Chapter 1
A routing/assignment problem in garden maintenance services

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Abstract We address a routing/assignment problem posed by Neoturf, which is a Portuguese company working in the area of project, building and garden's maintenance. The aim is to define a procedure for scheduling and routing efficiently its clients of garden maintenance services. The company has two teams available throughout the year to handle all the maintenance jobs. Each team consists of two or three employees with a fully-equipped vehicle capable of carrying out every kind of maintenance service. At the beginning of each year, the number and frequency of maintenance interventions to conduct during the year, for each client, are agreed. Time windows are established so that visits to the client should occur only within these periods. There are clients that are supposed to be always served by the same team, but other clients can be served indifferently by any of the two teams. Since clients are geographically spread over a wide region, the total distance traveled while visiting clients is a factor that weighs heavily on the company costs. Neoturf is concerned with reducing these costs, while satisfying agreements with its clients. We give a mixed integer linear programming formulation for the problem, discuss limitations on the size of instances that can be solved to guarantee optimality, present a modification of the Clarke and Wright heuristic for the vehicle routing with time windows, and report preliminary computational results obtained with Neoturf data.

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1.1 Introduction

In this paper we address a routing/assignment problem posed by Neoturf, which is a Portuguese company working in the area of project, building and garden's maintenance. One of the services provided by Neoturf is the maintenance of private gardens of residential customers (about 60), whose demands are mainly periodic short time interventions (usually 1 to 3 hours). In the beginning of each year, the number and the estimated frequency of maintenance interventions to conduct during the year are accorded with each client. That estimate on frequency is then used to settle, in regular conditions, a minimum and maximum periods of time separating two consecutive interventions on the same client. Consecutive days of irregular conditions (e.g., extreme weather conditions) may sporadically change those maximum (or minimum) values.

The amount of work highly depends on seasonality. The company allocates to this service two teams (each consists of two or three employees) during the whole year, which may be reinforced with an additional third team during summer. Each team has a van fully equipped with the tools needed to perform the maintenance jobs. There are customers who should be always served by the same team, while others can be served by any team.

Time windows were established so that visits to the client should occur only within these periods. The clients are geographically spread along an area around Oporto of approximately 10 000 km^2. In 2011, these teams traveled more than 60 000 km, with a significant impact on the costs.

Neoturf aims at finding a procedure to scheduling and routing clients efficiently so to reduce costs, while satisfying the agreements with the clients. The scheduling of clients for each day should be planed on a basis of short periods of time (say ten consecutive working days), since unforeseeable events (e.g., weather conditions, client not available at the time previously arranged) may force to postpone planned interventions and to re-settle the designed scheduling.

The routing of customers in each period is a vehicle routing problem (VRP). VRP designates a large class of problems that deals with the design of optimal routes for fleet of vehicles to serve customers. In part dictated by its practical relevance, VRPs have attracted intense research in Combinatorial Optimization expressed by some thousands of scientific and technical papers covering many aspects of the topic. The books [11, 6, 12] provide an insight into the huge variety of the research on this subject. The basic VRP is the problem of finding a set of routes minimizing the total cost or distance traveled for a number of identical vehicles, located at a depot, to supply a set of geographically dispersed customers with known demands subject to vehicle capacity constraints. A large number of variants and extensions of the basic VRP were proposed to model specific applications, including pickup-and-delivery, stochastic demands, online VRPs, multiple depots, ship routing. The VRP with time windows (VRPTW) is a special case/generalization of VRP where each customer can only be served within established time windows (see [3, 8, 5] for recent surveys on the VRPTW). The problem that we address here is a constrained version of the VRPTW where (i) some customers, but not all, are to be visited by a certain vehicle (team); (ii) no more than one route is assigned on each day to each vehicle (team) and (iii) each customer that is to be served in each period is assigned to exactly one route, in exactly one day of that period. We give a mixed integer linear programming formulation model for the problem, discuss limitations on the size of instances that can be solved to guarantee optimality, present a modification of the classic Clarke and Wright heuristic for the vehicle routing with time windows [4], and report computational results obtained with Neoturf data.

1.2 Formulation

We consider the year partitioned into consecutive short periods of time (say 10 consecutive working days) and, for each period P of m consecutive working days, we classify clients as

- mandatory, those for which an intervention has to take place during period P, i.e., the number of days since the last visit till the end of period P exceeds the maximum number of consecutive days which can elapse without any intervention taking place, according to what has been agreed with the client;
- discarded, those for which no intervention is expected to take place during period P, i.e., the number of days since the last visit till the end of period P is lower than the number of consecutive days that were agreed to elapse before a new intervention takes place;
- admissible, those for which an intervention may or may not take place during period P.

Let C be the set of clients to be served in period P. We start with C consisting of all mandatory and admissible clients. If no feasible scheduling is found, the decision maker may consider, among other options, to redefine C removing some or all admissible clients from the period P. If no feasible solution exists even when C only includes mandatory customers, then services to some of these customers have to be postponed to the next period. The customers to be removed from the current period may be selected according to some ranking on customers.

Our problem can be viewed as a VRPTW in which certain clients in C have to be visited by (vehicle) team E_0, other clients have to be visited by team E_1, and the remaining clients can be served indifferently either by team E_0 or by team E_1. We denote the sets of those clients by C_0, C_1 and $C_{0,1}$, respectively.

We based our formulation on the so-called *big M formulation* of the traveling salesman problem with time windows (model 1 in [2]).

We construct a directed weighted graph $G = (V, A, \rho)$ as follows (see Figure 1.2). The set of vertices V is equal to $C \cup B$, where each vertex b_i^k of B, with $i = 0, \cdots, m$ and $k = 0, 1$, is the i-th "day (fictitious) copy" of the depot for team E_k. There is an arc (u, v) linking client u to client v if there is any possibility to serve v immediately after visiting u, by a same team. Arcs with both directions link each vertex b_i^k, $i = 1, \cdots, m-1$, with every client of $C_k \cup C_{0,1}$, for $k = 0, 1$. There is an arc from b_0^k to every vertex in $C_k \cup C_{0,1}$, for $k = 0,1$, but there is no arc with head b_0^k. There is an arc from every vertex in $C_k \cup C_{0,1}$ to b_m^k, $k = 0,1$, but no arc with tail b_m^k. The other arcs in set A are $(b_0^k, b_1^k), (b_1^k, b_2^k), \cdots, (b_{m-1}^k, b_m^k)$, with $k = 0,1$, and no more arcs exist linking pairs of vertices in B. For $v \in V$, we use V_v^+ and V_v^- to denote the out-neighborhood and in-neighborhood of v, respectively, i.e., $V_v^+ = \{u \in V : (v,u) \in A\}$ and $V_v^- = \{u \in V : (u,v) \in A\}$.

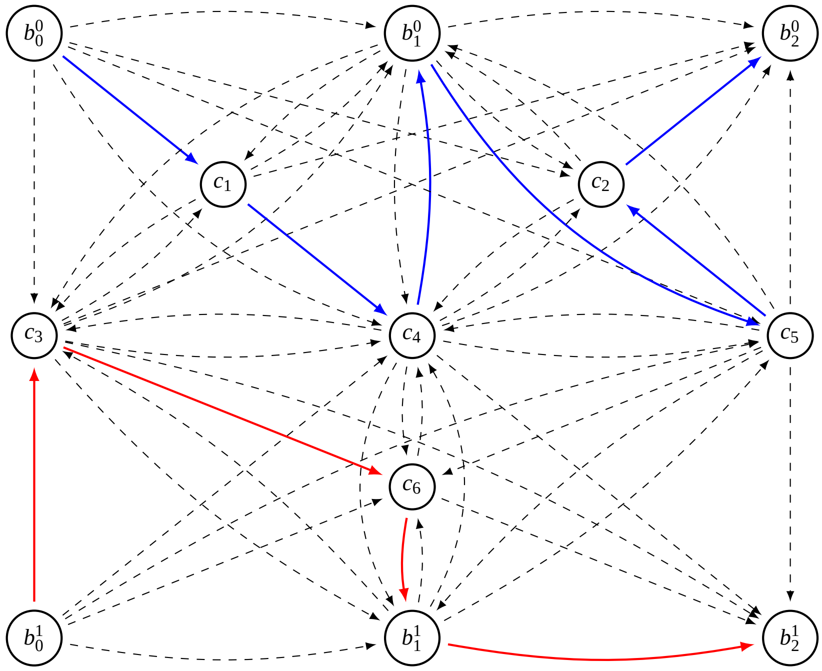

Fig. 1.1 An example of a directed graph G, and a feasible solution for a two days period. Vertices b_0^0, b_1^0, b_2^0 and b_0^1, b_1^1, b_2^1 are the "fictitious copies" of the depot for team E_0 and team E_1, respectively. The subsets of the set of clients $C = \{c_1, \ldots, c_6\}$ are $C_0 = \{c_1, c_2\}$, $C_1 = \{c_6\}$ and $C_{0,1} = \{c_3, c_4, c_5\}$. The scheduling of clients assigned to team E_0 is represented by the directed path consisting of continuous (blue) arcs $Q_0 = (b_0^0, c_1, c_4, b_1^0, c_5, c_2, b_2^0)$. Clients c_1, c_4 and c_5, c_2 are visited by that order on days one and two, respectively. The scheduling of clients assigned to team E_1 is represented by the directed path consisting of continuous (red) arcs $Q_1 = (b_0^1, c_3, c_6, b_1^1, b_2^1)$. Clients c_3, c_6 are visited by that order on day one and no client is visited on day two.

A scheduling of clients assigned to team E_k will be read on graph G as a directed path Q_k from b_0^k to b_m^k. The clients that are to be visited on day i are the vertices of C on the subpath of Q_k linking b_{i-1}^k to b_i^k. The order of vertices on that path specifies the order by which the corresponding clients should be visited. If arc (b_{i-1}^k, b_i^k) is included in path Q_k it means that no interventions on clients of set C will occur on day i for team E_k.

We define the weight ρ_{uv} of every arc $(u,v) \in A$ as the time to travel on arc (u,v), except when $u,v \in B$, where $\rho_{uv} = 0$.

For each vertex $v \in C$, let $T_v^j = [e_v^j, l_v^j]$ be the j-th time-window of client v, $j = 1,\cdots,nT_v$, where nT_v is the number of time-windows of vertex v, $e_v^j < l_v^j < e_v^{(j+1)}$, and e_v^j and l_v^j are the release time and the deadline time of the j-th time-window of client v, respectively. The release time and deadline time specify minimum and maximum instants for the start of the intervention at the client. For vertices of B, define $T^1_{b_0^k} = [ST, ST]$ and $T^1_{b_i^k} = [EN + 24(i-1), EN + 24(i-1)]$, for $i = 1,\cdots,m$ and $k = 0,1$, where ST and EN are, respectively, the daily service start hour and the daily service end hour.

For $v \in C$, let t_v be the processing time on client v, and set $t_{b_0^k} = 0$ and $t_{b_i^k} = ST + 24 - EN$, for $i = 1,\ldots,m$.

The formulation that we present below uses the following indices, sets, parameters and variables.

Indices

- i - days
- k - teams
- u,v - clients
- j - j-th time-windows

Sets

- C - clients
- C_k - clients to be visited by team k
- $C_{0,1}$ - clients served by any of the teams
- B - "day (fictitious) copies" of the depot, b_i^k
- V - vertices $C \cup B$ of the graph
- A - arcs in $V \times V$

Parameters

- m - number of days in the period
- ρ_{uv} - time to travel on arc (u,v)
- t_v - processing time on client v
- T_v^j - j-th time-window $[e_v^j, l_v^j]$ of client v
- nT_v - number of time windows of client v
- e_v^j - release time of the j-th time-window of client v
- l_v^j - deadline time of the j-th time-window of client v
- ST - daily service start hour
- EN - daily service end hour
- Δ_{uv} - weight to minimize the number of working days
- M - a large number

Variables

x_{uv} - binary variables that are equal to 1 if client v is served immediately after client u, by a same team
y_v^j - binary variables that are equal to 1 if client v is served in time-window T_v^j
a_v - binary variables that assigned client v to team E_{a_v}
s_v - time-instant in which the service starts at client v
w_v - waiting-time to start the service at client v

We deem minimize the sum of travel-time, waiting-time on clients, and number of working days. We thus have the following objective function.

$$\text{Min} \sum_{(u,v)\in A} (\rho_{uv} + \Delta_{uv}) x_{uv} + \sum_{v\in C} w_v \tag{1.1}$$

where $\Delta_{uv} = EN - ST$ if $u \in C$ and $v = b \in B \backslash \{b_0^0, b_0^1\}$, and $\Delta_{uv} = 0$ for the remaining arcs (u,v), to ensure that optimal solutions will have the minimum number of working days (i.e., the maximum number of arcs (b_{i-1}^k, b_i^k)).

The following equations

$$\sum_{u\in V} x_{vu} = 1, \quad \forall v \in V \backslash \{b_m^0, b_m^1\}, \tag{1.2}$$

$$\sum_{u\in V} x_{uv} = 1, \quad \forall v \in V \backslash \{b_0^0, b_0^1\}, \tag{1.3}$$

ensure there will be exactly one arc leaving every vertex $v \neq b_m^k$, and exactly one arc entering every vertex $v \neq b_0^k$.

To force that each client is visited exactly in one of its time-windows, we add equations

$$\sum_{j\leq nT_v} y_v^j = 1, \quad \forall v \in V. \tag{1.4}$$

To guarantee that the start time occurs within the selected time-window and that vehicle has enough time to travel from u to v, we use the following constraints

$$\sum_{j\leq nT_v} e_v^j y_v^j \leq s_v \leq \sum_{j\leq nT_v} l_v^j y_v^j, \quad \forall v \in V, \tag{1.5}$$

$$s_u + t_u + \rho_{uv} - (1 - x_{uv})M \leq s_v, \quad \forall (u,v) \in A, \tag{1.6}$$

where $M > 0$ is large enough (say $M = 24m$) to guarantee that the left hand side is non positive whenever $x_{uv} = 0$, and thus making constraint (1.6) not active when $x_{uv} = 0$.

Note that constraints (1.2), (1.3) together with (1.6), ensure that the set of selected arcs defines a directed path linking b_0^k to b_m^k, for $k = 0, 1$, where every vertex of V is included exactly once in exactly one of the two paths.

The following inequalities define upper bounds on the waiting-times on clients.

$$w_v \geq s_v - (s_u + t_u + \rho_{uv}) - (1 - x_{uv})M, \quad \forall (u,v) \in A, v \in C, \tag{1.7}$$

where $M > 0$ is large enough (say $M = 24m$) to guarantee that the right hand side is non positive whenever $x_{uv} = 0$, thus turning the constraint (1.7) redundant when $x_{uv} = 0$.

The following conditions guarantee that the team assigned to every client v in $C_{0,1}$ is the same team that has visited vertex u, whenever arc (u,v) is in the solution.

$$a_v \leq 1 - x_{uv} + a_u, \quad \forall (u,v) \in A \tag{1.8}$$

$$a_v \geq x_{uv} - 1 + a_u, \quad \forall (u,v) \in A \tag{1.9}$$

$$a_v = k, \quad \forall v \in C_k \cup \{b_0^k, b_1^k, \ldots, b_m^k\}, \ k = 0, 1 \tag{1.10}$$

Indeed, if $x_{uv} = 1$, $a_v = a_u$, and if $x_{uv} = 0$, the inequalities (1.8) and (1.9) are redundant.

The range of the variables is established as follows.

$$a_v \in \{0,1\}, \quad \forall v \in C_{0,1} \tag{1.11}$$

$$x_{uv} \in \{0,1\}, \quad \forall (u,v) \in A \tag{1.12}$$

$$y_v^j \in \{0,1\}, \quad \forall v \in V, \text{ and } j \leq nT_v \tag{1.13}$$

$$s_v \geq 0, \quad \forall v \in V \tag{1.14}$$

$$w_v \geq 0, \quad \forall v \in C \tag{1.15}$$

The above model (1.1)-(1.15) gives a mixed integer linear programming formulation for the problem of routing clients of C on a given period of m days, by two teams. The objective function (1.1) was defined to minimize travel-time and waiting-time on clients in the minimal number of days. Other alternative goals could be considered. For instance, minimizing the total completion-time, i.e., the time of the last service on period P. This could be achieved introducing variable F, imposing the constraints $F \geq s_v + t_v, \forall v \in C$, and defining as objective function: $\min F$. This would give solutions with a minimum number of consecutive working days, and leaving the non working days, if any, at the end of period P. Solutions that define a sequence of consecutive non working days finishing at the end of period P permit to anticipate the next period. However, the objective function (1.1) expresses the goals specified by Neoturf. The existence of intermittent non working days is not a issue for Neoturf, as it permits to assign the members of the team to other activities.

1.3 Heuristic approach

Given the limitations on the size of the instances that could be solved exactly with the formulation (1.1)-(1.15) above (see Section 1.4 below), we decided to waive from optimality guaranteed, and use an implementation of Clarke and Wright

(C&W) [4] heuristic for the vehicle routing problem with multiple time windows (VRPMTW) available in MATLAB [9] .

There are two main issues in applying C&W heuristic to our problem. First, C&W algorithm does not distinguish between clients from C_0, C_1 and $C_{0,1}$. Thus, solutions may include in the same routes clients from C_0 together with clients from C_1.

The second issue follows from the assumption behind C&W algorithm that there are enough vehicles available for the routes determined by the algorithm. Thus, the same team may be assigned, on the same day, to more than one route with incompatible time windows (i.e., services to clients in different routes overlap in time).

To handle the first issue we proceeded as follows.

- We duplicated the number m of days of period P.
- For all clients in C_1, we added $24 \times m$ hours to the release and deadline times of every time-window.
- For all clients in $C_{0,1}$ we duplicated the number of time-windows and, beside the original ones, we also added $24 \times m$ hours to the release and deadline times of every original time window.

Since each client is visited exactly once, in the whole period (now with $2m$ days), within one of its time-windows, setting the time windows of clients C_0 on the first m days and the time-windows of clients C_1 on days $m+1$ to $2m$, ensures that clients from C_0 will not be put together in the same routes with clients from C_1.

Duplicating as described above the number of time-windows of clients $C_{0,1}$, and given that each will be served exactly once, defines a partition of these clients into those that will be served in the first m days (together with clients of C_0), and those that will be served in days $m+1$ to $2m$ (together with clients of $C_{0,1}$).

To address the second issue we use matchings in bipartite graphs.

Suppose the number of routes assigned to a team is less than or equal to m, and there is more than one route on the same day. We consider a bipartite graph (see Figure 1.3) where vertices of bi-class R represent routes and vertices of the other bi-class D represent the m days. There is an edge $[r,d]$, with $r \in R$ and $d \in D$, if and only if route r can be done (w.r.t time-windows) in day d.

We then find the maximum matching [7] of this graph. If it has $|R|$ edges, then it indicates how routes should be distributed by the m days of the period, with no more than one route per day. If the maximum matching has less than $|R|$ edges, or $|R| > m$, we propose that the decision maker considers: assigning an extra-team for this period; increasing the number of days in the forecast period, or reducing the number of admissible clients for the period, and repeat the whole process. Quite often the matching obtained had cardinality $|R|$, which permitted to distribute the $|R|$ routes by the m days. Only in few cases the number of days and/or the set of customers of the period had to be redefined.

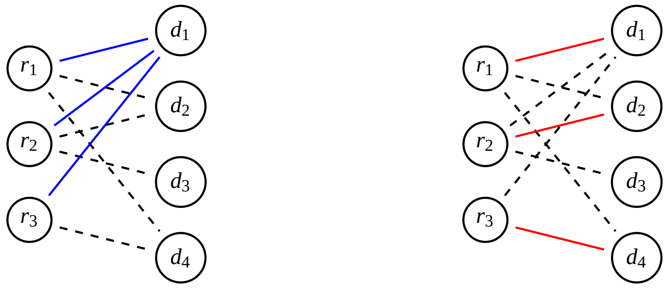

Fig. 1.2 Bipartite graph with $|R| = 3$ routes assigned to the same team for a period of four days. Blue edges (continuous lines on the left picture) indicate the assignment of routes to days on the solution obtained with the modified C&W heuristic. Edges $[r_i, d_j]$ indicate that route r_i can done (w.r.t time-windows) on day d_j. Red edges (continuous lines on the right picture) are the edges of a maximum matching. On the left picture, all routes were assigned to the first day. On the right, the maximum matching (red edges) defines a feasible assignment of the three routes to three days of the period.

1.4 Computational results

Here we report some computational experiments carried out with Neoturf data. We call total time to the sum of travel-time and waiting-times, i.e., the values of the objective function not accounting for parameters Δ.

We used the NEOS Server [10] platform to test the model (1.1)-(1.15). The implementation was made in AMPL [1] modeling language and ran using the commercial solver Gurobi. On the tests that we carried out, only for periods not exceeding five days Gurobi produced the optimal solutions. On two instances with periods of five days and thirteen customers, with $|C_{0,1}| = 2$ in one instance, and $|C_{0,1}| = 3$ in the other instance, the optimal solutions were obtained. However, on an instance with all parameters with the same size except $|C_{0,1}| = 4$, NEOS Server returned either “timeout” or “out of memory”. The same happen for all the instances that we considered with periods of more than five consecutive working days, and no improvements were achieved when we used different parameterization on *threads*, *mipgap* or *timelimit*.

For the small instances for which Gurobi determined optimal solutions, the gap of total routing times of the solutions obtained with C&W heuristic w.r.t. the optimal values (i.e., (T(C&W)-OPT)/OPT, where T(C&W) and OPT are the total time of the solution obtained with C&W heuristic and the optimal total time, respectively) did not exceed 5%.

We then compared the planning that Neoturf had established for a 14 days period (18-Feb-2013 till 3-Mar-2013) with the one produced with C&W heuristic. The solution produced with C&W has an total time of 8h54m (waiting-time = 0h00, and 105h24m if working time is also considered) to serve the 27 clients in 7 and 9 working days for teams E_0 and E_1, respectively. The planning of Neoturf consisted of 14h02m total time (waiting-time=1h00, and 110h32m considering working time), 8 days for team E_0 and 11 days for team E_1.

This gives a reduction on total time (100 $\times$ (14h02m - 8h54m)/14h02m) around 37%, that significantly decreases costs resulting from distances traveled, specially because the two teams travel around 60 000 km/year.

1.5 Conclusion

We considered a routing/location problem arising in the context of garden maintenance services. For each day of each period of time (consisting of some consecutive working days) routes are to be designed, starting and ending at a same point, so that every customer is visited only once during that period, by exactly one vehicle and within predefined time-windows. Customers may require a fixed team or be assigned indifferently to any team.

For this new variant of the VRPTW we constructed a directed graph and presented a compact formulation to minimize travel-time and waiting-time on clients that consists of finding vertex-independent paths of the graph, where every vertex is included in exactly one path, and vertices representing customers that require the same team are included in the same path.

The computational tests that we carried out showed that only for periods not exceeding five days we could obtain the optimal solutions. To deal with this limitation we presented a heuristic approach that uses an adaptation of the classic Clarke and Wright (C&W) heuristic for the VRPTW followed by a procedure to find a maximum matching in a bipartite graphs. The adaptation of the C&W heuristic was devised to satisfy the constraint that customers will be served by the team they required. The maximum matching will check, and possibly repair, infeasibilities on the solution obtained from the C&W heuristics regarding the existence of more than one route assigned to the same vehicle, in the same day. The procedure ran quickly on data provide by Neoturf and the solutions produced significantly improved the solutions that were conceived and implemented by Neoturf. Yet we believe that results may be improved using heuristics for routing more sophisticated than C&W, and exploring models alternative to (1.1)-(1.15). We intend to pursuit on this direction.

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References

1. AMPL modelling language, March 2013 Available online at:http://www.ampl.com/
2. Ascheuer, N., Fischetti, M., and Grötschel, M.: Solving the asymmetric travelling salesman problem with time windows by branch-and-cut. Mathematical Programming, Ser. **A 90** 475–506 (2001)
3. Bräysy, O. and Gendreau, M.: Vehicle Routing Problem with Time Windows, Part I: Route Construction and Local Search Algorithms. Transportation Science **39(1)** 104–118 (2005)
4. Clarke, G. and Wright, J. W.: Scheduling of Vehicles from a Central Depot to a Number of Delivery Points. Operations Research, **12(4)** 568–581 (1964)
5. Desaulniers, G., Madsen, O B. G. and Ropke, S.: Vehicle routing problems with time windows. In P. Toth and D. Vigo (eds.), Vehicle routing: Problems, methods, and applications, second edition, MOS-SIAM Series on Optimization, SIAM, Philadelphia, 119-159 (2014)
6. Golden, B. L., Raghavan, S. and Wasil, E. A.: The Vehicle Routing Problem: Latest Advances and New Challenges, Springer, New York, (2008)
7. Hopcroft, J. E., Karp, R. M.: An $n^{5/2}$ algorithm for maximum matchings in bipartite graphs. SIAM Journal on Computing **2(4)** 225–231 (1973)
8. Kumar, S. N. and Panneerselvam, R.: A Survey on the Vehicle Routing Problem and Its Variants. Intelligent Information Management **4** 66–74 (2012)
9. MATLAB, The MathWorks, Inc., Natick, Massachusetts, United States. http://www.mathworks.com/products/matlab/
10. NEOS Server, March 2013 Available online at: http://www.neos-server.org/
11. Toth, P. and Vigo, D.: The Vehicle Routing Problem. SIAM Monographs on Discrete Mathematics and Applications, Philadelphia (2002)
12. Toth, P. and Vigo, D.: Vehicle Routing: Problems, Methods, and Applications, second edition. MOS-SIAM Series on Optimization, SIAM, Philadelphia (2014)