DEVELOPING A CODE FOR AUTOMATIC SOLUTION OF HIGHER ORDER BOUNDARY VALUE PROBLEMS USING THE SHOOTING TECHNIQUE

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Introduction

Since the advent of computers, the numerical solution of boundary value problems for ordinary differential equations has been the subject of research. Boundary value problems manifest themselves in almost all branches of science, engineering and technology. In all the methods and codes developed using initial value techniques, the higher order equations are reduced to first order equations making it incompetent with other methods, which solve the equations directly. As there is a close relationship between boundary value problems and initial value problems, it is appropriate then, to construct a numerical method for a given boundary value problem by relating the problem to the corresponding initial value problems and solving the latter numerically. The code BVPDI is designed to solve mixed order systems of linear and nonlinear boundary value problems using direct integration method. This is in contrast to the other codes developed so far, which require conversion of the given problem to first order system, thereby increasing the number of equations and changing the algebraic structure of the discretized problem. Numerical results are compared with the existing code COLNEW (Bader and Ascher, 1987), which solves the higher order equations directly.

Materials and Methods

A generalised variable order variable stepsize direct integration method (Suleiman, 1989), a generalised backward differentiation method and shooting technique are used to solve the given boundary value problem. When using simple shooting technique, sometimes stability difficulties arise when the differential operator of the given ordinary differential equation contains rapidly growing and decaying fundamental solution modes. In order to decrease the bound of this error, the size of domains over which the initial value problems are integrated has to be restricted. This leads to the multiple shooting technique, which is a generalisation of the simple shooting technique. Multiple shooting technique for higher order ordinary differential equations with automatic partitioning is designed and successfully implemented in the code BVPDI, to solve the underlying initial value problem. Stiffness tests for the system of first order ordinary differential equations and the techniques of identifying the equations causing stiffness in a system are discussed. The analysis is extended for the higher order ordinary differential equations. By the application of Floquet theory (Eastham, 1973) and shooting algorithm, eigenvalues of Sturm-Liouvillie problems with periodic boundary conditions are determined without reducing to the first order system of equations.

Results and Discussion

The well-conditioning of a higher order boundary value problem is shown to be related to the bounds involving the boundary conditions and the green's function. It is shown how the conditioning of the multiple shooting matrix is related to the given boundary value problem. The conditioning constants for the given boundary value problem are derived for higher order boundary value problem. It is also shown that small perturbations in the boundary conditions induce small perturbations in the solution.

The multiple shooting technique is designed for higher order ordinary differential equations. A new strategy is developed for the automatic partitioning of the interval and has been successfully implemented in the code BVPDI. The functional values at the shooting nodes are determined by using a polynomial interpolation. New integration coefficients are derived for this purpose. For stiff boundary value problems, separate tests are undertaken to isolate the stiff equations and they are solved by using generalised BDF method. The stiffness test developed has been successful on the problems tested making it very efficient to solve higher order systems of equations. The numerical results compared with the only existing direct method code COLNEW, show that BVPDI performs better in all the aspects than COLNEW. The advantages in computational time and the accuracy of the computed solution when the range of interval is large is very significant. Also that BVPDI has no difficulty at all when the problem is higher than the fourth order, whereas COLNEW has to reduce to system of smaller order equations when the given problem is of order more than four.

The success of the BVPDI code applied to the general class of boundary value problems is the motivation to consider the same code for a special class of second order eigenvalue boundary value problems called Sturm-Liouville eigenvalue problems. The shooting algorithm developed for this purpose solves the eigenvalue of the given problem very efficiently. Some numerical examples are given to illustrate the success of the method. The results are compared, when the same problem is reduced to the first order system of equations and the advantages in storage and speed are more pronounced.

Conclusions

The results suggest that initial value methods when compared with other finite difference methods, can perform better by using the combination of multiple shooting and direct integration procedures. Our new approach not previously reported is flexible and efficient to solve the boundary value problems for ordinary differential equations directly.

References

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