Embedded explicit Runge–Kutta type methods for directly solving special third order differential equations y‴=f(x,y)

ABSTRACT

In this paper three pairs of embedded RungeóKutta type methods for directly solving special third order ordinary differential equations (ODEs) of the form $y=f(x,y)y=f(x,y)$ denoted as RKD methods are presented. The first is the RKD4(3) pair which is third order embedded in fourth-order method has the property first same as last (FSAL) whereby the last row of the coefficient matrix is equal to the vector output. The second method is the RKD5(4) pair followed by the RKD6(5) pair. The methods are derived with the strategies such that the higher order methods are very accurate and the lower order methods will give the best error estimates. Variables stepsize codes are developed based on the methods and used to solve a set of special third order problems. Numerical results are compared with the existing embedded RungeóKutta pairs which require the problems to be reduced into a system of first order ODEs. Numerical results have clearly shown the advantage and the efficiency of the new RKD pairs.

Keyword: RKD methods; Third-order ordinary differential equations; Embedded methods; Order conditions