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THE EVOLUTIONARY GAME OF POVERTY TRAPS*

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We study a coordination game, between a leader population and a follower population. Each individual of each population follows an imitative behavior in order to decide between being a high- or low-type economic agent. We show that individual behavior driven by imitation can lead to an economy that is either in a low-level equilibrium—a poverty trap—or a high-level equilibrium. We analyze how possible it is for an economy placed in the basin of attraction of the poverty trap to overcome it through the strategic action (limited on time) of a benevolent central planner.

1 Introduction

Standard growth theory teaches us that poverty traps are stable low-level balanced growth paths to which economies gravitate due to adverse initial conditions (shortage of human or physical capital, as in Azariadis and Drazen, 1990) or due to poor equilibrium selection by institutions (shallow financial markets as in Matsuyama (2007) or weak governance as in North (1990)) which do not allow coordinate investments successfully (see Azariadis and Starchuski, 2005).

This paper digs deeper into the microeconomic reasons why coordination of economic activity may fail on a scale that is large enough to cause persistent poverty. The economy is analyzed as an evolutionary game between economic agents, labeled as leaders and followers of high and low types that, respectively, can be viewed as R&D activities or innovative firms and human capital or skilled workers with R&D spending as a proxy for the state of 'scientific knowledge'. Rates of return of high-type leaders depend on average high-type followers (i.e. human capital), and rates of return on high-type followers depend on aggregate high-type leaders' investment (i.e. innovative firms or

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R&D). The outcome is a self-confirming equilibrium in evolutionary stable strategies in which unsuccessful players imitate successful ones. This equilibrium has the following property: *in developing economies with a large fraction of low-type followers or low-type leaders, imitative strategies do not support a take-off into sustained growth. To achieve that take-off, society should subsidize the cost of education and/or skill premia through a tax system on income until the economy builds a critical mass of high-type economic agents.*

Hence, the aim of this paper is to show how persistent states of underdevelopment can arise in strategic environments in which players are imitative rather than fully rational. The point of departure for an evolutionary model is the belief that people are not always rational. Rather than springing into life as the result of a perfectly rational reasoning process in which each player, armed with the common knowledge of perfect rationality, solves the game, evolutionary game strategies emerge from a trial-and-error learning process in which players find that some strategies perform better than others, and afterwards they decide to adopt or simply imitate such strategies. In the course of this learning process, the agents may simply take actions sometimes with great contemplation and sometimes with no thought at all. Their behavior is driven by rules of thumb, social norms, conventions, analogies to similar situations, or by other possibly more complex systems for converting stimuli into actions. In this direction, the imitative behavior is the way of copying strategies from a learning process (see Sanditov, 2006).

In some sense, the present paper can be related with the notion of institutional poverty traps and the membership theory of poverty traps. For instance, in Durlauf (2003), a model of incentives for wealthier families to segregate themselves into economically homogeneous neighborhoods is given, and the dynamics of these combinations explain persistent income inequality. Is precisely the concept of neighborhood effects that allows Durlauf to explain why poverty traps exist and persist. A poverty trap is defined as a community of economic agents initially composed by poor members with low types who remain in the low-level equilibrium over generations.

Samuel Bowles has built the seminal concept of ‘institutional poverty traps’, who emphasizes coordination failures and poverty traps as induced by the presence of specific institutions. Bowles defines institutions as conventions in which members of a population typically act in ways that maximize their own pay-offs, given the actions followed by others. Such process supports continued adherence to the conventions (Bowles *et al.*, 2006, p. 118). Polterovich (2008) pointed out that the formation of institutional traps due to economic agents with low types conforming specific strategies is one of the main obstacles to improve the economic performance. An important class of poverty traps is due to coordination failures. Such models are discussed in Cooper and John (1998) and Hoff (2001).

The remainder of the paper is organized as follows. Section 2 states the one-shot game and we analyze the Nash equilibria. Section 3 introduces the baseline model, namely behavioral rule to define an evolutionary dynamic. In Section 4, we develop the evolutionary game and we get the replicator dynamics driven by imitation. Section 5 presents the main result of the paper. Section 6 defines the notion of poverty trap and offers a clue to overcome such situation. Section 7 concludes the paper.

2 The Game

Let us allow economic agents being either leaders, (1), or followers, (2), with two different types: high or low type. The vectors (H, L) and (h, l) are the strategy spaces denoting high and low types of leaders and followers. Leader and follower games are described by imperfect information in which the leader moves first knowing the type of the follower, while followers move second without knowing the type of the leader (see Fudenberg and Tirole, 1991).

Let us assume that a contractual period is characterized by:

- **Strategic complementarities**. In this economy, looking to maximize profit the player 1 must employ player 2 under strategic complementarity, in the sense that the H-type agent matching with an h type is more profitable than matching with an l type. Analogously, the L-type agent matching with an l type is more profitable than matching with an h type.
- A gross income of 1 being an H type is U or u, and of being an L type is V or v, which depends on matching with high- or low-type followers.
- It is assumed that the leader knows the type of the follower (h or l), and the leader also decides the level of the 'wage' paid to the follower: W or w. We assume that the leader pays the same amount for any given type of follower.

That is, a gross income of 2 is W when hired by H and w when is hired by L, where $W > w > 0$. Notice that it seems to be more (or at least equally) natural to assume that the leader pays always W when the follower is h and always w when the follower is l. However, we consider that the H-type leader must give a signal, p, to the h-type follower. Such a signal can be viewed as giving some skill premium or bonus, which are perceived only by h-type followers.

- Agents 1 and 2 face income taxes $\gamma \in (0,1)$ and $\phi \in (0,1)$.
- The leader knows the types of the followers, but the followers do not know the type of the leaders and they assume with probability σ that they will be hired by a leader of H type and with probability $(1 - \sigma)$ by an L type.
- Choosing between high and low types does not have any cost for the leader. On the other hand, if the follower decides to become an h type they incur a training cost or cost of education C, while deciding to be an l type does not incur any costs. For instance, when engaged by a firm, a worker must present a certificate of expertise or skill.

A normal-form representation of this game is presented in the following pay-off matrix:

2\1	H	L
h	$(1-\phi)W+p-C, (1-\gamma)U-W-p$	$(1-\phi)w-C, (1-\gamma)v-w$
l	$(1-\phi)W, (1-\gamma)u-W$	$(1-\phi)w, (1-\gamma)V-w$

The following restrictions on the parameters hold:

- $p > C > 0$, incentives to be a high-type follower and it is pay-off dominant for a low-type follower, or,
  $0 < p \leq (1-\phi)(W-w)$

which means that h-type followers must get a positive bonus at most equal to the difference in wages (after taxes) to being high or low type.

- The H-type leaders cannot give a bonus or skill premium greater than the net difference of income for being high- or low-type leader plus the difference on wages to low- and high-type followers, i.e.
  $p < (1-\gamma)(U-v)+(w-W)$

Moreover, $(1-\gamma)(V-u) > (w-W)$ implies that a low-type leader prefers a low-type follower.

Therefore, the game has two pure Nash equilibria:

$$(\mathrm{H},\mathrm{h}) = ((1,0);(1,0)) \qquad \text{and} \qquad (\mathrm{L},\mathrm{l}) = ((0,1);(0,1))$$

the former is the pay-off dominant.

There is a mixed strategy Nash equilibrium given by

$$(\theta,(1-\theta);\sigma,(1-\sigma))$$

where θ is the leader's probability of matching a high-type follower, and σ is the follower's probability of matching a high-type leader, i.e.

$$\sigma = \frac{C}{p} \qquad \text{and} \qquad \theta = \frac{(1-\gamma)(V-u)+W-w}{(1-\gamma)(U-u+V-v)-p}$$

Note that the skill premium p is bounded so that $C < p < (1-\gamma)(U-u)$. This means the skill premium must be large enough to encourage followers to be high type, and small enough too as a leader always prefers to be a low type hiring a low-type follower.

The next section analyzes this game as an evolutionary process. Recall that the term evolutionary process means more successful types tend to proliferate while less successful types tend to disappear, an assumption that applies equally well to learning by imitation and cultural evolution as well as to literal population replacement by natural selection. The model applies as long as people tend to gravitate towards a type that does better than its alternatives.

3 Behavioral Rules

Considered an N-population strategic-form game, where individuals from each population $\tau = \{1, \ldots, N\}$ can choose between n_τ, different behaviors or pure strategies. We say that individuals playing according to the ith pure strategy belong to the ith club, so each population can split in different clubs or subpopulations. Individuals randomly matched with those of the other population to play the game. According with their characteristics (strategy) each individual will receive a pay-off, and the game is over. The players are allowed to change clubs or strategies, and then the game starts again.

This change from clubs or strategies is embodied in the so-called behavioral rules, which generate a system of differential equations, describing the evolution of the relative frequency at which some pure strategy occurs in a population.

The behavioral rules are characterized by two elements. First, the time rates at which agents in the populations review their strategy choice. This rate may depend on the current performance of the agent's pure strategy and on other aspects of the current population state. The second element specifies the choice of a reviewing agent. The probability that the i strategist in the population τ will switch to some pure strategy j in his or her own population may also depend on the current performance of these strategies and other aspects of the current population state. So there is one differential equation for each possible pure strategy. The ith differential equation describes the evolution on the population share represented by the number x_i^τ for all $1 \leq \tau \leq N$ and $1 \leq i \leq n_\tau$.

For each population τ we represent the set of mixed strategy by

$$\Delta^\tau = \left\{ x^\tau \in \mathbb{R}^{n_\tau} : \sum_{i=1}^{n_\tau} x_i^\tau = 1, x_i^\tau \geq 0, i = 1, \ldots, n_\tau \right\}$$

where n_τ is the cardinality of the set of pure strategies available for the population $\tau \in [1, \ldots, N]$.

Definition 1: A behavioral rule is a map from currently aggregate behavior to conditional switch rates. The map is given by two basic elements:

1. The time rate $r_i^\tau(x)$ at which agents review their strategy choice. This time rate depends on the performance of the agent's pure strategy and other aspects of the current population state.[1]
2. The probability $p_{ij}^\tau(x)$ that a reviewing i strategist will switch to some pure strategy j. The vector of this probability is written as

[1]This is the 'behavioral rule with inertia' (see Weibull, 1995; Björnerstedt and Weibull, 1996; Schlag, 1998, 1999) that allows an agent to reconsider his or her action with probability $r \in (0,1)$ each round.

$p_i^\tau(x) = (p_{i1}^1(x), \ldots, p_{ik}^N(x))$, where $p_i^\tau(x) \in \Delta^\tau$. This distribution may depend on the current performance of the strategies and other aspects of the population state.

In a finite population one may imagine that review times of an agent are the arrival time of a Poisson process with arrival time $r_i^\tau(x)$, and that at each time the agents select a pure strategy according to the probability distribution $p_i^\tau(x)$ over the set Δ^τ. As it is assumed that all the agents' Poisson processes are independent, the aggregate process in the subpopulation of *i* strategists is itself a Poisson process, with arrival rate $x_i^\tau r_i^\tau(x)$. Consider independence of switches across agents, and the process of switches from strategy *i* to strategy *j* as a Poisson process with arrival rate $x_i^\tau r_i^\tau p_{ij}^\tau$. Assuming a continuum of agents and by the law of large numbers, we model these aggregate stochastic process as a deterministic flow:

- The outflow from the *i* strategists thus is

$$\sum_{j \neq i} x_i^\tau r_i^\tau(x) p_{ij}^\tau(x)$$

- The inflow to this is

$$\sum_{j \neq i} x_j^\tau r_j^\tau(x) p_{ji}^\tau(x)$$

Behavioral rules allow to define an evolutionary dynamics as an inflow–outflow model. Rearranging terms, we obtain

$$\dot{x}_i^\tau = \sum_{j \in K} x_j^\tau r_j^\tau(x) p_{ji}^\tau(x) - \sum_{i \in K} x_i^\tau r_i^\tau(x) p_{ij}^\tau(x) \tag{1}$$

We say that this is a dynamic of replication, by extending the concept of replicator dynamics defined for the symmetric case in which a population is confronted with itself. Hence, every behavioral rule can generate an evolutionary dynamics and they take agents' behaviors as the starting point.

A population dynamics (1) will be called imitative if there are at least two different strategies and at least one agent following one of these strategies assesses, with a given probability, whether he or she should change his or her behavior. The final decision depends on the relationship between the benefits the agent obtains and the benefits obtained by agents following a different strategy.

Certainly there are some properties we would like to be verified by the rules of behavior. When these properties are verified by the behavioral rule we will say that the rule is good. A good behavioral rule is, for instance, the Lipschitz continuous function in pay-offs and social states. That is, to guarantee that this system of differential equations (1) induces a well-defined dynamic on the space $\Delta = \Pi_{\tau=1}^N \Delta^\tau$, we consider that $r_i^\tau : X \subseteq \Delta^\tau \to [0,1]$

and $p_i^\tau : X \rightarrow \Delta^\tau$ are Lipschitz continuous functions with a solution through any initial state $x_0 \in \Delta$ and such that a solution trajectory never leaves Δ. One such property can be seen technically, but also reflects a fact emerging from dynamics within large populations. The state space Δ is a forward invariant in this dynamic (1).

In fact, if agents had perfect information about all pay-offs yielded by other pure strategies, and if they knew the state of the population perfectly, behavioral adjustments would be much faster, possibly leading to discontinuous switches. Hence, Lipschitz continuity reflects an assumption that people have limited knowledge about pay-offs and population states, which has some appeal if we look at large populations.

The reviewing agents looking to maximize their own pay-offs will adopt the behavior of those agents perceived as successful. Other influences can also be worth imitating, including conformism or dissatisfaction.

In the next sections we show the replicator dynamics driven by imitation as a behavioral rule.

4 The Evolutionary Game

The simplest setting to study the learning is one in which agents' strategies are completely observed at the end of each round, and agents are randomly matched with a series of anonymous opponents so that the agents have no impact on what they observe. Hereafter, populations of leaders (1) and followers (2) are, X^τ, denoted by $X^1 = (X_{\mathrm{H}}^1 + X_{\mathrm{L}}^1)$ and $X^2 = (X_{\mathrm{h}}^2 + X_{\mathrm{l}}^2)$, and the populations are composed of a large number of agents who face the problem of selecting an i type: {H, L}; {h, l}. Let $X_{i(\tau)}^\tau$ be the total of i strategists, $i(1) \in$ {H, L} and $i(2) \in$ {h, l} and $\tau = \{1,2\}$. We denote a fraction of agents, $x_{i(\tau)}^\tau$, as the share of i-type strategists:

$$x_i^\tau = \frac{X_i^\tau}{X^\tau} \tag{2}$$

where X^τ is finite. Assume that both populations are normalized to 1, $x_{\mathrm{H}}^1 + x_{\mathrm{L}}^1 = 1$ and $x_{\mathrm{h}}^2 + x_{\mathrm{l}}^2 = 1$. Then, $x_{i(\tau)}^\tau$ denotes the percentage of individuals from the population τ belonging to the $i(\tau)$ club. Hence, the probabilities $\sigma = x_{\mathrm{H}}^1$ and $\theta = x_{\mathrm{h}}^2$, and thus the expected pay-offs can be written as

$$E_{\mathrm{h}}^2 = x_{\mathrm{H}}^1[(1-\phi)W + p] + (1 - x_{\mathrm{H}}^1)(1-\phi)w - C \tag{3}$$

$$E_{\mathrm{l}}^2 = x_{\mathrm{H}}^1(1-\phi)W + (1 - x_{\mathrm{H}}^1)(1-\phi)w \tag{4}$$

$$E_{\mathrm{H}}^1 = x_{\mathrm{h}}^2[(1-\gamma)U - W - p] + (1 - x_{\mathrm{h}}^2)[(1-\gamma)u - W] \tag{5}$$

$$E_{\mathrm{L}}^{1} = x_{\mathrm{h}}^{2}[(1-\gamma)v - w] + (1-x_{\mathrm{h}}^{2})[(1-\gamma)V - w] \tag{6}$$

So, the profile $(x_{\mathrm{H}}^{1},\ x_{\mathrm{L}}^{1};\ x_{\mathrm{h}}^{2},\ x_{\mathrm{l}}^{2})$ defines a mixed strategy of a two-population normal form game:

$$\Gamma = \langle \tau = \{1,2\}, ((\mathrm{H},\mathrm{L}); (\mathrm{h},\mathrm{l})), E_{i(\tau)}^{\tau}(\cdot) \rangle$$

as in Section 2, where $E_{i(\tau)}^{\tau}$ is the expected pay-off from following the $i(\tau)$ pure strategy.

From now on, we argue that, when the followers and leaders decide to imitate successful strategists and when the state of the economy is such that playing a high type is the best strategy, the economy converges to the high-level equilibrium. Otherwise, if the state of the economy is one in which being a low type is the best strategy, then the economy will be caught in a poverty trap.

4.1 Replication by Imitation

Let us consider the N-population replicator dynamics suggested by Weibull (1995, p. 172) and Taylor (1979). In our case $N = 2$. Consider an $i(\tau)$-type agent, $i(1) \in \{\mathrm{H}, \mathrm{L}\}$ and $i(2) \in \{\mathrm{h}, \mathrm{l}\}$.

By $s_{i(\tau)}^{\tau}$ we denote the $i(\tau)$ pure strategy from population $\tau = \{1,2\}$. To simplify the notation we will write only s_i. Suppose that an s_i strategist reviews his or her strategy with probability $r_i^{\tau}(x)$ to consider whether he or she should or should not change his or her current strategy, where $x = (x_i^1, x_i^2)$.

Assume that an agent's decision depends upon the pay-off associated with his or her own behavior, given the composition of the population, labeled as $E_i^{\tau}(s_i, x^{-\tau})$, $\forall i \in \{\mathrm{h}, \mathrm{l}; \mathrm{H}, \mathrm{L}\}$, of subpopulation τ, $-\tau = \{1,2\}$, $\tau \neq -\tau$.

Hence, $r_i^{\tau}(x)$ is the average time rate at which an agent who currently uses strategy i reviews his or her strategy choice. Then,

$$r_i^{\tau}(x) = f_i^{\tau}\left(E_i^{\tau}(s_i, x^{-\tau}), x\right) \in [0,1] \tag{7}$$

The function $f_i^{\tau}(\cdot)$ is the propensity for a member from the ith club to switch from one membership to another. This propensity is higher for individuals with a lower expected pay-off.

Having opted for a change, the agent will adopt a better strategy followed by the first person from his or her population to be encountered (his or her neighbor), i.e. for any $\tau = \{1,2\}$, $p_{ij}^{\tau}(x)$ is the probability that a reviewing i strategist changes to some pure strategy $j \neq i$, $\forall j \in \{(\mathrm{H}, \mathrm{L}); (\mathrm{h}, \mathrm{l})\}$.

The *outflow* from the i club in population τ is $x_i^{\tau} r_i^{\tau}(x) p_{ij}^{\tau}(x)$ and the *inflow* is $x_j^{\tau} r_j^{\tau}(x) p_{ji}^{\tau}(x)$. Then, $\forall j \neq i \in \{(\mathrm{H}, \mathrm{L}); (\mathrm{h}, \mathrm{l})\}$, $\tau = \{1,2\}$, and by the law of large numbers we model these processes as deterministic flows and, rearranging terms, we obtain

$$\dot{x}_i^\tau = x_j^\tau \left[f_j^\tau \left(E_j^\tau (s_j, x^{-\tau}) \right) p_{ji}^\tau (x) \right] - x_i^\tau \left[f_i^\tau \left(E_i^\tau (s_i, x^{-\tau}) \right) p_{ij}^\tau (x) \right] \tag{8}$$

System (8) represents the interaction between two groups of agents who imitate their neighbors.

By the normalization rule, $x_{\mathrm{H}}^1 + x_{\mathrm{L}}^1 = 1$ and $x_{\mathrm{h}}^2 + x_{\mathrm{l}}^2 = 1$, system (8) can be reduced to two equations with two independent state variables. Taking advantage of this property, we choose variables x_{H}^1 and x_{H}^1 with their respective equations.

Björnerstedt and Weibull (1996) studied a model where those agents who revise may imitate other agents in their player population, and show that a number of pay-off-positive selection dynamics, including the replicator dynamics, may be derived. In particular, if an agent's revision rate is linearly decreasing in the expected pay-off to his or her strategy (or to the agent's latest pay-off realization), then the intensity of each pure strategy's Poisson process will be proportional to its population share, and the proportionality factor will be linearly decreasing in its expected pay-off. If every revising agent selects his or her future strategy by imitating a randomly drawn agent in their own player population, then the resulting flow approximation is again the replicator dynamics.

Assume $f_i^\tau(\cdot)$ is linear in pay-off levels. Thus, the propensity to switch behavior is decreasing in the level of the expected utility, i.e. $\forall\, j \neq i \in \{(\mathrm{H}, \mathrm{L}); (\mathrm{h}, \mathrm{l})\}$, $\tau = \{1,2\}$, we get

$$f_i^\tau \left(E_i^\tau (s_i, x^{-\tau}) \right) = \alpha^\tau - \beta^\tau E_i^\tau (s_i, x^{-\tau}) \tag{9}$$

where α^τ, $\beta^\tau \geq 0$ and $\alpha^\tau / \beta^\tau \geq E_i^\tau (s_i, x^{-\tau})$ assures that $f_i^\tau(\cdot) \in [0,1]$. α^τ is interpreted as a degree of dissatisfaction and β^τ measures the performance of the own pay-off on reviewing the current strategy. As far as the pay-off level of the i strategist, $E_i^\tau(\cdot)$, increases his or her average reviewing rate, $r_i^\tau(x)$, will decrease.

Schlag (1998, 1999) pointed out an evaluation rule of simple imitation which is the 'average rule or proportional rule' where each strategy is evaluated according to the average pay-off observed in the reference group (see Apesteguia *et al.*, 2007).

Consider that economic agents do not know the exact pay-offs of their corresponding neighbors, but they can compute some average pay-offs in their neighborhoods and they can imitate the behavior that yields the highest average pay-off. Although an agent does not know all the true values of the pay-off of the others, he or she can take a sample of such true values in order to estimate the average. Let $\tilde{E}_i^\tau$ and $\tilde{E}_j^\tau$ be the estimators for the true values E_i^τ and E_j^τ. Hence, the process of copying successful behaviors exhibits *pay-off monotonic updating*, since strategies with above-average pay-offs are adopted by others and thus increase their share in the population, i.e. each i strategist changes his or her strategy if and only if $\tilde{E}_i^\tau < \tilde{E}_j^\tau$.

Let us apply the behavioral rule from Definition 1 where a reviewing strategist, i, who decides to change his or her current strategy must take into consideration: (i) a probability of imitating one strategy which performs better than his or her current strategy, $P[\tilde{E}_i^\tau(s_j, x^{-\tau}) < \tilde{E}_j^\tau(s_i, x^{-\tau})]$, and (ii) the probability of meeting the agent, x_j^τ, who currently uses such strategy.

Therefore, for any pair $i \neq j \in \{(\mathrm{H, L}); (\mathrm{h, l})\}$, $\tau = \{1,2\}$:

$$p_{ij}^\tau(x) = \lambda E_j^\tau(\cdot) x_j^\tau \qquad \text{if} \qquad E_j^\tau(\cdot) > 0$$

and

$$p_{ji}^\tau(x) = \lambda E_i^\tau(\cdot) x_i^\tau \qquad \text{if} \qquad E_i^\tau(\cdot) > 0$$

where

$$\lambda = \frac{1}{E_i^\tau(\cdot) + E_j^\tau(\cdot)}$$

Hence, by the above considerations, system (8) becomes the system of replicator dynamics driven by imitation, i.e.

$$\dot{x}_i^\tau = x_i^\tau(1 - x_i^\tau)\lambda[(\alpha^\tau - \beta^\tau E_j^\tau(\cdot)) - (\alpha^\tau - \beta^\tau E_i^\tau(\cdot))] \tag{10}$$

The term $x_i^\tau(1 - x_i^\tau)$ is the matching product when i meets j and the term $\lambda[\cdot] > 0$ measures the proportional growth of the j reviewers thought to be an i (a decrease if negative), from population τ. By substitution of the expected pay-offs (equations (3)–(6)), and after some algebraic manipulation, we get

$$\begin{cases} \dot{x}_\mathrm{h}^2 = -\dot{x}_\mathrm{l}^2 = x_\mathrm{h}^2(1 - x_\mathrm{h}^2)\left\{\dfrac{\beta^2(px_\mathrm{H}^1 - C)}{2[x_\mathrm{H}^1 W + (1 - x_\mathrm{H}^1)w](1 - \phi) + px_\mathrm{H}^1 - C}\right\} \\ \\ \dot{x}_\mathrm{H}^1 = -\dot{x}_\mathrm{l}^L = x_\mathrm{H}^1(1 - x_\mathrm{H}^1)\left[\dfrac{\begin{matrix}\beta^1\{x_\mathrm{h}^2[(v - U)(1 - \gamma) - p] - (1 - x_\mathrm{h}^2)(1 - \gamma)u \\ - V[(1 - x_\mathrm{h}^2) - (1 - x_\mathrm{h}^2)\gamma] - W + w\}\end{matrix}}{\begin{matrix}x_\mathrm{h}^2[(1 - \gamma)(U + v) - W - w - p] \\ +(1 - x_\mathrm{h}^2)[(1 - \gamma)(V + u) - W - w]\end{matrix}}\right] \end{cases} \tag{11}$$

The system $(\dot{x}_\mathrm{h}^2, \dot{x}_\mathrm{H}^1)$ describes the case where strategies propagate via imitation, and expected pay-offs drive the rate of imitation, reinforcement and inhibition of behaviors of the high-type agents from the populations of leaders 1 and followers 2.

As we know, the system $(\dot{x}_\mathrm{h}^2, \dot{x}_\mathrm{H}^1)$ admits five stationary states or dynamic equilibria, i.e.

$$(0,0), (0,1), (1,0), (1,1) \text{ and a positive interior equilibrium } (x_\mathrm{H}^{1*}, x_\mathrm{h}^{2*})$$

Let us denote as $\bar{P} = (x_H^{1*}, x_h^{2*})$ such interior equilibrium lying in the square $C = [0, 1] \times 0, 1]$, i.e.

$$\begin{cases} x_H^{1*} = \dfrac{C}{p} \\ \\ x_h^{2*} = \dfrac{(1-\gamma)(V-u)+W-w}{(1-\gamma)(U-u+V-v)-p} \end{cases} \tag{12}$$

The interpretation of the above equilibria is as follows. (i) The trivial equilibrium is one where leaders and followers are all low-type economic agents {L, l} or (0,0). (ii) At the opposite corner (1,1) is the case where all agents are high type {H, h}. (iii) The two remaining border equilibria, which are not Nash equilibria, show a different club dominating the two populations and in a sense a mismatch between strategies {L, h}, {H, l} or (0,1), (1,0). (iv) The interior equilibrium is composed of marriages among low- or high-type economic agents: $\bar{P} = (x_H^{1*}, x_h^{2*})$. In the next section we study the main dynamical properties of these equilibria.

5 Analyzing the Evolutionary Dynamics

To observe the dynamics of the game we calculate trajectories, i.e. how the mixed strategies change. We start with any pair of mixed strategies $(x_h^2(t_0)$, $x_H^1(t_0))$ for any initial time $t = t_0$, and calculate the dynamics given by the system $(\dot{x}_h^2,\ \dot{x}_H^1)$ as we progress along certain trajectory. In this vein, let us recall the standard definitions (not stated in formal mathematical terms) on:

Definition 2: A 'trajectory' is a path determined in the phase space of the solution of a dynamical system passing through a given point of the space at a given time.

Definition 3: An 'attractor' of a dynamical system is a subset of the state space to which orbits originating from typical initial conditions tend as time increases.

Definition 4: A 'saddle point' is a fixed point that has at least one positive eigenvalue and one negative eigenvalue in its linearization. More generally, a fixed point for which there are trajectories that tend to the fixed point in both positive and negative time. That is, a saddle point is a point whose stable and unstable manifolds have a dimension which is not zero.

Definition 5: The 'basin of attraction' of an attractor is the set of initial conditions as a region in the state space leading to long-time behavior that approaches to the attractor.

Thus the qualitative behavior of the long-time motion of a given system can be fundamentally different depending on which basin of attraction the initial condition lies in (e.g. attractors can correspond to periodic, quasi-periodic or chaotic behaviors of different types). Regarding a basin of attraction as a region in the state space, it has been found that the basic topological structure of such regions can greatly vary from system to system.

Another important concept is that of evolutionarily stable strategy (ESS). A population playing such a strategy is uninvadable by any other strategy. Uninvadability is a useful characterization of evolutionary stability, and indeed its original definition is that a strategy x^* is evolutionarily stable if and only if (i) it is a best response to itself and (ii) it is a better response to all other best responses than these are to themselves. By definition, no alternative best response exists for any player population if the profile in question x^* happens to be a strict Nash equilibrium, so such profiles should qualify. In other words, consider a two-population normal form game:

$$\Gamma = \langle \tau = \{1, 2\}, ((\mathrm{H}, \mathrm{L}); (\mathrm{h}, \mathrm{l})), E^{\tau}_{i(\tau)}(\cdot) \rangle$$

where each population has two possible behaviors (H, L) and (h, l) denoting high and low types of leaders (1) and followers (2) with expected pay-offs $E^{\tau}_{i(\tau)}(\cdot)$. Then:

Definition 6: A strategy $x \in \Delta^{\tau}$ is an ESS in asymmetric games, for a population τ if and only if

$$E^{\tau}(x_i, y_{-i}) \geq E^{\tau}(z_i, y_{-i})\, \forall z \in \Delta^{\tau}$$

and for all $y' \in \Delta$, $y' \neq y$ there exists some $\bar{\varepsilon}_{y'} \in (0,1)$ such that, for all $\varepsilon \in (0, \bar{\varepsilon}_{y'})$ and with $w_{\varepsilon} = \varepsilon y' + (1 - \varepsilon)y$,

$$E^{\tau}(x_i, w_{\varepsilon}) > E^{\tau}(y_i, w_{\varepsilon}) \qquad \forall \tau \in \{1, 2\}$$

Intuitively, we say that x is an ESS if and only if after a mutation in the population $-\tau$ continues to be a best response for the post-entry population, w_{ε}.

Zeeman (1992) shows that an ESS is asymptotically stable in the replicator dynamics such that trajectories do not necessarily have to settle at the equilibrium (which would be neutrally stable) to be an ESS (more details in Weibull, 1995), but the converse of this statement is not necessarily true, and being asymptotically stable does not imply ESS.

An ESS against the field is a mixed Nash equilibrium that is uninvadable by any alternative strategy into the basin of attraction. That is:

Definition 7: Consider that the profile distribution from population 1 is given by $x^1 = (x_H^1, x_L^1)$, then we say that the strategy $\bar{x}^2 = (\bar{x}_H^2, \bar{x}_L^2)$ is an **ESS against the field** of x^1 if there exists $\varepsilon_{x^1} > 0$ such that

$$E^2(\bar{x}^2, \breve{x}^1) \geq E^2(x^2, \breve{x}^1)$$

for all $x^2 \in S_2$ where $|x^1 - \breve{x}^1| \leq \varepsilon_{x^1}$.

Nevertheless, by complementarities it can happen that the field, defined by the strategy or the profile distribution from one population which the opponent takes as given, evolves at the same time under the pressures in the changes of the distribution generated by such opponents. Then, such interdependence of behaviors can generate ESS in both populations. For that reason, we should consider the definition of *ESS against the field for strategic profiles*:

Definition 8 (Evolutionarily stable strategic profile): A strategic profile $(x^1, \ldots, x^n)$ is evolutionarily stable, if x^i defines for all $I = 1, \ldots, n$ an *ESS* against the field x^{-i} being $x^{-i} = (x^1, \ldots, x^{i-1}, x^{i+1}, \ldots, x^n)$ generated by the strategic behavior of all the other opponents.

Hence, if an economic system evolves into a poverty trap, it is not enough that any small perturbation in the initial conditions can lead the system outside to the basin of attraction. This means that the poverty trap is an attractor of trajectories defined by probability distributions that are evolutionarily stable strategic profiles.

The following proposition summarizes our main results.

Proposition 1: By imitation of agents the evolutionary dynamics from the system $(\dot{x}_h^2, \dot{x}_H^1)$ is as follows:

(i) Equilibria (0,0) and (1,1) are asymptotically stable points (in 10) and define the strategic profiles (0,1; 0,1) and (1,0; 1,0), respectively, and they are ESS against the field into their basin of attraction.
(ii) Equilibrium $\bar{P} = (x_H^{1*}, x_h^{2*})$ is a threshold since it separates the basins of attraction of the low-level and high-level equilibria and, with the exception of a single curve through this point, all solution trajectories converge to the attractors.

Proof: See the Appendix. Figure 1 draws the evolutionary dynamics of the replicator dynamics driven by imitation (system (10)).

Recall that the location of the saddle point $\bar{P}$ depends on the parameters values: education costs, premia or bonus and income taxes. ■

In other words:

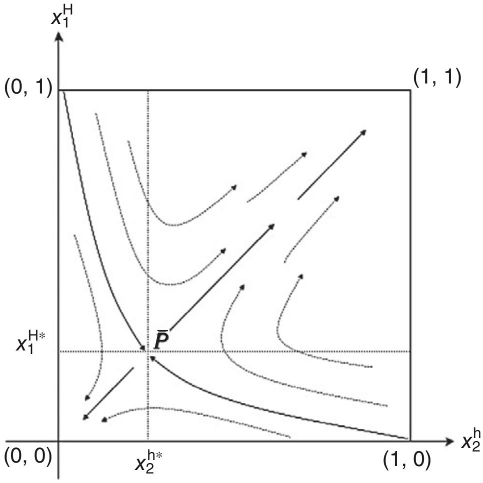

Fig. 1. Evolutionary Dynamics Driven by Imitative Behavior

1. Consider a pair of initial distributions $x_0 = (x_0^1, x_0^2) \in \Delta^1 \times \Delta^2$ such that the corresponding values of the high types are lower than $\bar{P} = (x_H^{1*}, x_h^{2*})$ then, the economy will evolve on trajectories defined by the solutions of the system (10), where $(0,1) \in \Delta^1$ is a best response for $x^2(t)$ and $(0,1) \in \Delta^2$ is a best response for $x^1(t)$, and $(x^1(t), x^2(t))$ is the solution for the system (10) with the initial conditions (x_0^1, x_0^2). Moreover, it holds that $x^1(t) \to (0,1)$ and $x^2(t) \to (0,1)$ with $t \to \infty$. Then, the whole economy converges to the attractor (0,1; 0,1).
2. But if the initial conditions of the economy defined by $(x_0^1, x_0^2) \in \Delta^1 \times \Delta^2$ are such that the initial values of profile distributions x_{H0}^1 and x_{h0}^2 are larger than $\bar{P}$, then we obtain trajectories such that the economy converges to (1,0; 1,0).

6 Overcoming the Poverty Trap

The following statement emphasizes our notion of poverty trap:

> Equilibrium (0,0) is a poverty trap in the sense that an economy starting with a (sufficiently) low number of high-type agents experiences a decreasing sequence of high-type economic agents that eventually leads to no high-type agents. This is due to the rationality of the economic agents that are facing a population state (or an economy) where low-type agents are dominant strategies. Then, for such an economy an imitative agent should decide to become a low-type economic agent.

In terms of game theory:

Definition 9: A **poverty trap** is a Nash equilibrium Pareto-dominated which is a steady state of the replicator dynamics defined by the rules of conduct (imitation) which define a local attractor, to which trajectories converge shaped by evolutionarily stable strategic profiles.

For a large initial number of high-type agents, greater than the level $\bar{P} = (x_1^{H*}, x_2^{h*})$, the economy converges over time towards the sample path $t \rightarrow (1,1)$ of high-type agents. To overcome the poverty trap we should reduce the basin of attraction of the low-level equilibrium (0,0) and either x_1^{H*} or x_2^{h*} should decrease, i.e.

1. The ratio of education costs–skill premia, C/P, decreases if the training costs C fall or the value of the premia p rises.
2. With fixed training costs, C, if the followers' probability of matching with a high-type leader, σ, decreases, then the number of high-type followers decreases also. To avoid this situation, the value of p must be larger than C. For instance, if $\sigma = \frac{1}{2} \geq C/p$ the bonus should be twice as large as the training costs, $p \geq 2C$. Hence, when the number of high-type followers is small, then the skill premia p should be large enough in order to encourage the other agents to switch their current behavior and to join the club of high-type followers.

Decreasing the value of x_1^{H*}: either training costs C should decrease, or the bonus p must increase, i.e.

$$\text{either} \quad \lim_{C \to 0} x_1^{H*} \quad \text{or} \quad \lim_{e \to \infty} x_1^{H*} \Rightarrow x_1^{H*} = 0$$

Hence, it fully expands the basin of attraction to (1,1) which is the high-level equilibrium.

6.1 Replicator Dynamics with Fiscal Incentives

We argue that fiscal incentives may encourage players to become high type. A short numerical example helps to understand the key role of taxes and subsidies for overcoming a poverty trap. Let us consider that the gross income of high-type followers is W = 10 units and w = 5 units for low types. Gross income of high-type leaders is U = 100 units when hiring high types while it is u = 50 units hiring low types. Gross income of low-type leaders is V = 60 units hiring low types and v = 40 units hiring high types.

2\1	**H**	**L**
h	$(1-\phi)10 + p - C, (1-\gamma)100 - 10 - p$	$(1-\phi)5 - C, (1-\gamma)40 - 5$
l	$(1-\phi)10, (1-\gamma)50 - 10$	$(1-\phi)5, (1-\gamma)60 - 5$

This is a coordination game and the following inequalities hold: (i) $p > C > 0$, (ii) $p < (1-\gamma)60 - 5$, and (iii) $\gamma < 1$. Then, still the two pure Nash equilibria are (H, h) and (L, l) and the former is the pay-off dominant while the latter is the risk dominant. Now, the threshold is

$$\begin{cases} x_{\mathrm{H}}^{1*} = \dfrac{C}{p} \\ \\ x_{\mathrm{h}}^{2*} = \dfrac{(1-\gamma)10+5}{(1-\gamma)80-p} \end{cases} \tag{13}$$

Recall that $p \le (1-\phi)(W-w) = (1-\phi)5$, and hence to overcome the poverty trap requires exogenous changes like fiscal policies on γ or reductions in training costs (or education costs) and increments of skill premia (or bonuses).

Consider that a central planner has implemented a policy characterized by income taxation and subsidies. The subsidies are awarded only to those who decide be a high-type economic agent.

Let $X_{i(\tau)}^{\tau}$ be the total of i strategists, $i \in \{(\mathrm{H}, \mathrm{L}); (\mathrm{h}, \mathrm{l})\}$, from the population $\tau = \{1,2\}$, and let $N = \sum_{i,\tau} X_{i(\tau)}^{\tau}$ be the total number of agents in the whole economy. The mass (or number) of leaders and followers that adopt a strategy i is given by

$$m_i = \frac{X_{i(\tau)}^{\tau}}{N} = \frac{x_{i(\tau)}^{\tau} X^{\tau}}{N} \tag{14}$$

where $X^{\tau} = \{(X_{\mathrm{H}}^{1} + X_{\mathrm{L}}^{1}); \quad (X_{\mathrm{h}}^{2} + X_{\mathrm{l}}^{2})\}$. Then, we denote by $\Delta = \left\{m \in R_{+}^{k} : \sum_{i=1}^{k} m_i = 1\right\}$ the simplex of R_k. In our case $k = 4$.

If such taxes are imposed on each population $\tau \in \{F, W\}$, then the total revenue collected in the economy is

$$T = m_{\mathrm{H}}[(1-\gamma)(U+u)] + m_{\mathrm{L}}[(1-\gamma)(V+v)] + (m_{\mathrm{h}} + m_{\mathrm{l}})[(1-\phi)(W+w)]$$

Consider that a proportion $\theta \in (0,1)$ of the total tax revenue collected, T, is shared with high-type followers and the rest by high-type leaders.

The new expected pay-offs, after transfers, for strategists h and l are now, respectively,

$$\begin{aligned} E_T(\mathrm{h}) &= E_{\mathrm{h}}^{2} + \theta T \\ E_T(\mathrm{l}) &= E_{\mathrm{l}}^{2} \end{aligned} \tag{15}$$

For strategist H and L, the new expected pay-offs are now, respectively,

$$\begin{aligned} E_T(\mathrm{H}) &= E_{\mathrm{H}}^{1} + (1-\theta)T \\ E_T(\mathrm{L}) &= E_{\mathrm{L}}^{1} \end{aligned} \tag{16}$$

Hence, the replicator dynamics (10), for all $i(\tau)$-type agent $i(1) \in \{\mathrm{H}, \mathrm{L}\}$ and $i(2) \in \{\mathrm{h}, \mathrm{l}\}$, is now substituted by the new replicator system with fiscal subsidies given by

$$\dot{x}_i^{\tau} = x_i^{\tau}(1-x_i^{\tau})[\lambda(\alpha^{\tau} + \beta^{\tau})(E_T(i) - E_T(j))] \tag{17}$$

Consider that the initial conditions are $z(t_0) = (x_S(t_0), y_I(t_0))$, then the solution of this system will be unique and symbolized by $\xi(t, t_0 z(t_0))$. Now, the threshold value corresponding to this system is given by the equations:

$$\begin{aligned} E_T(\mathrm{h}) &= E_\mathrm{l}^2 - \theta T \\ E_T(\mathrm{H}) &= E_\mathrm{L}^1 - (1-\theta)T \end{aligned} \tag{18}$$

From these equations we get the new threshold value with subsidies in terms of the share of high-type leaders and high-type followers: $G = (\hat{x}_\mathrm{H}^1, \hat{x}_\mathrm{h}^2) \le \bar{P} = (x_\mathrm{H}^{1*}, x_\mathrm{h}^{2*})$. This value is given by

$$\begin{aligned} \hat{x}_\mathrm{H}^1 &= x_\mathrm{H}^{1*} - \theta T \\ \hat{x}_\mathrm{h}^2 &= x_\mathrm{h}^{2*} - (1-\theta)T \end{aligned} \tag{19}$$

where x_H^{1*} and x_h^{2*} are the former threshold value defined in (12). So, if the initial value $z(t_0) = (x_\mathrm{h}^2(t_0), x_\mathrm{H}^1(t_0))$ in any given time, $t = t_0$ of the economy, is below the threshold value $\bar{P} = (x_\mathrm{H}^{1*}, x_\mathrm{h}^{2*})$, then the central planner needs to implement fiscal subsidies such that

$$\begin{aligned} x_\mathrm{H}^1(t_0) &\ge x_\mathrm{H}^{1*} - \theta T \\ x_\mathrm{h}^2(t_0) &\ge x_\mathrm{h}^{2*} - (1-\theta)T \end{aligned} \tag{20}$$

and therefore the initial conditions are outside from the basin of attraction of the low-level equilibrium (poverty trap), corresponding to the system (17). Hence, the economy moves towards a high-level equilibrium. Once the economy surpasses the threshold value $G = (\hat{x}_\mathrm{H}^1, \hat{x}_\mathrm{h}^2)$, the central planner may leave the economy to evolve by its own rules, i.e. the evolution of the economy will be determined by the system (10), but now the initial conditions are in the basin of attraction of the high-level equilibrium. This means that the central planner should withdraw fiscal subsidies once the economy, following a trajectory which corresponds to a solution of (17), surpasses G. Subsequently, the agents following their imitation rule will drive the economy to a high-level equilibrium. Therefore, the intervention by fiscal subsidies of the central planner in the economy, in a time $t = t_0$, could be justified for a given time period of $z(t_0)$ below the threshold value G.

7 Concluding Remarks

In this paper we developed and studied a framework which encapsulates a strategic coordination game between leaders and followers into an evolutionary dynamics based on the imitative behavior of two populations of leaders and followers. The economic interpretation of this set-up focus on the presence of poverty traps in economic development stemming from the equilibria multiplicity of the basic coordination game. The ‘favorable’ and the ‘unfavorable’ equilibrium are attractors of an evolutionary process of imitation and matching between leaders and followers populating the economy. The two basins of attraction are separated by a saddle path which depends upon the (unique) Nash equilibrium in mixed strategies of the coordination game. This latter equilibrium depends on the model's parameters, such as training or education costs, skill premia and the tax structure, so that the extension of the two basins of attraction is affected by these parameters. Even though the general motivation of this paper is mainly theoretical, the paper's policy proposal (i.e. to subsidize education, R&D etc.) is rather intuitive and common in poverty traps literature. Our most important point is the provision of a novel mechanism capable of explain the presence of poverty traps based on imitative behavior and bounded rationality coupled with a strategic coordination problem.

The central planner can change the initial conditions that define the future evolution of the economy. To do this, he or she must implement appropriate economic policy measures. For instance, some fiscal subsidies could encourage followers and leaders to become high types. The purpose of these policies is to withdraw the economy from a trajectory converging to an inefficient equilibrium (the poverty trap). This intervention of the central planner might stop once the solution $\xi(t, t_0, z(t_0))$ of the dynamical system given by (17) surpasses the threshold value G. From this period on, the economy will follow a trajectory, then according to a solution of the dynamical system (10) it converges to a high-level equilibrium. Then, the intervention of the central planner becomes superfluous. When this moment arrives is a question for future research.

Appendix

Proof: Consider the Jacobian associated to the system $(\dot{x}_h^2, \dot{x}_H^1)$ given by

$$J(\cdot) = \begin{pmatrix} (1-2x_h^2)(x_H^1 p - C) & x_h^2(1-x_h^2)p \\ x_H^1(1-x_H^1)(1-\gamma)(U-u+V-v) & (1-2x_H^1)\mathbb{k} \end{pmatrix} \quad (21)$$

where $\mathbb{k} = x_h^2[(1-\gamma)(\Delta U + \Delta V)] - (1-\gamma)(V+u) - W + w$. Equilibria for which it is determined that $\det(J) > 0$ and $\mathrm{tr}(J) < 0$ are asymptotically stable, thus from Definition 7 they are ESSs against the field. We evaluate $J(\dot{x}_h^2, \dot{x}_H^1)$:

1. $x_h^2 = x_H^1 = 0$. The evaluated Jacobian in this case is given by

$$J = \begin{bmatrix} -C & 0 \\ 0 & -[(1-\gamma)(V+u)+W-w] \end{bmatrix}$$

It yields $\det J = [W - w + (1-\gamma)(V+u)](C) > 0)$ and $\mathrm{tr} J < 0$. Hence this equilibrium point (0,0) is an attractor and therefore an ESS.

2. $x_h^2 = x_H^1 = 1$. The evaluated Jacobian is given by

$$J = \begin{bmatrix} -(p-C) & 0 \\ 0 & -[W-w+(1-\gamma)(U-2u-v)] \end{bmatrix}$$

Thus, det$J > 0$ and tr$J < 0$. Hence this equilibrium point (1,1) is an attractor and an ESS.

3. $x_{\text{h}}^2 = 1,\ x_{\text{H}}^1 = 0$. The evaluated Jacobian is

$$J = \begin{bmatrix} C & 0 \\ 0 & (1-\gamma)(U-2u-v)-W+w \end{bmatrix}$$

Thus, det$J > 0$ and tr$J > 0$. In this case, the equilibrium point (1,0) is a repulsor.

4. $x_{\text{h}}^2 = 0,\ x_{\text{H}}^1 = 1$. The evaluated Jacobian in this case is

$$J = \begin{bmatrix} p-C & 0 \\ 0 & (1-\gamma)(V+u)+W-w \end{bmatrix}$$

Thus, det$J > 0$ and tr$J > 0$. In this case, the equilibrium point (0,1) is a repulsor.

Since the C square is partitioned into four regions, the point $(x_{\text{h}}^{2*},\ x_{\text{H}}^{1*})$ is a saddle and the other four are local attractors or repulsors. The curve that converges to $\bar{P}$ is a set of critical values into the state of the C square with the following property: the optimal strategy is different depending on which side of the threshold the current state lies. Therefore, there is just a one-dimensional manifold (threshold level) which goes through $\bar{P}$. Such a threshold separates the basins of attraction into (0,0) and (1,1). Hence, if the initial distribution of high-type economic agents, $(x_{\text{H}0}^1,\ x_{\text{h}0}^2)$, is lower than the threshold $(x_{\text{h}}^{2*},\ x_{\text{H}}^{1*})$, then the strategic profile (L, l) = (0,1; 0,1) is an ESS against the field of $(\bar{x}_{\text{h}}^2,\ \bar{x}_{\text{H}}^1)$ for all $(\bar{x}_{\text{h}}^2, \bar{x}_{\text{H}}^1) \neq (x_{\text{h}}^{2*}, x_{\text{H}}^{1*}) \in \Delta^k$. Otherwise if it is upper to the threshold the equilibrium (H, h) = (1,0; 1,0) is an ESS against the field. ■

References

Apesteguia, J., Huck, S. and Oechssler, J. (2007). 'Imitation—Theory and Experimental Evidence', *Journal of Economic Theory*, Vol. 136, pp. 217–235.

Azariadis, C. and Drazen, A. (1990). 'Threshold Externalities in Economic Development', *Quarterly Journal of Economics*, Vol. 105, No. 2, pp. 501–526.

Azariadis, C. and Starchuski, H. (2005). 'Poverty Traps', in P. Aghion and S. Durlauf (eds), *Handbook of Economic Growth*, Oxford, Elsevier.

Björnerstedt, J. and Weibull, J. W. (1996). 'Nash Equilibrium and Evolution by Imitation', in K. Arrow, E. Colombatto, M. Perlman and C. Schmidt (eds), *The Rational Foundations of Economic Behavior*, London, Macmillan, pp. 155–171.

Bowles, S., Durlauf, S. and Hoff, K. (2006). *Poverty Traps*, Princeton, NJ, Princeton University Press.

Cooper, R. and John, A. (1998). 'Coordinating Coordination Failures in Keynesian Models', *Quarterly Journal of Economics*, Vol. 103, pp. 441–463.

Durlauf, S. N. (2003). 'Neighborhood Effects', Madison, University of Wisconsin, Department of Economics, *SSRI Working Paper 2003-17* (prepared for J. Vernon Henderson and Jacques-François Thisse (eds), *Handbook of Regional and Urban Economics*, Vol. 4, *Cities and Geography (Handbooks in Economics 7)*, Amsterdam, North-Holland).

Fudenberg, D. and Tirole, J. (1991). *Game Theory*, Cambridge, MA, MIT Press.

Hoff, K. (2001). 'Beyond Rosenstein–Rodan: the Modern Theory of Coordination Problems in Development', *Annual World Bank Conference on Development*, Washington, DC, The World Bank, pp. 145–176.

Matsuyama, K. (2007). 'Aggregate Implications of Credit Market Imperfections', in D. Acemoglu, K. Rogoff and M. Woodford (eds), *NBER Macroeconomics Annual*, Vol. 22, Cambridge, MA, National Bureau of Economic Research, and Chicago, University of Chicago Press.

North, D. (1990). *Institutions, Institutional Change and Economic Performance*, Cambridge, Cambridge University Press.

Polterovich, V. (2008). 'Institutional Trap', in S. N. Durlauf and L. E. Blume (eds), *The New Palgrave Dictionary of Economics*, London, Palgrave Macmillan.

Sanditov, B. (2006). 'Essays on Social Learning and Imitation', *PhD thesis*, Maastrictht University.

Schlag, K. H. (1998). '"Why Imitate, and If So, How?" A Boundedly Rational Approach to Multi-armed Bandits', *Journal of Economic Theory*, Vol. 78, No. 1, pp. 130–156.

Schlag, K. H. (1999). 'Which One Should I Imitate', *Journal of Mathematical Economics*, Vol. 31, No. 4, pp. 493–522.

Taylor, P. (1979). 'Evolutionarily Stable Strategies with Two Types of Player', *Journal of Applied Probability*, Vol. 16, pp. 76–83.

Weibull, W. J. (1995). *Evolutionary Game Theory*, Cambridge, MA, MIT Press.

Zeeman, A. C. (1992). 'Population Dynamics from Game Theory', in Z. Nitecki and C. Robinson (eds), *Global Theory of Dynamical Systems*, Heidelberg, Springer-Verlag, pp. 471–497, Lecture Notes in Mathematics 819.