Measurement of small forces in the physics of gravitation and geophysics (*)

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Summary. — The measurement of weak forces or accelerations is of fundamental importance in a variety of problems in science and technology. This topic is addressed in this paper with particular attention to gravity measurements. A general review of the current gravity measurement techniques in geophysics, general relativity and gravitation is first presented. Then, a general description of the instruments used for gravitational measurements and a brief analysis of the noise sources are discussed.

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1. – Introduction

The problem of measuring tiny forces or accelerations is of fundamental importance in a variety of contexts in science and technology; in general in all those experimental situations where extremely small mechanical or electrical effects are to be detected. This state of affairs, for example, is encountered in experiments to test general relativity and could arise, at least in principle, in the attempts to detect very low-energy neutrinos, like those of the cosmic relict background, through the mechanical effects following their coherent interactions with matter [1, 2].

A number of instruments have been built and are under continuous and active development: gravity gradiometers and gravimeters, displacement sensors and accelerometers, devices for inertial navigation and guidance systems.

We will limit our discussion to measurements of interest for the gravitational field, their significance and their relative instrumentation.

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It seems right to us to remind at this point the work of baron von Eötvös with his torsion balance which, in a sense, summarizes the subject of our paper. He, in fact, used his apparatus to perform experiments both for geophysical studies and for tests of fundamental principles in the theory of gravitation: the universality of free fall or equivalence principle [3]. Strictly speaking, torsion balances or torsion pendulums do not measure a force or an acceleration, but rather a difference between them at two distinct points in space, that is, in the idealised case, their space derivatives. It thus appears that both a gravimeter or accelerometer and a gradiometer may be used to measure the gravitational field: what is then the meaning of the data obtained from such conceptually different instruments and their relationship with the "true" gravitational field? The answer is: both results may be a measure of the field. In fact, contrary to the case, for example, of the electric field where one knows exactly how to measure it using test charges, for gravity the procedure is much vaguer due to the many mathematical quantities associated with gravitation like the metric, the connection coefficients, the Riemann curvature tensor and so on. As all these entities play equally a central role in the theory, it is obvious that many different measuring devices may be used, depending on the particular properties of the field one wants to stress. So sitting on a large body, like the Earth, the gravitational acceleration, g, is naturally a good and easy-to-measure field quantity, which, however, is completely meaningless in free fall, for example in an orbiting satellite. In the latter case the natural quantity to be associated with the field is the gradient, for which we give now some definitions and units.

The gravity gradient is a tensor formed, in Cartesian coordinates, by the second derivatives of the potential

$$T_{ij} = \frac{\partial^2 U}{\partial x_i \, \partial x_j} . \tag{1}$$

It has five independent components, by symmetries and Poisson equation. If measurements are performed with a gradiometer from a moving platform, linear acceleration noise is removed by common mode rejection, whereas rotational acceleration noise produces an additional gradient component due to angular velocities and angular accelerations. The resulting effects can be taken into account by measuring more than five components. In fact, three additional of them will give us the three components of angular motions which is all is needed to reconstruct the rotational behaviour.

2. – Geophysics

The problem of determining the Earth's gravitational field has engaged both the cleverness and the tenacity of natural scientists since the seventeenth century. The experimental methods and mathematical formulations employed have often been in the front ranks of current research; while the collection and analysis of the data is literally a monumental task.

A spherically symmetric Earth would have a spherically symmetric gravity field with potential simply given by $U = -GM/r$. Deviation from spherical symmetry in the mass distribution leads to a non-symmetric potential. Observing the external gravitational field does not uniquely determine the mass distribution, but combination with reasonable physical assumptions and other data provides valuable results. The detailed and accurate knowledge of the Earth gravity field is of fundamental importance to answer many crucial problems concerning the structure and evolution of the continents, the structure of the oceanic crust and of lithosphere, and, ultimately, the internal constitution of our planet. The Earth's gravitational field is commonly expressed by expanding the potential in spherical harmonics

$$U = \frac{Gm}{r} \sum_{m=0}^{\infty} \sum_{n=0}^{m} (R/r)^m Y_{mn}(\varphi, \lambda)\,, \tag{2}$$

where

$$Y_{mn} = P_{mn}(\sin\varphi)[C_{mn}\cos n\lambda + S_{mn}\sin n\lambda]\,. \tag{3}$$

P_{mn} is a Legendre function and R is the Earth's radius. Y_{mn} has wavelength $2\pi/m$ in φ and $2\pi/n$ in λ, and thus to resolve a feature of size x in the potential field at Earth's surface, one must retain terms through order $m = \pi R/x$. For $x = 100$ km, this leads to $m = 200$; *i.e.* 40 000 coefficients C_{mn} and S_{mn}.

Short-scale effects are better understood, based on more intuitive arguments, making reference to the "flat Earth" approximation.

The Earth is considered as an infinite horizontal plane. Spherical symmetry is now planar symmetry, that is uniform surface density σ, giving a gravity vector directed vertically with magnitude $g = -2\pi G\sigma$, independent of height above Earth. Departure from perfect planar symmetry will perturb the gravity field; however, one is now concerned directly with a local anomalous distribution rather than a global expansion such as (2). This planar formulation makes it easier to figure out the representation of departures of the gravity field from some reference field in terms of anomalies, a description particularly useful in the case of spacecraft observations. The quantity generally discussed is the vertical component of the anomalous gravitational acceleration, and the appropriate unit is the milligal ($1\ \text{gal} = 1\ \text{cm/s}^2$) corresponding to a surface density enhancement of $2.4 \cdot 10^3\ \text{g/cm}^2$. The scale of an anomaly is roughly the scale of the density variation. A long-term observational objective is a resolution of 50 km with an accuracy of 0.5–1 mgal.

2·1. *Current gravity measurement techniques.* – In sect. 4 a general description of basic instrumentation will be given in some detail. The methods used are extremely sophisticated refinements of simple concepts, such as timing a pendulum, measuring the displacement of a spring balance, observing the acceleration of a falling body. Such "gravimeters" provide precise local information, which, in practice, is however available only for a small portion of the Earth's surface, due to obvious practical difficulties.

A qualitative and quantitative jump in capability to refine and survey the gravity field on a global basis was determined by the advent of the space age and the development of satellite gravimetry. Since satellite orbits are uniquely determined by the forces acting on it, and gravity is by far the dominant one, the latter can be inferred from the observed orbits and an appropriate orbit theory. This approach has given successful information, with considerable accuracy, on the large-scale structure of the gravity field, thus complementing data obtained from ground measurements.

The complete state of the art is summarized in the models for the gravity field worked out by NASA (GEM's, Goddard Earth Models). Figure 1 gives an idea of the accuracy available.

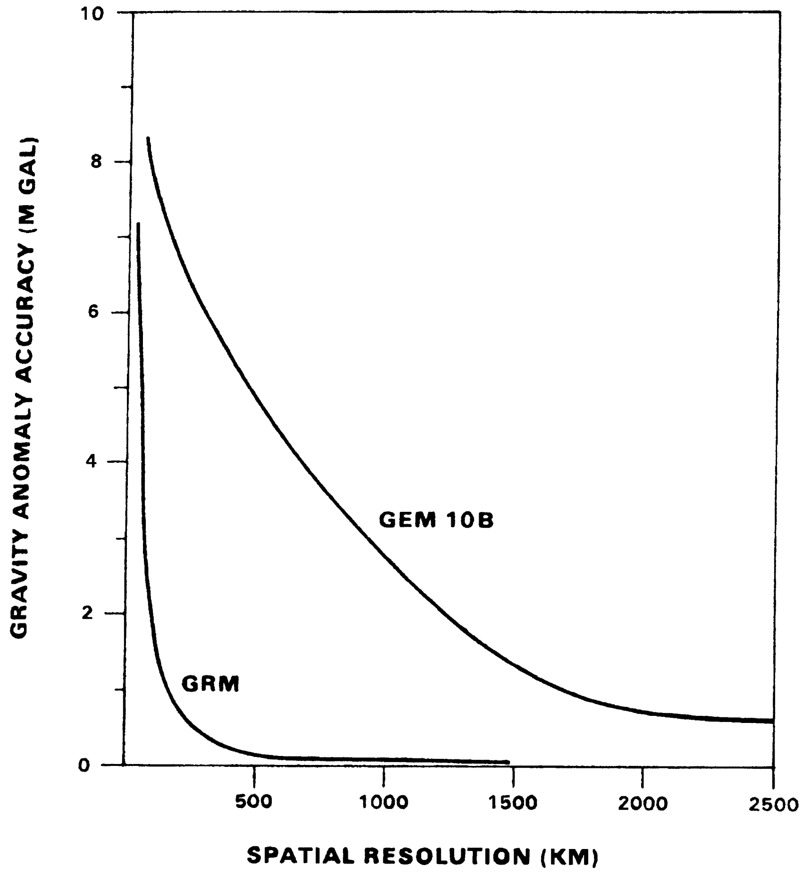

Fig. 1. – Accuracy in the Earth's gravity field from the Goddard Earth Model. The curve GRM represents the accuracy aimed at by geophysicists.

It has been realized for some time that significant improvements in the Earth's gravity field can be achieved by both satellite-to-satellite tracking and gravity gradiometry from space. Gradiometers provide a number of advantages compared to other techniques, in particular this method enhances high (spatial) frequencies because it measures the derivative of the acceleration.

2·2. *From gravity gradiometry to geometry.* – In the introduction we have briefly described the behaviour of a gravity gradiometer fixed on a moving platform. In a satellite in motion about the Earth, due to non-uniformity of the gravity field, gravitational effects are exactly balanced by the acceleration only at one point, the center of mass of the satellite. As we go away from this point, the field changes strength and direction, and small uncancelled components of the gravitational force can be detected. Within not very large sizes, these small forces are very nearly proportional to the distance from the center of the satellite, and have a quadrupole character. Figure 2a) shows the forces acting on four masses held fixed to Earth. Figure 2b) shows the situation if the four masses are in an orbiting satellite. These

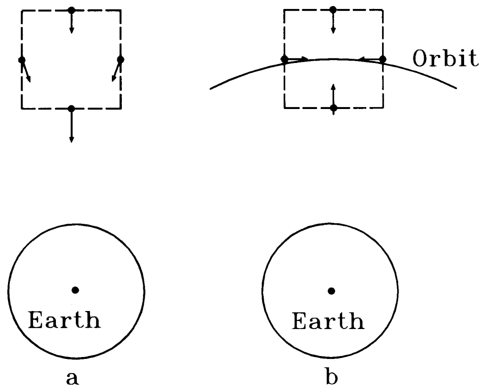

Fig. 2. – Gravity field of the Earth: a) seen by a stationary observer; b) seen by orbiting observers.

residual forces are the same as those causing tides on the Earth, so they are often called tidal forces. These are in fact the forces which are measured by a gravity gradiometer. In fig. 2b) the masses are in free fall and we see that they experience accelerations causing eventually changes in their relative distances. In a precise mathematical form this is called geodesic deviation, showing that the distance ξ_k between two "straight" lines (geodesics) changes in time as

$$\frac{d^2 \xi_k}{d\tau^2} = -R_{k0i0}\,\xi_i \,. \tag{4}$$

On the left is the (second) covariant derivative of distance with respect to proper time, which may incorporate possible inertial contributions (rotations). On the right we have some components of the Riemann tensor which expresses the geometry. In this particular case it is related to the gravity gradient tensor (1) by

$$T_{ij} = R_{i0j0} \,. \tag{5}$$

Thus, measuring the gravity gradients is also measuring a part of the geometry of the curved space. We distinguish between stationary or slowly varying fields, that are measured by a gradiometer, and rapidly varying fields, that are measured by a gravity wave antenna, which is in fact a one-axis in-line gradiometer. We end this section by regretting that no gradiometric satellite mission has yet been performed. The ESA project Aristoteles is under an advanced stage of study but still waiting for a final decision. Interesting data for geophysics are also expected from STEP [4], an ESA-NASA collaboration, now under phase-A study.

3. – General relativity and gravitation

We have seen, at the end of the previous section, the smooth connection between gravity gradiometry and geometry. It was indeed Einstein who interpreted gravity in terms of geometry with his General Theory of Relativity. General relativity was not motivated by the need to account for unexplained experimental results, but rather by a desire, in the mind of Einstein, of symmetry and simplicity, with all reference systems treated on the same footing. This was the way, together with the principle of equivalence, to incorporate gravity and special relativity. The principle of equivalence, a generalisation of Galileo's universality of free fall, states, in its weak form, that all non-gravitational laws of physics, such as electrodynamics, behave, in a freely falling frame, as if gravity were absent. Validity of this principle implies that gravitation must be a curved space-time phenomenon, and the resulting theory, which may be different from general relativity, is called metric. This gives a prescription on how matter and non-gravitational fields respond to gravity: the only coupling field is the metric itself. In other words, since energy gravitates, light will be deflected by the Sun or redshifted as it moves upward through a gravitational field. Assuming further (strong equivalence principle, proper of general relativity) that the gravitational energy also gravitates, we are led to the, non-linear, field equations where effects of gravitation are expressed in terms of space-time variables in place of mass energy variables. The consequences are well known. In addition to classical tests we remind: prediction of gravitational radiation; the Lense-Thirring frame dragging (which suggests a connection with Mach's principle and the origin of inertia); new celestial objects like black-holes; a new universe (expansion, closure, etc.).

The theory has survived all experimental tests which, since 1960, have been performed with increasing precision, including the gravitational-wave damping in binary pulsar.

3·1. *Most important issues in gravitation theory.* – In spite of all these triumphs, there are, however, crucial questions that general relativity seems to have failed to answer:

a) the problem of inertia and Mach's principle,

b) the incompatibility of general relativity and quantum mechanics.

A number of theories alternative to general relativity have been proposed. Broadly speaking they can be distinguished in metric and non-metric. Metric theories different from Einstein's theory violate at some level the strong equivalence principle. We remind in particular the Brans-Dicke theory as it was specifically conceived to solve problem *a*), *i.e.* to incorporate Mach's principle. All these alternatives, however, are contradicted in some way by experiment. On the other hand fragments of the principle do appear in general relativity, a particularly nice effect being demonstrated by the Lense-Thirring drag of inertial frames by a massive rotating body. Proposed measurements to detect this effect are the gyroscope experiment [5] and a variant of it where the gyroscope is the orbit of a satellite around the Earth [6, 7]. The object of the experiment is to measure the precession of a gyroscope's spin axis relative to the distant stars as the gyroscope orbits the Earth. An alternative view is to consider the above effect as a manifestation of the geometry of the field generated by a rotating body, and then determine it directly with a gravity gradiometer. This is, in fact, possible and proposals have been elaborated [8, 9].

Attempts to quantize gravity follow two different ideas. One approach is to start with gravity as a result of the geometrical curvature of four-dimensional space-time and to introduce more space-time dimensions in analogy with the early suggestion to geometrize electromagnetism in a theory with a fifth dimension. In this same approach an alternative view is to assume that gravity is connected with the gross structure of space-time but that at a smaller scale, further topologies, associated with other interactions, are present.

The second idea starts with a reverse approach. This is related to the attempts at a unified theory of interactions. Is it, in fact, possible that gravity is not quantized and all the rest of the world is? The so-called Standard Model of electroweak and strong interactions is in impressive agreement with experiments. In this model all matter is composed of fermions, whereas the fields responsible for their interactions are mediated by gauge bosons. A step beyond is to directly link matter and gauge fields. A symmetry directly associating particles of integer and half-integer spin is called a "supersymmetry", and there is hope that it can encompass the spin-2 fields of Einstein gravity.

This wide variety of theoretical scenarios naturally suggests new long-range interactions. Four-dimensional string theories almost inevitably predict dilatons that are scalar partners of the graviton and may couple to matter with roughly gravitational strength. For example, the existence of a massless dilaton entails a number of observable deviations from general relativity. In particular, a violation of the principle of equivalence, thus providing a motivation for trying to improve the precision of tests of the universality of free fall [10]. The principle was tested by Eötvös at an accuracy of 1 part in 10^8 [3]. Present accuracy is roughly between 10^{-11} and 10^{-12} [11, 12]. According to the calculations of ref. [10] deviations could be expected already at a level of 10^{-14}.

If the partners of the conventional spin-2 graviton have non-zero but sufficiently low mass the forces they generate have ranges long enough to originate macroscopic effects. The discussion of all this zoo is outside the scope of this brief paper. The interested reader may find a general review, both of theories and experiments in ref. [13]. The experiments relevant to this case are generally named "fifth force" experiments. At present only upper limits have been set, and they are reported in fig. 3.

We conclude by recalling that the STEP mission, we have previously mentioned, will push the precision of the principle of equivalence to 1 part in 10^{17}. In addition, it

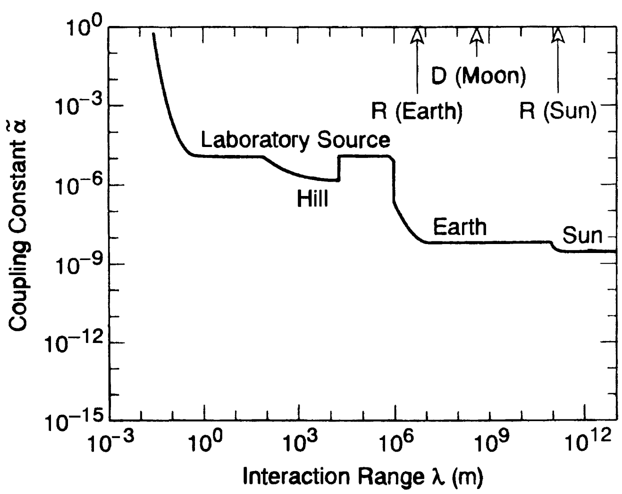

Fig. 3. – Upper limits of "fifth force" "strength" α as a function of range λ $[U_5 = (Gm/r)\,\alpha e^{-r/\lambda}]$.

will carry on board an experiment to detect spin-dependent forces of the Moody-Wilczek type [14] and is supposed to improve the limits in the value of G (constant of gravity) and in deviations from the inverse square law at short range (a few cm) by two orders of magnitude.

4. – Instruments

We have given in the previous sections a number of examples where the importance of performing measurements of feeble forces or accelerations is clearly demonstrated. We describe now the general principles underlying the functioning of the appropriate sensors. The basic instrument (accelerometer or gravimeter or displacement sensor) is essentially a mass-spring system plus a read-out circuit to detect motions of the sensing mass. Appropriate geometrical combinations of two or more of such accelerometers gives an in-line (T_{ii}) or cross-component (T_{ij}) gradiometer, depending on the relative orientation of the sensitive axes. Therefore we limit ourselves to a discussion of the single component, which we call in all generality the accelerometer. Before describing some practical realizations we want to point out what limitations are encountered as to the sensitivity of these instruments. Two kinds of noise are relevant: intrinsic noise and external noise.

Intrinsic noise sources are

a) Brownian motion of the sensing mass, proportional to the temperature T.

b) Current (back-action) and voltage noise of the amplifier (the influence of these quantities is reversed if a charge amplifier is used), whose combined action is summarized in the noise temperature T_{n}.

As a consequence, one can show that the minimum detectable acceleration in a bandwidth Δf is:

$$a_m^2 \geqslant \frac{\omega_0}{m} \Delta f k \left[\frac{4T}{Q} + 2T_{\mathrm{n}} \frac{\omega_0}{\Omega} \right], \tag{6}$$

where m, ω_0, Q are the mass, resonance frequency and merit factor, respectively, of the mechanical oscillator and Ω is the frequency at which the signal is detected. Thus, it is important to keep the frequency of resonance as low as possible compatible with the bandwidth of the detector, which generally is made to work at frequencies sensibly below the resonance.

Read-out systems are basically of two types: capacitive or inductive.

In the first case motion of the sensing mass modulates a capacitor which, inserted in an AC bridge, generates an unbalance resulting in a voltage signal around the carrier frequency Ω of the polarization of the bridge. As ω_0 is about 1 Hz and Ω 100 kHz, we see that in this scheme the noise temperature of the amplifier is relatively unimportant. Not so for the inductive case where modulation of an inductance having a persistent current (superconductive regimes are used) generates a current at the frequency of the mechanical signal. In this case $\Omega < \omega_0$, and very low noise amplifiers are to be used. The solution is in general in the use of a SQUID. Figure 4 gives a sketch of a capacitive sensor in a bridge configuration. Typical gradiometer sensitivities are 10^{-2} EU ($1\ \mathrm{EU} = 10^{-9}\,\mathrm{s}^{-2}$) in the non-cryogenic case and 10^{-4} EU in the cryogenic one. The

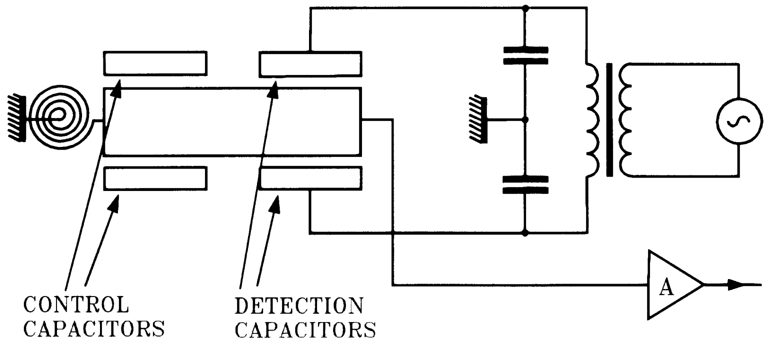

Fig. 4 – Scheme of accelerometer with capacitive sensor.

sensing mass suspension may be mechanical, or electromagnetic. A common requisite is that the equivalent springs be both soft and with very small losses in order to keep the level of thermal noise low. In the case of mechanical suspensions the proof mass, a parallelepiped or a cylinder, is suspended to a rigid frame by means of thin elastic bars, which may undergo flessural (Maryland [15]) or torsional (IFSI [16]) motions. The sensitive axis is thus well defined and orthogonal to the plane passing through the suspension and the center of mass of the proof mass. This solution has the advantage of simplicity of operation and is able to work at all temperatures. It is, however, difficult to get very low resonance and configurations suitable for a three axis instrument.

It is well known that one cannot suspend passively a body in an electric field (Earnshow theorem). The same is true for a magnetic field except if perfectly diamagnetic materials, *i.e.* superconductors, are used. Thus, in general, to introduce the necessary stability, a proper feed-back system must be exploited. An electrostatic suspension is used by ONERA [17] for room temperature accelerometers. The disadvantage of a certain complication is counterbalanced by the ability to use almost free masses and the capability, in principle, to allow three-axis instruments.

Diamagnetic suspensions, which are considered for STEP [4], are still in a very preliminary phase. They allow very low resonance frequencies and an almost ideal behaviour of the suspended mass.

A common problem of the electromagnetic suspensions is that the proof mass is electrically insulated and difficulties may arise, for example, with charging due to cosmic radiation.

External disturbances constitute a severe problem. On one side subtle backgrounds from the known and stronger interactions may induce false detection, on the other random motions, of seismic origin at ground or due to random accelerations originating from drag fluctuations in a spacecraft, may set stringent limits to the sensitivity. In principle many perturbing influences of non-gravitational origin can be eliminated by properly screening them with careful temperature control, magnetic or electrical shielding, etc. Gravity, however, cannot be shielded and *ad hoc* solutions, when possible, have to be adopted for each specific case, often making recourse to exploitation of some symmetries of the detector. So, for example, to minimize the influence of moving masses in the vicinity of the experiment, proper shapes of the sensor are selected, in order to take into account the contributions of higher multipoles.

Random motions of mechanical or seismic origin are difficult to damp because the great majority of experiments are performed at very low frequency. Even if in some particular case use of pendulums in quiet places [16] may help, the only general solution is the use of drag-free satellites where random accelerations as low as $10^{-13}\,g/\sqrt{\mathrm{Hz}}$ can be reached.

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