# Some Basic Statistical Properties of the Transmuted Burr X Distribution.

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## ABSTRACT

In this article, the Burr X distribution was extended using the transmuted family of distributions. Some basic statistical properties of the resulting Transmuted Burr X distribution were established while the method of maximum likelihood was proposed for parameter estimation.

(Key terms: Burr X distribution, estimation, properties, transmuted family of distributions)

## INTRODUCTION

Advances in distribution theory indicate that the Transmuted family of distributions has received appreciable consideration in recent years. Many compound distributions have been derived based on this approach, examples are Transmuted Weibull distribution (Aryal and Tsokos, 2011), Transmuted Rayleigh distribution (Merovci, 2013), Transmuted Pareto distribution (Merovci and Puka, 2014), Transmuted Exponential distribution (Owoloko et al., 2015), Transmuted Inverse Exponential distribution (Oguntunde and Adejumo, 2015) and many others which are contained in Tahir and Cordeiro (2016).

It is also interesting to note that the Transmuted Burr III distribution and Transmuted Burr XII distribution have been proposed and studied by Abdul-Moniem (2015) and Al-Khazaleh (2016) respectively but of interest to us in this research is to define and study the Transmuted Burr X distribution.

The transmuted family of distributions was proposed by Shaw and Buckley (2007) and its general properties were proposed in Bourguinon et al., (2016). Its cumulative density function (cdf) and the probability density function (pdf) are defined by:

$$F(x) = (1+\lambda)G(x) - \lambda\left[G(x)\right]^2 \quad \textbf{(1)}$$

and

$$f(x) = \left[(1+\lambda) - 2\lambda G(x)\right]g(x) \quad \textbf{(2)}$$

respectively, for $|\lambda| \leq 1$

where; $\lambda$ is the transmuted parameter

$G(x)$ and $g(x)$ are the cdf and pdf of the baseline distribution

On the other hand, the Burr distribution has different variants, whilst a general form of the distribution was proposed in Mudholkar and Srivastava (1993), the one-parameter Burr Type X distribution is defined by:

$$G(x) = \left[1 - e^{-x^2}\right]^{\theta} \quad \textbf{(3)}$$

and

$$g(x) = 2\theta x e^{-x^2}\left[1 - e^{-x^2}\right]^{\theta-1} \quad \textbf{(4)}$$

respectively, for $x > 0, \theta \geq 0$

where; $\theta$ is the shape parameter.

In Raqab and Kundu (2006), the one-parameter Burr X distribution can be seen to be a special case of the Exponentiated Weibull distribution proposed by Mudholkar and Srivastava (2006). Also, the one-parameter Burr X distribution can be called the Generalized Rayleigh or Exponentiated Rayleigh distribution (Surles and Padgett, 2001).

The purpose of this research is to generalize the Burr X distribution using the transmuted family of distributions due to Shaw and Buckley (2007).

The rest of this article is therefore structured as follows; the Transmuted Burr X distribution is defined and its basic statistical properties are derived including estimation of model parameters followed by a concluding remark.

## RESULTS AND DISCUSSION

The cdf of the Transmuted Burr X distribution is obtained by substituting the density in Equation (3) into Equation (1) as:

$$F(x)=(1+\lambda)\left[1-e^{-x^2}\right]^{\theta}-\lambda\left[\left(1-e^{-x^2}\right)^{\theta}\right]^{2}$$

This further gives:

$$F(x)=\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right] \quad \textbf{(5)}$$

for $x>0,\theta>0,|\lambda|\leq 1$

Its pdf is given by:

$$f(x)=2\theta xe^{-x^2}\left[1-e^{-x^2}\right]^{\theta-1}\left[(1+\lambda)-2\lambda\left(1-e^{-x^2}\right)^{\theta}\right] \quad \textbf{(6)}$$

for $x>0,\theta>0,|\lambda|\leq 1$

where; $\theta$ is a shape parameter

$\lambda$ is the transmuted parameter

A possible plot for the cdf of the Transmuted Burr X distribution at selected parameter values is shown in Figure 1.

### Reliability Analysis for the Transmuted Burr X Distribution

Here, expressions for the survival function, inverse survival function, hazard function, reversed hazard function and the odds function for the Transmuted Burr X distribution are explicitly derived.

***Survival Function:*** Survival function is expressed mathematically as:

$$S(x)=1-F(x)$$

Hence, the survival function of the Transmuted Burr X distribution can be expressed as:

$$S(x)=1-\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right] \quad \textbf{(7)}$$

for $x>0,\theta>0,|\lambda|\leq 1$

***Hazard Function:*** Hazard function is represented in mathematical form as:

$$h(x)=\frac{f(x)}{S(x)}$$

Hence, the hazard function for the Transmuted Burr X distribution is given by:

$$h(x)=\frac{2\theta xe^{-x^2}\left[1-e^{-x^2}\right]^{\theta-1}\left[(1+\lambda)-2\lambda\left(1-e^{-x^2}\right)^{\theta}\right]}{1-\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right]} \quad \textbf{(8)}$$

for $x>0,\theta>0,|\lambda|\leq 1$

***Reversed Hazard Function:*** The reversed hazard function is derived from:

$$r(x)=\frac{f(x)}{F(x)}$$

Hence, the reversed hazard function for the Transmuted Burr X distribution is given by;

$$r(x)=\frac{2\theta xe^{-x^2}\left[1-e^{-x^2}\right]^{\theta-1}\left[(1+\lambda)-2\lambda\left(1-e^{-x^2}\right)^{\theta}\right]}{\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right]} \quad \textbf{(9)}$$

for $x>0,\theta>0,|\lambda|\leq 1$

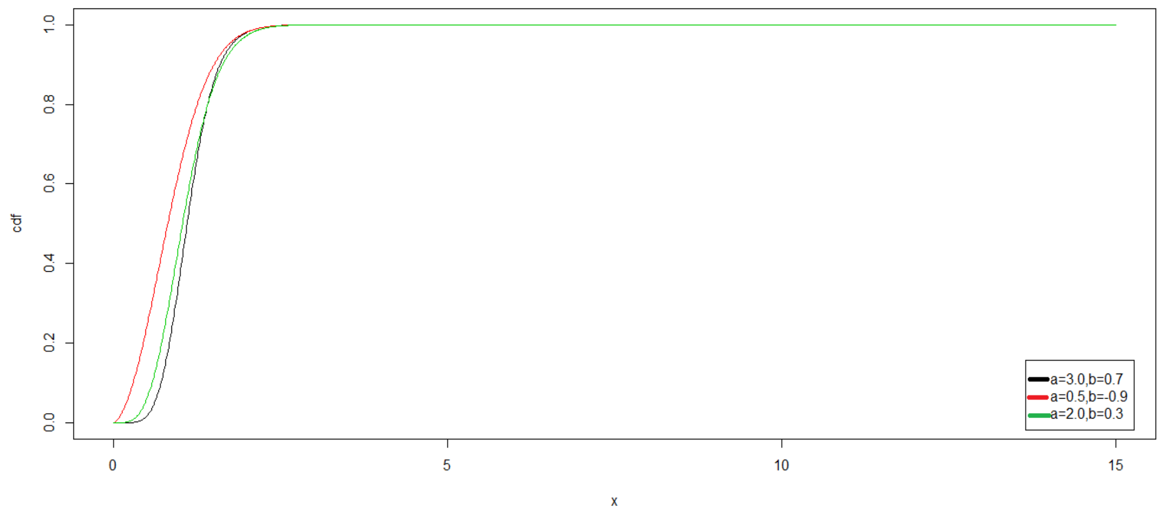


**Figure 1**: CDF of the Transmuted Burr X Distribution ( $a=\theta, b=\lambda$ ).

***Odds Function:*** The odds function is obtained from:

$$O(x)=\frac{F(x)}{S(x)}$$

Therefore, the odds function for the Transmuted Burr X distribution is given by:

$$O(x)=\frac{\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right]}{1-\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right]} \quad \textbf{(10)}$$

for $x>0, \theta>0, |\lambda| \leq 1$

**Distribution of Order Statistics**

Let $x_1, x_2, \ldots, x_n$ denote random samples from a continuous population with cdf and pdf denoted by $F(x)$ and $f(x)$, respectively. If $X_{(1)}<X_{(2)}<\ldots<X_{(n)}$ are the ordered statistics in ascending order, then the pdf of $X_{(j)}$ is given by:

$$f_{j:n}(x)=\frac{n!}{(j-1)!(n-j)!} f(x)\left[F(x)\right]^{j-1}\left[1-F(x)\right]^{n-j} \quad \textbf{(11)}$$

for $j=1,2,\ldots,n$

Therefore, the pdf of the order statistics for the Transmuted Burr X distribution is given by:

$$f_{j:n}(x)=\frac{n!}{(j-1)!(n-j)!} 2\theta x e^{-x^2}\left[1-e^{-x^2}\right]^{\theta-1} \times \left[(1+\lambda)-2\lambda\left(1-e^{-x^2}\right)^{\theta}\right] \times \left[\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right]\right]^{j-1} \times \left[1-\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right]\right]^{n-j} \quad \textbf{(12)}$$

The pdf of minimum order statistics for the Transmuted Burr X distribution is given by:

$$f_{1:n}(x) = 2n\theta x e^{-x^2}\left[1-e^{-x^2}\right]^{\theta-1} \times \left[(1+\lambda)-2\lambda\left(1-e^{-x^2}\right)^{\theta}\right] \times \left[1-\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right]\right]^{n-1} \quad \textbf{(13)}$$

The pdf of maximum order statistics for the Transmuted Burr X distribution is given by:

$$f_{n:n}(x) = 2n\theta x e^{-x^2}\left[1-e^{-x^2}\right]^{\theta-1} \times \left[(1+\lambda)-2\lambda\left(1-e^{-x^2}\right)^{\theta}\right] \times \left[\left[1-e^{-x^2}\right]^{\theta}\left[1+\lambda-\lambda\left(1-e^{-x^2}\right)^{\theta}\right]\right]^{n-1} \quad \textbf{(14)}$$

**Parameter Estimation**

Let $x_1, x_2, ..., x_n$ denote random samples of ‘n’ independently and identically distributed random variables from the Transmuted Burr X distribution as defined in Equation (6), the log-likelihood function denoted by ‘l’ is then given by:

$$l = n\log(2) + n\log(\theta) - \sum_{i=1}^{n} x_i^2 + (\theta-1)\sum_{i=1}^{n}\log\left[1-e^{-x_i^2}\right] + \sum_{i=1}^{n}\log\left[(1+\lambda)-2\lambda\left(1-e^{-x_i^2}\right)^{\theta}\right]$$

Differentiating the log-likelihood function with respect to $\theta$ and $\lambda$ gives:

$$\frac{\partial l}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^{n}\log\left[1-e^{-x_i^2}\right] + 2\lambda\sum_{i=1}^{n}\frac{\left(1-e^{-x_i^2}\right)^{\theta}\log\left(1-e^{-x_i^2}\right)}{(1+\lambda)-2\lambda\left(1-e^{-x_i^2}\right)^{\theta}} \quad \textbf{(15)}$$

and;

$$\frac{\partial l}{\partial \lambda} = \sum_{i=1}^{n}\frac{1-2\left(1-e^{-x_i^2}\right)^{\theta}}{(1+\lambda)-2\lambda\left(1-e^{-x_i^2}\right)^{\theta}} \quad \textbf{(16)}$$

Equating Equations (15) and (16) to zero and solving the resulting simultaneously gives the maximum likelihood estimates of parameters to $\theta$ and $\lambda$. Meanwhile, the solution cannot be derived analytical but it can be gotten numerically when data sets are available with the aid of software like R, SAS, etc.

**CONCLUSION**

The one-parameter Burr X distribution has been successfully extended to give the Transmuted Burr X distribution. Its statistical properties like the survival function, hazard function, reversed hazard function and the odds function were explicitly derived. The distribution of order statistics for the Transmuted Burr X distribution was also derived. The method of maximum likelihood estimation was proposed in estimating the model parameters. Further studies would involve applying the Transmuted Burr X distribution to data set to assess its flexibility over some other existing models.

**APPENDIX**

R-code for plotting the cdf of Transmuted Burr X distribution:

```
a=3

b=0.7

x=seq(0,15,0.01)

tb.cdf=function(x,a,b)

f=(1-exp(-x^2))^a*(1+b-b*(1-exp(-x^2))^a)

plot(x,tb.cdf(x,3,0.7),type='l',ylim=c(0,1.0),xlab="x
",ylab="cdf")

lines(x,tb.cdf(x,0.5,-0.9),col=2)

lines(x,tb.cdf(x,2,0.3),col=3)

legend("bottomright",inset=0.02,c("a=3.0,b=0.7","
a=0.5,b=-0.9","a=2.0,b=0.3")
```

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