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Dissipation of Alfvén waves in compressible inhomogeneous media(*)

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Summary. — In weakly dissipative media governed by the magnetohydrodynamics (MHD) equations, any efficient mechanism of energy dissipation requires the formation of small scales. Using numerical simulations, we study the properties of Alfvén waves propagating in a compressible inhomegeneous medium, with an inhomogeneity transverse to the direction of wave propagation. Two dynamical effects, energy pinching and phase mixing, are responsible for the small-scales formation, similarly to the incompressible case. Moreover, compressive perturbations, slow waves and a static entropy wave are generated; the former are subject to steepening and form shock waves, which efficiently dissipate their energy, regardless of the Reynolds number. Rough estimates show that the dissipation times are consistent with those required to dissipate Alfvén waves of photospheric origin inside the solar corona.

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1. – Introduction

The dissipation of the energy of magnetohydrodynamic (MHD) waves represents one of the mechanisms which have been proposed to explain the high temperature observed in the plasma of the solar corona. The waves have a probable origin in the photospheric motions and propagate along the magnetic field, which completely threads the corona. The corresponding energy flux [1] compares favorably with that required to heat the corona. However, in order to heat the plasma the wave energy must be significantly dissipated before it leaves the system, *i.e.* due to the smallness of the dissipative coefficients in the corona, the energy must be transferred to small scales. This can be achieved by the interaction between the wave and the inhomogeneities of the coronal structures. This process has been first studied analyzing the normal modes of inhomogeneous MHD structures.

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It has been found that these solutions develop small scales either localized in thin layers where resonance conditions are satisfied (resonant modes, [2-12]) or across the whole inhomogeneity region (resistive modes, [13, 14]).

The study of normal modes gives useful indications on the dissipation mechanisms and on the associated time scales, but it leaves some important questions open: how are these modes formed? How long does it take to form the small-scale structures? The answers to these questions require the study of the dynamical evolution of an initial wave in its propagation in a nonuniform medium. Using an asymptotic analysis Lee and Roberts [15] found two main dynamical effects responsible for small-scale formation: i) *energy pinching*: the energy of the wave concentrates at the resonance locations; ii) *phase mixing* [16], due to inhomogeneities of the Alfvén speed, transfers the disturbance energy to increasingly small-scale structures. Malara *et al.* [17], using numerical simulations of the MHD incompressible equations showed that the both mechanisms are at work, but phase mixing or energy pinching dominates, according to the disturbance wavelength. In the former case the small-scale formation time $\tau_{\rm ss}$ is roughly proportional to $S^{-1/3}$ [4], S being the Reynolds number. However, due to the large values of S, $\tau_{\rm ss}$ is too large to achieve an efficient dissipation within the corona.

In the present paper we will discuss, using a numerical simulation code, the properties of Alfvén waves propagation in a compressible medium, in oblique propagation. The aim is to elucidate the main differences in the mechanism of small-scale formation with respect to the incompressible case [17], and to see whether the coupling between compressible and incompressible modes can speed up the dissipation. Compressive fluctuations undergo steepening and can give origin to shocks; this process corresponds to the formation of infinitely small lengths in a finite time (*catastrophe*), and the dissipation time is independent of S. This process is of interest in a medium where S is very large, like the solar corona.

2. – Numerical model

The basic equations of our model are the compressible, dissipative, MHD equations, which can be written in the following form, using dimensionless variables:

$$\frac{\partial \rho}{\partial \tau} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla(\rho T) + \frac{1}{\rho}(\nabla \times \mathbf{b}) \times \mathbf{b} + \frac{1}{\rho S_\nu}\nabla^2 \mathbf{v}, \tag{2}$$

$$\frac{\partial \mathbf{b}}{\partial \tau} = \nabla \times (\mathbf{v} \times \mathbf{b}) + \frac{1}{S_\eta}\nabla^2 \mathbf{b}, \tag{3}$$

$$\frac{\partial T}{\partial \tau} + (\mathbf{v} \cdot \nabla)T + (\gamma - 1)T(\nabla \cdot \mathbf{v}) = \frac{\gamma - 1}{\rho}\left[\frac{1}{S_\kappa}\nabla^2 T + \frac{1}{S_\nu}\left(\frac{\partial v_i}{\partial x_j}\frac{\partial v_i}{\partial x_j}\right) + \frac{1}{S_\eta}(\nabla \times \mathbf{b})^2\right], \tag{4}$$

where ρ is the density normalized to a characteristic value ρ_0; $\mathbf{b}$ is the magnetic field normalized to a value B_0; $\mathbf{v}$ is the velocity normalized to the Alfvén velocity $c_{\rm A0}$ =

$B_0/(4\pi\rho_0)^{1/2}$; T is the temperature normalized to $\mu m_{\rm p} c_{\rm A0}^2/k_{\rm B}$ (with μ the mean molecular weight, $m_{\rm p}$ the proton mass, and $k_{\rm B}$ the Boltzmann constant). The space variables and the time τ are normalized to a characteristic length a and to $a/c_{\rm A0}$, respectively. The quantities $S_\nu = ac_{\rm A0}/\nu$ and $S_\eta = 4\pi ac_{A0}/(\eta c^2)$ represent, respectively, the kinetic and magnetic Reynolds numbers, while $S_\kappa = [\nu/(\mu m_{\rm p}\kappa)]S_\nu$, and γ is the adiabatic index.

Equations (1)-(4) are solved in a rectangular spatial domain $D = [-l, +l] \times [0, \pi Rl]$. The parameter l gives a measure of the domain width in units of the shear length a, while R determines the aspect ratio of the domain.

Since we will describe waves which essentially propagate along the y-direction, we impose periodic boundary conditions at $y = 0$ and $y = \pi Rl$, and free-slip boundary conditions at $x = \pm l$ (vanishing normal derivatives).

The initial condition is represented by the superposition of an (ideal) equilibrium structure and a perturbation. The equilibrium structure is defined by

$$\rho_{\rm eq} = 1\,, \qquad \mathbf{v}_{\rm eq} = 0\,, \tag{5}$$

$$\mathbf{b}_{\rm eq} = \left[1 + \Delta\left(\tanh x - \frac{x}{\cosh^2 l}\right)\right](\cos\psi\,\mathbf{e}_y + \sin\psi\,\mathbf{e}_z)\,, \tag{6}$$

$$T_{\rm eq}(x) = \frac{p_{\rm eq}}{\rho_{\rm eq}} - \frac{|\mathbf{b}_{\rm eq}(x)|^2}{2\rho_{\rm eq}}\,, \tag{7}$$

where ψ is the angle between $\mathbf{b}_{\rm eq}$ and the propagation direction y; Δ measures the amplitude of the inhomogeneity (which is in the x-direction). The temperature profile $T_{\rm eq}(x)$ satisfies the total (magnetic + kinetic) pressure equilibrium. The total pressure $p_{\rm eq}$ is determined by $p_{\rm eq} = \zeta b_{\rm eq}^2(l)/2$; the temperature $T_{\rm eq}$ is positive in the whole domain, provided that $\zeta > 1$. The parameter ζ determines the plasma β:

$$\beta(x) = \frac{c_{\rm s}^2(x)}{c_{\rm A}^2(x)} = \frac{\gamma}{2}\left(\zeta\frac{|\mathbf{b}_{\rm eq}(l)|^2}{|\mathbf{b}_{\rm eq}(x)|^2} - 1\right)\,, \tag{8}$$

where $c_{\rm s}(x) = \sqrt{\gamma T_{\rm eq}(x)}$ is the local sound velocity and $c_{\rm A}(x) = b_{\rm eq}(x)$ is the local Alfvén speed. Large values of ζ correspond to large average β values.

Since waves in the corona of photospheric origin are believed to be essentially Alfvénic [18,19], we have modeled such disturbances by superimposing on the above equilibrium structure the following perturbation:

$$\delta\mathbf{v}(x,y) = \delta\mathbf{b}(x,y) = \nabla\times[A_1\,{\rm Re}\,[(1/\alpha)\exp[i\alpha y]]\mathbf{e}_z]\,, \tag{9}$$

where A_1 is the amplitude of the perturbation and α is the wave number. This form corresponds to a plane wave front initially propagating along the y-direction, with $\delta\mathbf{v}$ and $\delta\mathbf{b}$ polarized along the x-axis.

Equations (1)-(4) have been numerically solved using a $2\frac{1}{2}$-D pseudospectral code [20]. At the initial time the equilibrium structure is homogeneous along the y-direction, while the perturbation has a vanishing space average along the same direction. Then we consider any variable f as the superposition of two contributions: the average in the y-direction f_0, which we ascribe to the equilibrium structure, and the fluctuating part $\delta f = f - f_0$.

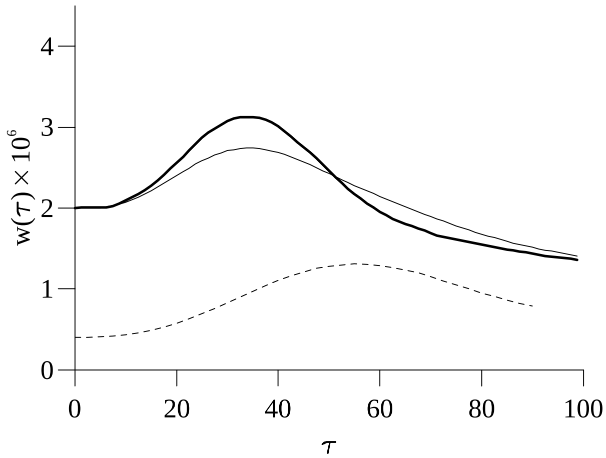

Fig. 1. – Power dissipated on fluctuations as a function of the time for: $S = 2000$, $\zeta = 1.01$ (thick line); $S = 10^4$, $\zeta = 1.01$ (dashed line); $S = 2000$, $\zeta = 3.01$ (thin line).

3. – Numerical results

In the first run we used the following values for the parameters of the model: $\alpha = 1$, corresponding to a wavelength in the y-direction $\lambda_y = 2\pi$, *i.e.* of the same order as the inhomogeneity scale length; the propagation angle is $\psi = \pi/4$; the initial normalized perturbation energy is $\varepsilon(\tau = 0) = 2.5 \times 10^{-5}$ (corresponding to a small amplitude perturbation); we chose $\zeta = 1.01$ and $\Delta = 0.25$, corresponding to β ranging between 8×10^{-3} and 1.49; the Reynolds numbers are $S_\nu = S_\eta = S_\kappa/(\gamma - 1) = 2000$; the domain size in the x-direction, determined by l, has been chosen sufficiently large ($l = 20$) to avoid that the boundary conditions affect the time evolution; finally, the domain length in the y-direction has been chosen equal to λ_y, in order to have the maximum spatial resolution in that direction (this corresponds to $R = 2/l$).

This configuration, in the case of homogeneous equilibrium structure, would correspond to an Alfvén monochromatic wave in oblique propagation, the wave vector being parallel to the y-axis. The interaction of such a disturbance with the equilibrium inhomogeneity generates both a modulation in the x-direction and an energy transfer to the y and z components, thus destroying the initial Alfvénic character of the disturbance. In fig. 1 we plotted the fluctuation dissipated power, which is defined by

$$(10)\quad w(\tau) = \frac{1}{E^{(0)}(0)} \int_D \left\{ \frac{1}{S_\nu} \left[\left(\frac{\partial v_i}{\partial x_j} \right)^2 - \left(\frac{\partial v_{0i}}{\partial x_j} \right)^2 \right] + \frac{1}{S_\eta} [(\nabla \times \mathbf{b})^2 - (\nabla \times \mathbf{b}_0)^2] \right\} \mathrm{d}x\, \mathrm{d}y,$$

where $E^{(0)}(0)$ is the initial equilibrium energy. It is seen that $w(\tau)$ increases up to a maximum and then it decreases, roughly exponentially, in time. This indicates that an effective generation of small scales takes place; the time τ_{ss} when $w(\tau)$ is maximum represents the small-scale formation time, and it gives a measure of the time necessary to dissipate a relevant part of the fluctuation energy. The dissipation time $\tau_{\mathrm{hom}} = S/\alpha$, which the same perturbation propagating in a homogeneous structure would have taken to be dissipated, is much longer than τ_{ss}. Increasing the Reynolds numbers by a factor 5 (fig. 1), we verified

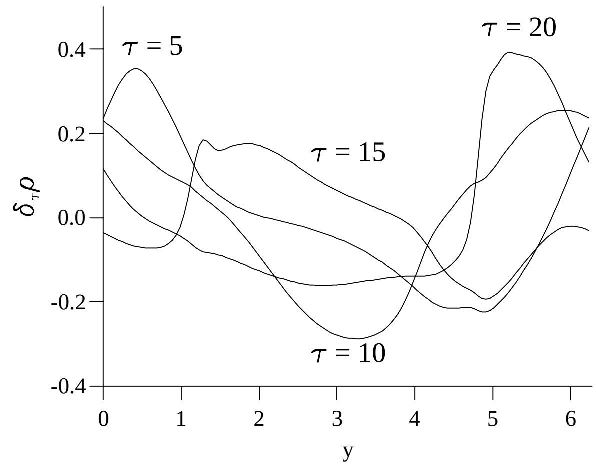

Fig. 2. – Profiles of the density fluctuation as function of y, in the large-amplitude run, at $x = 0$.

that $\tau_{ss} \propto S^{0.32}$, which is the scaling law typical of phase mixing [4]. Then, similarly to the incompressible case [17], for wavelengths of the order of the inhomogeneity length, phase mixing dominates in the small-scale formation process. However, increasing the value of ζ (fig. 1), we verified that larger values of β correspond to larger τ_{ss}, *i.e.* the dissipation process is faster in the more compressible case.

Since small scales form in the x-direction, the effective wave vector is quasi-perpendicular to $\mathbf{B}_0$. Then, the direction perpendicular to $\mathbf{B}_0$ and to the x-axis (z') correspond to the Alfvénic polarization, while the direction parallel to $\mathbf{B}_0$ (y') to the magnetosonic polarization. The time evolution shows that the perturbation energy, initially polarized in the x-direction, in the inhomogeneity region is completely transferred to the other directions (y' and z'); these two polarizations evolve in different ways. The y'-polarized fluctuations are formed by a superposition of two kinds of perturbations: 1) A slow magnetosonic disturbance; we verified that the fluctuations $\delta\rho$ and $\delta B_{y'}$ associated to such a disturbance are correlated as in a slow magnetosonic wave with quasi-perpendicular wave vector. Moreover, it oscillates with the local value of the cusp frequency $\omega_{\mathrm{cusp}} = c_{\mathrm{A}} c_{\mathrm{s}} / [\lambda_{\|} (c_{\mathrm{A}}^2 + c_{\mathrm{s}}^2)^{1/2}]$; since both the Alfvén velocity c_{A} and the sound velocity c_{s} vary across the inhomogeneity, the disturbance profile undergoes phase mixing which is responsible for small-scale formation in that polarization. The propagation direction is the same as that of the initial wave. 2) A static entropy wave, in which $\delta\rho$ and δT are anticorrelated, and the gas pressure fluctuation δp is vanishing. This perturbation is due to the coupling between the initial wave and the entropy transverse modulation associated to the equilibrium.

The z'-polarized fluctuation is essentially Alfvénic: $\delta B_{z'} \simeq \delta v_{z'}$. It oscillates at the local value of the Alfvén frequency $\omega_{\mathrm{A}} = c_{\mathrm{A}} / \lambda_{\|}$ and then it undergoes both energy pinching and phase mixing. The time evolution is similar to that observed in the incompressible case; actually, due to its Alfvénic character, this disturbance should not be affected by the compressibility of the medium. Globally, $\sim 10\%$–15% of the energy of the initial Alfvénic wave is converted into a compressive disturbance, while the remaining part keeps the non-compressive character.

We have performed one more run characterized by larger values of the initial amplitude ($\delta v_x = \delta b_x \simeq 0.29$). In this case, nonlinear effects play an important role in the wave dissipation. The dissipated power, rescaled to take into account that the amplitude is different with respect to the previous run, increases faster and its maximum value is larger by a factor ~ 1.5. This maximum is reached before the corresponding time of the small-amplitude case. The increase of $w(\tau)$ is due to the formation of compressive shocks in the simulation domain. In fig. 2 the profiles of $\delta\rho$ as functions of y at different times in the inhomogeneity region are shown. The small scales associated to the shock front contribute to dissipate the initial wave energy. This shock belongs to the slow mode, the density fluctuation being anticorrelated with $\delta B_{y'}$; its formation follows the steepening of the slow magnetosonic perturbation which form in the inhomogeneity region. The shock normal is strongly oblique with respect to the y-direction as a consequence of the dependence on x of both c_{A} and c_{s}.

4. – Discussion and conclusions

In this paper we studied the formation of small scales in the propagation of an initial Alfvénic disturbance in a compressible medium along an inhomogeneous equilibrium structure, in the case where the inhomogeneity direction is perpendicular both to the initial propagation direction and to the background magnetic field. The main differences between the compressible and incompressible [17] simulations can be summarized as follows:

i) In both cases small scales are efficiently formed by the same dynamical effects (phase mixing and, to a lower extent, energy pinching). However, the efficiency of these processes increases in the compressible case, *i.e.* decreasing β.

ii) In the compressible case the interaction between the Alfvénic disturbance and the inhomogeneous equilibrium structure generates slow and entropy waves. When the disturbance amplitude is sufficiently high the compressible perturbations steepen and then form shock waves.

As a consequence, the propagation of an Alfvénic perturbation in a compressible inhomogeneous medium could be more efficient in producing small scales than the corresponding propagation in an incompressible medium, in that in the former case a sort of *catastrophe* can be observed, due to the production and steepening of compressible fluctuations.

Now let us assume a wave period of say 10 s and an Alfvén velocity $c_{\mathrm{A}} \sim 10^3$ km/s, an inhomegeneity length $a \sim 1.6 \times 10^3$ km (corresponding to $\alpha = 1$). We have found for the typical time of small-scale formation $\tau_{\mathrm{ss}} \sim 55\ a/c_{\mathrm{A}}$ for $S = 10^4$. Using the scaling $\tau_{\mathrm{ss}} \propto S^{0.31}$, and assuming $S \sim 10^8$, we obtain a dissipation length $l_{\mathrm{ss}} \sim c_{\mathrm{A}}\tau_{\mathrm{ss}} \sim 1180\, a \sim 1.9 \times 10^6$ km, *i.e.* small scales are formed by phase mixing and the wave energy is efficiently dissipated when the wave propagates less than $3R_\odot$ above the photosphere.

For a fluctuating velocity to Alfvén velocity ratio of about 0.29 (corresponding to velocity fluctuations of the order of $\sim$ 290 km/s) we have found that the shock wave is formed at a time $\tau_{\mathrm{sh}} \simeq \tau_{\mathrm{ss}} \sim$ 20–25 a/c_{A}. The shock formation time is proportional to the perturbation amplitude, which, for our simulation, is larger than the velocity fluctuations observed in the corona by at least a factor 6. Multipling by the same factor τ_{sh}, in coronal conditions the shock formation length is $l_{\mathrm{sh}} \simeq c_{\mathrm{A}}\ \tau_{\mathrm{sh}} \simeq$ 120–150 $a \sim$ 1.9–2.4 $\times 10^5$ km, *i.e.* of the order of or smaller than one third $R_\odot$ and thus much smaller than the length for small-scale formation l_{ss} due to phase mixing. The energy associated with these shock waves can be as high as 10%–15% of the initial perturbation energy.

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