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GOODNESS-OF-FIT TESTS FOR EXTREME VALUE DISTRIBUTIONS

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This study concentrates on the Goodness-of-fit (GoF) test for the extreme value distributions. The distributions involved are Generalized Extreme Value (GEV) Type-I, Type-II and Type-III distributions. In this study, the types of GoF tests involved are the graphical plots as well as the statistical tests. In the graphical plot, the existing QQ plot is built based on the quantiles of the hypothetical and empirical distributions. However, the QQ plot suffers from the deviation at the tail of the distribution which particularly occurs very often for the case of heavy tailed distributions. In order to reduce the deviation, the conditional quantiles is recommended. The conditional quantiles plots the end points of the hypothetical and empirical distributions closer to each other. In addition, the alternative plot suggested is hybrid plot. Unlike the QQ plot which plots the original values of the quantiles, the hybrid plot illustrates the quantiles deviation between the hypothetical and empirical values. Moreover, the hybrid plot lets several statistical models to be plotted into a single graph. These plots are done in a graph because the

degree of fit for different statistical models can be visually compared. This is because the horizontal axis is restricted between 0 to 1 for any statistical distribution.

The parameters of GEV Type-I, Type-II and Type-III are estimated by maximum likelihood estimation (MLE). The statistical tests involved in the GoF test are Anderson-Darling (AD), Cramer-von Mises (CVM), Zhang Anderson-Darling (ZAD), Zhang Cramer-von Mises (ZCVM) and Shimokawa (L_n) tests. To determine the most powerful statistical test, the critical values of these statistical tests are generated first. Then, the reliability of the critical values are validated by the power study. If the rejection rate of the critical value is close to the respective significance level, that particular critical value is reliable. In addition, it is of interest to make use of the critical values developed by other researchers. These critical values are done for GEV distribution. These critical values were generated from AD, ZAD and Ahmad tests. For this study, they are labelled as AD-GEV, ZAD-GEV and Ahmad-GEV respectively. These critical values are tested for reliability as well. The power of the statistical tests are examined by the power study as well. Next, to evaluate the power, the alternative distributions are fitted to the extreme value distribution model. Based on the alternative distributions, the most powerful test should be able to produce the highest rejection rate.

The results for graphical plot show that conditional quantiles plot is better than the traditional quantiles plot to illustrate the agreement between two identical distributions as well as the discrepancy between two different distributions. Besides, for the statistical tests, the results state that the AD test is the most powerful test for GEV Type-I. For GEV Type-II, the most powerful test are devided according to the cluster of sample size n. The AD test can generally be used for cluster n=15 to 17, while the ZAD test is powerful for the cluster n=18 to 49. The cluster of

n=50 to 100 has AD-GEV test as the powerful test. Besides, for GEV Type-III, the ZAD test is generally powerful for cluster n= 18 to 100, but for cluster, n=15 to 17, the ZCVM test is more powerful.

In the application part, two types of data were used. The first type is the data that was collected from extreme value distribution while the second type is the data that is normally distributed. The extreme value distribution models are fitted to both types of data. The data that is distributed according to the extreme values distribution is used to verify the agreement between the extreme value distribution and the extreme value distribution model. On the other hand, data that is normally distributed is employed to verify that extreme value distribution model does not fit the non extreme value distribution. The result signifies that the findings in graphical and statistical method of GoF are applicable.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

UJIAN KEBAGUSAN KESESUAIAN BAGI TABURAN NILAI EKSTREM

Oleh

NAHDIYA ZAINAL ABIDIN

Julai 2013

Pengerusi: Mohd Bakri Adam, PhD

Fakulti: Institut Penyelidikan Matematik

Penyelidikan ini memfokuskan pada ujian kebagusan kesesuaian (GoF) bagi taburan nilai ekstrem. Taburan yang terlibat adalah taburan Nilai Ekstrem Teritlak (GEV) Jenis-I, Jenis-II dan Jenis-III. Dalam kajian ini, jenis GoF yang terlibat ialah plot grafik dan ujian statistik. Dalam plot grafik, plot QQ yang sedia ada dibina berdasarkan kuantil dari taburan hipotesis dan empirikal. Walaubagaimanapun, plot QQ mengalami sisihan pada bahagian ekor taburan dimana ianya kerap berlaku terutama bagi kes taburan berekor panjang. Bagi mengurangkan sisihan, kuantil bersyarat dicadangkan. Kuantil bersyarat memplot poin-poin akhir hipotesis dan empirical lebih hampir di antara satu sama lain. Di samping itu, plot alternatif yang dicadangkan ialah plot hibrid. Tidak seperti plot QQ yang memplotkan nilai asal kuantil, plot hibrid memaparkan sisihan kuantil di antara nilai hipotesis dan empirikal. Tambahan lagi, plot hibrid membenarkan beberapa model statistik untuk diplotkan di dalam satu graf. Plot ini dibuat di dalam satu graf kerana darjah kesesuaian bagi model statistik yang berbeza dapat dilihat secara perbandingan.Ini kerana paksi mendatar dihadkan diantara 0 dan 1 bagi

mana-mana taburan statistik.

Parameter GEV Jenis-I, Jenis-II dan Jenis-III dianggarkan oleh Kemungkinan Anggaran Maksimum (MLE). Ujian statistik yang terlibat dalam ujian GoF ialah ujian Anderson-Darling (AD), Cramer-von Mises (CVM), Zhang Anderson-Darling (ZAD), Zhang Cramer-von Mises (ZCVM) and Shimokawa (L_n). Untuk mendapatkan ujian statistik yang paling berkuasa, nilai kritikal bagi ujian statistik ini dibina terlebih dahulu. Kemudian, kebolehpercayaan nilai kritikal disahkan oleh kajian kuasa. Sekiranya kadar penolakan bagi nilai kritikal adalah hampir dengan aras keertian masing-masing, nilai kritikal tersebut adalah dipercayai. Di samping itu, adalah penting untuk menggunakan nilai kritikal yang dibina oleh penyelidik-penyelidik lain. Nilai-nilai kritikal tersebut dibuat untuk taburan GEV. Nilai-nilai kritikal tersebut dijana daripada ujian AD, ZAD dan Ahmad. Untuk kajian ini, mereka masing-masing dilabelkan sebagai AD-GEV, ZAD-GEV dan Ahmad-GEV. Nilai kritikal ini juga diuji kebolehpercayaannya. Kuasa ujian statistik diperiksa oleh kajian kuasa juga. Kemudian,untuk menilai kuasa, taburan alternatif dipadankan dengan model nilai ekstrem. Berdasarkan taburan alternatif, ujian yang paling berkuasa sepatutnya mampu menghasilkan kadar penolakan yang tertinggi.

Keputusan bagi plot grafik menunjukkan plot kuantil bersyarat adalah lagi bagus berbanding plot kuantil tradisional untuk menggambarkan kesefahaman di antara dua taburan yang sama serta percanggahan di antara dua taburan berbeza. Di samping itu, bagi ujian statistik, keputusan menyatakan bahawa ujian AD adalah paling berkuasa untuk GEV Jenis-I . Bagi GEV Jenis-II, ujian yang paling berkuasa dibahagikan mengikut kelompok saiz sampel n. Ujian AD secara umumnya digunakan untuk kelompok n=15 hingga 17, manakala ujian ZAD berkuasa untuk kelompok n=18 hingga 49. Kelompok n=50 hingga 100 mempunyai AD-

GEV sebagai berkuasa. Selain itu, untuk GEV Jenis-III, ujian ZAD secara umumnya berkuasa bagi kelompok n=18 hingga 100, tetapi bagi kelompok n=15 hingga 17, ujian ZCVM lebih berkuasa.

Dalam bahagian aplikasi, dua jenis data digunakan. Data yang pertama ialah data yang dikumpul daripada taburan nilai ekstrem sementara data jenis kedua ialah data yang bertaburan secara normal. Model taburan nilai ekstrem dipadankan dengan kedua-dua data. Data yang bertaburan mengikut taburan nilai ekstrem digunakan untuk mengesahkan kesefahaman diantara taburan nilai ekstrem dan model taburan nilai ekstrem. Selain itu, data yang bertaburan secara normal digunakan untuk mengesahkan bahawa model taburan nilai ekstrem tidak padan dengan taburan yang bukan nilai ekstrem. Keputusan mengesahkan bahawa penemuan dalam kaedah GoF grafik dan ujian statistik boleh diaplikasikan.

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I certify that a Thesis Examination Committee has met on **24 July 2013** to conduct the final examination of **Nahdiya Zainal Abidin** her thesis entitled **"Goodness-of-fit Tests for Extreme Value Distributions"** in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

Members of the Thesis Examination Committee were as follows:

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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DECLARATION

Declaration by graduate student

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