UNIVERSITI PUTRA MALAYSIA

VARIABLE STEP VARIABLE ORDER BLOCK BACKWARD DIFFERENTIATION FORMULAE FOR SOLVING STIFF ORDINARY DIFFERENTIAL EQUATIONS

SITI AINOR BINTI MOHD YATIM

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By

SITI AINOR BINTI MOHD YATIM

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

July 2013

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DEDICATIONS

to

my parents and families,

for their endless love and support.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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SITI AINOR BINTI MOHD YATIM

July 2013

Chairman: Zarina Bibi Binti Ibrahim, PhD

Faculty: Science

Block Backward Differentiation Formulae (BBDF) method with variable step variable order approach (VSVO) for solving stiff Ordinary Differential Equations (ODEs) is described in this thesis. The research on Variable Step Variable Order Block Backward Differentiation Formulae (VSVO-BBDF) method is divided into two parts where the first part attempts to solve first order stiff ODEs, whereby second order stiff ODEs are considered subsequently. Initially, the computation of D^{th}-order variable step BBDF (VS-BBDF) method of order three up to five is presented. The detailed algorithms of VSVO-BBDF method is discussed to show the crucial parts of the order and stepsize selections. Prior to getting the numerical results, the MATLAB's suite of ODEs solvers namely ode15s and ode23s is applied for the numerical comparison purposes. Meanwhile, the consistency and zero stability properties that lead to the convergence of the method are also discussed. Finally, the implementation of the VSVO-BBDF(2) method for the solution of

second order stiff ODEs is analyzed. The derivation of the method of order two up to four, as well as the strategies in choosing the order and stepsize are elaborated. Similarly, numerical results are obtained after a fair comparison is made between VSVO-BBDF(2) and stiff ODEs solvers in MATLAB. In conclusion, the results display positive trends in reducing the total number of steps and increasing the accuracy of the approximations. The results also show that VSVO-BBDF method reduces the time execution for solving first and second order stiff ODEs as compared to MATLAB's ODEs solvers. Therefore, these methods serve the purpose of significant alternatives for solving stiff ODEs.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Dokor Falsafah

LANGKAH BERUBAH PERINGKAT BERUBAH FORMULA PEMBEZAAN BLOK KEBELAKANG UNTUK PENYELESAI PERSAMAAN PEMBEZAAN BIASA KAKU

Oleh

SITI AINOR BNTI MOHD YATIM

Julai 2013

Pengerusi: Zarina Bibi Binti Ibrahim, PhD

Fakulti: Sains

Kaedah Formula Pembezaan Blok Kebelakang (FPBK) dengan pendekatan langkah berubah peringkat berubah (LBPB) bagi penyelesaian kepada Persamaan Pembezaan Biasa (PPB) kaku dibincangkan di dalam tesis ini. Kajian mengenai Langkah Berubah Peringkat Berubah Formula Pembezaan Blok Kebelakang (LBPB-FPBK) terbahagi kepada dua bahagian dengan bahagian pertama tertumpu kepada penyelesaian PPB kaku peringkat pertama, dan seterusnya PPB kaku peringkat kedua. Pada permulaannya, komputasi kaedah Formula Pembezaan Blok Kebelakang langkah berubah peringkat ke-*D* (LB-FPBK) yang terdiri daripada peringkat ke tiga hingga ke lima dipaparkan. Algoritma terperinci kaedah LBPB-FPBK dibincangkan untuk menunjukkan bahagian penting pemilihan langkah dan peringkat. Bagi mendapatkan keputusan berangka, set penyelesai untuk PPB oleh MATLAB iaitu ode15s dan ode23s digunakan untuk tujuan perbandingan berangka. Manakala, ciri ketetapan dan keseimbangan-sifar yang menjurus kepada penumpuan kaedah turut dibincangkan. Akhir sekali, implimentasi kaedah LBPB-FPBK(2) untuk

penyelesaian PPB kaku peringkat kedua dianalisa. Derivasi kaedah peringkat kedua sehingga keempat, juga strategi dalam pemilihan peringkat dan langkah dihuraikan. Sekali lagi, keputusan berangka diperoleh setelah perbandingan secara teliti dibuat antara penyelesai PPB oleh MATLAB. Kesimpulannya, keputusan mempamerkan tren positif dalam mengurangkan bilangan langkah dan meningkatkan ketepatan penganggaran. Keputusan juga menunjukkan kaedah LBPB-FPBK mengurangkan masa perlaksanaan untuk penyelesaian terhadap PPB kaku peringkat pertama dan kedua jika dibandingkan dengan penyelesai PPB oleh MATLAB. Oleh itu, kaedah ini amat sesuai untuk dijadikan alternatif bagi menyelesaikan PPB kaku.

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I certify that a Thesis Examination Committee has met on 2 July 2013 to conduct the final examination of Siti Ainor Binti Mohd Yatim on her thesis entitled "**Variable Step Variable Order Block Backward Differentiation Formulae for Solving Stiff Ordinary Differential Equations**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the degree of Doctor of Philosophy.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledge. I also declare that it has not been previously, and is not currently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

_____

SITI AINOR BINTI MOHD YATIM

Date: 2 July 2013

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