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Mathematical modelling of Portuguese hydroelectric energy system

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Abstract

Hydropower is one of the most traditional renewable energy source and a major contributor for renewable energy production in many countries. In Portugal it was the only renewable energy source for many years but nowadays wind presents similar production levels and for example in 2015 wind was the main source producing 45.5 % of the total renewable energy. However hydro energy will continue to be important in the renewable energy production and in this work ranking of nine models for hydro energy production with various numbers of parameters was done using adjusted R-squared and corrected Akaike information criterion (AICc).

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1. Introduction

Sustainable development relies on renewable energy sources and hydro power energy is recognized as being very important for many countries [1]. Hydropower energy is based on a renewable source, reduces pollution and emissions of greenhouse gas and has positive impacts on the quality of life of populations [2].

However it has also some environmental negative impacts usually related to fauna and flora [3]. Hydropower technology is considered mature but recent studies showed that there is still scope for development and

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optimization, as for example for small hydropower [4]. In Portugal hydropower energy was the only meaningful renewable energy for a long time its. Nowadays the electricity mix of Portugal has another significant source of renewable energy that is wind, which accounts for more or less 50% of total renewable energy. However energy produced from wind is characterized by being intermittent and with low controllability, which does not apply to hydro power energy. A 100% RES electricity scenario for Portugal will rely heavily on hydro energy [5]. Since hydropower energy will play a key role in future energy system it is important to have models that can help to calculate or forecast hydro energy production.

In this work several models were proposed to study hydropower energy production. Those models were then mathematical analyzed and the best model was selected using the R-squared and Akaike information criterion.

Nomenclature

A	model parameter
AIC	Akaike information criterion
AIC _c	Corrected Akaike information criterion
B	model parameter
C	modelparameter
D	modelparameter
E	energy
F	model parameter
G	model parameter
k	number of parameters
n	number of observations
P	installed power
P _c	precipitation
R ²	R-squared
R ² _{adj}	adjusted R-squared
SS	sum of squared differences
SST	total sum of squares

2. Method and data

2.1. Hydro energy real data

For this work the hydro energy production from 1995 to 2015 was considered [6]. After analyzing the problem, several variables were considered to explain the hydro energy produced namely hydro installed power, precipitation and the production of wind and photovoltaic energy [6, 7]. The inclusion of the first two factors above mentioned is easily understandable because they are directly linked to the production of hydro energy. The wind and photovoltaic energy production was considered because due to regulations the production of this type of energy has priority over the other ones in Portugal. Fig. 1 presents the data concerning hydro energy production and hydro installed power in Portugal over the period considered. From the analysis of this figure it is possible to conclude that there was an increase of around 40% in installed power over the years.

Fig. 2 presents the hydro energy production and the annual precipitation. From the analysis of the figure, it seems that there is a similar pattern for the variation of these two variables.

Fig. 3 presents the hydro energy production and the wind and photovoltaic energy production over the period considered. Analyzing the figure it is possible to conclude that the energy production from wind and photovoltaic systems has increase significantly in the last decade, mainly due to the wind energy production. If this factor affects the hydro energy production it will only be relevant in the last decade because from 1997 to 2005 it is residual.

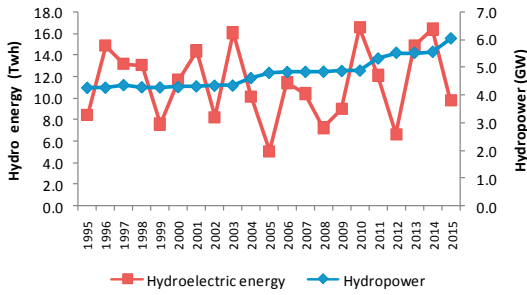


Fig. 1. Hydro energy production and installed power

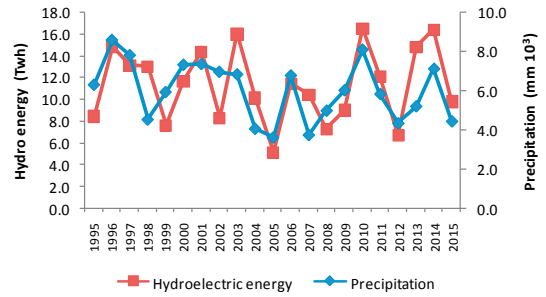


Fig. 2. Hydro energy production and precipitation

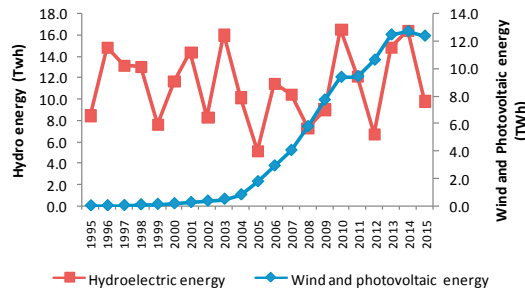


Fig. 3. Hydro energy production and wind and photovoltaic energy production

2.2. Mathematical models considered

The mathematical models proposed and studied are presented in table 1. In order to compare the models using the Akaike information criterion it is necessary that the data sets are equal (equal number of observations). Since some models calculated the hydro energy production based on the precipitation and installed power of previous years the period considered was reduced, being from 1997 to 2005, in order to obtain equal data sets and allow comparison of models.

Table 1. Mathematical models considered

Model	Models	Parameters
1	$E_i = APC_i$	A
2	$E_i = APC_i P_i$	A
3	$E_i = APC_i + BPC_{i-1}$	A,B
4	$E_i = APC_i P_i + BPC_{i-1} P_{i-1}$	A,B*
5	$E_i = APC_i P_i + BPC_{i-1} P_{i-1} + CE_w$	A,B,C*
6	$E_i = APC_i P_i + BPC_{i-1} P_{i-1} + DPC_{i-2} P_{i-2}$	A,B,D**
7	$E_i = A * PC_i * P_i + B * PC_{i-1} * P_{i-1} + D * PC_{i-2} * P_{i-2} + C * E_w$	A,B,C,D**
8	$E_i = APC_i^F P_i$	A,F
9	$E_i = APC_{i-1}^F P_i + BPC_{i-1}^G P_{i-1}$	A,F,G

*In this model the energy produced depends of the precipitation and installed power of the year and of the previous year (i-1).

**In this model the energy produced depends of the precipitation and installed power of the year and of the two previous years (i-1;i-2).

2.3. R-squared and Akaike information criterion (AIC)

To obtain the values of the parameters for the different models the sum of the squared differences, SS, was minimized and the models ranked according R-squared and Akaike information criterion. The model which gives the higher R-squared or minimum AIC is selected as the best model.

The R-squared can be calculated accordingly as given in equation 1 [8]:

$$R^2 = 1 - \frac{SS}{SST} \quad (1)$$

where SS is the sum of squared differences and SST is the total sum of squares.

Since the several proposed models have different number of parameters it is important to calculate the adjusted R-squared to compensate for different number of parameters. The adjusted R- squared can be calculated as shown in equation 2:

$$R_{adj}^2 = 1 - \frac{(n-1)}{(n-k)}(1 - R^2) \quad (2)$$

where n is the number of observations and k the number of parameters.

Some authors question the validity of using R-squared to select the best model when non linear models are [8] used and for that reason the Akaike information criterion (AIC) was also used. The Akaike information criterion is a measure that is usually accepted for model selection especially when dealing with nonlinear models in several domains [9,10,11]. AIC can be calculated as given in equation 3 [12]:

$$AIC = 2k - 2\ln(L) \quad (3)$$

where k is the number of parameters and $\ln(L)$ is the maximum log-likelihood of the estimated model. When the errors are normally distributed AIC can be calculated as shown in equation 4 [13]:

$$AIC = n\ln\left(\frac{SS}{n}\right) + 2k \quad (4)$$

where n is the number of observations , SS is the sum of squared differences and k the number of parameters.

Since in this work the sample size is relatively small, the corrected AIC was used as shown in equation 5 [14]:

$$AIC_C = AIC + \frac{2k(k+1)}{n-k-1} \quad (5)$$

where k is the number of parameters.

3. Results

For all models the sum of squares was minimized and the parameters determined. Table 2 presents these results and the values for AIC, AICc, R^2 , $R^2_{adjusted}$. Analyzing the data we can conclude that AICc is always higher and $R^2_{adjusted}$ always equal or lower as expected. Applying the criterion that the model with the minimum value of AICc is the most suitable model it is possible to conclude that model 4 is the best one (equation 6):

$$E_i = APc_iP_i + BPc_{i-1}P_{i-1} \quad (6)$$

where E_i is the hydropower energy produced in year i, P_{ci} is the precipitation in the year i, P is the installed power on

year i , P_{ci-1} is the precipitation in the previous year and P_{i-1} is the installed power in the previous year.

Applying the criterion that the model with the highest value for R^2_{adjusted} is the most suitable, conclusion is the same. Model 4 is the best according to both criteria.

Table 2. Values of the parameters, AIC, AICc, R2, R2adjusted

Model	Parameters	AIC	AICc	R ²	R ² _{adjusted}
1	A=1.904	41.0	41.2	0.309	0.309
2	A=0.399	38.3	38.5	0.400	0.400
3	A=1.366; B=0.544	40.2	40.9	0.404	0.369
4	A=0.295; B=0.106	37.7	38.4	0.478	0.447
5	A=0.302; B=0.108; C=-0.044	39.5	41.1	0.481	0.416
6	A=0.306; B=0.137; D=-0.043	39.3	40.9	0.488	0.424
7	A=0.313; B=0.139; D=-0.044; C=-0.046	41.1	44.0	0.493	0.391
8	A=0.467; F=0.913	40.2	40.9	0.404	0.369
9	A=0.006; F=2.688; B=0.674; G=0.478	40.3	43.1	0.514	0.417

Analyzing Fig. 2 we notice that in 2002 the pattern of energy produced and precipitation is very different and that year stands out because of that very different tendencies. In order to evaluate the impact of that year in the model fitting the calculations were repeated for all models after removing that year. The results improved because the AICc values are lower and the R^2_{adjusted} are higher for all models but model 4 is still the best one, presenting the lowest AICc and highest R^2_{adjusted} . The values of the parameters (A=0.297; B=0.113) did not change significantly (<10%). The model was then used to calculate the hydro energy produced in 2016 knowing the precipitation and installed power for the years 2015 and 2016 ($P_c = 6885.9$ mm and $P = 6.835$ GW for 2016), and the value obtained compared with the real value (16.684 TWh). The deviation between the real value and the value obtained by using the model is <2%, so the estimated value is quite close to the real value.

4. Conclusions

The hydro energy production is very important for many countries and this is also true for Portugal. During many years, hydro energy was the only renewable energy in Portugal. Nowadays the situation is very different because the production of energy using wind is reaching 50% of total renewable energy. However this RES presents low controllability, what is a disadvantage when comparing with hydro energy production systems. For this reason it is important to have models that can help to calculate or forecast the hydro energy production. In this work several models were proposed and studied. Although the models can give a real interpretation of hydropower energy production, they should be regarded as an empirical description rather than an explanation of the event, given the set of data considered (annual). The best model was selected using the R-squared and Akaike information criterion, and it showed that hydroelectric energy production of a given year depends on the precipitation and installed power of that year and of the previous year.

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