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Plasma transport in the interplanetary space: Percolation and anomalous diffusion of magnetic-field lines(*)

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Summary. — The magnetic fluctuations due to, *e.g.*, magnetohydrodynamic turbulence cause a magnetic-field line random walk that influences many cosmic plasma phenomena. The results of a three-dimensional numerical simulation of a turbulent magnetic field in plane geometry are presented here. Magnetic percolation, Lévy flights, and non-Gaussian random walk of the magnetic-field lines are found for moderate perturbation levels. In such a case plasma transport can be anomalous, *i.e.*, either superdiffusive or subdiffusive. Increasing the perturbation level a Gaussian diffusion regime is attained. The implications on the structure of the electron foreshock and of planetary magnetopauses are discussed.

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1. – Introduction

The plasma universe is shaped by the magnetic field, and, as a consequence, the magnetic turbulence influences plasma transport in many cosmic systems: in practice, a particle gyrating in the magnetic field follows a field line in its wandering due to the magnetic fluctuations. The concept of field line random walk was introduced by Jokipii [1] in connection with the confinement of cosmic rays in the galactic disc a long time ago, and contemporarily to the earliest studies of stochastic transport in plasma fusion devices [2]. Among the astrophysical problems to which magnetic-field line diffusion is relevant, we can mention the effect of magnetic fluctuations in the solar wind on the propagation of solar flare energetic particles [3], and the flaring of extragalactic radio jets, due to stochastic diffusion of the relativistic electrons [4]. In the space plasmas, the entry of solar-wind particles into the planetary magnetospheres is achieved by a sort of magnetic percolation [5].

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Common features of particle transport in astrophysical systems are: i) in the great majority of cases the particle mean free path is very large, so that collisions can be neglected; ii) in many cases the relevant particles are superalfvenic, so that the magnetostatic case is appropriate. Under these conditions, the particle diffusion coefficient perpendicular to the mean magnetic field is [6-8]

$$D_\perp = v_\parallel D_\mathrm{m}\,, \tag{1}$$

where D_m is the magnetic-field line diffusion coefficient and $v_\parallel$ the velocity along the magnetic field. On the other hand, the relevant perturbation amplitudes are large, $\delta B/B \sim 0.1$–1, so that a quasi-linear treatment is not appropriate to obtaining D_m.

2. – Numerical simulation of magnetic-field line transport

The magnetic-field line equations can be written as

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} = \frac{\mathbf{B}(\mathbf{r})}{|\mathbf{B}(\mathbf{r})|}, \tag{2}$$

where $\mathbf{B}(\mathbf{r})$ is the magnetic field at a generic point $\mathbf{r}$, and s is the field line length. Equation (2) constitutes a set of three non-linear equations which, in the strong turbulence limit, has to be solved by numerical integration. In particular, we consider a magnetic field $\mathbf{B} = B_0\hat{e}_z + \delta\mathbf{B}$, with three-dimensional magnetic fluctuations given by

$$\delta\mathbf{B}(\mathbf{r}) = \sum_{\mathbf{k},\sigma} \delta B_\sigma(\mathbf{k})\hat{e}_\sigma(\mathbf{k})\exp[i(\mathbf{k}\cdot\mathbf{r} + \varphi^\sigma_\mathbf{k})], \tag{3}$$

where $\delta B_\sigma(\mathbf{k})$ is the Fourier amplitude of the mode with wave vector $\mathbf{k}$ and polarization σ, $\hat{e}_\sigma(\mathbf{k})$ are the polarization unit vectors, and $\varphi^\sigma_\mathbf{k}$ are random phases. The field periodicity is that of a cube of side L, so that $\mathbf{k} = 2\pi(n_x, n_y, n_z)/L$. The Fourier amplitudes are given by [9]

$$|\delta\mathbf{B}(\mathbf{k})| = (2\pi/L)^{3/2}[C/(k^2\lambda^2 + 1)^{\gamma/4+1/2}], \tag{4}$$

where C is a normalization constant, and γ is the spectral index. The spectrum is truncated at $k_\mathrm{max} = 2\pi N/L$, where N is the ratio between L and the smallest turbulence wavelength present. For the present isotropic calculations, λ is set equal to L, while the spectral index is fixed as $\gamma = 3/2$. This spectrum can be representative of the solar-wind magnetic turbulence. The fluctuation amplitude is defined as $A = \sqrt{\langle\delta B^2\rangle}/B_0$, where $\langle\delta B^2\rangle$ is obtained from eq. (4) summed over the relevant wave vectors. More details on the numerical code are found in ref. [10].

3. – Lévy flights and anomalous diffusion

Extensive numerical simulations have shown that, grossly speaking, two regimes of transport can be identified, the first for $A \ll 1$ and the second for $A \sim 1$. For $A \ll 1$, *i.e.* in the weak-turbulence regime where good, nested magnetic surfaces exist, magnetic islands are found, and these are separated from each other by stochastic (percolating) layers. Increasing A the field line motion is more and more chaotic, until the last closed

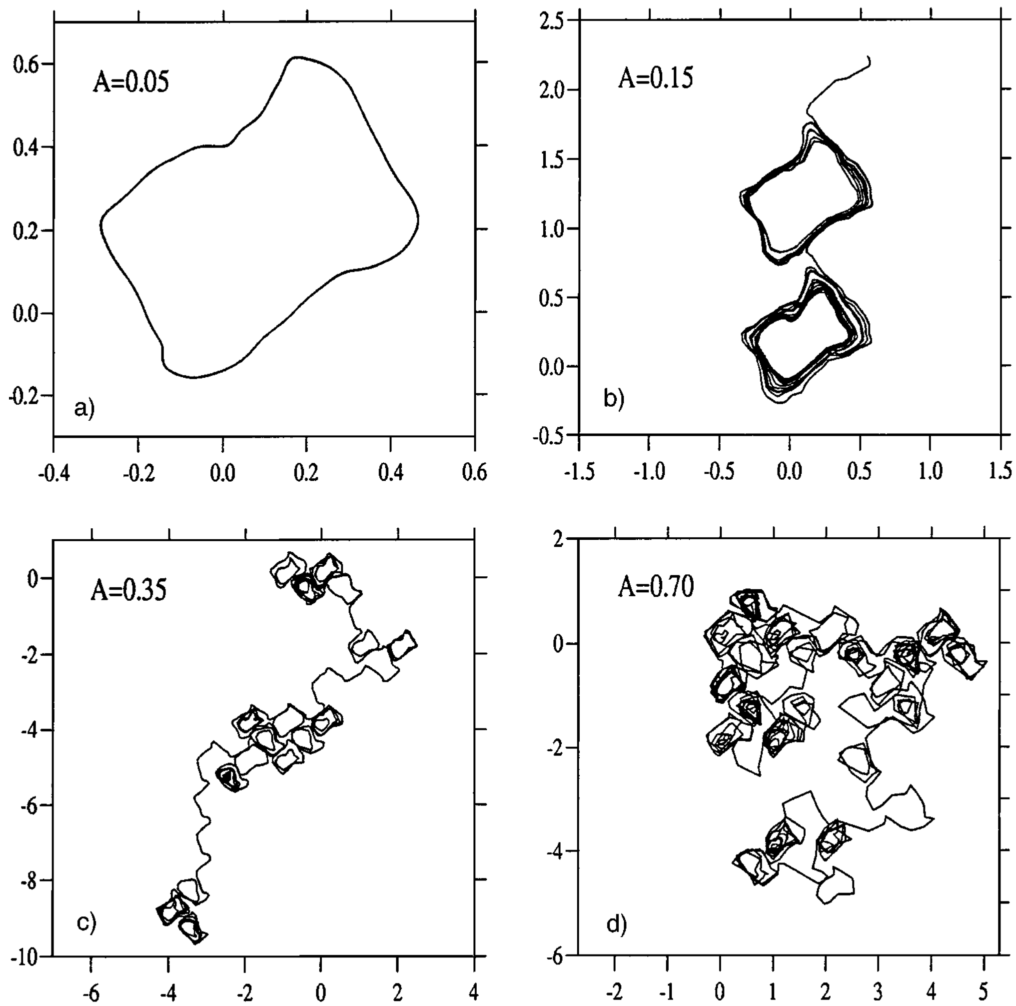

Fig. 1. – The projection on the xy plane of a field line with the same initial conditions is plotted for z varying from zero to 1000 (all lengths normalized to L) for different values of the fluctuation level A. a) For $A = 0.05$ good nested magnetic surfaces exist, and the field line evolves on the surface of a flux tube (a KAM torus). b) For $A = 0.15$ the magnetic surface opens, becoming a cantorus [14], and the field line trapped in a magnetic flux tube passes from one trap to the next. c) For $A = 0.35$ the random walk of the field line is composed of temporary trapping and of Lévy-like flights, as, in particular, the trajectory from about $y = -4$ to about $y = -8$. d) For $A = 0.70$ the random walk is more isotropic, tends to fill the plane like a Gaussian random walk, and both field line trapping and short flights happen frequently.

magnetic surface is destroyed when $A \sim 1$. For low A field lines can be trapped for some time in a region close to a good magnetic surface, a so-called cantorus, and then travel long paths, which are called "Lévy flights" [11-14]. These are legs of almost ballistic motion which occur with small but nonvanishing probability in the percolation layer [15]. This is shown in fig. 1, where a sample field line is plotted projected in the xy plane for $N = 12$ and $A = 0.05$, 0.15, 0.35, and 0.70. The trapping and subsequent Lévy flight of field lines is apparent in fig. 1c).

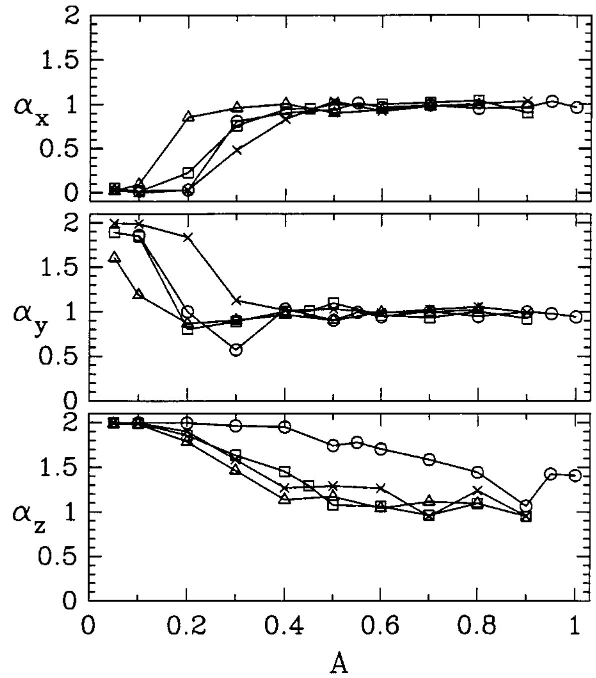

Fig. 2. – The anomalous diffusion exponents α_x, α_y, and α_z *vs.* the fluctuation amplitude A. Circles: $N = 3$; crosses: $N = 6$; squares: $N = 9$; triangles: $N = 12$.

The Lévy random walk corresponds to a non-Gaussian probability distribution $p(l)$ of making a jump of length l in a given time interval of the form $p(l) \sim l^{-(\mu+1)}$ for $l \to \infty$ [13]. The cases with $\mu > 2$ correspond to a normal random walk, whereas $\mu < 2$ gives rise to a random walk law of the form

$$\langle \Delta x^2 \rangle = 2Ds^{\alpha}, \qquad s \to \infty, \tag{5}$$

with $\alpha = 2/\mu$ [13]. Here, the exponent α characterizes the anomalous diffusion: $\alpha = 1$ in the diffusion regime, $\alpha = 2$ in the ballistic regime, $1 < \alpha < 2$ in the case of Lévy random walk [12,13] and $\alpha < 1$ in the case of trapping (subdiffusive regime). On the other hand, when $\alpha < 1$ the random walk is time intermittent and the microscopic dynamics is governed by long waiting times τ distributed according to $q(\tau) \sim \tau^{-(\nu+1)}$ for $\tau \to \infty$. For $\nu < 1$ subdiffusion results with $\alpha = \nu$. More generally, the anomalous diffusion exponent is given by $\alpha = 2\nu/\mu$ [10].

Figure 1 shows that, because of the increase of stochasticity of field lines with A, the probability of going from a "trapped" to a "percolating" path or vice versa increases with the amplitude of the fluctuating field, so that eventually the field line motion approaches a Gaussian random walk with a finite scale length. At last, for large values of A, the probability of long flights is very small, and the normal random walk is recovered (see below). We point out that the non-Gaussian nature of microscopic motion is particularly important in media which exhibit continuum percolation [16]. Indeed, many successive moves are in the same direction in the case of percolation, and this corresponds to a power law probability distribution $p(l)$.

Anomalous diffusion is studied by computing $\langle \Delta x^2 \rangle$, $\langle \Delta y^2 \rangle$, and $\langle \Delta z^2 \rangle$ as a function of s, rather than z, since in the case of strong turbulence the field line length s can depart considerably from its projection on the direction of the average field. The above mean square deviations are computed over an ensemble of 2000 lines of force integrated until an asymptotic regime is attained [10]. Here we give the results of the fit of eq. (5) by the numerical simulations, while further data are to be found in ref. [10]. The values for α_x, α_y, and α_z as a function of A are reported in fig. 2 for $N = 3, 6, 9$, and 12. Both superdiffusive and subdiffusive behaviour is found for the motion in the xy plane for low A, and a normal diffusion regime is obtained for $A \simeq 1$. In addition, the spreading of z around its mean value goes from a ballistic regime ($\alpha_z \simeq 2$) for low A to a diffusion regime for larger values of A. In particular, note that the diffusion regime is reached the sooner, the higher N. This matches with the fact that global stochasticity is attained earlier for higher N, an indication that is obtained from the Poincaré sections, too (not shown).

4. – Applications to space plasmas and conclusions

At the time of writing, two different astrophysical applications are under way, the first to the electron foreshock and the second to the Earth's magnetopause.

The upstream boundary of the electron foreshock is defined as the path of the fast electrons originating from the tangent point between the Interplanetary Magnetic Field (IMF) and the bow shock. When we consider that the magnetic turbulence in the solar wind can be strong, $A \sim 1$, we realize that the foreshock upstream boundary is not a very smooth surface. Rather, because of the magnetic fluctuations, this boundary will gradually become distorted and entangled, and will develop a very fine structure. Magnetic-field line diffusion implies that the upstream boundary is scattered in space, with a typical width given by $\langle \Delta x^2 \rangle = 2Ds^\alpha$, where s is the distance from the tangent point measured along the magnetic-field lines. Therefore, when analysing the experimental data collected by the spacecrafts crossing the foreshock, care must be used in determining the position of the bow shock: the broadening of the boundary tends to anticipate the detection of the tangent field line, for a spacecraft coming from upstream of the foreshock boundary [17].

The Earth's magnetopause, like the other planetary magnetopauses, is the boundary layer between the shocked solar wind and the actual magnetosphere. What amount of plasma can cross the magnetopause because of motion along magnetic-field lines? In the case that the IMF is northward, *i.e.*, parallel to the Earth's magnetic field at the subsolar point, magnetic reconnection (in the sense of a tearing instability) is not favoured at the low-latitude dayside magnetopause. Therefore, the magnetic connection between the magnetosheath and the magnetosphere could be due to the effect of magnetic fluctuations, which have been both observed [18, 19] and simulated numerically [20, 21], and which destroy the magnetic surfaces. On the other hand, when the IMF is southward, magnetic reconnection is strongly favoured at the subsolar magnetopause, and plasma transport along the reconnected magnetic-field line certainly plays an important role in the creation of the low-latitude boundary layer [5]. A numerical simulation dedicated to these configurations will be carried out in a future work.

The results of our numerical simulation of magnetic-field line transport in the presence of 3D fluctuations with cubic periodicity can be summarized as follows: for low fluctuation levels magnetic-field line transport is anomalous and the random walk is composed of trapping in the cantori and of long ballistic flights (Lévy flights) in the percolation layer. The transport law is $\langle \Delta x^2 \rangle \propto 2Ds^\alpha$ with $\alpha > 1$ and $\alpha < 1$. On the other hand, for higher fluctuation levels a Gaussian diffusion regime is attained; the fluctuation threshold for this regime is lowered as the number of modes, dependent on N, is increased. This suggests that many cosmic plasmas, where N is very large, are mostly diffusive.

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