"Structural Methods in the Study of Complex Systems"

Pre-conference Full-day Workshop at the

European Control Conference 2018

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Confirmed speakers:
József Bokor, Hungarian Academy of Sciences, Hungary
Kanat Camlibel, University of Groningen, The Netherlands
Sergio Galeani, University of Rome "Tor Vergata", Italy
Nicos Karcanias, City University of London, UK
Michel Malabre, Laboratoire des Sciences du Numérique de Nantes, France
Vladimir Rasvan, University of Craiova, Romania
Andrea Serrani, The Ohio State University, USA
Zoltán Szabó, Hungarian Academy of Sciences, Hungary

Extended Abstract:
The workshop provides a forum for a comprehensive exposition and discussion on how paradigmatic analysis and synthesis problems formulated for complex dynamical systems can be effectively investigated and solved through methodological approaches primarily grounded on structural views.

Complex dynamical systems emerge in a variety of disciplines and domains, ranging from those that deal with physical processes (biology, genetics, environmental sciences, etc.) to those that concern man-made systems (engineering, energy, finance, etc.). Indeed, in these fields, it is more and more common to refer to dynamical structures such as systems of systems, hybrid systems and multimodal systems. In brief, the former ones consist of many interconnected dynamical systems with various topological patterns and hierarchical relations; the second ones are dynamical systems that exhibit dynamics of a different nature, both continuous and discrete;

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the third ones are dynamical systems whose behavior may vary during their life-cycle owing to different operating conditions or depending on the occurrence of some events. The dynamical structures with these characteristics are currently modelled as, e.g., multi-agent systems, hybrid impulsive systems, switching systems, implicit switching systems, and so forth.

Consequently, control design techniques have changed to adapt to the ever-increasing system complexity. In this scenario, structural methodologies — i.e., those methods which have evolved from original graph theories, differential-algebraic techniques and geometric approaches to linear and nonlinear control — have proven to be particularly powerful for several reasons. Beforehand, the structural approaches privilege the essential features of dynamical systems and their interconnections, thus yielding abstractions that can fit a wide variety of situations. Meanwhile, the geometric perspective, which is often at the basis of the structural approaches, introduces a relevant visual and intuitive component which fosters research advancements. Nevertheless, the formalization of the structural and geometric concepts is rendered with algebraic tools, which, in turn, have a direct correspondence with computational algorithms, thus paving the way to actual implementation in engineering applications.

During the latest years, relevant theoretical achievements have been obtained within the scope of each of the methodologies encompassed in the sphere of the structural approaches (i.e., graph theoretic methods, geometric methods, differential-algebraic methods) in respect of fundamental control and observation problems stated for complex systems (e.g., multi-agent systems, hybrid impulsive systems, switching systems, implicit switching systems). Moreover, computational algorithms and specific applications have been developed together with the theoretical accomplishments. Thus, the corpus of consolidated results (both theoretical and practical/computational ones) presently available motivates the intention to share and disseminate the status of the art by a thorough presentation. Furthermore, the discussion among the participants is expected to establish fruitful relations between the various approaches, to emphasize pros and cons in connection with specific contexts or applications and to point out merging opportunities. Although these are definitely nontrivial tasks, the efforts made to these ends are expected to develop new sensibility towards the selection of the most suitable tools in relation to the problems at issue as well as to outline new directions for solving the open problems both in the theory and in the applications.

The workshop is organized in such a way that the discussion starts from a general description of complexity and structural attitudes towards it, then it focuses on some fundamental problems and, finally, it dwells on applications. In more detail, an overview on the complex systems emerging in the various fields, on the new challenges of engineering design and on how these can be mastered by means of the structural approaches is provided first. A novel geometric view, based on transformations which maintain the invariance of global properties, such as stability or H_∞ norm, is described and shown to provide new tools to investigate stability and to parametrize the set of the stabilizing controllers. A graph-theoretic-based approach is then shown to be an effective tool to analyse controllability in leader/follower multi-agent systems and to understand the effects of the underlying network structure. How solvability of the output regulation problem in hybrid linear systems with periodic state jumps can be investigated by structural methods is then illustrated. A mixed digraph-theory and geometric approach is exploited to solve another classic problem of system and control theory, i.e. disturbance decoupling, for a special class of hybrid switching systems. Furthermore, the synthesis of unknown-input state observers with minimum-complexity is tackled by structural tools in the context of linear impulsive systems: necessary and sufficient solvability conditions are derived once a set of essential requirements has been disentangled. The disturbance decoupling problem is also investigated for a class of implicit switching systems through geometric considerations inspired to the behavioral approach. In particular, the theoretical results are applied to the synthesis of a Beard-Jones filter. Finally, another meaningful application is considered in depth: a structural perspective is adopted to analyze Huygens synchronization over distributed media and it reveals a complex, but structured behavior, behind a seemingly chaotic one.

Keywords:
Complex Systems, Hybrid Dynamical Systems, Dynamical Systems Networks, Structural Methods, Algebraic/Geometric Methods, Graph Theory

Intended audience:
The workshop is intended for systems and control researchers who are interested in developing theoretical and practical computational tools for analysis and synthesis problems in the area of complex systems. Due to the emerging importance of dealing with and governing complexity of dynamical structures, we believe that the workshop may attract a large audience of conference attendees, including students as well as experienced researchers. Presentations aim at giving a comprehensive picture of available results together with a stimulating view of possible new directions of investigation and research in the field. Since the workshop will emphasize methodologies supported by a solid computational background and it will illustrate concrete applications, scientists who either focus on practical engineering aspects or are mainly involved in theoretical issues are equally welcome.

Program:
09:00 Opening
Elena Zattoni, Anna Maria Perdon, Giuseppe Conte

09:00–09:45
“Structure evolving systems and engineering design: Structural problems”
Nicos Karcanias (City University of London, UK)

09:45–10:30
“Stability and the Kleinian view of geometry”

Zoltán Szabó (Hungarian Academy of Sciences, Hungary)
József Bokor (Hungarian Academy of Sciences, Hungary)

10:30–11:00 Coffee break

11:00–11:45
"Controllability of networks"
Kanat Camlibel (University of Groningen, The Netherlands)

11:45–12:30
"Output regulation of hybrid linear systems: Solvability conditions and structural implications"
Sergio Galeani (University of Rome "Tor Vergata", Italy)
Mario Sassano (University of Rome "Tor Vergata", Italy)

12:30–14:00 Lunch break

14:00–14:45
"A stratified geometric approach to the disturbance decoupling problem with stability for switched discrete-time systems over digraphs"
Andrea Serrani (The Ohio State University, USA)
Junqiang Zhou (GE Global Research, USA)

14:45–15:30
"The unknown-input observation problem for hybrid dynamical structures"
Giuseppe Conte (Polytechnic University of Marche)
Anna Maria Perdon (Polytechnic University of Marche)
Elena Zattoni (Alma Mater Studiorum – University of Bologna, Italy)

15:30–16:00 Coffee break

16:00–16:45
"Advances of implicit description techniques in modelling and control of complex systems"
Moisés Bonilla (Control Automatico, CINVESTAV IPN and UMI LAFMIA CNRS, Mexico)
Michel Malabre (Laboratoire des Sciences du Numérique de Nantes, France)
Vadim Azhmyakov (Department of Basic Science, University of Medellin, Colombia)

16:45–17:30
"Huygens synchronization over distributed media – Structure versus complex behavior"
Vladimir Rasvan (University of Craiova, Romania)

17:30 Closing
Elena Zattoni, Anna Maria Perdon, Giuseppe Conte

Brief bios of the organizers:

Elena Zattoni received the Laurea degree cum laude in Electronics Engineering in 1995 and the Ph.D. in Systems Engineering in 1999 from the University of Bologna, Italy. Since 2001 she has been with the University of Bologna where she is an assistant professor. She has been a visiting professor at Aalto University, University of Michigan, Institut de Recherche en Communications et Cybernétique de Nantes, and Brown University. Her research interests are focused on geometric and structural approaches to control and observation of dynamical systems, including hybrid systems, switched systems, and time-delay systems, with applications to the synthesis of enhanced-reliability control systems. She has authored more than one hundred papers in the field. She is General Chair of the "15th European Workshop on Advanced Control and Diagnosis, ACD 2019". She serves as Associate Editor of "Nonlinear Analysis: Hybrid Systems". She is a member of the IFAC Technical Committee 2.2 "Linear Control Systems". She is a Senior Member of IEEE and has served in the "Member Activity" Board of the IEEE Control Systems Society.

Anna Maria Perdon received the Laurea in Mathematics from the University of Padua, Italy. She is Full Professor of Control Theory at the Università Politecnica delle Marche from 2000. She authored more than one hundred publications in journals, conference proceedings and books, she edited five books and she co-authored the research monographs Algebraic Methods for Nonlinear Control Systems. Theory and Applications (London, U.K.: Springer Verlag 2007). Her research interests include hybrid systems, switching systems, delay differential systems, algebraic, geometric and differential-algebraic techniques for linear and nonlinear dynamical systems. Dr. Perdon served in the editorial boards of several journals, as Program Chair or General Chair of systems and control conferences and as Associated Editor of System & Control Letters and SIAM Journal on Control and Optimization.

Giuseppe Conte received the Laurea in Mathematics from the University of Genoa, Italy, in 1974. He is currently Professor of Automation at the Polytechnic University of Marche, Ancona, Italy. His research interests are in algebraic and geometric system and control theory and in robotics. He has authored a number of publications in journals, conference proceedings and books, he edited two books and he co-authored the research monographs Algebraic Methods for Nonlinear Control Systems. Theory and Applications (London, U.K.: Springer Verlag 2007). He served in the editorial boards of several journals, as Program Chair and General Chair of systems and control conferences and currently serves as Associate Editor of IMA Journal of Mathematical Control and Information. Dr. Conte is Chairman of the IFAC Technical Committee on Linear Control Systems and he was Chairman of the Italian Chapter of the IEEE Control Systems Society.

Brief bios of the speakers:

József Bokor received the Dr. Eng. and Pd.D. degrees from the EE Department of Budapest University of Technology and Economics. He is Professor and former Head of the Automation Department, Faculty of Transportation and Vehicle Engineering, Budapest University of Technology and Economics. He is also Research Director of the Computer and Automation Research Institute, Hungarian Academy of Sciences. He was visiting the *Imperial College of Science and Technology, Computing and Control Department*, London, England, the *MIT Laboratory for Information and Decision Systems* as visiting Fulbright professor. He had a long collaboration with the Laboratory for Measurement and Control, *Technical University Delft* and with the University of Eindhoven, The Netherlands. He is an adjunct professor of the *Department of Aerospace and Mechanics, University of Minnesota, US, MN*. He is a Member of the Hungarian Academy of Sciences and a Fellow of IEEE and IFAC. His research interest includes linear and LPV systems, system identification, fault detection and isolation, robust control with application to automotive and flight control systems.

Kanat Camlibel received the Ph.D. degree in mathematical theory of systems and control from Tilburg University, Tilburg, The Netherlands, in 2001. He is an Associate Professor at the Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, Groningen, The Netherlands, where he served as an assistant professor between 2007 and 2013. From 2001 to 2007, he held postdoctoral/assistant professor positions with the University of Groningen, Tilburg University, and Eindhoven Technical University, Eindhoven, The Netherlands. His research interests include differential variational inequalities, complementarity problems, optimization, piecewise affine dynamical systems, switched linear systems, constrained linear systems, dynamical networks, multi-agent systems, energy systems, model reduction, and geometric theory of linear systems. Dr. Camlibel is an associate editor of the IEEE Transactions on Automatic Control, Systems & Control Letters, and SIAM Journal on Control and Optimization.

Sergio Galeani received the Laurea degree in 1998 and the Ph.D. in Computer Science and Control Engineering in 2002 from the University of Roma Tor Vergata, where he is currently a researcher. His research interests include periodic and multirate control systems, control systems with constraints, robust/adaptive anti-windup techniques, and hybrid, linear and nonlinear output regulation. He is an Associate Editor of IEEE Transactions on Control Systems Technology and in the Conference Editorial Board of the IEEE Control Systems Society (CSS) and of the European Control Association (EUCA).

Nicos Karcanias is a graduate of NTUA of Athens in Electrical Engineering and has M.Sc. and Ph.D. in Control Engineering from UMIST (UK) and the DSc from City University. During the period 1974 to 1980 he has carried out research in the Control and Management Systems Group of the University of Cambridge as a Research Assistant and then Research Fellow. In 1980 he joined the Department of Systems Science of City University as a Lecturer and then joined the Electrical Engineering Department of the same university where he was promoted to a personal chair in 1993 as Professor of Control Theory and Design. He is now Associate Dean for Research in the School of Engineering and Mathematical Sciences, and he is Director of the Systems and Control Centre. He is Fellow of IET (IEE), Fellow of IMA and senior member of IEEE. He is Editor of IMA Journal of Mathematical Control and Information, member of Editorial Board of IEEE Control Conferences (Associate Editor for CDC and ACC Conferences), Associate Editor for IFAC 2011 World Congress. His research has been in the development of the algebraic, geometric and algebra-geometric methods for Control Theory. His research on the Control fundamentals has been accompanied by an effort to migrate Systems and Control to Complex problems, such as the development of a Control based methodology to Systems Integration and developing Control based methodology for Complex Systems. His research has been supported by a number of EU projects and EPSRC. He has been the author/co-author of over 267 papers published in Scientific Journals and Conference Proceedings, the holder of a number of research grants including eight major EU grants and supervisor of over 35 completed PhD thesis. His research publications are in the areas of Linear Systems, Mathematical Systems Theory, Control Theory and Design, Algebraic Computations, Mathematical Methods for Control, Systems Theory of Measurement, Systems and Control to Complex Systems, Integrated Systems Design, and algebra-geometric methods. The main drive of his current research is the development of systems and control for complex systems, by developing the theory required for the new systems paradigms of "structure evolving systems" and "Systems of Systems".

Michel Malabre was born in Toulouse (France) in July 1954. He got the following diplomas: Mechanical and Control Engineering (1977), DEA, i.e. Master of Sciences, in Control (1979) both from Ecole Nationale Supérieure de Mécanique de Nantes (ENSM, which was the previous name of Ecole Centrale de Nantes), and PhD Doctorat Ingénieur (1981), and Doctorat d'Etat (1985), now replaced by "Habilitation á Diriger des Recherches", both at ENSM and Université de Nantes (co-habilitation). Michel Malabre has got a permanent position as a CNRS Researcher (Centre National de la Recherche Scientifique) since 1981. Michel Malabre is presently Directeur de Recherche CNRS in Automatic Control. From January 2008 to December 2016, he has been leading the Research Institute IRCCyN, "Institut de Recherche en Communications et Cybernétique de Nantes, a CNRS Joint Research Unit (n. 6597), with Centrale Nantes, Mines de Nantes (presently IMT Atlantique) and Université de Nantes. There were 11 research teams, with more than 270 members (counting permanent staff and PhD students), plus visitors for short stays. Since January 2017, this Joint Research Unit is now called LS2N (Laboratoire des Sciences du Numérique de Nantes, UMR CNRS 6004), with more than 20 teams and 430 members. Michel Malabre has been leading the Control team up to October 2006 (27 members: 13 permanent staff and 14 PhD students). His teaching (MS level) concerned Systems Structure and Control. His main research areas are analysis and control of linear time-invariant systems, possibly implicit (generalized, singular, of descriptor type), possibly with delays, with a particular attention to problems like non interaction (decoupling), disturbance rejection, model matching and failure detection. Michel Malabre is member of SIAM and of the IFAC Technical Committee on Linear Systems. He trained 8 PhD students, 7 Master students, and 3 Post-Docs (2 for 11 months, 1 for 2 years). He is author and co-author of 2 book chapters, more than 60 papers in international journals, and more than 110 contributions in international conferences with proceedings.

Vladimir Rasvan (b. 1945) obtained the *diplomed engineer* title and the *doctoral degree* (*Ph.D.*) both in Automatic Control from the Polytechnic Institute of Bucharest, Romania, in 1967 and 1972 respectively. After 10 years of service in applied research (control systems for power plants) he became *associate professor* in 1982 and *full professor* in 1990 at the University of Craiova, Romania, Department of Automatic Control. He is currently *Professor of Systems and Control* at the Department of Automation and Electronics of the University of Craiova. In almost 50 years of professional activity Dr. Vladimir Rasvan has published almost 100 journal papers, some 130 conference papers in conference proceedings, 7 books and other 20 book chapters. His main scientific interests are concerned with nonlinear systems stability and oscillations, synchronization, time delays and propagation distributed parameter systems. Senior Member IEEE, member SIAM and AMS, Dr. Rasvan is national vice-president of the Romanian Society of Automation and Technical Informatics (SRAIT) - Romania NMO at IFAC and member in two IFAC technical committees. As Associate Editor he has served or is still acting for several national and international journals. Dr. Vladimir Rasvan is full member of the Romanian Academy of Engineering Sciences.

Andrea Serrani received the Ph.D. degree from the University of Ancona, Italy, in 1997, and the D.Sc. degree from Washington University in Saint Louis in 2000. Since 2002, he has been with The Ohio State University, Columbus, Ohio, where he is currently a Professor. His research activity spans the field of control and systems theory, with applications to aerospace and vehicular systems. He is the co-author (with A. Isidori and L. Marconi) of the book Robust Autonomous Guidance: An Internal Model Approach, published by Springer Verlag. Prof. Serrani is a Fulbright Fellow and the recipient of four US Air Force Fellowships. He is currently the Editor-in-Chief for the IEEE Transactions on Control Systems Technology and an Associate Editor for the IEEE CSS and EUCA Conference Editorial Boards, and has served in the past on the editorial boards of Automatica and the International Journal of Robust and Nonlinear Control. He is a member of IEEE, AIAA, and IFAC. He is the Program Chair of the upcoming 2019 American Control Conference.

Zoltán Szabó is a doctor of Hungarian Academy of Sciences. As a research advisor of the Systems and Control Laboratory of the MTA SZTAKI he is head of the "Theory of Dynamical Systems" research group. His research interests include geometric methods in control, multivariable systems and robust control, system identification, fault detection with applications in power systems safety operations and also in the control of mechanical and vehicle structures. Currently he is involved in activities related to reconfigurable control and robust control of linear parameter varying (LPV) systems.

Abstracts of the talks:

Stability and the Kleinian View of Geometry

Zoltán Szabó[4] *and József Bokor*[5]

Geometry is one of the richest areas for mathematical exploration. The visual aspects of the subject make exploration and experimentation natural and intuitive. At the same time, the abstractions developed to explain geometric patterns and connections make the approach extremely powerful and applicable to a wide variety of situations. To put geometry and geometrical thought in a position to become a reliable engineering tool, a certain mechanism is needed that translates geometrical facts into a more accessible form for everyday algorithms. Klein proposed group theory as a tool of formulating and understanding geometrical constructions. The main concern of the presentation is to highlight the deep relations that exists between the seemingly different fields of geometry, algebra and control. While the Kleinian view creates the link between geometry and group theory, through different representations and homomorphism the abstract group theoretical facts obtain an algebraic (linear algebraic) formulation that opens the way to engineering applications. We demonstrate through examples of the basic control tasks that a natural framework to formulate various control problems is the world that contains the equivalence classes determined by the stabilizable plants as points and whose natural motions are the Möbius-transforms.

In contrast to traditional geometric control theory for the linear and for the nonlinear theory centering on a local view, the presented approach provides a global view. While the former uses tools from differential geometry, Lie algebra, algebraic geometry, and treats system concepts like controllability as geometric properties of the state space or its subspaces, the latter focuses on an input-output – coordinate free – framework where different transformation groups which leave a given global property invariant play a fundamental role. In the first case the invariants are the so-called invariant or controlled invariant subspaces, and the suitable change of coordinates and system transforms (diffeomorphisms), see, e.g. the Kalman decomposition, reveal these properties. In contrast, our interest is in the transformation groups that leave a given global property, e.g., stability or H_∞ norm, invariant.

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One of the most important consequences of the approach is that through the analogous of the classical geometric constructions it not only might give hints for efficient algorithms but the underlying algebraic structure, i.e., the given group operation, also provides tools for controller manipulations that preserves the property at hand, called controller blending. A detailed analysis is given for feedback stability: a blending operation is given under which well-posedness is a group while stability is a semigroup, in general. Moreover, a blending rule is provided that makes controllers with strongly stable property a group.
Youla-parametrization of stabilizing controllers is a fundamental result of control theory: starting from a special double co-prime factorization, as generalized projective coordinates of the plant we obtain a formula for the stabilizing controllers as a function of the elements of the set of stable systems. In this case, the set of parameters is universal, i.e., does not depend on the plant but only the dimension of the signal spaces. Based on the proposed geometric techniques our work provides an alternative, geometry based parametrization. In contrast to the Youla case, this parametrization is coordinate free: it is based only on the knowledge of the plant and a single stabilizing controller. While the parameter set itself is not universal, its elements can be generated by a universal algorithm. Moreover, it is shown that on the parameters of the strongly stabilizing controllers a simple group structure can be defined.
We would like to stress that it is a very fruitful strategy to try to formulate a control problem in an abstract setting, then translate it into an elementary geometric fact or construction. Finally, the solution of the original control problem can be formulated in an algorithmic way by transposing the geometric ideas into the proper algebraic terms. Our intention is to demonstrate the beauty of geometric interpretations in robust control. Besides its theoretical and educative value the presentation also provides a possible tool for the algorithmic development.

Controllability of Networks

Kanat Camlibel[6]

The study of networks of dynamical systems became one of the most popular themes within systems and control theory in the last two decades. This talk focuses on controllability of networks of dynamical systems. In particular, we will be interested in the so-called leader-follower multi-agent systems. A leader/follower multi-agent system consists of a collection of identical dynamical systems (agents) that interact each other via a given network topology. Each agent in the collection assumes one of two roles of leaders or followers and each leader can be influenced through an external input signal. The main goal of the talk is to understand the effect of the underlying network structure on the controllability of the entire network with an eye towards how one can choose leaders to render the entire network controllable.
The talk consists of three main parts. In the first part, we focus on the so-called

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diffusively coupled single-integrator leader/follower networks. Such networks can be considered as consensus dynamics with the presence of external inputs. For these systems, we will investigate controllability in terms of the (undirected) graph capturing the underlying network structure. In particular, we will introduce distance partitions with respect to a given node and almost equitable partitions. The distance partitions will provide a lower bound for the dimension of the controllable subspace. It turns out that almost equitable partitions lead to subspaces that are invariant under Laplacian matrix. This observation will give upper bounds (in terms of subspace inclusion) for the controllable subspace. By using the lattice structure of the set of all almost equitable partitions, we will provide the sharpest bounds that can be obtained via such partitions.
In the second part of the talk, we will discuss more general network dynamics for which the underlying (directed) graph is known but how the agents are coupled via this graph is not known. Such systems are called systems defined over graphs. Controllability of systems defined over graphs is intimately related to the notion of strong structural controllability. The study of controllability of such systems require different graph theoretical notions than graph partition. The main results of the second part will be necessary and sufficient conditions for controllability in terms of the so-called zero forcing sets of the underlying graph. In addition, we will discuss particular classes of graphs for which zero forcing sets can easily be obtained. Also, we will show that the (almost all) existing leader selection methods for Laplacian based dynamics are based on zero forcing sets.
The last part of the talk will be devoted to targeted controllability of systems defined over graphs. By targeted controllability, we mean controllability of the states of a given set of nodes in the graph. As such, targeted controllability is akin to strong structural output controllability. In order to study targeted controllability, we will focus on the so-called distance preserving matrices carrying a given graph structure. The main results of this part are necessary and sufficient conditions for targeted controllability in terms of zero forcing sets of subgraphs obtained from the original graph by considering the distances from the leader nodes to the targeted ones.

Output Regulation of Hybrid Linear Systems: Solvability Conditions and Structural Implications

Sergio Galeani[7] *and Mario Sassano*[8]

Output regulation is essentially the problem of ensuring that the output of a system under control converges to a desired reference, despite the influence of unknown initial conditions and unmeasured disturbances. The latter are typically described as the output of an *exosystem* with a known model and unknown initial state. Hybrid systems, on the other hand, are characterized by a combination of continuous-time and discrete-time dynamics, so that their state can evolve continuously (*flow*) according to a differential equation as well as evolve discontinuously (*jump*) according

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to a difference equation. One of the main reasons for the explosion of interest about hybrid systems is due to their ubiquitous appearance in cyber-physical applications, purely physical devices and natural phenomena.
The main objective of this talk consists in discussing the plethora of surprising and unprecedented phenomena arising in output regulation problems for linear hybrid systems in the presence of time-driven periodic jumps, which do not possess any counterpart in classical output regulation. Towards this end, it is firstly shown that, similarly to non-hybrid formulations, also in the hybrid context the solution hinges upon suitable differential equations – referred to as *Hybrid Francis Equations* - that replace the well-known (algebraic) Francis Equations. However, while in classical regulation there is no advantage in considering *fat plants* (namely characterized by a number of controlled inputs larger than the number of regulated outputs), by inspecting the Hybrid Francis Equations it is possible to conclude that *unsolvability* (rather than solvability) of the underlying equations is a generic and open property in the square case. Thus, as a rule the existence conditions are not satisfied, and even if satisfied at the nominal parameter values, arbitrarily small uncertainties cause them to be violated. The consequence of such a fact is that, lacking *well-posedness*, robustness, even in an arbitrarily small neighborhood of the nominal parameter values, of a solution to a hybrid output regulation problem for SISO plants may be achieved only for specially structured classes of systems. Therefore, we recognize the fundamental importance of redundant inputs in hybrid output regulation: by leveraging on input redundancy and the presence of a suitably large *hidden dynamics* (related to the zero dynamic of the flow equations), we propose modified constructive conditions that are shown to be both openly and generically satisfied.
Moreover, two key properties having high theoretical interest and deep practical implications are stated: the *heart of the hybrid regulator* (that is, the key portion of any regulator ensuring regulation during flows actually is necessarily constituted by the constant solution of a classic algebraic Francis equation) and the *flow zero dynamics internal model principle* (implying that, in order to achieve regulation, it is necessary for the regulator to contain not only a copy of the exosystem dynamics – as in the classic internal model principle – but also a copy of the hidden dynamics). The latter phenomenon, in particular, possesses dramatic consequences towards the design of an error feedback robust regulator. In fact, classical results in a similar setting for non-hybrid plants are based on assuming a suitable factorization of the time-varying components in the steady-state input in terms of some known functions of time multiplied by unknown, but constant, coefficients. Such assumption is always satisfied in classic output regulation, where the known functions of time are the natural modes of the exosystem, which are fixed. However, in the hybrid case, the flow zero dynamics internal model principle entails that in the square plant setting the above functions include also the natural modes of the flow zero dynamics, which generically change for arbitrarily small perturbations of the plant data (and since the effect of such small change is propagated along the whole flow interval, the

final effect on the regulated output can be quite large even for very small parameter perturbations).
To partially circumvent this issue, we identify a class of problems for which the flow zero dynamics internal model principle is trivially satisfied. These are characterized by the feature that the hybrid steady-state has a *semiclassical* form, namely it has a hybrid evolution (comprising nontrivial flows and jumps) but it only contains the natural modes of the exosystem, which are known and fixed even in an uncertain setting.

Structure Evolving Systems and Engineering Design: Structural Problems

Nicos Karcanias[9]

Complex Systems is a term that emerges in many disciplines and domains and has many interpretations, implications and problems associated with it. The specific domain provides dominant features and characterise the nature of problems to be considered. A major classification of such systems are to those linked with *physical processes* (physics, biology, genetics, ecosystems, social etc.) and those which are *man made* (engineering, technology, energy, transport, software, management and finance etc.) and deal with the *"macro level"* issues and technology. Each of the above classes has its own key paradigms, specific problems, concepts and methodologies. There exist however generic common issues amongst the different domains and this requires the need for developing generic methodologies and tools that can be applied across the different domains. For *man made systems*, Systems and Control concepts and tools are important in the development of methodologies aiming for the *Management of Complexity*.

Existing methods in Systems and Control deal predominantly with fixed systems, where components, interconnection topology, measurement-actuation schemes and control structures are specified. Two new major paradigms expressing forms of engineering complexity which have recently emerged are the new paradigms of:

- *Structure Evolving Systems (SES)*
- *Systems of Systems (SoS)*

Using the traditional view of the meaning of the system (components, interconnection topology, environment), the common element between those two new paradigms is that the interconnection topology may vary, evolve in the case of *SES*, whereas in the case of *SoS* the interconnection rule is generalised to a new notion of a "play" [1, 2]. The talk deals with the fundamentals as far as representation, structure, and properties of those two challenging classes, demonstrates the significance of traditional systems and control theory and introduces a new research agenda for the required control theory for those two complex systems paradigms. In fact:
Structure Evolving Systems [3]: Such a class of systems emerge in natural processes such as Biology, Genetics, Crystallography etc.; the area of man made processes

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includes Engineering Design, Power Systems under de-regulation, Integrated Design and Re-design of Engineering Systems (Process Systems, Flexible Space Structures etc.), Systems Instrumentation, Design over the Life-Cycle of processes, Control of Communication Networks, Supply Chain Management, Business Process Re-engineering, Data Processes etc. This family departs considerably from the traditional assumption that the system is fixed and its dominant features relate to:

- □ The topology of interconnections is not fixed but may vary through the life-cycle of the system *(Variability of Interconnection Topology Complexity)*.
- □ The overall system may evolve through the early-late stages of the design process *(Design Time Evolution)*.
- □ There may be Variability and/or uncertainty on the systems environment during the lifecycle requiring flexibility in organisation and operability *(Lifecycle Complexity)*.
- □ The system may be large scale, multi-component and this may impact on methodologies and computations *(Large Scale - Multi-component Complexity)*.
- □ There may be variability in the Organisational Structures of the information and decision making (control) in response to changes in goals and operational requirements *(Organisational Complexity Variability)*.

The above features characterise a new paradigm in systems theory and introduce major challenges for Control Theory and Design and Systems Engineering. There are different forms of structure evolution. Integrated System Design has been an area that has motivated some of the early studies on *SES*. The integration of traditional design stages [4], such as Process Synthesis *(PS)*, Global Instrumentation *(GS)* and finally Control Design *(CD)* is an evolutionary process as far model system formation and typical forms of evolution are the *structural design evolution*, *the early-late design evolution* and the *interconnection topology evolution* [3].

Addressing the issues of *SES* has important implications for the underpinning Control Theory and related Design methodologies. Control Theory and Design has developed considerably in the last forty years under the assumption that the system has been already designed and thus control has been viewed as the final stage of the design process on a system that has been formed. The new paradigm of *SES deviates from the "fixed system structure assumption"*. Methodologies and tools developed for *Fixed Structure Systems (FES)* cannot meet the challenges of the *SES* class and new developments on the level of concepts, modelling, analysis and synthesis methodologies are needed. The research is strongly influenced by the need to address life-cycle and re-design issues and such problems have a strong technological and economic dimension.

References

[1] N. Karkanias, A.G. Hessami, "Complexity and the notion of System of Systems: Part (i): General Systems and Complexity," Proceedings of 2010 World Automation Congress International Symposium on Intelligent Automation and Control (ISIAC), Kobe, Japan, September 19-23, 2010.

[2] N. Karkanias, A.G. Hessami, "Complexity and the notion of System of Systems: Part (ii): Defining the Notion of System of Systems," Proceedings of 2010 World Automation Congress International Symposium on Intelligent Automation and Control (ISIAC), Kobe, Japan, September 19-23, 2010.

[3] N. Karkanias "Structure Evolving Systems and Control in Integrated Design," IFAC Annual Reviews in Control, vol. 32, n. 2, pp. 161–182, December 2008, doi:10.1016/j.arcontrol.2008.07.004

[4] N. Karkanias "Integrated Process Design: A Generic Control Theory/Design Based Framework," Computers in Industry, vol. 26, pp. 291–301, 1995

Advances of Implicit Description Techniques in Modelling and Control of Complex Systems[10]

Moisés Bonilla[11], *Michel Malabre*[12] *and Vadim Azhmyakov*[13]

The aim of our talk is to review recent contributions related to the implicit linear systems and to the corresponding modelling approaches. We mainly analyze here the effective control design schemes for some classes of complex systems e.g., dynamic systems with switches. The main focus of our presentation is as follows: the *implicit systems representations* of Rosenbrock [10] in the context of a generalization of proper linear systems (see also [9]). Recall that this *implicit representation* $\Sigma^{imp}(E, A, B, C)$ is in fact a set of differential and algebraic equations of the generic form:

$$E\mathrm{d}x/\mathrm{d}t = Ax + Bu \quad \text{and} \quad y = Cx, \quad t \in \mathbb{R}^+, \tag{1}$$

where $E : \mathscr{X}_d \to \underline{\mathscr{X}}_{eq}$, $A : \mathscr{X}_d \to \underline{\mathscr{X}}_{eq}$, $B : \mathscr{U} \to \underline{\mathscr{X}}_{eq}$ and $C : \mathscr{X}_d \to \mathscr{Y}$ are maps. The spaces $\mathscr{X}_d$, $\underline{\mathscr{X}}_{eq}$, $\mathscr{U}$ and $\mathscr{Y}$ are usually called the "descriptor", "equation" and the "input and the output spaces", respectively.

In [1] it was shown that under the condition $\dim(\underline{\mathscr{X}}_{eq}) \leq \dim \mathscr{X}_d$ one can constructively describe a linear system with an internal Variable Structure. However, in the case $\dim(\underline{\mathscr{X}}_{eq}) < \dim \mathscr{X}_d$ and under the solvability condition for a system under consideration, solutions are generally non unique. In some sense there is a degree of freedom in (1) that can incorporate (by an implicit way) a *structure variation*. In [2] a non square implicit description was effectively used for modelling and control of various classes of linear systems. This effective control approach also includes systems with internal switches. Moreover, the necessary and sufficient conditions for a unique system behavior (expressed in terms of the overall implicit model) are developed. These conditions imply existence of the system parts which are associated with the common internal dynamic equation and also with the algebraic constraints.

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The last one are “controlled” (in a hidden way) by the degree of freedom. It was also shown how to include the variable internal structure representation into the common square implicit descriptions for a (A, E, B) invariant subspace generated by the kernel of the generic output map. The embedding mentioned above makes it possible to get an un-observable variable internal structure. As a consequence of this effect a proper closed loop system with a controllable pre-specified structure is obtained.

In [4] we have taken technical (descriptor) advantage from the analytic results obtained in [2]. In that paper, we have considered an important class of *time-dependent autonomous switched systems* formally represented by the following *state space representation*:

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{x} = A_\theta \bar{x} + Bu \quad \text{and} \quad y = C_\theta \bar{x} \tag{2}$$

where $\theta \in \mathscr{I} = \{\theta_1, \ldots, \theta_\eta\}$, $\bar{x}(T_{i-1}) = c_i$ and $t \in [T_{i-1}, T_i)$ for some $i \in \mathbb{N}$, $T_{i-1} \in \mathfrak{J} = \{T_i \in \mathbb{R}^+(i \in \mathbb{Z}^+) | T_0 = 0,\ T_{i-1} < T_i\ \forall i \in \mathbb{N}, \text{with } \lim_{i\to\infty} T_i = \infty\}$. The matrices A_θ and C_θ, $\theta \in \mathscr{I}$, have here a particular structure

$$A_\theta = \bar{A}_0 + \bar{A}_1 \bar{D}(\theta) \quad \text{and} \quad C_\theta = \bar{C}_0 + \bar{C}_1 \bar{D}(\theta). \tag{3}$$

We also have shown that this class of *time-dependent autonomous switched systems* can be studied using the structural concepts and properties of the so-called *linear time-invariant implicit systems theory*. For this aim we have represented (2) and (3) by the *implicit global representation* (developed in the recent works of Authors)

$$\begin{array}{l} \mathbb{E}\frac{\mathrm{d}}{\mathrm{d}t}x = \mathbb{A}_\theta x + \mathbb{B}u \quad \text{and} \quad y = Cx, \quad t \in (T_{i-1}, T_i) \\ \mathbb{E} = \begin{bmatrix} E \\ 0 \end{bmatrix}, \quad \mathbb{A}_\theta = \begin{bmatrix} A \\ D_\theta \end{bmatrix}, \quad \mathbb{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \end{array} \tag{4}$$

From (4) we next deduce the expected *implicit rectangular representation*

$$E\frac{\mathrm{d}}{\mathrm{d}t}x = Ax + Bu \quad \text{and} \quad y = Cx, \quad t \in \mathbb{R}^+ \backslash \mathfrak{J}. \tag{5}$$

Here

$$E = \begin{bmatrix} I & 0 \end{bmatrix}, \quad A = \begin{bmatrix} \bar{A}_0 & -\bar{A}_1 \end{bmatrix}, \quad C = \begin{bmatrix} \bar{C}_0 & -\bar{C}_1 \end{bmatrix}, \quad D_\theta = \begin{bmatrix} \bar{D}(\theta) & I \end{bmatrix}. \tag{6}$$

The additional *algebraic constraints* are given as follows:

$$0 = D_\theta x, \quad t \in [T_{i-1}, T_i) \tag{7}$$

In [5] we have next extended some reachability results previously obtained in [3]. The implicit PD feedback control law was comprehensively discussed in [5]. Recall that the necessary control

$$u^* = F_p^* x + F_d^* \mathrm{d}x/\mathrm{d}t + r, \tag{8}$$

is obtained here by the following “three steps” approach (see also [2]):

1. locate the supremal (A, E, B)-*invariant subspace contained in* $\ker C$; $\mathscr{V}^*$.

2. find a derivative feedback such that

$$\mathscr{V}^* \supset \ker(E - BF_d^*); \tag{9}$$

3. generate a proportional feedback with the property

$$(A + BF_p^*)\mathscr{V}^* \subset (E - BF_d^*)\mathscr{V}^*. \tag{10}$$

The recently obtained results from [5] also lead to the efficient proper approximation

$$\begin{array}{rcl} \mathrm{d}\bar{x}/\mathrm{d}t & = & -(1/\varepsilon)\bar{x} + (1/\varepsilon)F_d^* x \\ u & = & -(1/\varepsilon)\bar{x} + ((1/\varepsilon)F_d^* + F_p^*)x + r. \end{array} \tag{11}$$

One can prove that there exists $\varepsilon^* > 0$ such that

$$|y(t) - y^*(t)| \leq \delta \quad \forall \varepsilon \in (0, \varepsilon^*], \quad \forall t \geq t^*(\delta), \tag{12}$$

where $t^*(\delta)$ is a fixed transient time. This time instant strongly depends on the chosen parameters δ, y^*. In [6] we proposed a structural feedback linearization based on failure detection techniques. We have considered a nonlinear dynamic model represented by a nonlinear state representation

$$\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathrm{A}_o\mathbf{x} + \mathrm{B}_o\mathbf{u} + \mathrm{S}_o\mathbf{q}_o(\mathbf{x}, \mathbf{u}); \quad \mathbf{y} = C_o\mathbf{x}, \tag{13}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state variable, $\mathbf{y} \in \mathbb{R}^p$ is the output variable, $\mathbf{u} \in \mathbb{R}^m$ characterizes the input variable and $\mathbf{q}_o(\mathbf{x}, \mathbf{u})$ is an analytic vector field with the property $\mathbf{q}_o(0, 0) = 0$. Moreover, this implies that $\mathrm{C}_o \in \mathbb{R}^{p\times n}$ is an epic matrix, namely, $\ker \mathrm{C}_o^T = \{0\}$.
We next have shown that for a controllable pair $(\mathrm{A}_o, \mathrm{B}_o)$ there exists a structural differential operator $\mathrm{X}(\mathrm{d}/\mathrm{d}t)$

$$\mathrm{X}(\mathrm{d}/\mathrm{d}t) = -\mathrm{A}_o^T \mathcal{C}_n(\mathrm{S}_o)\Psi_n(\mathrm{d}/\mathrm{d}t), \tag{14}$$

$$\mathcal{C}_\kappa(\mathrm{S}_o) = \left[\begin{array}{cccc} \mathrm{S}_o & \mathrm{A}_o^T\mathrm{S}_o & \ldots & (\mathrm{A}_o^T)^{(\kappa-1)}\mathrm{S}_o \end{array}\right], \tag{15}$$

$$\Psi_\kappa(\mathrm{d}/\mathrm{d}t) = \left[\begin{array}{cccc} I & I\mathrm{d}/\mathrm{d}t & \ldots & I\mathrm{d}^{\kappa-1}/\mathrm{d}t^{\kappa-1} \end{array}\right]^T \tag{16}$$

that transforms (13) into the following constructive state description

$$\mathrm{d}\zeta/\mathrm{d}t = \mathrm{A}_o\zeta + \mathrm{B}_o(\bar{\mathbf{u}} + \mathbf{q}_*(\xi, \mathbf{u})); \quad \mathbf{y}(t) = \mathrm{C}_o\zeta(t). \tag{17}$$

Here the nonlinear components q_* are contained in the image of its constant input matrix B_o. Note that the above operator $\mathrm{X}(\mathrm{d}/\mathrm{d}t)$ depends the constant matrix S_o. As an example briefly consider the following disturbance rejector based on the Beard-Jones filter (*cf.* [7, 8, 11]):

$$\begin{array}{rcl} \mathrm{d}\mathbf{w}/\mathrm{d}t & = & (\mathrm{A}_\mathcal{B} + \mathrm{KC}_\mathcal{B})\mathbf{w} - \mathrm{K}\mathbf{y} + \mathrm{B}_\mathcal{B}\bar{\mathbf{u}}, \\ \bar{\mathbf{r}} & = & \mathrm{G}^\ell(\mathrm{C}_\mathcal{B}\mathbf{w} - \mathbf{y}), \quad \bar{\mathbf{u}} = \mathrm{F}\zeta + \bar{\mathbf{r}}, \end{array} \tag{18}$$

where $\mathrm{K} \in \mathbb{R}^{\hat{n}\times p}$ is an output injection

$$\sigma\{(\mathrm{A}_{\mathcal{B}} + \mathrm{K}\mathrm{C}_{\mathcal{B}})\} \subset \mathbb{C}^{-}. \tag{19}$$

By G^{ℓ} we denote a left inverse of the static gain $-\mathrm{C}_{\mathcal{B}}\mathrm{A}_{\mathcal{B}_K}^{-1}\mathrm{B}_{\mathcal{B}}$ associated with the remainder generator

$$\mathrm{d}\mathbf{e}/\mathrm{d}t = (\mathrm{A}_{\mathcal{B}_K}\mathbf{e} - \mathrm{B}_{\mathcal{B}}\mathbf{q}_{*}(\xi, \mathbf{u}); \quad \mathbf{r} = \mathrm{C}_{\mathcal{B}}\mathbf{e}, \tag{20}$$

where $\mathrm{A}_{\mathcal{B}_K} \doteq (\mathrm{A}_{\mathcal{B}} + \mathrm{K}\mathrm{C}_{\mathcal{B}})$, $\mathbf{e}(t) = \mathbf{w}(t) - \zeta(t)$ and $\mathbf{r} = \mathrm{C}_{\mathcal{B}}\mathbf{w} - \mathbf{y}$. Let us finally note that $F(s) = \mathrm{C}_{\mathcal{B}}(\mathrm{s}I - \mathrm{A}_{\mathcal{B}_K})^{-1}\mathrm{B}_{\mathcal{B}}$. Finally under the assumption $q_{*}(\xi, \bar{\mathbf{u}})$ constitutes a bounded limited band frequency signal, we only need to synthesize a Hurwitz low-pass filter $F(s)$ with a corner frequency ω_c in order to achieve an asymptotic feedback linearization

$$\|\bar{\mathbf{u}}_{*}(\omega) - \bar{\mathbf{u}}(\omega)\| \leq \|\mathrm{G}^{\ell}F(\jmath\omega) - I\| \; \|\mathbf{q}_{*}(\omega)\|. \tag{21}$$

Let us note that an evident advantage of this control schema is that it “enlarges” the linearity neighborhood around the equilibrium point $\mathbf{0}$. The possible uncertaintys model is absorbed by $\mathbf{q}_{*}$.

References

[1] Bonilla, M. and M. Malabre (1991). Variable structure systems via implicit descriptions. In: *1st European Control Conference*. Vol. 1. pp. 403-408.

[2] Bonilla, M. and M. Malabre (2003). On the control of linear systems having internal variations. *Automatica*, Vol. 39, 1989-1996.

[3] Bonilla, M., Lebret, G., Loiseau, J.J. and Malabre, M. (2013) Simultaneous state and input reachability for linear time invariant systems, *Linear Algebra and its Applications*, Vol. 439, 1425-1440.

[4] Bonilla, M., Malabre, M. and Azhmyakov, V. (2015a) An Implicit Systems Characterization of a Class of Impulsive Linear Switched Control Processes. Part 1: Modeling. *Nonlinear Analysis: Hybrid Systems*, Vol. 15, 157-170.

[5] Bonilla, M., Malabre, M. and Azhmyakov, V. (2015b) An Implicit Systems Characterization of a Class of Impulsive Linear Switched Control Processes. Part 2: Control. *Nonlinear Analysis: Hybrid Systems*, Vol. 18, 15-32.

[6] Blas L.A., Bonilla M., J.C., M. Malabre, V. Azhmyakov, S. Salazar (2017). Structural feedback linearization based on nonlinearities rejection. *20th IFAC World Congress*, pp. 945-950. July 9–14. Toulouse, France.

[7] Isermann, R. (1984). Process fault detection based on modeling and estimation methods–A survey, *Automatica*, Vol. 20, 387-404.

[8] Massoumnia. M-.A. (1986). A geometric approach to the synthesis of failure detection filters. *IEEE Trans. Automatic Control*, Vol. 31, n. 9, 1986, 839–846.

[9] Lewis, F.L. (1992). A tutorial on the geometric analysis of linear time-invariant implicit systems. *Automatica*, Vol. 28, n. 1, 119-137.

[10] Rosenbrock, H.H. (1970) *StateSpace and Multivariable Theory.* Nelson, London.

[11] Saberi, A., A.A. Stoorvogel, P. Sannuti and H.H. Niemann (2000). Fundamental problems in fault detection and identification. *International Journal of Robust and Nonlinear Control*, Vol. 10, pp. 1209–1236.

Huygens Synchronization over Distributed Media – Structure versus Complex Behavior

Vladimir Rasvan[14]

This analysis is concerned with Huygens synchronization over distributed media – vibrating strings or LC transmission lines.

If e.g. two oscillators with lumped parameters i.e. described by ordinary differential equations, which display self sustained oscillations, are coupled to some distributed medium, they interact in function of the structural properties of the resulting system. If this medium has infinite length then, according to the structural properties of the difference operator describing propagation, either synchronization with the external frequency or some "complex behavior" can occur.

If the coupling has a finite length, again the qualitative properties are determined by the structure of the aforementioned difference operator: either the self sustained oscillations are quenched, the system approaching asymptotically a stable equilibrium (opposite of the Turing coupling of two "cells" that is lumped oscillators) or again the aforementioned "complex behavior" can occur.

We state the *conjecture that this "complex behavior" is in fact some almost periodic oscillation and not a chaotic behavior*.

It is worth mentioning that the two types of qualitative behavior are in connection with the physical nature of the considered systems. Specifically, the difference operator is strongly stable for electrical systems and critically stable for mechanical systems. It is this aspect that explains proneness to standard or "complex" behavior. However, if the aforementioned conjecture will not be disproved, the difference between the corresponding oscillatory behaviors will consist only in the contents of harmonics given by the Fourier series attached.

A Stratified Geometric Approach to the Disturbance Decoupling Problem With Stability for Switched Discrete-Time Systems over Digraphs

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This contribution considers the disturbance decoupling problem with stability for

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switched discrete-time linear systems, where switching occurs within a set of admissible transitions defined via a weighted directed graph. The concept of subspace arrangement, as a collection of linear subspaces, is employed as a main tool for the definition of geometric properties tailored to switched linear systems on digraphs. Stratified geometric concepts are developed as natural extensions of familiar ones in linear systems theory as applicable to the framework of switched discrete-time linear systems. Based on the definition of a subspace arrangement as a finite set of linear subspaces corresponding to each single switched subsystem, invariance concepts are specifically characterized when the system is switched from one mode to the next. For this reason, this study employs a weighted directed graph to describe the switching rules of the switched systems. Moreover, the consideration of two types of feedback control, namely a mode-dependent piecewise constant control and a graph-based time-varying control, is seen to induce different notions of controlled invariance, leading to strong and weak controlled invariant subspace arrangements, respectively. Sufficient conditions for solvability of the Disturbance Decoupling Problem with Stability (DDPS) under certain classes of admissible switching signals are expressed in terms of the proposed invariance notions. Conditions for eigenvalue assignment and stabilizability are provided under the assumptions of arbitrary dwell-time and sufficiently large dwell-time switching strategies, respectively. These results specialize previous approaches based on robust controlled invariance, while providing a constructive solution in cases where previous methodologies fail.

The Unknown-Input Observation Problem for Hybrid Dynamical Structures

Giuseppe Conte[17], *Anna Maria Perdon*[18] *and Elena Zattoni*[19]

This talk illustrates how the general problem of observing a linear function of the state of a given system in the presence of unknown inputs can be effectively tackled in the context of hybrid multivariable systems by adopting a structural approach. The hybrid systems considered exhibit a continuous-time behavior except at isolated point of the time axis, where they are subject to abrupt state discontinuities (in the literature, such dynamical structures are also referred to as linear impulsive systems). A solution of the observation problem is represented by an hybrid observer system, whose output is an asymptotic estimate of the considered function. After clarifying the features an observer must have to be of practical interest, suitable structural notions and geometric tools for the devised class of systems are introduced and thoroughly discussed. Their study and characterization are shown to be functional to the solution of the considered observation problems. In particular, the results they provide make it possible to establish necessary and sufficient conditions for the existence of globally asymptotically stable observers, whose complexity is reduced as much as possible (in relation to the specific problem) by minimizing

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their order. The results obtained in this way by the structural geometric approach are compared with others that can be found in the literature, so as to gain further insight into the problem and to appreciate better the efficacy and the potential of the employed methodology. Then, applications of the results on the unknown input observation problem to fault detection are discussed. Also in this case, the structural point of view is shown to be able to analyze the problem in depth and to provide satisfactory solutions. An important point to remark is that, in this approach, the theoretical results are supported by simple and effective computational algorithms. By exploiting such algorithms it is possible, in practical applications, to check conditions for the existence of solutions to observation and to fault detection problems and to derive clear procedures for synthesizing solutions, if they exist.