UNIVERSITI PUTRA MALAYSIA

BAYESIAN INFERENCE OF WEIBULL DISTRIBUTION FOR RIGHT AND INTERVAL CENSORED DATA

CHRIS BAMBEY GUURE

IPM 2013 3

BAYESIAN INFERENCE OF WEIBULL DISTRIBUTION FOR RIGHT AND INTERVAL CENSORED DATA

By

CHRIS BAMBEY GUURE

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

June 2013

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DEDICATION

To Syntyche Guure

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

BAYESIAN INFERENCE OF WEIBULL DISTRIBUTION FOR RIGHT AND INTERVAL CENSORED DATA

By

CHRIS BAMBEY GUURE

June 2013

Chair: Professor Noor Akma Ibrahim, Ph.D.

Faculty: Institute for Mathematical Research

The main purpose of this work is to draw comparisons between the classical maximum likelihood and the Bayesian estimators on the parameters, the survival function and hazard rate of the Weibull distribution when the data under consideration are right and interval censored. We have considered the survival data to follow Weibull distribution due to its adaptability in fitting time-to-failure of a very widespread multiplicity to multifaceted mechanisms in the field of life-testing and survival analysis.

In Bayesian estimations, prior distributions as well as loss functions need to be specified. The prior distributions can be obtained via previous study in relation to the current study or by soliciting information from experts. We have considered in this study, different types of priors, such as, Jeffreys prior, extension of Jeffreys' prior information, gamma priors and have also proposed a generalised non-informative prior. The loss functions considered in this study are asymmetric and symmetric loss functions.

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Lindley's approximation procedure is used in the Bayesian estimation approach to reduce the ratio of integrals in the posterior distributions which cannot be obtained in close forms. When we consider both the scale and shape parameters under the right and interval censored data, we observed that the estimate of the shape parameter under the maximum likelihood method cannot be obtained in close form; therefore, a numerical approach known as Newton-Raphson has been employed to estimate the shape parameter.

The mean squared errors and mean absolute biases of the estimates under Bayes and its maximum likelihood counterpart are examined through simulation study under several conditions to evaluate the performance of both methods. Overall, it has been observed that, the proposed Bayesian estimation under the generalised non-informative prior performed better than the other estimators for the scale and shape parameters, the survival function and hazard rate. The Bayesian estimator via the generalised non-informative prior occurred largely with the linear exponential loss function followed by general entropy loss function.

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Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PENTAKBIRAN BAYES BAGI TABURAN WEIBULL UNTUK DATA MANDIRAN TERTAPIS KEKANAN DAN SELANG

Oleh

CHRIS BAMBEY GUURE

Jun 2013

Pengerusi: Profesor Noor Akma Ibrahim, Ph.D.

Fakulti: Institut Penyelidikan Matematik

Tujuan utama kajian ini adalah untuk membuat perbandingan di antara kebolehjadian maksimum klasik dengan penganggar Bayesian terhadap parameter, fungsi mandirian dan kadar bahaya taburan Weibull apabila data di bawah pertimbangan adalah tertapis kekanan dan tertapis berselang. Kami telah mempertimbangkan data mandirian bertaburan Weibull kerana kebolehsuaiannya dalam suaian masa-terhadap-kegagalan dalam mekanisme pelbagai aspek dalam bidang ujian hayat dan analisis mandirian.

Dalam penganggaran Bayes, taburan prior serta fungsi kehilangan perlu dinyatakan. Taburan prior boleh diperolehi melalui kajian sebelumnya berhubung dengan kajian semasa atau dengan meminta maklumat daripada pakar. Kami telah mempertimbangkan dalam kajian ini, jenis prior yang berbeza, seperti prior Jeffreys, lanjutan kepada prior Jeffreys, prior gamma dan juga telah mencadangkan prior tak bermaklumat teritlak. Fungsi kehilangan yang dipertimbangkan dalam kajian ini adalah asimetri dan fungsi kehilangan simetri.

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Tatacara penghampiran Lindley digunakan dalam pendekatan penganggaran Bayes untuk mengurangkan nisbah kamiran dalam taburan posterior yang tidak boleh diperolehi dalam bentuk mudah. Apabila kita mempertimbangkan parameter skala dan parameter bentuk bagi data tertapis kekanan dan tertapis berselang, kita perhatikan bahawa anggaran parameter bentuk yang di bawah kaedah kebolehjadian maksimum tidak boleh boleh diperolehi dalam bentuk mudah. Oleh itu, satu pendekatan berangka yang dikenali sebagai Newton-Raphson telah digunakan untuk menganggar parameter bentuk.

Purata ralat kuasa dua dan kepincangan mutlak bagi penganggar di bawah Bayes dan kebolehjadian maksimum diteliti melalui kajian simulasi di bawah beberapa syarat untuk menilai prestasi kedua-dua kaedah. Dapat diperhatikan bahawa prior tak bermaklumat teritlak yang dicadangkan bagi Bayes menunjukkan yang prestasinya adalah lebih baik daripada penganggar kaedah lain bagi parameter skala dan parameter bentuk, fungsi mandirian dan kadar bahaya. Ini berlaku kebanyakannya dengan fungsi kehilangan linear eksponen diikuti oleh dengan fungsi kehilangan entropi teritlak.

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I certify that a Thesis Examination Committee has met on 13 June 2013 to conduct the final examination of Chris Bambey Guure on his thesis entitled "Bayesian Inference of Weibull Distribution for Right and Interval Censored Data" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

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Date: 13 June 2013

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