GEOMETRICAL STRUCTURES IN QUANTUM HALL EFFECT AND RELATED TOPICS

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Introduction

Quantum Hall effect presents an area that hinges upon fundamental aspects of quantum theory. Of particular interest is the topological aspects related to the Hall conductance being integer/fractional multiples of the ratio of fundamental constants e^2/h. While many explanations have been given to its topological origins, a coherent explanation on how this crops up in the quantisation process building up all possible states, is still of interest. Traditionally, quantisation involves looking at irreducible representations of a Lie algebra or group, but in order to incorporate topological ingredients, further arguments are needed. There is however, a quantisation programme that is readily geometric in nature but whose approach stays close to the conventional quantisation, namely Isham's group-theoretic quantisation programme (Isham, 1984).

Materials and Methods

Isham's group-theoretic quantisation is originally being used to study systems of particle/field on a nonlinear configuration space (Isham, 1984). The method, however, has been extended to systems involving external fields when investigating anomalies (Zainuddin, 1989). Such extension allows us to study Hall systems comprising of particles moving on two-dimensional surfaces in a transverse magnetic field. The underlying configuration spaces for integer Hall systems being studied are the two-torus and the two-sphere, which can arise effectively by imposing the usual boundary conditions on the wave functions. For fractional Hall systems, one has to introduce punctures onto these spaces to effectively 'simulate' the many-body effects, which must be present for such systems. Given these models, one proceeds to apply the group-theoretic quantisation scheme which involves finding a suitable canonical group that describes the global symmetries of the system's phase spaces. Once the canonical group is identified, its inequivalent representations give the different possible quantum Hall systems and hence its classification.

Results and Discussion

Investigations of Zainuddin, (1989) can be adapted to find the usual normal quantum Hall states being characterised by the radii of the torus as well as the magnetic monopole charge attributed to the U(1)-bundle presence. One can in fact observe the intertwining of topology with group representations from the correspondence between the inequivalent line bundles counted from the dimension of the second cohomology group of the torus with the number of possible automorphisms found in the canonical group given by the (nested) semi-direct product structure of Euclidean groups (Zainuddin et al. 1998). One also finds an extra anomalous class of representations characterised by a continuous parameter whose wave functions are functions of a single-variable. There seems to be a connection between these representations and those found by Gotay (1995) in his full quantisation of the torus. Group-theoretic quantisation of the integer Hall system on the sphere can also be similarly carried out. The same canonical algebra as the case without the magnetic field is obtained (Zainuddin et al. 1998). While this is so, indications of a nontrivial modification can be seen by the necessity of inclusion of the angular momentum of the magnetic field to make the algebra closed (Bouketir and Zainuddin, 1998). This can checked by constructing the lifting problem of group action from the configuration space of S^2 over to its line bundle. It is found there is indeed an obstruction of lifting the SO(3) part of the canonical group action onto the line bundle and thus implying the necessity of going to its universal cover SU(2). Thus the canonical group is simply the universal cover of the Euclidean group. The classification for this S^2 turns out to be simpler in that it only reproduces the normal quantum Hall states characterised by the radius of the sphere and the magnetic monopole charge (Zainuddin et al. 1998). For fractional Hall systems, we have chosen to focus on the S^2 case as it has a simpler parental group structure. On the algebraic level, the punctured system obeys the same kinematical symmetries as its parents. But on the global level, its canonical group must incorporate a braid group structure that arises by virtue of the exchange symmetry of the punctures. This has been the subject of an ongoing structure by the group but we expect to make connections with modular groups and vector bundle representations allowing for fractional filling classifications which is yet not fully understood. As a spin-off study, we have also been experimenting around with computational knot theory (anticipating some usage) by looking at knot codes and their behaviour under skein relations which has been done to knots of up to 10 crossings.

Conclusions

In group-theoretic quantising Hall systems, we have brought up an intricate mathematical scheme moulding topological information of the underlying system into the canonical group which is reflected in its classification of representations. The scheme successfully reproduces the entire known integer quantum Hall states and possibly even more. For fractional Hall systems, the structures involved are more intricate indicating much richer structures and connections whose origin previously are not well understood.

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