact the condition

$$\left[- \frac{\partial \psi_{\mathrm{B}}}{\partial y} \right]_{y = L} \leqslant \frac{\beta}{\widehat{F}} , \qquad \forall x \in [x_{\mathrm{w}}, x_{\mathrm{e}}]$$

takes explicitly the form

$$\frac{f_0}{\beta H} \left[\frac{\partial w_{\mathrm{E}}}{\partial y} \right]_{y = L} (x_{\mathrm{e}} - x) \leqslant \frac{\beta}{\widehat{F}} , \qquad \forall x \in [x_{\mathrm{w}}, x_{\mathrm{e}}] ,$$

that just coincides with inequality (11). The same analysis but for the reverted condition (10) is reported in [8].

Finally, we note from inequality (11) that the criterion is sensitive to the east-west extension of the basin in the sense that a large extension does not favour a resting abyss.

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