

Find solution using Simplex(BigM) method

MIN $Z = 7560x_1 + 1680x_2 + 4636.8x_3 + 1478.4x_4$

subject to

$x_1 + x_2 \geq 110$

$x_1 + x_3 \geq 100$

$x_1 + x_4 \geq 80$

$x_1 \leq 90$

and $x_1, x_2, x_3, x_4 \geq 0$

Solution:

Problem is

Min $Z = 7560x_1 + 1680x_2 + 4636.8x_3 + 1478.4x_4$

subject to

$x_1 + x_2 \geq 110$

$x_1 + x_3 \geq 100$

$x_1 + x_4 \geq 80$

$x_1 \leq 90$

and $x_1, x_2, x_3, x_4 \geq 0$;

$\therefore \text{Max } Z = -7560x_1 - 1680x_2 - 4636.8x_3 - 1478.4x_4$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropriate

1. As the constraint 1 is of type ' \geq ' we should subtract surplus variable S_1 and add artificial variable A_1
2. As the constraint 2 is of type ' \geq ' we should subtract surplus variable S_2 and add artificial variable A_2
3. As the constraint 3 is of type ' \geq ' we should subtract surplus variable S_3 and add artificial variable A_3
4. As the constraint 4 is of type ' \leq ' we should add slack variable S_4

After introducing slack,surplus,artificial variables

Max $Z = -7560x_1 - 1680x_2 - 4636.8x_3 - 1478.4x_4 + 0S_1 + 0S_2 + 0S_3 + 0S_4 - MA_1 - MA_2 - MA_3$

subject to

$x_1 + x_2 - S_1 + A_1 = 110$

$x_1 + x_3 - S_2 + A_2 = 100$

$x_1 + x_4 - S_3 + A_3 = 80$

$x_1 + S_4 = 90$

and $x_1, x_2, x_3, x_4, S_1, S_2, S_3, S_4, A_1, A_2, A_3 \geq 0$

Iteration-1		C_j	-7560	-1680	-4636.8	-1478.4	0	0	0	0	-M	-M	-M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	A_1	A_2	A_3	MinRatio X_B/x_1
A_1	-M	110	1	1	0	0	-1	0	0	0	1	0	0	$110/1 = 110$
A_2	-M	100	1	0	1	0	0	-1	0	0	0	1	0	$100/1 = 100$
A_3	-M	80	(1)	0	0	1	0	0	-1	0	0	0	1	$80/1 = 80 \rightarrow$
S_1	0	90	1	0	0	0	0	0	0	1	0	0	0	$90/1 = 90$
Z = 0		Z_j	-3M	-M	-M	-M	M	M	M	0	-M	-M	-M	
		$Z_j - C_j$	$-3M + 7560 \uparrow$	$-M + 1680$	$-M + 4636.8$	$-M + 1478.4$	M	M	M	0	0	0	0	

Negative minimum $Z_j - C_j$ is $-3M + 7560$ and its column index is 1. So, the entering variable is x_1 .

Minimum ratio is 80 and its row index is 3. So, the leaving basis variable is A_3 .

\therefore The pivot element is 1.

Entering = x_1 , Departing = A_3 , Key Element = 1

$R_3(\text{new}) = R_3(\text{old})$

$R_1(\text{new}) = R_1(\text{old}) - R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - R_3(\text{new})$

$R_4(\text{new}) = R_4(\text{old}) - R_3(\text{new})$

Iteration-2		C_j	-7560	-1680	-4636.8	-1478.4	0	0	0	0	-M	-M	-M	

B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	A_1	A_2	A_3	MinRatio $X_B \cdot S_3$
A_1	$-M$	30	0	1	0	-1	-1	0	1	0	1	0	-1	$30 \cdot 1 = 30$
A_2	$-M$	20	0	0	1	-1	0	-1	1	0	0	1	-1	$20 \cdot 1 = 20$
x_1	-7560	80	1	0	0	1	0	0	-1	0	0	0	1	---
S_1	0	10	0	0	0	-1	0	0	(1)	1	0	0	-1	$10 \cdot 1 = 10$
$Z = -604800$		Z_j	-7560	$-M$	$-M$	$2M - 7560$	M	M	$-2M + 7560$	0	$-M$	$-M$	$2M - 7560$	
		$Z_j - C_j$	0	$-M + 1680$	$-M + 4636.8$	$2M - 6081.6$	M	M	$-2M + 7560 \uparrow$	0	0	0	$3M - 7560$	

Negative minimum $Z_j - C_j$ is $-2M + 7560$ and its column index is 7. So, the entering variable is S_3 .

Minimum ratio is 10 and its row index is 4. So, the leaving basis variable is S_1 .

∴ The pivot element is 1.

Entering = S_3 , Departing = S_1 , Key Element = 1

$$R_4(\text{new}) = R_4(\text{old})$$

$$R_1(\text{new}) = R_1(\text{old}) - R_4(\text{new})$$

$$R_2(\text{new}) = R_2(\text{old}) - R_4(\text{new})$$

$$R_3(\text{new}) = R_3(\text{old}) + R_4(\text{new})$$

Iteration-3		C_j	-7560	-1680	-4636.8	-1478.4	0	0	0	0	$-M$	$-M$	$-M$	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	A_1	A_2	A_3	MinRatio $X_B \cdot x_2$
A_1	$-M$	20	0	(1)	0	0	-1	0	0	-1	1	0	0	$20 \cdot 1 = 20 \rightarrow$
A_2	$-M$	10	0	0	1	0	0	-1	0	-1	0	1	0	---
x_1	-7560	90	1	0	0	0	0	0	0	1	0	0	0	---
S_3	0	10	0	0	0	-1	0	0	1	1	0	0	-1	---
$Z = -680400$		Z_j	-7560	$-M$	$-M$	0	M	M	0	$2M - 7560$	$-M$	$-M$	0	
		$Z_j - C_j$	0	$-M + 1680 \uparrow$	$-M + 4636.8$	1478.4	M	M	0	$2M - 7560$	0	0	M	

Negative minimum $Z_j - C_j$ is $-M + 1680$ and its column index is 2. So, the entering variable is x_2 .

Minimum ratio is 20 and its row index is 1. So, the leaving basis variable is A_1 .

∴ The pivot element is 1.

Entering = x_2 , Departing = A_1 , Key Element = 1

$$R_1(\text{new}) = R_1(\text{old})$$

$$R_2(\text{new}) = R_2(\text{old})$$

$$R_3(\text{new}) = R_3(\text{old})$$

$$R_4(\text{new}) = R_4(\text{old})$$

Iteration-4		C_j	-7560	-1680	-4636.8	-1478.4	0	0	0	0	$-M$	$-M$	$-M$	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	A_1	A_2	A_3	MinRatio $X_B \cdot x_3$
x_2	-1680	20	0	1	0	0	-1	0	0	-1	1	0	0	---
A_2	$-M$	10	0	0	(1)	0	0	-1	0	-1	0	1	0	$10 \cdot 1 = 10 \rightarrow$
x_1	-7560	90	1	0	0	0	0	0	0	1	0	0	0	---
S_3	0	10	0	0	0	-1	0	0	1	1	0	0	-1	---
$Z = -714000$		Z_j	-7560	-1680	$-M$	0	1680	M	0	$M - 5880$	-1680	$-M$	0	
		$Z_j - C_j$	0	0	$-M + 4636.8 \uparrow$	1478.4	1680	M	0	$M - 5880$	$M - 1680$	0	M	

Negative minimum $Z_j - C_j$ is $-M + 4636.8$ and its column index is 3. So, the entering variable is x_3 .

Minimum ratio is 10 and its row index is 2. So, the leaving basis variable is A_2 .

∴ The pivot element is 1.

Entering = x_3 , Departing = A_2 , Key Element = 1

$$R_2(\text{new}) = R_2(\text{old})$$

$$R_1(\text{new}) = R_1(\text{old})$$

$$R_3(\text{new}) = R_3(\text{old})$$

$$R_4(\text{new}) = R_4(\text{old})$$

Iteration-5		C_j	-7560	-1680	-4636.8	-1478.4	0	0	0	0	-M	-M	-M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	A_1	A_2	A_3	MinRatio X_B/S_4
x_2	-1680	20	0	1	0	0	-1	0	0	-1	1	0	0	---
x_3	-4636.8	10	0	0	1	0	0	-1	0	-1	0	1	0	---
x_1	-7560	90	1	0	0	0	0	0	0	1	0	0	0	90/1 = 90
S_3	0	10	0	0	0	-1	0	0	1	(1)	0	0	-1	10/1 = 10 →
$Z = -760368$		Z_j	-7560	-1680	-4636.8	0	1680	4636.8	0	-1243.2	-1680	-4636.8	0	
		$Z_j - C_j$	0	0	0	1478.4	1680	4636.8	0	-1243.2 ↑	$M - 1680$	$M - 4636.8$	M	

Negative minimum $Z_j - C_j$ is -1243.2 and its column index is 8. So, the entering variable is S_4 .

Minimum ratio is 10 and its row index is 4. So, the leaving basis variable is S_3 .

∴ The pivot element is 1.

Entering = S_4 , Departing = S_3 , Key Element = 1

$$R_4(\text{new}) = R_4(\text{old})$$

$$R_1(\text{new}) = R_1(\text{old}) + R_4(\text{new})$$

$$R_2(\text{new}) = R_2(\text{old}) + R_4(\text{new})$$

$$R_3(\text{new}) = R_3(\text{old}) - R_4(\text{new})$$

Iteration-6		C_j	-7560	-1680	-4636.8	-1478.4	0	0	0	0	-M	-M	-M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	A_1	A_2	A_3	MinRatio
x_2	-1680	30	0	1	0	-1	-1	0	1	0	1	0	-1	
x_3	-4636.8	20	0	0	1	-1	0	-1	1	0	0	1	-1	
x_1	-7560	80	1	0	0	1	0	0	-1	0	0	0	1	
S_4	0	10	0	0	0	-1	0	0	1	1	0	0	-1	
$Z = -747936$		Z_j	-7560	-1680	-4636.8	-1243.2	1680	4636.8	1243.2	0	-1680	-4636.8	-1243.2	
		$Z_j - C_j$	0	0	0	235.2	1680	4636.8	1243.2	0	$M - 1680$	$M - 4636.8$	$M - 1243.2$	

Since all $Z_j - C_j \geq 0$

Hence, optimal solution is arrived with value of variables as :

$$x_1 = 80, x_2 = 30, x_3 = 20, x_4 = 0$$

$$\text{Max } Z = -747936$$

$$\text{Min } Z = 747936$$