

1 A note on the analysis of two-stage task results: how changes in
2 task structure affect what model-free and model-based
3 strategies predict about the effects of reward and transition on
4 the stay probability

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Abstract

Many studies that aim to detect model-free and model-based influences on behavior employ two-stage behavioral tasks of the type pioneered by Daw and colleagues in 2011. Such studies commonly modify existing two-stage decision paradigms in order to better address a given hypothesis, which is an important means of scientific progress. It is, however, critical to fully appreciate the impact of any modified or novel experimental design features on the expected results. Here, we use two concrete examples to demonstrate that relatively small changes in the two-stage task design can substantially change the pattern of actions taken by model-free and model-based agents. In the first, we show that, under specific conditions, computer simulations of purely model-free agents will produce the reward by transition interactions typically thought to characterize model-based behavior on a two-stage task. The second example shows that model-based agents' behavior is driven by a main effect of transition-type in addition to the canonical reward by transition interaction whenever the reward probabilities of the final states do not sum to one. Together, these examples emphasize the benefits of using computer simulations to determine what pattern of results to expect from both model-free and model-based agents performing a given two-stage decision task in order to design choice paradigms and analysis strategies best suited to the current question.

1 Introduction

The brain contains multiple systems that interact to generate decisions, among them model-free systems, which reinforce rewarded actions and create habits, and model-based systems, which build a model of the environment to plan toward goals. Model-free and model-based influences on behavior can be dissociated by multi-stage behavioral tasks. In such tasks, agents predict different state-action-reward contingencies depending on whether or not they employ a model of the task, i.e., whether or not they know how the transitions between task states most often occur [1]. Since the original two-stage task was first proposed and reported by Daw et al. [1], it or one of its variations has been employed by many studies on decision making (e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]).

In the original two-stage task [1], each trial takes the participant sequentially through two different environmental states, where they must make a choice (Fig 1). Typically, at the initial state, the participant makes a choice between two actions, which we will refer to as “left” or “right.” Each initial-state action has a certain probability of taking the participant to one of two final states, which will be called “pink” and “blue.” Importantly, each initial-state action has a higher probability (for example, 0.7) of taking the participant to one of the final states, the “common” transition, and a lower probability (for example, 0.3) of taking the participant to the other final state, the “rare” transition. Let us assume that the left action commonly transitions to the pink state and the right

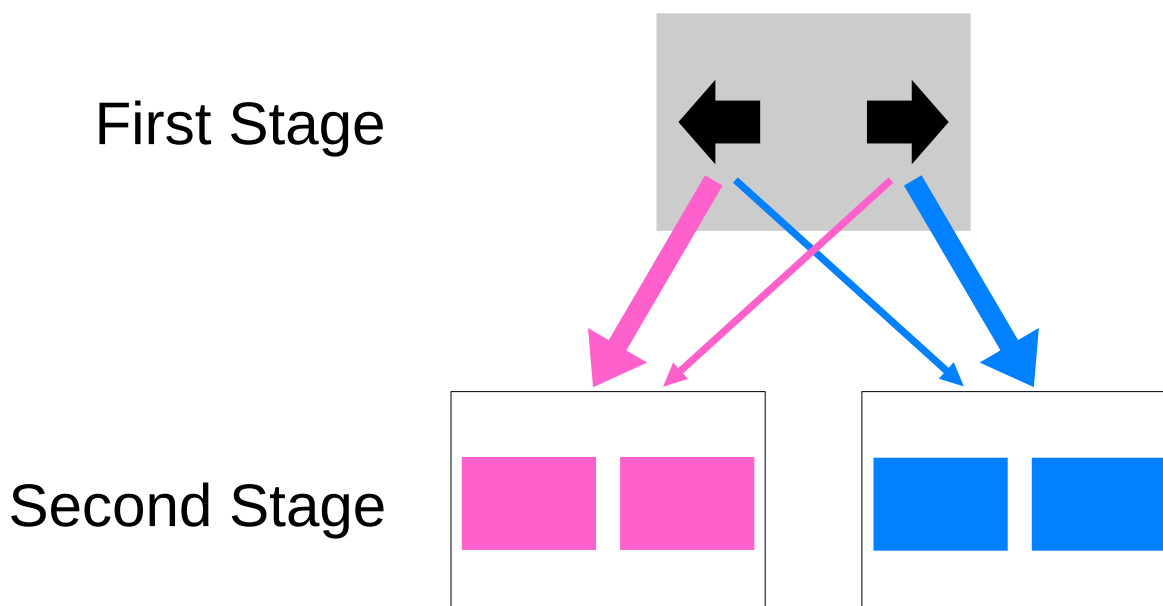


Fig 1: **Scheme of a typical two-stage task.** The thicker arrow indicates the common transition and the thinner arrow indicates the rare transition.

44 action commonly transitions to the blue state. A participant should thus choose left if they want to
45 maximize the probability of reaching the pink state and right if they want to maximize the probability
46 of reaching the blue state. At the final state, the participant makes another choice between one or more
47 actions (typically two), and each final-state action may or may not result in a reward with a certain
48 probability. Typically, the probability of reward, or in some cases the reward magnitude, changes from
49 trial to trial in order to promote continuous learning throughout the experiment.

50 Daw et al. [1] proposed that, to analyze the results of this task, each initial-state choice is coded
51 as 1 if it is a stay, that is, the participant has repeated their previous choice, or as 0 otherwise. Then,
52 the participant's stay probability is calculated depending on whether the previous trial was rewarded
53 or not and whether the previous transition was common or rare. This analysis involves performing a
54 logistic regression in which the stay probability is a function of two factors, reward and transition.

55 Applying this analysis to results obtained from simulated model-free or model-based agents pro-
56 duces a plot similar to that shown in Fig 2. (Note that the exact stay probability values depend on the
57 simulated agents' parameters.) It is observed that for model-free agents, only reward affects the stay
58 probability, and for model-based agents, only the interaction between reward and transition affects the
59 stay probability. This difference allows us to distinguish between model-free and model-based choices.

60

61 The choice patterns of model-free and model-based agents in Fig 2 are different because model-
62 based reinforcement learning algorithms take into account the task structure and model-free algorithms
63 do not, with the result that they make different predictions about which action agents will choose at

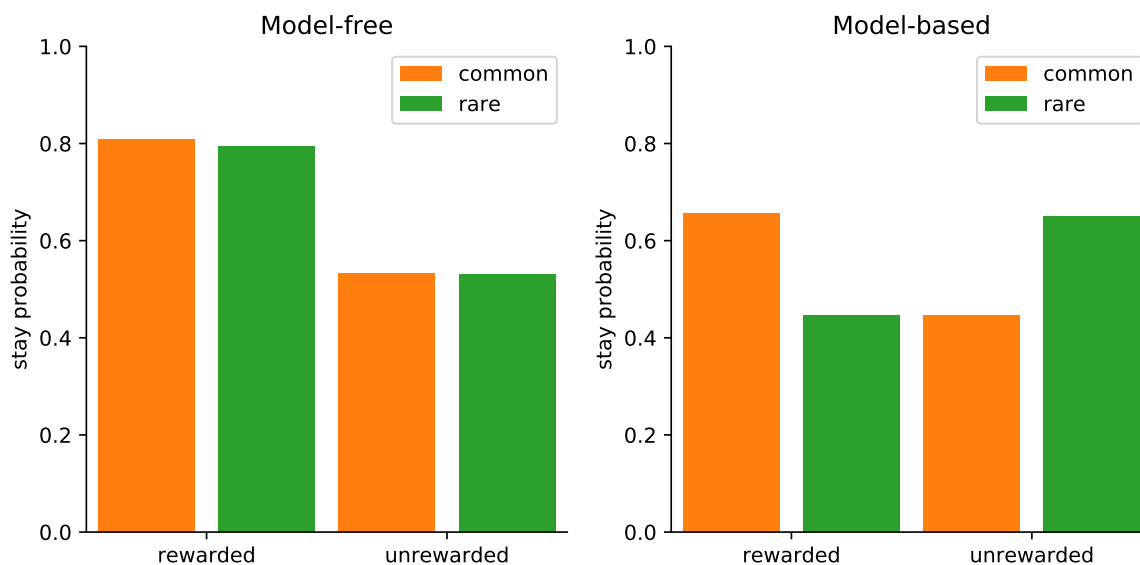


Fig 2: Results of a typical two-stage task, obtained by simulating model-free and model-based agents.

64 the initial stage. Here, we use “agent” as a general term to refer to either a computer simulation or a
65 human or non-human participant. The model-free SARSA($\lambda = 1$) algorithm predicts that if an agent
66 makes a certain initial-state choice in a trial, they are more likely to repeat it on the next trial if it was
67 rewarded, whether the transition was common or rare. A model-based algorithm [1], however, predicts
68 that the agent is more likely to repeat the previous choice if, in the previous trial, it was rewarded
69 *and* the transition was common, or if it was unrewarded *and* the transition was rare. For example,
70 suppose an agent chooses left, is taken to the blue state through the rare transition, and receives a
71 reward. In this case, the model-free prediction is that the agent is more likely to choose left again in
72 the next trial, while the model-based prediction is that the agent is instead more likely to switch and
73 chose right. The model-based agent is predicted to choose to go right, instead of left, at the initial
74 state because the right action maximizes the probability of reaching the blue state, where the agent
75 received the reward on the previous trial.

76 One might assume that even if the two-stage task structure is slightly changed to suit a particular
77 research goal, model-free-driven actions will remain unaffected by transition-types because the model-
78 free algorithm predicts that rewarded actions are more likely to be repeated regardless of transition.
79 Similarly, one might assume that model-based choices will not be affected by reward because reward
80 effects are characteristic of model-free actions. However, the general danger of relying on untested
81 assumptions is well-known, and our work here aims to highlight the particular dangers of assuming
82 fixed relationships between reward, transition-types, and model-free or model-based processing in two-
83 stage tasks. It has already been demonstrated that these assumptions do not hold for a simplified

84 version of the two-step task, optimized for animal subjects [15]. Here, we demonstrate by means
85 of computer simulation that even seemingly small changes in task design can change the resulting
86 choice patterns for model-based and model-free agents. For example, depending on the task details,
87 it is possible that the stay probability of model-free agents is larger for common transitions than for
88 rare transitions (i.e. that there is an interaction between reward and transition of the type thought
89 to characterize model-based behavior). Below, we demonstrate two concrete examples of how slight
90 changes in task design strongly affect the results of model-free and model-based agents. We also
91 explain why these task features change the *expected* behavior of model-free and model-based agents
92 and offer some further thoughts on how to analyze data from these modified tasks. Together, these
93 examples emphasize the importance of simulating the behavior of model-free and model-based agents
94 on any two-stage task, especially novel modifications, in order to determine what pattern of behavior
95 to expect.

96 **2 Results**

97 **2.1 Unequal reward probabilities make model-free agents indirectly sensi-** 98 **tive to transition probabilities**

99 Contrary to the assumptions of many researchers, it is not universally true that the stay probability of
100 model-free agents is only affected by reward or that the stay probability of model-based agents is only
101 affected by the interaction between reward and transition. Therefore, the stay probability plot will
102 not necessary follow the “classic” pattern shown in Fig 2; alterations in this pattern can stem from
103 seemingly small and innocuous variations in the properties of the two-stage task.

104 The behavior of model-free agents is indirectly sensitive to the relative reward probabilities of the
105 final states. If, for instance, we set the reward probabilities of the actions at the pink state to a fixed
106 value of 0.8 and the reward probabilities of the actions at the blue state to a fixed value of 0.2, we
107 obtain the results shown in Fig 3 instead of those shown in Fig 2. (Similar results have already been
108 observed by Smittenaar et al. [6] and Miller et al. [15].) Recall that these are computer-simulated
109 model-free agents, who cannot use a model-based system to perform the task because they do not
110 have one. Thus, this pattern cannot result from a shift between model-free and model-based influences
111 on behavior.

112 The reason for this change is not that the reward probabilities are now fixed rather than variable.
113 If we fix the reward probabilities to 0.5, we obtain the original pattern again, as shown in Fig 4. In
114 their original paper, Daw et al. [1] noted that the reward probabilities drift from trial to trial because
115 this encourages participants to keep learning. Continued learning is a critical feature for testing many

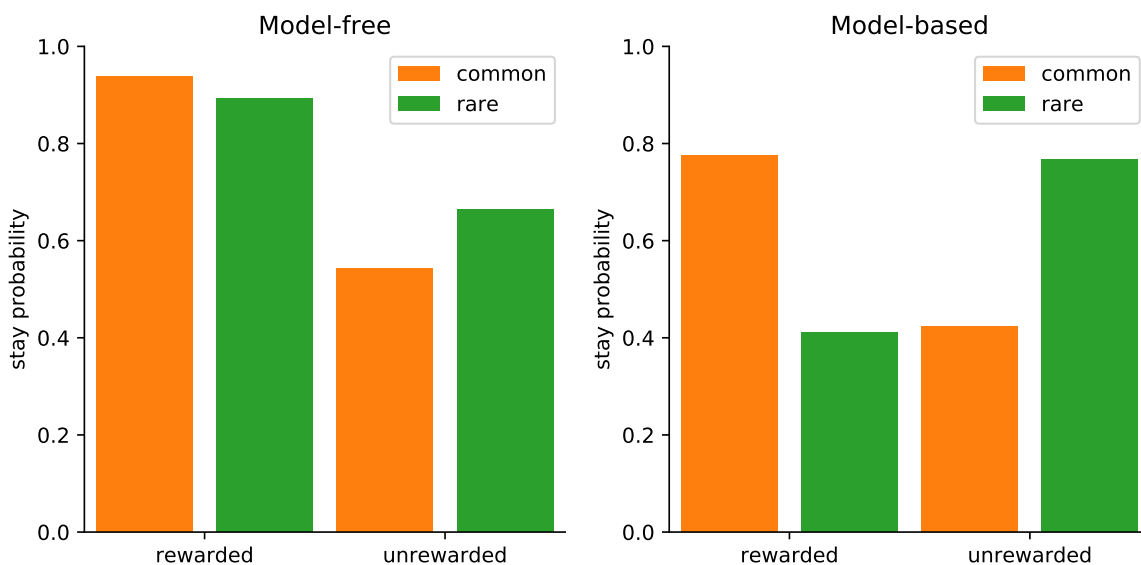


Fig 3: **Results of a two-stage task wherein one final state has a higher reward probability than the other and $\lambda = 1$.**

116 hypotheses, but it is not the feature that distinguishes model-free from model-based behavior.

117 The different model-free pattern in Fig 3 versus Fig 2 is caused by one final state being associated
118 with a higher reward probability than the other. If actions taken at one final state are more often
119 rewarded than actions taken at the other final state, the initial-state action that commonly leads to the
120 most frequently rewarded final state will also be rewarded more often than the other initial-state action.
121 This means that in trials that were rewarded after a common transition or unrewarded after a rare
122 transition, corresponding to the outer bars of the plots, the agent usually chose the most rewarding
123 initial-state action, and in trials that were rewarded after a rare transition or unrewarded after a
124 common transition, corresponding to the inner bars of the plots, the agent usually chose the least
125 rewarding initial-state action. Since one initial-state action is more rewarding than the other, model-
126 free agents will learn to choose that action more often than the other, and thus, the stay probability
127 for that action will be on average higher than the stay probability for the other action. This creates
128 a tendency for the outer bars to be higher than the inner bars, and alters the pattern of model-free
129 results relative to the canonical pattern by introducing an interaction between reward and transition.
130 It does not alter the pattern of model-based results because model-based results already have higher
131 outer bars and lower inner bars even if all reward probabilities are 0.5 (or stochastically drifting around
132 0.5).

133 Furthermore, unequal final-state reward probabilities will have an even greater effect on model-free
134 agents with an eligibility trace parameter $\lambda < 1$ (Fig 5). This is because the values of the initial-state
135 actions are updated depending on the values of the final-state actions, which causes the action that

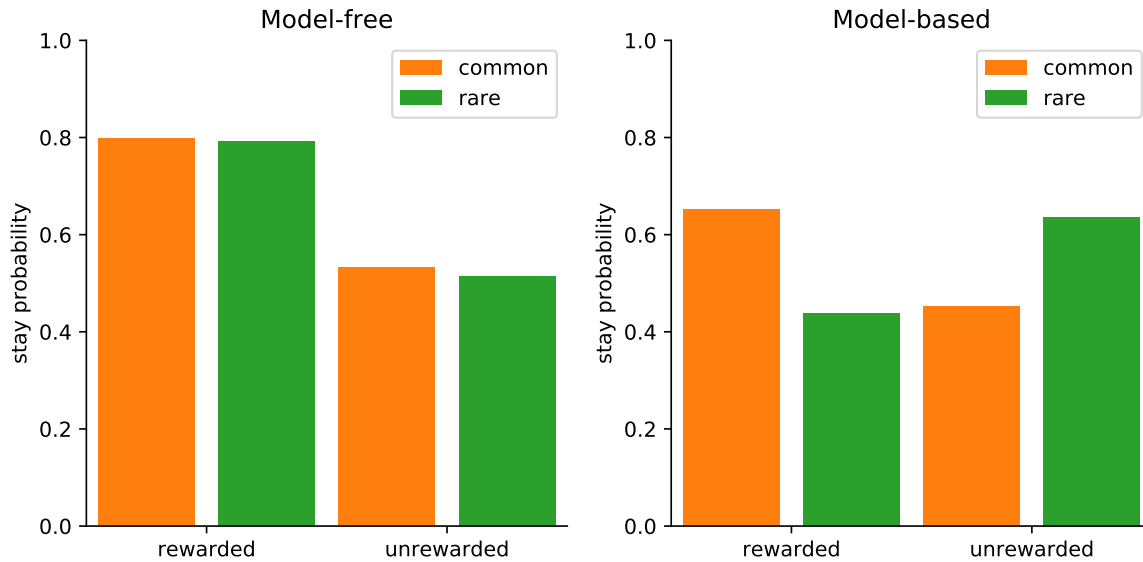


Fig 4: **Results of a two-stage task wherein both final states have 0.5 reward probability.**

136 takes the agent to the most rewarding final state to be updated to a higher value than the action that
 137 takes it to the least rewarding final state (see Equation 6 in the Methods section for details).

138 It also follows that if the reward probabilities of the final state-actions drift too slowly relative to
 139 the number of trials, model-free results will also exhibit an interaction between reward and transition.
 140 This is why the simulated results obtained by Miller et al. [15] using a simplified version of the two-step
 141 task do not exhibit the expected pattern; it is not because the task was simplified by only allowing
 142 one action at each final state. In that study, there was a 0.02 probability that the reward probabilities
 143 of the two final-state action (0.8 and 0.2) would be swapped, unless they had already been swapped
 144 in the previous 10 trials. If the swap probability is increased to 0.2 for a task with 250 trials, the
 145 canonical results are obtained instead (results not shown).

146 Despite changes in the expected pattern of model-free choices, it is still possible to use this mod-
 147 ification of the task to distinguish between model-free and model-based agents based on reward and
 148 transition. In order to do so, we simply need to include two more features in the data analysis. As
 149 previously discussed, experimental data from two-stage tasks are typically analyzed by a logistic re-
 150 gression model, with p_{stay} , the stay probability, as the dependent variable, and x_r , a binary indicator
 151 of reward (+1 for rewarded, -1 for unrewarded), x_t , a binary indicator of transition (+1 for common,
 152 -1 for rare), and $x_r x_t$, the interaction between reward and transition, as the independent variables:

$$p_{\text{stay}} = \frac{1}{1 + \exp[-(\beta_0 + \beta_r x_r + \beta_t x_t + \beta_{r \times t} x_r x_t)]}. \quad (1)$$

153 The levels of the independent variables were coded as +1 and -1 so that the meaning of the coefficients

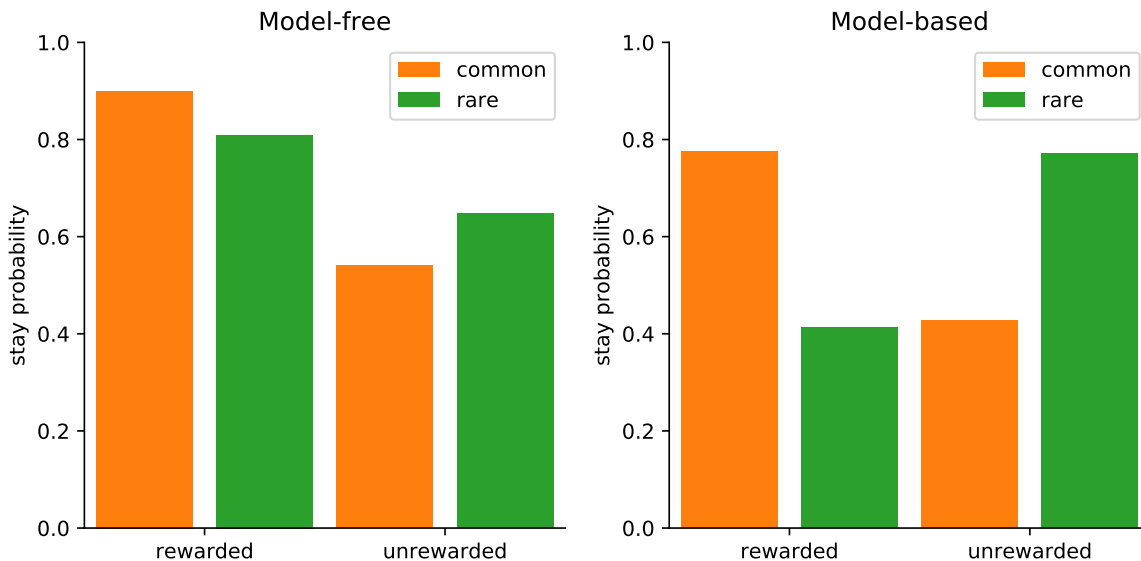


Fig 5: Results of a two-stage task wherein one final state has a higher reward probability than the other and $\lambda < 1$.

154 are easy to interpret: β_r indicates a main effect of reward, β_t indicates a main effect of transition, and
 155 $\beta_{r \times t}$ indicates an interaction between reward and transition. We applied this analysis to create all the
 156 plots presented so far, which can also be created from raw simulation data with similar results. In the
 157 modified task we just discussed, the $\beta_{r \times t}$ coefficient is positive for model-free agents, which does not
 158 allow us to distinguish between purely model-free and hybrid model-free/model-based agents.

159 We can, however, obtain an expected null $\beta_{r \times t}$ coefficient for purely model-free agents if we add
 160 two control variables to the analysis: x_c , a binary indicator of the initial-state choice (+1 for left, -1
 161 for right), and x_f , a binary indicator of the final state (+1 for pink, -1 for blue):

$$p_{\text{stay}} = \frac{1}{1 + \exp[-(\beta_0 + \beta_r x_r + \beta_t x_t + \beta_{r \times t} x_r x_t + \beta_c x_c + \beta_f x_f)]} \quad (2)$$

162 These two additional variables control for one initial-state choice having a higher stay probability than
 163 the other and for one final state having a higher reward probability than the other, respectively. The
 164 variable x_f is only necessary for model-free agents with $\lambda < 1$, because only in this case are the values
 165 of the initial-state actions updated depending on the values of the final-state actions.

166 By applying this extended analysis to the same data used to generate Fig 5 and setting $x_c = x_f = 0$,
 167 we obtain Fig 6, which is nearly identical to Figs 2 and 4. This result demonstrates that even though
 168 the original analysis fails to distinguish between model-free agents and hybrid agents, other analyses
 169 may succeed if they can extract more or different information from the data. Another analysis that
 170 can be applied for this task is to fit a hybrid model to the data and estimate the model-based weight
 171 (see [1] for details), although fitting a reinforcement learning model is much more computationally

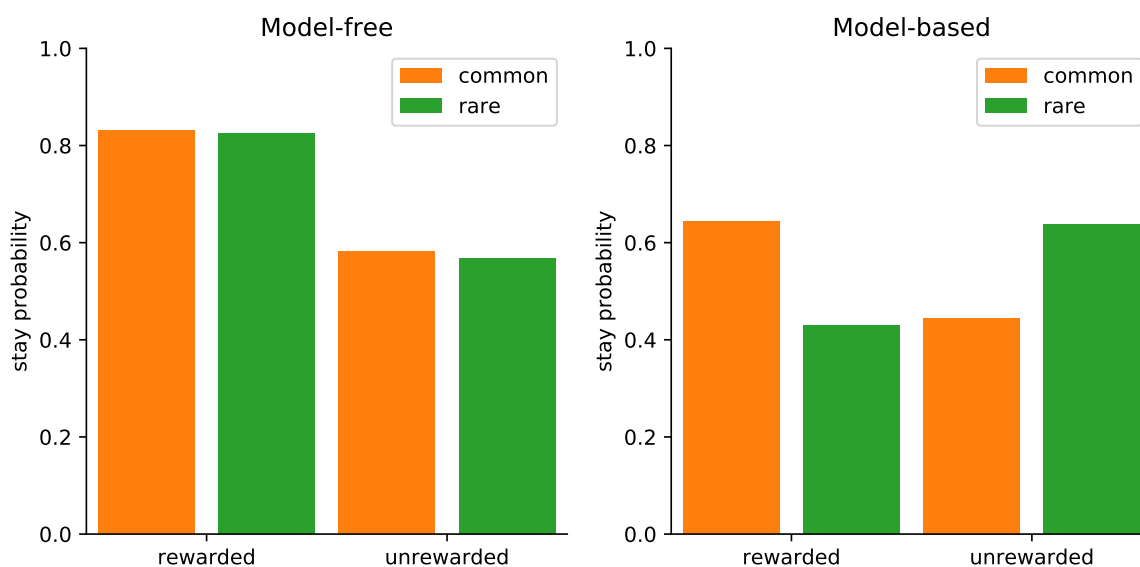


Fig 6: **Results using the extended analysis.** Results of a two-stage task wherein one final state has a higher reward probability than the other and $\lambda < 1$, obtained by adding additional control variables to the logistic regression model.

172 intense than fitting a logistic regression model such as that of Equation 2, and the results will be
173 sensitive to the details of the reinforcement learning implementation.

174 **2.2 Model-based agents will show main effects of transition in addition to** 175 **transition by reward interactions under specific task conditions**

176 When the final state probabilities do not sum to one, model-based agents will show both a main effect
177 of transition and a transition by reward interaction. An example of these combined influences on
178 model-based behavior can be seen in Fig 7. This pattern was generated by modifying the original
179 two-stage task so that the reward probability of all actions available at the pink and the blue states
180 was 0.8. In this case, the reward probabilities of both final states are the same, and therefore, the stay
181 probability of model-free agents is only affected by reward. On the other hand, the stay probability of
182 model-based agents is not only affected by the interaction between reward and transition, but also by
183 transition type itself. This main effect of transition can be seen in the right panel of Fig 7 by comparing
184 the two outermost and innermost bars, which show that the common transitions (orange bars) lead
185 to a lower stay probability relative to the corresponding rare transitions (green bars). This negative
186 effect of common transitions on stay probabilities is because the sum of the reward probabilities of the
187 final states, 0.8 and 0.8, is 1.6, which is greater than 1.

188 Fig 8 shows the relative extent to which the stay probabilities of model-based agents are influenced
189 by transition type as a function of the sum of the reward probabilities at the final state. Let p be

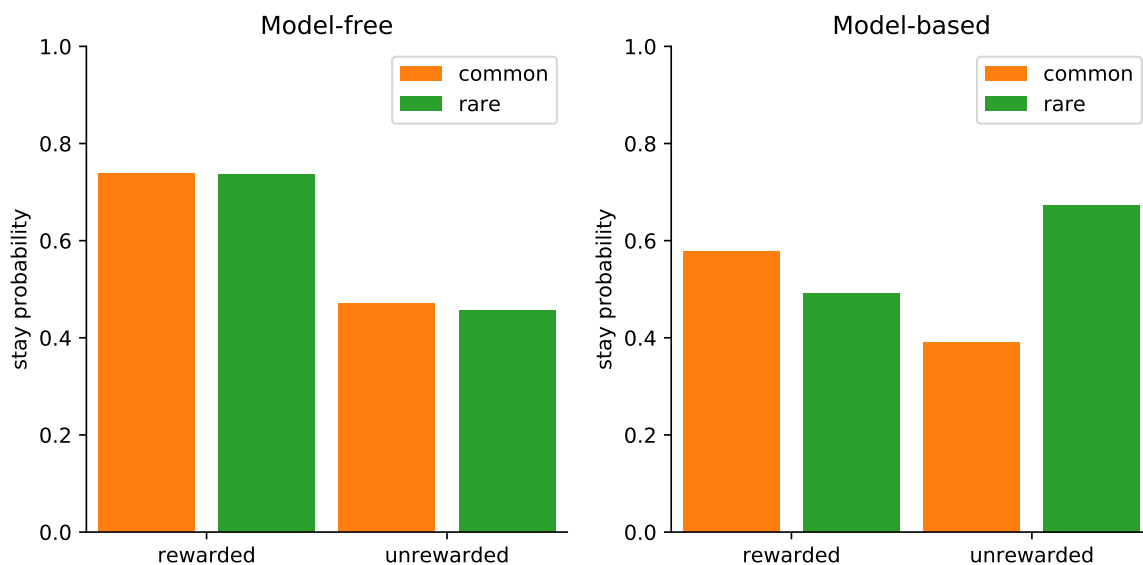


Fig 7: Results of a two-stage task wherein both final states have 0.8 reward probability.

190 the value of the most valuable action at the pink state and b the value of the most valuable action at
191 the blue state. The relative stay probabilities for model-based agents will be lower following common
192 than rare transitions when $p + b > 1$. Conversely, relative stay probabilities for model-based agents
193 will be higher following common than rare transitions when $p + b < 1$. Fig 8 shows the difference
194 in stay probabilities between common and rare transitions as a function of both the sum of the final
195 state reward probabilities and learning rate, α . Indeed, this graphic shows that model-based agents
196 will show a main effect of transition in all cases except when $p + b = 1$. We explain the intuition and
197 algebra behind this characteristic of our model-based agents in the following paragraphs.

198 Model-based agents make initial-state decisions based on the difference, $p - b$, between the values of
199 the most valuable actions available at the pink and blue states (this is a simplification; further details
200 are given in the Methods section). As $p - b$ increases, the agent becomes more likely to choose left,
201 which commonly takes it to pink, and less likely to choose right, which commonly takes it to blue. This
202 difference increases every time the agent experiences a common transition to pink and is rewarded (p
203 increases) or experiences a rare transition to blue and is not rewarded (b decreases). Analogously, this
204 difference decreases every time the agent experiences a common transition to blue and is rewarded (b
205 increases) or experiences a rare transition to pink and is not rewarded (p decreases). This is why the
206 model-based agent's stay probabilities are affected by the interaction between reward and transition.
207 But $p - b$ may change *by different amounts* if the agent experiences a common transition and is rewarded
208 or if it experiences a rare transition and is not rewarded. If the agent experiences a common transition

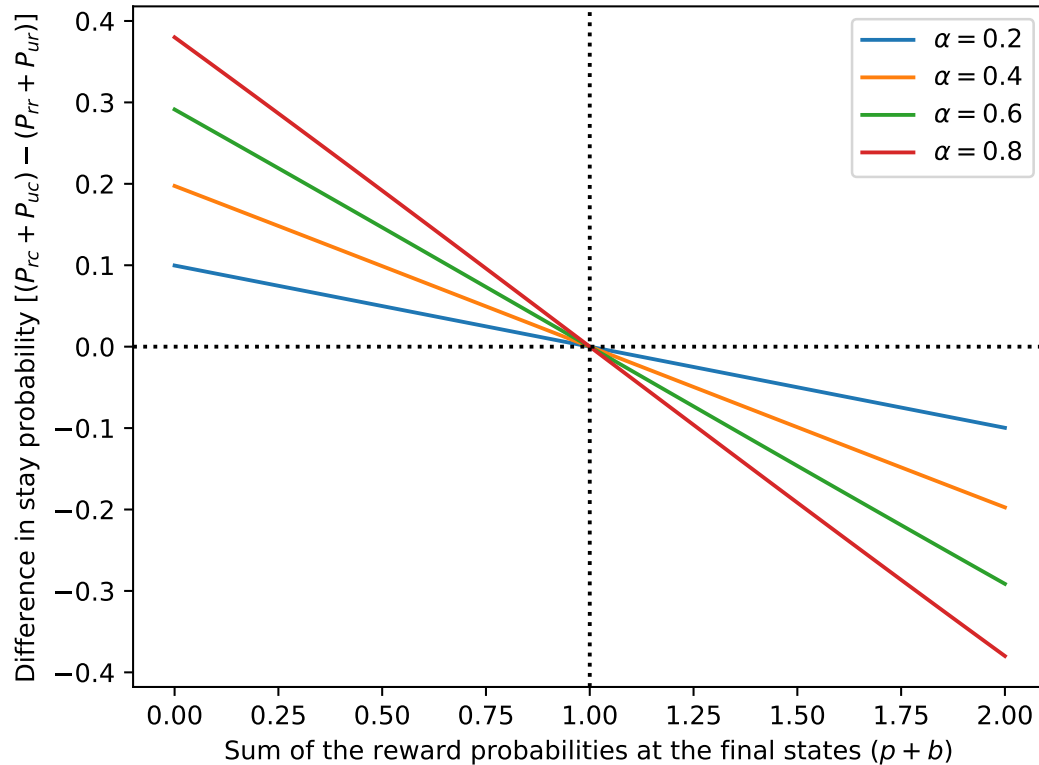


Fig 8: Difference in stay probability for model-based agents. Differences between the sum of the stay probabilities for model-based agents following common versus rare transitions (i.e., the sum of the orange bars minus the sum of the green bars) as a function of the sum of the reward probabilities at the final state ($p + b$). This example plot was generated assuming that final state reward probabilities are equal ($p = b$) and that the exploration-exploitation parameter in Equation 13 is $\beta = 2.5$. When computing the differences in stay probability on the y-axes, P_{rc} stands for the stay probability after a common transition and a reward, P_{uc} is the stay probability after a common transition and no reward, P_{rr} is the stay probability after a rare transition and a reward, and P_{ur} is the stay probability after a rare transition and no reward.

209 to pink and receives 1 reward, the difference between the final-state values changes from $p - b$ to

$$[(1 - \alpha)p + \alpha \cdot 1] - b, \quad (3)$$

210 where $0 \leq \alpha \leq 1$ is the agent's learning rate. If, on the other hand, the agent experiences a rare
211 transition to blue and receives 0 rewards, the difference between the final-state values becomes

$$p - [(1 - \alpha)b + \alpha \cdot 0]. \quad (4)$$

The two values are the same only if

$$\begin{aligned} [(1 - \alpha)p + \alpha \cdot 1] - b &= p - [(1 - \alpha)b + \alpha \cdot 0] \\ p - \alpha p + \alpha - b &= p - b + \alpha b \\ -\alpha p + \alpha - \alpha b &= 0 \\ \alpha(1 - p - b) &= 0 \\ 1 - p - b &= 0 \text{ (assuming } \alpha > 0) \\ p + b &= 1 \end{aligned} \quad (5)$$

212 that is, when the sum of the final-state action values is 1. This is expected to occur when the actual
213 reward probabilities of the final states sum to 1, as p and b estimate them. Thus, when the reward
214 probabilities do not sum to 1, the outer bars of the stay probability plots may not be the same height.
215 Similarly, $p - b$ may change by different amounts if the agent experiences a common transition and
216 is not rewarded or if the agent experiences a rare transition and is rewarded, which also occurs when
217 the reward probabilities do not sum to 1 (calculations not shown) and causes the inner bars of the
218 stay probability plots to be different heights. In the S1 Appendix to this paper, we prove that this
219 specifically creates a transition effect.

220 The end result is that the model-based behavior is not solely a function of the interaction between
221 reward and transition, but also of the transition in many cases. Unlike our previous example, the main
222 effect of transition cannot be corrected for by adding the initial-state choice and the final state as
223 control variables. Fortunately, however, the original analysis can still be used to distinguish between
224 model-free and model-based agents on this task because model-free agents exhibit only reward effects
225 while model-based agents exhibit only transition and reward by transition interaction effects.

226 **3 Discussion**

227 The class of two-stage tasks pioneered by Daw et al. [1] has been instrumental in advancing efforts in the
228 behavioral, computational, and biological sciences aimed at teasing apart the influences of model-free
229 and model-based behavior and how the relative influences of these systems may change as a function of
230 environmental context, biological development, and physical or mental health ([2, 3, 4, 5, 6, 7, 8, 9, 10,
231 11, 12, 13, 14, 15, 16, 17] among many others). The continued and expanded utilization of such tasks
232 will require design modifications to better address specific new hypotheses and such efforts currently
233 constitute an active and productive line of research across multiple scientific disciplines.

234 In the current paper, we have shown that modifications to established versions of the two-stage task
235 design may deviate substantially from the expected patterns of results for both model-free and model-
236 based agents. Specifically, the canonical pattern of stay probabilities being driven solely by rewards for
237 model-free agents versus reward by transition interactions in model-based agents is not as universal as
238 is often assumed. Instead, the patterns of behavior produced by model-free and model-based agents
239 are rather sensitive to changes in task features or learning algorithms. Indeed, it is important to keep
240 in mind that the examples we discuss here also rely on specific task features and parameterizations of
241 the model-free and model-based learning algorithms.

242 Fortunately, there is a very straightforward means of avoiding potential design flaws or misinter-
243 pretations created by incorrect assumptions about the nature of model-free and model-based behavior
244 in a given context —*test* how any changes in task design affect model-free and model-based agents’
245 choice patterns. Specifically, researchers who plan to use customized two-stage-style tasks in their work
246 should check by computer simulation of model-free and model-based agents what patterns each type of
247 agent will produce in the new paradigm. Such simulation exercises will allow researchers to better un-
248 derstand both the intended as well as potential unintended consequences of their design modifications
249 *before* spending the time, effort, and money to acquire data from human participants or non-human
250 animals. This will lead to better experimental designs that in turn yield more readily interpretable
251 and informative conclusions about the question(s) of interest.

252 **4 Methods**

253 The code used to generate the results discussed in this paper is available at Github: [https://github.](https://github.com/carolfs/note_analysis_2stage_tasks)
254 [com/carolfs/note_analysis_2stage_tasks](https://github.com/carolfs/note_analysis_2stage_tasks)

255 4.1 Task

256 The results were obtained by simulating model-free and model-based agents performing the two-stage
257 task reported by Daw et al. [1] for 250 trials. In each trial, the agent first decides whether to perform
258 the left or right action. Performing an action takes the agent to one of two final states, pink or blue.
259 The left action takes the agent to pink with 0.7 probability (common transition) and to blue with 0.3
260 probability (rare transition). The right action takes the agent to blue with 0.7 probability (common
261 transition) and to pink with 0.3 probability (rare transition). There are two actions available at final
262 states. Each action has a different reward probability depending on whether the final state is pink or
263 blue. All reward probabilities are initialized with a random number in the interval $[0.25, 0.75]$ and drift
264 in each trial by the addition of random noise with distribution $\mathcal{N}(\mu = 0, \sigma = 0.025)$, with reflecting
265 bounds at 0.25 and 0.75. Thus, the expected reward probability of final-state actions is 0.5.

266 4.2 Model-free algorithm

267 Model-free agents were simulated using the SARSA(λ) algorithm [18, 1]. The algorithm specifies that
268 when an agent performs an initial-state action a_i at the initial state s_i , then goes to the final state
269 s_f , performs the final-state action a_f and receives a reward r , the model-free value $Q_{MF}(s_i, a_i)$ of the
270 initial-state action is updated as

$$Q_{MF}(s_i, a_i) \leftarrow (1 - \alpha)Q_{MF}(s_i, a_i) + \alpha[(1 - \lambda)Q(s_f, a_f) + \lambda r], \quad (6)$$

271 where α is the learning rate and λ is the eligibility trace parameter [1]. Since λ is a constant, this
272 means that the value of an initial-state action is updated depending on the obtained reward and the
273 value of the performed final-state action. If $\lambda = 1$, the equation becomes

$$Q_{MF}(s_i, a_i) \leftarrow (1 - \alpha)Q_{MF}(s_i, a_i) + \alpha r, \quad (7)$$

274 that is, the updated value depends only on the reward. The value $Q_{MF}(s_f, a_f)$ of the final-state action
275 is updated as

$$Q_{MF}(s_f, a_f) \leftarrow (1 - \alpha)Q_{MF}(s_f, a_f) + \alpha r. \quad (8)$$

276 The values of all actions a for all states s are initialized at $Q_{MF}(s, a) = 0$.

277 The probability $P(a|s)$ that an agent will choose action a at state s is given by

$$P(a|s) = \frac{\exp[\beta Q_{MF}(s, a)]}{\sum_{a' \in \mathcal{A}} \exp[\beta Q_{MF}(s, a')]}, \quad (9)$$

278 where \mathcal{A} is the set of all actions available at state s and β is an exploration-exploitation parameter [18].

279 Unless indicated otherwise, in the simulations discussed in this paper, the learning rate of the
 280 model-free agents is $\alpha = 0.5$, the eligibility trace parameter is $\lambda = 0.6$, and the exploration parameter
 281 is $\beta = 5$.

282 4.3 Model-based algorithm

283 Model-based agents were simulated using the algorithm defined by Daw et al. [1]. Model-based agents
 284 make initial-state decisions based on the estimated value of the most valuable final-state actions and
 285 the transition probabilities. The value $Q_{MB}(s_i, a_i)$ of an initial-state action a_i performed at the initial
 286 state s_i is

$$Q_{MB}(s_i, a_i) = P(\text{pink}|s_i, a_i) \max_{a \in \mathcal{F}} Q(\text{pink}, a) + P(\text{blue}|s_i, a_i) \max_{a \in \mathcal{F}} Q(\text{blue}, a), \quad (10)$$

287 where $P(s_f|s_i, a_i)$ is the probability of transitioning to the final state s_f by performing action a_i and
 288 \mathcal{F} is the set of actions available at the final states [1].

289 When the agent receives a reward, it will update the value of the final-state action a_f performed
 290 at state s_f , $Q_{MB}(s_f, a_f)$, according to the equation

$$Q(s_f, a_f) \leftarrow (1 - \alpha)Q(s_f, a_f) + \alpha r, \quad (11)$$

291 where α is the learning rate and r is the reward. The values of all final-state actions a_f for all final
 292 states s_f are initialized at $Q_{MB}(s_f, a_f) = 0$.

293 Let $p = \max_{a \in \mathcal{F}} Q(\text{pink}, a)$ and $b = \max_{a \in \mathcal{F}} Q(\text{blue}, a)$. The probability $P(\text{left}|s_i)$ that the agent
 294 will choose the left action at the initial state s_i is given by

$$P(\text{left}|s_i) = \frac{1}{1 + \exp[\beta(P(\text{pink}|s_i, \text{left})p + P(\text{blue}|s_i, \text{left})b - P(\text{pink}|s_i, \text{right})p - P(\text{blue}|s_i, \text{right})b)]}, \quad (12)$$

295 where β is an exploration-exploitation parameter. If each initial-state action transitions to a different fi-
 296 nal state with the same probability, e.g., $P(\text{pink}|s_i, \text{left}) = P(\text{blue}|s_i, \text{right})$ and hence $P(\text{pink}|s_i, \text{right}) =$
 297 $P(\text{blue}|s_i, \text{left})$, this equation is simplified to

$$P(\text{left}|s_i) = \frac{1}{1 + \exp[\beta(P(\text{pink}|s_i, \text{left}) - P(\text{blue}|s_i, \text{left}))(p - b)]}. \quad (13)$$

298 Hence, the agent's probability of choosing left, the action that will take it more commonly to the pink
 299 state, increases with $p - b$.

300 In all simulations presented in this paper, the learning rate of model-based agents is $\alpha = 0.5$ and
301 the exploration parameter is $\beta = 5$.

302 4.4 Analysis

303 The simulation data were analyzed using the logistic regression models described in the Results section.
304 1,000 model-free and 1,000 model-based agents were simulated for each task modification discussed.
305 The regression models were fitted to the data using scikit-learn's regularized logistic regression classifier
306 with the liblinear algorithm [19].

307 5 Acknowledgments

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391 S1 Appendix

392 We will prove that if $p + b \neq 1$, then there is a transition effect on the results of model-based agents.
393 As explained in the Methods, if each initial-state action transitions to a different final state with the
394 same probability, then the probability $P(\text{left}|s_i)$ of choosing left at the initial state s_i is given by

$$P(\text{left}|s_i) = \frac{1}{1 + \exp[-K(p - b)]} = \text{logit}^{-1} K(p - b), \quad (14)$$

395 where $K \geq 0$ is a constant that depends on the transition probabilities and the exploration-exploitation
396 parameter.

397 According to the model-based reinforcement learning rule (Equation 11), if the agent chooses left,
398 then experiences a common transition to pink and receives 1 reward, the stay probability p_{stay} (of
399 choosing left again in the next trial) is given by

$$p_{\text{stay}} = \text{logit}^{-1} K[(1 - \alpha)p + \alpha - b]; \quad (15)$$

400 if instead the agent experiences a rare transition to blue and receives 1 reward, p_{stay} is given by

$$p_{\text{stay}} = \text{logit}^{-1} K[p - (1 - \alpha)b - \alpha]; \quad (16)$$

401 if the agent experiences a common transition to pink and receives 0 rewards, p_{stay} is given by

$$p_{\text{stay}} = \text{logit}^{-1} K[(1 - \alpha)p - b]; \quad (17)$$

402 and if the agent experiences a rare transition to blue and receives 0 rewards, p_{stay} is given by

$$p_{\text{stay}} = \text{logit}^{-1} K[p - (1 - \alpha)b]. \quad (18)$$

403 The logistic regression model, on the other hand, determines p_{stay} as a function x_r ($x_r = +1$ for 1
404 reward, $x_r = -1$ for 0 rewards in the previous trial) and x_t ($x_t = +1$ for a common transition, $x_t = -1$
405 for a rare transition in the previous trial):

$$p_{\text{stay}} = \text{logit}^{-1}(\beta_0 + \beta_r x_r + \beta_t x_t + \beta_{r \times t} x_r x_t). \quad (19)$$

Since logit^{-1} is a one-to-one function, this implies that

$$K[(1 - \alpha)p + \alpha - b] = \beta_0 + \beta_r + \beta_t + \beta_{r \times t}, \quad (20)$$

$$K[p - (1 - \alpha)b - \alpha] = \beta_0 + \beta_r - \beta_t - \beta_{r \times t}, \quad (21)$$

$$K[(1 - \alpha)p - b] = \beta_0 - \beta_r + \beta_t - \beta_{r \times t}, \quad (22)$$

$$K[p - (1 - \alpha)b] = \beta_0 - \beta_r - \beta_t + \beta_{r \times t}. \quad (23)$$

Solving this system for β_0 , β_r , β_t , and $\beta_{r \times t}$ yields

$$\beta_0 = K \left(1 - \frac{\alpha}{2} \right) (p - b), \quad (24)$$

$$\beta_r = 0, \quad (25)$$

$$\beta_t = K \frac{\alpha}{2} (1 - p - b), \quad (26)$$

$$\beta_{r \times t} = K \frac{\alpha}{2}, \quad (27)$$

406 which implies that if $\alpha > 0$, $K > 0$ and $p + b \neq 1$, then $\beta_t \neq 0$. This proof assumes that the agent chose
407 left, but the same can be proved if the agent chose right, as in this example “left,” “right,” “pink,”
408 and “blue” are arbitrary.