Bistability Effects
in the Strong Coupling Regime of
Cavity and Circuit Quantum Electrodynamics

Theses of the PhD Dissertation

ANDRÁS DOMBI

Supervisor: Dr. Peter Domokos

2016

I. INTRODUCTION

In classical optics, macroscopic matter, such as lenses, prisms, birefringent crystals, etc., is used to manipulate light. In atomic spectroscopy, classical coherent laser light is used to probe the electronic structure of the atoms and, even more, to manipulate the quantum state of the electron cloud, for example, by means of optical pumping. **Cavity quantum electrodynamics** unites these complementary aspects, since it studies the mutual action of atoms and light on each other. This interaction combines the material and radiation degrees of freedom into a single dynamical system, which is in the focus of the present thesis.

The research fields addressed in this thesis can be split into two very similar, yet significantly different, parts: atomic cavity quantum electrodynamics (also called in general cavity QED) and circuit quantum electrodynamics (circuit QED). These two are closely related in their theoretical and phenomenological frameworks but they are realized in vastly different experimental realms.

- A cavity QED system consists of a Fabry–Pérot-type resonator and particles trapped within or passing through the cavity.
- A circuit QED system is made up of microwave stripline resonators, superconducting circuit elements and Josephson junctions (artificial atoms or qubits) on a chip.

Both fields show promising potential for the development of quantum information processing, quantum computing and quantum communication, and both can already be used to study exciting physical phenomena. These include, among others, nonlinear optical devices at low photon numbers and many–body effects with controllable interacting objects like atoms and photons. The main subject in this thesis, the bistability effects, are inherently related to both of these directions.

II. OPTICAL BISTABILITY

Optical bistability is a benchmark of nonlinear light-matter interaction. Initially it has been demonstrated for a macroscopic nonlinear medium, typically a saturable absorber or a Kerr-type dispersive medium, contained in a Fabry–Pérot-type optical resonator [1]. The effect consists in the multi-valued solution and hysteresis in the transmitted mean-field intensity through the resonator for a certain range of the input power and frequency. Subsequently, owing to the development of high-finesse optical microresonators, the bistability effect could be observed at much lower light excitation level, with the medium size also reduced to hundreds of atoms [2].

With the advent of the strong coupling regime of cavity QED, however, a much more refined picture of the non-linearity in the matter-light coupling must be conceived. The figure of merit is the saturation photon number n_{sat} which defines that intracavity intensity where the atomic response to excitation becomes nonlinear according to the classical theory. Today, cavity QED extends to the range of $n_{\mathrm{sat}} < 1$ which indicates an obviously quantum regime in the light-matter interaction.

Microwave circuit QED systems reached an unprecedented strong coupling regime of cavity QED, with saturation photon number $n_{\mathrm{sat}} \ll 1$. The ratio of g (coupling parameter between a single mode of the stripline resonator and the artificial atom) to the loss rates is far larger than in atomic cavity QED realizations: typically $\gamma, \kappa \lesssim g/100$, where κ is the cavity mode linewidth, and γ is the characteristic decay rate of the electronic dipole system [3].

In this thesis work, we revisited the semiclassical optical bistability effect, and studied the intensity transmission of a cavity QED system in the very strong coupling regime, where the saturation photon number is smaller and much smaller than unity, i.e., $n_{\mathrm{sat}} < 1$ and $n_{\mathrm{sat}} \ll 1$.

III. SHORT DESCRIPTION OF THE WORK

The Jaynes–Cummings model (JC) is fundamental to cavity QED and describes the interaction between a single atomic dipole transition and a single mode of the radiation field sustained by a high-finesse resonator. In the course of our work, this basic model was extended to describe the current system in all cases. In order to reproduce, understand and explain the experimental results of such schemes, both a semi–classical mean-field approximation and exact numerical solution was used.

The semi–classical mean-field approximation consisted of deriving a set of Heisenberg-Langevin equations equivalent with the master equation, and further splitting the cavity mode amplitude and the collective spin variables to mean-field and quantum-fluctuation components. First-order correlation functions of the quantum fluctuations could be analytically calculated.

For the full quantum solution of the problem, the corresponding master equation was unraveled into quantum trajectories using the Monte-Carlo wavefunction method for the numerical solutions. This was done within the C++QED framework, which is an open-source C++/Python application-programming framework for efficient simulations of open quantum dynamics of interacting quantum systems [4].

With the above described approaches it was possible to study systems consisting of a number of individually decaying atoms, both two and three level atoms, coupled to the cavity via a single mode. Backed with a high-performance computing facility, we were able to investigate the transmission as a function of different parameters like pump strength, coupling strength, various decay rates, and detuning of the mode and that of the atom with respect to the external drive frequency, from the resonance to far detuned regimes.

The thesis work consists of the study of three different systems which lead to different phenomena, though all of them take place within the same realm of atomic dipole transitions strongly coupled to continuously driven resonator.

1. Absorptive bistability

In the strong coupling regime of cavity QED, the interplay of quantum fluctuations with nonlinear coupling at low intracavity photon number is expected to inherently modify the optical bistability effect. Remarkably, the remnants of the semi–classical bistability have been recorded by means of a single atom coupled to the single mode of a high-finesse microresonator in the regime of $n_{\text{sat}} \lesssim 1$ [5, 6]. Today, cavity QED allows for the controlled variation of the size of the atomic medium by single atom resolution. It is thus a suitable platform to explore the quantum corrections in a finite-size system to the semi–classical mean-field results in the vicinity of a critical point.

The semi–classical Maxwell–Bloch equations, usually adopted to describe optical bistability, were derived in the limit of a large ensemble of independent atoms and weak atom-mode coupling. A linearized fluctuation analysis around the mean field solutions was also performed analytically. The dimensionless parameters characterising the system are the cooperativity and the saturation photon number which were set $C \approx 10$ and $n_{\text{sat}} \approx 10^{-2}$, respectively. While varying the number of atoms and the strength of the coupling, the cooperativity C was kept constant, which allows for exploring systematically the crossover from the semiclassical to the quantum regime. The prior corresponds to many atoms weakly coupled to a single mode, whereas in the quantum regime a few atoms, – or eventually a single one, – are coupled strongly to the cavity mode. When the number of atoms was decreased, the coupling was increased such that the cooperativity remained invariant. Besides the solution given by Maxwell–Bloch equations, a numerical study was carried out directly for the cases of 2, 4, 6 and 8 atoms, and the crossover regime was explored in which the semi–classical solution gradually emerges from the exact solution of a quantum model.

2. Nonlinearities in the circuit QED system

When the saturation photon number (n_{sat}) is dramatically decreased, microwave circuit QED systems are needed for an experimental realization. The formal semi–classical solution for these parameters is obviously invalid. Unlike the case of absorptive bistability at n_{sat} where the transition from the quantum to the semi–classical solution can be recovered, in this regime there is no continuous path to the semi–classical solution. An exception is the special case of very large detuning between the mode and the resonance of the artificial atom [7] since the large detuning reduces the effective coupling between the two quantum systems. In this case, dispersive bistability can be expected and interpreted semiclassically [8].

In the regime $n_{\mathrm{sat}} \ll 1$, nonlinear quantum effects show up in the spectrum in the form of discrete, well-resolved multi-photon resonances. Since these resonances correspond to transitions towards excited states of the coupled system, they are hit by the driving field frequency if it is considerably detuned from the frequency of the bare atom and cavity mode. In order to gain insight into the nonlinear behaviour of the system, the input-output relation of a resonator-driven circuit QED system was explored in a broad frequency range from large-detuning towards resonance.

We found a certain driving intensity and frequency range in which the simple JC model gives rise to a quantum bistability effect. Interestingly, such a small quantum system could exhibit bimodal steady-state which corresponds to having simultaneously two distinct quasi-classical attractors. On the one hand, the anharmonicity of the low-lying levels in the JC spectrum impedes the excitation of the system by an external driving tuned away from the resonances associated with resolved multi-photon transitions. The JC system is then fully reflective and stays in a dim state close to the ground state. On the other hand, for large excitation numbers, the spectrum approaches to a pair of equidistant ladders with step size equal to the bare resonator frequency. This harmonic part of the otherwise substantially anharmonic spectrum can host coherent states which are stationary states of the damped-driven system. Let us emphasize again that this bimodal solution is not a remainder of the well-known semi–classical optical bistability. It is a purely quantum effect that the strongly-driven open system develops a small but non-vanishing probability amplitude to be in the highly-excited harmonic part of the spectrum, and the continuous measurement provided by the transmission can project the system state into this attractor.

3. Multi-level atoms

In an experiment carried out by Johannes Fink at the ETH Zürich, in the group of Andreas Wallraff, a similar quantum bistability has been observed. Interestingly, it appeared for pumping the cavity at resonance which is not included in the range of quantum bistability we had identified before. Instead, the calculation presented for the JC model suggests that the waiting time to reach the excited attractor diverges close to resonance. We carefully checked the lack of the bimodal steady-state for a broad range of parameters. For the parameter set corresponding to that of the experiment, the numerical simulations on the JC model confirmed that the ground state is expected to be the steady-state and we cannot see the presence of the highly excited other attractor in reasonable measurement time.

This discrepancy motivated us for a collaborative work with the experimental group. We realized the importance of taking into account a more realistic level scheme for the artificial atom, the so-called transmon qubit. We extended the Jaynes–Cummings model to describe three level atoms. The numerical calculation led then to a very good agreement between theory and experiment. By exploiting the additional tunability of parameters in the simulation, – inaccessible in a real experiment –, we could unambiguously demonstrate that the observed bistability effect is closely related to the one distilled in the case of the simpler JC model. However, using the slightly modified quantum system that has three relevant states, the same mechanism can be implemented in an easily accessible parameter range. As opposed to the case of the JC system, the actually observed quantum bistability process requires much less pump intensity and a biased mixture of the two semi–classical states can be obtained.

IV. RESULTS IN THE FORM OF THESES

Thesis I. I studied the intensity transmission of a resonantly driven single-mode, high-finesse resonator containing a low number of atoms. I proved by means of determining numerically the full quantum mechanical solution that an atomic medium composing of only 6 to 8 atoms reproduces the semiclassical absorptive optical bistability effect. It can be the basis for the development of signal processing devices, e.g., optical switches, in the low-photon number regime.

Thesis II. By comparing the linearized fluctuations around the semiclassical mean to the numerically exact quantum solution, I found significant quantum correlations between the atomic and the radiation field degrees of freedom.

Thesis III. I explored the driving strength and frequency dependent transmission of the Jaynes-Cummings model in a novel regime of the parameters, which occurs in circuit QED setups, where the coupling between the resonator mode and the qubits (atoms) is so strong that the saturation photon number is by orders of magnitude below 1. I showed that any remnant of the dispersive as well as the absorptive semiclassical optical bistability effects vanishes and gives place to the photon blockade and multi-photon resonance effects which reflect the quantised low-energy spectrum of the Jaynes-Cummings system.

Thesis IV. I identified a domain of the driving amplitude and frequency, called quantum bistability domain, where the driven Jaynes–Cummings system evolves into a bimodal distribution. The steady-state is a mixture of two well-separated semiclassical attractors of the small quantum system, i.e., the ground state and a highly excited quasi-classical state with well-defined phase and amplitude. The corresponding time evolution of the outgoing intensity is a telegraph signal alternating between two attractors. This solution cannot be connected by some limiting procedure to the semiclassical optical bistability, it is of intrinsically different nature.

Thesis V. I considered the transmission spectrum of the driven cavity when it contains a three-level atom in Ξ configuration instead of the usual two-level qubit, which is more appropriate model for the actual level scheme of the nonlinear transmon system in the circuit QED experiments. I found that the presence of the third level drastically modifies the response to external driving. When the highest, third level is weakly coupled to the middle one, the stationary transmission of the cavity-atom system is identical to that of an empty cavity. Surprisingly, the weakly coupled third level annihilates apparently the two-level atom in the stationary state. In the opposite limit, when the coupling is very strong, the full spectrum is very anharmonic and the transmission is hindered, up to very high power driving, by the lack of resonances. In between the two limits, I identified a range of coupling strengths for which the transmission is multivalued and corresponds to a new kind of quantum bistability.

Thesis VI. I solved numerically the theory using the three-level Ξ atom model, and fitted the results to experimental records obtained at the Quantum Device Lab at the ETH Zürich. In the experimental realization, the dipole coupling constants of the upper and the lower atomic transitions, $g_2 \sim \sqrt{2} g_1$, falls within the predicted range of quantum bistability. By fitting to the experimental bistability curve of the input-output intensity relation, I found an excellent agreement between experiment and theory, which applies also to the bimodal Q-functions of the stationary state and to the observed telegraph-signal-like temporal evolution of the system.

V. PUBLICATIONS RELATED TO THE THESES

1. A. Dombi, A. Vukics, and P. Domokos, Journal of Physics B: Atomic, Molecular and Optical Physics 46, 224010 (2013).
2. A. Dombi, A. Vukics, and P. Domokos, Eur. Phys. J. D 69, 60 (2015).
3. J. M. Fink, A. Dombi, A. Vukics, A. Wallraff, and P. Domokos, arXiv, 04892v1 (2016).

[1] L. A. Lugiato, *II Theory of Optical Bistability* (Elsevier, 1984), vol. 21, pp. 69–216, ISBN 9780444867612, URL `http://dx.doi.org/10.1016/s0079-6638(08)70122-7`.

[2] G. Rempe, R. J. Thompson, R. J. Brecha, W. D. Lee, and H. J. Kimble, Physical Review Letters **67**, 1727 (1991), URL `http://dx.doi.org/10.1103/PhysRevLett.67.1727`.

[3] A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Physical Review A **69**, 062320 (2004).

[4] A. Vukics (2015), URL `http://cppqed.sourceforge.net`.

[5] M. A. Armen and H. Mabuchi, Physical Review A **73**, 063801+ (2006), URL `http://dx.doi.org/10.1103/physreva.73.063801`.

[6] J. Kerckhoff, M. A. Armen, and H. Mabuchi, Opt. Express **19**, 24468 (2011), URL `http://dx.doi.org/10.1364/oe.19.024468`.

[7] M. Boissonneault, J. M. Gambetta, and A. Blais, Phys. Rev. A **77**, 060305 (2008), URL `http://link.aps.org/doi/10.1103/PhysRevA.77.060305`.

[8] L. S. Bishop, E. Ginossar, and S. Girvin, Physical review letters **105**, 100505 (2010).